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Start-Up Investment With Scarce Venture Capital Support

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Abstract

Venture capitalists, representing informed capital, screen, monitor and advise start-up entrepreneurs. The paper reports three new results on venture capital (VC) finance. First, there is an optimal number of companies in the VC portfolio with a trade-off between the number of companies and the value of managerial advice. Second, dilution of advice plays an important role in the adjustment of the VC industry. Third, as a welfare result, a VC tends to provide too little advisory effort but takes too many companies to be advised. Testable implications are discussed.[1]

Keywords

Venture capital, company portfolio, managerial advice, economic rents

JEL Classification

D82, G24, G32, L19

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1 Introduction

A professionally managed pool of venture capital funds with active involvement in managerial advice has increasingly supplemented or even replaced more traditional forms of financial intermediation in funding start-ups. Venture capitalists (VCs) represent informed capital, trying carefully to screen, select and monitor the projects they fund (Kaplan and Strömberg 2001). Sahlman (1990) observed that a typical large VC firm received up to 1.000 proposals each year but invested in only a dozen or so new companies[1]. VC is not a new financial innovation. Zerox, Microsoft, and Netscape, among many, all obtained seed capital from outside sources in their early state. The seminal papers (Gorman and Sahlman (1989), Sahlman (1990), Gompers (1995), Lerner (1995)) indicated that VC investors take a rather different role from other intermediaries not to mention passive equity participations. Given the often limited business competence of the founding entrepreneur, VC advice in building business relations, hiring the right personnel and marketing the product etc. becomes a key complementary expertise to entrepreneurial efforts. There are probably few industries where experience matters as much as in VC investing. Such competence arises from active business involvement in the respective industry. It cannot be acquired in short order, nor is it easily transferable to other persons.[2] Citing Gompers and Lerner (1999, p.4): *"Not only is it difficult to raise a new venture capital fund without a track record, but the skills needed for successful venture capital investing are difficult and time-consuming to acquire".* It is expected that the limited supply of informed VCs, rather than the availability of financial capital, is the scarce factor in launching young innovative firms.

The boom of stock markets in the advent of the internet economy and the initial

[1]While there were 331 VC firms in the US in 1982 and 658 in 1988, the VC industry boomed along with the rise of the "new economy" towards the end of the next decade. While VC finance in the US amounted to no more than 2-5 billion USD annually in the early 90s, recent estimates put it at 103 billion in 2000. Lagging behind for a long time, Europe has somewhat been catching up as well. In 1999, its VC was 25 billion Euros, five times what it was in the early 90s, c.f. www.evca.com and www.nvca.com.

[2]In a changing business environment, such competence cannot be permanent either and can depreciate.

success of telecommunication and other high-tech industries was, however, followed by a dramatic collapse of stock prices of high-tech shares in western economies since early 2000. Such a turmoil has cast serious doubts on the quality of project assessment and the proper scope of the VC finance. Not only is the informed capital scarce but seems also to be subject of bad assessment and capable of making mistakes. In early 2001, the US VC saw its first negative return for any 12-month period with younger funds suffering most.[3]

In markets with substantial risks, VC companies play a valuable role in screening and guiding new start-ups and in monitoring the progress of projects and devoting resources to enhance their success.[4] But in times of high expected returns, more VC funding becomes available and VCs seem keen to finance more companies. This creates a problem of how to allocate attention and advisory effort between portfolio companies. In consequence, the value of advice may be diluted.

The recent debates on VC, both in the popular press and in economic and financial journals, point to insufficient understanding of the mechanisms associated with company portfolios and industry equilibrium in VC finance.[5] What is the relationship between expected returns and investment portfolio choices of a VC? The company portfolio becomes relevant as it is inefficient to concentrate advisory effort on a single start-up only. It becomes also important because a loss in one project can be compensated by a gain in another. As loss offset provisions in company taxation typically are incomplete, a company portfolio can also be used to create an internalized loss offset mechanism. However, the gains from expanding the size of the company portfolio start to diminish as more companies are involved.

Casual reading suggests that in booming markets VCs may be diluting their support by taking on too many companies. Such a concern arises in the light of the disappointing

[3] For all ventures, the six month rate of return was as low as -21.1 and the overall number of funds dropped considerably (www.nvca.com).

[4] Of the new jobs created in the US in 2000, 4.3 million were originally backed by VC (www.nvca.com).

[5] The required capital in the earliest stage is often infused by some wealthy individuals, typically called business angels. We do not make here a distinction between business angels and VCs.

recent performance of many internet companies. It is not the case that these questions only have academic importance; they also are of vital interest to the business community and to welfare. A related question is how the accumulated experience of VCs affects the quality of their investments, leading to potential differences, say between European and US VC markets. Indeed, the availability of high quality VC is probably still a considerable bottleneck especially in Europe. Using econometric techniques to compare the performance of VC backed and other young firms listed on the Euro.nm stock markets, Bottazzi and Da Rin (2001) found that European VCs had little effect on growth, corporate strategy and job creation of their portfolio firms. Their results indicate that "*the quality of European venture capital might be a more urgent issue than its sheer quantity*". This contrasts sharply with the recent findings on the role of U.S. VCs. In the U.S., VC backed firms engage in more radical product or process innovations and are found to be faster in developing their products and in bringing them to market, as compared to other start-ups. They have a higher rate of CEO turnover, reflecting faster managerial professionalization (Hellmann and Puri 2000, 2001). And they produce more and more valuable patents (Kortum and Lerner 2000).

The early literature recognized and emphasized the need to monitor and limit opportunistic behavior under profit sharing arrangements (Gorman and Sahlman (1989), Admati and Pfleiderer (1994), Barry (1994)). Many studies have by now scrutinized optimal contracts that create proper incentives. A substantial literature explains what type of contracts may arise and how they deal with the double-sided moral hazard problem (Berglöf (1994), Cornelli and Yosha (1997), Trester (1998), Marx (1998), Schmidt (1999), Repullo and Suarez (1999)). The main task has been to explain the observed extensive use of convertibles. Staging of capital infusion creates an option to abandon a disappointing project (Sahlman (1990)) and obtain new information (Gompers (1995), Bergemann and Hege (1998)). Empirical work (Kaplan and Strömberg (2000)) has also established the use of direct control rights (boards, voting rights), analyzed by Hellman (1998)).

Evidence on VC contracts so far exists only for the US.[6] Most of the analytic work has focused on the form of the optimal incentive contract. Our paper has a rather different focus. It suggests that the analysis of a single VC contract is not sufficient to understand the optimal size and the effects of company portfolios and how they shape the evolution of the VC sector. Our paper addresses these two issues and the related welfare implications. We contribute to the theory of VC finance with an analysis of company portfolio and industry equilibrium. It is assumed that in the short-run there is shortage of informed VC but that rents attract new VCs in the long-run.

The idea of informed capital advising and overseeing a portfolio of independent enterprises has not yet been analyzed formally, though it has been brought into attention repeatedly in the literature (Gorman and Sahlman (1989), Sahlman (1990), Gompers (1995), Norton and Tenenbaum (1993) and Reid, Terry and Smith (1997)). Recently, Cumming (2000) has provided information on 231 Canadian VC firms investing in 5323 companies. In his data, the average number of companies in first-round investments ranges from 5.4 for private VC firms up to 29.5 for labor sponsored VC corporations. Allowing for all investments in 1991-1998, the average portfolio size is 13.7 and 59.9 respectively.

No previous study asked how many companies can effectively be financed when consulting is a costly activity. In spite of initial scale effects, a VC investor should avoid advising too many companies as its advisory input might subsequently deteriorate and become too thin. Moreover, there is no previous study on the equilibrium of the VC industry. Since specialized managerial competence is acquired only through active business experience, the emergence of experienced VCs tends to be a slow process, making the supply of VCs rather inelastic in the short-run.

Among our results, we show how optimal second-best profit shares respond to economic rents and costs faced by the contracting partners. Most importantly, we prove that

[6] Kaplan and Stromberg (2000) provide detailed empirical data in fourteen US VC partnerships having invested in 118 portfolio companies. They find that with increasing performance of the enterprise, the entrepreneur's equity stake rises. They report that more than 94 per cent of the VC backed enterprises are financed with convertible preferred stock.

a unique optimal company portfolio exists, subject to a trade-off between the number of firms advised and the advisory effort provided. We also carry out a full-scale comparative static analysis of industry equilibrium, showing how some important demand and supply side shocks determine how the industry adjusts. We show that dilution of advice plays an important role in this adjustment. We also show how past business experience, determining the quality of advice, interacts with the expansion of the VC industry.

Our paper has important welfare and policy implications. In particular, we show that a VC tends to provide too little advisory effort but takes too many companies to be advised. Then, while the second best equilibrium is subject to welfare losses, it also points to difficulties in constructing, say appropriate tax policies to allow for and provide appropriate tax shields on inputs which cannot be observed. Moreover, a substantial fraction of failing projects raises the question of proper tax policies to deal with loss offsets[7]. The paper is rich of testable implications which are also discussed in the concluding section.

The paper is organized as follows. In section 2, we derive the incentive compatible contract and solve for the optimal managerial advice when a VC finances a portfolio of companies. Section 3 studies the optimal number of portfolio companies and emphasizes the key trade-off between portfolio size and riskiness of projects. Section 4 turns to the industry equilibrium. In the short-run, the number of VCs is fixed and each one finances a variable number of firms to satisfy demand for VC support. We discuss how the presence of rents attracts more VCs and then explore the industry equilibrium when free entry makes the supply of VCs elastic and the managerial resource becomes more abundant. Section 5 provides a short summary with some concluding comments.

2 The Model

Basic Assumptions: Our model of venture capital focuses on the managerial contribution of the financier and emphasizes the importance of profit sharing arrangements to

[7]We explore these issues in Kanniainen and Keuschnigg (2001).

realign the incentives of entrepreneurs and financiers. We keep the model simple in other respects. Agents are risk neutral. All potential projects require a uniform start-up cost $I > 0$ and have identical stochastic structure ex ante. An entrepreneur is able to handle one project only which we associate with a start-up firm. Starting a firm requires the entrepreneur to give up an income $w \geq 0$ which she could earn elsewhere. Having no own resources, she must ask a VC to pay for the start-up cost. Her main capital is a business idea and her technological knowledge. Since the entrepreneur also tends to be commercially inexperienced in the early stages of her career, the managerial expertise of the VC becomes valuable. A VC, in turn, finances and advises a portfolio of n firms and must allocate her limited managerial capacity appropriately across firms to maximize value.

Start-up entrepreneurship is highly risky. We thus assume that a project or firm is either successful and yields a return $R > 0$, or is a failure and yields nothing. Complete failure is quite common among innovative start-up firms with yet untested products. The probability of success is independent across projects. It depends on effort $e_i \in \{0, 1\}$ of the entrepreneur which can be either high or low, and on advice $a_i \geq 0$ that the VC allocates to the i-th project:

$$P(e_i, a_i) = e_i p(a_i), \qquad p''(a_i) < 0 < p'(a_i), \qquad p(a_i) < 1, \qquad A = \sum_{i=1}^{n} a_i. \tag{1}$$

We assume diminishing returns to advice per project over the relevant range of a_i. Because of the assumption of symmetry, $A = an$. The special form of the success probability implies that entrepreneurial effort is the critical input. Without her effort and full commitment, the project is always a failure. Active managerial consulting by a VC adds value and enhances survival chances, if the entrepreneur's effort is high.

Expanding her total consulting activity is increasingly costly to the financier. We assume an increasing and convex cost of the VC's advising effort $\gamma c(A)$ satisfying $c'(A) > 0 > c''(A)$, where higher values of $\gamma > 0$ indicate increased effort cost.[8] Given discrete effort choice, the entrepreneur's effort cost is simply $l(e) = \{0, \beta\}$, $\beta > 0$. The subsequent

[8]Lerner (1995) claims that the VC's oversight of new firms and monitoring their progress involves substantial costs, as enterprises have to be periodically reevaluated. He finds, for example, that organi-

analysis will be greatly simplified in assuming isoelastic functional forms:

$$p(a) = \alpha \cdot \frac{a^{1-\theta}}{1-\theta}, \qquad c(A) = \frac{A^{1+\varepsilon}}{1+\varepsilon}, \qquad 0 < \theta < 1, \qquad \varepsilon > 0. \tag{2}$$

We interpret the parameter α as reflecting the VC's experience and industry knowledge. We take it for granted that a more experienced VC endowed with superior knowhow is more productive in advising her portfolio companies and achieves with the same effort a higher survival probability than a novice. The parameter θ measures the degree of decreasing returns to advice, and ε the degree of cost progression.

Venture Capital Investing: Since entrepreneurs have no own resources, the VC must finance the entire start-up cost of each project in exchange for a profit share $1 - s_i$. She also provides managerial advice to the entrepreneur and thereby further adds value. The VC's activities are decomposed into a sequence of decisions which is illustrated by the following time line:

——|————|—————|—————|—————|——

entry cost c_f choose n contract $s_i, b_i + I$ effort e_i, a_i, A risk resolved

First, a VC decides whether to pay a fixed entry cost c_f. Having established a fund, she then chooses the number of projects that she wants to finance and advise. Next, she proposes to buy a share $1 - s_i$ of the firm at a price $b_i + I$ that covers the start-up cost I but also includes an upfront payment b_i to the entrepreneur. In structuring the deal, the VC anticipates how the terms of the contract subsequently determine incentives for her own and the entrepreneur's effort. Given the terms of the contract, both parties next choose their effort levels which determines the survival probability. The level of effort is private information, is not verifiable and cannot be contracted. The fact that the contract is fixed prior to effort choice creates a double sided moral hazard problem. Since effort is costly, the entrepreneur may be tempted to shirk if her profit share is too low. But also

zations with offices within 5 miles of the firm's headquarters are twice as likely to be board members as those more than 500 miles distant.

the VC's willingness to actively advise and support her portfolio firms depends on the size of her equity stake and, thus, on her ability to share in the upside potential of the firm.[9] Finally, risk is resolved and payments are made.

The VC's overall problem is to maximize expected profits,

$$\pi = \max_{s_i, b_i} \sum_{i=1}^{n} \left[e_i p(a_i)(1 - s_i) R - b_i - I \right] - \gamma c(A) - c_f, \tag{3}$$

subject to participation and incentive compatibility constraints,

$$PC^E : \quad \Pi_i = e_i p(a_i) s_i R - l(e_i) + b_i - w \geq 0, \qquad i = 1, ..., n \tag{3.i}$$

$$IC^E : \quad p(a_i) s_i R - \beta \geq 0, \qquad i = 1, ..., n \tag{3.ii}$$

$$IC^F : \quad \{a_i\} = \arg\max \textstyle\sum_i \left[e_i p(a_i)(1 - s_i) R \right] - \gamma c(A). \tag{3.iii}$$

Condition (3.i) is the participation constraint of entrepreneurs arising from their occupational choice. In opting for an entrepreneurial career, she gives up her alternative wage income w. The contract must be generous enough to make entrepreneurship a worthwhile option. Conditions (3.ii) and (3.iii) reflect the *ex post* incentive constraints. Given that investments are sunk and the contract is already specified, agents choose effort to maximize the remaining income that is still at their discretion. The minimum profit share s_i that satisfies (3.ii) and makes entrepreneurs willing to provide high effort, $e_i = 1$, depends on the success probability and, thus, on the extent of managerial advice. On the other hand, shirking by the entrepreneur destroys any return to the VC's advisory effort in (3.iii). Thus, the actions of entrepreneurs and the VC are importantly interactive.

We close our model of the VC industry by endogenizing the return R of new ventures. We assume sufficiently large numbers such that the individual success probability p is equal to the fraction of all start-ups that are successful in equilibrium. Focussing on symmetric cases with N VCs in the industry and each one mentoring n firms, we have an

[9] Straight debt finance would seriously impair the VC's incentives for consulting since she wouldn't participate in the extra value created by this costly effort. For this reason, equity, or equity like instruments such as convertible debt, are important in VC finance.

aggregate of pnN start-up firms. We assume that venture returns decline as more start-ups successfully mature to production stage. We also assume that there is no shortage of potential entrepreneurs as long as the reward from starting a business is sufficiently attractive. Neither is there a shortage of financial resources. In our view, the critical resource in the development of the VC industry is the managerial expertise and knowledge of experienced VCs. The specialized human capital of VCs isn't easily acquired but takes time to develop. To emphasize this point, we consider a short-run equilibrium where the number of VCs is fixed. The scarcity of the managerial resource should create rents which eventually attract new VCs who compete them away in the long-run equilibrium. Industry equilibrium will be analyzed in section 4. In particular, we are interested how certain structural parameters shape the nature and evolution of the VC industry. We first start with the decisions of a representative VC.

3 Venture Capital Investing

The financier's and entrepreneurs' decisions are characterized by means of backward solution of the sequence of events described in the preceding section.

3.1 Managerial Advice and Profit Sharing

Effort: The necessary conditions for the VC's optimal advice to each firm are[10]

$$\Omega_i \equiv e_i p'(a_i)(1-s_i)R - \gamma c'(A) = 0, \qquad i = 1, \ldots, n. \tag{4}$$

The efforts of the VC and all n entrepreneurs are determined interactively. If $e_i^* = 1$, then the VC provides a positive level of advice, $a_i^* > 0$. If the entrepreneur shirks, $e_i^* = 0$, the return to advice is negative, and the VC would not want to waste any effort, $a_i^* = 0$. Our assumptions on the success probability make efforts complementary.

[10] The second order conditions can be characterized more precisely in terms of the Hessian matrix in the usual way. We have $\Omega_{ii} \equiv e_i p''(1-s_i)R - \gamma c'' < 0$. All cross derivatives are identical and negative as well, $\Omega_{ij} \equiv -\gamma c'' < 0$.

Financial Contract: Optimal managerial advice as well as the entrepreneur's effort depend on the agreed profit shares. Anticipating how profit sharing shapes incentives for joint efforts, the VC chooses s_i to maximize her profits. Suppose the entrepreneur's share is sufficiently high such that her incentive constraint (3.ii) is slack. Applying the envelope theorem in (3), the VC is seen to raise her profits by cutting the entrepreneur's profit share, $\frac{d\pi}{ds_i} = -p\left(a_i^*\right)R < 0$. She will do so until she hits the IC^E constraint which must therefore be binding in optimum. Therefore, conditions (3.ii) and (4) jointly determine the optimal profit shares and managerial advice such that entrepreneurial effort is assured to be high. Since all projects are identical ex ante, we may concentrate on the symmetric solution. Figure 1 which is developed fully in appendix A.1, depicts the entrepreneur's and the financier's incentive constraints, respectively. As appendix A.1 demonstrates, the constraints intersect exactly two times. The profit maximizing solution is the lowest s and, correspondingly, the highest a as in point A of Figure 1. With (3.ii) binding, the entrepreneur's participation constraint (3.i) reduces to $b_i \geq w$. The VC, of course, strives to purchase the stake $1 - s_i$ at the lowest possible price $b_i + I$ that just succeeds to make the entrepreneur start the firm. Hence, the purchase price pays for the full investment cost but must also include an upfront payment $b_i = w$ that compensates the entrepreneur for her foregone outside opportunities, but no more than that.[11]

Figure 1

Comparative Statics: Figure 1 demonstrates that a larger company portfolio tightens the VC's incentive constraint. In allocating her attention to a larger number of firms, she spends less time on each single project, even if she raises her overall managerial effort. Since less advice implies higher risk, she must cede a larger equity stake to entrepreneurs to enlist their full effort. With a lower share for her own, the VC will be able to extract a smaller profit from each firm in her portfolio. These will be important considerations in

[11] Keuschnigg and Nielsen (2000) assume risk averse entrepreneurs. The fixed payment b_i must then include a risk premium to compensate for risk bearing, on top of the outside wage.

deciding about how many firms to finance and advise. Characterizing optimal portfolio size thus requires knowledge of the comparative statics of advice and incentive compatible equity shares. We are also interested in how venture returns, experience, effort costs and other parameters affect the nature of VC investing. We obtain the comparative statics in terms of logarithmic differentials, $d \log a = da/a \equiv \hat{a}$, which are interpreted as percentage changes relative to an initial equilibrium position such as point A in Figure 1.[12] Using (2), the linearized incentive constraints (3.ii) and (4) are

$$IC^E : \hat{s} = -\hat{R} - \hat{\alpha} - (1-\theta)\,\hat{a}, \qquad IC^F : (\theta+\varepsilon)\,\hat{a} = \hat{R} + \hat{\alpha} - \frac{s}{1-s}\hat{s} - \hat{\gamma} - \varepsilon\hat{n}. \qquad (5)$$

Since effort costs increase progressively with the overall consulting activity $A = an$, the VC optimally diverts attention and managerial support from the existing firms in her portfolio by $\hat{a} = -\frac{\varepsilon}{\theta+\varepsilon}\hat{n}$, when she decides to advise and finance additional start-ups. Higher effort cost γ and a smaller equity share $1-s$ also discourage the consulting activity. As venture returns increase, however, the financier chooses to advise more intensively. The entrepreneur's equity share $\hat{s}$ may be cut if her incentives are strengthened by higher returns or by a higher success probability $\hat{p} = \hat{\alpha} + (1-\theta)\,\hat{a}$. Taking account of the simultaneity in (5), the equilibrium adjustment of advice is

$$\hat{a} = \frac{1}{(1-s)\,\Psi}\left[\hat{R} + \hat{\alpha} - (1-s)\,(\hat{\gamma} + \varepsilon\hat{n})\right], \qquad \Psi \equiv \theta + \varepsilon - (1-\theta)\,\frac{s}{1-s} > 0. \qquad (6)$$

According to (5), a higher project value directly raises the VC's incentives for consulting. In reducing risk, more advice allows to cut the incentive compatible profit share s. The

[12] Using the hat notation, we obtain, for example, $\hat{p} = \frac{a}{p}\frac{\partial p}{\partial a} \cdot \hat{a}$ instead of the simple linearization $dp = \frac{\partial p}{\partial a} \cdot da$. Logarithmic differentiation thus yields percentage changes $da/a \equiv \hat{a}$ instead of absolute changes da. It gives the magnitude of the change in terms of elasticities such as $\frac{a}{p}\frac{\partial p}{\partial a}$, thus answering the question, by how much an endogenous variable changes in percentage terms as an exogenous variable changes by one per cent. The computations follow simple rules such as $\widehat{ab} = \hat{a} + \hat{b}$, $\widehat{a/b} = \hat{a} - \hat{b}$, or $\widehat{a^b} = b\hat{a}$. Constants drop out. We compute $\hat{p} = \hat{\alpha} + (1-\theta)\,\hat{a}$ from (2), for example. Logarithmic linearization allows us to uncover complicated nonlinear interactions by solving linear equations. In Figure 1, we thus replace the nonlinear curves by their linear counterparts with the same slope at point A, and explicitely solve for the comparative static effects.

implied increase in the financier's share reinforces her incentives for consulting which then allows for a further reduction in s. When this cycle converges, the total effect is positive, $\Psi > 0$, and exceeds the direct effect.[13] If consulting effort becomes more costly, the overall managerial activity A and the advice per firm a decline. The solution (6) points to a fundamental trade-off that a VC faces in financing additional start-up firms:

Lemma 1 ***(Dilution of Advice)*** *Increasing the number of portfolio companies dilutes managerial advice and raises individual project risk.*

Proof. By (6), $\hat{a} = \frac{-\varepsilon}{\Psi}\hat{n}$ which reduces the success rate by $\hat{p} = (1-\theta)\,\hat{a}$, see (2). ■

Substituting (6) back into the entrepreneur's constraint in (5) yields

$$\hat{s} = \frac{1}{\Psi}\left[(1-\theta)\,(\hat{\gamma}+\varepsilon\hat{n}) - (1+\varepsilon)\left(\hat{R}+\hat{\alpha}\right)\right]. \tag{7}$$

The intuition is immediately revealed upon inspection of the entrepreneur's incentive constraint. A higher project value directly strengthens her incentives for high effort, allowing the VC to squeeze the profit share s. More interestingly, when the VC's effort cost or portfolio size increase, she must cede higher profit shares to entrepreneurs. In both cases, the VC advises less and thereby reduces the expected revenue of the project on account of a higher downside risk. She must then offer a higher profit share to enlist the entrepreneurs' full effort which is critical to retain survival chances.

Proposition 1 ***(Advice and Profit Share)*** *(a) Advice per firm falls with portfolio size n and the VC's effort cost γ, but rises with return R and productivity (experience) α.*
(b) The entrepreneur's profit share rises with portfolio size n and effort cost γ, but falls with return R and productivity α.

Proof. Equations (6) and (7). ■

[13] Appendix A shows that $\Psi > 0$ is equivalent to IC^E being steeper than IC^F at point A of Figure 1.

3.2 How Many Portfolio Companies?

The number of portfolio companies is optimal when the contribution of the marginal firm to expected overall profits is zero. Adding another firm to the portfolio shifts down the financier's incentive constraint as is illustrated in Figure 1. Expanding the portfolio thus dilutes advice per firm, see Lemma 1, because the overall effort cost is increasing progressively with total managerial activity $A = an$.[14] In adding another firm, the VC thus raises project risk. Figure 1 shows that she must therefore offer a higher profit share to entrepreneurs to enlist their full effort. The erosion of her own equity share thus impairs profits from inframarginal firms and eventually offsets the extra profit added by the marginal project. For a more formal proof, substitute $b_i = w$, differentiate (3) and impose symmetry to get[15]

$$\pi_n \equiv \frac{d\pi}{dn} = [p(a)(1-s)R - w - I] - a\gamma c'(na) - np(a)R\frac{\partial s}{\partial n}. \tag{8}$$

Although a larger portfolio dilutes advice, the marginal effect on profits is zero by the envelope theorem applied to (3.iii). The first term in square brackets indicates the contribution of an extra firm to expected VC profits. The second term reflects the additional effort cost from extending managerial support to the marginal firm. These two terms will be consolidated to a *profit creation effect*, net of the advising cost. The last term is a *profit destruction effect*. Having more firms leads the VC to advise each one less which erodes survival chances. To preserve incentives in face of higher risk, the VC must cede a higher profit share to all her partners.[16]

Proposition 2 ***(Optimal Portfolio)*** *A unique optimal number of portfolio companies exists,* $0 < n < \infty$.

[14] Parameter ε in (6) reflects the convexity of the effort cost function. If effort cost were linear, i.e. $\varepsilon = 0$, advice per firm would no longer be diluted.

[15] To avoid integer problems, we now treat n as continuous and replace the summation by an integral.

[16] There are some interesting parallels to the industrial organization literature on the efficiency of firm entry in imperfectly competitive markets. While new firms create value, they also depress profits of incumbent firms. Since private entry decisions typically ignore such "business stealing effects", private entry tends to be excessive, see Mankiw and Whinston (1986).

Proof. Use $ap' = (1-\theta)\,p$ from (2) and write (4) as $a\gamma c' = (1-\theta)(1-s)\,pR$. Substitute into (8) and replace pR by β/s from (3.ii) to obtain the net profit creation effect. Expand the last term in (8), $npR\frac{\partial s}{\partial n} = spR\frac{\hat{s}}{\hat{n}}$, use the elasticity noted in (7), $\hat{s} = \frac{(1-\theta)\varepsilon}{\Psi}\hat{n}$, and make use of (3.ii) to also rewrite the profit destruction effect, the last term in (8). Noting that the equity share depends on the number of firms, $s(n)$ as in (7), the necessary condition for optimal portfolio size is

$$\pi_n = z_1(s) - z_2(s) = 0, \quad z_1(s) \equiv \theta\beta\frac{1-s}{s} - w - I, \quad z_2(s) \equiv \frac{(1-\theta)\,\varepsilon\beta}{\Psi(s)}. \tag{8.a}$$

We have $z_1'(s) < 0$ and $z_2'(s) > 0$ since $\Psi'(s) = -\frac{1-\theta}{(1-s)^2} < 0$. Noting that s increases with n according to (7) gives the sufficient condition $\pi_{nn} < 0$. We need to show that the functions $z_1(s)$ and $z_2(s)$ cross in the admissible interval $[\underline{s}, \bar{s}]$ where $\underline{s} = \beta/R$ and $\bar{s} = \frac{\theta+\varepsilon}{1+\varepsilon}$ are defined in appendix A.1. $z_1(s)$ turns negative for $s \to 1$ while $z_2(s) \to +\infty$ for $s \to \bar{s}$ since $\Psi(\bar{s}) = 0$. Hence, $\pi_n < 0$ near the upper end of the admissible range. Moving towards the lower end, $\Psi(s)$ becomes larger and $z_2(s)$ relatively small. On the other hand, $z_1(s) \to +\infty$ for $s \to 0$. Since we can make $\underline{s} = \beta/R$ arbitrarily small by scaling up the return function, we can always guarantee $\pi_n > 0$ for small s. Hence, the two curves intersect, and π_n crosses zero, within the admissible interval $[\underline{s}, \bar{s}]$. The portfolio condition thus determines a share s which is uniquely related to n by proposition 1b. ■

Noting that the equity share s is an increasing function of n, Figure 2 illustrates the solution for the optimal number of firms. The *profit destruction effect* becomes more severe as more firms are added to the portfolio and higher equity shares must be ceded to entrepreneurs. When the VC consolidates her portfolio, she advises rather more intensively and is able to appropriate a larger profit share without loosing the entrepreneur's effort. With small n, the *profit creation effect* (net of effort cost) of expanding the portfolio is then relatively high. It rapidly declines as more firms are included and the equity shares of all entrepreneurs in the portfolio have to be raised. The optimal number of portfolio firms is attained when the profit creation and destruction effects just cancel and the marginal firm fails to raise the VC's overall profits.

Figure 2

If effort cost were linear ($\varepsilon = 0$), the VC's total effort cost would be $\gamma c(A) = \gamma \cdot \sum_i a_i$. Instead of (4), the VC's consulting incentives are then determined by $p'(a)(1-s)R = \gamma$ which solves together with (3.ii) for advice and profit shares independently of n. With s constant, the profit destruction effect z_2 disappears which makes the marginal benefits of expanding the portfolio a constant $\pi_n = \theta p(a)(1-s)R - w - I \gtrless 0$ and, thus, leaves the portfolio problem indeterminate. Consider alternatively a linear success probability with $\theta = 0$, giving $p = \alpha a$. The VC's advice now yields a constant marginal value added equal to $p'(a) = \alpha$, giving an IC^F of $\alpha(1-s)R = \gamma c'(an)$. An expansion of portfolio size continues to reduce advice and requires a higher profit share of the entrepreneur [set $\theta = 0$ in (5), (6) and (7)] which gives rise to the profit destruction effect. The marginal effort cost in (8) of advising an extra firm is now $a\gamma c' = a\alpha(1-s)R$. With $p = \alpha a$, it fully absorbs the extra revenue $p(1-s)R$ contributed by the additional firm, leaving a negative profit creation effect equal to $z_1 = -w - I$. Consequently, the VC consolidates her portfolio as much as possible and advises only a few firms, subject to the constraint that $p(a)$ cannot exceed one.[17]

Both curvatures are important in determining the size of the company portfolio. With decreasing returns to advice ($0 < \theta < 1$), it is better to advise more firms since a small amount of advice yields a big marginal effect p' on the success rate. When the VC concentrates on a few firms only and advises very intensively, she becomes less effective in further raising the success probability since the marginal effect of advice rapidly disappears. The concavity of the success probability works to expand portfolio size. With a convex cost function, however, the VC's effort cost increases progressively as more firms are added. Advice is easily stretched too thin. Risks inevitably increase, forcing the VC

[17] However, the success probability might realistically include a constant term p_0 that is independent of the VC's contribution but results from the entrepreneur's effort only, $p(a) = p_0 + \alpha a$. Write $\kappa = \alpha a/p$ and $1 - \kappa = p_0/p$ for the shares that the VC and the entrepreneur contribute to the overall success rate. If $p = \alpha a$, $\kappa = 1$ as in the previous case. Suppressing other shocks, we obtain in (5) $IC^E : \hat{s} = -\kappa \hat{a}$ and $IC^F : \varepsilon \hat{a} = -\frac{s}{1-s}\hat{s} - \varepsilon \hat{n}$ which solve for $\hat{s} = (\varepsilon/\Psi)\hat{n}$ in (7) where $\Psi = \frac{\varepsilon}{\kappa} - \frac{s}{1-s} > 0$. The profit creation effect is then $z_1 = p_0(1-s)R - w - I$. Using (3.ii), $z_1 = (1-\kappa)\beta\frac{1-s}{s} - w - I$. This compares with z_1 as in (8) and allows for an interior solution of optimal portfolio size.

to cede higher profit shares to entrepreneurs which erodes her own profits. This profit destruction effect eventually makes a further expansion of the portfolio unattractive.

Proposition 3 ***(Effects on Portfolio)*** *Optimal portfolio size rises with venture returns R and managerial productivity α but falls with effort cost γ and start-up cost I,*

$$\hat{n} = \zeta_R \left(\hat{\alpha} + \hat{R}\right) - \zeta_\gamma \hat{\gamma} - \zeta_I \hat{I}, \tag{9}$$

where ζ are positive coefficients defined in (9.b).

Proof. Note that n enters the optimality condition $\pi_n = 0$ in proposition 2 only via its effect on s. Because of (7), the sufficient condition $\pi_{nn} = \frac{\partial \pi_n}{\partial s}\frac{\partial s}{\partial n} < 0$ requires $\frac{\partial \pi_n}{\partial s} = -\frac{1}{s}\Omega < 0$ with Ω given in (9.a). Using this fact and taking the differential of $\pi_n = z_1(s) - z_2(s) = 0$ yields (9.a),

$$\hat{s} = -\frac{I}{\Omega}\hat{I}, \qquad \Omega = \frac{\beta(\theta+\delta)}{s}, \qquad \delta = \varepsilon \cdot \left[\frac{s(1-\theta)}{(1-s)\Psi}\right]^2. \tag{9.a}$$

$$\zeta_R = \frac{1+\varepsilon}{(1-\theta)\varepsilon}, \qquad \zeta_\gamma = \frac{1}{\varepsilon}, \qquad \zeta_I = \frac{I\Psi}{(1-\theta)\varepsilon\Omega}. \tag{9.b}$$

Substituting (9.a) into (7) gives (9) with coefficients listed in (9.b). ■

Figure 3 illustrates the solution s^* and n^* by the intersection of the contract line given by (7) and the portfolio condition given by (8.a). The latter is horizontal since it autonomously solves for the profit share $s^* = s(I)$. Suppose the VC starts with a number of firms $n > n^*$, and must thus offer an equity share $s > s^*$ on the contract line. Since $d\pi_n/ds < 0$ by the sufficient condition, an equity share larger than s^* impairs the VC's net marginal benefits from adding an extra firm and makes her consolidate the portfolio which is illustrated by the directional arrows in Figure 3. By Lemma 1, she advises more intensively when she is concentrating on fewer firms and thereby reduces the risk of project failure. Smaller risk allows to cut the entrepreneur's incentive compatible profit share. Moving along the contract line to the South West eventually stops the incentive to consolidate the portfolio as the profit share approaches s^*.

Figure 3

Figure 3 is also useful in demonstrating comparative static effects. A larger start-up cost I shifts down the horizontal portfolio condition as noted by (9.a) because it directly weakens the marginal benefits of portfolio expansion in (8.a). The initial equity share is now too high. The VC reduces the number of firms and offers a smaller profit share to entrepreneurs in exchange for more advice and lower risk. Moving along the contract line to the South West then leads to the new intersection point. Increases in venture returns R or in managerial productivity α shift down the contract line. Higher returns strengthen incentives for advice. Reducing risk allows the VC to offer a smaller share s and still enlist the entrepreneur's effort. Since the horizontal portfolio line is unaffected, the directional arrow indicates that the VC wants to expand her portfolio. With a larger profit share of her own, she is keen on funding more firms. In adding firms, she moves North East along the new contract line until the dilution of advice forces her to raise the equity share again to s^*.[18] A higher effort cost γ shifts up the contract line and leads the VC to fund and advise a smaller number of portfolio companies.

Finally, the results of this section also help to explain how cost and productivity differences affect portfolio size, managerial support, and profits among VC firms. Suppose that effort cost increases, $\hat{\gamma} > 0$, implying that one firm has higher managerial cost than another. Substituting (9) back into (6) gives $\hat{a} = -\frac{1}{\Psi}(\hat{\gamma} + \varepsilon\hat{n}) = 0$. A high cost firm will keep advice per firm constant but simply advises a smaller number of them. A high cost firm also earns smaller profits as can be checked from (B.7) in appendix B. In contrast, a VC with higher ability chooses a larger portfolio in (9) and puts less hours to each firm. By the same steps, we get $\hat{a} = \frac{-1}{1-\theta}\hat{\alpha}$ and $\hat{p} = \hat{\alpha} + (1-\theta)\hat{a} = 0$. Since she is more effective, she achieves with fewer hours the same success rate as a less experienced VC. According to (B.7), a more productive VC achieves higher profits.

[18] In the end, advice per firm is reduced by $\hat{a} = -\hat{R}/(1-\theta)$ [substitute (9) into (6) and use $(1-s)\Psi = (1-s)(1+\varepsilon) - (1-\theta)$]. This keeps expected revenue pR constant and satisfies (3.ii) for a fixed s.

3.3 Efficiency

Is private VC investing efficient? If profit maximizing VCs acquire too many portfolio companies, their managerial capacity is stretched too thin. If such a bias is present, it would be detrimental to the extent of advice and mentoring that start-up companies may expect. It would diminish the quality of private VC investing as compared to a socially efficient trade-off between advice and portfolio size. Lack of involvement undermines the prospects of start-up firms since new entrepreneurs are often commercially inexperienced. An unfavorable bias would also imply that VC backed companies are not significantly different from other start-up firms that rely on traditional sources of finance. To investigate the existence of potential distortions in VC financing, we compare private decisions with an efficient allocation that maximizes the joint surplus of a VC and a variable number of entrepreneurs. By (3) and (3.i), joint surplus is

$$V = \pi + \sum_{i=1}^{n} \Pi_i = \sum_{i=1}^{n} \left[p\left(a_i\right) R - \beta - w - I \right] - \gamma c\left(A\right) - c_f. \tag{10}$$

Maximization of joint surplus internalizes the mutual externalities resulting from separate effort choices of the VC and the entrepreneurs. We thus consider a first best situation of full information where the incentive constraints become irrelevant. Maximizing with respect to advice per firm and the number of entrepreneurs per VC yields

$$\begin{array}{llll} (a) & \frac{dV}{da_i} & = & p'\left(a_i\right) R - \gamma c'\left(A\right) = 0, \\ (b) & \frac{dV}{dn} & = & \left[p\left(a_n\right) R - \beta - w - I \right] - \gamma a_n c'\left(A\right) = \theta p\left(a\right) R - \beta - w - I = 0. \end{array} \tag{11}$$

From (11.a), we have $\gamma a c' = a p' R = (1-\theta) pR$ which gives the second equality in (11.b).

Proposition 4 ***(Efficiency)*** *Suppose the VC has optimally chosen a, s and n. Joint surplus increases if the VC can be induced to marginally consolidate her portfolio.*

Proof. Comparing (11.a,b) with (4) and (8.a), the private choices imply

$$\begin{array}{llll} (a) & \frac{dV}{da} & = & p'\left(a\right) R - \gamma c'\left(A\right) = sp'\left(a\right) R > 0, \\ (b) & \frac{dV}{dn} & = & \theta p\left(a\right) R - \beta - w - I = -\left(\theta - s\right) \frac{(1-\theta)\beta}{(1-s)\Psi(s)} < 0. \end{array} \tag{12}$$

Using (3.ii), the profit creation effect in (8.a) is related to (11.b) by $\frac{dV}{dn} = \theta pR - \beta - w - I = z_1 - (1-\theta)\beta$. Substitute the private condition $z_1 = z_2 = \frac{(1-\theta)\varepsilon\beta}{\Psi(s)}$ for an optimal portfolio and get, after some minor manipulations, the second equality in (12.b). The portfolio condition $z_1 = z_2$ in (8.a) also implies $w + I = (\theta - s)\frac{(\theta+\varepsilon)\beta}{s\Psi}$ and, therefore, $\theta - s > 0$, which establishes the sign in (12.b). Marginally consolidating the portfolio raises joint surplus by (12.b). With a smaller portfolio, the VC advises more intensively, see (6), which further boosts joint surplus by (12.a). ■

The proposition reveals that privately optimal advice is smaller and portfolio size larger than what would be required to maximize joint surplus. The intuition is as follows. In private equilibrium, the entrepreneur must receive a share s of project returns so that she supplies high effort. Otherwise, the venture would fail for sure. While profit sharing is incentive compatible, it also implies that the VC bears the full marginal cost but obtains only a share $1-s$ of the marginal benefit $p'(a)R$ of her consulting activity. The rest accrues to the entrepreneur. This external benefit is internalized when maximizing joint surplus which considers the income gains to both parties. For a given number of firms n, the VC therefore provides a level of advice per firm smaller than what is efficient, $a < a^*$.

Consider next the implications for the number of firms in a VC's portfolio. With advice per firm smaller, this shortfall is in fact multiplied and implies a *much* smaller level of total consulting effort $A = an < A^*$. Consequently, the marginal effort cost $c'(A) < c'(A^*)$ of advising an extra firm is also *much* smaller than in the efficient case where the difference is further magnified by the convexity of the effort cost. For this reason, the VC tends to expand her portfolio beyond the efficient level. On the other hand, the success rate in private equilibrium is lower than the efficient one and gives a lower level of expected revenue $p(a)R$ which retards the VC's willingness to finance the marginal firm. Moreover, the presence of the profit destruction effect also tends to restrict private portfolio size. From a social perspective, the profit destruction effect is a mere redistribution from the VC to entrepreneurs and, therefore, doesn't appear in the condition for an efficient portfolio in (11.b). However, the large savings in marginal effort cost, as compared to the efficient level, dominate the incentives to expand the number of portfolio companies such that $n > n^*$. If VCs could be brought to consolidate their company portfolios, they would advise more intensively and contribute more value added to start-up firms.

4 Industry Equilibrium

We now investigate how VCs react when market conditions for start-up investment change. A demand shift for industry output may boost market prices and returns to new ventures, making business start-ups more profitable. Higher start-up costs I, on the other hand, may slow down the rate of business formation. VCs may find it more costly to put up the required effort, i.e. γ increases. The productivity α of VCs in providing advice may also increase, reflecting increased experience, industry knowledge and a better understanding of the key factors that determine the success and failure of innovative start-ups. Finally, we consider entry barriers to VC investing in terms of a fixed cost c_f of setting up a fund. We now investigate how such shocks shape the nature of VC investments and the evolution of the industry.

4.1 Market Demand and Supply: How Rents Promote Entry

Successful VC investing requires much experience and rather specialized knowhow as well as detailed industry knowledge, all of which are difficult to acquire in short order. These requirements represent formidable entry barriers which make the short-run supply of VCs inelastic. Only gradually will VC activity become more widespread through entry of knowledgeable financiers. We therefore take both a short- and long-run perspective. Keeping the number N of VCs fixed in the short-run, we investigate how they adjust their portfolios to changing market conditions. The fixed supply of VCs may create rents that should attract other VCs to pay the entry cost c_f. As ever more VCs set up a fund, the expansion of the industry will eventually erode profit opportunities and make further entry unprofitable. A zero profit condition then determines the number of VCs in the long-run. The questions of how many VCs the industry is able to sustain, and how entry affects the nature of VC investing, have eluded an analysis in the earlier literature.

We close our model of the VC industry by showing that the equilibrium return R to new ventures declines as more start-ups successfully mature to production stage. Given sufficiently many VCs, we may abstract from strategic interactions and assume that they behave as competitive price-takers. With large numbers, the probability p is equal to the fraction of all start-ups that actually succeed in equilibrium. Focusing on symmetric cases with N VCs in the industry and each one mentoring n firms, we have an aggregate of pnN successful firms. Each firm is assumed to produce one unit of output, if successful. The market output (supply) is thus given by $p(a)nN$. We endogenize venture returns by interpreting them as the market-clearing price R. If output sells at a price R, then this is the return to a firm producing one unit of output. We are left to specify the demand of consumers for these products. We deliberately keep the demand function simple and abstract from market uncertainty, network externalities and other aspects of demand for innovative goods. Demand is simply a downward sloping function $D = \phi R^{-\eta}$ of the market price R where $\phi > 0$ is a demand shift parameter and $\eta > 0$ is the price elasticity of market demand. Some of the comparative static results will depend on whether the price elasticity is smaller or larger than one. In industry equilibrium, price must adjust to equate market supply and demand,

$$\phi R^{-\eta} = p\,(a)\,nN. \tag{13}$$

To identify how the short-run equilibrium with a fixed number of VCs is affected by certain supply and demand shocks, we log-linearize (13) at the initial equilibrium position,

$$\eta\hat{R} = \hat{\phi} - \hat{N} - \hat{\alpha} - (1-\theta)\,\hat{a} - \hat{n}. \tag{14}$$

In short-run equilibrium, the number of VCs is exogenous. The comparative statics is then determined by three simultaneous equations (14), (9) and (6) in three unknowns, $\hat{R}$, $\hat{n}$, and $\hat{a}$. We now discuss how an increase in venture returns due to an exogenous demand shock affects the nature and evolution of the VC sector. The following propositions also report the results for various supply shocks to which we turn in subsequent subsections.

Proposition 5 ***(Short-run Equilibrium)*** *With an exogenous number of VCs, the comparative static effects of demand and supply shocks are*

Table 1: Exogenous Number of VCs

Equations	*(B.1)*	*(B.3)*	*(B.4)*	*(9.a)*	*(B.8,9)*
Dep. Var.	$\hat{R}$	$\hat{n}$	$\hat{a}$	$\hat{s}$	$\hat{\pi}$
$\hat{\phi} - \hat{N}$	$+$	$+$	$-$	0	$+$
$\hat{\alpha}$	$-$	$+^*$	$-^*$	0	$+^*$
$\hat{\gamma}$	$+$	$-^*$	$-$	0	$-^*$
$\hat{I}$	$+$	$-$	$+$	$-$	$-?$
$\hat{c}_f$	0	0	0	0	$-$

*Note: A * indicates that sign changes if* $\eta < 1$.

Proof. See Appendix B. A column gives the short-run equilibrium response of a dependent variable to the shocks listed in the first column. The first line locates the corresponding equation which also allows to retrieve the magnitude of the elasticities. ■

Consider now an exogenous demand shock which directly raises returns by $\eta\hat{R} = \hat{\phi}$, see (14). Higher returns feed back on supply since VCs find it profitable to attract more firms which boosts supply by $\hat{n} = \zeta_R \hat{R}$ as noted in (9). A higher return tends to encourage managerial support if the number of firms in the portfolio is given, see (6). The dilution of advice on account of a larger number of portfolio firms dominates, however, and makes projects more risky in the end (Lemma 1). Substituting $\hat{n} = \zeta_R \hat{R}$ into (6) gives, indeed, $\hat{p} = (1-\theta)\hat{a} = \frac{1-\theta}{(1-s)\Psi}\left[\hat{R} - (1-s)\varepsilon\hat{n}\right] = -\hat{R}$, and shows that a smaller fraction of firms survives the start-up phase. Including these supply reactions in (14), the equilibrium return eventually settles at $(\eta - 1 + \zeta_R)\hat{R} = \hat{\phi}$ where $\eta - 1 + \zeta_R > 0$ as noted in (B.1). In competitive equilibrium with an exogenous number of VCs, the immediate effect of a demand shift is, thus, to boost venture returns. Although a demand shock does not directly affect individual decisions, subject to given returns, a VC does respond to increased venture returns by adding firms to her portfolio. With more firms to attend, she advises each single entrepreneur less intensively and accepts a higher risk of project failure. It turns out that the terms of the contracts are not affected in equilibrium. While increased risk calls for a higher profit share of the entrepreneur, higher returns allow for a lower share such that the net effect just cancels.

Since funding and mentoring start-up companies requires much experience, new VC funds cannot be established in short order to accommodate an accelerating rate of business formation. With a fixed number of VCs, a demand shock makes VC investing rather profitable and creates rents. With sustained profit opportunities, new VCs should appear and promote the expansion of the industry. According to the first column relating to equation (B.1), entry spoils returns on new ventures which, in turn, cuts into profits and makes further entry increasingly unattractive. Entry stops when profit opportunities are exhausted. In solving for the long-run equilibrium with free entry, we first derive in (C.1) the effect on the return R which satisfies the zero profit condition (B.7). We then investigate how such changes to venture returns affect managerial advice, profit sharing, and portfolio size. Finally, we substitute the price effect $\hat{R}$ into (B.1) and obtain the number of VCs that are sustained in the free entry equilibrium.

Proposition 6 ***(Long-run Equilibrium)*** *With free entry of competitive VCs, the comparative static effects of demand and supply shocks are*

Table 2: Free Entry of VCs

Equations	*(C.1)*	*(C.4)*	*(C.5)*	*(9.a)*	*(C.2,3)*
Dep. Var.	$\hat{R}$	$\hat{n}$	$\hat{a}$	$\hat{s}$	$\hat{N}$
$\hat{\phi}$	0	0	0	0	+
$\hat{\alpha}$	−	0	0	0	$+^*$
$\hat{\gamma}$	+	0	−	0	$-^*$
$\hat{I}$	+	?	?	−	−?
$\hat{c}_f$	+	+	−	0	−

*Note: A * indicates that sign changes if* $\eta < 1$*.*

Proof. See Appendix C. The first line in Table 2 locates the corresponding equations which also reveal the magnitude of the elasticities. ■

The difference between short- and long-run equilibrium is illustrated in Figure 4. For this purpose, we refer to (B.2) and define excess market demand for the innovative good by $\hat{D} = -\lambda\hat{R} + \hat{\phi} - \hat{N}$. In the absence of a shock, there is no deviation from the initial equilibrium and the excess demand function runs through the origin, see the dashed line. In the short-run, the number of VCs is fixed, $\hat{N} = 0$. A demand shock shifts up the excess demand function by $\hat{\phi}$. The goods price, or venture returns, must increase up to point 1 to eliminate excess demand which gives the short-run increase $\hat{R} = \hat{\phi}/\lambda$. More valuable projects boost profits in (B.7), $\hat{\pi} = (1 - s)\, npR\hat{R}$. In the long-run equilibrium with free entry, however, profits must be zero, implying $\hat{R} = 0$, i.e. venture returns must return to their initial level at the origin. The "zero profit line" therefore is identical with the vertical axis in Figure 4.[19] To the right of the zero profits line, higher returns generate positive profits and attract new VCs, $\hat{\pi} = \hat{N} > 0$, while the area on the left hand side is identified with negative profits and exit of VCs. The short-run equilibrium at point 1 clearly boosts profits. The subsequent entry of VCs shifts down the excess demand function. With

[19] With other shocks, the zero profit line is different from the vertical axis, see (C.1).

ever more VC backed projects flooding the market, entry depresses venture returns and squeezes VC profits which eventually stops further entry. In the long-run equilibrium at point 2, the excess demand function must run through the origin again so that the equilibrium venture return, or goods price, is compatible with zero profits. The demand shift is thus fully offset by entry of $\hat{N} = \hat{\phi}$ new VCs which keeps returns at their initial level. Since a demand shift affects VC investing exclusively via the price channel but has otherwise no direct implications for managerial advice, portfolio size and profit shares, these variables must also return to their initial values as noted in the $\hat{\phi}$-line of Table 2.

Figure 4

The analysis of demand shocks in propositions 5 and 6 demonstrates an important adjustment pattern. When new and innovative businesses are started at an accelerating rate, short-run VC investing can expand only if incumbent VCs finance more firms. With a fixed number of VCs, the quality of VC finance must first deteriorate on account of Lemma 1. As advice gets diluted, the rate of business failure is excessive in the short-run. Once entry of VCs occurs, each individual one is able to consolidate her portfolio and concentrate on a smaller number of firms which she advises more carefully. Only in the long-run is the quality of VC investing restored again. Note, however, that VC support is inefficiently low and VC portfolios are overly large even in the long-run equilibrium with free entry. Proposition 4 holds irrespective of whether VC profits are zero or not.

4.2 VC Experience

Experienced VCs are more successful in adding value to their firms and score a higher success rate with the same number of consulting hours. As more start-ups turn out successful, market supply expands by $\hat{\alpha}$ in (14). More sophisticated VCs also like to include a larger number of firms in their portfolio, $\hat{n} = \zeta_R \hat{\alpha}$ in (9). For a given number of portfolio firms, more experience would encourage further advisory effort. The expansion of the portfolio, however, dilutes advice per firm to an extent that dominates the overall reaction. To see this, substitute $\hat{n} = \zeta_R \hat{\alpha}$ into (6) and use (9.b) and (B.2), $(1-\theta)\,\hat{a} = \frac{1-\theta}{(1-s)\Psi}\left[\hat{\alpha} - (1-s)\,\varepsilon\hat{n}\right] = -\hat{\alpha}$. Since each consulting hour is more effective, the VC actually cuts back the pure quantity of advice to each firm, but achieves the same success probability $\hat{p} = \hat{\alpha} + (1-\theta)\,\hat{a} = 0$. With these results, supply expands by $\hat{n} = \zeta_R \hat{\alpha}$ in (14) and the market price must fall. Declining venture returns on account of market saturation discourage VC activity, however, and result in the end in a more moderate reduction of returns equal to $\hat{R} = -(\zeta_R/\lambda)\,\hat{\alpha}$, see (B.1) and Table 1. While more experience, subject to a given return, induces VCs to expand portfolios and restrict the level of advice (although each consulting unit is now more effective), declining equilibrium returns work in the opposite direction. If venture returns fall only to a minor degree in equilibrium, the VC will acquire a larger number of portfolio companies and allocate a smaller number of consulting hours to each one, as indicated in the $\hat{\alpha}$-line of Table 1. This is the case if the price elasticity of demand is very large, $\eta > 1$. A small price reduction then suffices to generate a large increase in demand which easily absorbs the increased supply from more experienced VCs. If the price elasticity is rather small, venture returns must fall a lot to restore equilibrium which changes the signs noted in Table 1.

The same case distinction carries over to short-run VC rents and, therefore, to the incentives for entry and exit. More sophisticated financiers succeed to bring a larger fraction of start-ups to the market. A higher success rate on account of more experience directly boosts VC profits but on the other hand depresses the market price. The net effect on short-run profits is shown to be positive if the demand elasticity is large, $\eta > 1$, such that equilibrium prices, or venture returns, fall to a minor extent only, see (B.7) and (B.8). As $\hat{N} = (\eta - 1)\,\hat{\alpha}$ new VCs emerge, prices must fall even further, i.e. by a total amount of $\hat{R} = -\hat{\alpha}$ in the long-run, see (C.1). Although more experience encourages, for given project returns, larger portfolios, more intensive advice and smaller equity stakes for entrepreneurs, such incentives are completely offset in the long-run by the erosion of venture returns.

4.3 Cost of Effort

VC investing may become a more difficult business due to government regulations and red tape, unpredictable and rapidly changing market conditions, and other reasons. For a given number of portfolio companies, VCs will then find it more costly to achieve a desired success rate. Quite intuitively, higher effort cost leads financiers to consolidate their portfolio, $\hat{n} = -\zeta_\gamma \hat{\gamma}$ in (9), and to cut back on advice per firm as well. On the other hand, the quality of advice benefits from VCs concentrating on fewer firms. Substituting $\hat{n} = -\zeta_\gamma \hat{\gamma}$ into (6) and using $\zeta_\gamma = 1/\varepsilon$ as given in (9.b), the net effect on managerial support is seen to be exactly zero, $(1-\theta)\,\hat{a} = -\frac{1-\theta}{\Psi}(\hat{\gamma} + \varepsilon\hat{n}) = 0$. Consequently, VC investing contracts on account of smaller company portfolios, leading to an increase in market price and venture returns. The feed back from a higher equilibrium return encourages portfolio expansion at the cost of advice. In the end, each VC finances fewer firms and also provides less managerial support to each one, see Table 1. If the price elasticity η is small, however, venture returns must increase strongly to restore equilibrium which overturns the short-run effect on portfolio size while the effect on advice is robust. Again, the equilibrium profit share remains unaffected due to offsetting influences. Since VCs tolerate higher project risk by cutting back advice, entrepreneurs ask for a higher profit share. On the other hand, a higher equilibrium price makes them accept a lower profit share according to (3.ii). The two effects cancel in (7).

Higher effort costs result in smaller profit margins while higher equilibrium returns boost profits. The net effect depends on the magnitude of the demand elasticity. With $\eta > 1$, equilibrium returns rise only moderately, leaving effort costs to erode profits. The subsequent exit of VCs leads to a supply contraction which magnifies the initial price increase. While higher effort cost per se causes VCs to advise a smaller number of start-up firms as indicated by (9), more profitable projects on account of rising venture returns lead them to restore the initial portfolio size, leaving the number of portfolio companies unchanged in the long-run, see Table 2. With the dilution of advice effect being eliminated, the reduction in managerial support is still negative but less pronounced than in the short-run. Reflecting the adjustment of managerial advice, start-up investment is excessively risky in the short-run and moderately more risky in the long-run. The incentive compatible equity share of entrepreneurs is affected neither instantaneously nor after entry is completed.

4.4 Start-up Investment Cost

Start-up investment may become more capital intensive. When financiers must incur higher start-up costs, they respond by cutting the number of portfolio companies. As they concentrate on fewer firms, managerial advice per firm is higher and thereby helps to contain the failure rate. With lower risk, the entrepreneurs' profit share can be reduced. The net effect on industry supply of smaller portfolios but larger survival rates is negative and boosts the market price as in Table 1. VC investing is likely to be less profitable in the short-run, see the discussion of (B.9). Since entrepreneurs are assumed to have no own funds, the VC must finance the entire start-up cost. On the other hand, higher venture returns and a larger own profit share props up profits. This effect, however, is unlikely to dominate.

Responding to diminishing rents, part of the VCs will leave the sector, see (B.9) and (C.3). Exit further boosts the market price until the remaining investors break even. Venture returns in the free entry equilibrium must thus exceed short-run values. Via this effect, exit of VC firms makes the remaining investors expand their portfolios at the cost of less intensive advice which reverses the short-run adjustment to higher start-up cost in Table 1. For this reason, the net effects on managerial advice and portfolio size in the long-run equilibrium become ambiguous in Table 2 [see (C.4) and (C.6)].

We can now discuss our simplifying assumption that entrepreneurs are in perfectly elastic supply. In section 3.1, we stated that the VC must pay $w + I$ to buy a stake in the firm since the upfront payment b_i must cover the entrepreneur's outside option. Inelastic supply means that it would become more costly to attract additional entrepreneurs when the industry expands. A shortage of entrepreneurs would imply a better outside option w.

Since the solution depends only on $w + I$, a better outside option influences equilibrium VC activity exactly the same way as higher start-up cost I.

4.5 Entry Cost

Finally, more rapid innovation, faster industry restructuring, government regulations and other uncertainties can increase the cost of setting up a VC fund and can also inflate general overhead expenses for market studies and read tape. Such cost is not specifically related to any individual start-up company but must be covered by revenues from the entire portfolio of firms. Since fixed costs are sunk at later stages of the VC cycle, they cannot directly affect managerial advice, profit sharing and portfolio size in the short-run. In cutting profits, however, inflated fixed costs restrain entry of financiers. Fewer VC backed investments drive up venture returns until VCs break even again. From (B.1) we infer that the industry sustains a smaller number of VCs, i.e. $\hat{N} = -\lambda \hat{R}$, see Table 2. By (9), VCs expand their portfolio of firms when projects become more valuable, $\hat{n} = \zeta_R \hat{R}$. Even though a higher return encourages advice, the larger portfolio dilutes advice to an extent that managerial support per firm declines.[20] Projects become more risky. Profit sharing remains invariant since increased risk and higher project value exactly offset each other in affecting incentives of entrepreneurs.

5 Conclusions

Our results advocate the view that the critical resource in the development of the VC industry is the managerial expertise and knowledge of experienced VCs. The specialized human capital of VCs isn't easily acquired but takes time to develop. To emphasize this point, we consider a short-run equilibrium where the number of VCs is fixed. The scarcity of the managerial resource should create rents which eventually attract new VCs who compete them away in the long-run equilibrium.

[20] Substituting $\hat{n} = \zeta_R \hat{R}$ into (6) and using (9.b) and (B.2), we get $\hat{a} = \frac{\hat{R} - (1-s)\varepsilon \hat{n}}{(1-s)\Psi} = -\frac{\hat{R}}{1-\theta}$.

The results obtained lead to a number of testable implications. For example, controlling for the effects of economic rents, an economy with a more experienced VC sector should have a better quality of advice, hence less failures, and hence less dramatic stock price reactions to disturbances. From this perspective, there might be differences between the performance of the US and the European VC sectors. Comparing the size of company portfolios and the speed of market entry should provide light into the dynamics of innovative industries in those economies. The results on the interaction of adjustment of rents and market entry predict that VC finance is subject and sensitive to macroeconomic effects, like changes in behavior of consumers. The data that are available so far, do not allow for an analysis of the many implications that arise from our results. For example, how does the number of portfolio companies vary between years with booming markets and recession periods? What is the quality of advice, as measured for example by visits of a VC to a start-up firm or by the number of contacts, and how does it depend on economic conditions? Moreover, though the behavior of consumers was analyzed in a straightforward model, herding behavior in the VC sector cannot be excluded. It is hoped that, in the future, more time series data on VC portfolios become available to facilitate the empirical investigation of the implications which arise from our results.

Appendix

A Figure 1

We first show how the solution of a and s of (3.ii) and (4) depends on n. Since we must impose $p \leq 1$, the form of $p(a)$ in (2) implies an upper limit $\bar{a}$ for advice. By the same argument, (3.ii) implies a minimum profit share $\underline{s}$,

$$\bar{a} \equiv [(1-\theta)/\alpha]^{1/(1-\theta)}, \qquad \underline{s} = \beta/R. \tag{A.1}$$

We thus write $p(a) = (a/\bar{a})^{1-\theta}$. Figure 1 plots the IC^Ecurve in (3.ii) in s, a-space:

$$IC^E: \quad a = E(s) \equiv \bar{a} \cdot (\underline{s}/s)^{1/(1-\theta)}, \qquad E' < 0 < E''. \tag{A.2}$$

This curve hits the upper limit at $\underline{s}$, i.e. $\bar{a} = E(\underline{s})$, see Figure 1. Since the profit share cannot exceed one, it is bounded below by $\underline{a} = E(1) = \bar{a}\underline{s}^{1/(1-\theta)}$.

The VC's incentive constraint (4) is

$$IC^F: \quad a = F(s) \equiv \left[\frac{(1-s)\,\alpha R}{\gamma n^{\varepsilon}}\right]^{\frac{1}{\theta+\varepsilon}}, \quad F'(s) < 0, \quad F''(s) \gtrless 0. \tag{A.3}$$

This curve satisfies $F(1) = 0$. Since $F''(s) = \frac{-F'(s)(1-\theta-\varepsilon)}{(1-s)(\theta+\varepsilon)}$, it is concave for $1-\theta < \varepsilon$ and convex otherwise. For an interior solution with IC^E binding, we must impose that IC^F evaluated at $\underline{s}$ gives a value of a smaller than $\bar{a}$, such as it is drawn in Figure 1:

$$F(\underline{s}) \leq \bar{a} \qquad \Leftrightarrow \qquad (1-\theta)^{\theta+\varepsilon}(n^{\varepsilon}\gamma)^{1-\theta}/\alpha^{1+\varepsilon} \geq (R-\beta)^{1-\theta}. \tag{A.4}$$

For a solution to exist with both constraints binding, they must intersect as in Figure 1. By $F(s) = E(s)$, we get $H(s) \equiv (1-s)^{1-\theta} s^{\theta+\varepsilon} = [(1-\theta)\beta]^{\theta+\varepsilon}(\gamma n^{\varepsilon})^{1-\theta}/(\alpha R)^{1+\varepsilon} \equiv X$. The H-schedule satisfies $H'(s) = \frac{H(s)}{s}\Psi(s)$ with $\Psi(s) = \theta + \varepsilon - (1-\theta)\frac{s}{1-s}$. The H-schedule is non-negative over the admissible range and satisfies $H(0) = H(1) = 0$ and attains a maximum at $\bar{s} = \frac{\theta+\varepsilon}{1+\varepsilon} < 1$ which is defined by $H'(\bar{s}) = \Psi(\bar{s}) = 0$ and the fact that $H'(s) \gtrless 0 \Leftrightarrow \Psi(s) \gtrless 0 \Leftrightarrow s \lessgtr \bar{s}$, i.e. the Ψ-schedule declines monotonically with slope $\Psi'(s) = -(1-\theta)/(1-s)^2 < 0$. This implies that there are exactly two intersections satisfying $H(s) = X$, if they exist at all. Existence requires $H(\bar{s}) > X$ or

$$H(\bar{s}) = \frac{(1-\theta)^{1-\theta}(\theta+\varepsilon)^{\theta+\varepsilon}}{(1+\varepsilon)^{1+\varepsilon}} > \frac{[(1-\theta)\beta]^{\theta+\varepsilon}(\gamma n^{\varepsilon})^{1-\theta}}{(\alpha R)^{1+\varepsilon}}. \quad \text{(A.5)}$$

Since $H(s) = X$ has two solutions, the incentive constraints in Figure 1 intersect exactly twice. For the solutions to be admissible, (A.5) and (A.4) must be satisfied simultaneously. Multiplying (A.5) by $(\alpha R)^{1+\varepsilon}/\beta^{\theta+\varepsilon}$ and comparing with (A.4) gives

$$(1-\theta)^{1-\theta}\left(\frac{\theta+\varepsilon}{\beta}\right)^{\theta+\varepsilon}\left(\frac{\alpha R}{1+\varepsilon}\right)^{1+\varepsilon} > (1-\theta)^{\theta+\varepsilon}(\gamma n^{\varepsilon})^{1-\theta} \geq \alpha^{1+\varepsilon}(R-\beta)^{1-\theta}. \quad \text{(A.6)}$$

Choosing R large and β small opens a wide wedge, allowing placement of the middle term in this interval by 'calibrating' appropriate values for γ and n.

Of the two intersection points in Figure 1, A is the solution. All combinations to the north east of the IC^E-schedule are admissible. For any given s, the IC^F-curve gives optimal advice according to (4). Applying the envelope theorem to (3), the VC is seen to raise her profits by reducing s, $\frac{d\pi}{ds_i} = -p(a_i^*)R < 0$. Starting from B, the VC maximizes profit by increasing her own profit share (i.e. reducing s) and moving along the IC^F-curve to the north west. She continues until the entrepreneur's incentive constraint binds at A. Equations (5) linearize the constraints at the solution A. The condition $\Psi > 0$ in (6) reflects the fact that IC^E is steeper than IC^F at A.[21]

B Fixed Supply of VCs: Table 1

Venture Return: The comparative statics with fixed N is determined by three simultaneous equations (14), (9) and (6) in three unknowns, $\hat{R}$, $\hat{n}$, and $\hat{a}$, relating to venture returns, portfolio size and managerial advice. We solve the system by using (9) and (6) to replace advice $\hat{a}$ and portfolio size $\hat{n}$ in (14). Using (9.b) and $(1-s)\Psi$ as given in (B.2)

[21] Since $E'(s) = \frac{-a}{(1-\theta)s} < 0$ and $F'(s) = \frac{-a}{(\theta+\varepsilon)(1-s)} < 0$, we have $E'(s) < F'(s) \Leftrightarrow \Psi(s) > 0$.

in various places, we find how equilibrium venture returns R respond to various shocks:[22]

$$\lambda\hat{R} = \hat{\phi} - \hat{N} - \zeta_R\hat{\alpha} + \zeta_\gamma\hat{\gamma} + \frac{\theta - s + (1-s)\,\theta\varepsilon}{(1-s)\,\Psi}\zeta_I\hat{I}, \qquad \lambda \equiv \eta - 1 + \zeta_R > 0. \qquad \text{(B.1)}$$

A higher output price R raises demand by $\eta\hat{R}$ and supply by $(\zeta_R - 1)\,\hat{R}$, where $\zeta_R - 1 = \frac{1+\theta\varepsilon}{(1-\theta)\varepsilon} > 0$ by use of (9.b). The slope of the excess demand function $-\lambda$ is therefore negative and guarantees stability. For later use, we report

$$\Psi = \frac{(1-s)\,(1+\varepsilon) - (1-\theta)}{(1-s)}, \quad (1-s)\,npR = \frac{1+\varepsilon}{1-\theta}\gamma c. \qquad \text{(B.2)}$$

The last equality uses (2) and (4), $Ac' = (1+\varepsilon)\,c$, $ap' = (1-\theta)\,p$ and $(1-s)\,Rp' = \gamma c'$.

Portfolio Size: To take account of the effects of equilibrium venture returns on portfolio size, we multiply (9) by λ, substitute (B.1), use (B.2) and finally obtain

$$\lambda\hat{n} = \zeta_R\left(\hat{\phi} - \hat{N}\right) + (\eta-1)\,\zeta_R\hat{\alpha} - (\eta-1)\,\zeta_\gamma\hat{\gamma} - \left[\eta + \frac{1-\theta}{(1-s)\,\Psi}\right]\zeta_I\hat{I}. \qquad \text{(B.3)}$$

Advice and Profit Shares: Using (B.1), (B.3) and (9.b) in (6) gives

$$\lambda\hat{a} = -\frac{1}{1-\theta}\left(\hat{\phi} - \hat{N}\right) - \frac{\eta-1}{1-\theta}\hat{\alpha} - \frac{1}{1-\theta}\zeta_\gamma\hat{\gamma} + \frac{1+(1-s)\,\varepsilon\eta}{(1-s)\,\Psi}\zeta_I\hat{I}. \qquad \text{(B.4)}$$

The profit share is fixed autonomously by the portfolio condition, see (9.a).

Venture Capital Profits: Parameters affect profits in (3) directly and via their effect on profit share s. Variations of advice a and portfolio size n, however, do not affect profits at the margin because of the envelope theorem. We note the following partial derivatives:

$$\begin{array}{lll} \frac{d\pi}{ds_i} = -p_i R, & \frac{d\pi}{dR} = \sum_i (1-s_i)\,p_i, & \frac{d\pi}{dI} = -n, \\ \frac{d\pi}{d\gamma} = -c\,(A)\,, & \frac{d\pi}{d\alpha} = \sum_i (1-s_i)\,Rp_i/\alpha, & \frac{d\pi}{dc_f} = -1. \end{array} \qquad \text{(B.5)}$$

By (9.b), the profit share s depends on start-up cost I only. Higher investment cost directly cuts into profits but, at the same time, strengthens them by raising the VC's

[22] The proof of proposition 4 shows $\theta - s > 0$.

equity share, i.e. by reducing s. Using (3.ii) and (9.a), and noting $\theta - s > 0$ from the proof of proposition 4, the net effect is negative:

$$\frac{d\pi}{dI} = -n - \frac{npRs}{I}\frac{\hat{s}}{\hat{I}} = -\frac{\theta - s + \delta}{\theta + \delta} \cdot n. \tag{B.6}$$

Using (B.5) and (B.6), we state the profit differential in elasticity form:

$$\hat{\pi} \equiv d\pi = (1-s)\,npR\left(\hat{R} + \hat{\alpha}\right) - \gamma c\hat{\gamma} - \frac{\theta - s + \delta}{\theta + \delta} In\hat{I} - c_f\hat{c}_f. \tag{B.7}$$

Replace $\hat{R}$ with (B.1) and use (9.b) and (B.2),

$$\lambda\hat{\pi} = (1-s)\,npR\left(\hat{\phi} - \hat{N}\right) + (\eta - 1)(1-s)\,npR\hat{\alpha} - (\eta - 1)\,\gamma c\hat{\gamma} - \lambda c_f\hat{c}_f \tag{B.8}$$

By the same steps, and using additionally (3.ii) and (9.a,b), we also obtain the effect of investment cost on profits

$$\lambda\hat{\pi} = [s - (\eta - 1)(\theta - s) - \lambda\delta]\,\frac{nI}{\theta + \delta}\hat{I} < 0. \tag{B.9}$$

Higher start-up cost raises the price which feeds back positively on profits. We assume that this feedback does not overturn the directly negative effect of higher cost, i.e. we assume the square bracket to be negative.

C Free Entry of VCs: Table 2

Venture Return: Entry of VCs expands supply and, thereby, squeezes returns as in (B.1) which, in turn, cuts into VC profits as noted in (B.7). Entry occurs and returns continue to fall until profits are gone. If we start from a zero profit equilibrium, we compute from (B.7) the long-run equilibrium return that sets $\hat{\pi} = 0$ and sustains zero profits. With $\hat{R}$ fixed this way, we invert the short-run equilibrium condition in (B.1) to solve for the implied entry of VCs. Use (B.2) and set $\hat{\pi} = 0$ in (B.7) to get

$$\hat{R} = -\hat{\alpha} + \frac{1-\theta}{1+\varepsilon}\hat{\gamma} + \frac{\theta - s + \delta}{\theta + \delta}\frac{I}{(1-s)\,pR}\hat{I} + \frac{c_f}{(1-s)\,npR}\hat{c}_f. \tag{C.1}$$

Number of VCs: Substituting (C.1) into (B.1) and solving for $\hat{N}$ shows how many VCs the industry sustains in the zero profit equilibrium. Use λ in (B.1) and (9.a),

$$\hat{N} = \hat{\phi} + (\eta - 1)\,\hat{\alpha} - \frac{(\eta - 1)(1 - \theta)}{1 + \varepsilon}\hat{\gamma} - \frac{\lambda c_f}{(1 - s)\,pRn}\hat{c}_f. \tag{C.2}$$

Using (3.ii), (9.a,b) and (B.1,2), higher investment cost is seen to reduce entry,

$$\hat{N} = [s - (\theta - s)(\eta - 1) - \lambda\delta]\,\frac{nI}{(\theta + \delta)\,\zeta_R \varepsilon\gamma c}\hat{I} < 0, \tag{C.3}$$

where the effect is negative by the argument noted in (B.9).

Portfolio Size: Substitute (C.1) into (9) and note (B.2) as well as (9.a,b) and (3.ii). The effect of start-up cost I is ambiguous:

$$\hat{n} = 0 \cdot \hat{\alpha} + 0 \cdot \hat{\gamma} + \frac{c_f \zeta_R}{(1 - s)\,npR}\hat{c}_f + \frac{\delta\zeta_R - 1}{(\theta + \delta)(1 - s)\,pR} I\hat{I}. \tag{C.4}$$

Advice and Profit Share: Substitute (9) into (6) and use $1 - (1 - s)\,\varepsilon\zeta_R = \frac{-(1-s)\Psi}{1-\theta}$,

$$(1 - s)\,\Psi\hat{a} = -\frac{(1 - s)\,\Psi}{1 - \theta}\left(\hat{\alpha} + \hat{R}\right) + (1 - s)\,\varepsilon\zeta_I \hat{I}, \tag{C.5}$$

where the effect of $\hat{\gamma}$ cancels. Insert (C.1), use (9.b) and (B.2), and rearrange:

$$\hat{a} = 0 \cdot \hat{\alpha} - \frac{1}{1 + \varepsilon}\hat{\gamma} - \frac{c_f}{(1 + \varepsilon)\,\gamma c}\hat{c}_f + \frac{(1 - \theta - \delta)\,nI}{(\theta + \delta)(1 + \varepsilon)\,\gamma c}\hat{I}. \tag{C.6}$$

The coefficient of $\hat{I}$ is again ambiguous. The effect on the profit share is given by (9.a).

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Figures

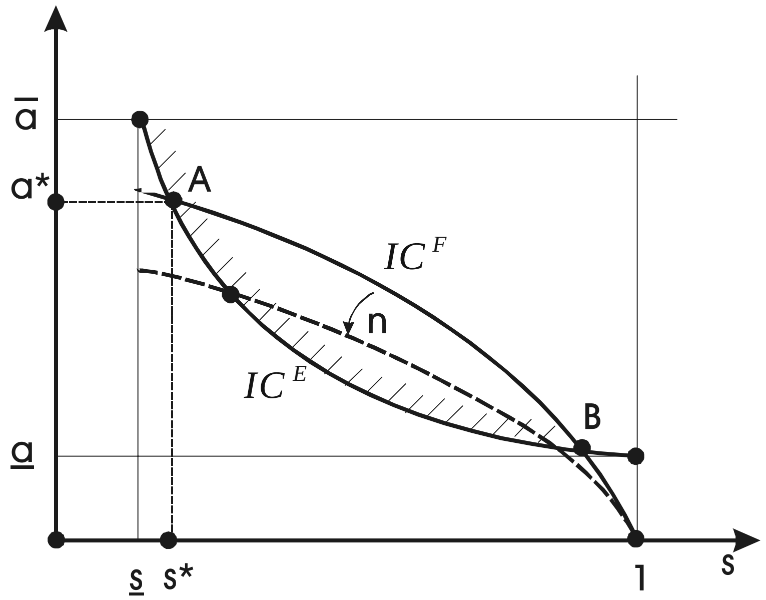

Fig. 1: Optimal Advice and Profit Share

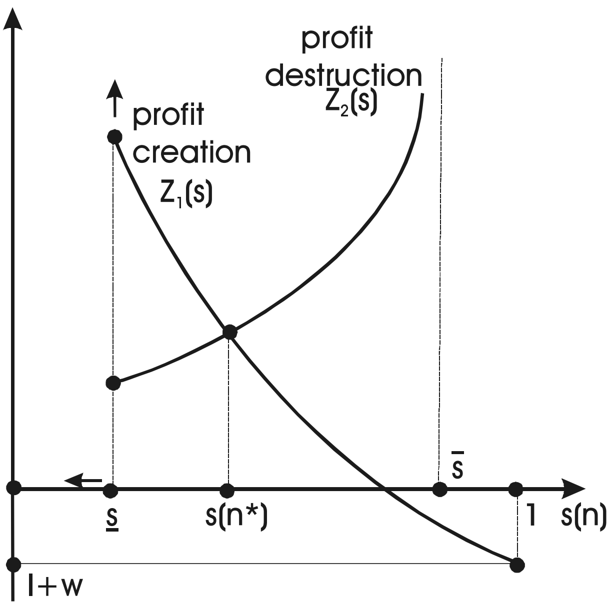

Fig. 2: Optimal Number of Firms

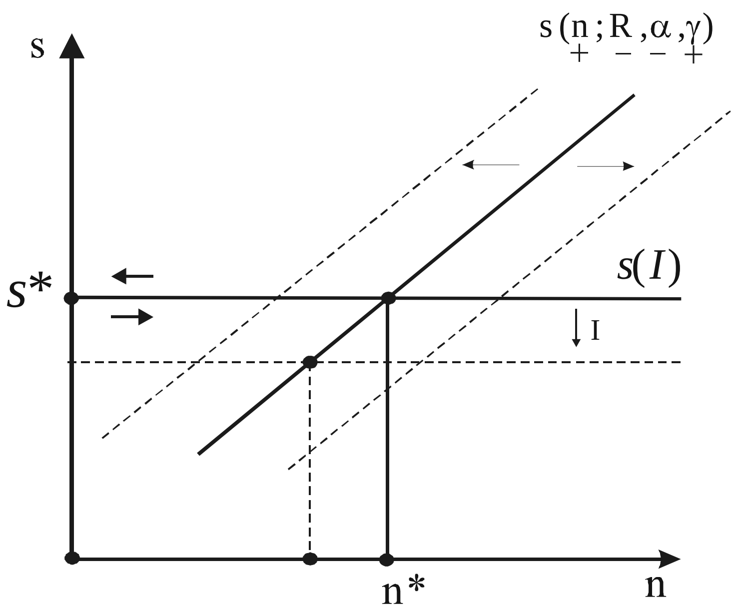

Fig. 3: Number of Portfolio Companies

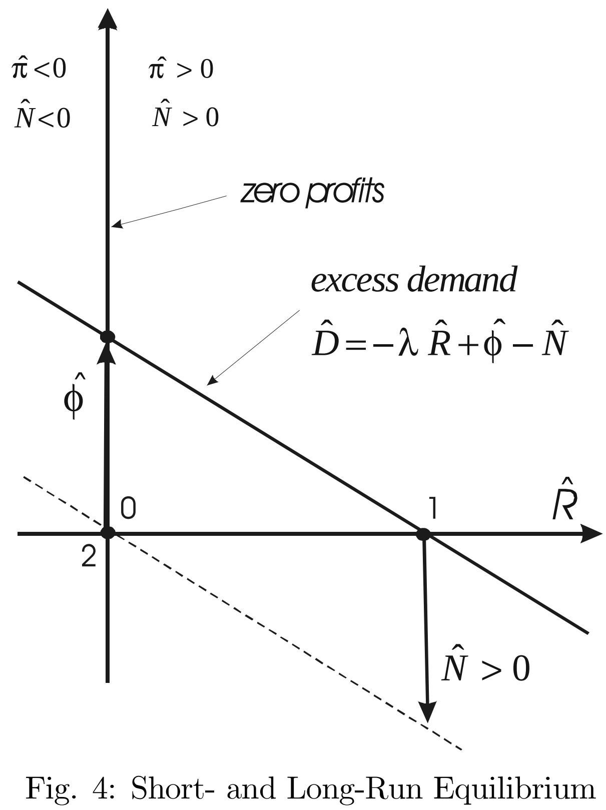

Fig. 4: Short- and Long-Run Equilibrium