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Abstract

We start by reviewing the graphical approach to teaching the real business cycle model introduced in Barro (1984). We then look at where this approach cuts corners and suggest refinements. Finally, graphical and exact models are compared by means of impulse response functions. The graphical models yield reliable qualitative results. Sizable quantitative differences exist, but these can partly be remedied by adding appropriate refinements. Used by experienced instructors the graphical analysis of the real business cycle equips students with a first understanding of the economy‘s supply side and generates results that will survive closer scrutiny later in the curriculum.

Keywords

Undergraduate teaching, macroeconomics, real business cycles, fluctuations

JEL Classification

A22, E32

1. Introduction

The real business cycle (RBC) model[1] has enhanced our understanding of macroeconomic fluctuations. While it eventually fell short of its initial goal to convince the profession that the ups and downs of the business cycle can be properly understood even without reference to nominal disturbances, its lasting legacy will be to have drawn attention to the fact that much more is happening on the economy‘s supply side than its demand-driven predecessors had been prepared to concede.

Given the importance and richness of the insights it can generate, students should not have to wait for graduate classes in order to be introduced to the mechanisms emphasized by the RBC model. Unfortunately, the mathematical and computational tools needed to solve and explore the properties of such models are not yet available at the undergraduate level. (And if they were, it would be questionable whether this would really be helpful in fostering a first understanding of the economic substance and mechanisms of the RBC model.) Instructors, therefore, typically resort to some graphical apparatus, similar to the one introduced in Barro‘s (1984) innovative text, when introducing the RBC approach to business cycles[2], or choose to refrain from developing any kind of explicit model at all.[3]

This paper has three purposes:

■ It briefly describes the graphical approach to teaching the basic RBC model in its simplest form, drawing on standard microeconomic concepts, as it can be used in second or third year macroeconomics courses.

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[1] The seminal papers on the real business cycle model are Black (1982), Kydland and Prescott (1982), and Long and Plosser (1982).

[2] A number of other writers have embraced Robert Barro‘s graphical apparatus. In Gregory Mankiw‘s *Macroeconomics*, probably today‘s leading intermediate macroeconomics text, real business cycle theory initially was taught in terms of the graphical analysis discussed in this paper. It was dropped after the second edition, however, and replaced by a simpler verbal discussion with a lot less depth. The most elaborate description of the graphical approach we are aware of is due to Braumann (undated).

[3] Examples of intermediate macroeconomics textbooks that do not feature an explicit model of the real business cycle are Blanchard (2003), Burda and Wyplosz (2001), DeLong (2002), Dornbusch, Fischer and Startz (2003), or Farmer (2001).

■ It points out the weak spots of this basic graphical approach to teaching the RBC model to undergraduates, revealing where it cuts corners in order to remain tractable at this level, and suggests ways to refine the employed diagrams if so desired.

■ Finally, it compares the simplified, approximate solution to the RBC model suggested by the suggested graphical approaches with the exact solution obtained by rigorously applying established solution algorithms from RBC research. This reveals the seriousness and size of the errors, qualitatively and quantitatively, that we are making by cutting corners using the simplifying graphical approach.

2. Real-business-cycle textbook analysis for undergraduates

The RBC model is a model with explicit microfoundations in which the involved agents are driven by optimizing behaviour. It assumes that all individuals are the same. This sidesteps potential aggregation problems and, in fact, makes the entire economy behave very much like an individual person behaves.

2.1. The basic RBC model

The basic RBC model comprises the following equations:

Households derive utility U from present and future consumption C and from present and future leisure time F. Future utility streams are discounted at the rate σ. This can be summed up in the intertemporal utility function

$$(1) \qquad U_t = \sum_{i=t}^{\infty} \left(\frac{1}{1+\sigma_i}\right)^{i-t} u_i(C_i, F_i)$$

which includes the well-behaved period or instantaneous utility function $u_t(C_t, F_t)$ in which both arguments generate positive but decreasing marginal utility. The subscripts on σ and u allow for the possibility of stochastic shocks to preferences.

Firms generate output, or aggregate supply Y^s, drawing on current technology T, by employing capital K and labour L.

$$(2) \qquad Y_t^s = T_t f(K_t, L_t)$$

Marginal products are positive and decreasing. The novel feature here is that technology is stochastic, displaying unforeseeable changes from one period to the next.

The capital stock is driven by net investment *I*, which depends positively on the marginal product of capital *MPK* and negatively on the real interest rate *r*.

(3) $$K_{t+1} = K_t + I_t(MPK_{t+1}, r_t)$$

Since current investment only increases *next* period's capital stock and output, the time-to-build assumption, the investment decision must be based on the marginal product of capital foreseen (or expected) for next period. When only one-time shocks or impulses are considered, we may sidestep issues of expectations formation and directly assume perfect foresight to apply during the adjustment process following the shock.

Keeping the economy closed and ignoring the public sector, aggregate demand Y^d is the sum of consumption and investment:

(4) $$Y_t^d = C_t + I_t$$

Since *I* denotes *net* investment, the real-world counterpart to income *Y* must be *net* domestic product.

The goods-market equilibrium condition $Y_t^s = Y_t^d$, which holds permanently, and the time constraint $L_t + F_t \leq 24$ complete the model.

Households maximize utility by deciding, on the one hand, on how much leisure time to enjoy, today and in each future time period during their infinite lifetime. Because of the time constraint, this is tantamount to deciding on its intertemporal pattern of labour supply and income. Simultaneously, they must decide on how much of this income they want to spend, today and in each future time period. This intertemporal pattern of consumption need not coincide with the income pattern, but it must be financed by it.

The exact formal solution to this complex, intertemporal optimization problem requires dynamic optimization algorithms, typically from the classical *calculus of variations* (a generalization of the Lagrange approach), the *theory of optimal control*, or *dynamic programming*.[4]

[4] One of the best introductions to methods of intertemporal optimization – with comparatively little emphasis on dynamic programming, though – is Chiang (1992). Frequently, those methods do not yield an explicit analytical solution. Then approximate optimality conditions

Whenever these methods are not available, approximate solutions can be derived by means of graphical analysis, during which we employ a partial perspective, pretending that the complex, simultaneous dynamic optimization process can be broken into a sequence of smaller, digestible steps that are independent of each other.

It is useful to look at *static optimization* within a given time period and at *intertemporal optimization* over time separately. We start with the former.

2.2. Current-period optimization

Suppose the decision of how much to work this period could be taken independently of other decisions that are of intertemporal nature. This is, of course, not an assumption that is generally valid. But it would apply, for example, in a long-run equilibrium, in which we do not expect technology to change (in a foreseeable way) and all adjustments have taken place. In this sense, the current-period optimum we derive here will yield some sort of *long-run anchor* to which we can tie the intertemporal adjustments to be discussed below.

When it maximizes utility during the current period alone, the representative household maximizes the current utility function $u(C, F)$, which can be represented by convex indifference curves in a diagram with consumption C and leisure time F on the axes. The relevant macroeconomic constraint is the production function $Y = f(\bar{K}, L)$, with capital being fixed in this period. Indifference curves and constraint can be merged into a single diagram under two conditions. First, the time constraint must be binding, so that $L = 24 - F$. This is uncontroversial. Second, consumption must equal income, leaving no room for net saving and net investment. This certainly will not hold in general. But, reflecting the permanent income hypothesis of consumption, it would hold in long-run equilibrium. The current-period optimum may thus be interpreted to constitute such a *long-run equilibrium*.

So, after letting $L = 24 - F$ and $C = Y$, the current-period utility function becomes $u(Y, 24 - L)$. Indifference curves can now be graphed in the same diagram as the production function. Optimal employment and income is determined by the point of tangency between the production function and an indifference curve, denoted by A (see Figure 1). This optimum changes, of course, if preferences change or if technological progress tilts the production function upward. The latter's effect on income is always positive. Its effect on employment depends on whether the

are derived and numerical methods are used to gain insights into the model's dynamic behaviour through impulse response functions. We will see examples of this in section 4.

substitution effect or the income effect dominates, making it an empirical matter. Empirical experience suggests that the two roughly cancel, leaving employment very much unchanged and bringing the economy into a point such as B.

Figure 1 near here

2.3. Intertemporal optimization

The nature of the intertemporal optimum is best brought out by assuming that households‘ horizons extend to two periods only. In this case equation (1) simplifies to

$$(5) \qquad U = u(C_0, F_0) + \frac{1}{1+\sigma} u(C_1, F_1)$$

As we see, households must make two related but nevertheless independent intertemporal decisions, one regarding consumption and the other regarding work or leisure time. Looking at those one at a time, intertemporal consumption preferences may be characterized by convex indifference curves as shown in Figure 2.

Figure 2 near here

The slope of these indifference curves is obtained by setting the total differential of (5) equal to zero and letting F_0 and F_1 be given. This yields

$$(6) \qquad \frac{dC_1}{dC_0} = -\frac{\partial u/\partial C_0}{\partial u/\partial C_1}(1+\sigma)$$

It is interesting to note that on the 45° line the marginal utilities of consumption are the same in both periods, because $C_0 = C_1$. Hence

$$\frac{dC_1}{dC_0} = -(1+\sigma) \qquad \text{if } C_0 = C_1.$$

The household‘s budget constraint states that the present value of consumption spending in periods 1 and 2 must equal the present value of income, i.e. $C_0 + C_1/(1+r) = Y_0 + Y_1/(1+r)$. This rewrites

(7) $$C_1 = (1+r)Y_0 + Y_1 - (1+r)C_0$$

Equation (7) is a straight line with slope $-(1+r)$ which passes through the endowment point $C_0 = Y_0$ and $C_1 = Y_1$, which we assume to be in A, that sums up the intertemporal income profile. Perfect consumption smoothing, that is $C_0 = C_1$, results whenever the interest rate equals the time discount rate. Then the point of tangency between the constraint line and an indifference curve lies on the 45° line (Figure 2, point A) and households do not save. When $r > \sigma$, households reduce current consumption in exchange for higher consumption next period, as indicated by point B. When $r < \sigma$, the opposite happens..

By analogous though slightly more complicated reasoning, an analogous diagram can be drawn and corresponding arguments can be advanced for the intertemporal choice of leisure (see Figure 3). Based on the utility function used in the simulations below, indifference curves in the leisure-choice diagram are being kept linear [see equation (1')]. This suffices for a unique optimum to obtain, since the constraint is concave. This is somewhat labourious to show formally. The intuition is as follows: consider the optimal consumption pattern C_0^*, C_1^* given. This can be paid for by an infinite number of period 0 and period 1 combinations of income and, hence, of work or leisure time. One option would be to work the same hours both periods, as in point A. This substitution is not linear, however, because of decreasing marginal productivity of labour. One hour less leisure time in period 0 will buy less and less added leisure time in period 1, making the constraint concave. Here again, an increase in the interest rate makes the constraint steeper, with the turning point being on the 45° line.

Figure 3 near here

2.4. The graphical real business cycle model

The insights gained so far can be brought together in a single diagram which features the interest rate on the vertical axis and income on the horizontal axis (see Figure 4). Suppose parameters, including technology and preferences, have not changed for a while. So the economy has settled into a long-run equilibrium in point A, in which $r = MPK = \sigma$. Associated levels of income, consumption, employment and leisure time reflect current-period utility maximization as discussed in section 2.2. There is no intertemporal substitution. In terms of the intertemporal-decision diagram discussed in section 2.3. the economy is on the 45° line.

Figure 4 near here

Of course, output only equals the level indicated by point A (along with the associated level of employment) if $r = \sigma$. Whenever $r \neq \sigma$ intertemporal substitution kicks in. Should, hypothetically, the interest rate rise above the time discount factor, households would decide to take advantage of these unusually high interest rate by working more today in exchange for more leisure time tomorrow. The opposite would occur if r were to drop below σ. This gives rise to a *positively sloped aggregate supply curve* Y_0^s, which shows how, given the capital stock and technology available in period 0, the level of output produced reflects the intertemporal substitution of work time in response to a changing interest rate.

What about demand, which comprises consumption and investment? Well, in the initial equilibrium point A the capital stock does not change. So net investment must be zero, which means consumption must equal income: $C_0 = Y_0$. From section 2.3. we know that consumption is also subject to intertemporal substitution when $r \neq \sigma$. When r exceeds σ, households reduce current consumption in order to save more. When r drops below σ, current consumption is spurred. This *negative relationship* between consumption and the interest rate is captured by a negatively sloped line.

Since investment is zero in A, the investment line passes through the ordinate at $r = \sigma$. The cheaper the financing of investment projects becomes, the more firms will invest. So this demand component as well depends negatively on the interest rate, which is reflected in a negatively-sloped investment line. The aggregate-demand line, $Y^d = C + I$, reflects the sum of consumption and investment.

2.5. The real business cycle

Any improvement in production technology raises income via the production function. If the improvement is permanent, there is a lasting income hike. If the improvement is temporary, there is a corresponding income effect. What the RBC model has added to these unexciting insights is that any such changes in technology may induce drawn out responses of key macroeconomic variables, including income, investment, consumption, the interest rate, wages and employment, which look a lot like some segment taken out of a traditional business cycle. The exact path of the income response, for example, depends on the nature of the shock. We will analyse on a permanent shock here, for ease of graphical exposition.[5]

The immediate consequence of a permanent technology shock is a shift of the aggregate-supply curve to the right, which signals that more output can be produced at all employment levels. Figure 5 shows this movement. Since households recognize this shock as permanent, their long-run equilibrium response would be to increase consumption by just as much as income increased - as suggested by the permanent-income hypothesis. Since this would only happen, however, if the interest rate remained at $r = \sigma$, making the new level of income a new long-run equilibrium, the new consumption curve intersects the new aggregate-supply curve exactly at $r = \sigma$.

Figure 5 near here

Improved technology has made the capital stock more productive, both on average and at the margin. This raises investment at all interest rates, moving the investment line to the right also. The shift of the aggregate-demand line to the right reflects the aggregated movements of the investment line and the consumption line. Since the latter alone already moves as far as the Y^s line, the Y^d line moves further than the Y^s line. This means, however, that there would be a disequilibrium at the old interest rate, the equilibrium interest rate: aggregate demand would exceed aggregate supply. The new goods market equilibrium obtains where the Y_1^d and Y_1^s lines intersect, which is at a higher interest rate.

How does this new equilibrium come about? The key word is intertemporal substitution. When the interest rate goes up, households take advantage of this by working more today. In the graph this particular type of intertemporal substitution results in a movement along the Y_1^s line, from B‘ towards B. On the demand side there is intertemporal substitution as well. Households consume less and save more when the interest rate goes up. Since investment falls as well, a rising interest rate results in a demand-side movement up the Y_1^d curve, from B“ towards B. Both movements

[5] A perfectly transitory, one-time shock, marking the case at the other extreme, is discussed in the appendix. Intermediate cases, shocks of an autoregressive nature, could also be dealt with graphically. The price is a much more crowded diagram, however, with little in terms of added substantial insights being gained.

reduce the initial gap between supply and demand that would have materialized at an unchanged interest rate, eliminating it as we reach point B. So B is the new temporary equilibrium in period 1.

Point B cannot be the new long-run equilibrium, however. First, we know that the interest rate must equal the time preference rate in long-run equilibrium. Otherwise there would still be intertemporal substitution. Second, since there was positive net investment in period 1, the economy operates with an increased capital stock in period 2. This moves the Y^s curve still further to the right. When households consider this increase in the capital stock permanent, the consumption curves follows the aggregate-supply curve‘s move to the right. Regarding investment we must note that the MPK has fallen because of the increase in the capital stock. This shifts the investment line slightly down left. As a consequence, period 2 equilibrium still features an unusually high interest rate, but the interest rate has already begun to ease back down toward its long-run equilibrium value (point C).

For the same reasons described in the previous paragraph, income will continue to rise and the interest continues to fall during subsequent periods. A new long-run equilibrium will eventually be reached in a point such as D. Since we again have $r = MPK = \sigma$, there is no net saving. So all income is being consumed. Income is higher, both due to improved technology and because of a higher capital stock. Employment is about the same, if the conditions suggested in Figure 1 apply. This would lead to a much higher real wage, since both better technology and more capital have boosted labour‘s marginal productivity.

3. Weak spots of the graphical textbook model

As conceded from the very beginning, there is quite some hand waving involved when the real business cycle story is told in terms of diagrams that draw on and combine basic microeconomic concepts. When time permits, these weaknesses may be pointed out, and more refined solutions may be offered.

3.1. Horizon reduced to two periods

The most obvious weak spot in our graphical analysis is the inconsistency between the intertemporal decisions and expectations described in Figure 2 on the one hand, and the business-cycle dynamics summed up in Figure 5 on the other hand. According to Figure 5, a lasting improvement in the production technology triggers an initial increase in the real interest rate and gives rise to positive net savings in period 1, in period 2, and for quite some time after. This is in open conflict with what happened in Figure 2, where a rising interest rate also drives up savings in period 1, but causes households to *dissave* in period 2. What is causing this inconsistency?

The culprit is obviously the arbitrary limitation of the planning horizon to two periods which, through the budget constraint given by equation (7), makes the second period very much a mirror image of the first. When we extended the planning horizon to more than two periods in section 2.5, this had to lead to inconsistencies. A first, not yet complete answer to how to resolve this goes as follows:

The budget constraint in the infinite-horizon case reads

$$(8) \qquad C_0 + C_1/(1+r_0) + C_2/[(1+r_0)(1+r_1)] + \ldots = Y_0 + Y_1/(1+r_0) + Y_2/[(1+r_0)(1+r_1)] + \ldots$$

Employing the definitions

$$(9) \qquad C_1^+ \equiv C_1 + \frac{C_2}{1+r_1} + \frac{C_3}{(1+r_1)(1+r_2)} + \ldots$$

and

$$(10) \qquad Y_1^+ \equiv Y_1 + \frac{Y_2}{1+r_1} + \frac{Y_3}{(1+r_1)(1+r_2)} + \ldots$$

where C_1^+ and Y_1^+ denote the current value of all future consumption and income, respectively, we may rewrite the constraint to be depicted in a C_1^+/C_0 diagram as

$$(7') \qquad C_1^+ = (1+r_0)\,Y_0 + Y_1^+ - (1+r_0)\,C_0$$

which, again, is a straight line with slope $-(1+r_0)$ which passes through the endowment point $C_0 = Y_0$ and $C_1^+ = Y_1^+$.

The general formula for indifference curves in C_1^+/C_0 space looks a bit messy. However, from the total differential

$$(11) \qquad dU = 0 = \frac{\partial u}{\partial C_0} dC_0 + \frac{1}{1+\sigma}\left(\frac{\partial u}{\partial C_1} dC_1 + \frac{1}{1+\sigma}\frac{\partial u}{\partial C_2} dC_2 + \ldots\right)$$

we easily see, that in steady state, where all marginal utilities are the same and, thus, cancel out, and all interest rates employed in (9) and (10) equal the time discount rate σ, indifference curves have slope (see Figure 6)

$$(12)\qquad \frac{dC_1^+}{dC_0} = -(1+\sigma)$$

Figure 6 near here

This permits an infinite horizon-perspective which makes intertemporal substitution compatible with the real business cycle story told in section 3. If the interest rate rises above its steady state value, households raise current savings. This is balanced by dissavings at some point in the future, but not necessarily in the near future and most likely not in period 2.

A second answer, one that also fits with the simulations presented below, starts from a generalized budget constraint. Up to now we assumed that households eventually consume everything that has been saved plus any interest income that derives from past savings. According to Figure 5, however, a lasting productivity shock leads to a permanently higher capital stock. So incomes may either be consumed, or they may materialize in an increase in the steady-state capital stock. Denoting the additions to the capital stock accumulated in period 1 and after by ΔK_1^+, the budget constraint becomes

$$C_0 + \frac{C_1}{1+r_0} + \frac{C_2}{(1+r_0)(1+r_1)} + \ldots + \Delta K_1^+ = Y_0 + \frac{Y_1}{1+r_0} + \frac{Y_2}{(1+r_0)(1+r_1)} + \ldots$$

After rearranging terms to obtain

$$C_1^+ - Y_1^+ + (1+r_0)\Delta K_1^+ = (1+r_0)(Y_0 - C_0),$$

we see that current net savings need not give rise to dissavings ever, but may instead be kept in the form of a higher capital stock in the new long-run equilibrium. This is what, in fact, happens in Figure 5, where the visual build-up of the capital stock during the first few periods after the productivity shock is not followed by any time of capital consumption ever.

3.2. Identifying permanent income

The graphical analysis repeatedly referred to *permanent income*. We used it to justify the derivation of the current-period optimum in Figure 1 and to interpret it as a steady state, a long-run equilibrium. We used it to nail down the endowment point for the analysis of intertemporal substitution in Figure 2. And it played a key role in positioning the consumption line in the course of business cycle dynamics in Figure 5. In order to keep things simple, permanent income, used interchangeably with steady state income, was estimated by a simple adaptive process. Households always expected income to remain where it currently was. This mechanism is not completely off in the case of a permanent shock[6], but we can certainly improve on it.

Take a second look at the current-period or steady-state optimum, repeated for convenience in Figure 7. Assuming that employment would not change, we argued that a positive shock to technology, turning the production function upward, would raise income to the level associated with point B. Since this is the effect on income that occurs in the absence of intertemporal substitution, we could use it to position the aggregate supply curve for period 1. When we argued that the consumption line would move the same distance to the right as the supply curve, invoking the permanent income hypothesis, we implied that households expected income to remain at this level.

Figure 7 near here

Subsequently we learned that this was not really the case. Positive net savings made the capital stock and income grow until the latter finally settled into its long-run equilibrium at point D. In terms of Figure 7 the rising capital stock turned up the production function still further, revealing that the movement into point B was only a short-run effect. Now if households anticipate this correctly, the consumption line in period 1 shifts further to the right as the supply curve. In fact, this shift is almost as large as the long-run shift of the supply curve.

Does this affect our results? Quantitatively – yes, of course. But qualitatively – no. While, technically, we could position the curves in the graph so as to yield no positive net investment in period 1, this would be inconsistent with households anticipating income to rise in the long-run due to capital accumulation. Therefore, economic logic requires consumption to fall short of income in

[6] Though it would be, if the shock was transitory. For a discussion of this case, see the Appendix.

period 1.

3.3. Simultaneity of decisions

A final point worth looking into is the neglect of the simultaneity of decisions by the suggested stepwise graphical analysis. This is, of course, related to what we discussed in the previous section, but it is also worth considering as a separate issue.

Reconsider the intertemporal consumption and employment decisions discussed in Figures 2 and 3. Figure 2 assumed leisure choice had already been made and resulted in an initial intertemporal income profile that gave us an endowment point *on* the 45° line (point A). The preliminary assumption was that any permanent technological improvement would raise income permanently, today and during each future period, pushing the endowment point further out on the 45° line into a point such as B (Figure 8).

Figure 8 near here

After what we discussed in section 3.2 we know that this graphical interpretation is not entirely correct. Since income continues to rise due to capital accumulation even after the shock period, until it finally settles into its higher long-run equilibrium level, we must reposition the new endowment point north of the original one. This has nothing to do yet with the interdependency of decisions. But it does provide the interesting insight that there would be intertemporal substitution even if $r_0 = \sigma$, because the consumption point would be located on the 45° line. Now, since the productivity shock raises the *MPK* and the interest rate, thus turning the constraint steeper, intertemporal substitution pushes current savings up, as discussed previously, and the economy ends up in C.

Remember, though, that this interest rate hike at the same time entices households to work more today, as shown in Figure 3, raising today‘s income. This means that an increase in *r* not only turns the constraint steeper, it also moves the endowment point, the point around which the constraint turns, to the right (and probably even up) into B“. Intertemporal substitution lets households optimize utility in D.

While taking this interdependency of decisions into account does not appear to have dramatic effects on how the model behaves, it may be worthwhile noting how it fits in. In fact, with the same arguments, what happens in the consumption diagram affects intertemporal leisure choices, and as those are being modified they have second-order effects on consumption choices, which feed back into leisure choices, and so on.

4. The errors we make: are they large?

Section 3 gave proof of the limits to how far we can push the precision of graphical analysis. Weaknesses may be pointed out, and we looked at some of the major ones. And we may often be able to say in what direction the exact result must be expected to deviate from the one obtained by graphical approximation. But if we are interested in a definitive statement about how far off the mark the graphical textbook model really is, quantitatively or qualitatively, we must resort to numerical simulation. This is what this section is about.

As a first step towards obtaining and comparing quantitative results, we need to calibrate the model(s). Regarding the household utility function, the following functional form will be used:

$$(1') \qquad U_t = \sum_{i=t}^{\infty} \left(\frac{1}{1+\sigma}\right)^{i-t} [\ln C_i + \beta(1-F_i)]$$

The production function is Cobb-Douglas, namely

$$(2') \qquad Y_t^s = T_t K_t^{\alpha} L_t^{1-\alpha}$$

Parameter values are set to $\beta = 2.85$, $\sigma = 0.05$, $\alpha = 0.36$, and the initial technology level is assumed to equal $T_t = 1$.

We will be comparing three versions of the basic RBC model:[7]

[7] We are actually discussing three *solutions* to the basic real business cycle model. But since the solution algorithms make different assumptions about the optimizing behavior of the involved agents, we are, in fact, dealing with three different *models*.

The exact model was solved with the dynamic programming techniques described in Judd (1999). For the two versions of the graphical textbook model we wrote programs that replicate the sequential decision making process which households follow.

■ **The exact model**.[8] In this version we are applying the very intertemporal optimization algorithms used in real-business-cycles research. Households and firms display perfect rationality. The behaviour of the exact model serves as a benchmark which the graphical solutions will be compared to.

■ **The graphical model**. This is the simplest graphical textbook version of the RBC model as described in sections 2.1-2.4. Households and firms commit all the errors pointed out in Section 3. In particular, household planning extends to 2-period horizon only and their consumption decisions are based on a naive version of the permanent income hypothesis (rather than going through a full-fledged optimization procedure), ignoring the effect of investments on future capital stock and income.

■ **The refined graphical model**. This version of the graphical model is the same as above except that households optimize with an infinite horizon as described in section 3.1 rather than with a 2-period horizon.

4.1. Simulation results from the exact model

The six panels of Figure 9 show the dynamic behaviour of key macroeconomic variables by means of impulse response functions to an unexpected, permanent improvement of production technology. Solid lines indicate the behaviour of the exact model, reflecting well-known properties of the real business cycle model.

Figure 9 near here

First, the boost to technology pushes up the marginal productivity of capital and the interest rate, triggering intertemporal substitution: households raise their labour supply at the cost of less time of leisure. Part of the income increase is saved, permitting a boost in investment. The latter makes the capital stock grow, which gradually lowers the interest rate. So intertemporal substitution eventually peters out as the economy settles into its new long-run equilibrium. While all this happens, income grows. Initially in the form of a large leap due to improved technology and an

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[8] Calling this the exact model is a bit misleading in the sense that no analytical solution exists, as is usually the case in real business cycle research. Following established procedures in academic research, we therefore take recourse to approximate solutions obtained by numerical methods.

increased labour supply. Subsequently because of the gradual build up of the capital stock. Because the latter is counterbalanced by an increase in depreciations and the gradual reduction in the household labour supply (which had initially leaped up), the effect on net income is in fact negative[9]. So, despite what is happening behind the curtain under the heading of intertemporal substitution, the path of income very much reflects what happened to technology.

4.2. Simulation results from the textbook models

Dotted lines represent the behaviour of the unrefined version of the *graphical model* which still possesses all the weaknesses spelled out in section 3. In spite of these, however, the qualitative behaviour compares favourably with the exact model:

- The initial or impact response of all variables goes in the right direction.
- The subsequent dynamic adjustment paths, as driven by intertemporal substitution, always leads towards the same long run equilibrium as well.

This means that as long as we refrain from making any quantitative claims and settle for pointing out the directions in which variables move initially, in the medium and in the long run, we are fine. So even the simplest graphical model teaches the right propagation forces being at work and points out where the economy is heading.

All this also holds for the refined graphical model with an infinite horizon, the behaviour of which is depicted in dashed lines. In qualitative terms, this model variant behaves just as the simple graphical and the exact models do. In quantitative terms it produces results that are in between those generated from the other two versions, though somewhat closer to the exact model.

4.3. Comparing quantitative results

Quantitative claims we obviously should not make, however. Here the errors really kick in and, in many cases, heavily distort the magnitudes and speed by which variables change and adjust.

As a rule, adjustments in the exact model are much quicker than in the graphical versions: After the initial drop, consumption recovers much faster. This, of course, is reflecting the fact that the capital stock approaches its new long run equilibrium much faster. For the same reason, the

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[9] This might be surprising at the first glance, but remember that we are looking at *net* income here. Whereas *gross* income increases over time, with the productivity effect of the increase in the capital stock outweighing the reduction in labor supply, net income decreases because of increased depreciation.

labour supply also returns to its normal level rather quickly, and capital stock growth, after initially very high rates, approaches its new long-run equilibrium much earlier. In our simulation, the movements of the two production factors (capital stock growth and employment decline) just seem to cancel out so that not much movement in income is left after the initial boost.

The central reason for why the dynamics in the naive baseline model is so subdued in quantitative terms is the 2-period horizon of households, which implies that what is being saved today must be consumed tomorrow. This drastically reduces the incentives for intertemporal substitution. The benefits from saving and investing today - having a higher capital stock and higher income in the future - can only be reaped for one single period. So the incentive to save today is rather weak. The same applies to the labour-supply decision. The fruits from working more today can only be enjoyed in the form of higher leisure time for one period only. This is how far the planning horizon extends.

Extending the planning horizon from those artificially short 2 periods to infinity greatly spurs the economy‘s dynamics, as the simulations show, moving it much closer to what it should look like according to the exact model. Differences remain, however. So what are the factors that explain these prevailing differences?

The most obvious error that remains is that households still fail to see that the positive shock to technology will lead to a build up in capital, entailing further growth in income. Households do not see this, since they consider permanent income to equal current income, and thus always consume all of current income when the interest rate equals the time discount rate. Note, however, that this systematic underestimation of permanent income does not explain why adjustment is so slow. By contrast: if households were to act on a correct estimate of permanent income, rather than taking current income as a proxy for permanent income, they would actually increase consumption. This would lead to a slower accumulation of capital and, hence, a lower speed of convergence to the new steady state.

A more relevant omission from the perspective of too low a convergence speed is the following: households do not take the term structure of the interest rate into account. The term structure does not play a role in the basic graphical model, since households optimize over two periods only. It does play an important role, however, once we extend the horizon to infinity. To see why, note that the real interest rate has a high degree of persistence and remains significantly above its equilibrium rate long after the positive technology shock. This guarantees higher income from owning capital for many periods after the permanent technology shock has occurred, and it thus gives households an added incentive to build up capital through reduced consumption and increased work hours.

There are several options of dealing with the term structure. The one chosen in our simulations of the two graphical versions of the RBC model was to let households expect that the interest rate would drop back towards the time discount rate after one period. As the simulations show, this assumption prolongs the adjustment process. Another option would be to provide households with the knowledge that the interest rate follows an AR(1) process. In effect, by adding the knowledge that of the interest rate and wages follow an AR(1) process to the information set of households, the refined graphical model can, in fact, almost perfectly replicate the behaviour of the exact model. The high price to pay would be the added and probably unreasonable complication this would introduce into an undergraduate lecture.

5. Summary and concluding comments

The results of this paper's numerical exercises are quite comforting: A diagrammatic analysis of the real business cycle model that combines fairly standard building blocks from intermediate microeconomics (static and two-period optimization under a constraint) with the permanent income hypothesis of consumption and an equation describing capital formation serves well in bringing out the innovative features attributed to real business cycle theory. It permits students to accurately, in qualitative terms, observe and follow the causes and consequences of intertemporal substitution of consumption and labour and appreciate why under such circumstances one time shocks may generate a complex, smooth adjustment in most macroeconomic variables, reminiscent of old-fashioned business cycles, rather than a simple, one-time jump.

The evident discrepancy between the quantitative behaviour of the accurately solved baseline model and the graphical textbook versions deserves notion but not worry. After all, throughout undergraduate macroeconomics we normally and quite happily settle for mainly qualitative discussions of those models and concepts that have become workhorses of business cycle and growth analysis on that level, including such mainstays as the Keynesian cross, IS-LM, Mundell-Fleming, aggregate supply and demand, and the Solow model.

Employing the described graphical apparatus requires some flexibility and caution on behalf of the instructor, however, particularly in the use of the permanent-income building block. The approximation suggested in the main part of this paper, that households' estimates of permanent income simply adapt to current actual income, only then works well when shocks are permanent or highly persistent. If the economy is exposed to shocks that are mostly transitory, for obvious reasons the preferable option is to let the estimate of permanent income remain where last period's actual income was.[10] With this kind of flexibility in its application, the graphical real business cycle model constitutes an important and valuable teaching tool in the hands of an experienced instructor even on the undergraduate level.

[10] The case of a transitory shock to productivity is briefly discussed in the appendix, along with a comparison of how the three types of model variants, on which we had focused above, compare in terms of qualitative and quantitative performance.

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Appendix: Looking at temporary instead of permanent shocks

This appendix serves to illustrate by means of an example that the proposed graphical framework is flexible enough to permit adaption to different environments. Here we leave the permanent-shock scenario employed in the main part of the paper and consider a one-time, purely transitory productivity shock, one that improves technology for one period only.

The immediate consequence of this productivity shock is a movement of the aggregate-supply curve to the right, signalling that more output can be produced at all levels of employment (see Figure 10). Households are aware of the transitory nature of the shock. Hence it does not make sense to assume their estimate of permanent income to follow actual income, as was a reasonable first approximation in the presence of a permanent shock. In the presence of a temporary shock we may instead assume that permanent income remains where income was before the shock and to where it will eventually return after the shock. As a consequence, the consumption line stays put in its initial position.

Figure 10 near here

The position of the investment line is determined by the (expected) marginal productivity of capital. Firms are also aware of the temporary nature of the productivity shock. The recognize that any investment projects undertaken today will have to start production tomorrow using old technology again. As a first approximation, therefore, we may assume that the investment line remains in its original position as well. With both the investment and consumption lines unmoved, the aggregate-demand line also stays put.

Since the aggregate-supply line move right, while the aggregate-demand line doesn‘t, there would be an excess supply in the goods market at the pre-shock, the equilibrium interest rate. This is avoided by intertemporal substitution kicking in, both on the supply and on the demand side. On the supply side, a fall in the interest rate reduces the supply of labour and, hence, output. This materializes as a slide down the aggregate-supply curve. On the demand side two things happen: first, households expand consumption at the cost of lower savings in the face of a drop in the interest rate; second, the drop in the interest rate stimulates investment. Both effects constitute a slide down the aggregate-demand line, into the point of intersection between Y_1^d and Y_1^s , the period-1 equilibrium point, denoted by B.

Period 2 sees the aggregate supply and demand lines deviate in opposite directions from their pre-shock positions. The reversal of the technology shock alone would lead the aggregate-supply curve back into its original position. But since there was positive net investment in period 1, the capital stock has grown, placing Y_2^s just a little bit to the right of Y_0^s. On the demand side, permanent- income considerations would make the consumption line intersect the aggregate supply line at $r = \sigma$. But because the increased capital stock lowers the marginal productivity of capital, the investment line shifts down and thus, with the consumption line given, the aggregate demand line as well. Period 2 equilibrium obtains in point B, where income has roughly returned to its original level, and net disinvestment begins to make the capital stock shrink again. This process continues until the economy is back in its initial equilibrium in A.

Figure 11 compares the development of key variables as described by the adapted basic graphical model, its refined graphical version, and by the correctly solved version of the model. Results are similar to what we obtained for the case of a permanent shock discussed in the text. While there are notable, in a few cases sizable differences between the graphical models and the baseline model, the graphical models never lead users astray with their description of qualitative behaviour.

Figure 11 near here

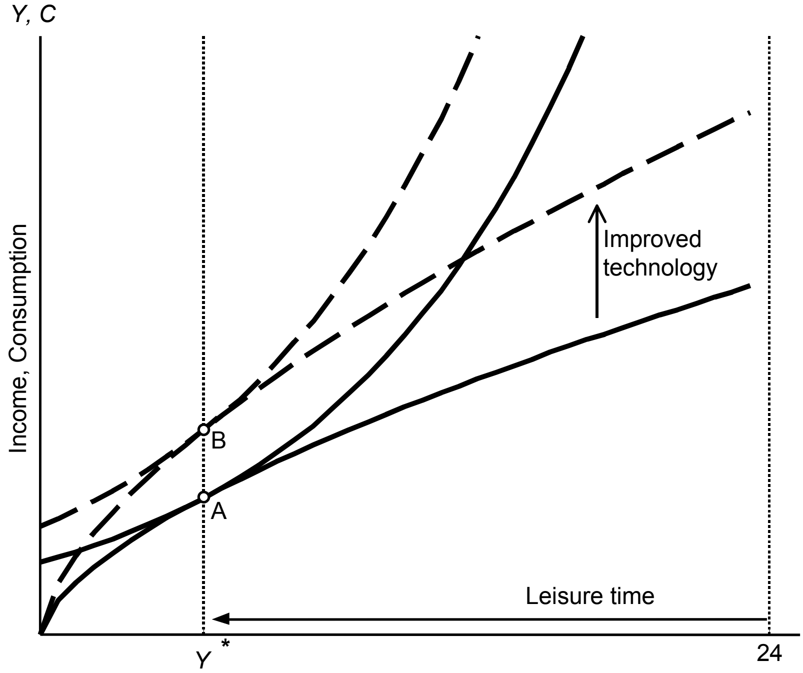

Figure 1. Households' current-period optimization regarding work and leisure time.

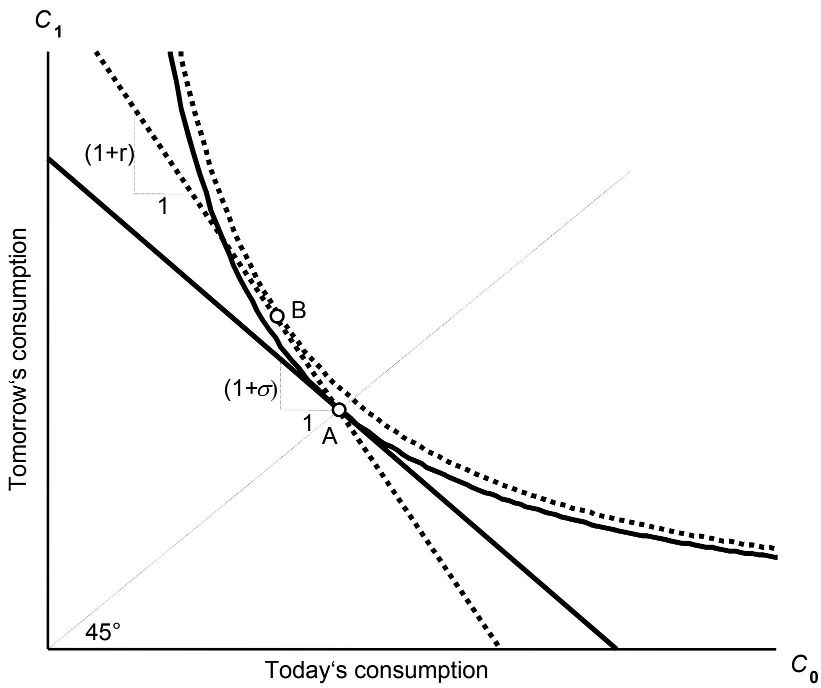

Figure 2. Households' intertemporal optimization regarding consumption and saving.

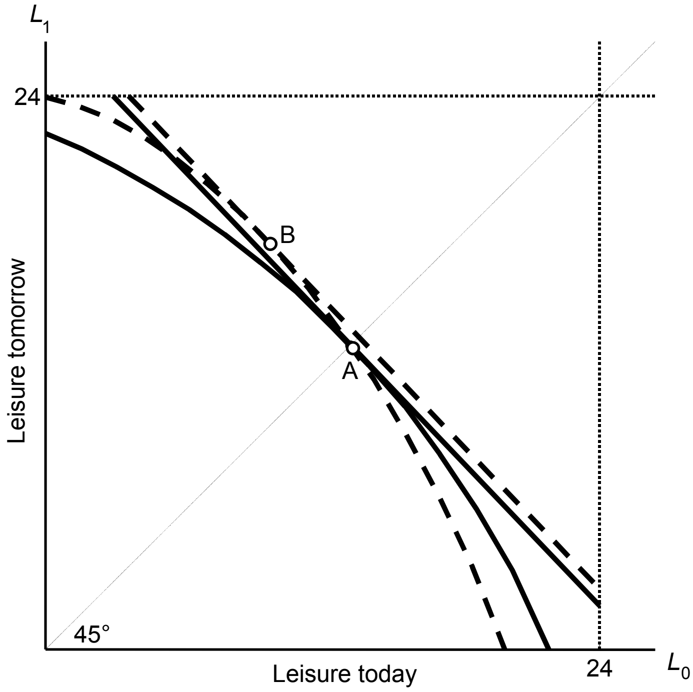

Figure 3. Households‘ intertemporal optimization regarding work and leisure time.

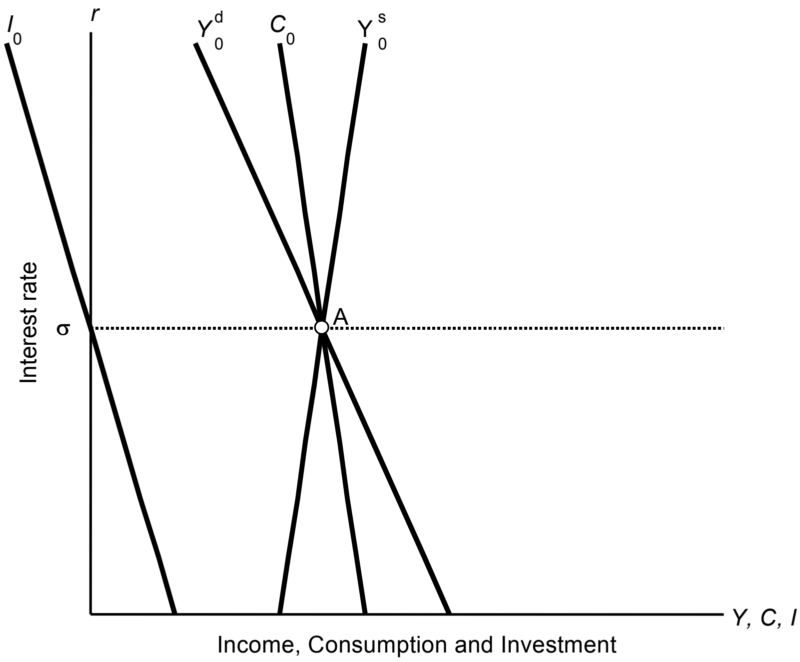

Figure 4. Aggregate demand and supply in an interest rate/income diagram.

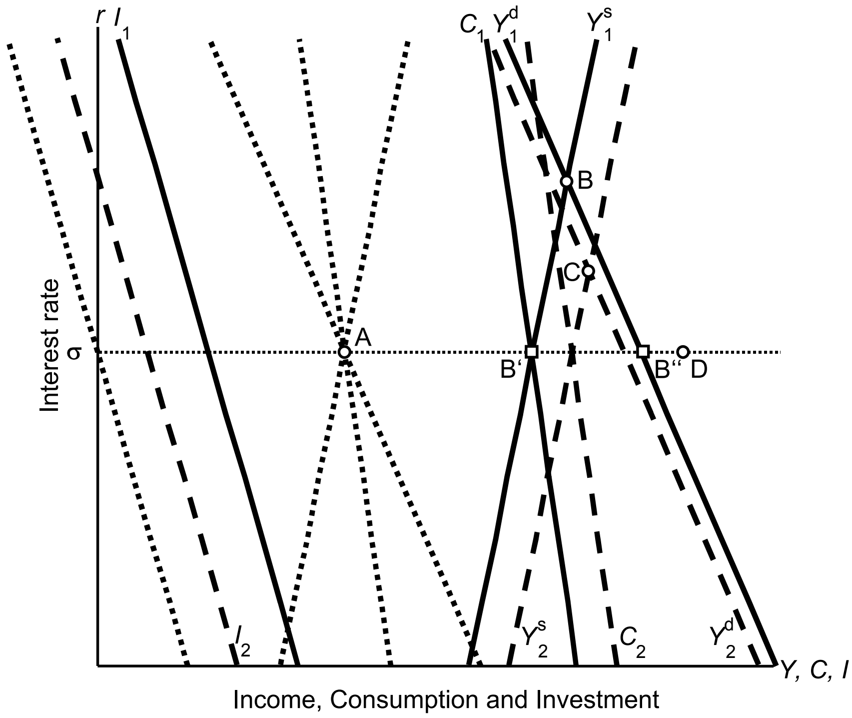

Figure 5. Dynamic response to a permanent productivity shock.

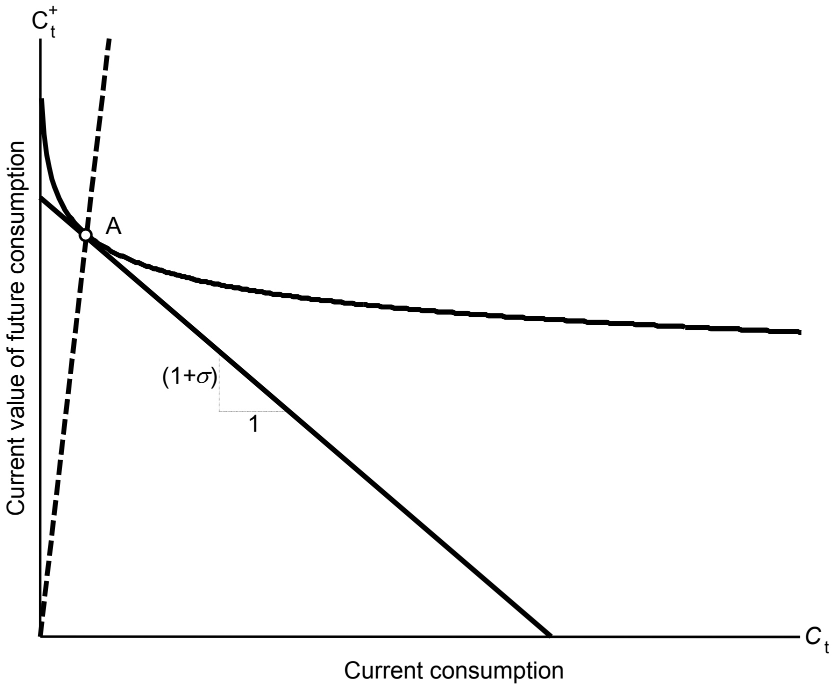

Figure 6. The intertemporal diagram with an infinite horizon.

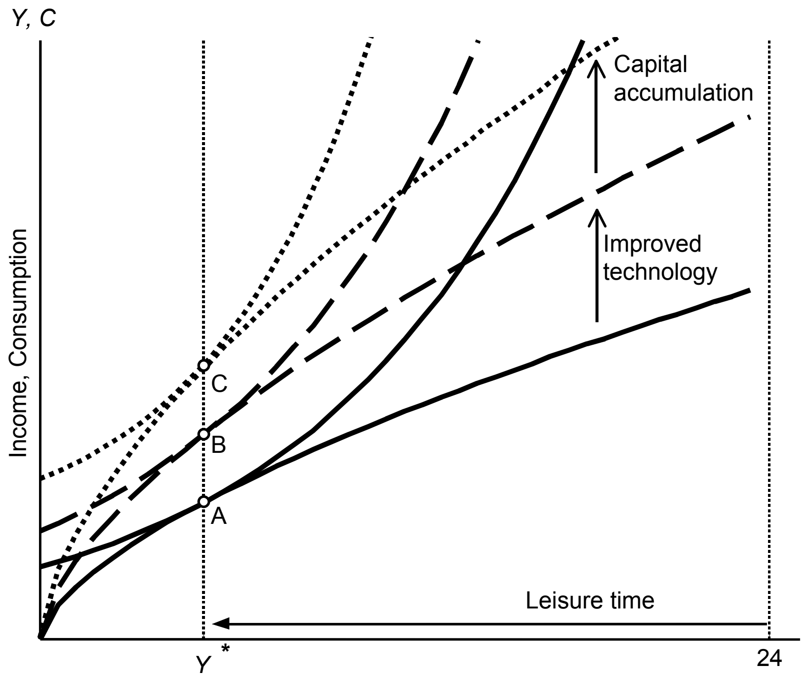

Figure 7. The current-period optimum with an infinite horizon.

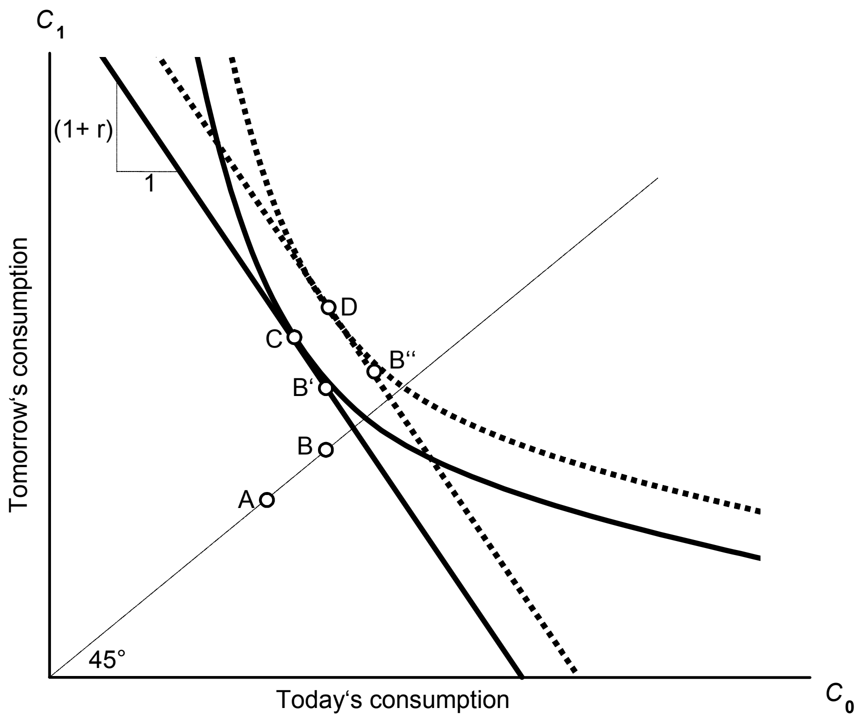

Figure 8. A refined perspective of households' intertemporal optimization.

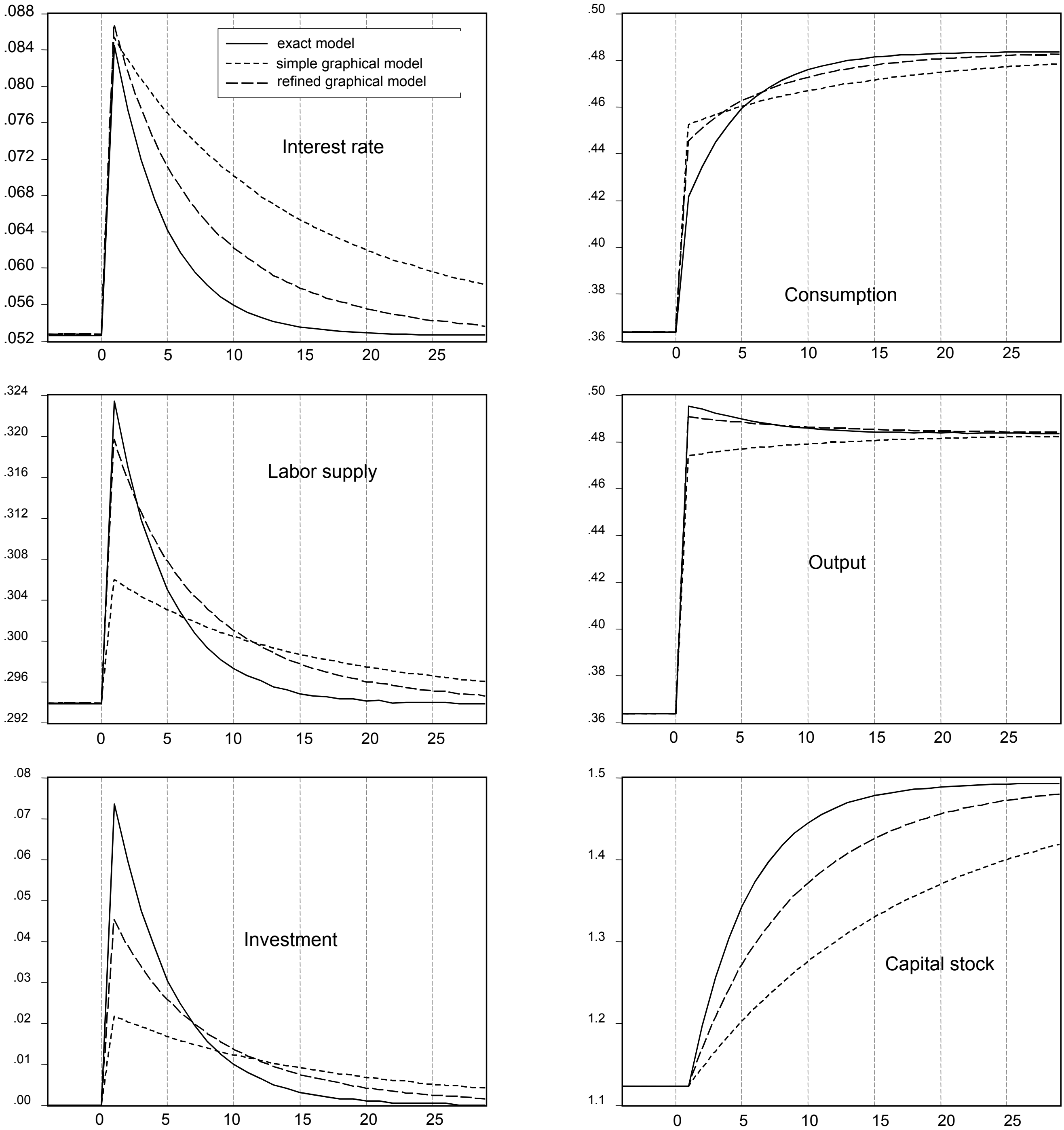

Figure 9. Impulse responses of all three models to a permanent productivity shock.

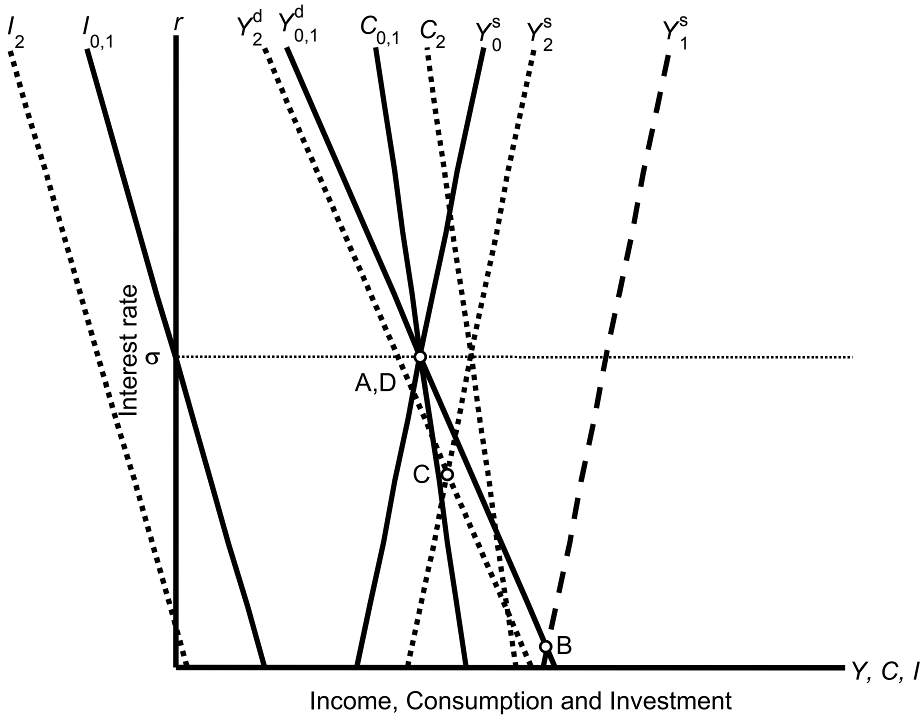

Figure 10. Dynamic response to a transitory productivity shock.

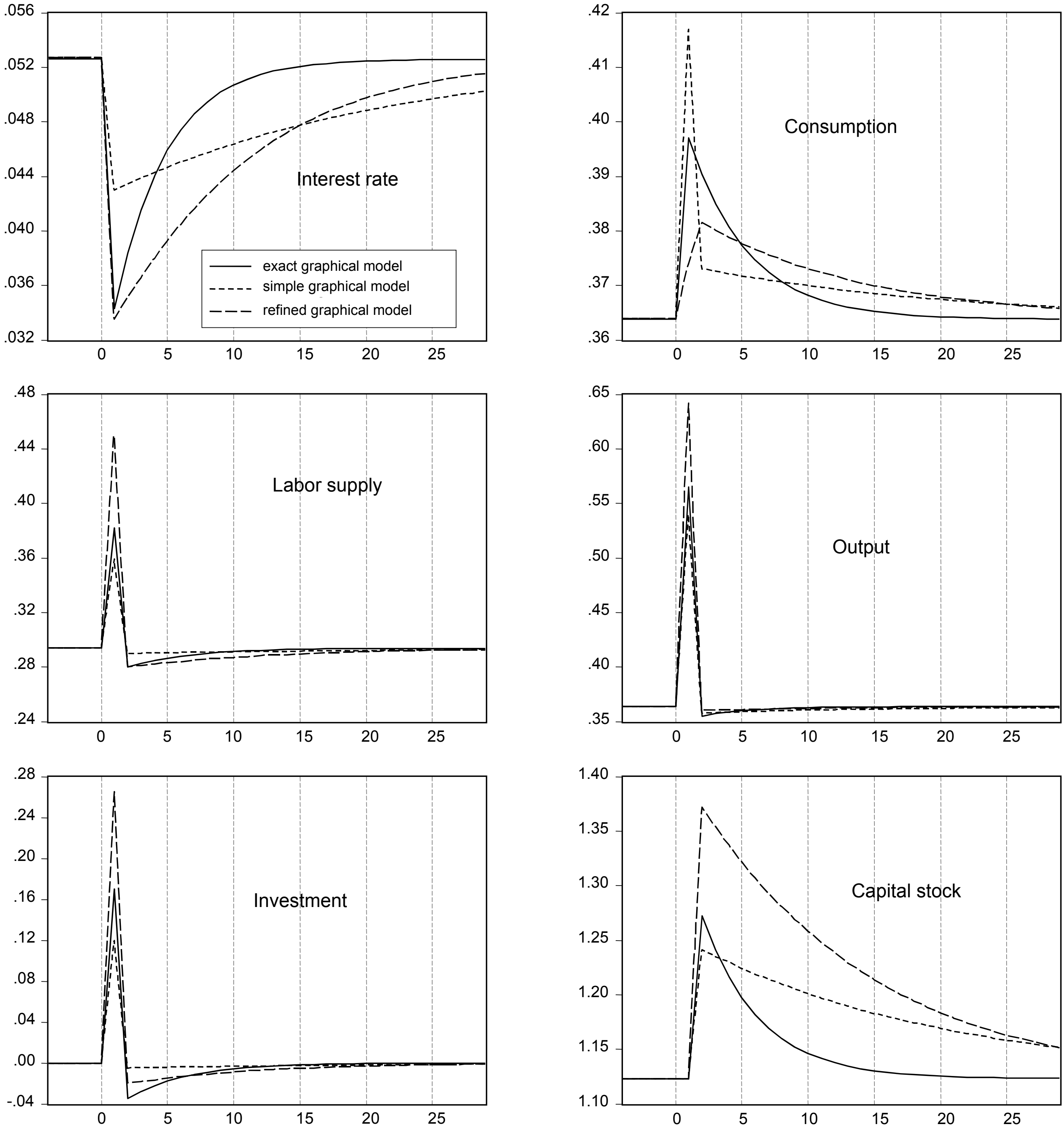

Figure 11. Impulse responses of all three models to a transitory productivity shock.