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Abstract

In this paper I analyze firm training in the dynamic context of a Blanchard-model with infinite periods. Firms provide their workers with training due to wage compression caused by frictions in the labor market. I do not only describe the stationary solution but as well the transition to long-run equilibrium. It turns out that after a positive shock to the stock of physical capital, training investments overshoot and then slowly converge to the new equilibrium level.

Furthermore, I discuss some aspects of public policy like a tax on capital income and a subsidy for firm training or the combination of both policies. It turns out that a capital tax influences training investments via two opposing effects. On the one hand, it lowers the stock of physical capital and thereby the productivity of training. On the other hand, the bargaining power of workers is diminished because there are fewer firms active in the market. This leads to a higher degree of wage-compression improving the incentives to train. Principally, both effects can dominate. However, for empirically justified values for the elasticity of substitution between capital and labor, the productivity effect is more likely to prevail, implying that a tax on physical capital discourages firm training too. Since underinvestment in training is more severe than underinvestment in physical capital, it is possible that a tax on capital income increases welfare.

Keywords

Firm Training, Public Policy, Capital Taxation, Overlapping Generations

JEL Classification

E24, J24, M53

1 Introduction

The purpose of this paper is twofold. On the one hand it incorporates the two-period model of Acemoglu and Pischke (1999a) into a model of infinite horizon in order get a more realistic distinction between short-run and long-run effects in a growth setting. On the other hand, it analyzes the consequences of public policy on the provision of firm training.

The approach of Acemoglu and Pischke (1999a) is the most influential explanation for the empirical fact that firms typically bear a large share of their workers' training, even if this training is general and can thereby be used in other firms as well (see for instance Loewenstein and Spletzer (1998, 1999) for evidence on the US, Booth and Bryan (2002) for the UK or Gerfin (2003a, 2003b) for Switzerland). According to traditional theory firms should not pay at all for the general training of workers since this would lead to one-to-one wage increases (see Becker (1962)). The approach of Acemoglu and Pischke is based on the idea of wage-compression. The wage structure is compressed if wages react less sensitively to training than productivity does. Explanations for this phenomenon, ranging from frictions in the labor market, specific human capital, efficiency wages to minimum wages and unions, are discussed in detail in Acemoglu and Pischke (1999a, 1999b).

In the appendix of Acemoglu and Pischke(1999a) the standard model workers who live for two periods is extended to a continuous-time infinite horizon model. To keep things simple, workers do not die and they have no children. Unemployment is allowed for but when the model starts, all workers are trained no matter whether they do have a job or not. This procedure avoids heterogeneity among workers so that all workers are trained and have the same amount of human capital. Yet the training decision is based on profit-maximization of firms, which seems a bit obscure, because even unemployed workers receive on-the-job training. So indeed, the structure of this appendix-version is very different from Acemoglu's standard-model.

Therefore, I build an infinite-horizon model that is more consistent with the earlier versions of the model. I do so by considering a Blanchard-model[1] in which new workers are born at a constant rate. These workers are untrained and unemployed but once they find a job they will be trained by the firm. Thus, my model is more in line with the earlier contributions of Acemoglu and Pischke, since training is really done on-the-job. This comes at the cost of more heterogeneity because unemployed workers can both be trained and untrained and even among trained workers different levels of human capital are possible.

The theoretical literature on the consequences of capital taxation for human capital investments is not so much in unison as one might think. Heckman (1976) noticed that taxation of physical capital might lead to overinvestment in human capital. The reason is as simple as the result is surprising: A tax on capital diminishes the after-tax returns on investments. Consequently, an individual facing the decision whether to invest her savings in physical or human capital invests too little in physical and too much in human capital. In a similar way, Nielsen and Sörensen (1997) analyze the Nordic system of dual income taxation. In this system the income from capital is taxed proportionally while labor-income is taxed progressively. Nielsen and Sörensen are able to show that progressive taxation of labor can be justified on pure efficiency grounds. As in Heckman (1976) a tax on physical capital will lead to overinvestment in human capital if it is combined with proportional taxation of labor-income. They assume that the only investments necessary to produce human capital are time inputs, i.e. the costs are forgone earnings. Then a proportional tax on labor-income would not only decrease the returns to human capital but also its costs. At the margin the two effects will cancel out and therefore human capital investments are not affected by a proportional tax on labor-income. However, the tax on capital income will decrease the returns on investments in physical capital and thereby the investment decision is distorted, leading to overinvestment in human and underinvestment in physical capital. This distortion can be avoided by a progressive tax

[1] See for instance Blanchard (1985).

on labor-income. In this case the returns to human capital are affected more than the costs: The incentives to invest in human capital are lowered and the distortion can be diminished or even avoided.

On the other hand, models of endogenous growth typically arrive at the result that even proportional taxation of labor income diminishes investments in human capital. Therefore, the tax-rate on labor income as well as on capital income should be zero in the long run - see for instance Jones et al. (1997) or Milesi-Ferretti and Roubini (1998). In a model that is similar to the one by Nielsen and Sörensen (1997), Nerlove et al. (1993) argue that an important difference between physical and human capital is the fact that the depreciation of human capital is not tax-deductible. Therefore, the wage tax would apply to both the yield on, and the principal of investments in human capital. Consequently, in their model a universal tax on income (applied to both capital- and labor-income) would discriminate against human capital but not against physical capital as in Nielsen and Sörensen (1997). A progressive tax on labor income would of course increase the distortion even further.

To my knowledge there are no contributions in the literature relating the taxation of capital to training provided by firms. Therefore, let me compare my model to the work of Nielsen and Sörensen (1997), which is methodologically closest to my own work. In Nielsen and Sörensen - as well as in Heckman (1976) - the only connection between physical and human capital investments is that they are both investment-possibilities of households. If the returns on these investments are distorted through taxation, then the investment decisions will be distorted as well. In this work I offer two alternative channels of interaction between the taxation of capital and investments in human capital which work in opposite directions: The complementarity of labor and capital in the production function and the bargaining positions of workers and firms.

- Since labor and capital are joined in the production of final goods, it appears only natural that the taxation of one factor should affect the marginal productivity of the other factor. If human and physical capital are complements, an increase in the stock of physical capital will increase the profitability of training and thereby tend to increase human capital as well. This channel is ruled out in Nielsen/Sörensen by the assumption that firms can rent their capital on the international market, paying an exogenous rental-rate that is not affected by the taxation in the home country. Therefore, firms always use the optimal amount of physical capital. Taxation does only affect the savings decision of households but not the investment decision of firms. Although I do assume an exogenous interest-rate given by an international market, in my model it is the direct return on investment that is taxed and not just the income of the household. Therefore, a tax on physical investment leads to a lower usage of physical capital affecting the returns to training as well.

- The second channel works through the effect of capital on the bargaining positions of workers and firms in a model where wages are determined by Nash-bargaining. Since physical capital has no direct effect on the fall-back position of workers and firms, one might be tempted to think that capital affects wages only linearly. However, there is an indirect effect via the hazard-rates at the labor market. With more capital in the market, production and thereby rents are increased. This tends to attract new firms and thus the probability of an unemployed worker finding a job increases. Consequently, the value of unemployment is higher and thereby the threat-point of the worker increases. Since this effect is more important for more productive workers wage compression tends to decrease, lowering the incentives for training-investments.

The two effects point in opposite directions. Which effect prevails depends primarily on the complementarity between labor and capital. If the production technology is described by a function with constant elasticity of substitution between the two inputs capital and labor, only elasticities above 1.1 can create a dominating tightness effect. Since the empirical literature typically finds lower values,[2] this result seems rather unlikely if not impossible. Furthermore, a subsidy on firm training is analyzed and it turns out that

[2] See for instance Chirinko (2002).

both, investments in human and physical capital, are encouraged. A combination of both policy instruments is clearly welfare-enhancing.

The remainder of the paper is organized as follows. The proceeding section gives a general description of the model. In section three the technical details follow. Section four discusses the steady-state equilibrium of the model while section five illustrates the dynamics out of the steady-state. In section six some aspects of public policy are discussed. Finally, section seven concludes the work.

2 General Description

I concentrate on the decisions of firms to analyze their investment incentives into the training of their workers and the effects of capital taxation on these decisions. To keep things simple I treat workers merely as a black box: I neither model their consumption-behavior nor their search-effort on the labor market and I do not consider education. Utility is such that searching for a job is always efficient when a worker is unemployed. The workers face an exogenous and constant death-risk d, as in a standard Blanchard-model (see for instance Blanchard (1985)). At the same rate new workers are born. These young workers are unemployed and untrained and immediately engage in job-search. The total mass of the population is normalized to one. The production technology is described by a standard CES-function f with capital K and labor g as arguments. The returns to physical capital shall be taxed at an exogenous tax-rate κ.

Whenever a worker holds a job, she can be trained on-the-job (τ) to increase her stock of human capital g. Human capital increases the productivity of the worker but it does so at a declining rate ($\frac{\partial f(K,g)}{\partial g} > 0$, $\frac{\partial^2 f(K,g)}{\partial g^2} < 0$). As is usually assumed in the literature,[3] training takes one period to be completed so that the training of period t improves the output of all periods from $t+1$ onwards. Additionally, I assume that no production takes place during the first period of a match. This assumption can be motivated by arguing

[3] See for instance Acemoglu (1999a).

that at the beginning of a match more extensive training is necessary and therefore no time for production left. However, it is merely a technical simplification that is useful to compute the solution of the model. It does not change any results of the analysis qualitatively. The only costs of training are the direct monetary costs, which are assumed to be linear ($c(\tau) = \tau$). These costs have to be paid by the training firm but in principle the worker can indirectly bear some part of the costs by accepting lower wages. The stock of human capital of an employed worker is assumed to depreciate at the rate δ_e, while the human capital of the recently fired or unemployed workers is depreciating at rates δ_f and δ_u.[4]

The effect of physical capital is equivalent to the one of human capital, i.e. it increases output at a declining rate. Its depreciation rate is given by δ_K. Once the worker and the firm separate, the firm is able to sell the capital at the market-price. According to Pissarides (2000), it would be equivalent to assume that the firm is renting the capital instead of buying it.

I assume that wage-negotiations take place before the firm decides about the magnitude of training it is going to provide. This is the exact opposite to what Acemoglu and Pischke(1999a) assume. However, it is in line with empirical evidence: For instance, Parrent (1999) finds strong evidence in favor of delayed training. Actually, in a model of infinite periods the timing of wage-negotiations does not matter that much as in a two-period model. It does not affect the investment decision of the firm but only the distribution of the match's rent.

At the end of each period the worker-firm pair might be hit by a match-specific productivity shock. This happens with a constant probability s and will lead to a separation,[5] but the shock does neither affect the productivity of the worker in another firm nor the productivity of another worker within the same firm. I assume that the human capital of

[4] In accordance with Ljungqvist and Sargent (1998, 2000) this allows that human capital depreciates faster during unemployment spells.

[5] Here we assume that s is independent of training. For a model with endogenous separations where training has an influence on the likelihood of separations see the first two chapters.

workers is perfectly general, so that it can be used without any restrictions in firms other than the training firm.

As a consequence, there are always different types of workers on the labor market: Trained and untrained workers. Trained workers typically have different stocks of human capital, depending on how long they have been unemployed. I assume that the human capital of a worker is not observable so that firms which are searching for a worker cannot concentrate their search on any of these groups. When a firm gets matched to a worker it will learn the worker's stock of human capital and decide whether to provide further training. Actually, the firm will always do so since human capital constantly depreciates.

The labor market is modelled by a standard matching-function $m(u,v)$ with constant returns to scale and the number of vacancies (v) and unemployed workers (u) as arguments.[6] This results in hazard-rates, which depend on the tightness of the labor market (defined as $\theta = \frac{v}{u}$) and are taken as given by a single agent.

3 The Model

3.1 Description of Firms

In this section I describe the value-functions for the representative firm. By paying an exogenous search-cost c per period any firm can open up a vacancy V. The value of this vacancy is governed by:[7]

$$V = -c + \rho \int P(g)J(g)dF(g) + \rho(1 - P_f)V \tag{1}$$

where $\rho = \frac{1}{1+r}$ is an exogenously determined discount factor - given by a foreign capital market which is not influenced by domestic decisions, which is typical for a small open

[6] See Pissarides(2000).

[7] The notation is very much in line with Pissarides (2000) or the Appendix in Acemoglu and Pischke (1999a).

economy. The value of the vacancy consists of the prospect of finding a worker $P(g)$ with a certain stock of human capital g - in this case the firm gets the value of a trained worker $J(g)$. The probability of finding a worker of a certain kind $P(g)$, depends on the share of this type in the pool of all unemployed and on the probability of finding a worker of any kind (P_f). This probability in turn depends on the tightness of the labor market. For a more detailed discussion see further below. The integral sums up over all the different types of workers. At the same time the firm has to pay an exogenous search-cost c for every period of active search, i.e. as long as the vacancy is not filled.[8] Finally, in case the search was not successful (which happens with probability $1 - P_f$), the firm continues to search.

As is standard I impose a zero-profit condition,[9] which restricts the value of a vacancy to be zero. This is, in essence, an equilibrium condition. Whenever V is larger than zero it would be profitable to open up new vacancies - this increases the competition for unemployed workers, decreases the firms' probability of finding a worker and thereby the value of a vacancy. This procedure continues until the value of a vacancy has dropped to zero so that there is no incentive for further market entries. Conversely, if V is smaller than zero, firms drop out of the market, thereby increasing the probability of finding a worker for the remaining firms and hence increasing the value of a vacancy.

Every matched worker comes with a certain stock of human capital g - if the worker did not yet have any training her stock of human capital is assumed to be zero. At the beginning of any period the firm has the possibility of further enhancing the productivity of the worker by providing some additional training τ. This is illustrated in the value-function for a filled job:

[8] We assume that each firm can employ only one worker.

[9] In other words we assume free market entry. See for instance Pissarides (2000).

$$\begin{aligned} J(K_t, g_t) &= f(K_t, g_t) - w - \tau - I + \\ &\quad \rho(1-\pi)J(K_{t+1}, g_{t+1}) + \rho\pi(K_{t+1} + V) \\ s.t. &: g_{t+1} = (1-\delta_e)g_t + \tau \\ K_{t+1} &= (1-\delta_K)K_t + I \end{aligned} \tag{2}$$

For the firm it does not matter for what reason it got separated from a worker, i.e. whether there was a negative productivity-shock or whether the worker died. To keep notation simple, we therefore define the rate of job destruction as $\pi \equiv s + d - sd$.

The function $f(K, g)$ denotes the value of production, which is dependent on the training of the worker and the stock of physical capital while w is the wage paid to the worker. The third term denotes the costs for the additional training the firm wants to provide. With probability $(1-\pi)$ the match continues and the value of the job will be $J(K_{t+1}, g_{t+1})$. This value can be different form the value of the current period for three reasons: Human as well as physical capital might change and the tightness in the labor market θ might change, affecting the threat-point of the worker and thereby the wage. Of course, in the steady state all three variables are stationary and consequently the value of the firm does not change from one period to the other (as long as the match survives). With probability π the job is destroyed and the firm is left with the value of a vacancy - however in that case the firm is able to sell its capital (last term).

The second line of equation (2) shows the law of motion for the stock of human capital of the worker. As already discussed in the non-technical description of the model, training takes one period to be completed and human capital is subject to depreciation. Finally, the third line of equation (2) gives the law of motion for the stock of physical capital which is equivalent to the one for human capital.

3.2 Description of Workers

The value of an unemployed worker with human capital g is given by:

$$U = P_w(1-d)W + \rho(1-P_w)(1-d)U \tag{3}$$

With a certain probability P_w the worker finds a new job, but this will only add to her value if the worker does not die. In this case the worker gains the value of an employed worker W. With probability $(1-P_w)(1-d)$ the worker survives but stays unemployed. Since human capital is assumed to depreciate the value of being unemployed diminishes even in the steady-state. With probability d the worker dies and looses the value of being unemployed and since the value of being dead is zero this term drops out.

In turn, the value of a job is determined by:

$$W = w + \rho s(1-d)U + \rho\,(1-\pi)\,W \tag{4}$$

The value of the job is equal to current earnings (wage w) plus the expected future value. Again, three outcomes are possible:

- With probability $s(1-d)$ she gets fired but survives and is left with the value of being unemployed with human capital $g_{t+1} = (1-\delta_f)g_t + \tau$.[10]
- With probability $(1-\pi)$ the worker stays employed - yet the value of being employed might change due to the change in human capital and market tightness.
- Finally, with probability d the worker dies and looses all value.

3.3 Wages

Wages maximize according to Nash-bargaining:[11]

[10] The depreciation rate δ_f is potentially larger than δ_e to account for skill loss due to job loss as in Ljungqvist and Sargent (1998, 2002).

[11] This is standard with the only destinction that the possibility to sell capital improves the bargainig position of the firm. See for instance Shaked and Sutton (1984) for a game-theoretic foundation of Nash-bargaining or Pissarides (2000) for an application to the matching framework.

$$w(g) = Arg\max(W-U)^{\beta}(J-(K+V))^{(1-\beta)} \tag{5}$$

which results in the following standard rule:

$$W - U = \beta(W + J - U - V - K) \tag{6}$$

where β denotes the bargaining power of workers. Thus, Nash-bargaining assures that the surplus of the worker is a share β of the joint surplus of the match. Plugging in equations (2) and (4) yields after some manipulations:[12]

$$w = \frac{r+d}{1+r}U + \beta\left[f(K,g) - \tau - I - \frac{r}{1+r}K - \frac{r+d}{1+r}U\right] \tag{7}$$

This equation is valid for all periods although of course training in the first period is larger than in the proceeding periods. In fact, as will become clear further below, in later periods the firm does just reinvest what is lost due to depreciation.

The wage is set in such a way that the worker receives at least the value of her best alternative (the threat-point), which is being unemployed. The value of unemployment has to be adjusted by the interest-rate and the death-risk d. The threat-point in our model is different from Pissarides (2000) with respect to two points: First, the additional d and second the discounting by $\frac{1}{1+r}$ which is due to the fact that in our discrete-time framework the wage-payment and the expected future values accrue at different points in time (for instance in equation (4)).

In addition to the threat-point the worker gets a share of the joint surplus of the match, according to her bargaining strength β, where the interest on capital payments and investments in both human and physical capital have to be deducted. The first

[12] For a more precise derivation see Appendix A.

(deduction of interest-payments) is due to the nature of physical capital: It belongs to the firm and in contrast to human capital it is not embodied in the worker. Thus, in case of a separation the firm has the possibility of selling it - this possibility improves the bargaining position of the firm as can be seen from equation (5). This is the reason why the worker bears part of the costs of capital investments which are the interest-rates that could be earned by investing the same amount of money at the foreign capital market. In fact, capital improves the threat-point of the firm in a similar way, as the value of unemployment improves the threat-point of the worker. As we will see further below, this sharing of capital costs is important for the investment decision.

In contrast to physical capital, human capital is embodied in the worker and will be lost to the firm in case of a separation. For this reason it does not improve the bargaining position of the firm but instead the threat-point of the worker: Her prospective wage in a new job is increased by training and therefore the value of unemployment rises. This explains why the worker does not bear any of the interest-payments caused by the training-investment. Nevertheless, the worker does share in the costs of depreciation. This is due to the fact that depreciation leads to reinvestments. These reinvestments are anticipated at the time of wage negotiations and therefore the costs shared. The worker even bears part of the initial training-costs. However, since training takes place after the first wage-negotiations this is irrelevant for training investments. In fact, the sequencing of training and negotiations is irrelevant for the provision of training. It only has consequences for the distribution of rents: The worker does only share in the initial training costs if wages are negotiated before training takes place.

The wage structure implied by wage setting as in equation (7) is clearly compressed. The worker receives only a share β of the output and thereby of any increases in productivity. The fact that training is general tends to decrease the degree of wage compression but not enough to offset it.[13] Moreover, the sharing of capital costs and depreciation tend

[13] Due to the risk of unemployment, depreciation and Nash-bargaining in other firms, the value of unemployment reacts less to training than the wage in the current firm.

to increase wage compression.

It can be shown analytically that wage compression decreases with labor market tightness (see Appendix B). A tighter labor market improves the chances of unemployed workers finding a new job. Consequently, the bargaining position of workers is improved and wages increase. However, what drives the effect of tightness on wage compression is the fact that the value of an unemployed worker is dependent on her human capital as well. A stronger bargaining position of a worker is the more useful the more productive this worker is. Thus a trained worker is not only able to bargain a higher wage because she is producing more but also because her bargaining position is better. Consequently, a more productive worker gets a higher share of the match's rent than an untrained worker. In other words, the wage structure is less compressed. It should be noted that market tightness would not increase wage compression if human capital were specific because in that case training would not influence the bargaining position of the worker. An increase in θ would still increase the wage of the worker but it would not lead to higher wage compression.

3.4 Unemployment

The change of unemployment over time is governed by:

$$\Delta u = d + (1-u)s(1-d) - (1-d)P_w u - du \tag{8}$$

The first two terms denote the inflow into unemployment and the last two terms the outflow. The first term stands for the new-born which are unemployed by definition. The second term are the workers who got fired last period and are still alive. The third term denotes formerly unemployed workers who found a new job and the forth term stands for unemployed workers who did not survive into the current period.

Since the mass of the population is normalized to one, u does indicate the total number of unemployed workers as well as the share of unemployed workers in the population, i.e.

the unemployment rate.

3.5 Probabilities

In this section the hazard rates of the labor market are discussed in more detail. As mentioned above, a firm's probability of finding a worker is denoted by P_f. The labor market is modelled by a standard matching function $m(u, v)$ with constant returns to scale. Given a number of vacancies v and unemployed workers u, the matching function tells us how many pairs get matched every period. Consequently, P_f is defined as the ratio of the number of matches and the number of vacancies:

$$P_f = \tfrac{m(u,v)}{v}$$

Due to constant returns to scale, this equation can be rearranged in such a way that the probability of finding a worker is only dependent on market tightness θ, so that:

$$P_f = \frac{m(u,v)}{v} = m(\frac{u}{v}, 1) = q(\theta) \tag{9}$$

Finally, the probability of finding a certain type of worker depends on the probability of finding a worker of any type P_f and on the share of this type in the pool of all unemployed:

$$P(g) = P_f \frac{u(g)}{u} \tag{10}$$

where $u(g)$ denotes the total number of unemployed workers who have a stock of human capital g and u is the number of all unemployed workers.

The probability of a worker finding a job is given by the ratio of the number of matches and the number of searching workers. By using the above definition of $q(\theta)$ (equation (9)) we get:

$$P_w = \frac{m(u,v)}{u} = \frac{v}{u}\frac{m(u,v)}{v} = \theta q(\theta) \tag{11}$$

As discussed in the section above, in equilibrium θ will be endogenously determined by free entry of firms.

3.6 Training

Since it is the firms' decision whether to train the worker, training maximizes the value of the firm $J_t(K_t, g_t)$ as given in equation (2). For convenience, this equation shall be repeated here:

$$\begin{aligned} J(K_t, g_t) &= \underset{\tau, I}{Max}\{f(K_t, g_t) - w(K_t, g_t) - \tau - I + \\ & \quad \rho(1-\pi)J(K_{t+1}, g_{t+1}) + \rho\pi(K_{t+1} + V) \\ s.t. &: g_{t+1} = (1-\delta_e)g_t + \tau \\ K_{t+1} &= (1-\delta_K)K_t + I\} \end{aligned}$$

The firm maximizes this value by choosing the optimal amounts of training τ and investment I. The first-order condition (FOC) for human capital is:

$$-1 + \rho(1-\pi)q_{t+1} = 0$$

where q_{t+1} denotes the shadow value of human capital. The negative 1 stands for the marginal cost of training. With probability $(1-\pi)$ the match survives into the next period. The remaining value of human capital is denoted by the shadow value q_{t+1}. This shadow value is found by taking the derivative of the value function with respect to the state variable g:

$$q_t = \frac{\partial J_t(g_t, K_t)}{\partial g_t} = \frac{\partial f(g_t, K_t)}{\partial g_t} - \frac{\partial w(g_t, K_t)}{\partial g_t} + \rho(1-\pi)(1-\delta_e)q_{t+1}$$

After iterating we get:

$$q_t = \sum_{i=t} [\frac{\partial f(g_i,K_i)}{\partial g_i} - \frac{\partial w(g_i,K_i)}{\partial g_i}] \rho^{i-t} (1-\pi)^{i-t} (1-\delta_e)^{i-t}$$

By plugging in the shadow-value q_{t+1} the FOC yields:

$$1 = \sum_{i=t+1} [\frac{\partial f(g_i, K_i)}{\partial g_i} - \frac{\partial w(g_i, K_i)}{\partial g_i}] [\rho (1-\pi)]^{i-t} (1-\delta_e)^{i-t-1} \tag{12}$$

On the left-hand side of this equation we find the marginal costs and on the right-hand side the marginal benefits of an extra unit of human capital. The marginal cost is equal to one due to our assumptions about training costs. The marginal benefit consists of the additional rents $(\frac{\partial f(g_i,K_i)}{\partial g_i} - \frac{\partial w(g_i,K_i)}{\partial g_i})$ that the firm can accrue. These rents stem from the fact that - due to wage compression - the output reacts more sensible to changes in productivity than wages do. The firm can reap such a rent in any future period. However, the rents have to be discounted due to the risk of separation (π), due to depreciation (δ) and due to time preferences (r).

As in Acemoglu and Pischke (1999a), an increase in π implied by an increase in the separation rate s (or the death rate d) leads to a decrease in firm-training. This is not surprising because a higher risk of losing the worker decreases the future chances to benefit from the higher productivity of the worker.

From the FOC it is clear that a firm underinvests in training due to two reasons. It subtracts the increase in wages in its calculations and therefore does not take into account the benefits of training to its workers. Furthermore, from the training firm's perspective it does not matter whether the job is destructed due to the death of the worker or due to a productivity shock. However, whenever a worker gets fired she can find another job and be productive again - this socially relevant effect is neglected by the training firm. Summarizing, we can state that there are two kinds of externalities: One on the worker and one on potential future employers of the worker. A more formal discussion follows in the section on welfare.

It is obvious that every employed worker gets the same stock of human capital. No matter whether she was unemployed or working during the previous period, equation (12) defines a unique value of optimal human capital. Since there are no adjustment costs, human capital jumps immediately to the optimal level. Thus, every employed worker has the same stock of human capital.

3.7 Market tightness and firm training

One of the important novelties of this paper is the fact that the taxation of physical capital influences firm training not only via the production function but also through its effect on wage bargaining and market tightness. Therefore this section provides a short discussion of how market tightness affects the wage negotiation and thereby the compression of the wage structure.

First of all it is useful to see what happens in a traditional setup where changes in job-finding rates shift the threat point of the worker parallel. This situation might arise if firm training were completely specific. In such a setup the threat point of a worker is independent of training since the wage in alternative firms is unaffected. This is illustrated in the left part of figure (1). Of course this picture is rather stylized, for instance training increases output linearly which is not true in the present model. However, for illustrative purposes this is just fine.

The difference between the output and the threat point U of the worker is the surplus of the match. Nash-bargaining assures that this surplus is shared between the two parties, so the wage must lie in between the two lines. For the picture it was assumed that the bargaining power of both parties is equal to one half. It can be seen that Nash-bargaining leads to wage compression, the difference between output and wages becomes larger and larger ($\frac{\partial f(g,K)}{\partial g} > \frac{\partial w(g,K)}{\partial g}$). It follows that not only the worker gains from higher productivity but also the firm and thus the firm will invest in training. This is just a replication of the well known result by Acemoglu and Pischke (1999).

The right part of the picture shows what happens if for some reason market tightness increases. As a consequence the probability of workers to find a job increases and thus

Figure 1: Effect of Tightness when Training is specific

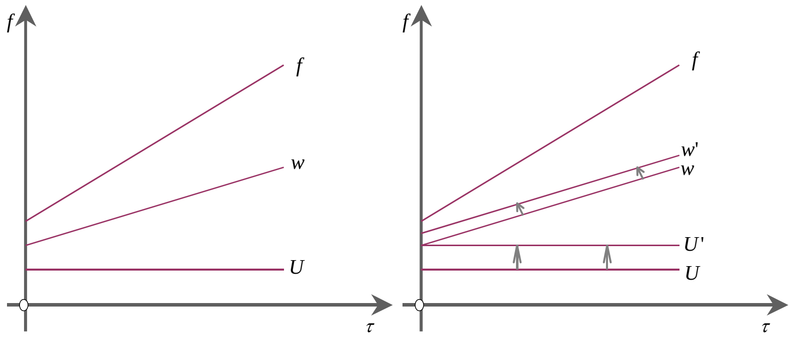
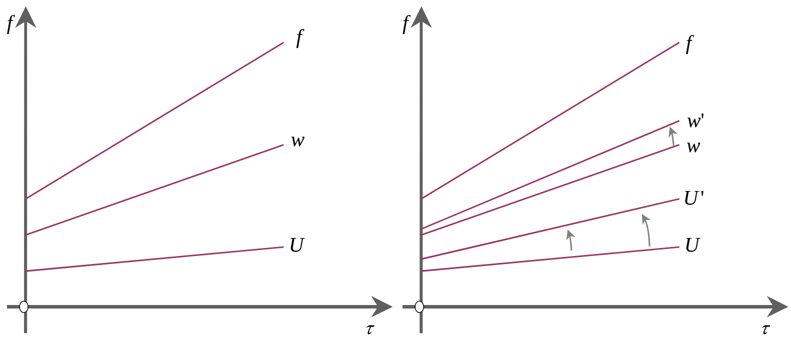


Figure 2: Effect of Tightness when Training is general



the value of unemployment improves. Since all workers are affected in the same way, the threat point U shifts up parallel. The wage increases as well but just like the threat point it shifts parallel upwards. Although the rent of firm diminishes, the change in rents due to training does not change ($\frac{\partial f(g,K)}{\partial g} - \frac{\partial w(g,K)}{\partial g}$ is still the same) and this is what matters for firm training. In consequence, the amount of training provided by the firm does not change at all.

Let´s turn back to our model with general training. The threat point of the worker is no longer a horizontal line because workers with higher training will also earn higher wages in alternative firms. In other words the value of unemployment is higher for better trained workers or $\frac{\partial U}{\partial g} > 0$. This is shown in the left part of figure (2).

Of course in such a situation the threat point of a worker with more human capital depends stronger on the tightness of the labor market. The value of unemployment no longer shifts parallel but also turns upwards or put formal $\frac{\partial^2 U}{\partial g \partial \theta} > 0$ as is proven in the Appendix. The same is true for the wage curve which means that not only the firm´s rent becomes smaller, it is also less responsive to training (i.e. $\frac{\partial f(g,K)}{\partial g} - \frac{\partial w(g,K)}{\partial g}$ gets smaller). In other words wage compression decreases and thereby the incentives to train. This is illustrated in the right part of figure (2).

3.8 Capital Investment

The optimal amount of capital investments is found by taking the derivative of the value of the firm $J(K_t, g_t)$ (equation (2)) with respect to investments I. This yields the first order condition:

$$-1 + \rho(1-\pi)\lambda_{t+1} + \rho\pi = 0$$

where λ_{t+1} denotes the shadow value of physical capital. The only substantial difference to the FOC for human capital is the last term which stems from the fact that the firm can sell the physical capital in case of a separation (as is clear from equation (2)). The shadow value is again given by the envelope condition:

$$\lambda_t = \frac{\partial J(g_t,K_t)}{\partial K_t} = \frac{\partial f(g_t,K_t)}{\partial K_t} - \frac{\partial w(g_t,K_t)}{\partial K_t} + \rho(1-\pi)(1-\delta_K)\lambda_{t+1} + \rho\pi(1-\delta_K)$$

After iterating we find that λ_t is given by:

$$\lambda_t = \sum_{i=t} [\frac{\partial f(g_i,K_i)}{\partial K_i} - \frac{\partial w(g_i,K_i)}{\partial K_i} + \rho\pi(1-\delta_K)]\rho^{i-t}(1-\pi)^{i-t}(1-\delta_K)^{i-t}$$

which leads to the following optimality condition:

$$1 = \sum_{i=t+1} [\frac{\partial f(g_i, K_i)}{\partial K_i} - \frac{\partial w(g_i, K_i)}{\partial K_i} + \frac{\pi}{1-\pi})] [\rho(1-\pi)]^{i-t} (1-\delta_K)^{i-t-1} \qquad (13)$$

Again we have marginal costs on the left-hand side and marginal returns on the right-hand side. The marginal returns are made up by the difference between productivity and wage increases of all future periods. These rents have to be discounted due to risk of separation and the interest-rate. The last term inside the square-brackets accounts for the fact that the firm will sell the capital in case of a separation.

3.9 The human capital of unemployed workers

Because the stock of human capital in the pool of unemployed workers is so essential for the entry decision of firms - and thereby for the solution of the model - we have a closer look at it in this section. After all it is the expected value of human capital of unemployed workers that is relevant for a firm's decision whether to enter the market and post a vacancy. The average training per unemployed worker is nothing else but the ratio of the stock of human capital of all unemployed workers and the number of unemployed workers:

$$\overline{g}_t = \frac{(1-\delta_u)\overline{g}_{t-1}u_{t-1}(1-P_w)(1-d) + (1-\delta_f)g^*_{t-1}(1-u_{t-1})s(1-d)}{u_{t-1}(1-P_w)(1-d) + (1-u_{t-1})s(1-d) + d} \tag{14}$$

Three different types of workers can be distinguished: New-born workers, workers who were unemployed in the last period and workers who got fired and the end of last period. These three types make up the total number of unemployed workers and therefore the denominator of equation (14). The first term in the denominator are the unemployed of last period who did not find a job and did not die, so they are still unemployed. The second term are all the employed workers of last period who got separated and did not die and finally the last term are the new-born workers.

Except for the new-born workers all unemployed workers are endowed with a certain stock of human capital. This is illustrated by the numerator of equation (14). The first term again stands for all the unemployed of the last period who are still unemployed in the current period ($u(1-P_w)(1-d)$). Since search in the labor market cannot be directed towards more able workers, all workers have the same chances to find a job. It follows that those who are still unemployed this period have the same average training as the unemployed of the previous period, adjusted for depreciation: $(1-\delta_u)\overline{g}_{t-1}$. The second term stands for those workers who did have a job in period $t-1$ but lost it at the end of the period. Since they had a job in period $t-1$ they have the optimal level of human capital of that period which is equal for all workers, as was discussed in the section on training. Again we have to adjust for depreciation: $(1-\delta_f)g^*_{t-1}$. Since, by definition, the new-born have no human capital at all, it is the only group of unemployed workers that does not show up in the numerator.

4 Steady State

In the steady state things simplify a lot since the market-conditions relevant for the firms (market tightness θ and the stock of human capital of unemployed workers $\overline{g}$) are constant. In the symmetric equilibrium all firms face similar decisions and therefore the optimal level of investment in each firm is the same. However, there is still heterogeneity among the unemployed workers, because some of them have never had a job and therefore do not have any human capital at all while others are already trained. But even among the latter group there is heterogeneity because human capital depreciates from one period to the other. Thus the longer a worker has been unemployed, the lower is her stock of human capital. Nevertheless, for a firm with a vacancy this heterogeneity is irrelevant since from its perspective only the aggregate and the expected value of a worker's human capital are important. [14]

As we have seen in the section above, the only endogenous variable relevant for the

[14] In fact, it is the assumption that no production takes place during the first period of a match that makes the exact distribution of human capital irrelevant to the firm. Due to this assumption, to the firm the only difference between workers with different stocks of human capital is the training cost that has to be paid to reach the optimal value of human capital. Because training costs are linear, from the perspective of a firm with a vacancy it is only the expected value of these costs that is important for the entry-decision.

training decision of the firm is market tightness. Whenever tightness is stable the optimal level of human capital will be stable as well. If a firm engages a worker whose human capital lies below the optimal level, the firm will invest just as much to reach that level. Indeed, since human capital is constantly depreciating, every employed worker is trained at the beginning of each period to make up for the loss.

From the law of motion for human capital it is clear that human capital can only be stable if an employed worker is trained every period what she would loose due to depreciation. Thus training equals depreciation:

$$\tau = \delta_e g$$

for every insider.

The same is true for investments in physical capital:

$$I = \delta_K K$$

This implies that wages (equation (7)) of all periods except the first are given by:

$$w = \frac{r+d}{1+r}U + \beta \left[f(K,g) - \delta_e g - (\frac{r}{1+r} + \delta_K)K - \frac{r+d}{1+r}U \right] \tag{15}$$

The optimal stock of human capital in the steady state is found by using equation (12). As mentioned above, in the steady state θ is stationary and therefore the rents of all periods are identical. Thus we can take the rent out of the sum-operator and the first order condition (equation (12)) simplifies to:

$$\left[\frac{\partial f}{\partial g} - \frac{\partial w}{\partial g} \right] (1 - \pi) = r + \delta_e + \pi - \delta_e \pi \tag{16}$$

If production took place during the first period of a match, the value of that output would be different for workers with different stocks of human capital. Since this difference is non-linear in training, the exact distribution of human capital would become relevant.

On the left-hand side we have the marginal returns of training investments: The additional profits of the firm per period ($\frac{\partial f}{\partial g} - \frac{\partial w}{\partial g}$) multiplied with the probability of survival. On the right-hand side are the marginal costs, which consist of discount-rate, depreciation and the risk of separation.

By following the same reasoning the FOC for physical capital (equation (13)) simplifies to:

$$\left[\frac{\partial f}{\partial K} - \frac{\partial w}{\partial K}\right](1-\pi) = r + \delta_K - \delta_K \pi \tag{17}$$

where again marginal revenues are on the left-hand side and marginal costs on the right-hand side. The equation is very similar to the FOC of human capital. However, there are two important distinctions. The first one is obvious: The risk of separation does not show up in equation (17). This is due to the fact that the firm can sell the physical capital when it does not need it anymore, whereas human capital is totally lost. The other difference is hidden in the wage-equation (15) but relates to the same fact. The possibility of selling physical capital improves the bargaining-position of the firm and therefore the worker bears a share of the capital costs r. Therefore, the wage of the workers reacts less to changes in physical capital compared to human capital. The stationary unemployment rate is found by setting the change in unemployment as given in equation (8) equal to zero. By rearranging we get:

$$u = \frac{d + (1-d)s}{(1-d)s + P_w + (1-P_w)d} \tag{18}$$

Except for the workers' probability of finding a job P_w, all variables in equation (18) are exogenous. Probability P_w is defined as in (11) and is determined by the free-entry condition which assures zero profits for the representative firm. It follows that unemployment, as well as the optimal stock of human capital, is unambiguously determined by θ.

More tricky is the derivation of the stable stock of human capital among unemployed workers. Those workers are also loosing skills due to depreciation. At the same time the human capital of the unemployed is diminished by the outflow of unemployed workers finding a new job and by dying workers. Those are replaced by newborn workers who have no human capital by definition and by workers who have been laid off. Only the latter group has a positive effect on the stock of human capital of unemployed workers since employed workers typically have more human capital.

In the stationary equilibrium the outflow of human capital of unemployed workers has to be equal to the inflow. This stationary stock of human capital is found by using equation (14) and setting $\overline{g}_{t-1}$ equal to $\overline{g}_t$. By rearranging we get:

$$\overline{g} = \frac{(1-\delta_f)(1-u)s(1-d)}{\delta_u u(1-P_w)(1-d)+(1-u)s(1-d)+d} g^* \tag{19}$$

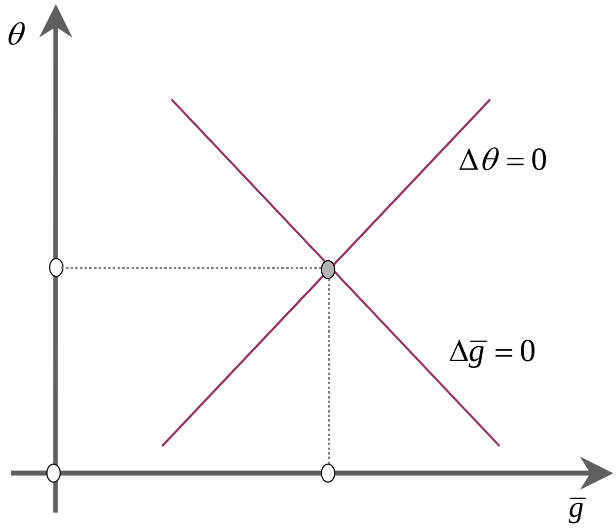
with three endogenous variables in it. Two of these variables (u and P_w) are dependent on θ alone as can easily be seen from equations (11) and (18). The optimal investment levels g^* and K^* are jointly determined for a given θ and therefore uniquely determined as well. Finally, stationary market tightness is found by solving the zero-profit condition and setting the value of a vacancy (equation (1)) equal to zero.

Thus we arrive at a system of five endogenous variables (g^*, $\overline{g}$, θ,K and u) and five equations. As just discussed, three of these variables are only dependent on labor market tightness (u, g^* and K^*). Therefore, we can use equations (18), (16) and (17) and plug them into equation (19). In this way the system can be reduced to two unknown variables in two equations (19 and 1).

It is clear that equation (1) describes a positive relationship between θ and $\overline{g}$. An increase in the stock of human capital increases the value of a worker and therefore the profitability of a firm. Consequently, new firms enter the market and drive down the probability of finding a worker. In other words, labor market tightness increases.[15]

[15] For a more formal argument see the Appendix.

Figure 3: Stationary Equilibrium



What is less clear is the effect of θ on $\overline{g}$ described in equation (19). Here we have to distinguish three separate effects:

- We already know that an increase in θ decreases the optimal level of human capital g^*, tending to decrease the stock of human capital of unemployed workers $\overline{g}$.[16]
- At the same time the unemployment rate is increased as can be seen from equation (18). It can be shown analytically that a decrease in unemployment tends to increase $\overline{g}$ (see Appendix C). This is so because every period more employed workers (with a high stock of human capital) loose their jobs: A lower unemployment rate implies more employed workers and - since the probability of separations is exogenous - more layoffs. To keep unemployment in equilibrium the outflow out of unemployment has to increase as well. However, on average these workers have a lower stock of human capital than employed workers (respectively workers that have just been laid off). Since the additional inflow of workers into unemployment has a higher stock of human capital than the outflow, the average human capital of unemployed workers has to increase.

[16] See the section above on firm training or Appendix B.

- Additionally an increase in P_w (implied by an increase in θ) directly increases the value of $\overline{g}$ - see the negative sign of P_w in the denominator of equation (19) - because the outflow of unemployed workers increases.

Numerical simulations clearly suggest that the first effect outweighs the other two effects. This is the reason why I restrict myself to the more realistic cases where the effect of θ on g^* prevails.[17] In these cases $\overline{g}$ depends negatively on tightness θ. The stationary equilibrium is determined by the intersection of both equations as illustrated in figure (3).

5 Out-of-steady-state dynamics

In the section above we have discussed the stationary equilibrium of the model. In this section we turn our attention to the transition to the steady-state. To do so we need to analyze what drives the two most important variables of the model, θ and $\overline{g}$.

Lets turn first to the distribution of human capital in the pool of unemployed. The change in average human capital is found by subtracting the average human capital of the last period from both sides of equation (14):

$$\begin{aligned} \Delta\overline{g}_{t-1} &= \overline{g}_t - \overline{g}_{t-1} = \\ &= \frac{-\delta_u \overline{g}_{t-1} u(1-P_w)(1-d) + \left[(1-\delta_f)g^*_{t-1} - \overline{g}_{t-1}\right](1-u)s(1-d) - \overline{g}_{t-1}d}{u(1-P_w)(1-d) + (1-u)s(1-d) + d} \end{aligned} \quad (20)$$

Human capital of unemployed workers changes due to three reasons. On the one hand, the inflow of workers into the pool of unemployed changes the average training, since workers who had a job have a different stock of human capital than unemployed workers (see the second term in the numerator of equation (20)). Besides that, the newborns lower the average since they have received no training at all (third term in the numerator) and

[17] For an illustration of the degenerate case see the end of the next section.

finally the stock of human capital of unemployed workers depreciates at rate δ_u (first term).

It is clear that the change in human capital as given in equation (20) depends negatively on the average of training in the previous period. As discussed in the section above, an increase in labor market tightness decreases the compression of the wage structure and thus lowers the optimal investments in training (g^*). But at the same time unemployment falls which tends to decrease $\overline{g}$. Just as in the section above I concentrate on the more sensible cases where the effect of optimal training prevails. In this case $\Delta\overline{g}_t$ depends negatively on θ_t as well as $\overline{g}$:

$$\Delta\overline{g} = \Phi(\overset{(-)}{\theta}, \overset{(-)}{\overline{g}})$$

As illustrated in figure (3) $\Delta\overline{g}_t = 0$ slopes downwards in the $\theta, \overline{g}$-space.

The analysis of the dynamics of market tightness is analogous to Pissarides (2000),[18] with the only difference that the value of a vacancy depends not only on one condition of the labor market but on two: It is not only dependent on the probability of finding a new worker (and thereby market tightness), but also on the stock of human capital of unemployed workers where the latter is also given exogenously to the firm and can thereby be interpreted as a variable given by the market. These conditions affect the firm in opposite ways: An increase in labor market tightness decreases the possibilities of the firm while an increase in the stock of human capital enhances them.

As is clear from introspection of equation (1) and is noted in Pissarides, the change in market tightness $\Delta\theta$ is determined by the change in the value of a vacancy ΔV which in turn is determined by the change in the value of a filled job ΔJ. Whenever the value of a job increases, the value of a vacancy will do so as well. A positive value of vacancies attracts new firms to the market so that market tightness goes up as well.

The change in job-value ΔJ is found by rearranging equation (2):

[18] See chapter 1, page 26 and the following.

$$\Delta J = \frac{(r+\pi)J - (1+r)\left[f - w - \tau - I\right] - \pi K}{1-\pi} \tag{21}$$

It is clear that the term in square brackets is decreasing in θ (since θ decreases wage compression and thereby the rent of the firm $f(g_t) - w(g_t)$) and increasing in the stock of human capital. But what about the first term, the value of a filled job? Free entry of firms assures that the value of a vacancy is equal to zero at any time. Therefore, we can rearrange equation (1) to see what the market demands the value of a job to be:

$$c = \int P(g)J(g)dF(g)$$

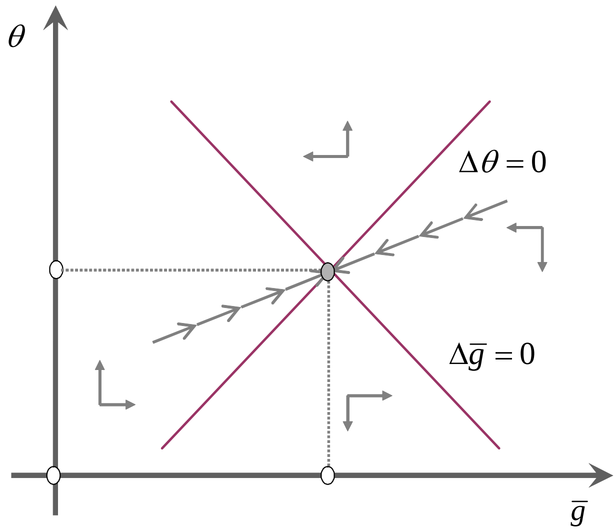
On the left-hand side are the marginal costs of searching for a worker while on the right-hand side we can find the marginal return, the probability of finding a worker with a certain stock of human capital. As already mentioned above, this equation is different to Pissarides (2000) only because we have an additional market-value here: The stock of human capital. So what happens when market tightness increases? Because there are more firms searching for a worker, the probability of any single firm being successful is diminished. To bring the market back to equilibrium, the firms have to demand a higher value of a job, so that J has to go up. It is the exact opposite with human capital: A higher value of human capital increases the profitability of firms so that the market demands a lower job-value. Thus, the demanded value of a job J depends positively on labor market tightness θ and negatively on the stock of human capital $\overline{g}$.

It follows that the change in job value ΔJ as given in equation (21) is a decreasing function of human capital and an increasing function of market tightness, so that:

$$\Delta J = \Phi(\overset{(+)}{\theta}, \overset{(-)}{\overline{g}})$$

As in Pissarides (2000), the number of vacancies is a forward-looking and unstable variable. This can be interpreted in the following way. If θ is too high, firms will demand a higher job-value. However, since θ is improving the bargaining position of the worker current profits of the firm diminishes. The only way to increase the value of a job is therefore by expecting a further increase in the value of a job (since the change in job value is a determinant of the job-value itself). But if the value of a job increases, market tightness will increase as well, driving up the demands for job-value even further. This necessitates an even larger increase in the job-value and so on - a classical bubble.

Figure 4: Transition to Steady State



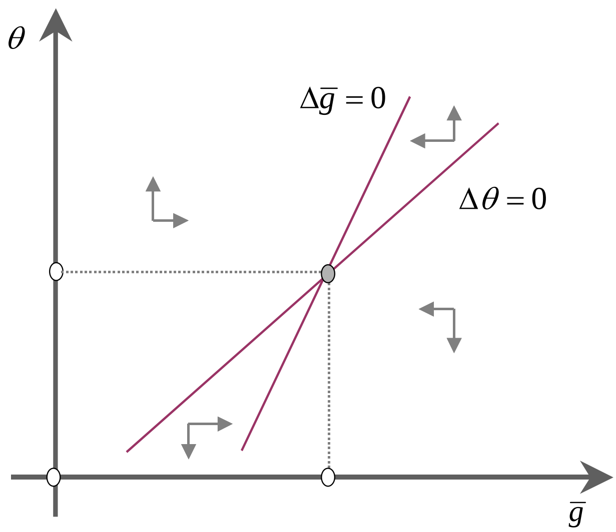
Summarizing, the sign pattern of the two difference-equations can be illustrated by:

$$\begin{pmatrix} \Delta\theta \\ \Delta\overline{g} \end{pmatrix} = \begin{pmatrix} + & - \\ - & - \end{pmatrix} \begin{pmatrix} \theta \\ \overline{g} \end{pmatrix} \tag{22}$$

with negative determinant - thus the necessary and sufficient conditions for a saddle-point equilibrium are satisfied. This equilibrium is illustrated in figure (4), showing the phase-diagram and the saddle path.

As was discussed in the section above, the effect of θ on $\overline{g}$ described in equation (19) is not unambiguous. Of course this ambiguity has consequences for the transition as well. The locus $\Delta\overline{g} = 0$ will slope downwards (as in figure (3)) if and only if the effect of θ on $\Delta\overline{g}$ is negative. However, in principle it is possible that the effect of g^* on $\overline{g}$ is dominated by the effect of unemployment. As the effect of unemployment becomes more important, the $\Delta\overline{g} = 0$ locus is getting steeper and steeper and finally the slope might become even positive. This special case is illustrated in figure (5) where the system still has a stable saddle-path. As the unemployment effect becomes even more important, the $\Delta\overline{g} = 0$ locus might get flatter than the $\Delta\theta = 0$ locus and the saddle-path would vanish. However, these cases are rather artificial.

Figure 5: Degenerate Case



6 Welfare Analysis

6.1 Training

This section provides a formal derivation of the welfare measure used in the following numerical simulations as well as the first-best solution to identify inefficiencies and externalities.

A central planner would not only maximize the value of the firm but the value of both worker and firm - and therefore both value functions (equations (2) and (4)) - taken together. This yields the following optimization problem:

$$
\begin{aligned}
W^*(K_t, g_t) &= \underset{\tau,I}{Max}\{f(K_t, g_t) - \tau - I + \\
&\quad \rho(1-\pi)\left[W^*(K_{t+1}, g_{t+1})\right] + \rho s(1-d)(K_{t+1} + U^*(g_{t+1}) + V) \\
&\quad + \rho d(K_{t+1} + V) \\
s.t. &: g_{t+1} = (1-\delta_e)g_t + \tau \\
g_{t+1} &= (1-\delta_f)g_t + \tau \qquad \text{in case of a separation} \\
K_{t+1} &= (1-\delta_K)K_t + I\}
\end{aligned}
\tag{23}
$$

There are two obvious differences to the firm´s maximization problem: First of all the wage is not subtracted in the profit of the current period and second the value U^* is added for the case of a separation that is not due to death. In analogy to W^* the value U^* does not only include the value of the worker but also the value of potential future employers:

$$
\begin{aligned}
U^*(g_t) &= P_w(1-d)W^*(g_{t+1}) + \rho(1-P_w)(1-d)U^*(g_{t+1}) \\
s.t. &: g_{t+1} = (1-\delta_u)g_t
\end{aligned}
\tag{24}
$$

The first-order condition for optimal training investments is:

$$-1 + \rho(1-\pi)Q_{t+1} + \rho s(1-d)\Pi_{t+1} = 0$$

where Q_{t+1} stands for the impact of human capital on the value of the existing match and Π_{t+1} for its impact on the value in case of a separation. The negative 1 stands for the marginal cost of training. With probability $(1-\pi)$ the match survives into the next period. The remaining value of human capital is denoted by the shadow value Q_{t+1}. This shadow value is found by taking the derivative of the social planner´s value function with respect to the state variable g:

$$Q_t = \frac{\partial W^*(g_t, K_t)}{\partial g_t} = \frac{\partial f(g_t, K_t)}{\partial g_t} + \rho(1-\pi)(1-\delta_e)Q_{t+1} + \rho s(1-d)(1-\delta_f)\Pi_{t+1}$$

After iterating we get:

$$Q_t = \sum_{i=t} [\frac{\partial f(g_i, K_i)}{\partial g_i} + \rho s(1-d)(1-\delta_f)\Pi_{t+1}]\rho^{i-t}(1-\pi)^{i-t}(1-\delta_e)^{i-t}$$

and the FOC for the social optimum becomes:

$$\begin{aligned} 1 &= \rho s(1-d)\Pi_{t+1} + \\ &+ \sum_{i=t+1} [\frac{\partial f(g_i, K_i)}{\partial g_i} + \rho s(1-d)(1-\delta_f)\Pi_{t+1}]\rho^{i-t}(1-\pi)^{i-t}(1-\delta_e)^{i-t-1} \end{aligned} \quad (25)$$

It is not very surprising that this equation differs from the FOC of a private company (see equation (12)) with respect to two points, both differences working in the same direction, namely making private investments inefficiently low. A private company has no interest in increasing the wage of the worker and therefore subtracts these benefits in its calculations. Of course a central planner cares about both the firm and the worker and therefore the term $\frac{\partial w(g_i, K_i)}{\partial g_i}$ in equation (12) does not show up in equation (25) - this is one channel which leads to underinvestment in the private economy. The second difference is even more obvious. Its the term including Π_{t+1} which stands for the effect of training on the value of unemployment. This shadow value is found by taking the derivative of the value of unemployment (equation (24)) with respect to firm training:

$$\Pi_t = \frac{\partial P_w}{\partial g}(1-d)(W^* - \rho U^*) + P_w(1-d)\frac{\partial W^*}{\partial g} + \rho(1-P_w)(1-d)\Pi_{t+1}$$

Again we can iterate this equation forward to arrive at:

$$\Pi_t = \sum_{i=t} [\frac{\partial P_w}{\partial g}(1-d)(W^* - \rho U^*) + P_w(1-d)\frac{\partial W^*}{\partial g}]\rho^{i-t}(1-P_w)^{i-t}(1-d)^{i-t}$$

Thus the value of unemployment is affected by training in two ways:

- The first term in square brackets illustrates the effect of training on unemployment via its effect on a worker´s probability to find a job. This term is positive since an increase in training investments implies that a potential new employer would need to invest less in the training of the worker to reach the optimal level. Thereby the profitability of firms is increased, new firms are attracted to the market and the number of vacancies goes up (see equation (1) giving the definition of the value of a vacancy or the discussion of the steady state for a more formal derivation). Consequently, a worker´s probability to find a job (given in equation (11)) increases. Clearly, the value of a filled job is higher than the value of unemployment and so this term is positive.

- The second term in square brackets illustrates the effects of firm training on potential matches in the future, combining the effect on the wage of the worker and the effect on profits of the future employer. This term is clearly positive and therefore works in the same direction as the effect just discussed above.

Summarizing, we can state that a private firm would underinvest due to three reasons: It does not take into account the effect of training on wages (as well in the present match as in future jobs), the effect on profits of other firms and the effect on job-finding rates (and thereby unemployment).

6.2 Investment

The optimal degree of capital investments is derived from equation (23) as well, yielding:

$$-1 + \rho(1-\pi)\Lambda_{t+1} + \rho\pi = 0$$

as the first order condition, where Λ is the central planner´s shadow value of physical capital. It is given by the derivative of the central planner´s value function (equation (23)) with respect to capital:

$$\Lambda_t = \frac{\partial W^*(g_t,K_t)}{\partial K_t} = \frac{\partial f(g_t,K_t)}{\partial K_t} + \rho(1-\pi)(1-\delta_K)\Lambda_{t+1} + \rho\pi(1-\delta_K)$$

Iteration yields:

$$\Lambda_t = \sum_{i=t} \left[\frac{\partial f(g_t,K_t)}{\partial K_t} + \rho\pi\left(1-\delta_K\right)\right]\rho^{i-t}\left(1-\pi\right)^{i-t}\left(1-\delta_K\right)^{i-t}$$

We can plug this into the FOC and it becomes:

$$1 = \sum_{i=t} \left[\frac{\partial f(g_t,K_t)}{\partial K_t} + \frac{\pi}{1-\pi}\right]\rho^{i-t}\left(1-\pi\right)^{i-t}\left(1-\delta_K\right)^{i-t-1} \tag{26}$$

In the steady state this equation simplifies to:

$$\frac{\partial f}{\partial K}(1-\pi) = r + \delta_K - \delta_K\pi \tag{27}$$

This is almost the same as equation (17) given the optimal investment of a firm but there is one little difference: The firm subtracts the increase in wages from the increase in output. Only if the wage were irresponsive to physical capital (at least at the optimal level), the firm would invest efficiently. From introspection of equations (15) and (27) it is clear that this would be the case if the wage equation included the term $\frac{r+\delta_K-\delta_K\pi}{1-\pi}K$ instead of $\frac{r+\delta_K+\delta_K r}{1+r}K$. With a little algebra it is easy to show that the difference between these two terms is $\frac{-r(\pi+r)}{(1+r)(1-\pi)}$, which is clearly negative. This implies that the wage responds positively (if not much) to changes in physical capital and the firm will underinvest. The intuition for this results lies in the fact that most costs and returns of physical investments are shared between worker and firm but not the risk of separation which is solely born by the firm.[19]

6.3 Welfare Measure

A measure of overall welfare should aggregate the incomes of all individuals and institutions active in the market. These are the workers (employed and unemployed), the firms that are searching for a worker, the firms actually producing and finally the government. By weighing these groups with their shares we arrive at the following equation:

$$mw_0 + (1-u-m)w - vc + m(-w_0 - (\tau^* - \overline{\tau}) - I^*) + (1-u-m)(f - w - \delta_e g - \delta_K K - tax) + (1-u-m)tax$$

The first two terms are the incomes of employed workers - remember that there are no unemployment benefits in this model. The third term are the costs of actively searching firms. The third and fourth term are the earnings of firms who found a worker and the last term is the income of the government. The variable m stands for the number of matches per period. Since these workers earn a different wage (they pay for a share of the training but do not produce anything), they need to be taken into account separately. The remaining employed workers $(1-u-m)$ all earn the same wage. The latter group only receives training to make up for the regular loss due to depreciation $(\delta_e g)$. However, those workers who are in the first period of their job will receive more training to reach the optimal level $(\tau^* - \overline{\tau})$. Since every firm employs only one worker, the number of producing firms is equal to the number of employed workers. The wages of workers and

[19] Another (minor) reason for underinvestment lies in the fact, that the worker´s share of interest costs is discounted by $1+r$.

the income of the government cancel out[20] and what remains is the value of production minus all costs, including search and investment costs:

$$(1-u-m)(f-\delta_e g-\delta_K K)-vc-m(\tau^*-\overline{\tau}+I^*) \tag{28}$$

Of course this does not mean that taxation is irrelevant for welfare since it affects investments and thereby production. Equation (28) is the welfare-measure used in the numerical simulations that follow.

7 Discussion of fiscal policy

7.1 Calibration

In this section public policy in the form of a tax on capital income and a subsidy of training costs is introduced. Since counteracting effects are likely and it is not possible to determine analytically which effects dominate, numerical simulations are needed to demonstrate likely outcomes. The first part of this section begins with a discussion of the calibration of the model which is assumed to operate at a monthly basis.

Estimates for the elasticity of substitution between capital and labor (ε) vary widely in the empirical literature.[21] Therefore I have chosen an elasticity of 0.7 as the baseline, but simulated the model for higher and lower elasticities as well. The monthly interest-rate is 0.05/12 and the depreciation rate for physical capital is 0.1/12, values which are commonly used in the literature.[22] The depreciation rates for human capital are $\delta_e = \delta_f = 0.05/12$ and $\delta_u = 0.1/12$. This setup is roughly in line with the models of Ljungqvist and Sargent (1998 and 2002), who assume that human capital does only

[20] As well as training subsidies which are not listed explicitly.

[21] See for instance Chirinko (2002) for a detailed survey of the empirical literature or Hamermesh (1993) and Krusell (2000) on capital-skill complementarity.

[22] See for instance Altig and Carlstrom (1999) or Baxter and King (1993).

depreciate during unemployment but not during employment or when a worker is fired - this assumption does not seem very plausible. Therefore I use positive depreciation rates for all states but a larger one for unemployed workers.

The calibration of the labor market is very much in line with the recent papers by Hall (2005), Hall and Milgrom (2005) and Shimer (2005). The matching function used is:

$$q(\theta) = \mu\theta^{-\gamma}$$

where γ is the elasticity of the matching function and μ a parameter describing the efficiency of the labor market.

Table 1: Parameters of the Numerical Model

Parameter	Value	Comment
r	0.05/12	
s	0.034	Hall (2005)
d	1/540	Working life 45 years
γ	0.765	Hall (2005)
θ	0.767	Hall (2005)
u	0.04	Assumed
P_{find}	0.877	Calibrated to match u
μ	0.937	Calibrated to match θ
sc	0.43	Calibrated to get $V = 0$
ε	0.7	Baseline
δ_K	0.1/12	
δ_e, δ_f	0.05/12	
δ_u	0.1/12	

Using US-data Hall (2005) estimates an elasticity of 0.765 for the matching function and a market tightness of 0.767 (year 2000). To avoid inefficient unemployment rates I assume that the Hosios condition is fulfilled (see Hosios (1990)) and set the bargaining power of workers equal to the elasticity of the matching function (i.e. $\beta = \gamma$).

As in Hall (2005) I assume that the model operates at a monthly frequency. For this setup he estimates a separation rate of 3.4 % which I use as well. The probability of dying d can also be interpreted as the probability of leaving the labor market i.e. retiring. I choose d in such a way that the average working life lasts 45 years.

Similar to Hall (2005) I target an unemployment rate of 4% as the baseline. Given the separation rate of 3.4 % this implies a job-finding rate of 0.86. Given the estimated market-tightness of 0.767 and the chosen job-finding rate of 0.86, I calibrate 0.92 as the efficiency parameter μ of the matching function. Finally, I calibrate the search-cost c so that the parameters above fulfill the zero-profit condition. The resulting c is 0.72 or 90% of monthly output per trained worker. Table (1) gives an overview of the parameters chosen.

7.2 Introduction of capital taxes

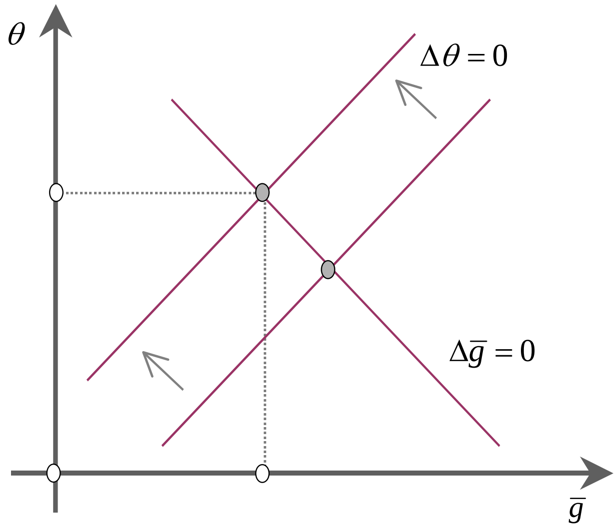
In this section a tax on capital income κ is introduced and its effects on human capital analyzed. The FOC for investments (equation (17)) modifies to:

$$(1-\kappa)\left[\frac{\partial f}{\partial K} - \frac{\partial w}{\partial K}\right](1-\pi) = r + \delta_K \tag{29}$$

It is immediately clear that the tax lowers the investments in physical capital since the return to these investments is diminished. However, the effect on human capital is not so clear, since there are two countervailing effects:

- First of all, a change in the stock of physical capital most likely affects the productivity of workers - since it is usually assumed that labor and capital are complements, according to this channel physical and human capital move in the same direction. In the following I will call this the productivity effect.

Figure 6: Isolated Tightness Effect

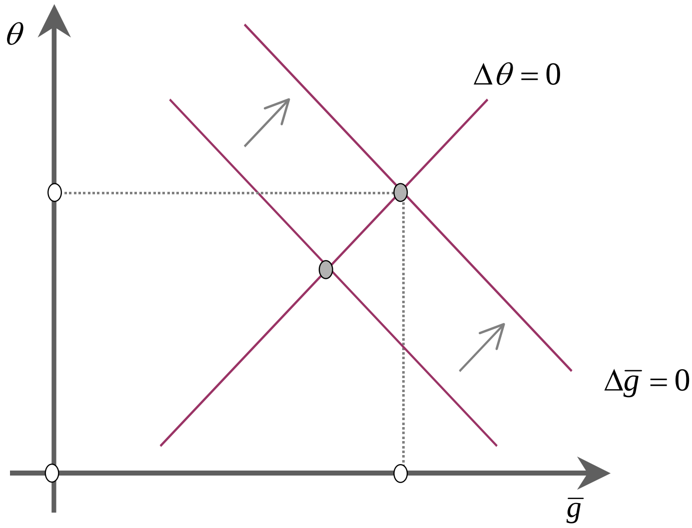


- On the other hand, capital also changes the degree of wage compression on the labor market: A higher tax on capital makes business less profitable. There are smaller rents to be shared and consequently some firms drop out of the market and labor market tightness goes down. As was already discussed is in an own section and proven in Appendix B, an increase in tightness tends to decrease the degree of wage compression and vice versa. Thus according to this channel physical and human capital move in opposite directions. In the following this will be called the tightness effect.

Which of the two effects prevails, depends most crucially on the interaction of physical and human capital in the production function. It is useful to isolate the two effects and see how they affect the market outcome. For instance, by assuming that the two kinds of capital are additively separable in the production function,[23] we can rule out the productivity effect altogether. This case is illustrated in Figure (6), where we can see the effects of a decrease in the capital tax κ. In this case only one of the curves is directly affected, namely the zero-profit condition. An increase in the stock of capital increases

[23]Or more formally that the cross-derivative is zero ($\frac{\partial^2 f}{\partial g \partial k} = 0$).

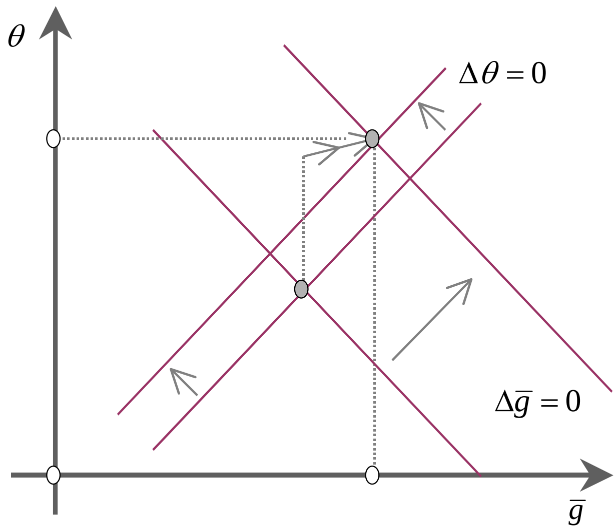
Figure 7: Isolated Productivity Effect



the value of output. Since there is more output, firms are more profitable (even though they have to give a part of the additional revenue to the worker). To bring the labor market back into equilibrium, tightness θ has to increase for every given stock of human capital of unemployed workers, i.e. the locus $\Delta\theta = 0$ has to shift upwards, as illustrated in figure (6). As the picture makes clear, this leads to a higher θ (and thereby lower unemployment) but a lower $\overline{g}$ in equilibrium. It might seem surprising that a positive capital shock diminishes the stock of human capital but the reason lies in the already discussed effect of capital on wage compression. Thus figure (6) describes the working of the tightness effect very well.

It is also possible to isolate the productivity effect. This can be done by assuming that human capital is entirely specific so that it does not affect the productivity in firms other than the training firm. Alternatively, we can assume that the capital tax has no direct influence upon the profitability of the firm (by some lump some transfers). Of course, these assumptions do not make much sense in the current model but they are useful to clarify the working of the productivity channel. Both assumptions assure that market tightness - and thereby the compression of the wage structure - are not directly affected

Figure 8: Shock with both Effects



by the capital tax. In such a case the only impact of a cut in capital taxes would be to shift the $\Delta\overline{g} = 0$ locus upwards and to the right since an increase in physical capital enhances the productivity of labor. This situation is depicted in figure (7), where it is assumed that profitability is not influenced by the tax.[24] It is clear that in such a setup, a tax on capital would not only lower the stock of physical capital but also the stock of human capital.

Figure (8) illustrates a possible outcome when both effects are combined and the complementarity between human and physical capital is large enough to lead to an increase in $\overline{g}$.

The figure not only illustrates the new equilibrium but as well the transition to the new equilibrium. Right after the shock the stock of physical capital jumps upwards since there are no adjustment costs for physical capital. This implies a jump in market tightness θ because new firms are attracted to the market. The optimal level of training jumps up as well (due to the higher productivity of labor that outweighs the effect of a tighter market).

[24] The assumption that training is specific would make the $\Delta\theta = 0$ locus horizontal but the implications were the same.

Table 2: Effect of Capital Income Tax on Average Human Capital

		Tax on Capital Income						
		0	5	10	15	20	25	30
	0,30	1,0000	0,9954	0,9906	0,9855	0,9802	0,9746	0,9684
	0,40	1,0000	0,9941	0,9881	0,9819	0,9755	0,9688	0,9618
	0,50	1,0000	0,9927	0,9854	0,9781	0,9706	0,9630	0,9550
Elasticity	0,60	1,0000	0,9913	0,9828	0,9745	0,9661	0,9577	0,9491
	0,70	1,0000	0,9902	0,9809	0,9718	0,9629	0,9541	0,9453
of	0,80	1,0000	0,9898	0,9803	0,9712	0,9625	0,9540	0,9456
	0,90	1,0000	0,9907	0,9822	0,9742	0,9668	0,9596	0,9527
Substitution	1,00	1,0000	0,9936	0,9879	0,9828	0,9781	0,9738	0,9697
	1,10	1,0000	0,9992	0,9988	0,9988	0,9990	**0,9995**	**1,0002**
	1,20	1,0000	**1,0081**	**1,0161**	**1,0240**	**1,0319**	**1,0398**	**1,0477**
	1,30	1,0000	**1,0205**	**1,0402**	**1,0594**	**1,0782**	**1,0968**	**1,1154**
	1,40	1,0000	**1,0362**	**1,0711**	**1,1052**	**1,1388**	**1,1721**	**1,2055**
	1,50	1,0000	**1,0546**	**1,1081**	**1,1608**	**1,2133**	**1,2661**	**1,3194**

The table illustrates the effects of various rates of capital income taxes on overall welfare when the tax-income is used to subsidize training.
The status quo of zero taxes is the baseline and normalized to one, so that all numbers illustrate the relative deviation from the baseline.

However, the average of human capital among unemployed workers is determined by the training decisions of the past (when those workers had a job) and therefore cannot jump to its new equilibrium-value. Instead it adjusts only slowly as workers with the new optimal value of human capital get fired. Due to this reaction in average human capital more firms are attracted to the market and θ increases even further. Since this increase in tightness is not accompanied by an increase in capital (as it was right after the shock) the optimal value of human capital now diminishes. This does not mean that firms have to deinvest in human capital but that they will reinvest less than what is lost due to depreciation. In a sense the slow reaction of average human capital among unemployed workers implies an overshooting of training over the new equilibrium level. The new optimal level of human capital is still above the old optimal level (before the shock) and so fired workers are going to have a higher level of training than fired workers had before the shock. As a result the average human capital of unemployed workers goes up even though training goes down. This explains the positive slope of the saddle path in figure (8).

For the picture it was assumed that the productivity effect dominates the tightness effect. Analytically, it is not clear which of the two effects will prevail. However, numerical simulations suggest that an outcome like the one depicted in figure (8) is most likely.

Table (2) shows the average human capital of unemployed workers for different elasticities of substitution and different tax-rates. The status-quo with zero taxes acts as the baseline and is normalized to one, so that all values show average human capital as a share of average human capital with zero taxes. The table illustrates that for most cases an increase of the tax-rate would imply a decrease in human capital. Only for an elasticity of substitution of 1.2 or higher, the effect is positive - these cases are printed bold. Since econometric studies[25] typically arrive at elasticities below 1.2, it appears rather likely that the taxation of physical capital will hurt human capital investments as well. A curious phenomenon appears in line 9 of table (2) showing the outcomes for $\varepsilon = 1.1$: First human capital decreases and then it increases. This is due to the fact that the tightness effect becomes stronger for higher unemployment rates.

As can be seen from table (2), the higher the elasticity becomes i.e. the more substitutable the production factors become, the less important gets the productivity effect of taxation on average human capital in equilibrium. This is indicated by the increase of human capital for lower columns and makes perfect sense. The more substitutable physical capital and labor are, the easier it is to avoid a tax on one factor by shifting to the other. If, on the contrary, two production factors are strong complements, a decrease in the usage of one factor will hurt the productivity of the other factor considerably. The fact that the decrease in human capital in the uppest lines of the table is smaller than in the following lines might seem to contradict this statement. The explanation for this is that for low values of ε the decrease in capital induced by taxation is lower than for large values.

So far we have seen that a capital tax might increase firms' training investments under certain circumstances but what does that mean for overall welfare? This question in answered by table (3) which shows relative changes in welfare where welfare is measured as shown in the section on welfare analysis, namely production minus search-, training- and investment costs. It is immediately clear that a positive effect of the capital tax on training is not sufficient to increase welfare.

[25] See for instance Chirinko (2002) for a detailed survey of the empirical literature or Hamermesh (1993) and Krusell (2000) on capital-skill complementarity.

Table 3: Effect of Capital Income Tax on Welfare

		Tax on Capital Income						
		0	5	10	15	20	25	30
	0,30	1,0000	0,9995	0,9990	0,9985	0,9978	0,9972	0,9964
	0,40	1,0000	0,9994	0,9987	0,9979	0,9971	0,9962	0,9953
	0,50	1,0000	0,9992	0,9983	0,9973	0,9962	0,9951	0,9938
Elasticity	0,60	1,0000	0,9990	0,9978	0,9966	0,9952	0,9937	0,9921
	0,70	1,0000	0,9988	0,9973	0,9957	0,9940	0,9921	0,9902
of	0,80	1,0000	0,9985	0,9968	0,9948	0,9927	0,9904	0,9880
	0,90	1,0000	0,9984	0,9963	0,9939	0,9913	0,9885	0,9856
Substitution	1,00	1,0000	0,9982	0,9958	0,9930	0,9899	0,9866	0,9831
	1,10	1,0000	0,9981	0,9954	0,9922	0,9885	0,9846	0,9805
	1,20	1,0000	0,9981	0,9951	0,9913	0,9871	0,9826	0,9778
	1,30	1,0000	0,9980	0,9946	0,9904	0,9856	0,9805	0,9750
	1,40	1,0000	0,9979	0,9941	0,9894	0,9840	0,9782	0,9721
	1,50	1,0000	0,9976	0,9935	0,9883	0,9824	0,9759	0,9691

The table illustrates the effects of various rates of capital income taxes on overall welfare when the tax-income is used to subsidize training.
The status quo of zero taxes is the baseline and normalized to one, so that all numbers illustrate the relative deviation from the baseline.

Table 4: Effect of Capital Income Tax on Human Capital - lower ε

		Tax on Capital Income						
		0	5	10	15	20	25	30
	0,30	1,0000	0,9974	0,9946	0,9917	0,9886	0,9854	0,9820
	0,40	1,0000	0,9967	0,9933	0,9898	0,9862	0,9824	0,9785
	0,50	1,0000	0,9962	0,9923	0,9883	0,9842	0,9801	0,9758
Elasticity	0,60	1,0000	0,9958	0,9916	0,9873	0,9831	0,9788	0,9746
	0,70	1,0000	0,9957	0,9915	0,9874	0,9834	0,9794	0,9755
of	0,80	1,0000	0,9962	0,9925	0,9890	0,9857	0,9825	0,9796
	0,90	1,0000	0,9973	0,9949	0,9927	0,9908	0,9892	0,9880
Substitution	1,00	1,0000	0,9994	0,9991	0,9991	0,9996	**1,0004**	**1,0018**
	1,10	1,0000	**1,0026**	**1,0056**	**1,0090**	**1,0129**	**1,0174**	**1,0225**
	1,20	1,0000	**1,0071**	**1,0147**	**1,0228**	**1,0315**	**1,0410**	**1,0513**
	1,30	1,0000	**1,0130**	**1,0267**	**1,0410**	**1,0561**	**1,0723**	**1,0896**
	1,40	1,0000	**1,0204**	**1,0416**	**1,0638**	**1,0871**	**1,1117**	**1,1380**
	1,50	1,0000	**1,0290**	**1,0594**	**1,0910**	**1,1244**	**1,1597**	**1,1973**

The table illustrates the effects of various rates of capital income taxes on overall welfare when the tax-income is used to subsidize training.
The status quo of zero taxes is the baseline and normalized to one, so that all numbers illustrate the relative deviation from the baseline.

The results just discussed are rather robust. For instance changes in the interest-rate or the rate of depreciation have only minor influence. However, what is important is the elasticity of the matching function and the rate of unemployment. This is not surprising since the tightness effect is strongly connected to these parameters: The elasticity of the matching function ε tells us how strong the firm´s probability to find a worker is influenced by relative changes in market tightness and $1 - \varepsilon$ tells us how strong the worker´s matching-rate reacts. A decrease in ε thus increases the response of P_w to θ and this implies stronger reactions in the bargaining position of workers.

A very common assumption about the elasticity of the matching function is the value

Table 5: Effect of Capital Income Tax on Welfare - lower ε

		Tax on Capital Income						
		0	5	10	15	20	25	30
	0,30	1,0000	0,9999	0,9998	0,9997	0,9995	0,9994	0,9992
	0,40	1,0000	0,9998	0,9996	0,9994	0,9992	0,9989	0,9986
	0,50	1,0000	0,9998	0,9995	0,9992	0,9988	0,9984	0,9979
Elasticity	0,60	1,0000	0,9997	0,9994	0,9989	0,9984	0,9979	0,9972
	0,70	1,0000	0,9997	0,9993	0,9987	0,9981	0,9974	0,9966
of	0,80	1,0000	0,9997	0,9992	0,9986	0,9979	0,9970	0,9960
	0,90	1,0000	0,9997	0,9992	0,9986	0,9977	0,9967	0,9955
Substitution	1,00	1,0000	0,9998	0,9993	0,9986	0,9977	0,9965	0,9951
	1,10	1,0000	0,9999	0,9995	0,9987	0,9977	0,9964	0,9948
	1,20	1,0000	1,0000	0,9997	0,9989	0,9977	0,9963	0,9946
	1,30	1,0000	**1,0002**	0,9998	0,9990	0,9978	0,9962	0,9943
	1,40	1,0000	**1,0003**	1,0000	0,9992	0,9978	0,9961	0,9939
	1,50	1,0000	**1,0004**	1,0001	0,9992	0,9977	0,9958	0,9934

The table illustrates the effects of various rates of capital income taxes on overall welfare when the tax-income is used to subsidize training.
The status quo of zero taxes is the baseline and normalized to one, so that all numbers illustrate the relative deviation from the baseline.

0.5.[26] As table (4) illustrates a positive impact on average training becomes more likely. What is even more important is the fact that it is now possible that an increase in the capital tax enhances overall welfare even though this is only true for very low tax rates and high values of elasticities as shown by table (5).

The dominating tightness effect for high elasticities of substitution might seem to confirm at least partially the papers by Heckman (1976) or Nielsen and Sörensen (1997) discussed in the introduction, who argue that a capital tax would lead to overinvestment in human capital. Although it is true that in my model human capital might increase with a tax on capital income, overinvestment does not occur, because there is underinvestment in the status quo without taxes. Moreover, since underinvestment in human capital is more severe than in physical capital, it is in principle even possible that a tax on capital income increases welfare as table (5) demonstrated.

7.3 Training Subsidy

In this section a subsidy on all training investments is introduced. I assume that the government bears a share of all training costs so that the firm only pays $(1-\mu)\tau$. Such a subsidy modifies the FOC of firm training to:

[26] See for instance Hall and Milgrom (2005).

Table 6: Effect of Training Subsidies on Physical Capital

		Training subsidy						
		0	5	10	15	20	25	30
	0,30	1,0000	1,0442	1,0933	1,1478	1,2089	1,2780	1,3568
	0,40	1,0000	1,0381	1,0801	1,1264	1,1781	1,2360	1,3014
	0,50	1,0000	1,0320	1,0670	1,1054	1,1478	1,1950	1,2480
Elasticity	0,60	1,0000	1,0259	1,0540	1,0847	1,1183	1,1555	1,1968
	0,70	1,0000	1,0199	1,0414	1,0647	1,0900	1,1179	1,1485
of	0,80	1,0000	1,0142	1,0294	1,0457	1,0635	1,0828	1,1039
	0,90	1,0000	1,0088	1,0183	1,0284	1,0393	1,0512	1,0640
Substitution	1,00	1,0000	1,0042	1,0086	1,0134	1,0185	1,0240	1,0300
	1,10	1,0000	1,0004	1,0008	1,0013	1,0017	1,0022	1,0028
	1,20	1,0000	**0,9976**	**0,9951**	**0,9924**	**0,9895**	**0,9863**	**0,9830**
	1,30	1,0000	**0,9959**	**0,9915**	**0,9868**	**0,9817**	**0,9763**	**0,9703**
	1,40	1,0000	**0,9951**	**0,9898**	**0,9841**	**0,9779**	**0,9713**	**0,9640**
	1,50	1,0000	**0,9950**	**0,9895**	**0,9836**	**0,9772**	**0,9702**	**0,9625**

The table illustrates the effects of various rates of capital income taxes on overall welfare when the tax-income is used to subsidize training.
The status quo of zero taxes is the baseline and normalized to one, so that all numbers illustrate the relative deviation from the baseline.

Table 7: Effect of Training Subsidies on Welfare

		Training subsidy						
		0	5	10	15	20	25	30
	0,30	1	1,0056	1,0115	1,0179	1,0246	1,0318	1,0396
	0,40	1	1,0055	1,0113	1,0174	1,0240	1,0311	1,0386
	0,50	1	1,0053	1,0109	1,0169	1,0233	1,0302	1,0375
Elasticity	0,60	1	1,0051	1,0106	1,0164	1,0225	1,0291	1,0363
	0,70	1	1,0049	1,0102	1,0157	1,0217	1,0280	1,0349
of	0,80	1	1,0047	1,0097	1,0150	1,0207	1,0268	1,0334
	0,90	1	1,0045	1,0092	1,0143	1,0197	1,0255	1,0318
Substitution	1,00	1	1,0042	1,0087	1,0135	1,0186	1,0242	1,0301
	1,10	1	1,0040	1,0082	1,0127	1,0175	1,0227	1,0284
	1,20	1	1,0037	1,0076	1,0118	1,0164	1,0213	1,0266
	1,30	1	1,0034	1,0070	1,0109	1,0152	1,0197	1,0247
	1,40	1	1,0031	1,0064	1,0100	1,0139	1,0181	1,0227
	1,50	1	1,0028	1,0058	1,0090	1,0126	1,0164	1,0207

The table illustrates the effects of various rates of capital income taxes on overall welfare when the tax-income is used to subsidize training.
The status quo of zero taxes is the baseline and normalized to one, so that all numbers illustrate the relative deviation from the baseline.

$$\left[\frac{\partial f}{\partial g} - \frac{\partial w}{\partial g}\right](1-\pi) = (r + \delta_e + \pi)(1-\mu) \tag{30}$$

It is immediately clear that the subsidy tends to increase firm training since marginal costs are directly reduced.

Concerning the effect on physical capital the same is true as was discussed in the previous section. Again a productivity effect and a tightness effect can be distinguished. However, since the wage does in general react less to physical capital than to human capital, the tightness effect is of lower importance. This is confirmed by the numerical simulations illustrated in table (6): For high values of the elasticity of substitution (1.2 or higher) the tightness effect can prevail. In all other cases, human and physical capital move in the same direction, implying that a subsidy for firm training does not only increase human capital but as well the stock of physical capital. Due to underinvestment in both kinds of capital, the subsidy clearly increases welfare if the subsidy can be financed via a lump sum tax. This is illustrated in table (7). Over the whole range of parameter values and subsidies an increase in subsidies implies an increase in welfare. Since the assumption of a lump sum tax is not very realistic, the next section combines both, the tax on labor income and the subsidy of firm training.

7.4 Combination of Taxation and Subsidy

In this section I analyze a policy in which revenue is raised through a tax on capital income and the whole tax income is redistributed as a subsidy on firm training. In the sections above we learned that the subsidy can increase welfare when financed through a lump sum tax and that the distortionary effects of a capital tax are not too severe. Thus, it is not surprising that the training subsidy financed by a capital tax raises overall welfare. What is surprising is the result that the capital tax necessary to finance the subsidies is so small[27] that the welfare effects are negligible and the outcome almost identical to table (7). This is due to the fact that the base of the capital tax is so large compared to the base of the training subsidy.

8 Conclusion

The appendix in Acemoglu and Pischke (1999a) tries to embed the standard two period model into the framework of a model with infinite horizon. However, their approach is not very convincing since it is assumed that all workers are trained by firms, no matter whether they actually have a job or are unemployed.

This paper has exactly the same purpose of putting the model into a more realistic framework of multiple periods. However, it does so in a way that is more consistent with

[27] Below 1% to finance a subsidy of 30%.

the two-period versions of the model, i.e. only workers that really have a job receive firm-training. This implies major heterogeneity among unemployed workers. They can be trained or have no human capital at all, and among trained workers different levels of human capital are possible. In this model I try to cope with the heterogeneity by assuming that no production takes place in the first period of the match. This assumption is not unusual (for instance, Acemoglu and Pischke (1999a) assume the same) and can be justified by arguing that the training of the first-period is so substantial that there is no time left for producing output. In later periods, training is just used to refresh what is lost due to depreciation and therefore not so time-consuming as in the first period. The assumption taken together with a linear cost-structure for training implies that the single firm does not care about the exact distribution of human capital among unemployed workers but only about its expected value. Thus, the steady state solution can be illustrated in a diagram of labor market tightness and average human capital among unemployed workers. After a shock the economy converges only slowly to the new equilibrium values of market tightness and average human capital. This is a major difference to the dynamic search-model in Pissarides (2000). There human capital is not modelled at all and consequently workers are homogenous. This implies that market tightness immediately jumps to its new equilibrium level after a shock. In my model market tightness depends on the average human capital of workers whereas the latter depends on the former. This kind of interaction results in a slow convergence after a shock.

An advantage of a model with infinite horizon is that it allows a distinction between short- and long-run effects of shocks and policy changes. So I was able to show that a shock that increases both training and market tightness in equilibrium will lead to overshooting of training investments, i.e. training jumps up by a large amount and decreases slowly during the transition to the new equilibrium.

The model is applied to analyze the effects of a change in the taxation of physical capital in a model of a small, open economy where the interest rate is exogenously given by the international capital market. I am able to distinguish two opposing effects: One works via the production function and one via the bargaining position of workers and firms in wage negotiations. The first effect is clear and not unusual. A tax that lowers the stock of physical capital causes the productivity of labor and thereby training investments to decline. The other effect is not so straight-forward. An increase in the stock of capital increases output and thereby the rents to be shared. This temporarily allows positive profits but attracts new firms to enter the market until the possibility of accruing profits vanishes. This has consequences for the labor market as well. More firms are searching for workers and thus the probability of an unemployed worker finding a job increases. The value of unemployment - which is an important parameter in wage negotiations - rises. Consequently, the bargaining position of the worker is improved and as a result wage compression decreases.

Thus, we have two opposing effects and it is not clear per se which of the two effects prevails. However, numerical simulations suggest that the productivity effect is more likely to dominate and a tax on capital income hurts both human and physical capital. Nevertheless, for higher elasticities of substitutions between capital and labor it is possible that training increases with capital taxation and in such a case even welfare might increase.

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10 Appendix A: Derivation of wage

By plugging in equations (2) and (4) into equation (6) we get:

$w + \rho s(1-d)U + \rho(1-\pi)W - U =$

$= \beta(f - \tau - I + \rho s(1-d)(U+K) + \rho(1-\pi)(W+J) + \rho d K - U - K)$

By noting that equation (6) is valid for all periods this equation can be written as:

$w + \rho s(1-d)U + \rho(1-\pi)U - U =$

$= \beta(f - \tau - I + \rho s(1-d)(U+K) + \rho(1-\pi)(U+K) + \rho d K - U - K)$

This transformation takes account of the fact that all future changes of rents in the match will be taken account of in the following negotiations. Therefore, the negotiations of this period need only care about what happens in between these two negotiations. By joining the terms with a U and a K we arrive at:

$w + \rho(1-d)U - U =$

$= \beta(f - \tau - I + \rho(1-d)U - U) + \rho K - K)$

Finally, we can put all U on the right-hand side and use the definition of $\rho \equiv \frac{1}{1+r}$ to get equation (7):

$w = \frac{r+d}{1+r}U + \beta\left[f - \tau - I - \frac{r}{1+r}K - \frac{r+d}{1+r}U\right]$

11 Appendix B: Effect of labor market tightness on wage compression

The degree of wage compression is found by taking the derivative of the wage (equation (7)) with respect to the productivity of the worker i.e. her human capital:

$$\frac{\partial w(K,g)}{\partial g} = (1-\beta)\frac{r+d}{1+r}\frac{\partial U(g)}{\partial g} + \beta\left[\frac{\partial f(K,g)}{\partial g} - \frac{\delta_e}{1+r}\right] < 1 \quad (31)$$

The further this equation deviates from one the more compressed is the wage structure. This implies that any increase in the above equation lowers the degree of wage compression. The effect of market tightness on wage compression is found by taking the derivative of equation (31) with respect to market tightness:

$$\frac{\partial^2 w(K,g)}{\partial g \partial \theta} = (1-\beta)\frac{r+d}{1+r}\frac{\partial^2 U(g)}{\partial g \partial \theta} > 0 \tag{32}$$

Given that the value of unemployment is made up by the probability of finding a new job and the expected value of that job, it is clear that the first term of this equation is positive, since θ improves the chances to find a job and g increases future wages and thereby the value of unemployment. The positive sign in equation (32) implies that wage compression decreases with labor market tightness.

12 Appendix C: Effect of unemployment on average HC

The effect of unemployment on average human capital is found by taking the derivative of $\overline{g}$ (equation (19)) with respect to u:

$$\frac{\partial \overline{g}}{\partial u} = \frac{-(1-\delta)s(1-d)[\delta u(1-p)(1-d)+(1-u)s(1-d)+d]}{[\delta u(1-p)(1-d)+(1-u)s(1-d)+d]^2} - \frac{[\delta(1-p)(1-d)-s(1-d)](1-\delta)(1-u)s(1-d)}{[\delta u(1-p)(1-d)+(1-u)s(1-d)+d]^2}$$

After some manipulations this equation simplifies to:

$$-\frac{(1-\delta)s(1-d)[\delta(1-p)(1-d)+d]}{[\delta u(1-p)(1-d)+(1-u)s(1-d)+d]^2} < 0$$

which is clearly negative. Thus an increase in unemployment tends to decrease the average human capital of unemployed workers.

13 Appendix D: Effect of average human capital on market tightness

As discussed in the section on the stationary solution, a firm does not care about the exact distribution of human capital among unemployed workers but only about the average of human capital. This section gives a more formal proof for this statement. The assumption that no production takes place during the first period of a match implies that the value of a vacancy (equation (1)) can be rewritten in the following way:

$$V = -c + \rho \int P(g) J(g) dF(g) + \rho(1 - P_f)V =$$

$$= -c + \rho \int P(g) \left[-(g^* - g) - K^* - w + \rho(1-\pi) J(g^*) + \rho\pi K^* \right] dF(g) + \rho(1 - P_f)V$$

The firm just invests so much in the worker´s human capital that it reaches the optimal level g^*. According to equation (7) the worker bears part of the training cost by accepting a lower wage. All the remaining parts of the wage are independent of the actual investment. To save notation all those terms which are independent of investments shall be summarized by the newly defined variable Ω:

$$V = -c + \rho \int P(g) \left[-(g^* - g)(1-\beta) + \Omega \right] dF(g) + \rho(1 - P_f)V =$$

$$= -c - \rho P_f((1-\beta)g^* - \Omega) + \rho(1-\beta) \int P(g) g dF(g) + \rho(1 - P_f)V$$

In the last equation I have put all constant terms outside of the integral. The remaining term in the integral is nothing else but the average of human capital ($\int P(g) g dF(g) = \overline{g}$). Thus the value of a vacancy simplifies to:

$$V = -c + \rho P_f \left[\Omega - (g^* - \overline{g})(1-\beta) \right] + \rho(1 - P_f)V$$

From this equation it is immediately clear that an increase in average human capital among unemployed workers increases the value of a vacancy since the necessary (re)investments are smaller. This increase in V will induce new firms to enter market. In consequence, market tightness goes up and the firm´s probability to find a worker P_f goes down until the market is back in equilibrium and $V = 0$.