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Information Quality and Stock Returns Revisited

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Abstract

Building on the seminal work of Veronesi (2000), we investigate the relationship between the quality of information on the state of the economy and equity risk premium. In this, we use a setup where investors have Epstein-Zin preferences and the economy switches between booms and recessions at random intervals (as proposed by Hamilton, 1989).

Calibrating the model to fit the business cycle patterns in the US postwar data, we are able to establish two key results: First, as conjectured in the existing literature, we demonstrate that investors with high intertemporal elasticity of substitution will require lower excess returns for holding stocks if they are provided with better information on the state of the economy. Second, and even more interesting (since not predicted in the literature), we find that this will also hold for investors with a moderate elasticity of intertemporal substitution if they are moderately risk averse.

Both our main results are due to the uncoupling of the tight link between risk aversion and the elasticity of intertemporal substitution that Epstein-Zin preferences allow us.

Keywords

Information quality, Regime switching, Epstein-Zin preferences, Learning, Asset pricing.

JEL Classification

E32, E37, G10, G12

1 Introduction

One of the key challenges for investors in modern financial markets is to convert the flood of news they are constantly facing into updated projections on the state of the economy. Obviously, publicly available signals might contain more or less information on the underlying state of the economy. High quality signals will enable investors to make high quality forecasts on the state of the economy, so it is natural to expect risk premia to vary with the amount of information they contain.

In a seminal contribution, Veronesi (2000) studies the link between information quality and risk premia within the framework of an exchange economy populated by a continuum of agents with identical power utility preferences. He finds several intriguing and quite surprising relationships between signals and stock returns, including: (i) the risk premium is increasing in the amount of information contained in the signals and (ii) unless the signals contain complete information on the state of the economy, the equity is *bounded above* independently of investors' risk aversion. The second result has the strong theoretical implication that, even assuming extremely risk averse investors, the model would not be capable of replicating the empirically observed risk premium. The first result is maybe even more intriguing, since it is so much at odds with what economic intuition tells us.

In this article, we revisit the relation between information quality and stock returns by introducing Epstein-Zin's preferences to Veronesi's model. As we will show, both results are overturned for a resonable calibration.

While there is a large literature exploring the asset-pricing implications of alternative preference specifications, we are not aware of anyone addressing specifically the topic of information quality. One particularly prominent line of research looks at the asset-pricing implications of including habits in the utility function (see e.g. Constantinides, 1990; Abel, 1990; Gali, 1994; Jermann, 1998, Campbell and Cochrane, 1999; and Boldrin et al., 2001.) Another line of research, started by Epstein and Zin (1989), Epstein and Zin (1991), and Weil (1989), looks at generalizations of the power utility function that allows us to relax the link between risk aversion and the elasticity of intertemporal substitution. This type of utility function, referred to as Epstein-Zin preferences, is particularly well-suited for our purposes. First, this type of utility function has been underlying much of the important recent research in asset-pricing (see e.g. Campbell and Viceira, 2001; Campbell et al., 2003; Bansal et al., 2002; Bansal and Yaron, 2004; Guvenen, 2005; and many others), including some featuring the same kind of Bayesian learning we are assuming (Brandt et al., 2004 and Lettau et al., 2004). Second, since Epstein-Zin preferences nest the power utility function as a special case, we are able to build on Veronesi's work and provide a direct comparison with his results.

The main finding of our paper is that for a wide range of plausible parameterizations of the utility function, the required risk premium on equity is decreasing in the quality of information available to investors. This range covers both a domain where this reversal has been predicted in the literature ($\psi > 1$), as well as a domain where it comes as a completely new result ($\psi < 1$). Both sides are important, since there is a considerable controversy with respect to the appropriate parameter value for the EIS parameter.[1]

Grasping the mechanisms that govern the relation between information quality and equity returns is far from trivial. On a technical level, it is related to how stock prices move with the state of the economy. We show that the premium, equities command over risk-free bonds, depends on: 1) the variance of consumption, 2) the volatility of equity prices and 3) the covariance of returns with consumption growth rates.[2] There is no link between information quality and variance of consumption growth rates, so we will focus on the last two factors. Furthermore, we will generally focus on the case of investors with an higher RA parameter relative to their EIS. (Or a preference for early resolution of uncertainty, in the language of Kocherlakota, 1990.)

As to the second factor, the variance of returns is increasing in the signal quality. Assuming a preference for early resolution of uncertainty, higher variance translates into a higher required equity premium if investors have a high EIS (> 1) and a lower required equity premium if investors have a

[1]The empirical estimates vary strongly with the assumptions made on the structure of the economy. One strand of empirical research uses representative agent setup and estimates the EIS parameter using aggregate consumption data. This approach typically leads to estimated EIS coefficients in the range of 0-0.3 (see e.g. Hall, 1988; Campbell and Mankiw, 1989, 1991; Hahm, 1998; Campbell, 2003; and Yogo, 2004). Another strand of research seeks to avoid potential biases, introduced using aggregate data, relying on microeconomic survey data. For stockholders, these studies find EIS parameters around or above 1. (See Beaudry and van Wincoop, 1996; Vissing-Jørgensen, 2002; Vissing-Jørgensen and Attanasio, 2003; and Guvenen, 2005). Recent asset-pricing literature relies on the high EIS estimates of the latter literature (e.g. Bansal and Yaron, 2004 or Lettau et al., 2004 both calibrate their models with an EIS greater than one.)

[2]See also equation (8.3.7) on page 320 of Campbell et al. (1997).

moderate EIS (< 1). As we argue below, the market price of equity is a projection of the true underlying value of equity. If investors can only access low quality information on the state of the economy, their estimate of the underlying value will not change much over time, and the volatility of prices will be low. By providing investors with better information, we enable them to better estimate the underlying value. This will increase the volatility of prices because changes in the underlying value of equity will translate directly into changes in its projected value.

As to the third factor, the covariance of consumption and returns turns out to be decreasing (in absolute terms) in the quality of information available to investors. For the parameterizations we focus on, this characteristic translates into a lower required equity premium. The finding that the covariance is decreasing in the signal quality is somewhat surprising at the first glance. Better information on the state of the world will enable investors to better identify the state of the economy, making prices more cyclical. However, the stronger cyclicality does not translate into a higher covariance of equity returns with consumption. The intuition is that without an informative external signal, prices will only move with the information available in realized consumption growth rates. This leads to a relatively high covariance. If investors can access informative external signals, the tight link between agents' beliefs and consumption growth rates is relaxed. This will reduce the comovement of consumption and returns. Because better information quality will generally bring the covariance down, the impact of information quality on the required risk premium will depend on whether the covariance is positive or negative in the first place. In one way an EIS of 1 constitutes a watershed with procyclical prices for higher EIS and countercyclical prices for lower EIS.[3] However, for the parameter configurations we consider, the coefficient on the covariance term also changes sign depending on whether the investors have an EIS greater or smaller than 1. Summarizing, the influence of better information quality on the covariance term will be such that it reduces the required risk premium.

The impact of increased information quality on the required excess return to equity is found by adding up the impact on the variance and the covariance term. For the parameters range that we deem most relevant:

- if investors have an EIS moderately smaller than 1, then both terms change in such a way that the equity premium decreases;

[3] Bansal and Yaron (2004) demonstrate an analogous result within a log-linear model.

- if investors have an EIS > 1, then the increased variance of returns due to better information quality will push the required equity premium up, while the reduced covariance between returns and consumption innovations will push it down. Our numerical results show that the second effect dominates.

Hence, both for EIS < 1 and EIS > 1, we obtain the intuitive result that the equity premium is decreasing in the signal quality.

The implied cyclicality of equity prices provides a metric for comparing the empirical relevance of various parameterizations. A well known feature of the power utility function is that it imposes the restriction that the EIS parameter equals the inverse of the coefficient of relative risk aversion γ. Effectively, since asset-pricing models are almost exclusively parametrized with a γ larger than one, it would predict countercyclical equity prices within our setting. This is not true with Epstein-Zin preferences, where a RA parameter larger than 1 does not impose an EIS parameter less than 1. We show that empirical equity prices are strongly procyclical, lending strong support to the assumption that investors have a high EIS.

Another important finding is that there is no global maximum for the required equity as a function of the RA of investors. This is different from what obtains in a regime-switching economy where investors are power utility maximizers (Veronesi, 2000, proposition 3b). Unless the EIS parameter is very low, increasing investors' risk aversion leads monotonously to a higher required equity premium. The key intuition behind this result is that the cyclicality of returns is mainly governed by investors' EIS. Under power utility, increasing investors RA automatically decreases their EIS, making prices and hence returns increasingly countercyclical. For high levels of RA, returns actually become countercyclical, making equity a good hedge against consumption risk. Thus investors are willing to accept expected returns lower than the risk-free rate to hold it. With Epstein-Zin preferences, investors' RA is uncoupled from their EIS. Hence we can generate as high a procyclicality of equity prices as we like by increasing ψ and, at the same time, make the investors as averse to procyclical payoffs as we want by increasing γ.

While we differ from Veronesi in the choice of preferences, we remain close to his model in terms of the dynamics of our model economy. We assume that the underlying state of the economy follows an ergodic, two-state

Markov switching process. The state of the economy is a hidden variable, so investors have to rely on the information embedded in dividend growth rates and other signals for pricing equity and bonds. Since this type of model is able to capture non-linearities found in the data that are missed by more traditional models (see discussion in Hamilton, 2005), it has been widely used in economics since its introduction by Hamilton (1989).[4]

The paper is organized as follows: Section 2 introduces the general model and the properties of the external signal, Section 3 discusses the cyclical properties found in the postwar US time series, and Section 4 presents formulae for equity prices, returns and the risk-free rate. A numerical analysis is provided in Section 5 and Section 6 concludes. Proofs, algebraic derivations, and additional results are provided in Appendix A.

2 Model

We assume a Lucas (1978) type exchange economy, populated by a continuum of identical agents with Epstein-Zin preferences given by

$$U(c_t, \mathrm{E}_t(U_{t+1}) = \left[(1-\beta)c_t^{\frac{1-\gamma}{\kappa}} + \beta(\mathrm{E}_t(U_{t+1}^{1-\gamma}))^{\frac{1}{\kappa}}\right]^{\frac{\kappa}{1-\gamma}}, \tag{1}$$

where $\kappa \equiv (1-\gamma)/(1-1/\psi)$ and $\gamma > 0$.

The parameter γ is the coefficient of relative risk aversion, while the EIS is given by ψ. The function reduces to a monotone transformation of the standard power utility function for $\psi = \gamma^{-1}$. Dividends (the endowment good) grow according to the process

$$C_t = C_{t-1}\mathrm{e}^{\mu_{c,i}+\sigma_{c,i}\epsilon_{c,t}}, \tag{2}$$

[4]In particular, in the asset-pricing literature, the implications of a Markov switching process in the conditional mean of the endowment process are analyzed by Cecchetti et al. (1990); Kandel and Stambaugh (1991); Cecchetti et al. (1993); Abel (1994); Abel (1999). Time series behavior of the second moments are recently studied in a regime switching framework: by setting up an equilibrium economy where the endowment process follows a latent two state regime switching process, Veronesi (1999) shows a better explanatory power of volatility clustering than a model without regimes. In the same setting, Whitelaw (2000) introduces time-varying transition probabilities between regimes, finding a complex nonlinear relation between expected returns and stock market volatility. A recent contribution that studies the impact of regime switches in the volatility of the endowment process is in Lettau et al. (2004).

where $\mu_{c,i}$ and $\sigma_{c,i}$ denote the mean log consumption growth rate in state i, and its standard deviation, respectively, and ϵ_t is an i.i.d. standard normal variable.

The underlying state of the economy s_t follows an ergodic, two-state Markov chain with transition probability matrix between time t and $t+1$ given by

$$\Theta = \begin{pmatrix} \theta_1 & (1-\theta_2) \\ (1-\theta_1) & \theta_2 \end{pmatrix} \tag{3}$$

where $\theta_i > 0.5$. For identification, we assume $\mu_{c,1} > \mu_{c,2}$, so that the first state has the natural interpretation of a boom state, while the second state is a recession state.

The state of the economy is not directly observable, but agents have various sources of information at hand for inferring it. The most obvious such source is the growth rate of dividends themselves. Given the structure of the economy, which is assumed known to the agents, high growth rates will indicate a high probability of being in the boom state, while the reverse is true for low growth rates.

All information in addition to that contained in dividend growth rates, is aggregated as an independent noisy signal. For convenience, we let this signal take the form

$$y_t = 1_{\{s_t=1\}}\mu_{y,1}(h) + \epsilon_{y,t}, \tag{4}$$

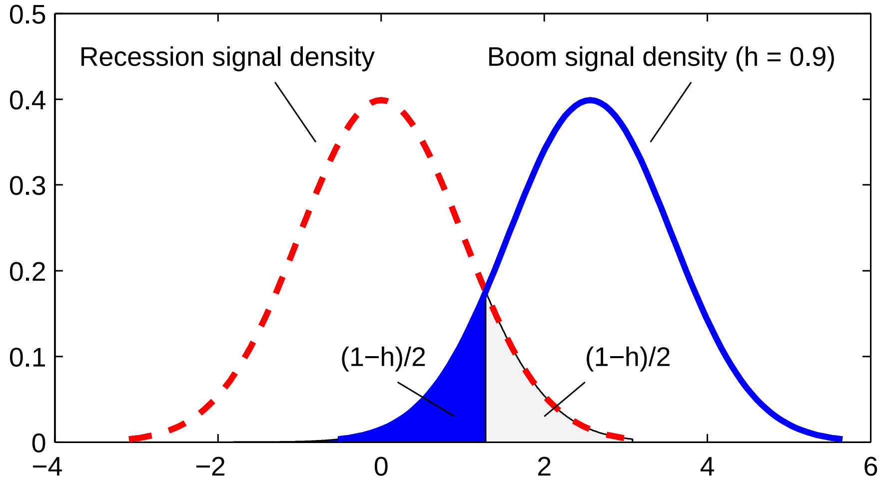
where $\epsilon_{y,t}$ is an i.i.d. white noise term, and $1_{\{s_t=1\}}$ is an indicator function which equals one if we are in the second state and zero otherwise. The strength of the signal is determined by $h \in [0,1]$. An h of zero implies that the signal contains no information, while an h of one implies that the signal is strong enough to reveal the state of the economy with certainty. The mean value of the signal in state one is normalized to one, so the information contained in the signal is solely a function of its mean in the second state.

In the case where the external signal is almost pure noise, the probability of assigning a lower posterior probability to the true state based on only one realization of the signal (i.e. of making a type I error) is 50 %. For intermediate levels of signal quality, we let h be the percentage reduction in this probability relative to the pure noise case.

Figure 1 illustrates this: Based on a single observation, a higher probability is assigned to the state where the denisity is highest for that observation.

Figure 1: Signal precision and state densities

Plotted are the densities of the signal in the two states when the signal has strength $h = 0.9$.



Thus, the probability of a type one error is given by the area of the shaded region. It probability converges to 0.5 as the means of the two distributions converge.

Normalizing the mean of the signal in the recession state to 0, and assuming without loss of generality that the signal has a positive mean in the boom state, the mean that generates a h percent reduction of the probability of making a type I error is given by

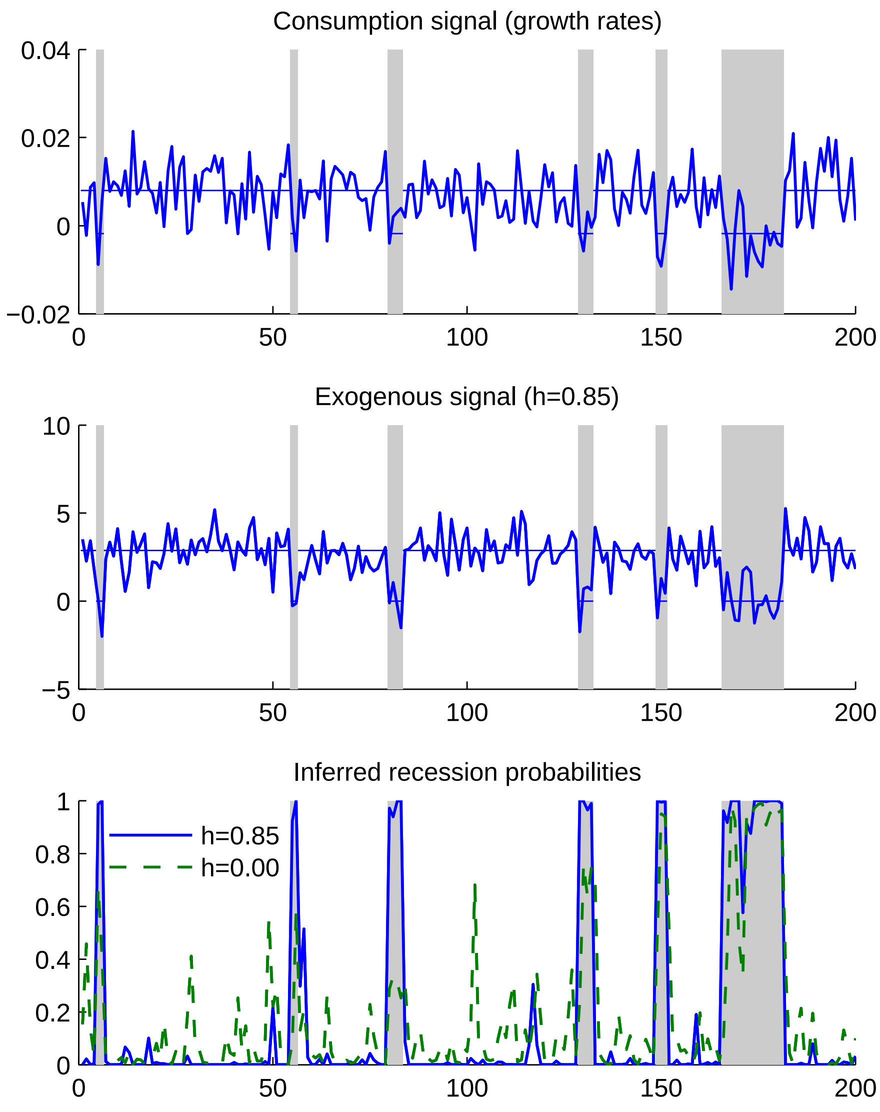
$$\mu_{y,1}(h) = -2F^{-1}\left(\frac{1-h}{2}\right), \tag{5}$$

where F denotes the cumulative distribution function of a standard normal. To see this, note that $F^{-1}\left((1-h)/2\right)$ gives the point diametrically opposite to the intersection of the two densities in Figure 1 and that the mean signal in the boom state is twice as far from zero as the point of intersection.

Figure 2 illustrates how the information embedded in dividend growth rates is combined with that of the external signal to infer state probabilities. The figure is created by generating a series of states, growth rates, and signals using the estimated process parameters from the next section. Realizations

Figure 2: Simulated signals and inferred probabilities

Plotted are simulated consumption and signal series for a Markov-switching economy, given estimated process parameters from section 3 and a signal strength of $h = 0.85$. The solid line in the lower panel are inferred state probabilities given both the consumption and external signal; the dashed line those given only by the consumption signal (with $h = 0$, the external signal is pure noise and investors give it no weight.)



of the low growth state are marked as shaded areas. Given an intial draw of states, dividend growth rates (top panel) and signals (mid panel) were generated. Applying Hamilton's 1989 filter to the realized dividend growth rates *only* produces the dashed line in the bottom panel. By passing also the external signal through the filter, investors are able to establish with much larger certainty the current state of the economy, as illustrated by the solid line in the bottom panel.

3 Data, estimation, and calibration

The sample period chosen for calibrating the model, spans from the first quarter of 1952 to the last quarter of 2003. The data-set is expressed in real terms with a quarterly frequency. Prices and dividends are on the S&P 500 composite, while the risk-free rate is the yield on 1 year treasury bills, all downloaded from Robert J. Shiller's webpage.[5] Consumption is quarterly real total personal consumption expenditures (NIPA table 2.3.6, line 1) and GDP is quarterly real Gross Domestic Income (NIPA table 1.1.6, line 1), both downloaded from the Bureau of Economic Analysis' website.[6] Finally, we use the official recession dates as reported on the website of the National Bureau of Economic Research.[7]

Our data-set is a standard one and descriptive statistics are similar to those reported elsewhere in the literature. The average return on equity is 8.8% on an annual basis with a standard deviation of 16.1%. Compared with the mean risk-free rate of 1.8%, this yields an equity premium around 7.0%. As to macroeconomic variables, GDP grew during the postwar sample at a mean annual rate of 2.2%, while the consumption had an annual growth rate of 2.3%. These numbers are summarized Table 1.

Financial market lore contends that prices move procyclically with the business cycle. To verify this conjecture, we calculate the correlation matrix amongst the cyclical components of the US economic and financial series. For this, all series were expressed in real terms, logged, and then filtered with the HP-filter (Hodrick and Prescott, 1997). As it is shown in Table 2, the cyclical compentents of all the series are strongly positively correlated, with a correlation coefficient ranging from 0.40 for GDP and the price-dividend ratio

[5]http://www.econ.yale.edu/ shiller/data.htm.
[6]http://www.bea.doc.gov/.
[7]http://www.nber.org/cycles.html/.

Table 1: Descriptive statistics

This table summarizes empirical means and standard deviations in the US postwar data. (Q1:1952–Q4:2003; sources: BEA and Robert Shiller's webpage.)

	Equity return	Risk-free rate	GDP growth	Consumption growth
Mean	0.088	0.018	0.022	0.023
Std. deviation	0.161	0.030	0.020	0.017

Table 2: Cyclical correlations

This table reports the correlation matrix for the cyclical component of main US financial and economic series. (Q1:1952–Q4:2003; sources: BEA and Robert Shiller's webpage.)

	S&P 500	PD ratio	GDP	Consumption
S&P 500	1.000			
PD ratio	0.965	1.000		
GDP	0.487	0.397	1.000	
Consumption	0.545	0.469	0.877	1.000

to 0.97 for stock prices and the price-dividend ratio. Of particular interest is the correlation between the cyclical component of GDP and stock prices. The scatter plot in Figure 3 illustrates this for the cyclical components of GDP and the price-dividend ratio. From table 2, we see that the slope of the regression line (i.e. the correlation coefficient) is close to 0.5.

Parameter estimates for the regime switching model was found by using a Markov-Chain Monte-Carlo (MCMC) procedure on the total consumption series. In this, we closely followed the algorithm described in section 9.1 of Kim and Nelson (1999). The resulting estimates are given in Table 3.

Some preliminary intuition on the model economy can be inferred from the regime switching estimation: from the perspective of a risk averse agent, the boom state is preferable to the recession state because of the higher mean

Figure 3: Correlation between detrended stock prices and GDP

This figure plots the cyclical component of real GDP versus the cyclical component of real stock prices, both estimated with the HP-filter.

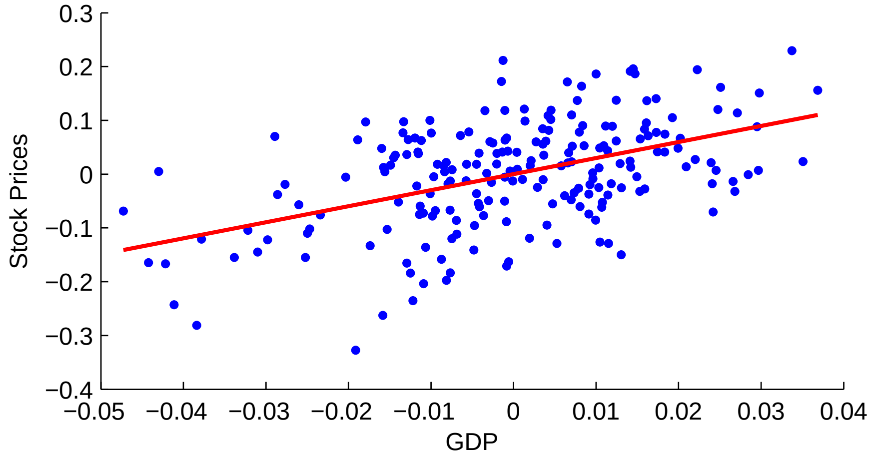


Table 3: Estimated regime parameters

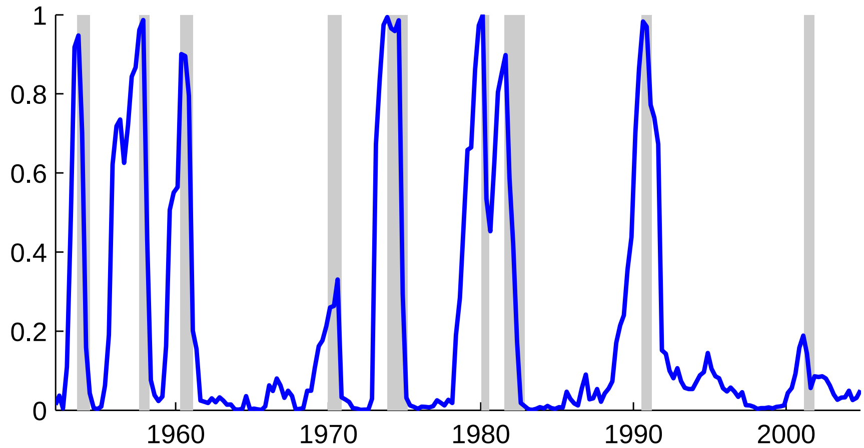
Reported are the estimated parameters of the regime switching model for the US postwar data. (Q1:1952–Q4:2003; source: BEA)

State	$\mu_c(s)$	$\sigma_c(s)$	θ_{ij}
Boom (s=b)	0.0080	0.0061	0.0586
	(0.0007)	(0.0003)	(0.0262)
Recession (s=r)	-0.0018	0.0061	0.1809
	(0.0020)	(0.0003)	(0.0704)

growth rates. A crucial variable for our analysis is the high persistence of both states, especially the boom state. The probabilities of switching from the two states are 5.86% and 18.09%, respectively. These probabilities imply an average duration of 17.1 quarters for booms and 5.5 quarters for recessions. Hence, if we find ourselves in either of the two states, we expect to stay in it for several periods, leading to a higher expected utility in the boom state than in the recession state.

Figure 4: Model recession probabilities

Plotted are our estimated posterior probabilities of being in a recession coupled with the official NBER recession dates (shaded areas).



The regime switching characteristics of the consumption series is assessed using the test by Carrasco et al. (2005), following the approach proposed by Hamilton (2005). Results are comforting, with a probability value of 2.5% for Markov switching. This lends support to modeling the data according to Equation (2).[8]

For the analysis of the model's properties, we calibrate the utility function with parameters in line with those used in leading asset-pricing models. In particular, we use the parametrization of Lettau et al. (2004) as a benchmark and set $\gamma = 25$, $\psi = 1.5$, and $\beta = 0.9925$.

[8]In order to test whether some autocorrelation in the data could lead to not rejecting the regime switching specification, we also test our specification against a first order autoregressive process. The obtained probability value of 19% does not allow us to reject the AR(1) specification at conventional levels, but is still in line with our regime switching specification.

4 Prices and returns

In this section we introduce the equilibrium price-dividend ratio formula for the stock market and the relevant rate of returns in the model economy.

One of the key results of Epstein and Zin (1989) is that the stochastic discount factor for the recursive utility function in Equation (1) can be expressed as

$$M_{t+1} = \beta^{\kappa} \left(\frac{c_{t+1}}{c_t} \right)^{-\frac{\kappa}{\psi}} (R_{e,t+1})^{\kappa - 1} \tag{6}$$

where $R_{e,t+1}$ is the equilibrium gross return to aggregate wealth between t and $t+1$. Using this result, we can find expressions for the equity premium, as well as the one period risk-free rate. As usual, the gross risk-free rate is given by the inverse of the expected value of the stochastic discount factor, or

$$R_{b,t+1} = \mathrm{E}_t \left[\beta^{\kappa} \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left(\frac{1 + w_{t+1}}{w_t} \right)^{\kappa - 1} \right]^{-1}. \tag{7}$$

Thus, in this setting, the interest rate will fluctuate not only with the expected growth rate of consumption, but also with the expected changes in the price-consumption ratio.

Proposition 4.1. *Given a boom probability of $\xi_{t|t}$, the price-consumption ratio is given by*

$$w_t = [\xi_{t|t} w_b^{\kappa} + (1 - \xi_{t|t}) w_r^{\kappa}]^{\frac{1}{\kappa}}, \tag{8}$$

where w_b and w_r denote the price-consumption ratio when the investors know, with certainty, that they are in a boom or a recession respectively:

$$w_j = \mathrm{E}_t \left[\beta^{\kappa} \left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} (1 + w_{t+1})^{\kappa - 1} \middle| s_t = j \right]^{1/\kappa}.$$

Proof. see Appendix A. □

Since we are in an endowment economy, equilibrium conditions require that consumption is always equal to dividends so the price-dividend ratio is also given by Proposition 4.1. We will use the two terms interchangeably.

It is easy to verify that the price-consumption formula is strictly convex in the state probability if $\kappa < 1$, and strictly concave if $\kappa > 1$. Hence, for $\kappa < 1$, as it is in our calibration, the average price-dividend ratio under uncertainty is lower than the linear combination of the certainty price-dividend ratios weighted by the state beliefs. This non-linearity rules out the typical, linear-algebra closed form solutions, that we obtain under power utility. To solve for the state prices w_j, we relied on numerical integration using a Gauss-Hermite Quadrature. By using a large number of nodes, we assured that our prices are arbitrarily close to the true values. (See e.g. Judd, 1998, chapter 7). We tested our solution algorithm against the closed form solutions for the power-utility case. The differences we found were only a few orders of magnitude away from the machine's precision.

One key concern is to determine whether prices are moving pro or counter-cyclically with the state of the economy. We restrict our attention to the relevant case where the probability of the economy remaining in the same state, on a period-on-period basis, is higher than the probability that we have a regime switch. In the degenerate case where $\theta_1 = \theta_2 = 0.5$, the information that the current state of the economy contains is irrelevant for forming expectations about future payoffs, and the price-dividend ratio will not move with the state probabilities.

As we show in the Appendix, decreasing the transition probability in each state will increase the extent to which variations in expected discounted payoffs over the two states translate into variations in state prices. Furthermore, we use this result to show that procyclical prices follow whenever the EIS parameter ψ is greater than 1. This is established in the following proposition:

Proposition 4.2. *For θ_1 and θ_2 greater than $\frac{1}{2}$, when $\psi > 1$ ($\psi < 1$) the price-dividend ratio is higher (lower) in booms than in recessions.*

Proof. see Appendix A. □

The intuition behind this finding is that variations in the expected growth rate of dividends not only influence expected payoffs, but also the rate at which they are discounted. We can better understand this by log-linearizing the Euler equation for equity, and solving it for expected returns. This gives us

$$\mathrm{E}\left[r_e\right] = -\log\beta + \frac{1}{\psi}\,\mathrm{E}\left[\frac{c_{t+1}}{c_t}\right] - \frac{1}{2}\kappa\left[\frac{1}{\psi^2}\sigma_c^2 + \sigma_r^2 - 2\frac{1}{\psi}\sigma_{g,r}\right]. \tag{9}$$

The main difference between booms and recessions is that the expected growth rate of dividends is higher in booms than in recessions, so that the second term of the equation will be higher in booms. On the one hand, an upward revision of the expected growth rate of dividends increases the expected payoffs of equity, increasing its value to investors. On the other hand, investors prefer consumption profiles which are smooth over time. Given an upward revision in the expected dividend growth rate, investors would like to smooth their intertemporal consumption profile by shifting consumption from the future to the present. Since the model does not allow for any aggregate saving or dissaving, an equilibrium can only be obtained if the expected return on all assets increase enough check investors' desire to sell them off in order to finance consumption increases. The amount expected returns will have to increase to maintain an equilibrium depends on how tolerant investors are to consumption variations over time (i.e. on their elasticity of intertemporal substitution). If $\psi < 1$, an upward adjustment of the expected dividend growth rate causes an even larger upward adjustment of the required return to equity. This leads to a drop in prices. If $\psi > 1$, an upward adjustment of expected consumption growth rates is matched by less than a one-to-one adjustment of the required return to equity; hence prices would be increasing in the boom probability. Because Equation (9) does not hold strictly for our non-linear economy, we have also included a (tedious) proof in the Appendix that does not rely on loglinearization.

The key relation we need to analyze in our model, is the one between unconditional equity premium and different signal quality. The following proposition provides an approximate analytical expression for assessing this relation.[9]

Proposition 4.3. *If consumption and asset returns are homoskedastic and jointly lognormal, the equity premium can be expressed as*

$$EP = \gamma\sigma_c^2 + (1-\kappa)\,\sigma_\omega^2 + ((1-\kappa)+\gamma)\,\sigma_{\omega,c} \tag{10}$$

where σ_c^2 is the variance of the log consumption growth, σ_ω^2 is the variance of $\log\frac{1+w_{t+1}}{w_t}$, and $\sigma_{\omega,c}$ is their covariance.

Proof. see Appendix A. □

[9] Is worth noting that the quantitative results in the next section do not rely on this linear approximation, but on precise numerical algorithms.

The first part of the equation, $\gamma\sigma_c^2$, is the familiar textbook formula for the equity premium. There is no link between the quality of the signal and the volatility of consumption, so we can safely ignore this term for our purposes. Turning to the second term of the equation, we know that with perfect signals prices will move in step with the state of the economy, entailing a relatively large jump whenever it switches from one state to the another. By analogy to Shiller (1981), this implies that the price volatility is increasing in the signal quality. The price-dividend ratio is a forecast of the true value of equity given knowledge on the state of the economy. Denoting this value by $w_t^\star$, we can use standard regression theory to express $w_t^\star$ as the sum of forecast and an orthogonal forecasting error term u_t:

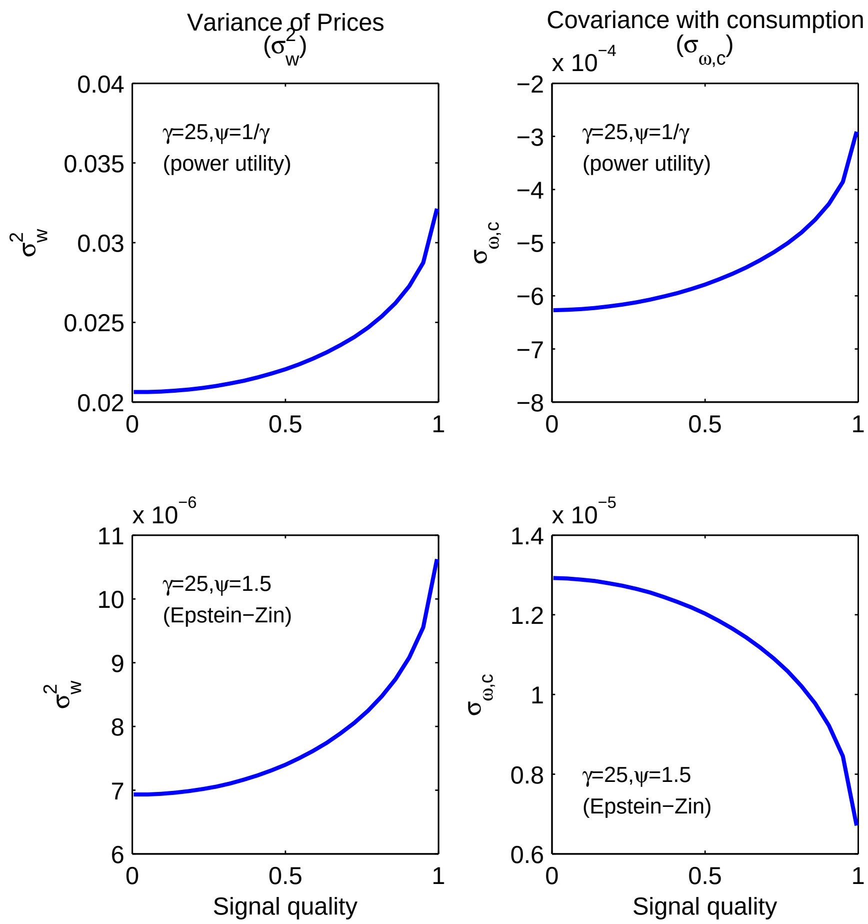
$$w_t^\star = w_t + u_t.$$

Since u_t is orthogonal to w_t, we can rely on the basic statistics rule that the variance of the sum of two uncorrelated variables equals the sum of their variances. It follows that $\text{var}(w_t) = \text{var}(w_t^\star) - \text{var}(u_t)$. Since variances cannot be negative, the variance of the price-dividend ratio must be lower than the one of the underlying value. The expected effect of better information quality is to bring the price-dividend ratio toward $w_t^\star$. This reduces the variance of u_t and increases that of w_t. The variance of w_t reaches its upper bound $\text{var}(w_t^\star)$ when the signal is strong enough to pin down the state of the economy with certainty.

Regarding the last term of Equation (10), it follows from Proposition 4.2 that the covariance of prices with consumption will be positive whenever ψ is greater than one, and negative otherwise. Moreover, increasing the signal quality pushes the covariance toward zero in both the power utility and Epstein and Zin specification. The intuition is that, without an informative external signal, prices will only move with the information available in realized consumption growth rates. This leads to a relatively high covariance. If investors can access informative external signals, the tight link between agents' beliefs and consumption growth rates is relaxed. This will dampen the covariance. Because better information quality will generally bring the covariance down, the impact of information quality on the required risk premium will depend on whether the covariance is positive or negative in the first place. In order to verify this claim, we performed a Monte Carlo analysis on the model economy by simulating prices and consumption growth for 10000 periods for various signal strengths. Results are just as expected and

Figure 5: Signal quality and second moments

Plotted are the predicted variance of ex cum-dividend equity returns σ_ω^2 and their predicted covariance with consumption growth rates $\sigma_{\omega,c}$ for two parameter constellations over the whole range of signal strengths.



are displayed in Figure 5.

Given the above analysis, we can now disentangle the movements in the equity premium with respect to variations in the signal quality. Taking the derivative of equation (10) with respect to signal quality yields

$$\frac{d\text{EP}}{dh} = (1-\kappa)\underbrace{\frac{d\sigma_\omega^2}{dh}}_{>0} + ((1-\kappa)+\gamma)\underbrace{\frac{d\sigma_{\omega,c}}{dh}}_{\substack{>0 \text{ if } \psi<1 \\ <0 \text{ if } \psi>1}} \tag{11}$$

In the power utility case, Equation (11) simplifies to $\gamma\frac{d\sigma_{\omega,c}}{dh}$. Consequently, the influence of the signal quality is entering only through the covariance between consumption and price innovations. Both of Veronesi's key results follow directly: First, if $\gamma > 1$, then ψ, which is its inverse under power utility, must be smaller than one; thus, the better the signal quality, the higher the risk premium will be. Second, the bound on the risk premium follows from how the shape of the price function changes as we increase γ. By the chain-rule, the derivative of the equity premium is given by

$$\frac{dEP}{d\gamma} = \sigma_c^2 + \sigma_{w,c} + \gamma\frac{d\sigma_{w,c}}{d\gamma}. \tag{12}$$

In the immediate proximity of $\gamma = 1$, both the covariance term and its derivative with respect to γ are zero.[10] Accordingly, increasing γ will lead to an increase in the required equity premium. As we let γ approach infinity, the two last terms become more and more negative. At some point, the whole right hand side becomes negative and increasing γ beyond this point decreases the equity premium. If we increase γ enough, we will enter a domain where the equity premium turns negative. As we show in the next section, even moderate values for γ lead us into this domain.

In the Epstein and Zin case things are more complicated. All the terms in Equation (11) are fully contributing to the equity premium. Moreover, their direction of influence depends on how the utility function is parameterized.

[10] In the power utility case, it follows from the Euler equation that

$$w_b = \theta_1 \,\mathrm{E}\left[\beta e^{(1-\gamma)(\mu_{c,1}+\sigma_{c,1}\epsilon_{t+1})}\right] + (1-\theta_1)\,\mathrm{E}\left[\beta e^{(1-\gamma)(\mu_{c,2}+\sigma_{c,2}\epsilon_{t+1})}\right].$$

Hence $dw_b/d\gamma = -w_b$. Analogously, $dw_r/d\gamma = -w_r$. The derivative of the spread between boom and recession prices is given by $d(w_b - w_r)/d\gamma = w_r - w_b$. Since $w_b = w_r$ for $\gamma = 1$, an incremental increase of γ, from $\gamma = 1$, does not entail any increase in σ_ω from its value of 0. For the same reason $\left.\frac{\sigma_{w,c}}{d\gamma}\right|_{\gamma=0} = 0$.

Table 4: Summary table

This table summarizes the relation between information quality and the terms governing the equity premium. Columns in bold typeface cover main scenarios in our analysis.

	$0<\psi<0.5$		$0.5<\psi<1$			$1<\psi$		
	$\gamma \in (0, \frac{1}{\psi})$	$\gamma > \frac{1}{\psi}$	$\gamma \in (0, \frac{1}{\psi})$	$\gamma \in (\frac{1}{\psi}, \frac{1}{2\psi-1})$	$\boldsymbol{\gamma > \frac{1}{2\psi-1}}$	$\gamma \in (0, \frac{1}{2\psi-1})$	$\gamma \in (\frac{1}{2\psi-1}, \frac{1}{\psi})$	$\boldsymbol{\gamma > \frac{1}{\psi}}$
$(1-\kappa)$	+	−	+	−	**−**	−	−	**+**
$\frac{d\sigma_\omega^2}{dh}$	+	+	+	+	**+**	+	+	**+**
$(1-\kappa)\frac{d\sigma_\omega^2}{dh}$	+	−	+	−	**−**	−	−	**+**
$(1-\kappa)+\gamma$	+	+	+	+	**−**	−	+	**+**
$\frac{d\sigma_{\omega,c}}{dh}$	+	+	+	+	**+**	−	−	**−**
$((1-\kappa)+\gamma)\frac{d\sigma_{\omega,c}}{dh}$	+	+	+	+	**−**	+	−	**−**
EP	+	?	+	?	**−**	?	−	**?**

Table 4 gives a breakdown of the analytical implications of Equation (11) for different parameter configurations. Notice that for certain configurations, we cannot determine unambiguously the effect of improved signal quality based on this expression alone. For such cases, we rely on numerical results.

We focus on the scenario where investors have a high risk aversion relative to their elasticity of intertemporal substitution.[11] Consider first the case $\psi > 1$ and $\gamma > 1$. This covers our baseline calibration and, as we argue above, is the most realistic case: 1) asset-pricing models are hardly ever calibrated with a coefficient of relative risk aversion less than one; 2) a ψ larger than one is both necessary to replicate the procyclicality of equity prices and concurs with recent microeconomic studies. Looking at Equation (10), κ is negative for such a parametrization, leading the term $(1-\kappa)\sigma_\omega^2$ to be positive and increasing in the signal quality, and the term $(1-\kappa+\gamma)\sigma_{\omega,c}$ to be decreasing. Consequently an analysis of the relative magnitude of the two terms is necessary. It turns out that, over a large span of possible parameterizations, the two second moments are almost identical, while the coefficient is always higher by γ for the covariance term. Thus the influence of the last term is stronger, and the impact of information quality on the required return to equity will be determined by the sign of $\sigma_{\omega,c}$. The outcome is that the required equity premium is decreasing in the quality of information available on the state of the economy.

The result for $\psi > 1$ is important but maybe not that surprising, since $\psi > 1$ is the condition for procyclical prices.[12] The more surprising result is that we obtain the same effect in an even more clear cut manner for some realistic constellations with $\psi < 1$. As we argue above, the variance of returns is always increasing in the signal quality while the covariance between returns and consumption innovations goes to zero. For $\psi < 1$, the covariance term is negative, so when it goes to zero, it actually increases. This means that both derivatives in Equation (11) are positive, and that the required risk premium will be unambiguously decreasing in the signal quality if their coefficients are both negative. For ψ in the range 0.5 to 1.0, this will be the case as long as investors are sufficiently risk averse. Specifically, we need $\gamma > 1/(2\psi - 1)$. In contrast, if investors have a sufficiently low degree of risk aversion that $\gamma \leq 1/\psi$ (i.e. they have a preference for late resolution

[11]In Table 4, these columns are set off in a bold typeface.

[12]In fact, Veronesi had conjectured that this result in a set of lecture notes he has made available to us.

of uncertainty), both coefficients in Equation (11) turn positive, pushing us back to a situation where investors require higher excess returns if they can access to better information. Notice that the power utility function forms the borderline case where $\gamma = 1/\psi$. For intermediate levels of risk aversion, the effects of the variance and the covariance term are of opposite signs, and the total effect cannot be derived from Equation (11) alone. If γ is close to the lower bound of this interval, the variance term will dominate; if γ is close to the upper bound, the covariance term will dominate and the risk premium will be decreasing in the signal quality.

If $\psi < 0.5$, i.e. if agents are very averse to substituting consumption over time, there is no case where all terms of Equation (11) are negative. However, as long as investors have a preference for early resolution, we are in the ambiguous case where the relative sizes of the two terms determine the total impact. For high levels of risk aversion, such as the one we use in our baseline calibration, we maintain a negative relation between the quality of information and required equity premium, even for very low levels of the EIS. For example, with a $\gamma = 25$, the required equity premium is decreasing in the signal quality even if the EIS is as low as 0.075.

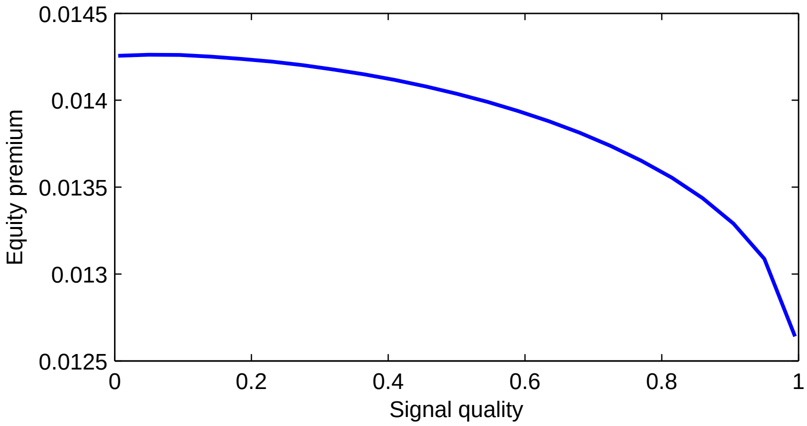
5 Numerical analysis

In this section we underscore and quantify the qualitative results from Section 4. Our first result is that less precise signals increase the equity premium and derives directly from Proposition 4.3. Using the benchmark parametrization described in Section 3, we use numerical integration to calculate the equity premium with different levels of signal quality. Figure 6 shows that the more precise the signal, the lower the required equity premium. The relative increase in the equity risk premium from a perfect signal ($h = 1$) to a signal which is pure noise ($h = 0$) is significant and equal to 11.3%. The result concurs with the analysis on the equity premium provided in Section 4, since both relevant parameters, γ and ψ, are greater than 1.

Using the same analysis, we can investigate the influence of the EIS on the relation between signal quality and equity premium. Figure 7 shows the results of a sensitivity analysis with respect to the EIS. The upper left panel displays the case of a power utility investor, illustrating the monotonous positive relation between signal quality and equity premium (i.e. better signal quality increases the required risk premium). In the other three panels, we

Figure 6: Baseline: signal quality and expected risk premium

Plotted is the relationship between signal quality and the implied equity premium for our baseline calibration ($\gamma = 25$, $\psi = 1.5$, and $\beta = 0.9925$.)

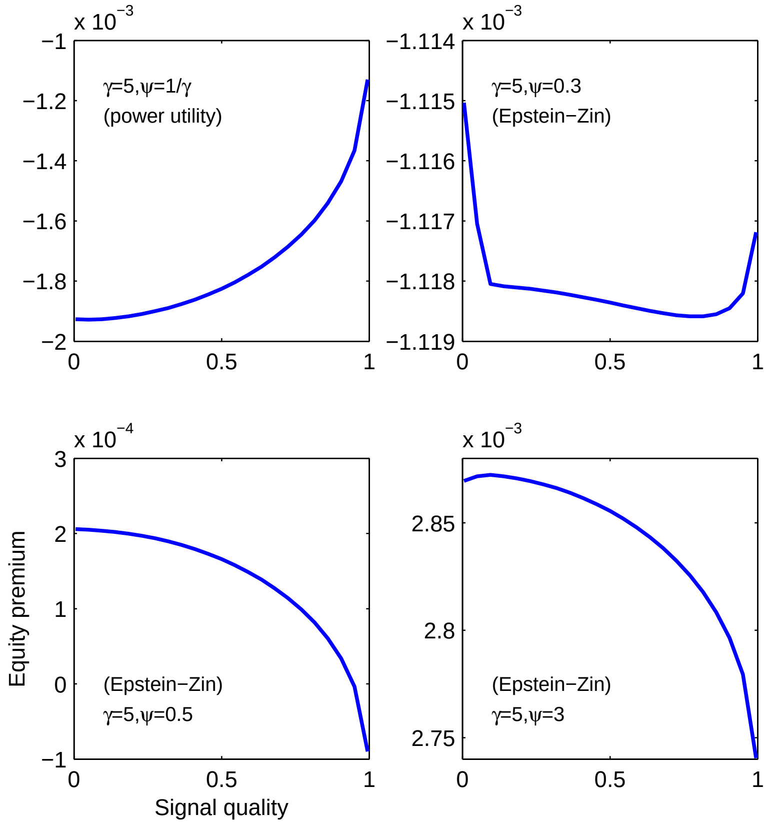


trace out the same relation for various EIS parameters. Clearly the relation is not always unambiguous, even showing cases with a non linear relation (upper right panel).

As pointed out in Proposition 4.2, an increase in the ψ parameter increases the required equity premium regardless the signal quality (see the two lower panels). Moreover, the interplay between the RA and the EIS that determines the coefficients of the relevant second moments for the equity premium, allows the model to reverse the relation between signal quality and equity premium, even with values of ψ less than 1 (lower left panel). As noted in the Introduction, the other key counterintuitive prediction of the power utility setup is that, when signals are noisy, the equity premium is bounded above in the parameter γ. Figure 8 contains a replication of Panel B of Figure 2 from Veronesi (2000). This is the solid line, displaying a well defined global maximum. Such a maximum makes it even more difficult to solve the equity premium puzzle of Mehra and Prescott (1985), since increasing the risk aversion parameter would not do any good beyond this point. As a means of comparison, Figure 8 also plots the equity risk premium with Epstein-Zin preferences. In this setup, increasing the RA parameter does not affect the EIS and hence leaves the cyclicality of returns largely unaffected. The result is that for most values of ψ an almost linear and increasing rela-

Figure 7: Sensitivity analysis: variations in the EIS

This figure shows how the relation between signal quality and predicted equity premium varies with the EIS parameter.

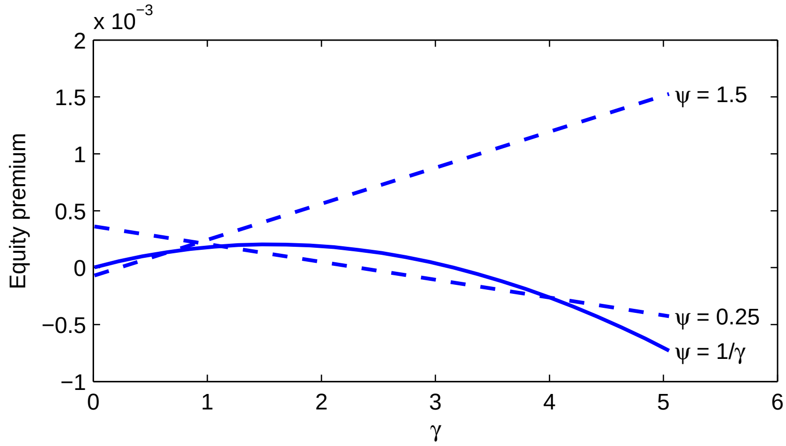


tion between equity risk premium and risk aversion parameter obtains. Only for very small values of ψ is this relation reversed. For such small values, returns become negatively correlated with the pricing kernel. Thus, the more risk averse investors are, the more return they are willing to give up to hold such a claim.

A striking feature of Figure 8 is that, in general, the equity premium is not zero even when investors have a risk aversion parameter of 0. The exception is the curve graphing the power utility case where the required

Figure 8: Required equity premia and risk aversion

The figure shows how the required equity premium responds to variations in agents' RA for different assumptions on their EIS.



equity premium at $\gamma = 0$ is exactly zero. Mathematically we can see that this holds by noting that Equation (10) simplifies to $\gamma(\sigma_c^2 + \sigma_{w,c})$. No matter what the values of the second moments, a risk neutral agent would not require any risk premium. Again the Epstein-Zin allows for richer dynamics. For $\gamma = 0$, Equation (10) simplifies to $-\frac{1}{\psi-1}(\sigma_w^2 + \sigma_{w,c})$. Thus, if ψ is greater than one, the required equity premium will be negative.

It is somewhat puzzling that risk neutral investors should require a premium (or be willing to pay a discount) to hold equity. Again, this has to do with the peculiarities of the Epstein-Zin preferences. The pricing kernel of a power utility maximizer reduces to a constant if he is risk neutral. In contrast, the pricing kernel of an investor with Epstein-Zin preferences is given by $\beta^{\psi/(1-\psi)}((1 + w_{t+1})/w_t)^{1/(\psi-1)}$, which will fluctuate with the state of the economy. It is straightforward to show that this expression is always procyclical.[13] Equity will command a premium if its returns are concentrated in periods where the kernel is low; or, in this situation, if its returns are countercyclical. Factorizing the gross return to equity into the gross dividend growth rate, c_{t+1}/c_t, and the gross return to equity net of dividends,

[13] For $\psi < 1$, $((1+w_{t+1})/w_t)$ is countercyclical, but the exponential $1/(\psi-1)$ is negative, so the whole expression becomes procyclical; for $\psi > 1$, both terms are of opposite sign, again making the whole expression procyclical.

Table 5: Equity premium with E-Z Preferences

This table reports the unconditional risk premium calculated for a range of RA (γ) and EIS (ψ) parameters. (The discount factor is fixed at 0.9925 and the signal quality is kept at a completely noisy value.)

	ψ				
γ	0.75	1.20	1.75	2.50	3.00
0.50	0.00	0.01	0.01	0.01	0.01
2.00	0.03	0.05	0.06	0.07	0.07
4.00	0.08	0.11	0.13	0.14	0.14
10.00	0.20	0.28	0.32	0.35	0.36
25.00	0.51	0.69	0.79	0.86	0.89
50.00	—	—	1.58	1.70	1.75
75.00	—	—	—	2.50	2.57
100.00	—	—	—	—	3.30

$((1+w_{t+1})/w_t)$, we know that the former will always be procyclical, while the cyclicality of the latter will depend on the EIS parameter. For returns to be countercyclical, we need prices to be sufficiently countercyclical to compensate the procyclicality of dividends. This requires a ψ which is substantially below 1. Given the moments in our consumption data, and our calibrated value for the time discount factor, we find a threshold value of around 0.42. Risk neutral investors with a lower EIS will require a positive premium to hold equity; risk neutral investors with a higher EIS will be willing to accept an expected return to equity which is lower than the risk-free rate.

Finally, we calculate equity premia for different parameterizations of the Epstein-Zin utility function. Table 5 illustrates how variations in investors' RA and EIS result in variations in the predicted equity premia. It clearly displays an increasing relation between γ and the unconditional risk premium. As argued above, an increase of the elasticity of intertemporal substitution increase the unconditional risk premium by making returns more procyclical.

6 Conclusion

This paper focuses on the implications of changes in the quality of information on asset prices in a pure exchange economy. Matching empirical figures with model predictions in such a setting has been a challenging aim since the seminal contribution of Mehra and Prescott (1985). When variations in information quality are introduced, the model predictions become even more puzzling. Veronesi (2000) has shown that if investors investors maximize a power utility function, the required risk premium is increasing in the quality of information. He also shows that, in this case, there is a strict and small upper bound for the attainable equity premium.

By allowing agents to have a general recursive utility function, we show cases where the relation between information quality and required equity premia is reversed with respect to previous literature, even with moderate values of elasticity of intertemporal substitution (i.e. $\psi < 1$). Using a realistic parametrization of the model economy, we obtain an equity premium which is decreasing in the quality of information and has no local maximum on the equity premium itself.

When the EIS is less than one, the interplay between the utility's parameters is switching signs of the relevant second moments, allowing the model to predict an equity premium which is decreasing in the signal quality. When the EIS is greater than one, the results are mainly driven by the capability of the model to replicate the procyclicality of prices over the business cycles. The degree to which this procyclicality translates into a positive covariance between consumption and returns, and hence high risk premia, depends on the quality of the signals available to investors. The better the external information available, the less prices will be driven by the information embedded in consumption growth rates, and the smaller the covariance will be.

A Proofs and derivations

Proof of proposition 4.1. Using the expression for the stochastic discount factor given by Equation ((6)), it follows that the Euler equation for the claim to aggregate consumption is given by

$$\mathrm{E}_t \left[\beta^{\kappa} \left(\frac{c_{t+1}}{c_t} \right)^{-\frac{\kappa}{\psi}} R_{c,t+1}^{\kappa} \right] = 1 \tag{13}$$

Substituting for $R_{c,t+1}$ in Equation ((13)) using the definition $R_{c,t+1} \equiv (c_{t+1} + c_{t+1} w_{t+1})/(c_t w_t)$, and multiplying both sides of the equation by w_t^{κ} (which is known at time t):

$$w_t^{\kappa} = \mathrm{E} \left[\beta^{\kappa} \left(\frac{c_{t+1}}{c_t} \right)^{\kappa(1-\frac{1}{\psi})} (1 + w_{t+1})^{\kappa} \right] \tag{14}$$

From the definition of κ, it follows that the exponential term on consumption growth is equal to $(1-\gamma)$. The solution for w_t is found by solving forward

$$\begin{aligned} w_t^{\kappa} &= \mathrm{E}_t \left[\beta^{\kappa} \left(\frac{c_{t+1}}{c_t} \right)^{\kappa(1-\frac{1}{\psi})} \left(1 + \mathrm{E}_{t+1} \left[\beta^{\kappa} \left(\frac{c_{t+2}}{c_{t+1}} \right)^{\kappa(1-\frac{1}{\psi})} (1 + w_{t+2})^{\kappa} \right]^{\frac{1}{\kappa}} \right)^{\kappa} \right] \\ &= \mathrm{E}_t \left[\beta^{\kappa} \left(\frac{c_{t+1}}{c_t} \right)^{\kappa(1-\frac{1}{\psi})} \left(1 + \beta \left(\frac{c_{t+2}}{c_{t+1}} \right)^{1-\frac{1}{\psi}} + \beta^2 \left(\frac{c_{t+3}}{c_{t+1}} \right)^{1-\frac{1}{\psi}} + \cdots \right)^{\kappa} \right] \\ &= \mathrm{E}_t \left[\left(\beta \left(\frac{c_{t+1}}{c_t} \right)^{1-\frac{1}{\psi}} + \beta^2 \left(\frac{c_{t+2}}{c_t} \right)^{1-\frac{1}{\psi}} + \cdots \right)^{\kappa} \right] \end{aligned} \tag{15}$$

Applying the law of iterated expectations to Equation ((15)), it follows that:

$$w_t^{\kappa} = \sum_{j=1}^{n} \xi_{t|t}(j)\, \mathrm{E}_t \left[\left(\beta \left(\frac{c_{t+1}}{c_t} \right)^{1-\frac{1}{\psi}} + \beta^2 \left(\frac{c_{t+2}}{c_t} \right)^{1-\frac{1}{\psi}} + \dots \right)^{\kappa} \middle| s_t = j \right] \tag{16}$$

Since w_j^{κ} is defined by the expectations term

$$w_j^\kappa = \mathrm{E}_t \left[\left(\beta \left(\frac{c_{t+1}}{c_t} \right)^{\kappa\left(1-\frac{1}{\psi}\right)} + \beta^2 \left(\frac{c_{t+2}}{c_t} \right)^{\kappa\left(1-\frac{1}{\psi}\right)} + \dots \right)^\kappa \middle| s_t = j \right], \qquad (17)$$

the proposition follows directly. □

Proof of proposition 4.2. We start from the simplified case of $\theta_1 = \theta_2 = \frac{1}{2}$. The price-dividend ratio in states 1 and 2, denoted by $w_1 = w_2 + \Delta w$ and w_2, respectively, are given as by the following system of implicit functions:

$$\begin{aligned}
F_1 =& (w_2 + \Delta w)^\kappa - \theta_1(w_2 + \Delta w + 1)^\kappa E_1 - (1 - \theta_1)(w_2 + 1)^\kappa E_2 = 0 \\
F_2 =& (w_2)^\kappa - (1 - \theta_2)(w_2 + \Delta w + 1)^\kappa E_1 - \theta_2(w_2 + 1)^\kappa E_2 = 0,
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
E_1 &= \mathrm{E}_t \left[\beta^\kappa G_{t+1}^{1-\gamma} \mid s_t = 1\right] \\
E_2 &= \mathrm{E}_t \left[\beta^\kappa G_{t+1}^{1-\gamma} \mid s_t = 2\right].
\end{aligned}$$

We can define

$$\begin{aligned}
J =& \begin{bmatrix} \frac{\partial F_1}{\partial \theta_1} & \frac{\partial F_1}{\partial \theta_2} \\ \frac{\partial F_2}{\partial \theta_1} & \frac{\partial F_2}{\partial \theta_2} \end{bmatrix} \\
=& \begin{bmatrix} -(w_2 + \Delta w + 1)^\kappa E_1 + (w_2 + 1)^\kappa E_2 & 0 \\ 0 & +(w_2 + \Delta w + 1)^\kappa E_1 - (w_2 + 1)^\kappa E_2 \end{bmatrix}
\end{aligned}$$

By applying Cramer's rule we get: $\frac{\partial \Delta w}{\partial \theta_1} = \frac{|J_{\Delta w}|}{|J|}$
where

$$\begin{aligned}
J_{\Delta w} =& \begin{bmatrix} -\frac{\partial F_1}{\partial \Delta w} & \frac{\partial F_1}{\partial \theta_2} \\ -\frac{\partial F_2}{\partial \Delta w} & \frac{\partial F_2}{\partial \theta_2} \end{bmatrix} \\
=& \begin{bmatrix} -\kappa((w_2 + \Delta w)^{\kappa-1} + \kappa\theta_1((w_2 + \Delta w + 1)^{\kappa-1} E_1 & 0 \\ \kappa(1 - \theta_2)((w_2 + \Delta w + 1)^{\kappa-1} E_1 & +((w_2 + \Delta w + 1)^\kappa E_1 - (w_2 + 1)^\kappa E_2 \end{bmatrix}
\end{aligned}$$

Hence it is sufficient to investigate the sign of the determinant of $J_{\Delta w}$ in order to assess the relation between prices and states. In the same fashion we can derive the relation when θ_1 and θ_2 are bigger than $\frac{1}{2}$.
in this case the determinant of $J_{\Delta w}$ obeys

$$|J_{\Delta w}| \propto -\kappa \left((w_2 + \Delta w)^\kappa - \overbrace{\frac{w_2 + \Delta w}{w_2 + \Delta w + 1}}^{<1} \theta_1 \left(w_2 + \Delta w + 1\right)^\kappa E_1 \right)$$
$$\left(\left(w_2 + \Delta w + 1\right)^\kappa E_1 - \left(w_2 + 1\right)^\kappa E_2\right)$$

Table 6: Summary table

This table reports the relation between prices and utility parameters.

	$\gamma \in <0,1>$		$\gamma \in <1,\infty>$	
	$\psi \in <0,1>$	$\psi \in <1,\infty>$	$\psi \in <0,1>$	$\psi \in <1,\infty>$
Panel A: $\theta_1 = \theta_2 = \frac{1}{2}$				
κ	$-$	$+$	$+$	$-$
$\left(\theta_1 E_1 - \bar{E}^{(1-1/\kappa)}\right)$	$-$	$-$	$-$	$-$[1]
$E_1 - E_2$	$+$	$+$	$-$	$-$[2]
$\lvert J_{\Delta w}\rvert$	$+$	$-$	$+$	$-$
$\left\lvert \frac{\partial \Delta w}{\partial \theta_1} \right\rvert_{\theta_1=\theta_1=0.5}$	$-$	$+$	$-$	$+$
Panel B: $\theta_1, \theta_2 > \frac{1}{2}$				
κ	$-$	$+$	$+$	$-$
a_1	$-$	$+$	$-$	$+$
$(w_2+\Delta w)^\kappa - (w_2)^\kappa$	$+$	$+$	$-$	$-$
$\lvert J_{a_1}\rvert$	$+$	$-$	$+$	$-$
$\left\lvert \frac{\partial \Delta w}{\partial \theta_1} \right\rvert_{\theta_1=\theta_1=0.5}$	$-$	$+$	$-$	$+$

[1] At $\theta_1 = \theta_2 = 0.5$, $w_2 = (1-\bar{E}^{1/\kappa})^{-1}\bar{E}^{1/\kappa}$, so $\bar{E}^{1/\kappa} < 1$.
Hence, $\bar{E}^{1-1/\kappa} > \bar{E} > \theta_1 E_1$

[2] $\gamma > 1 \iff E_2 > E_1$ since $E[G_{t+1} \mid s_t = 1] > E[G_{t+1} \mid s_t = 2)]$

By noticing that $\frac{w_2+\Delta w}{w_2+\Delta w+1}\theta_1 \left(w_2+\Delta w+1\right)^\kappa E_1 < (w_2+\Delta w)^\kappa$ and that the term in the last parenthesis can be rewritten as $\frac{1}{\theta_1+\theta_2+1}\left((w_2+\Delta w)^\kappa - (w_2)^\kappa\right)$ we can asses the sign of the determinant via:

$$J_{\Delta w} \propto -\kappa \left(\left(w_2+\Delta w\right)^\kappa - \left(w_2\right)^\kappa\right)$$

Consulting Table 6 it then follows that our claim is satisfied. $\square$

Proof of proposition 4.3. Let us first introduce some useful definitions. Here and henceforth we define $g_{t+1} = \log \frac{c_{t+1}}{c_t}$ and $\omega_{t+1} = \log \frac{1+w_{t+1}}{w_t}$.

Using the expression for the stochastic discount factor given by equation (6), we can apply the Euler equation, $\mathrm{E}_t \left[M_{t+1} R_{t+1} = 1 \right]$, to both the return on the consumption claim, $R_{c,t+1}$, and to the return on a risk free claim, $R_{b,t+1}$. Assuming that consumption and asset returns are homoskedastic and jointly lognormal, we can log-linearize the two Euler equations obtaining

$$
\begin{aligned}
(R_{c,t+1}) \Rightarrow 0 = \kappa log(\beta) - \frac{\kappa}{\psi} \mathrm{E}_t \left[g_{t+1} \right] + \kappa \, \mathrm{E}_t \left[r_{e,t+1} \right] \\
+ \frac{1}{2} \left[\left(\frac{\kappa}{\psi} \right)^2 \sigma_g^2 + \kappa^2 \sigma_r^2 - \frac{2\kappa^2}{\psi} \sigma_{g,r} \right]
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
(R_{b,t+1}) \Rightarrow 0 = \kappa log(\beta) - \frac{\kappa}{\psi} \mathrm{E}_t \left[g_{t+1} \right] + (\kappa - 1) \, \mathrm{E}_t \left[r_{e,t+1} \right] + r_b \\
+ \frac{1}{2} \left[\left(\frac{\kappa}{\psi} \right)^2 \sigma_g^2 + (\kappa - 1)^2 \sigma_r^2 - \frac{2\kappa \left(\kappa - 1 \right)}{\psi} \sigma_{g,r} \right]
\end{aligned}
\tag{20}
$$

Subtracting Equation (20) from Equation (19), we get

$$
\mathrm{E}_t \left[r_{e,t+1} \right] - r_b + \frac{1}{2} \sigma_r^2 = (1 - \kappa) \, \sigma_r^2 + \frac{\kappa}{\psi} \sigma_{g,r}. \tag{21}
$$

Now we can use the definition of log returns, $r_{c,t+1} = \omega_{t+1} + g_{t+1}$, in order to calculate the second moments in Equation (21). Using the linear properties of the variance and covariance operators we obtain

$$
\begin{aligned}
\sigma_r^2 &= \sigma_\omega^2 + \sigma_g^2 + 2\sigma_{\omega,g} \\
\sigma_{g,r} &= \sigma_g^2 + \sigma_{\omega,g}
\end{aligned}
. \tag{22}
$$

Proposition 4.3 follows after substituting Equation (22) in Equation (21). □

Equation (7): We start from the definition of stochastic discount factor in Epstein and Zin (1989):

$$M_{t+1} = \beta^{\kappa} \left(\frac{c_{t+1}}{c_t} \right)^{-\frac{\kappa}{\psi}} (R_{c,t+1})^{\kappa-1} \tag{23}$$

we can rewrite it as:

$$\begin{aligned} M_{t+1} &= \beta^{\kappa} \left(\frac{c_{t+1}}{c_t} \right)^{-\frac{\kappa}{\psi}} \left(\frac{p^c_{t+1} + c_{t+1}}{c_t} \right)^{\kappa-1} \\ &= \beta^{\kappa} \left(\frac{c_{t+1}}{c_t} \right)^{-\frac{\kappa}{\psi}} \left(\frac{p^c_{t+1}}{c_{t+1}} + 1 \right)^{\kappa-1} \left(\frac{p^c_t}{c_t} \right)^{1-\kappa} \left(\frac{C_{t+1}}{C_t} \right)^{\kappa-1} \\ &= \beta^{\kappa} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\kappa}{\psi}+\kappa-1} (w_{t+1}+1)^{\kappa-1} (w_t)^{1-\kappa} \end{aligned} \tag{24}$$

with some further manipulation we get:

$$\begin{aligned} M_{t+1} &= \beta^{\kappa} \left(\frac{c_{t+1}}{c_t} \right)^{\kappa(1-\frac{1}{\psi})-1} (w_{t+1}+1)^{\kappa-1} (w_t)^{1-\kappa} \\ &= \beta^{\kappa} \left(\frac{c_{t+1}}{c_t} \right)^{\frac{1-\gamma}{1-1/\psi}(1-\frac{1}{\psi})-1} (w_{t+1}+1)^{\kappa-1} (w_t)^{1-\kappa} \\ &= \beta^{\kappa} \left(\frac{w_{t+1}}{w_t} \right)^{-\gamma} (w_{t+1}+1)^{\kappa-1} (w_t)^{1-\kappa} \end{aligned} \tag{25}$$

From Equation (25) and the definition of the gross risk free return, $R_{b,t+1} = \frac{1}{\mathbb{E}_t[M_{t+1}]}$, Equation (7) follows immediately.

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