Universität St.Gallen

Analyzing Mergers under Asymmetric
Information:
A Simple Reduced-Form Approach

Thomas Borek, Stefan Bühler, Armin Schmutzler

Department of Economics   University of St. Gallen

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Corresponding Author's address:

Prof. Dr. Stefan Bühler
Institute of Public Finance and Fiscal Law
University of St. Gallen
Varnbüelstr. 19
9000 St. Gallen
Phone   +41 71 224 2303
Fax   +41 71 224 2670
Email   Stefan.buehler@unisg.ch

Abstract

This paper provides a simple reduced-form framework for analyzing merger decisions in the presence of asymmetric information about firm types, building on Shapiro's (1986) oligopoly model with asymmetric information about marginal costs. We employ this framework to examine what types of firms are likely to be involved in mergers. While we give sufficient conditions under which only low-type firms merge, as a lemons rationale would suggest, we also argue that these conditions will often be violated in practice. Finally, our analysis shows how signaling considerations affect merger decisions.

1 Introduction

In spite of the paramount role that mergers play in the process of restructuring firms and markets,[1] a number of interesting theoretical questions have received scarce attention. For instance, little is known about the kinds of firms that are likely to be involved in a merger, even though a large body of research provides reasons why mergers might occur, including synergies, market power, market discipline, etc. (see, e.g., Andrade et al. 2001, 103). Should we expect mergers mostly between inefficient firms? Or between efficient firms? Or, more generally, how does the market environment determine what kind of firms are likely to merge?

At first glance, one might expect that mergers involve relatively inefficient firms: Since inefficient firms realize low stand-alone profits, they have little to lose from a merger. Other things equal, they should therefore be more likely to consent to a merger. This line of argument is supported by an important paper by Hansen (1987), who considers the choice of exchange medium—that is, whether a transaction should be paid using cash or stock. He argues that, with two-sided asymmetric information about firm values, a double lemons problem (Akerlof 1970) emerges:

> "With cash offers, and when the target has proprietary information on the state of its assets, a "lemons" problem arises: the target will sell only when its value is less than the offer made. [...] Allowing the acquiring firm to have proprietary information on its own value sets up a double lemons problem in that the acquiring firm will not offer stock when the target seriously underestimates the value of the offer [...]." (Hansen 1987, 76)

This paper investigates the limitations of the lemons rationale in the con-

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[1] For 2007, the Mergerstat Factset reports the following M&A activity around the world (`https://www.mergerstat.com`): U.S.: 10,574 deals with a value of $1,345.3 billion; Europe: 6,010 deals with a value of $946.4 billion; Asia: 4,072 deals with a value of $239.6 billion.

text of mergers under two-sided asymmetric information. More specifically, we provide a simple reduced-form framework for studying merger decisions in an oligopoly setting where firms may have private information about their types. Building on Shapiro's (1986) analysis of Cournot oligopoly with asymmetric information about marginal costs c_i, we develop a two-stage merger game. In stage 1, before product-market competition takes place, two out of $N \geq 2$ firms are matched, whose types $z_i = -c_i$, $i = 1, ..., N$, are private knowledge. Observing their own types and given their beliefs about the other firms' types, these two firms can either consent to a merger or reject it. A merger takes place if and only if both firms consent. The merger decisions of both firms become publicly known. In particular, when a merger fails to take place, it is known which party, if any, consented to the merger. This information can be used by all parties, including those not involved in the merger, to update their beliefs about types. In case of a merger, the merged firm's profits are distributed to the constituent firms, using stock or cash payments. The distribution of profits is the outcome of some mechanism that may depend on the types of firms. Consenting to a merger means consenting to this particular mechanism. In stage 2 of the game, the remaining firms compete under asymmetric information about marginal costs in the Cournot market.

We characterize the perfect Bayesian equilibrium of the two-stage game. In doing so, it is instructive to proceed in three steps. In a first step, we ignore asymmetric information altogether, asking merely what types of firms consent to a merger under *perfect information*. Clearly, mergers will only arise if merger returns, defined as the difference between post-merger profits and stand-alone profits, are positive for both firms. We want to emphasize that, even in this simple setting, it is less than obvious that mergers will typically involve low-type firms:

(i) Even though firm i's stand-alone profits are increasing in own type z_i, merger returns are not necessarily decreasing in own type. To see this, suppose, quite naturally, that an efficient firm i will also contribute

to higher joint profits in the event of a merger. If the owners of firm i benefit from these higher profits of the joint entity—for instance, because they hold shares in the new firm—then high types may, in fact, be more likely to consent to a merger than low types.

(ii) Even if the stand-alone profit effect dominates, so that higher types do face lower merger returns, this does not imply that mergers necessarily involve low-type firms. Rather, as we illustrate below, under a set of reasonable assumptions, firms need to be relatively similar to make sure that both firms benefit from the merger.[2]

In a second step, we allow for *asymmetric information* in stage 1 when firms take merger decisions, but we sustain our assumption of perfect information in stage 2 when firms compete in the product market. In this setting, the properties of the merger-returns function from the perfect information case translate directly into properties of the equilibrium under two-sided asymmetric information, provided that firm types are uncorrelated. For instance, if merger returns are decreasing in own type, a *two-sided lemons equilibrium* emerges, where only low types below a certain cut-off value (if any) consent to the merger.[3] If merger returns are increasing rather than decreasing, this result is reversed, and a *two-sided peaches equilibrium* emerges, where only high types above a certain cut-off value consent. We show that there may also be so-called *lemons-and-peaches equilibria* where low-type firms merge with high-type firms.[4]

In a third step, we relax all simplifying informational assumptions, allowing for *asymmetric information in both stages*. In this setting, merger

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[2]Another caveat to the idea that inefficient firms are most likely to merge has been explored in the context of vertical mergers. Buehler and Schmutzler (2005) show that, under quite natural assumptions, more efficient firms are more likely to integrate vertically.

[3]Note, however, that this structure may break down with (positively) correlated types: If high types believe that the other firms are also likely to be of a high type, then they will be more likely to consent to a merger if they expect higher profits in the event of a merger or lower profits in the absence of a merger.

[4]Again, these equilibrium structures may break down if types are correlated.

decisions carry an informational value which depends on the qualitative properties of the strategies. When firms are expected to play cut-off strategies with only low types consenting to a merger, then *signaling* considerations make parties more reluctant to consent to a merger: In the event that the other party rejects the merger, then having consented will signal a low type to competitors. They will therefore expect the consenting firm to produce a low output in the product market and act more aggressively (i.e., choose higher outputs) themselves. To avoid this unfavorable outcome, at least types who are almost indifferent between merging and not merging in the absence of signaling may want to refrain from consenting. In contrast, if firms are expected to play cut-off strategies with only high types (rather than low types) consenting to a merger, the previous signaling result is reversed: Then firms may announce a merger to create the impression of being a high type.

This paper contributes to the literature on mergers under asymmetric information. Our analysis is perhaps most closely related to Hviid and Prendergast (1993), who study the signaling effect of an unsuccessful merger proposal in a Cournot duopoly, assuming both one-sided asymmetric information about the target's type and take-it-or-leave-it offers. These authors argue that the information transmission effect of merger rejection extends to the case of two-sided asymmetric information, but they do not provide a formal analysis. Our paper extends the analysis to an oligopoly setting with $N \geq 2$ firms and two-sided asymmetric information, abstracting from the details of the negotiation process. A key difference to Hviid and Prendergast (1993) is our finding that consenting to a rejected merger does not necessarily convey bad news about the profitability of the consenting firm.

A related strand of the literature analyzes mergers from a mechanism design perspective. The bulk of this literature focuses on revenue-maximizing mechanisms, assuming that the stand-alone profits of firms are known.[5] Notable exceptions are Brusco et al. (2007) and Gärtner and Schmutzler (2006),

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[5]Important contributions include Hansen (1985), Crémer (1987), Samuelson (1987), Rhodes-Kropf and Viswanathan (2000), and DeMarzo et al. (2005).

who allow for private information about stand-alone profits. An important insight of these papers is that, with private information about stand-alone profits, it is impossible to find either efficient or regret-free merger mechanisms, except for very restrictive settings.[6] It thus seems natural to assume that in practice merger decisions are not necessarily based on efficient merger mechanisms. Our reduced-form setting accounts for this by allowing for arbitrary (potentially inefficient) mechanisms determining the profit sharing rule and transfer payments. We deliberately chose the simplest possible setting where merger negotiations are not modeled explicitly to increase the transparency of our analysis. Without having to worry about the details of the negotiation process, we can identify what determines the set of merging firms for general classes of merger mechanisms.

The remainder of the paper is organized as follows. In Section 2, we introduce the analytical framework. Section 3 analyzes the full information benchmark. Section 4 considers the case with asymmetric information in stage 1 and full revelation in stage 2. Section 5 examines the signaling case with asymmetric information about firm types in both stages. Section 6 discusses some limitations and extensions. Section 7 concludes.

2   Analytical Framework

We consider an adapted version of Shapiro's (1986) Cournot oligopoly model with private information about marginal costs. The key difference to Shapiro is that, in our model, endogenous merger decisions precede product market competition. More specifically, we consider a two-stage game where, initially, there are $N \geq 2$ suppliers of a homogenous good. Each firm $i = 1, ..., N$ is characterized by its privately known type $z_i \in [\underline{z}_i, \overline{z}_i]$. Throughout the

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[6]Efficient mechanisms ensure that mergers occur if and only if the profits of the merged entity exceed the sum of profits of the merging firms ex post. Regret-free mechanisms guarantee for each of the merging firms that, ex post, their profits exceed their stand-alone profits.

analysis, we suppose that the type of firm i is the negative of marginal cost, i.e., $z_i \equiv -c_i$. We assume that firm types $\mathbf{z} = (z_1, ..., z_N)$ are distributed according to a commonly known cumulative distribution function $F(\mathbf{z})$. We denote the derived distribution of the other firms' types as $F_i(\mathbf{z}_{-i} | z_i)$. This distribution reflects firm i's interim beliefs about the types of competitors, after revelation of its own type, but before merger decisions have been made.

Even though this specific model is helpful for concreteness, the following considerations also pertain to more general oligopoly models. Except for the arguments in Section 5, which rely on the strategic-substitutes property of the Cournot model, nothing of substance depends on the particular formulation.

2.1 Stage 1: Merger Decisions

In stage 1, two firms denoted by $i = 1, 2$ are matched to decide about a possible merger. Having observed their own type z_i and given their belief $F_i(\mathbf{z}_{-i} | z_i)$ about the types of the other firms, $\mathbf{z}_{-i}$, they simultaneously announce whether they are willing to merge.[7] The decision of firm i is represented by a variable s_i such that $s_i = 1$ if it consents to the merger and $s_i = 0$ if it rejects it. The firms' merger decisions are summarized by the vector $\mathbf{s} = (s_1, s_2)$, which is publicly observable. We impose the following natural assumption on the type of the merged entity:

Assumption 1 *The type of the merged entity is a non-decreasing function $z_M(z_1, z_2)$ of the types of the constituent parts.*

The function $z_M(\cdot)$ reflects the relevant merger technology. We do not specify its properties except that we suppose that a merged entity is (weakly) more efficient the more efficient its constituent parts. Further, we shall assume that, if a merger occurs, the merged firm indexed by M not only knows

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[7]Allowing for endogenous matching and merger negotiations does not affect the qualitative results of our analysis, provided that firms are unable to fully reveal their types before merger decisions (see Section 6).

its own type z_M, but also the types of the constituent firms (z_1, z_2).[8]

2.2 Stage 2: Product Market Competition

Denote the number of firms remaining after merger decisions have been made as n. In stage 2, these firms compete à la Cournot in the product market. Let $p(X)$ denote the inverse demand function, where $X = \sum_{i=1}^{n} x_i$ is the sum of individual outputs. Then, if the output vector is given by $\mathbf{x} = (x_1, ..., x_n)$, firm i's profit is

$$\pi_i(z_i, \mathbf{x}) = (p(X) + z_i)x_i. \tag{1}$$

When firms take their output decisions, they know their own type z_i and the merger decisions $\mathbf{s}$ from stage 1, such that firm i's strategy in stage 2 is a function $x_i = x_i(z_i, \mathbf{s})$. Conditional on the available information, each firm forms beliefs $F_i(\mathbf{z}_{-i}| z_i; \mathbf{s})$. At this stage, we do not yet specify how beliefs are derived. The expected stage-2 profits of a firm i that is not part of a merged entity, given the merger announcements $\mathbf{s}$ from stage 1 and its output x_i, are

$$\int_{\mathbf{z}_{-i}} \pi_i(z_i, x_i, \mathbf{x}_{-i}(\mathbf{z}_{-i}, \mathbf{s}))dF_i(\mathbf{z}_{-i}| z_i; \mathbf{s}), \tag{2}$$

where $\mathbf{x}_{-i}(\mathbf{z}_{-i}, \mathbf{s}) \equiv (x_1(z_1, \mathbf{s}), ..., x_{i-1}(z_{i-1}, \mathbf{s}), x_{i+1}(z_{i+1}, \mathbf{s}), ..., x_n(z_n, \mathbf{s}))$ denotes the vector of the other firms' outputs as a function of their types $\mathbf{z}_{-i}$ and the announcements $\mathbf{s}$ from stage 1. Moreover, recall that we assumed that the merged entity $i = M$ not only knows its type z_M, but also the types of both constituent firms (z_1, z_2). Thus, the expected stage-2 profits of the merged entity, given the merger announcements $\mathbf{s} = (1,1)$, are

$$\int_{\mathbf{z}_{-M}} \pi_M(z_M, x_M, \mathbf{x}_{-M}(\mathbf{z}_{-M}, (1,1)))dF_M(\mathbf{z}_{-M}| z_1, z_2). \tag{3}$$

We write $x_i^*(z_i, \mathbf{s})$ for $i \neq M$ and $x_M^*(z_1, z_2)$ to denote the Cournot equilibrium outputs corresponding to beliefs $F_i(\mathbf{z}_{-i}| z_i; \mathbf{s})$ for $i \neq M$ and

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[8]This assumption will turn out to be important for the implementation of the profit-sharing rule.

$F_M(\mathbf{z}_{-M}|z_1, z_2)$.[9] These outputs satisfy the sequential rationality conditions

$$x_i^*(z_i, \mathbf{s}) \quad \in \quad \arg\max \int_{\mathbf{z}_{-i}} \pi_i(z_i, x_i, \mathbf{x}_{-i}^*(\mathbf{z}_{-i}, \mathbf{s}))dF_i(\mathbf{z}_{-i}|z_i; \mathbf{s}), \text{ for all } i \neq M,$$

$$x_M^*(z_1, z_2) \quad \in \quad \arg\max \int_{\mathbf{z}_{-M}} \pi_M(z_M, x_M, \mathbf{x}_{-M}^*(\mathbf{z}_{-M}))dF_M(\mathbf{z}_{-M}|z_1, z_2),$$

where the equilibrium outputs of firms not part of the merged entity are denoted as $\mathbf{x}_{-M}^*(\mathbf{z}_{-M})$. For future reference, it is useful to introduce the following notation:

Definition 1 (stage-2 profits) *The expected equilibrium profits of*

(i) a firm i that is not part of a merged entity are given by

$$\Pi_i(z_i; \mathbf{s}) = \int_{\mathbf{z}_{-i}} \pi_i(z_i, x_i^*(z_i, \mathbf{s}), \mathbf{x}_{-i}^*(\mathbf{z}_{-i}, \mathbf{s}))dF_i(\mathbf{z}_{-i}|z_i; \mathbf{s}); \qquad (4)$$

(ii) the merged firm M are given by

$$\Pi^M(z_1, z_2) = \qquad\qquad\qquad (5)$$

$$\int_{\mathbf{z}_{-M}} \pi_M(z_M(z_1, z_2), x_M^*(z_1, z_2), \mathbf{x}_{-M}^*(\mathbf{z}_{-M}))dF_M(\mathbf{z}_{-M}|z_1, z_2);$$

(iii) a firm i participating in the merged firm M are given by

$$\Pi_i^M(z_1, z_2) = \lambda_i(z_1, z_2) \cdot \Pi^M(z_1, z_2) + t_i(z_1, z_2) \qquad (6)$$

where $\lambda_i(z_1, z_2) \in [0, 1]$ denotes firm i's share of the merged firm's profits and $t_i(z_1, z_2)$ indicates a cash transfer received ($t_i \geq 0$) or paid ($t_i < 0$) by firm i.

Note that, according to Definition 1, expected stage-2 profits are denoted by $\Pi_i(\cdot)$, whereas ex-post profits are denoted by $\pi_i(\cdot)$. Part (iii) of Definition 1 needs some explanation. While both the expected stand-alone profits

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[9]For notational ease, we suppress the dependence of outputs on beliefs in $x_i^*(z_i, \mathbf{s})$ and $x_M^*(z_1, z_2)$.

Π_i and the expected profits of the merged entity Π^M depend exclusively on the properties of product market competition and the beliefs about the competitors' types, the profits of the constituent firms Π_i^M in the event of a merger also depend on the profit sharing rule λ_i, which splits the merged entity's profits into the profit shares of the constituent firms, and possible cash payments t_i between the constituent firms.

Our reduced-form specification is general enough to account for several possibilities. In the simplest case, the merger decision might assign a fixed share to each firm, irrespective of types, and abstract from cash payments. More generally, however, the combination of λ_i and t_i could be interpreted as the outcome of an arbitrary mechanism assigning profits to the constituent firms.[10] In any case, consenting to a merger also means consenting to a combination of λ_i and t_i.

2.3 Perfect Bayesian Equilibrium

We now characterize the properties that a set of strategies $s_i(z_i)$, $x_i(z_i, \mathbf{s})$, and $x_M(z_1, z_2)$, as well as beliefs $F_i(\mathbf{z}_{-i}| z_i; \mathbf{s})$ and $F_M(\mathbf{z}_{-M}| z_1, z_2)$ must satisfy to be part of an equilibrium. Let $B_i \equiv B_i(s_i) \equiv \{z_i | s_i(z_i) = 1\}$ denote the set of types z_i for which firm i consents to a merger. Further, let $\mathbb{P}[B_j| z_i]$ denote the probability that player j consents to a merger. Then, the *expected merger returns* for firm i with type z_i, when its belief about the distribution of z_j is given by $F_i(z_j| z_i)$ and only types $z_j \in B_j$ consent to a merger, is

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[10]See Brusco et al. (forthcoming) for a recent study of the efficiency of various merger mechanisms.

given by

$$G_i(z_i; B_j, F_i) \equiv \tag{7}$$

$$\mathbb{P}[B_j | z_i] \int_{z_j \in B_j} \Pi_i^M(z_i, z_j) dF_i(z_j | z_i)$$

$$+ (1 - \mathbb{P}[B_j | z_i]) \int_{z_j \notin B_j} \Pi_i(z_i; (1, 0)) dF_i(z_j | z_i)$$

$$- \mathbb{P}[B_j | z_i] \int_{z_j \in B_j} \Pi_i(z_i; (0, 1)) dF_i(z_j | z_i)$$

$$- (1 - \mathbb{P}[B_j | z_i]) \int_{z_j \notin B_j} \Pi_i(z_i; (0, 0)) dF_i(z_j | z_i).$$

Intuitively, (7) states that the expected returns to consenting to a merger are given by the difference between the expected profits when consenting (the first two terms) and rejecting the merger (the last two terms). The matched firm j accepts the merger with probability $\mathbb{P}[B_j | z_i]$ and rejects it with probability $1 - \mathbb{P}[B_j | z_i]$, giving rise to four different possible outcomes with (generally) different expected profits. It is important to note that all but the first term correspond to profits in the absence of a merger; they differ only with respect to the firm (if any) that consented to the merger. Differences between the integrals in the second, third and fourth term are thus exclusively the result of differences in updated beliefs about types resulting from different merger announcements.

The expected merger returns given in (7) are well-defined no matter how players arrive at the beliefs used for calculating Π_i and Π_i^M and $\mathbb{P}[B_j | z_i]$. However, we shall think of them as obeying standard consistency requirements as follows.

Definition 2 (PBE) *A Perfect Bayesian Equilibrium of the Merger Game is a set of strategies $s_i^*(z_i)$, $x_i^*(z_i, \mathbf{s})$, and $x_M^*(z_1, z_2)$, with $i \neq M$, as well as posterior beliefs $F_i(\mathbf{z}_{-i} | z_i; \mathbf{s})$ and $F_M(\mathbf{z}_{-M} | z_1, z_2)$, such that,*

    (i) outputs $x_i^*(z_i, \mathbf{s})$ and $x_M^*(z_1, z_2)$ in stage 2 are given by the Cournot equilibrium quantities corresponding to posterior beliefs $F_i(\mathbf{z}_{-i} | z_i; \mathbf{s})$ and $F_M(\mathbf{z}_{-M} | z_1, z_2)$.

(ii) merger decisions $s_i^*(z_i)$ in stage 1 are optimal for the given posterior beliefs and the corresponding outputs, i.e.,

$$s_i^*(z_i) = \begin{cases} 1, & \text{if } G_i(z_i; B_j, F_i) \geq 0 \\ 0, & \text{if } G_i(z_i; B_j, F_i) < 0 \end{cases} , \quad i = 1, 2. \tag{8}$$

(iii) posterior beliefs $F_i(\mathbf{z}_{-i}| z_i; \mathbf{s})$ and $F_M(\mathbf{z}_{-M}| z_1, z_2)$ are derived from the prior distribution $F_i(\mathbf{z}_{-i}| z_i)$ and equilibrium strategies using Bayes' rule (when applicable). The probabilities $\mathbb{P}[B_j| z_i]$ used to calculate $G_i(z_i; B_j, F_i)$ are derived from the requirement that player i has interim beliefs $F_i(z_j| z_i)$ and expects each type j to follow his strategy $s_j(z_j)$.

We now characterize the properties of the equilibrium step-by-step, starting with two special cases.

3   Full Information

In a first step, we consider the full information benchmark where all types are common knowledge. In this case, the equilibrium profits of a firm i not participating in the merged entity can be written as $\pi_i(z_i, \mathbf{z}_{-i})$, whereas the profits of the merged entity can be written as $\pi^M(z_M, \mathbf{z}_{-M})$. Hence, the profits of firm i participating in a merger are given by

$$\pi_i^M(z_i, \mathbf{z}_{-i}) = \lambda_i(z_1, z_2)\pi^M(z_M(z_1, z_2), \mathbf{z}_{-M}) + t_i(z_1, z_2). \tag{9}$$

We introduce the following definition of merger returns:

Definition 3 (merger returns) *Under full information, firm i's merger returns are given by*

$$g_i(z_i, \mathbf{z}_{-i}) \equiv \pi_i^M(z_i, \mathbf{z}_{-i}) - \pi_i(z_i, \mathbf{z}_{-i}). \tag{10}$$

13

Using (10), differentiating merger returns with respect to own type yields

$$
\begin{aligned}
\frac{\partial g_i(z_i, \mathbf{z}_{-i})}{\partial z_i} &= \frac{\partial \pi_i^M(z_i, \mathbf{z}_{-i})}{\partial z_i} - \frac{\partial \pi_i(z_i, \mathbf{z}_{-i})}{\partial z_i} \\
&= \left(\frac{\partial \lambda_i}{\partial z_i} \pi^M + \lambda_i \frac{\partial \pi^M}{\partial z_i} + \frac{\partial t_i}{\partial z_i} \right) - \frac{\partial \pi_i}{\partial z_i}.
\end{aligned}
\tag{11}
$$

Inspection of (11) indicates that a marginal increase in own type affects firm i's merger returns through four quantities:

(i) the profit share λ_i,

(ii) the profits of the merged entity π^M,

(iii) the cash transfer t_i, and

(iv) the stand-alone profits π_i.

By definition, the partial derivative of stand-alone profits π_i with respect to own type is non-negative: In the Cournot model firms with lower marginal costs earn higher profits. At first glance, this suggests a lemons problem: As better firms have more to lose from consenting to a merger, they are reluctant to consent. Yet, taking the remaining effects into account, this observation is not sufficient to guarantee that merger returns are non-increasing in own type. Because of Assumption 1 and the fact that the profits of the merged firm M must be increasing in z_M, *total* merger profits π^M are non-decreasing in own type. It is quite natural to expect that the mechanism, as captured by λ_i and t_i, rewards high-type firms by letting them benefit from the increase in π^M that they generate, so that π_i^M should be increasing in z_i. If so, it is not clear that merger returns are decreasing in types: While high types forego higher stand-alone profits than low types, they may also earn greater profits as part of the merged entity.

It is easy to understand, however, under which circumstances the effect on stand-alone profits dominates, so that merger returns are decreasing in own type. For instance, if neither the type of the merged entity nor the

14

components of the mechanism, λ_i and t_i, depend substantially on types, then merger returns will be decreasing in own type.

Finally, note that, in the subgame-perfect equilibrium of the perfect information game, merger decisions $\hat{s}_i$ satisfy the following condition:

$$\hat{s}_i(\mathbf{z}) = \begin{cases} 1, & \text{if } g_i(z_i, \mathbf{z}_{-i}) \geq 0 \\ 0, & \text{if } g_i(z_i, \mathbf{z}_{-i}) < 0 \end{cases}, \quad i = 1, 2. \tag{12}$$

That is, firms consent to a merger if and only if their merger returns are (weakly) positive. It is well known that, in the linear Cournot model, merger returns can only be positive for both firms (i.e., $g_i(\cdot) \geq 0, i = 1, 2$) if the merger leads to a sufficiently strong rationalization effect (see Salant et al. 1983 and Barros 1998) or to synergies (Farrell and Shapiro 1990). In our setting, this implies that a merger will only occur if $z_M(z_1, z_2)$ is sufficiently large. Otherwise, there will be no merger in the SPE.

Under full information, the properties of the merger-returns function translate straightforwardly into properties of the set of merging types. In the Appendix, we use this to illustrate that, even if, in spite of our earlier reservations, merger returns are decreasing in own types, so that low types are more eager to participate in mergers, it is by no means clear that mergers involve low types. Rather, under a set of reasonable assumptions, firms need to be relatively similar to make sure that both firms benefit from the merger.

This leads to the second important caveat to the idea that inefficient firms are most likely to merge. Even in settings where inefficient firms have the strongest incentives to join, under full information, this does not necessarily lead to equilibria where only the least efficient firms merge. The equilibrium requirement that both parties must consent may imply, for instance, that equilibria involve relatively similar firms, irrespective of their efficiency.

4   Full Revelation in Stage 2

Let us now assume that firm types are revealed *after* merger decisions, but *before* firms compete in the product market. This has two immediate implications:

(i) Merger decisions $\mathbf{s} = (s_1, s_2)$ must be taken under uncertainty;

(ii) In spite of ex-ante uncertainty about firm types, merger decisions $\mathbf{s} = (s_1, s_2)$ cannot serve as signaling devices.

The assumption that types are revealed before firms compete in the product market is a convenient simplification. It allows us to characterize some key properties of the Bayesian equilibrium under two-sided asymmetric information without having to worry about signaling, as beliefs are not updated by assumption.[11]

In this setting, the second stage game corresponds exactly to the full information case. It is important to note that, in stage 2, it is irrelevant why a merger did not occur in stage 1: Given the type vector $\mathbf{z}$, the profits of all firms $i = 1, ..., n$ are independent of whether firm 1 and/or 2 rejected the merger. That is, in the general formulation of expected second-stage profits (2), the dependence of posterior beliefs on merger decisions is superfluous. As a result, Cournot outputs in the absence of a merger are independent of which of the two firms, if any, consented to the merger. In (7), therefore, the second and fourth term cancel out, so that expected merger returns are given by

$$
\begin{aligned}
G_i(z_i; B_j, F_i) &\equiv \mathbb{P}\left[B_j|\, z_i\right] \mathbb{E}_{\mathbf{z}_{-i}} \left[\pi_i^M\left(z_i, \mathbf{z}_{-i}\right) | z_i, z_j \in B_j\left(s_j\right)\right] \qquad (13) \\
&\quad - \mathbb{P}\left[B_j|\, z_i\right] \mathbb{E}_{\mathbf{z}_{-i}} \left[\pi_i(z_i, \mathbf{z}_{-i}; (0, 1)) | z_i, z_j \in B_j\left(s_j\right)\right] \\
&= \int_{\mathbf{z}_{-i}} g_i(z_i, \mathbf{z}_{-i}) dF_i(\mathbf{z}_{-i}|\, z_i, z_j \in B_j\left(s_j\right)),
\end{aligned}
$$

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[11] We will consider the effects of signaling in Section 5.

where the expectation is taken over the types of all other firms $\mathbf{z}_{-i}$.

Inspection of (13) indicates that, if firm types are correlated, a marginal increase in own type z_i affects merger returns not only through the four quantities mentioned above—the combination of profit share λ_i and cash payment t_i, the profits of the merged entity π^M, and the stand-alone profits π_i—but also through the probability $\mathbb{P}[B_j | z_i]$ of firm j consenting to the merger, and the belief $F_i(\mathbf{z}_{-i} | z_i, z_j \in B_j(s_j))$ about the other firms' types $\mathbf{z}_{-i}$. Under the plausible assumption that types are positively correlated, higher types put more weight on the possibility that the other firms have high types. As a result, there is a third potential reason why, contrary to the initial intuition, higher types might be more likely to merge: They expect to be facing better competitors if they stay alone, and they expect to be matched with a better partner in the event of a merger.

We now introduce the following terminology:

Definition 4 *The function* $G_i : [\underline{z}_i, \overline{z}_i] \to \mathbb{R}$ *satisfies* **strong downward single crossing** (**SSC⁻**) *if, for all* $z_i', z_i'' \in [\underline{z}_i, \overline{z}_i]$ *such that* $z_i' > z_i''$, $G_i(z_i') \geq 0$ *implies* $G_i(z_i'') \geq 0$ *and* $G_i(z_i') > 0$ *implies* $G_i(z_i'') > 0$.

This definition is closely related to the familiar single-crossing property of incremental returns (Milgrom and Shannon 1994).[12]

Next, we characterize the Bayesian equilibrium of the merger game with revelation of types before product market competition. The equilibrium is given by $s_i^R(z_i)$, $x_i^R(z_i, \mathbf{s})$ and $x_M^R(z_i, z_j, \mathbf{s})$, where the superscript indicates that types are revealed before product market competition. We first give a cut-off condition in terms of expected merger returns, and then consider more primitive conditions on merger returns without asymmetric information.[13]

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[12]Let $\Pi_i(s_i, z_i; B_j, F_j)$ define the expected payoff from strategy s_i for a firm with type z_i facing competitors characterized by B_j and F_j. Then $\Pi_i(s_i, z_i; B_j, F_j)$ satisfies the Milgrom-Shannon Single-Crossing Property in $(-s_i, z_i)$ if and only if G_i satisfies SSC^-.

[13]Using the equivalence between SSC^- and the Milgrom-Shannon condition, Lemma 1 is a special case of Theorem 1 in Athey (2001).

Lemma 1 (cut-off property) *Suppose $G_i(z_i; B_j, F_i)$ satisfies SSC^- in z_i for all $B_j \subset \mathcal{Z}_j$, $i = 1, 2$, $j \neq i$, with beliefs $F_i = F_i(\mathbf{z}_{-i} | z_i, z_j \in B_j(s_j))$ and players j adhering to strategy $s_j(z_j)$. Then every Bayesian Equilibrium (s_1^*, s_2^*) in pure strategies with $\mathbb{P}[B_i(s_i^*)] > 0$ for $i = 1, 2$ satisfies the cut-off-property, that is, there are cut-off values $z_i^* \in \mathcal{Z}_i$ such that*

$$
s_i^*(z_i) = \begin{cases} 1, & \text{if } z_i \leq z_i^*; \\ 0, & \text{if } z_i > z_i^*; \end{cases} \quad i = 1, 2.
$$

Proof. See Appendix. ∎

The intuition for Lemma 1 is as follows: SSC^- states that, for any distribution $F_i(\mathbf{z}_{-i} | z_i, z_j \in B_j(s_j))$, if some type z_i consents to a merger, so will any lower type $z_i' < z_i$. The proof then shows that this property of best responses translates into the cut-off property of the equilibrium. One might be concerned because the assumptions in Lemma 1 refer to endogenous equilibrium beliefs. However, as we now show, the SSC^- condition of Lemma 1 often holds for arbitrary beliefs and thus, by implication, for equilibrium beliefs.

Using Lemma 1, the next result follows immediately:

Proposition 1 (two-sided lemons) *Suppose that, under full information, firm i's merger returns $g_i(z_i, \mathbf{z}_{-i})$ are monotone **decreasing** in own type z_i for $i = 1, 2$. Then, if firm types are independently distributed, every Bayesian Equilibrium satisfies the cut-off property.*

Proof. Since types are independently distributed by assumption, expected merger returns are monotone decreasing by (13) for arbitrary beliefs, and the result follows directly from Lemma 1. ∎

The intuition for Proposition 1 is straightforward: If higher types face lower merger returns for arbitrary realizations of competitor types, then they must gain less in expectation for arbitrary beliefs, provided that types are independently distributed. This is what we call a *two-sided lemons equilibrium*. The result crucially relies on the assumption that firm types are

independently distributed: It avoids the possibility that better firms opt for a merger simply because they expect the other firms to have higher types.

Note the effect of introducing uncertainty on the qualitative properties of the equilibrium. As we illustrate in the Appendix, if merger returns are monotone decreasing in own type (and a set of reasonable assumptions hold), only relatively similar types will merge (if any) in the absence of uncertainty. In contrast, if (expected) merger returns are monotone decreasing in own type under uncertainty, the fact that high types have a smaller propensity to merge is reflected in the cut-off property of the equilibrium.[14]

Next, we consider two simple implications of Proposition 1.

Corollary 1 *Suppose that firm types are independently distributed. Then,*

(i) *if firm i's merger returns $g_i(z_i, \mathbf{z}_{-i})$ are monotone **increasing** in z_i for $i = 1, 2$, every Bayesian Equilibrium satisfies a reversed cut-off property where only high types consent (**two-sided peaches**).*

(ii) *if $g_1(z_1, \mathbf{z}_{-1})$ is monotone **decreasing** in z_1 and $g_2(z_2, \mathbf{z}_{-2})$ is monotone **increasing** in z_2, there exist z_1^L and z_2^H such that $s_1(z_1) = 1$ iff $z_1 \leq z_1^L$ and $s_2(z_2) = 1$ iff $z_2 \geq z_2^H$ (**lemons-and-peaches**).*

Proof. Follows from redefining types and applying Proposition 1. ∎

Corollary 1 reflects our first argument that the lemons rationale may be misleading in the context of mergers. As argued above, even if a high type foregoes higher stand-alone profits than a low type firm when entering a merger, it may also gain more from the merger, as it performs better in the merged entity. If the latter effect dominates, a firm's merger returns are increasing in own type, and a merger is only profitable for high types ("peaches"). As a result, we obtain equilibria where only high-type firms

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[14] As usual, there is also a degenerate cut-off equilibrium where no types are willing to merge. However, this *no-merger* equilibrium is Pareto-dominated in terms of expected profits whenever a cut-off equilibrium exists where firms consent to a merger with strictly positive probability.

(*two-sided peaches*) or at least some high-type firms (*lemons-and-peaches*) consent to the merger.

5 Signaling

We finally discuss the case where merger decisions are taken under uncertainty and types are not revealed before firms compete in the product market. In this setting, a firm's merger decision may carry an informational value, because this decision allows other firms to make inferences about its type in situations where no merger occurs because the other firm declines. That is, a firm's merger decision serves as a signaling device. Formally, observing $\mathbf{s}$ will allow firm i to update its prior belief $F_i(\mathbf{z}_{-i}|z_i)$ about the other firms' types to $F_i(\mathbf{z}_{-i}|z_i; \mathbf{s})$. The payoffs are therefore given by (7).

We now characterize the PBE in the presence of signaling. Let us first note that the game always has a *pooling* equilibrium where no types are willing to merge.[15] This result is very intuitive: If both firms believe that the other firm will not consent to the merger—no matter what its type is—it is a (weakly) best response not to consent, and beliefs are correct in equilibrium.

The preceding analysis suggests, however, that there may also be (partially) *separating* equilibria. Recall that in settings where firm types are revealed before product market competition and merger returns (as defined in (13)) satisfy SSC^-, cut-off equilibria arise (Lemma 1). Without type revelation in stage 2, a similar result holds, with a single-crossing condition on (7) rather than (13). Let us therefore consider a setting where the game has a cut-off equilibrium. How is the structure of this equilibrium affected by signaling considerations?

To answer this question, consider (7). The second and the fourth term jointly reflect the informational value of firm i's merger decision to competitors. They highlight that the expected profits of firm i in the case where both firms decline may differ from the case where only firm j declines, as

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[15]The same statement is true for the game of the preceding section; see footnote 14.

firm i's decision reveals information about its type. In principle, consenting to a merger can signal both high or low type, depending on competitors' beliefs. If merger returns are SSC^-, competitors expect low types to merge, so that consenting signals a low type. This immediately implies that signaling considerations provide incentives to decline. Intuitively, if competitors take declining as a sign of strength, they will react by reducing their Cournot outputs, reflecting the strategic-substitutes property of product market competition. In such a setting, signaling considerations will make firms more reluctant to consent to a merger, leading to lower cut-off values.

Conversely, if merger returns are upwards (rather than downwards) single-crossing (SSC^+), competitors expect high types to merge, so that consenting signals a high type. In this case, a type who is just indifferent between merging and not merging without signaling wants to consent to a merger to signal his strength. Thus, more types will consent to a merger than without signaling, meaning that cut-off values are lower.

Our next proposition summarizes these results.

Proposition 2 (signaling) *Suppose that, both with and without signaling, the game has a cut-off equilibrium.*

(i) *If there exist (z_1^*, z_2^*) such that player $i = 1, 2$ consents to a merger iff $z_i \leq z_i^*$, cut-off values are lower with signaling than without, so that **less** players consent.*

(ii) *If there exist (z_1^*, z_2^*) such that player $i = 1, 2$ consents to a merger iff $z_i \geq z_i^*$, cut-off values are lower with signaling than without, so that **more** players consent.*

Proof. See Appendix. ■

6   Limitations and Extensions

So far, we have analyzed a simple two-stage game where, in stage 1, two firms are matched to announce their merger decisions, and, in stage 2, the

remaining firms compete in a Cournot market. This is arguably the simplest setting that allows us to study the question of which types of firms consent to a merger if firms have private information about both stand-alone and post-merger profits. We now consider a number of limitations and extensions and discuss to what extent the latter are likely to affect the properties of merger returns.

Matching   Let us first consider stage 1 of the game. Assume that rather than being matched and forced to communicate their merger decisions, firms can endogenously choose (i) whether they are willing to enter merger negotiations, and (ii) with whom they want to negotiate. While this is clearly a very different (and perhaps more realistic) institutional setting, the reduced-form representation of expected merger returns will not be affected, provided that firms are unable to fully reveal their types before the merger decision, and $F_i(\mathbf{z}_{-i} | z_i)$ is interpreted to represent the residual uncertainty after merger negotiations.[16]   As a result, the qualitative results of our analysis remain unaffected.

Product Market Competition   Consider now another form of product market competition in stage 2 of the game. More specifically, let $x_i, i = 1, ..., n$, represent prices rather than quantities and assume that firms produce horizontally differentiated goods. In this case, competition in the product market is in *strategic complements* rather than substitutes, so that the signaling effect characterized in Proposition 2 is reversed. Intuitively, if competitors take declining as a sign of strength (i.e., low cost), they will now react by reducing their prices (rather than reducing their quantities), reflecting the strategic-complements property of product market competition. As a result, the profit differential associated with signaling a low type must be positive, so that the cut-off value must be *higher* (rather than lower) than in

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[16]Modelling the negotiations explicitly would require strong assumptions on the details of the negotiation process.

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the reference case without signaling.

Properties of Merger Mechanisms Our reduced-form analysis suggests that merger decisions crucially rely on the properties of the merger-returns function. In particular, we find that it is decisive whether merger returns are increasing or decreasing in own type. In part, this depends on the properties of the merger mechanism determining λ_i and t_i that we take as exogenous. It would therefore be desirable to explicitly characterize the properties of merger mechanisms that are associated with decreasing and increasing merger returns, respectively. While this is beyond the scope of this paper, it is clearly an interesting subject for future research.

Principal-Agent Problems Our analysis abstracts from principal-agent problems within firms by assuming that merger decisions are taken by firm owners. We are well aware, though, that managers may have a strong impact on merger decisions, and that their interests are likely to diverge from those of firm owners.[17] By focusing on owner-decisions, we restrict attention to mergers that are conditionally efficient ex ante in the sense that, conditional on λ_i and t_i, they occur if and only if they weakly increase expected profits for both firms.

7 Conclusions

This paper provides a simple reduced-form framework for analyzing merger decisions in the presence of asymmetric information about firm types. We employ this framework to examine what types of firms are likely to be involved in mergers. We show that the lemons rationale has severe limitations in the context of mergers. It is true that there are circumstances where only low types (if any) consent to a merger, so that a two-sided lemons equilib-

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[17]See Schleifer and Vishny (2003) for a recent theory of market-driven acquisitions, and Ang and Cheng (2006) for empirical evidence.

rium emerges. There are, however, alternative settings where only high types consent (two-sided peaches equilibrium), and even settings where low types merge with high types (lemons-and-peaches equilibrium). For such alternative equilibria to emerge, it is necessary that the merger-returns function of at least one of the firms involved in a merger is increasing rather than decreasing in own type, which is perfectly conceivable. An important implication of this insight is that consenting to a merger that gets rejected by the other firm does not necessarily signal a low type.

Our paper sheds new light on the likely pattern of merging firms when merger mechanisms may be inefficient. A natural extension of our paper would be to explicitly characterize the properties of merger returns for alternative merger mechanisms and profit functions. This is an interesting subject for future research.

Appendix

Merger Set with Full Information and Decreasing Merger Returns

We want to illustrate that, even if merger returns are monotone decreasing in own type, so that low types are more eager to participate in a merger, it is not clear that mergers involve low types. To this end, we make the following two additional assumptions.

(i) The merger returns $g_i(\cdot)$ are increasing in z_j for $i = 1, 2$; $j \neq i$.

(ii) The merger-return functions $g_i(\cdot)$ are identical for both firms.

The first condition is quite natural: By definition, stand-alone profits are non-increasing in the competitor's type. Also, joint merger profits are non-decreasing in the competitor's type. Unless the mechanism $\lambda_i(z_1, z_2)$ is such that the share of a firm is strongly decreasing in its own type, property (i) should therefore be satisfied.

Condition (i) implies that there is a critical type $C_j(z_i)$ of firm j for which type z_i would consent to a merger. Since $g_i(z_i, \mathbf{z}_{-i})$ is decreasing in z_i and increasing in $z_j, j \neq i$, the functions $C_j(z_i)$ are increasing in (z_1, z_2)-space for $i, j = 1, 2, i \neq j$. For profiles (z_1, z_2) to the left of $C_2(z_1)$, firm 1 consents to the merger; for profiles (z_1, z_2) to the right of $C_1(z_2)$ firm 2 consents. The merger set consists of the intersection of these sets.

Under the symmetry condition (ii), the merger set will either be empty or consist of relatively similar types (see Figure 1).

<center><Figure 1 about here></center>

Proof of Lemma 1

Firm i's expected merger return, facing firm j with strategy s_j, is $G_i(z_i; B_j, F_i)$. If $G_i(z_i; B_j, F_i)$ is positive, firm i will consent to the merger, otherwise it will reject the merger. By assumption, $G_i(z_i; B_j, F_i)$ satisfies SSC^- in z_i. Thus, if a $z_i \in \mathcal{Z}_i$ exists such that $G_i(z_i; B_j, F_i) > 0$, then there exists a $z_i^{\circ}(s_j) \in [\underline{z}_i, \overline{z}_i]$ such that $G_i(z_i; B_j, F_i) \geq 0$ if and only if $z_i \leq z_i^{\circ}(s_j)$. Now define

$$
\tilde{R}_i(s_j) = \begin{cases} z_i^{\circ}(s_j), & \text{if } z_i^{\circ}(s_j) \leq \overline{z}_i; \\ \overline{z}_i, & \text{if } z_i^{\circ}(s_j) \geq \overline{z}_i \text{ or if } z_i^{\circ}(s_j) \text{ does not exist.} \end{cases}
$$

Then firm i's optimal reaction is

$$
R_i(z_i, s_j) = \begin{cases} 1, & \text{if } z_i \leq \tilde{R}_i(s_j); \\ 0, & \text{if } z_i > \tilde{R}_i(s_j). \end{cases}
$$

In particular, for an equilibrium strategy s_j, the best reply has the required cut-off structure.

Proof of Proposition 2

(i) By assumption, in the PBE of a game with cut-off values (z_1^*, z_2) such that player $i = 1, 2$ consents iff $z_i \leq z_i^*$, beliefs must be such that any

player that has consented to a merger has a lower type than any player that has not. Therefore, players $j \neq i$ choose lower outputs in product market competition when firm i has declined the merger, as they expect this firm to choose a higher output. Thus, the profit differential associated with signaling a low type must be negative, so that the cut-off values with signaling must be smaller than in the reference case without signaling.

(ii) By assumption, in the PBE of a game with cut-off values (z_1^*, z_2) such that player $i = 1, 2$ consents iff $z_i \geq z_i^*$, beliefs must be such that any player that has consented has a higher type than any player that has not. Therefore, players $j \neq i$ choose lower outputs when firm i has consented. Thus, the profit differential associated with signaling a high type must be positive, so that the cut-off values with signaling must be smaller than in the reference case without signaling.

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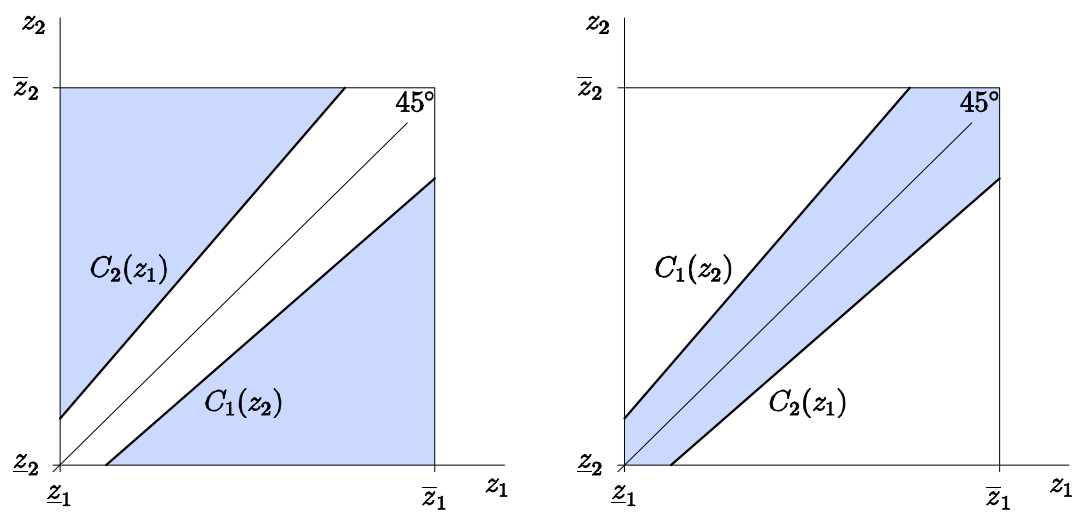Shleifer, A., Vishny, R. (2003), Stock Market Driven Acquisitions, *Journal of Financial Economics* 70, 295-311.

Figure 1: Consenting types under full information.