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## **Abstract**

We propose a simple but effective estimation procedure to extract the level and the volatility dynamics of a latent macroeconomic factor from a panel of observable indicators. Our approach is based on a multivariate conditionally heteroskedastic exact factor model that can take into account the heteroskedasticity feature shown by most macroeconomic variables and relies on an iterated Kalman filter procedure. In simulations we show the unbiasedness of the proposed estimator and its superiority to different approaches introduced in the literature. Simulation results are confirmed in applications to real inflation data with the goal of forecasting long-term bond risk premia. Moreover, we find that the extracted level and conditional variance of the latent factor for inflation are strongly related to NBER business cycles.

## **Keywords**

Macroeconomic variables; Exact factor model; Kalman filter; Heteroskedasticity; Forecasting bond risk premia; Inflation measures; Business cycles.

## **JEL Classification**

C13; C33; C53; C82; E31; E47.

# 1 Introduction

In their highly influential paper, using a reduced form no-arbitrage framework with time-varying risk premia, Ang and Piazzesi (2003) conclude that macroeconomic variables have an important explanatory power for yields and that the inclusion of such variables in term structure models can improve their forecasting performances significantly. More recently, many other studies (see, among others, Ludvigson and Ng (2009b), Joslin et al. (2009), Duffee (2009) for the U.S. or Wright (2009) in an international context) have documented that macroeconomic variables capture significant predictive power for bond excess returns over and above the standard financial factors. In order to avoid relying on specific macro series, Ang and Piazzesi (2003) and Ludvigson and Ng (2009a), measure different macroeconomic fundamentals as the first principal components of blocks of large numbers of macroeconomic series.

In this paper we propose considering as possible macroeconomic factors relevant for modeling the dynamics of the bond risk premia process (and therefore the whole term-structure). We take into account not only the level of a macroeconomic variable, but also its volatility. Moreover, we also propose a different method for reconstructing the level and volatility dynamics of the latent macro-factor from a bunch of observable indicators. Our approach is considerably simpler from a computational perspective than the classical ones introduced in the literature and at the same time performs better in simulations as well as in a real data applications.

In macroeconomics, it is common to have a large set of indexes that measure or are highly dependent on a latent macroeconomic variable. Given the pervasiveness of heteroskedasticity in macroeconomic variables, we model the observable set of proxies using a multivariate conditionally heteroskedastic exact factor model, i.e. a linear factor model where the heteroskedastic conditional variance is a function of the past values of the latent factor (see for instance, Diebold and Nerlove 1989). In such a type of model, the conditional density, depending on unobservable variables, is generally unknown. As a consequence, the log-likelihood function cannot be obtained explicitly and hence standard maximum likelihood estimators cannot be employed (Harvey et al. 1992). To overcome this problem, alternative estimation procedures have been proposed in the literature: the Bayesian Markov chain Monte Carlo (MCMC) estimation methods introduced by Fiorentini et al. (2004) and the indirect inference estimators introduced by Sentana et al. (2008).

However, following the direction proposed by Diebold and Nerlove (1989) and Sentana (2004), in this study we introduce a (computationally) simple estimation approach that relies on filtering the latent factor from a panel of data via an iterated Kalman filter procedure. This approach hinges on recent results about efficient estimation of the macro-parameters in dynamic panel data models with a common factor. In particular, Gagliardini and Gourieroux (2009) showed that substituting the true factor values by their cross-sectional approximations does not lead to any asymptotic efficiency loss. For the cross-sectional reconstruction of the latent factor we propose an iterated process in which we estimate the volatility dynamics of the factor from the time series of a first (time-invariant) Kalman filter approximation of the factor and use it in a new cross-sectional conditional (time-varying) Kalman filter estimation. New volatility dynamics can

be estimated from the dynamics of the new estimated factor and the procedure can be iterated until convergence.

Simulation results based on different data-generating processes and the same amount of data that are available in the empirical application show the unbiasedness of the proposed estimator for the conditional variance parameters and its superiority to other simple alternative methods, in particular, to the principal component approach used by Ludvigson and Ng (2009a).

The superiority of our approach is also confirmed by a real data application. Using a panel of 21 monthly inflation time series, we filter the level and the volatility of inflation via several different techniques. We test the ability of the estimated factors in forecasting long-term bond risk premia and find that both the level and the volatility of inflation obtained via an iterated Kalman filter significantly outperform the other competitors. Moreover, by analyzing the correspondence between the different factors and National Bureau of Economic Research (NBER) business cycles, we show that our inflation estimates are not only statistically but also economically significant.

The remainder of the paper is organized as follows. Section 2.1 describes in detail the procedure of reconstructing the level and volatility dynamics of a latent factor. Section 2.2 shows the performance of the latent macroeconomic variable and its volatility in a simulation study. In Section 3 we apply our estimation technique on real macroeconomic data. Section 4 concludes.

## 2 Reconstructing the dynamics and volatility of the latent factor

Our purpose in this section is to reconstruct the underlying time series dynamics of a latent macroeconomic variable and its volatility process from the observations of a certain number of proxies. We propose a simple estimation approach that exploits the possibility of filtering the latent factor from cross-sectional information via an iterated Kalman filter procedure.

### 2.1 Model and estimation procedure

We model the latent factor dynamics at time  $t$  through a factor model for the  $N$ -dimensional vector of the observed index returns  $r_t = (r_{t,i})_{i=1}^N$

$$r_t = Bf_t + e_t, \quad \text{for} \quad t = 1 \dots T \quad (1)$$

with  $B$  the  $N \times k$  matrix of factor loadings,  $e_t$  the  $N \times 1$  vector of idiosyncratic noises, and the latent factor  $f_t$  being the variables of interest, which are assumed to follow a general GARCH type dynamic with (for simplicity) mean zero and (for identifiability) unconditional unit variance i.e.  $f_t \sim N(0, \Delta_t)$  with  $\mathbb{E}[\Delta_t] = \Delta = I_k$  the identity matrix of order  $k$ . Assuming the vector of idiosyncratic noises  $e_t$  is conditionally orthogonal to  $f_t$  and has a positive semidefinite diagonal variance matrix  $\Phi$ , the distribution of  $r_t$  conditional on the information set  $\mathbb{I}_{t-1}$  containing  $r_{t-1}$  and  $f_{t-1}$  is  $N(0, \Sigma_t)$  where  $\Sigma_t = B\Delta_t B' + \Phi$  has the usual exact factor structure.

In the literature this type of model is called a multivariate conditionally heteroskedastic exact factor model and nests several models widely used in empirical finance (for instance, Diebold

and Nerlove 1989). When the variance of the factor is a function of lagged values of  $f_t$ , as in the GARCH case, the exact form of the conditional density of  $r_t$  given its past is generally unknown and, hence, the log-likelihood function cannot be explicitly obtained (Harvey et al. 1992). To overcome this problem, Bayesian Markov chain Monte Carlo (MCMC) estimation methods (Fiorentini et al. 2004) and indirect inference estimators (Sentana et al. 2008) have been proposed in the literature.

Here, instead, we propose a simpler approach in which we iterate between filtering the factor with a Kalman filter in the cross-sectional dimension and estimating its variance dynamics in the time series dimension. This approach hinges on the recent theories of efficient estimators of the macro-parameters in dynamic panel data models with a common factor that show how substituting the true factor values by their cross-sectional approximations does not lead to any asymptotic efficiency loss (Gagliardini and Gourieroux 2009). These studies show that, under certain speed of convergence assumptions,<sup>1</sup> estimating the macro-parameter on the cross-sectional approximations of the factors is root- $T$  consistent, asymptotically normal and achieves the same asymptotic efficiency bound as the one obtained with an observable factor (i.e. the Cramer-Rao bound in linear Gaussian models). Therefore, the estimators built on the approximated factor are asymptotically equivalent to the unfeasible estimator that uses the true factor values.

Different approaches can be used to approximate  $f_t$ : simple cross sectional averaging, principal component analysis (PCA) or factor analysis (FA). In this study we propose a reconstruction of the  $f_t$  factor by an iterative procedure in which the factor is first estimated with a Kalman filter using the cross-section of the observable indicators at our disposal. From the time series of this first approximation of the factor, the variance dynamics are estimated in a classical GARCH framework. The estimated GARCH dynamics of the factor conditional variance are then used in a conditional Kalman filter estimation to obtain new factor estimates. This iterative procedure is run until convergence.

Before starting the procedure, we need an estimate of the factor loading matrix  $B$ . Given that in these types of models the factor loadings are assumed to be constant over time, they can be conveniently estimated from unconditional quantities. Moreover, conditionally heteroskedastic factor models also imply unconditional covariance matrices that have an exact  $k$  factor structure as in the traditional factor models. Hence, recalling that  $\Delta = I_k$ , the unconditional covariance matrix  $\Sigma$  can be written as

$$\Sigma = BB' + \Phi \quad (2)$$

Clearly, the correlation matrix  $R = D^{-1}\Sigma D'^{-1}$  with  $D = \text{diag}(\Sigma)$  will also have the same factor structure

$$R = B^*B^{*'} + \Phi^* \quad (3)$$

with  $B^* = D^{-1}B$  and  $\Phi^* = D^{-1}\Phi D'^{-1}$ .

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<sup>1</sup>When  $N, T \rightarrow \infty$  and  $T/N \rightarrow c > 0$  the fixed effects estimator is consistent, while if  $N, T \rightarrow \infty$  such that  $T^b/N = O(1), b > 1$  the estimator is efficient.

Given that in our case all the observed indexes are mainly driven by a single latent macroeconomic variable they are supposed to measure, we assume a factor structure with only one common factor (i.e.  $k = 1$ ). Then, the correlation matrix takes the following simple structure.

$$R = \begin{bmatrix} 1 & b_1^* b_2^* & \dots & b_1^* b_N^* \\ b_2^* b_1^* & 1 & \dots & b_2^* b_N^* \\ \vdots & & \ddots & \vdots \\ b_N^* b_1^* & b_N^* b_2^* & \dots & 1 \end{bmatrix}$$

where  $[B^*]_i = b_i^*$  is the generic element of the  $N \times 1$  vector  $B^*$ . This structure, together with the fact that the factor loadings of the proxy are assumed to be all positive, suggests the possibility to estimate the vector of standardized factor loadings  $B^*$  by simply minimizing the difference between any generic off diagonal element of the matrix  $B^* B^{*'} with the corresponding element of the sample unconditional correlation matrix  $[S^*]_{ij} = s_{ij}^*$ , that is$

$$\hat{b}^* = \underset{b^*}{\operatorname{argmin}} \sum_{i=1}^N \sum_{j \neq i} (b_i^* b_j^* - s_{i,j}^*)^2. \quad s.t. \quad 0 < b_i^* < 1 \quad \forall i \quad (4)$$

The minimization algorithm in (4) projects the sample correlation matrix into the space spanned by single factor models.

Having the estimated standardized factor loadings  $\hat{B}^*$ 's, we can estimate the elements of the diagonal matrix  $\hat{\Phi}^*$  as  $[\hat{\Phi}^*]_{ii} = 1 - (\hat{b}_i^*)^2$ . Then the original idiosyncratic variance matrix and factor loadings are simply obtained as  $\hat{\Phi} = D\hat{\Phi}^*D'$  and  $\hat{B} = D\hat{B}^*$  respectively.

With  $\hat{B}^*$  and  $\hat{\Phi}^*$  at hand, we can now start the Kalman filter iteration. If the joint conditional distribution of  $r_t$  and  $f_t$  given  $\mathbb{I}_{t-1}$  is normal, the model (1) has a natural conditionally Gaussian linear state-space representation. In fact, considering the common factor  $f_t$  as state variable, equation (1) could be seen as a standard measurement equation. Hence, the Kalman filter would coincide with the conditional expectation of  $f_t$  given  $r_t$ , which is optimal in the conditional mean squared error sense. Actually, the optimality of the Kalman filter extraction of the factor holds under the more general assumption that  $f_t$  and  $r_t$  follow a conditional joint distribution that is elliptically symmetric (Sentana 1991). Thus, the conditional Kalman filter estimate of the common factor would be given by the (unfeasible) updating equation of the filter

$$f_t^{CK} = \Delta_t B' \Sigma_t^{-1} r_t = \Delta_t B' (B \Delta_t B' + \Phi)^{-1} r_t. \quad (5)$$

In order to have a feasible conditional Kalman filter, we propose to start the iterative procedure from the unconditional Kalman filter estimates with time-invariant weights

$$\hat{f}_t^{(0)} = \hat{B}' \hat{\Sigma}^{-1} r_t = \hat{B}' (\hat{B} \hat{B}' + \hat{\Phi})^{-1} r_t \quad (6)$$

using the estimates  $\hat{B}$  and  $\hat{\Phi}$  obtained from the unconditional information.

Having this first reconstruction of the dynamics of the latent macro-variable, we then get an estimate of the dynamics of its volatility by estimating a GARCH model on  $\hat{f}_t$ . In this way we



obtain a first estimate of the dynamics of the conditional variance of the factor i.e.  $\hat{\Delta}_t^{(0)}$  which is then used in the conditional Kalman filter estimation of the factor

$$\hat{f}_t^{(1)} = \hat{\Delta}_t^{(0)} \hat{B}' \hat{\Sigma}_t^{-1} r_t = \hat{\Delta}_t^{(0)} \hat{B}' \left( \hat{B} \hat{\Delta}_t^{(0)} \hat{B}' + \hat{\Phi} \right)^{-1} r_t \quad (7)$$

from which a new reconstruction of the latent factor can be computed and a new conditional variance dynamics  $\hat{\Delta}_t^{(1)}$  estimated. Iterating this procedure provides our proposed estimator for the dynamics of the latent factor and its conditional variance. Note that in practice, only a small number of iterations is necessary to reach converge and the algorithm is very fast.

## 2.2 Simulations

We first judge the performance of the proposed approach on the accuracy in the reconstruction of the time series of the latent factor  $f_t$ . The first employed data generating process (DGP) is a one factor model with the latent factor following a GARCH type dynamics with zero mean and unconditional unit variance. We simulate 1000 paths and for each path we assume 49 years of monthly observations ( $T = 588$ ). Similarly to our real data application, we assume to have 20 observable indicators for the latent macroeconomic variable ( $N = 20$ ). The true  $\beta$ s in the DGP are randomly chosen within a range of values analogous to that estimated on the empirical data. For comparison purposes we also include the result obtained with a simple cross-sectional average of the indexes, the PCA and the FA with one factor.

To judge the accuracy in reconstructing the  $f_t$  series with the various approaches, we compute the Root Mean Square Error (RMSE) for each simulated path between the true path of the latent factor and the estimated one. For each simulation path we also compute the correlation coefficient between the two series. Results are reported in the first two rows of Table 1.

[Table 1 about here.]

According to both metrics, our proposed procedure for the latent  $f_t$  process turns out to be the most precise; it is the one with, on average, the smallest RMSE and the highest correlation coefficient.

We then evaluate the ability of the different approaches to reconstruct the volatility dynamics of the true factor by computing the RMSE and correlation coefficient between the true series of simulated volatilities and the reconstructed ones obtained by fitting a GARCH(1,1) process to the estimated  $f_t$  series. Again, the Iterated Kalman filter provides the reconstruction of the latent factor volatility with, on average, the lowest RMSE and the highest correlation coefficient, as shown in the last two rows of Table 1.

Finally, in Figure 1, we plot the distributions of the estimated parameters of the GARCH process for the volatility.

[Figure 1 about here.]

The figure clearly shows that the estimates of the true parameters  $\alpha$  and  $\beta$  of the GARCH process in the factor DGP are both unbiased and reasonably accurate.

We also test the procedure on two more challenging volatility DGP processes: in the first one the variance matrix of the idiosyncratic noise  $\Phi$  is also time-varying, with each idiosyncratic component following a different GARCH process. The second one consists of a two-regime process with lagged return as the threshold variable where the local conditional variance evolves according to a FIGARCH(1,d,1) model (see Baillie et al. 1996) in one regime and a model that is not of a GARCH type in the second regime. Results for the two more complex volatility DGPs are reported in Tables 2 and 3.

[Table 2 about here.]

[Table 3 about here.]

The results confirm in both cases the more accurate reconstruction of the latent process by the proposed iterated Kalman filter method. Finally, as in Figure 1, in Figure 2 we plot the distribution of the  $\alpha$  and  $\beta$  parameter estimates in the case of DGP process with time varying (GARCH type) idiosyncratic noise  $\Phi_t$ .

[Figure 2 about here.]

GARCH parameter estimates seem to remain unbiased even in this misspecified context.

### 3 Real data application: bond risk premia forecasting

Economic theory suggests that (a great portion of) bond term premia variation is driven by macroeconomic fundamentals. Yet, the link between macroeconomic activity and risk premia might be hard to detect. Using different modeling setups, many recent studies (see, among others, Ludvigson and Ng (2009b), Joslin et al. (2009), or Duffee (2009)) document that macroeconomic variables capture significant predictive power for excess returns over and above the standard financial factors. In this section we assess the performance of our iterated Kalman filter technique in forecasting long-term bond excess returns.

#### 3.1 Data and estimated inflation levels and variances

In our empirical study two different datasets are used.

##### *Bond Data*

We use monthly data (June 1961 onward) from the Federal Reserve Board constructed as in Gürkaynak et al. (2006).<sup>2</sup> Following Cochrane and Piazzesi's (2005) procedure, bond excess returns are calculated as 12-month holding period returns in excess of the one-year risk-free

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<sup>2</sup>The data are available under <http://www.federalreserve.gov/econresdata/researchdata.htm>.

rate.<sup>3</sup> Furthermore, we construct our tent-shape bond-return forecasting factor described in Cochrane and Piazzesi (2005) (hereafter CP factor) as a linear combination of forward rates. The inclusion of the CP factor is motivated simply by the fact that it has high explanatory power for bond excess returns.

### *Macroeconomic Data*

The second dataset consists of monthly observations for 21 U.S. inflation time series. Exact description of the data is given in Appendix A. The panel spans the period January 1959 – December 2007 and has already been used as a part of other studies: see, among others, Stock and Watson (2005), Ludvigson and Ng (2009b) and Ludvigson and Ng (2009a). We build two alternative pairs of estimates for inflation levels and variances. First, similar to Ludvigson and Ng (2009a), we extract the first principal component (PC) as a measure for inflation’s level. PC volatility is computed from fitting a GARCH(1,1) on monthly data. Our second approach for reconstructing the level and the variance of inflation is based on the iterated Kalman filter procedure described in Section 2.1.

For our analysis we take the largest common period of the two datasets and split it into two parts. We consider June 1961 to December 2003 as in-sample period. The rest of the data (January 2004 - December 2008) has been left to evaluate the out-of-sample forecasting performance of the different predictors. Summary statistics of the data are reported in Table 4.

[Table 4 about here.]

Figure 3 illustrates the difference between the level and the volatility of the two inflation measures.

[Figure 3 about here.]

## **3.2 Financial variables, inflation measures, and business cycles**

To begin with, we analyze the correspondence between the NBER business cycles and the different financial and inflation measures. The last row of Table 4 reports the results. The weak correlation (around 0.04) between the NBER recession and CP factor confirms Ludvigson and Ng (2009b) finding that, without macro factors, bond risk premia appear virtually acyclical. Yet, theory says that risk premia have a marked counter-cyclical behavior, compensating the investors for macroeconomic risks. The almost two times higher correlation between the NBER business cycles indicator and the iterated Kalman filter inflation variables in comparison to those estimated with the PC approach assures more pronounced cyclical fluctuations in bond risk premia. By its iterated nature, our measures for inflation seem to better capture perceptions of risks looming on the investors horizon. Thus, they convey valid and timely information over and above that

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<sup>3</sup>Let  $rx_{t+1}^{(n)}$  denote the continuously compounded log excess return on an  $n$  year bond at time  $t + 1$ . Then bond excess returns are defined as  $rx_{t+1}^{(n)} = r_{t+1}^{(n)} - y_t^{(n)}$ , where  $r_{t+1}^{(n)}$  is the log holding period return from buying an  $n$  year bond at time  $t$  and selling it at time  $t + 1$ , and  $y_t^{(1)}$  is the log yield on a one year bond.

contained in other financial and PC inflation fundamentals. These findings make the inflation factors obtained by the iterated Kalman filter approach highly economically significant.

### 3.3 Long-term bond risk premia forecasting results

To assess the impact of the two different pairs of inflation factors on bond excess returns, we run the following regressions:

$$\begin{aligned}
\text{Model } \mathcal{M}_1 : \quad & rx_{t+12}^{(n)} = \gamma_0 + \gamma_1 CP_t + \varepsilon_{t+12}^{(n)} \\
\text{Model } \mathcal{M}_2 : \quad & rx_{t+12}^{(n)} = \gamma_0 + \gamma_1 CP_t + \gamma_2 \pi_t^{IK} + \varepsilon_{t+12}^{(n)} \\
\text{Model } \mathcal{M}_3 : \quad & rx_{t+12}^{(n)} = \gamma_0 + \gamma_1 CP_t + \gamma_2 \text{vol} \pi_t^{IK} + \varepsilon_{t+12}^{(n)} \\
\text{Model } \mathcal{M}_4 : \quad & rx_{t+12}^{(n)} = \gamma_0 + \gamma_1 CP_t + \gamma_2 \pi_t^{PC} + \varepsilon_{t+12}^{(n)} \\
\text{Model } \mathcal{M}_5 : \quad & rx_{t+12}^{(n)} = \gamma_0 + \gamma_1 CP_t + \gamma_2 \text{vol} \pi_t^{PC} + \varepsilon_{t+12}^{(n)} \\
\text{Model } \mathcal{M}_6 : \quad & rx_{t+12}^{(n)} = \gamma_0 + \gamma_1 CP_t + \gamma_2 \pi_t^{IK} + \gamma_3 \text{vol} \pi_t^{IK} + \varepsilon_{t+12}^{(n)} \\
\text{Model } \mathcal{M}_7 : \quad & rx_{t+12}^{(n)} = \gamma_0 + \gamma_1 CP_t + \gamma_2 \pi_t^{PC} + \gamma_3 \text{vol} \pi_t^{PC} + \varepsilon_{t+12}^{(n)}, \\
\text{Model } \mathcal{M}_8 : \quad & rx_{t+12}^{(n)} = \gamma_0 + \gamma_1 CP_t + \gamma_2 \pi_t^{IK} + \gamma_3 \pi_t^{PC} + \varepsilon_{t+12}^{(n)} \\
\text{Model } \mathcal{M}_9 : \quad & rx_{t+12}^{(n)} = \gamma_0 + \gamma_1 CP_t + \gamma_2 \text{vol} \pi_t^{IK} + \gamma_3 \text{vol} \pi_t^{PC} + \varepsilon_{t+12}^{(n)},
\end{aligned}$$

where  $rx_{t+12}^{(n)}$  are the excess returns on an  $n$  year nominal bond ( $n = 5, 10, 20, 30$ ) at time  $t + 12$ .  $CP_t$  represents the CP factor,  $\pi_t$  and  $\text{vol} \pi_t$  denote the inflation level and inflation volatility factors, estimated by the two different approaches: iterated Kalman filter (denoted by  $\pi_t^{IK}$  and  $\text{vol} \pi_t^{IK}$ ) and principal component analysis (denoted by  $\pi_t^{PC}$  and  $\text{vol} \pi_t^{PC}$ ), respectively. To this end, we estimate nine different models. First, we regress the excess returns only on CP factor (Model  $\mathcal{M}_1$ ). This regression should serve as a benchmark model. Then, in Model  $\mathcal{M}_2$  and Model  $\mathcal{M}_3$  we add one more predictor, the level and the volatility of inflation, each estimated by the iterative Kalman filter approach. We repeat the same procedure for the next two models (Model  $\mathcal{M}_4$  and Model  $\mathcal{M}_5$ ), where we add once again the level and the volatility of inflation, this time estimated by the PC technique. In Model  $\mathcal{M}_6$  and Model  $\mathcal{M}_7$  we take into consideration all three predictors: CP factor, level and volatility of inflation. The only difference between Model  $\mathcal{M}_6$  and Model  $\mathcal{M}_7$  is in the way the inflation variables are measured. In particular, in Model  $\mathcal{M}_6$  the inflation variables are derived by the iterated Kalman filter procedure, whereas in  $\mathcal{M}_7$  PCA has been used. In contrast to the previous models, where the main idea is to assess performance, the individual filtering techniques, the last two models (Model  $\mathcal{M}_8$  and Model  $\mathcal{M}_9$ ) provide a direct comparison between the two level (Model  $\mathcal{M}_8$ ) and the two volatility (Model  $\mathcal{M}_9$ ) factors. All coefficients are estimated with ordinary least squares, and standard errors are corrected for autocorrelation and heteroskedasticity. Table 5 and Table 6 present the results.

[Table 5 about here.]

[Table 6 about here.]

The estimated coefficients for the CP factor are positive and highly significant for predicting bond risk premia at all maturities. Fully in line with the literature, the CP factor accounts for around 28% of the excess returns variation. The strength of the predictive power of the inflation factors changes with time to maturity of a bond, explaining up to 6% of the variation in addition to the CP factor. The estimated coefficients for level and volatility of inflation are negative, and they are significant most of the time. The negative correlation between the different inflation measures and excess returns is quite intuitive, as higher inflation decreases the value of the nominal bond. Including both level and volatility of the inflation factor (see Models  $\mathcal{M}_6$  and  $\mathcal{M}_7$ ) in the regression does not seem to improve the accuracy, and both predictors become statistically not significant.<sup>4</sup>

Although, at first glance, both filtering techniques seem to perform equally well, the ability of our approach to reconstruct in a more accurate way both the level and the volatility of inflation has empirical merits. First, in-sample results providing direct comparison between the individual level and volatility factors (see Model  $\mathcal{M}_8$  and Model  $\mathcal{M}_9$  presented in Table 5 and Table 6) reveal that the iterated Kalman filter variables are significant most of the time, whereas the impact of the PC measures is always negligible. Second, the dominance of our approach is confirmed by an out-of-sample study. Forecasting results covering the period January 2004 to December 2008 are shown in Table 7.

[Table 7 about here.]

The superior predictive ability test of Hansen (2005) (see Table 7) reveals that our inflation's level and volatility measures on top of the CP factor matter for forecasting bond risk premia, significantly outperforming other alternatives. Importantly, however, their impact can differ, depending on the time to maturity of a bond.

We also test the performance of the two filtering techniques in a more challenging framework. Without making any additional assumptions, we create a pool of predictors, including the two different pairs of inflation measures and the CP factor, and let the data themselves choose the most informative variables. This is achieved by finding for each possible number of predictors the subset of the corresponding size that gives the smallest residual sum of squares.<sup>5</sup> Then, we use the Bayesian Schwarz Information Criterion (BIC) to select the best model. We find that regressing the excess returns on the CP factor and the volatility of inflation obtained by the iterated Kalman filter i.e. Model  $\mathcal{M}_3$  leads to optimal results.

Finally, we discuss the overall impact of the individual inflation factors in forecasting bond risk premia. Based on the in-sample fit, out-of-sample forecasting, and economic significance, we document that the most important macroeconomic variable for bond excess returns represents the volatility of inflation estimated via the iterated Kalman filter technique. Yet, our inflation volatility measure is no longer a statistically significant predictor of long-term bond risk premia

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<sup>4</sup>This result is a consequence of the high correlation between the two variables (and both series are very persistent) together with the necessary Newey-West correction that substantially lowers the t-statistics.

<sup>5</sup>This procedure is known in the literature as best subset selection. See Hastie et al. (2001) for more details.

once the level of inflation is in the same regression. The reason for this is the high correlation between the two iterated Kalman filter factors. However, their impact varies with the time to maturity of a bond. In general, we may conclude that the iterated Kalman filter technique allows us to extract in a more accurate way the investors' perceptions of inflation risk in comparison with alternative approaches.

## 4 Conclusions

In this paper we propose a new, computationally simple approach for reconstructing the level and volatility dynamics of a latent macroeconomic factor from a large panel of macroeconomic indices. Our estimation procedure is based on the iterated Kalman filter technique in which we iterate between filtering the unobservable factor with a Kalman filter in the cross-sectional dimension and estimating its variance dynamics in the time series dimension.

We assess the performance of our iterated Kalman filter approach on a set of empirical studies. Extensive simulation results reveal the accuracy of our latent factor volatility estimates and its superiority in comparison with other alternative approaches. Encouraged by those results, we test the ability of our approach to reconstruct in a more accurate way the unobservable macroeconomic driver and its volatility on a real data application – bond risk premia forecasting. Using a panel of a large number of inflation time series, we filter the level and the volatility of inflation via different techniques. We find that in predicting long-term bond risk premia, our inflation estimates significantly outperform the other competitors. In addition, looking at the correspondence between NBER business cycles and inflation fundamentals, we conclude that our estimates are not only statistically but also economically significant.

Our analysis could be taken a step further by studying the performance of bond risk premia in a term structure modeling framework. The iterated Kalman technique could also be used to obtain more accurate estimates for other important macroeconomic predictors such as real activity. However, those extensions are left for future research.

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# A Data Appendix

This appendix presents U.S. inflation data used in our real data analysis. The first column lists the short name of the inflation variable, followed by its mnemonic in column 2, and a brief data description in column 4. All data series are from Global Insights Basic Economic Database. The third column shows the transformations used to assure stationarity of the individual time series. In particular,  $\Delta \ln$  and  $lv$  denote the first difference of the logarithm and the level of the series, respectively. These data span the period January 1959 - December 2007 for a total of 588 monthly observations.

Short Name	Mnemonic	Tran	Description
PPI: fin gds	pwfsa	$\Delta \ln$	Producer Price Index: Finished Goods (82=100,Sa)
PPI: cons gds	pwfcsa	$\Delta \ln$	Producer Price Index: Finished Consumer Goods (82=100,Sa)
PPI: int materials	pwimsa	$\Delta \ln$	Producer Price Index:Intermed Mat.Supplies & Components(82==100,Sa)
PPI: crude matls	pwcmsa	$\Delta \ln$	Producer Price Index: Crude Materials (82=100,Sa)
Spot market price	psccom	$\Delta \ln$	Spot market price index: bls & crb: all commodities(1967=100)
PPI: nonferrous materials	pw102	$\Delta \ln$	Producer Price Index: Nonferrous Materials (1982=100, Nsa)
NAPM com price	pmcp	$lv$	Napm Commodity Prices Index (Percent)
CPI-U: all	punew	$\Delta \ln$	Cpi-U: All Items (82-84=100,Sa)
CPI-U: apparel	pu83	$\Delta \ln$	Cpi-U: Apparel & Upkeep (82-84=100,Sa)
CPI-U: transp	pu84	$\Delta \ln$	Cpi-U: Transportation (82-84=100,Sa)
CPI-U: medical	pu85	$\Delta \ln$	Cpi-U: Medical Care (82-84=100,Sa)
CPI-U: comm.	puc	$\Delta \ln$	Cpi-U: Commodities (82-84=100,Sa)
CPI-U: dbles	pucd	$\Delta \ln$	Cpi-U: Durables (82-84=100,Sa)
CPI-U: services	pus	$\Delta \ln$	Cpi-U: Services (82-84=100,Sa)
CPI-U: ex food	puxf	$\Delta \ln$	Cpi-U: All Items Less Food (82-84=100,Sa)
CPI-U: ex shelter	puxhs	$\Delta \ln$	Cpi-U: All Items Less Shelter (82-84=100,Sa)
CPI-U: ex med	puxm	$\Delta \ln$	Cpi-U: All Items Less Medical Care (82-84=100,Sa)
PCE defl	gmde	$\Delta \ln$	Pce, Impl Pr Defl:Pce (2000=100) (AC) (BEA)
PCE defl: dlbes	gmdcd	$\Delta \ln$	Pce, Impl Pr Defl:Pce; Durables (2000=100) (AC) (BEA)
PCE defl: nondble	gmdeu	$\Delta \ln$	Pce, Impl Pr Defl:Pce; Nondurables (2000=100) (AC) (BEA)
PCE defl: service	gmdeu	$\Delta \ln$	Pce, Impl Pr Defl:Pce; Services (2000=100) (AC) (BEA)



# PERFORMANCE COMPARISON - SIMULATIONS

	Simple Average	Factor Analysis	Principal Component	Iterated Kalman
Average correlation on $f_t$	0.9634	0.9892	0.9422	0.9899
Average RMSE on $f_t$	1.7594	0.1470	0.3372	0.1394
Average correlation on $\sigma_t$	0.9467	0.9671	0.9247	0.9687
Average RMSE on $\sigma_t$	0.6506	0.1296	0.1466	0.0545

Table 1: Performance comparison of different filtering methods for the factor dynamics and its conditional volatility over 1000 simulation paths. The methods are: simple cross-sectional averages, Factor Analysis, Principal Component, and Iterated Kalman filter. The performance measures are the average correlation and the average Root Mean Square Error (RMSE).

	Simple Average	Factor Analysis	Principal Component	Iterated Kalman
Average correlation on $f_t$	0.9627	0.9893	0.9404	0.9899
Average RMSE on $f_t$	1.7405	0.1475	0.3421	0.1397
Average correlation on $\sigma_t$	0.9491	0.9698	0.9240	0.9707
Average RMSE on $\sigma_t$	0.6444	0.1283	0.1458	0.0533

Table 2: Performance comparison of different filtering methods for the factor dynamics and its conditional volatility over 1000 simulation paths of a DGP model with time-varying  $\Phi_t$ . The methods are: simple cross-sectional averages, Factor Analysis, Principal Component, and Iterated Kalman filter. The performance measures are the average correlation and the average Root Mean Square Error (RMSE).

	Simple Average	Factor Analysis	Principal Component	Iterated Kalman
Average correlation on $f_t$	0.9629	0.9891	0.9414	0.9898
Average RMSE on $f_t$	1.7582	0.1472	0.3394	0.1403
Average correlation on $\sigma_t$	0.6159	0.6371	0.5951	0.6419
Average RMSE on $\sigma_t$	0.6908	0.2213	0.2285	0.2049

Table 3: Performance comparison of different filtering methods for the factor dynamics and its conditional volatility over 1000 simulation paths of a DGP model with two-regime processes. The methods are: simple cross-sectional averages, Factor Analysis, Principal Component, and Iterated Kalman filter. The performance measures are the average correlation and the average Root Mean Square Error (RMSE).

# SUMMARY STATISTICS OF DATA

	$rx^{(5)}$	$rx^{(10)}$	$rx^{(20)}$	$rx^{(30)}$	CP	$\pi^{IK}$	$\text{vol}\pi^{IK}$	$\pi^{PC}$	$\text{vol}\pi^{PC}$
Panel A:									
Mean	0.011	0.013	0.011	0.009	0.006	0.838	1.034	0.034	0.876
Std	0.056	0.104	0.198	0.325	0.019	0.579	2.109	1.014	0.694
AC1	0.931	0.921	0.880	0.798	0.916	0.989	0.993	0.973	0.953
Panel B:									
$rx^{(5)}$	1.00								
$rx^{(10)}$	0.96	1.00							
$rx^{(20)}$	0.82	0.92	1.00						
$rx^{(30)}$	0.62	0.72	0.90	1.00					
CP	0.43	0.48	0.49	0.44	1.00				
$\pi^{IK}$	-0.22	-0.25	-0.26	-0.26	-0.06	1.00			
$\text{vol}\pi^{IK}$	-0.31	-0.36	-0.31	-0.29	-0.15	0.89	1.00		
$\pi^{PC}$	-0.30	-0.29	-0.30	-0.29	-0.41	0.55	0.42	1.00	
$\text{vol}\pi^{PC}$	-0.23	-0.24	-0.26	-0.26	-0.38	0.49	0.49	0.69	1.00
NBER					0.04	0.46	0.45	0.17	0.24

Table 4: Panel A reports summary statistics for the following variables: 5, 10, 20, 30 year bond excess returns (denoted by  $rx^{(5)}$ ,  $rx^{(10)}$ ,  $rx^{(20)}$ ,  $rx^{(30)}$ , respectively), Cochrane and Piazzesi (2005) factor (denoted by CP), inflation level and inflation volatility factors estimated by iterated Kalman filter (denoted by  $\pi_t^{IK}$  and  $\text{vol}\pi_t^{IK}$ ), inflation level and inflation volatility factors estimated by principal component technique (denoted by  $\pi_t^{PC}$  and  $\text{vol}\pi_t^{PC}$ ). NBER is a binary variable, where one indicates month designated as recessions by the National Bureau of Economic Research. AC1 denotes the first autocorrelation coefficient. Panel B reports cross-correlations.

PANEL A: PREDICTIVE REGRESSION ANALYSIS 5 YEAR EXCESS RETURNS

	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$
Intercept	-0.002 (-0.310)	0.013 (1.168)	0.005 (0.678)	-0.001 (-0.174)	0.002 (0.239)	0.004 (0.339)	-0.007 (-0.640)	0.009 (0.748)	0.001 (0.088)
CP Factor	1.533*** (4.749)	1.474*** (4.555)	1.410*** (4.219)	1.351*** (3.982)	1.472*** (4.173)	1.408*** (4.102)	1.383*** (4.022)	1.403*** (4.192)	1.465*** (4.245)
Inflation Level (Iterated Kalman)		-0.016 (-1.433)				0.001 (0.059)		-0.012 (-0.869)	
Inflation Vol (Iterated Kalman)			-0.005 (-1.545)			-0.005 (-0.826)			-0.006 (-1.629)
Inflation Level (PCA)				-0.009 (-1.430)			-0.012 (-1.510)	-0.004 (-0.626)	
Inflation Vol (PCA)					-0.004 (-0.548)		0.006 (0.615)		0.005 (0.694)
$R^2$	0.257	0.285	0.294	0.279	0.259	0.294	0.282	0.289	0.297
Adjusted $R^2$	0.256	0.283	0.292	0.277	0.257	0.290	0.279	0.285	0.293

PANEL B: PREDICTIVE REGRESSION ANALYSIS 10 YEAR EXCESS RETURNS

	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$
Intercept	-0.013 (-1.164)	0.018 (0.991)	0.001 (0.088)	-0.011 (-1.094)	-0.002 (-0.165)	0.002 (0.119)	-0.016 (-0.901)	0.0152 (0.708)	-0.005 (-0.351)
CP Factor	3.025*** (4.649)	2.899*** (4.521)	2.772*** (4.251)	2.707*** (4.077)	2.869*** (4.146)	2.778*** (4.203)	2.733*** (4.077)	2.841*** (4.331)	2.857*** (4.271)
Inflation Level (Iterated Kalman)		-0.035* (-1.751)				-0.002 (-0.078)		-0.031 (-1.278)	
Inflation Vol (Iterated Kalman)			-0.011* (-1.917)			-0.010 (-0.960)			-0.012* (-1.928)
Inflation Level (PCA)				-0.015 (-1.410)			-0.018 (-1.371)	-0.003 (-0.284)	
Inflation Vol (PCA)					-0.011 (-0.748)		0.005 (0.287)		0.008 (0.587)
$R^2$	0.283	0.320	0.328	0.303	0.288	0.328	0.303	0.320	0.330
Adjusted $R^2$	0.282	0.317	0.325	0.300	0.285	0.324	0.299	0.316	0.326

Table 5: Results for ordinary least squares regressions for nine different models (labeled as  $\mathcal{M}_1, \mathcal{M}_2, \dots, \mathcal{M}_9$ ) utilizing annual returns on 5- and 10-year Treasury bonds. Standard errors are corrected for autocorrelation and heteroskedasticity. t-statistics are reported in parenthesis. Asterisks \*, \*\*, \*\*\* indicate statistical significance at the 10%, 5%, and 1% level, respectively. The data span the period June 1962 to December 2003. See text for more details.

PANEL A: PREDICTIVE REGRESSION ANALYSIS: 20 YEAR EXCESS RETURNS

	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$
Intercept	-0.042** (-1.976)	0.017 (0.634)	-0.016 (-0.972)	-0.039** (-2.045)	-0.011 (-0.449)	-0.005 (-0.157)	-0.036 (-1.114)	0.009 (0.248)	-0.002 (-0.624)
CP Factor	5.901*** (4.366)	5.659*** (4.439)	5.438*** (4.300)	5.2422*** (4.235)	5.458*** (4.016)	5.482*** (4.298)	5.224*** (4.129)	5.481*** (4.456)	5.438*** (4.176)
Inflation Level (Iterated Kalman)		-0.066* (-1.964)				-0.019 (-0.362)		-0.056 (-1.359)	
Inflation Vol (Iterated Kalman)			-0.019** (-2.348)			-0.015 (-1.169)			-0.012** (-2.123)
Inflation Level (PCA)				-0.032 (-1.542)			-0.031 (-1.437)	-0.011 (-0.466)	
Inflation Vol (PCA)					-0.032 (-0.983)		-0.003 (-0.103)		-5.3e - 05 (-0.001)
$R^2$	0.299	0.336	0.340	0.322	0.310	0.341	0.322	0.338	0.340
Adjusted $R^2$	0.297	0.334	0.338	0.319	0.307	0.337	0.318	0.334	0.336

PANEL B: PREDICTIVE REGRESSION ANALYSIS: 30 YEAR EXCESS RETURNS

	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$
Intercept	-0.072** (-2.027)	0.029 (0.798)	-0.031 (-1.215)	-0.065** (-2.113)	-0.009 (-0.218)	0.008 (0.177)	-0.049 (-0.905)	0.007 (0.170)	-0.016 (-0.368)
CP Factor	8.779*** (3.830)	8.372*** (4.013)	8.057*** (3.911)	7.564*** (3.901)	7.874*** (3.460)	8.213*** (3.886)	7.477*** (3.692)	7.928*** (4.092)	7.845*** (3.560)
Inflation Level (Iterated Kalman)		-0.112** (-2.292)				-0.070 (-0.808)		-0.085* (-1.690)	
Inflation Vol (Iterated Kalman)			-0.030*** (-2.602)			-0.013 (-0.751)			-0.027** (-2.260)
Inflation Level (PCA)				-0.061 (-1.639)			-0.053 (-1.325)	-0.027 (-0.708)	
Inflation Vol (PCA)					-0.066 (-1.113)		-0.017 (-0.262)		-0.021 (-0.353)
$R^2$	0.246	0.285	0.283	0.275	0.262	0.286	0.276	0.288	0.285
Adjusted $R^2$	0.244	0.282	0.280	0.272	0.259	0.282	0.271	0.284	0.280

Table 6: Results for ordinary least squares regressions for nine different models (labeled as  $\mathcal{M}_1, \mathcal{M}_2, \dots, \mathcal{M}_9$ ) utilizing annual returns on 20- and 30-year Treasury bonds. Standard errors are corrected for autocorrelation and heteroskedasticity. t-statistics are reported in parenthesis. Asterisks \*, \*\*, \*\*\* indicate statistical significance at the 10%, 5%, and 1% level, respectively. The data span the period June 1962 to December 2003. See text for more details.

PANEL A: OUT-OF-SAMPLE MEAN SQUARED ERRORS

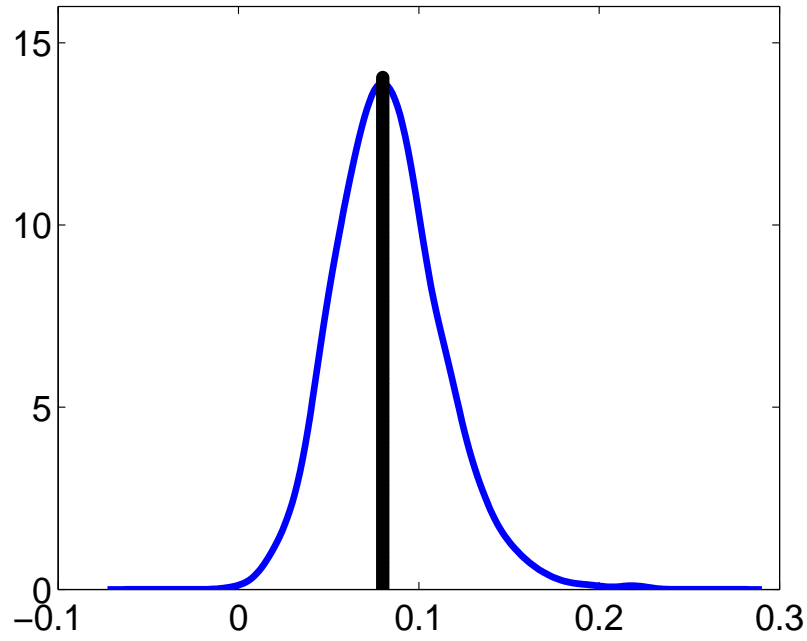
	5Y Bond Exret	10Y Bond Exret	20Y Bond Exret	30Y Bond Exret
Model $\mathcal{M}_1$	0.0033 (0.0972)	0.0091 (0.0079)	0.0320 (0.0000)	0.0873 (0.0000)
Model $\mathcal{M}_2$	0.0029 (0.1385)	0.0074 (0.1387)	0.0256 (0.0479)	0.0706 (0.0644)
Model $\mathcal{M}_3$	0.0028 (0.6321)	0.0070 (0.3957)	0.0241 (0.3365)	0.0674 (0.3459)
Model $\mathcal{M}_4$	0.0035 (0.0602)	0.0103 (0.0000)	0.0378 (0.0000)	0.1044 (0.0000)
Model $\mathcal{M}_5$	0.0033 (0.0789)	0.0094 (0.0000)	0.0343 (0.0000)	0.0947 (0.0000)
Model $\mathcal{M}_6$	0.0028 (0.4037)	0.0070 (0.6307)	0.0242 (0.6062)	0.0683 (0.5857)
Model $\mathcal{M}_7$	0.0035 (0.0473)	0.0104 (0.0000)	0.0377 (0.0000)	0.1042 (0.0000)

PANEL B: OUT-OF-SAMPLE MEAN ABSOLUTE ERRORS

	5Y Bond Exret	10Y Bond Exret	20Y Bond Exret	30Y Bond Exret
Model $\mathcal{M}_1$	0.0418 (0.0834)	0.0786 (0.0000)	0.1576 (0.0000)	0.2473 (0.0000)
Model $\mathcal{M}_2$	0.0400 (0.3525)	0.0697 (0.1293)	0.1381 (0.1790)	0.2146 (0.0910)
Model $\mathcal{M}_3$	0.0391 (0.6545)	0.0676 (0.3945)	0.1349 (0.3555)	0.2120 (0.3186)
Model $\mathcal{M}_4$	0.0439 (0.0672)	0.0862 (0.0000)	0.1738 (0.0000)	0.2768 (0.0000)
Model $\mathcal{M}_5$	0.0422 (0.1149)	0.0812 (0.0000)	0.1643 (0.0000)	0.2612 (0.0000)
Model $\mathcal{M}_6$	0.0390 (0.4887)	0.0676 (0.6905)	0.1348 (0.6384)	0.2117 (0.6722)
Model $\mathcal{M}_7$	0.0440 (0.0315)	0.0861 (0.0000)	0.1737 (0.0000)	0.2767 (0.0000)

Table 7: Results (mean squared errors (Panel A) and mean absolute errors (Panel B)) of out-of-sample forecasting performance of seven different models for 5-, 10-, 20- and 30-year Treasury Bond excess returns, as described in detail in the text. p-values of the superior predictive ability (SPA) test of Hansen (2005) are reported in parenthesis. The results are based on out-of-sample period, January 2004 - December 2008.

PDF OF THE ESTIMATION ERROR OF  $\alpha = 0.08$



PDF OF THE ESTIMATION ERROR OF  $\beta = 0.90$

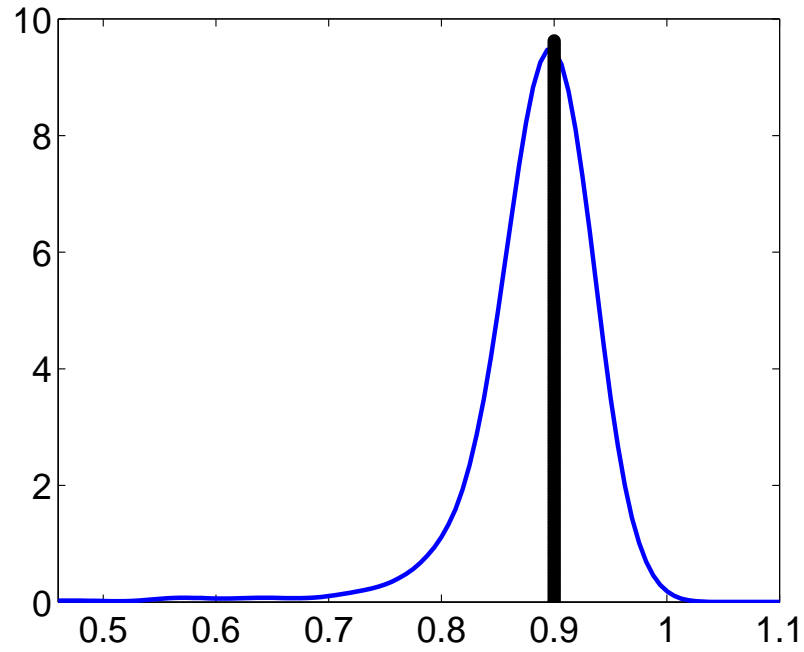


Figure 1: Probability distribution function (pdf) of the estimation error over 1000 simulation paths of the parameters of the GARCH(1,1) process for the factor conditional variance  $\sigma_t^2 = c + \alpha f_{t-1}^2 + \beta \sigma_{t-1}^2$ .



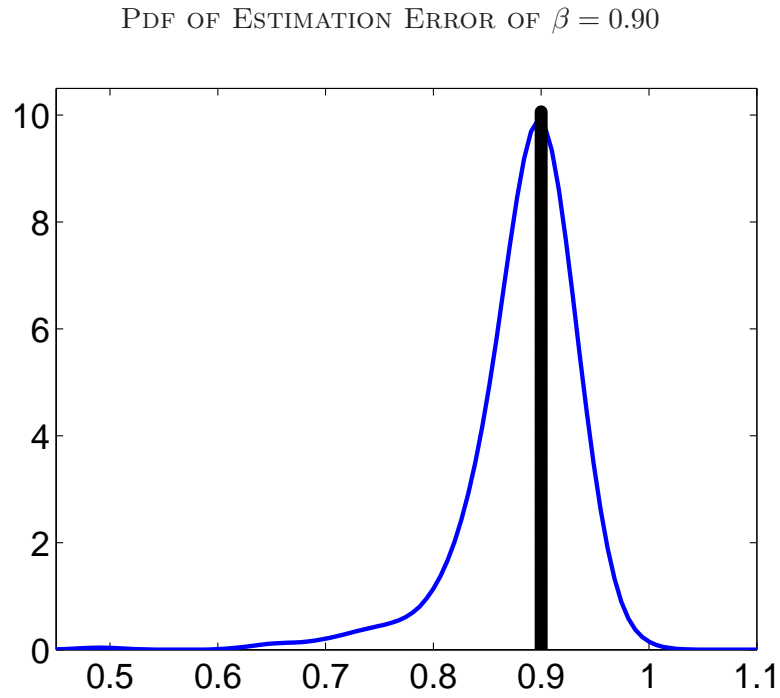
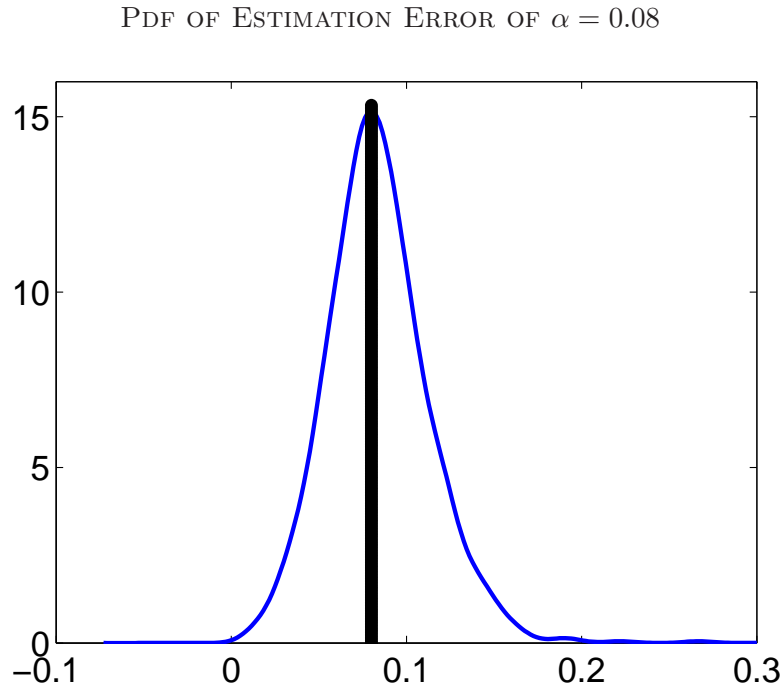
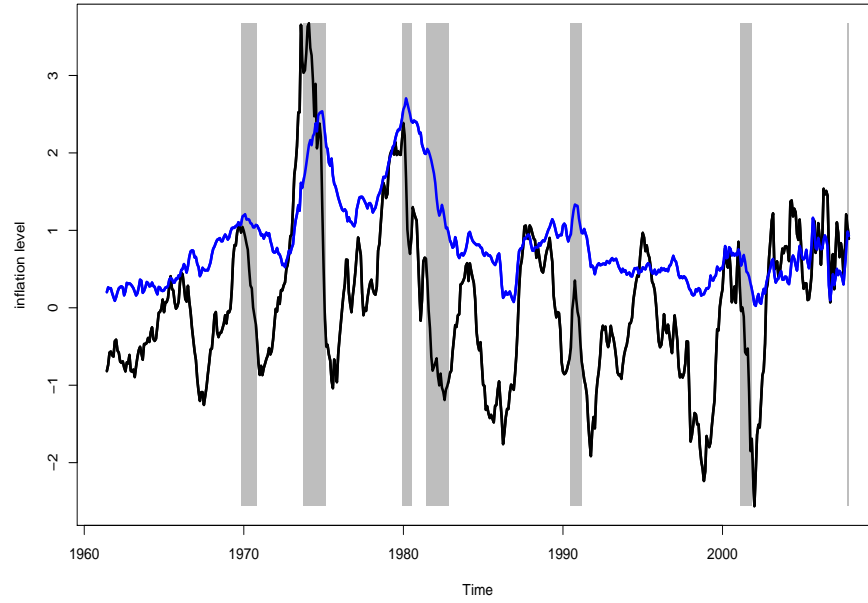


Figure 2: Probability distribution function of the estimation error over 1000 simulation paths of the parameters of the GARCH(1,1) process for the factor conditional variance  $\sigma_t^2 = c + \alpha f_{t-1}^2 + \beta \sigma_{t-1}^2$  in a DGP with time varying (GARCH type) idiosyncratic noise  $\Phi_t$ .

### INFLATION LEVEL



### INFLATION VOLATILITY

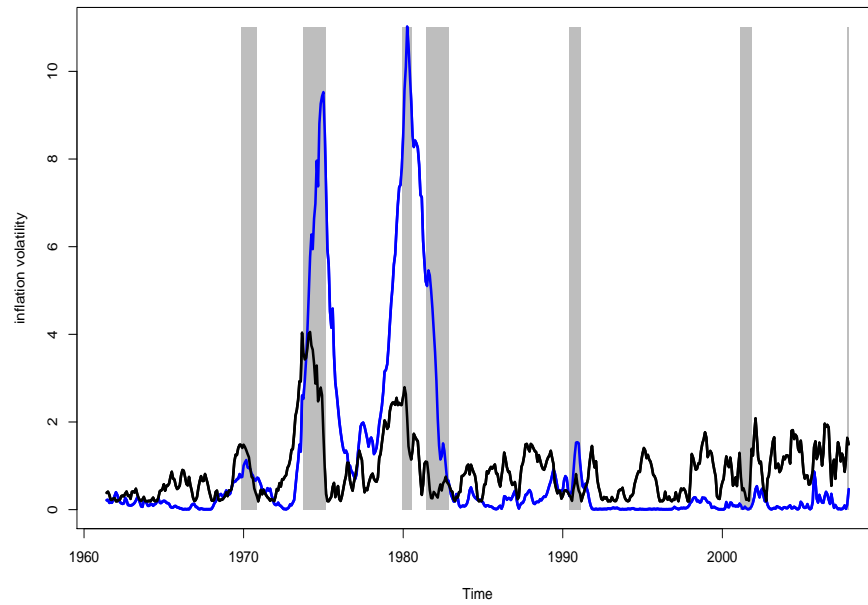


Figure 3: The upper panel plots the two estimates of inflation level: iterated Kalman filter (blue line) and PC (black line) based on a panel of 21 inflation time series, as described in text. The lower panel plots the inflation volatility filtered by the two techniques. Once again the blue line indicates the iterated Kalman filter estimate, whereas the black line represents the dynamics of the PC volatility. The shaded bars denote months designated as recessions by NBER.