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**Abstract**

This paper examines demand-enhancing investment and pricing in mixed duopoly. We analyze a model with differentiated products and reduced-form demand, making no assumptions on the relative efficiency of the public firm. First, we derive sufficient conditions for public investment to crowd out private investment. Second, we characterize the conditions under which individual investments (prices, respectively) in the mixed duopoly are higher (lower) than in the standard duopoly. Third, we show that with linear demand the public firm effectively disciplines the private firm, inducing an improvement in its price-quality ratio relative to the standard duopoly.

# 1  Introduction

In many markets, state-owned public firms compete with private firms. Well-known examples include public utilities (e.g., telecommunications, electric power, water, and gas), armaments, automobiles, banking, insurance, education, and medical care. The mixed-oligopoly literature has analyzed the functioning of such markets extensively, assuming that public firms maximize welfare rather than profits.[1] The public debate on the role of state-owned firms, however, conveys a less favorable view of public firms. In particular, there is a concern that public firms might *crowd out* (potentially more efficient) private firms because of their non-profit-oriented investment and pricing decisions. This concern is particularly relevant in industries where firms must make large demand-enhancing investments (e.g., in building network infrastructure, enhancing product design, ramping up advertising campaigns, etc.) before competing in the product market.[2] The empirical evidence on the impact of public investment on private investment is arguably mixed. David et al. (2000) conclude from a survey of the empirical evidence accumulated over the past 35 years that it is ambivalent whether public R&D is a complement or substitute for private R&D. It is thus surprising that the mixed-oligopoly literature has largely ignored demand-enhancing investment.

   In this paper, we introduce demand-enhancing investment by a public and a private firm into a mixed-oligopoly model. Specifically, we analyze a duopoly model with differentiated products and reduced-form demand functions, making no assumptions on the relative efficiency of the private and the public firm. We consider three different market configurations. In the *welfare benchmark*, the social planner chooses the prices and investments of

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[1]Important contributions to this literature include De Fraja and Delbono (1989), Cremer et al. (1991), Anderson et al. (1997), Matsumura (1998), Matsumura and Matsushima (2004), and Ishibashi and Matsumura (2006). We will provide a more detailed discussion of the related literature below.

[2]See, e.g., the articles "Roads to nowhere" (December 11, 2009) and "Paved with good intentions" (January 29, 2009) in *The Economist*.

both firms so as to maximize social welfare. In the *standard duopoly*, both firms maximize profits (i.e., the public firm "mimics" the private firm) and play a two-stage game where they simultaneously choose investments in stage 1 and prices in stage 2. In the *mixed duopoly*, firms play a two-stage game and simultaneously choose investments in stage 1 and prices in stage 2, but the public firm maximizes social welfare rather than profits.

We characterize equilibrium investments and pricing in each market configuration and derive the following main results: *First*, for public investment to crowd out private investment, it is sufficient that public investment reduces (i) the equilibrium price of the private firm, and (ii) the demand-enhancing effect of private investment. These effects both reduce the private firm's marginal returns to investment and therefore dampen its investment incentive.[3] *Second*, we demonstrate that the effect of welfare (rather than profit) maximization by the public firm on equilibrium investments and prices is generally ambiguous. In the linear demand model, for instance, the changes in investments and prices crucially depend on the substitutability among products. *Third*, to further study the role of the public firm for market performance, we examine the price-quality ratios offered by the public and the private firm in the linear demand model. We find that the public firm effectively disciplines the private firm in the mixed duopoly. In particular, we show that the price-quality ratios offered in the mixed duopoly are more favorable than those in the standard duopoly. In fact, the public firm's price-quality ratio in the mixed duopoly is even better than in the welfare benchmark (except for very high substitutability) to correct for the private firm's profit-maximization.

This paper contributes to the mixed-oligopoly literature initiated by Merrill and Schneider (1966). One strand of this literature focuses on imperfect price competition with differentiated products. Cremer et al. (1991) ana-

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[3]In the linear demand model, crowding out may occur only if public investment has a direct negative effect on the demand for the private firm's product.

lyze a Hotelling model with quadratic transportation costs and show that a mixed oligopoly with one public firm is socially preferable to a standard oligopoly only for two or more than six firms. Employing a CES model with endogenous entry, Anderson et al. (1997) study the effect of privatizing the public firm. These authors show that privatization increases welfare if the public firm makes a loss and suggest that profitable public firms should not necessarily be privatized. None of these papers analyzes demand-enhancing investments or considers reduced-form demand functions.

Another strand of the literature focuses on R&D investments by private and public firms. Delbono and Denicolò (1993) consider a mixed duopoly with an R&D race. Their key result is that the public firm can mitigate the standard overinvestment problem in R&D races, leading to higher social welfare. Poyago-Theotoky (1998) considers a setting where innovation is easily imitated such that free riding leads to an underinvestment problem. She shows that the public firm can alleviate the underinvestment problem but finds ambiguous welfare effects. Matsumura and Matsushima (2004) employ a Hotelling model where production costs are endogenous and firms can engage in cost-reducing activities. These authors show that the private firm has lower costs because it undertakes excessive cost-reducing activities. Ishibashi and Matsumura (2006) investigate a setting where a public research institute competes against profit-maximizing private firms. They use a patent race model where each firm chooses both its innovation size and R&D expenditure. These authors show that the innovation size (R&D expenditure) chosen by the public institute is too small (too large) from a social welfare perspective. It is important to note that none of these papers analyzes demand-enhancing investments.

The remainder of the paper is structured as follows. In Section 2, we introduce the analytical framework. In Section 3, we characterize equilibrium pricing and investment in the various market configurations. In Section 4, we derive our key results for the reduced-form model. In Section 5 we provide

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an extensive analysis of the linear demand model. Section 6 concludes.

## 2   Analytical Framework

We consider a duopoly model with a public firm 1 and a private firm 2 which produce horizontally differentiated products indexed by $i = 1, 2$. Each firm faces a reduced-form demand $D_i(\mathbf{p}, \boldsymbol{\theta})$, where $\mathbf{p} = (p_i, p_j), i \neq j$, is the vector of prices and $\boldsymbol{\theta} = (\theta_i, \theta_j), i \neq j$, reflects the respective product qualities.[4] Firms face constant marginal cost $c_i$ and can make demand-enhancing investments into quality at cost $F_i(\theta_i)$.

Throughout the analysis, we suppose that the following assumptions hold:

[**A1**] Products are demand substitutes and prices are strategic complements, i.e., $\partial D_i/\partial p_i < 0$, $\partial D_i/\partial p_j \geq 0$, $\partial D_i^2/\partial p_i^2 \leq 0$, and $\partial^2 D_i/(\partial p_i \partial p_j) \geq 0$, $i, j = 1, 2, i \neq j$.

[**A2**] Higher quality strictly increases own demand and weakly decreases demand for the other product, i.e., $\partial D_i/\partial \theta_i > 0$ and $\partial D_j/\partial \theta_i \leq 0$, $i, j = 1, 2, i \neq j$.

[**A3**] Firms face constant marginal costs $c_i \geq 0$ and investment costs $F_i(\theta_i)$, with $\partial F_i/\partial \theta_i > 0$ and $\partial F_i^2/\partial \theta_i^2 > 0$.

For later reference, we note that reduced-form firm profits are given by

$$\pi_i(p_i, p_j, \theta_i, \theta_j) = (p_i - c_i) \, D_i(p_i, p_j, \theta_i, \theta_j) - F_i(\theta_i), \quad i, j = 1, 2. \quad (1)$$

## 3   Alternative Market Configurations

We consider three market configurations that differ in terms of the firms' objective functions and the sequence of events. The benchmark configuration is

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[4]If product $i$'s quality encompasses multiple dimensions, $\theta_i$ should be interpreted as a real-valued index summarizing the various aspects of quality (cf. Buehler et al. (2006)).

the *welfare optimum*, where the social planner chooses prices and investments in markets 1 and 2 so as to maximize welfare. In the *standard duopoly*, firms 1 and 2 play a two-stage game, where investments are simultaneously chosen in stage 1 and prices are determined in stage 2. Both firms are assumed to maximize profits, that is, the public firm behaves as if it were a private firm. In the *mixed configuration*, firms also play a two-stage game, but the public firm 2 determines its choice variables $(p_2, \theta_2)$ so as to maximize social welfare.

## 3.1 Welfare Optimum

Let

$$\mathcal{W}(p_1, p_2, \theta_1, \theta_2) = \int_{p_1}^{\infty} D_1(\tilde{p}_1, p_2, \theta_1, \theta_2) d\tilde{p}_1 + \int_{p_2}^{\infty} D_2(\tilde{p}_2, p_1, \theta_2, \theta_1) d\tilde{p}_2$$
$$+ \pi_1(p_1, p_2, \theta_1, \theta_2) + \pi_2(p_2, p_1, \theta_2, \theta_1)$$

denote the welfare function, where the first two terms represent consumer surplus in markets 1 and 2, respectively, and the third and fourth term represent firm profits.

The first-order conditions for welfare-maximizing prices $\mathbf{p}^W = (p_1^W, p_2^W)$ and quality levels $\boldsymbol{\theta} = (\theta_1^W, \theta_2^W)$, respectively, are given by

$$\frac{p_i^W - c_i}{p_i^W} = \frac{\int_{p_j^W}^{\infty} \frac{\partial D_j^W}{\partial p_i} d\tilde{p}_j}{D_i^W \varepsilon_{ii}^W} - \frac{(p_j^W - c_j)\varepsilon_{ij}^W D_j^W}{R_i^W \varepsilon_{ii}^W} \tag{2}$$

and

$$(p_i^W - c_i)\frac{\partial D_i^W}{\partial \theta_i} + \int_{p_i^W}^{\infty} \frac{\partial D_i^W}{\partial \theta_i} d\tilde{p}_i + (p_j^W - c_j)\frac{\partial D_j^W}{\partial \theta_i} + \int_{p_j^W}^{\infty} \frac{\partial D_j^W}{\partial \theta_i} d\tilde{p}_j = \frac{\partial F_i^W}{\partial \theta_i}, \tag{3}$$

with $i, j = 1, 2, i \neq j$, where own- and cross-price elasticities are defined as

$$\varepsilon_{ii} \equiv -\frac{(\partial D_i/\partial p_i)p_i}{D_i} > 0 \qquad \text{and} \qquad \varepsilon_{ij} \equiv -\frac{(\partial D_j/\partial p_i)p_i}{D_j} \leq 0,$$

and the revenue in market $i$ is given by $R_i \equiv p_i D_i$. The superscript $W$ indicates welfare-maximizing quantities.

Inspection of condition (2) indicates that marginal-cost pricing ($p_i^W = c_i$) maximizes social welfare if markets $i$ and $j$ are independent ($\partial D_j^W/\partial p_i = \varepsilon_{ij}^W = 0$). If markets $i$ and $j$ are interdependent ($\partial D_j^W/\partial p_i > 0, \varepsilon_{ij}^W < 0$), however, optimal pricing in market $i$ must account for its effects on market $j$, leading to deviations from marginal-cost pricing. Specifically, a marginal increase in $p_i$ increases the demand for product $j$, affecting both consumer surplus (the first term on the r.h.s. of (2)) and firm profit in market $j$ (the second term). Since both the first and the second term are positive, welfare-optimal prices are strictly higher than marginal costs in the respective markets ($p_i^W > c_i$).[5] For later reference, we rewrite the first-order condition (2) as

$$\frac{p_i^W - c_i}{p_i^W} = \frac{1}{\varepsilon_{ii}^W} \left( Y_{ij}^W + X_{ij}^W \right), \tag{4}$$

where

$$Y_{ij} \equiv \frac{\int_{p_j}^{\infty} \frac{\partial D_j}{\partial p_i} d\tilde{p}_j}{D_i} \geq 0 \qquad \text{and} \qquad X_{ij} \equiv -\frac{(p_j - c_j)\varepsilon_{ij} D_j}{R_i} \geq 0,$$

summarize the mark-up of $p_i$ over $c_i$ because of the positive impact of $p_i$ on consumer and producer surplus in market $j$ (conditional on $\varepsilon_{ii}$), respectively.

According to condition (3), welfare-maximizing investment requires that the social benefits of investment equal social costs. If the investment in market $i$ does not directly affect demand in market $j$ (i.e., $\partial D_j^W/\partial \theta_i = 0$), the social benefits relate to the demand-enhancing effects in market $i$ only (the first two terms on the l.h.s. of (3)). If the investment in market $i$ also reduces the demand for product $j$ (i.e., $\partial D_j^W/\partial \theta_i < 0$), the welfare-maximizing investment is smaller because of the adverse effect on the other

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[5]Note that the second term on the r.h.s. of (2) is similar to the upward correction that a profit-maximizing multi-product monopolist applies to its Lerner index in market $i$ relative to a single-product monopolist (see, e.g., Tirole, 1988, p. 70).

product.

## 3.2  Standard Duopoly

In this configuration, both the private and the public firm maximize profits (i.e., the public firm 'mimics' the private firm). Firms simultaneously make demand-enhancing investments in stage 1 and compete in the product market in stage 2. The first-order condition for profit-maximizing pricing in stage 2 is given by

$$\frac{p_i^S - c_i}{p_i^S} = \frac{1}{\varepsilon_{ii}^S}, \quad i = 1, 2, \tag{5}$$

where the superscript $S$ denotes the standard duopoly market configuration.

Given the vector of investment levels $\boldsymbol{\theta} = (\theta_1, \theta_2)$ from stage 1, equilibrium prices in stage 2 are functions of these investments and characterized by the best-response functions $p_i^S(\boldsymbol{\theta}, p_j^S) = p_i^S, i \neq j$. With equilibrium prices denoted as $\mathbf{p}^S(\boldsymbol{\theta}) = (p_1^S(\boldsymbol{\theta}), p_2^S(\boldsymbol{\theta}))$, the profit-maximizing investment solves the problem

$$\max_{\theta_i} \pi_i(\boldsymbol{\theta}) = \left(p_i^S(\boldsymbol{\theta}) - c_i\right) D_i(\mathbf{p}^S(\boldsymbol{\theta}), \boldsymbol{\theta}) - F_i^S(\theta_i).$$

For the characterization of the first-order condition, it is useful to introduce the following notation.

**Notation 1 (demand effect)**  *The total differential of demand in market $i$ with respect to a marginal quality change in market $j$ is denoted as*

$$\widehat{D}_{i,j}^k = \frac{\partial D_i^k}{\partial p_i} \frac{\partial p_i}{\partial \theta_j} + \frac{\partial D_i^k}{\partial p_j} \frac{\partial p_j}{\partial \theta_j} + \frac{\partial D_i^k}{\partial \theta_j},$$

*with $k$ indicating the relevant market configuration.*

Using Notation 1 and applying the envelope theorem, the first-order condition

can be written as

$$(p_i^S - c_i)\widehat{D}_{i,i}^S = \frac{\partial F_i^S}{\partial \theta_i}. \tag{6}$$

## 3.3  Mixed Duopoly

In the mixed duopoly, the private firm 1 maximizes profits, whereas the public firm 2 chooses its price and investment so as to maximize social welfare. The key difference to the welfare benchmark in Subsection 3.1 is that the social planner *cannot* determine firm 1's pricing and investment.

We first consider pricing in stage 2. Note that firm 1's pricing rule is similar to (5) in the standard duopoly, whereas firm 2's pricing rule is similar to (4) under welfare maximization. More formally, we have

$$\frac{p_1^M - c_1}{p_1^M} = \frac{1}{\varepsilon_{11}^M}, \quad i = 1, 2, \tag{7}$$

and

$$\frac{p_2^M - c_2}{p_2^M} = \frac{1}{\varepsilon_{22}^M} \left( Y_{21}^M + X_{21}^M \right), \tag{8}$$

where the superscript $M$ indicates the mixed duopoly configuration.

Next, consider investment in stage 1. The first-order condition of the private firm is again similar to the standard duopoly,

$$(p_1^M - c_1)\widehat{D}_{1,1}^M = \frac{\partial F_1^M}{\partial \theta_1}. \tag{9}$$

Using Notation 1 and applying the envelope theorem for the public firm, the first-order condition for welfare-maximizing public investment can be written as

$$(p_2^M - c_2)\widehat{D}_{2,2}^M + \int_{p_2^M}^{\infty} \widehat{D}_{2,2}^M d\tilde{p}_2 + (p_1^M - c_1)\widehat{D}_{1,2}^M + \int_{p_1^M}^{\infty} \widehat{D}_{1,2}^M d\tilde{p}_1 = \frac{\partial F_2^M}{\partial \theta_2}. \tag{10}$$

The key difference to (3) in the welfare benchmark is that a change in the pub-

lic firm's investment $\theta_2$ now also generates price-mediated demand effects via $p_1$, whereas there are only direct demand effects in the welfare benchmark.

# 4    Results

In this section, we present our key results for the model with reduced-form demand. First, we derive sufficient conditions for public investment to crowd out private investment. Second, we examine how the public firm's welfare (rather than profit) maximizing behavior affects equilibrium prices and investments in quality, respectively.

Our first result gives sufficient conditions for public investment to crowd out private investment.

**Proposition 1 (crowding out)** *Consider market configuration $k = S, M$. For the crowding out of private investment $(d\theta_1^k/d\theta_2 < 0)$, it is sufficient that public investment*

(i) *decreases the equilibrium price of the private firm $(\partial p_1^k/\partial\theta_2 < 0)$, and*

(ii) *(weakly) decreases the demand-enhancing effect of private investment $(\partial\widehat{D}_{1,1}^k/\partial\theta_2 \leq 0)$.*

**Proof.** By the implicit function theorem, public investment crowds out private investment if and only if $d\theta_1/d\theta_2\big|_k = -\frac{\partial^2\pi_1^k/(\partial\theta_1\partial\theta_2)}{\partial^2\pi_1^k/\partial\theta_1^2} < 0$. Since the denominator is negative in a profit maximum, this condition is equivalent to

$$\frac{\partial^2\pi_1^k}{\partial\theta_1\partial\theta_2} = \frac{\partial p_1^k}{\partial\theta_2}\widehat{D}_{1,1}^k + (p_1^k - c_1)\frac{\partial\widehat{D}_{1,1}^k}{\partial\theta_2} < 0, \quad k = S, M.$$

Now, observe that $(p_1^k - c_1) > 0$ from (5) or (7), respectively, and $\widehat{D}_{1,1}^k > 0$ from $\partial F_i^k/\partial\theta_i > 0$ by [A3] and (6) or (9). Conditions (i) and (ii) thus jointly guarantee that $\partial^2\pi_1^k/(\partial\theta_1\partial\theta_2) < 0$. ■

Conditions (i) and (ii) of Proposition 1 jointly guarantee that the marginal returns to private investment are decreasing in public investment, such that investments are strategic substitutes from the private firm's point of view $(\partial^2 \pi_1^k / (\partial \theta_1 \partial \theta_2) < 0)$.[6]

Proposition 1 highlights that public investment is likely to crowd out private investment if it (i) reduces the equilibrium price that the private firm can charge for its differentiated product, and (ii) undermines the effectiveness of private investment in generating demand for its own product. These effects both reduce the private firm's marginal returns to investment and therefore dampen its investment incentive. Intuitively, conditions (i) and (ii) are likely to be satisfied if products are close substitutes and investments lead to business stealing.

Next, consider how the public firm's welfare (rather than profit) maximizing behavior affects equilibrium prices.

**Proposition 2 (pricing)** *Changing the market configuration from S to M*

(i) *reduces the private firm's price if $\varepsilon_{11}^M / \varepsilon_{11}^S > 1$;*

(ii) *reduces the public firm's price if $\varepsilon_{22}^M / \varepsilon_{22}^S > Y_{21}^M + X_{21}^M > 0$.*

**Proof.** (i) Rewriting $p_1^S > p_1^M$ in terms of Lerner indices yields

$$\frac{p_1^S - c_1}{p_1^S} = \frac{1}{\varepsilon_{11}^S} > \frac{p_1^M - c_1}{p_1^M} = \frac{1}{\varepsilon_{11}^M}.$$

The claim now follows immediately.

(ii) Rewriting $p_2^S > p_2^M$ yields

$$\frac{p_2^S - c_2}{p_2^S} = \frac{1}{\varepsilon_{22}^S} > \frac{p_2^M - c_2}{p_2^M} = \frac{1}{\varepsilon_{22}^M} \left( Y_{21}^M + X_{21}^M \right).$$

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[6]Note that Proposition 1 does not place any restrictions on the public firm's profit function.

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The result follows immediately. ∎

Condition (i) highlights that the welfare maximization of the public firm reduces the private firm's price (relative to the standard duopoly) if the price elasticity—evaluated at the relevant equilibrium quantities—is higher. For a reduction of the public firm's price, the increase in the price elasticity must dominate any price-increasing externalities to market 1. Proposition 2 thus suggests that the impact of the public firm's welfare maximization on equilibrium prices is not clear-cut and crucially depends on the properties of the demand functions.[7]

Finally, consider how the public firm's welfare (rather than profit) maximizing behavior affects equilibrium investment.

**Proposition 3 (investment)** *Changing the market configuration from $S$ to $M$*

*(i) increases private investment if*

$$(p_1^M - c_1)\widehat{D}_{1,1}^M > (p_1^S - c_1)\widehat{D}_{1,1}^S; \tag{11}$$

*(ii) increases public investment if*

$$\int_{p_2^S}^{\infty} \widehat{D}_{2,2}^S d\tilde{p}_2 + (p_1^S - c_1)\widehat{D}_{1,2}^S + \int_{p_1^S}^{\infty} \widehat{D}_{1,2}^S d\tilde{p}_1 > 0. \tag{12}$$

**Proof.** (i) The investment incentive of firm 1 is given by $(p_1^k - c_1)\widehat{D}_{1,1}^k, k = S, M$. Condition (11) guarantees that the investment incentive increases with a change from $S$ to $M$.

(ii) From (10), firm 2's first-order condition in market configuration $M$ is

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[7]In Section 5 below, we will show that the price effects of changing the market configuration from $S$ to $M$ are subtle even in the linear demand case.

given by

$$(p_2^M - c_2)\widehat{D}_{2,2}^M - \frac{\partial F_2^M}{\partial \theta_2} + \int_{p_2^M}^{\infty} \widehat{D}_{2,2}^M d\tilde{p}_2 + (p_1^M - c_1)\widehat{D}_{1,2}^M + \int_{p_1^M}^{\infty} \widehat{D}_{1,2}^M d\tilde{p}_1 = 0.$$

The investment incentive is higher than under market configuration $S$ if, evaluated at $S$ quantities,

$$(p_2^S - c_2)\widehat{D}_{2,2}^S - \frac{\partial F_2^S}{\partial \theta_2} + \int_{p_2^S}^{\infty} \widehat{D}_{2,2}^S d\tilde{p}_2 + (p_1^S - c_1)\widehat{D}_{1,2}^S + \int_{p_1^S}^{\infty} \widehat{D}_{1,2}^S d\tilde{p}_1 > 0,$$

where $(p_2^S - c_2)\widehat{D}_{2,2}^S - \partial F_2^S/\partial \theta_2 = \partial \pi_2^S/\partial \theta_2 = 0$ from (6). This completes the proof. ∎

Condition (i) of Proposition 3 states that private investment in the mixed duopoly is strictly higher than in the standard duopoly if the marginal investment incentive—evaluated at $M$ rather than $S$ quantities—is strictly higher. Condition (ii) follows from the argument that public investment in $M$ must be strictly higher than in $S$ if the marginal investment incentive—evaluated at $S$ quantities (such that $\partial \pi_2^S/\partial \theta_2 = 0$)—is strictly positive.

Beyond Propositions 1–3, little can be said about equilibrium pricing and investment in the reduced-form demand model. In the next section, we therefore analyze the linear demand model where we can derive closed-form solutions for these variables.

## 5   The Linear Demand Model

Let us now consider the linear model and suppose, for simplicity, that the demand for product $i$ does not directly depend on firm $j$'s demand-enhancing investment (i.e., $\partial D_i/\partial \theta_j = 0$).[8] Specifically, we assume that demand is given by

$$D_i(p_i, p_j, \theta_i) = \alpha - \beta p_i + \gamma p_j + \theta_i, \quad \alpha, \beta, \gamma > 0 \tag{13}$$

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[8]We will discuss below how allowing for such a direct effect affects the results.

where $\alpha, \beta$ and $\gamma$ are exogenous parameters and $\beta > \gamma$, that is, demand is more responsive to a change in own price than to a change in the competitor's price. For simplicity, we assume that marginal costs are constant and normalized to zero ($c_1 = c_2 = 0$), whereas investment costs are given by $F_i(\theta_i) = \theta_i^2$. Note that this model satisfies assumptions [A1]–[A3].

Table 1 summarizes the equilibrium prices $p_i^k$ and qualities $\theta_i^k$, as well as the corresponding price-quality ratios $r_i^k \equiv p_i^k/\theta_i^k, k = W, S, M$, as functions of the model parameters.

<center>&lt;Table 1 around here&gt;</center>

We now derive a number of results for the linear demand model that illustrate Propositions 1-3 above.

**Result 1 (crowding in)** *Suppose demand is linear and given by (13). Then,*

(i) *in the standard duopoly, public investment enhances private investment* $(\partial^2 \pi_1^S/(\partial\theta_1\partial\theta_2) > 0)$.

(ii) *in the mixed duopoly, public investment does not affect private investment* $(\partial^2 \pi_1^M/(\partial\theta_1\partial\theta_2) = 0)$.

**Proof.** (i) Using (13), straightforward calculations yield $\partial p_1^S/\partial\theta_2 = \gamma/(4\beta^2 - \gamma^2) > 0$ and $\partial \widehat{D}_{1,1}^S/\partial\theta_2 = 0$, implying $\partial^2 \pi_1^S/(\partial\theta_1\partial\theta_2) > 0$. (ii) Similarly, $\partial p_1^M/\partial\theta_2 = 0$ and $\partial \widehat{D}_{1,1}^M/\partial\theta_2 = 0$ yield $\partial^2 \pi_1^M/(\partial\theta_1\partial\theta_2) = 0$ (see Proposition 1). ∎

To understand the intuition for Result 1, first note that public investment cannot affect the demand-enhancing effect of private investment in the linear demand model $(\partial \widehat{D}_{1,1}^k/\partial\theta_2 = 0, k = S, M)$. Therefore, the only effect that public investment may have on private investment is price-mediated: Because of strategic complementary in prices by Assumption [A1], public investment (weakly) increases the prices of both the public firm $(\partial p_2^M/\partial\theta_2 > 0)$ and

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the private firm $(\partial p_1^M / \partial \theta_2 \geq 0)$. Part (i) of Result 1 shows that the strategic complementarity in prices carries over to investments in the standard duopoly $(\partial p_2^S / \partial \theta_2 > 0)$. Part (ii) of Result 1, in turn, highlights that public investment does not affect private investment in the mixed duopoly. This follows from the fact that, for a welfare-maximizing public firm, the direct extra revenues from a marginal price increase, $D_2^M$, cancel against the direct extra expenses by consumers, $D_2^M$, such that its first-order condition for optimal pricing does not depend on $\theta_2$ in the linear demand model. As a result, the price of the private firm does not react to changes in public investment $(\partial p_1^M / \partial \theta_2 = 0)$, leaving the private firm's investment incentive unaffected.

Before proceeding, it is worth noting that Result 1 crucially relies on the assumption that demand-enhancing investment in market $j$ does not directly affect the demand for product $i$ (i.e., $\partial D_i / \partial \theta_j = 0$). Depending on parameter values, a negative cross-effect $(\partial D_i / \partial \theta_j < 0)$ might dominate the (weakly) positive price-mediated effect of public investment, leading to crowding out both in the standard and the mixed duopoly.

Let us now consider the price changes associated with changes in market configuration. For the linear demand model, we can directly compare prices across all three market configurations, accounting for the associated changes in investments.[9] Figure 1 plots the closed-form solutions for the equilibrium prices $p_i^k, i = 1, 2; k = W, S, M$, reported in Table 1, using the parameter values $\alpha = 1/2$ and $\beta = 1$.[10]

<Figure 1 around here>

We first study the price changes associated with a change in market configuration from $S$ to $M$. Figure 1 highlights that the effects on the prices of the private and the public firm crucially depend on the level of $\gamma$. Changing

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[9]We will discuss these changes in investments below.

[10]Choosing other parameter values does not affect the qualitative results of the analysis. Since we focus on positive equilibrium prices and investments, we restrict attention to $\gamma \in [0, 0.7]$.

from $S$ to $M$ decreases (increases) both prices for low (high) values of $\gamma$. For intermediate values of $\gamma$, the public firm's price falls, whereas the private firm's price increases. Similarly, there is no clear-cut relation between $p_1^M$ and $p_2^M$: We find $p_2^M < p_1^M$ for low values of $\gamma$ and the reversed inequality for high values of $\gamma$.

Next, consider the welfare-maximizing price in the mixed duopoly and the welfare optimum, respectively. Figure 1 indicates that $p_2^M \leq p_2^W$ for any admissible $\gamma$. That is, in the mixed duopoly, the public firm's price is consistently below the benchmark price in the welfare optimum. This result follows from the need of the welfare-maximizing firm to distort its pricing downwards to correct for the profit-maximizing behavior of its competitor in the mixed duopoly.

The following result summarizes these findings.

**Result 2 (pricing)** *Suppose demand is linear and given by (13). Then,*

(i) *changing the market configuration from $S$ to $M$ may increase or decrease the prices of both the private and the public firm, depending on $\gamma$.*

(ii) *in the mixed duopoly, the public firm distorts the welfare-maximizing price downwards to correct for the private firm's profit-maximizing behavior.*

Next, let us compare the firms' investments across market configurations. Figure 2 plots the closed-form solutions for the respective quality levels from Table 1.

<Figure 2 around here>

Inspection of Figure 2 indicates that firms consistently underinvest in the standard duopoly relative to the welfare optimum ($\theta_i^S < \theta_i^W, i = 1, 2$). In the mixed duopoly, only the private firm consistently underinvests ($\theta_1^M <$

$\theta_1^W$), whereas the public firm overinvests ($\theta_2^M \geq \theta_2^W$) for low values of $\gamma$ and underinvests ($\theta_2^M < \theta_2^W$) for high values of $\gamma$. That is, in addition to distorting its pricing downwards, the public firm distorts its quality upwards (downwards) for low (high) $\gamma$. It is also worth noting that the private firm's quality in the mixed duopoly ($\theta_1^M$) tends to be higher than its quality in the standard duopoly ($\theta_i^S$). The next result summarizes these findings.

**Result 3 (investment)** *Suppose demand is linear and given by (13). Then,*

(i) *in the standard duopoly, both firms strictly underinvest ($\theta_i^S < \theta_i^W, i = 1, 2$).*

(ii) *in the mixed duopoly, the private firm strictly underinvests ($\theta_1^M < \theta_1^W$), whereas the public firm distorts investment upwards (downwards) for low (high) $\gamma$ to correct for the private firm's profit-maximizing behavior.*

Finally, we consider the price-investment ratios $r_i^k = p_i^k/\theta_i^k$ across market configurations. Figure 3 plots the corresponding closed-form solutions from Table 1.

<Figure 3 around here>

We first focus on the price-quality ratio $r_i^W$ offered in the welfare optimum. Figure 3 illustrates that this ratio is linearly increasing in $\gamma$, that is, the price-quality ratio gets worse for closer substitutes. This is in marked contrast to the price-quality ratio $r_i^S$ in the standard duopoly, which is monotone decreasing (i.e., "getting better") in $\gamma$. Intuitively, the result follows from the social planner's internalization of the externalities between the two markets.

Next, consider the impact of a change from $S$ to $M$ on the ratios offered by the private and the public firm (as functions of $\gamma$): The locus of the private firm's price-quality ratio $r_1^M$ is rotated downwards, whereas the locus of the public firm's price-quality ratio $r_2^M$ is becoming strictly convex in $\gamma$.

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More specifically, we find that the change from $S$ to $M$ leads to an improvement of the price-quality ratio offered by the private firm for any admissible $\gamma$. In addition, it distorts the price-quality ratio offered by the public firm downwards (upwards) for low (high) values of $\gamma$. That is, for low values of $\gamma$, the mixed duopoly offers price-quality ratios that are even better than in the welfare optimum. The intuition is, again, that the welfare-maximizing public firm must correct for the profit-maximizing behavior of the private firm in the mixed duopoly.

The next result summarizes these findings.

**Result 4 (price-quality ratios)** *Suppose demand is linear and given by (13). Then,*

(i) *changing the market configuration from $S$ to $M$ induces the private firm to offer a better price-quality ratio ($r_1^M \leq r_1^S$) for any admissible $\gamma$.*

(ii) *in the mixed duopoly, the public firm offers an even better price-quality ratio than the welfare benchmark ($r_1^M \leq r_1^W$) for low $\gamma$ to correct for the private firm's profit-maximizing behavior.*

# 6    Conclusion

This paper has introduced demand-enhancing investment into a mixed-duopoly model with reduced-form demand functions. Analyzing a model with product differentiation and making no assumptions on the relative efficiency of the public firm, we have derived the following key results. First, public investment crowds out private investment if it (i) reduces the equilibrium price of the private firm, and (ii) undermines the demand-enhancing effect of private investment. These effects reduce the private firm's marginal returns to investment and therefore dampen its investment incentive. Second, the effect of the public firm's welfare (rather than profit) maximization on equilibrium investments and prices is generally ambiguous and depends on the details of

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the demand functions. In the linear demand case, for instance, the sign of these effects depends on the value of the substitutability parameter. Third, with linear demand, the presence of a public firm effectively disciplines the private firm. The price-quality ratios offered by the private and the public firm are better than in the standard duopoly, and the public firm's price-quality ratio is even better than in the welfare optimum to discipline the profit-maximizing private firm.

Our analysis indicates that, depending on demand conditions, the impact of public investment on private investment and market performance may vary considerably across markets. This is consistent with the ambivalent empirical findings discussed in David et al. (2000). The model also suggests that public investment is more likely to crowd out private investment if products are close substitutes or public investment has a direct negative (business-stealing) effect on the demand of the private firm (as illustrated for the case with linear demand). These insights provide guidance for the practical assessment of whether public investment is likely to crowd out private investment in a specific market.

Let us conclude by noting that, in a standard mixed-oligopoly setting, it is not clear why the crowding out of private investment should be prevented. In fact, the crowding out (if any) is a very consequence of the public firm's welfare-maximizing behavior. Crowding out is arguably less harmless if the public firm does adhere to some political agenda rather than maximize social welfare. We hope to address this issue in future research.

# References

Anderson, S. P., A. de Palma, and J.-F. Thisse (1997), "Privatization and efficiency in a differentiated industry." *European Economic Review*, 41, 1635–1654.

Buehler, S., D. Gaertner, and D. Halbheer (2006), "Deregulating Network Industries: Dealing with Price-Quality Tradeoffs." *Journal of Regulatory Economics*, 30, 99–115.

Cremer, H., M. Merchand, and J.-F. Thisse (1991), "Mixed oligopoly with differentiated products." *International Journal of Industrial Organization*, 9, 43–53.

David, P. A., B. H. Hall, and A. A. Toole (2000), "Is public R&D a complement or substitute for private R&D? A review of the econometric evidence." *Research Policy*, 29.

De Fraja, G. and F. Delbono (1989), "Alternative strategies of a public enterprise in oligopoly." *Oxford Economic Papers*, 41, 302–311.

Delbono, F. and V. Denicolò (1993), "Regulating innovative activity: The role of a public firm." *International Journal of Industrial Organization*, 11, 35–48.

Ishibashi, I. and T. Matsumura (2006), "R&D competition between public and private sector." *European Economic Review*, 50, 1347–1366.

Matsumura, T. (1998), "Partial privatization in mixed duopoly." *Journal of Public Economics*, 70, 473–483.

Matsumura, T. and N. Matsushima (2004), "Endogenous Cost Differentials between Public and Private Enterprises: A Mixed Duopoly Approach." *Economica*, 71, 671–688.

Poyago-Theotoky, J. (1998), "R&D Competition in a Mixed Duopoly under Uncertainty and Easy Imitation." *Journal of Comparative Economics*, 26, 415–428.

Tirole, J. (1988), "The Theory of Industrial Organization." *Massachusetts Institute of Technology.*

|  | **Welfare Benchmark** $(k = W)$ | **Standard Duopoly** $(k = S)$ | **Mixed Duopoly** $(k = M)$ |
|---|:---:|:---:|:---:|
| **Prices** | | | |
| Private Firm: $p_1^k$ | $\dfrac{2\alpha\gamma}{2(\beta^2-\gamma^2)-\beta}$ | $\dfrac{\alpha\left(4\beta^2-\gamma^2\right)}{8\beta^3-2\beta^2(2\gamma+1)-\gamma^2(2\beta-\gamma)}$ | $\dfrac{2\alpha\beta\left(\beta^2-\gamma^2\right)}{4(\beta^2-\gamma^2)^2-\beta^3}$ |
| Public Firm: $p_2^k$ | $\dfrac{2\alpha\gamma}{2(\beta^2-\gamma^2)-\beta}$ | $\dfrac{\alpha\left(4\beta^2-\gamma^2\right)}{8\beta^3-2\beta^2(2\gamma+1)-\gamma^2(2\beta-\gamma)}$ | $\dfrac{4\alpha\gamma\left(\beta^2-\gamma^2\right)}{4(\beta^2-\gamma^2)^2-\beta^3}$ |
| **Qualities** | | | |
| Private Firm: $\theta_1^k$ | $\dfrac{\alpha\beta}{2(\beta^2-\gamma^2)-\beta}$ | $\dfrac{2\alpha\beta^2}{8\beta^3-2\beta^2(2\gamma+1)-\gamma^2(2\beta-\gamma)}$ | $\dfrac{\alpha\beta^3}{4(\beta^2-\gamma^2)^2-\beta^3}$ |
| Public Firm: $\theta_2^k$ | $\dfrac{\alpha\beta}{2(\beta^2-\gamma^2)-\beta}$ | $\dfrac{2\alpha\beta^2}{8\beta^3-2\beta^2(2\gamma+1)-\gamma^2(2\beta-\gamma)}$ | $\dfrac{\alpha\left(4\left(\beta^2-\gamma^2\right)^2+2\beta\gamma\left(\beta^2-\gamma^2\right)-\beta^3\right)}{\left(4(\beta^2-\gamma^2)^2-\beta^3\right)(2\beta-1)}$ |
| **Price-Quality Ratios** | | | |
| Private Firm: $r_1^k = p_1^k/\theta_1^k$ | $\dfrac{2\gamma}{\beta}$ | $\dfrac{4\beta^2-\gamma^2}{2\beta^2}$ | $\dfrac{4\left(\beta^2-\gamma^2\right)}{2\beta^2}$ |
| Public Firm: $r_2^k = p_2^k/\theta_2^k$ | $\dfrac{2\gamma}{\beta}$ | $\dfrac{4\beta^2-\gamma^2}{2\beta^2}$ | $\dfrac{4\gamma(2\beta-1)\left(\beta^2-\gamma^2\right)}{4(\beta^2-\gamma^2)^2+2\beta\gamma(\beta^2-\gamma^2)-\beta^3}$ |

Table 1: Linear Demand $(c_1 = c_2 = 0)$

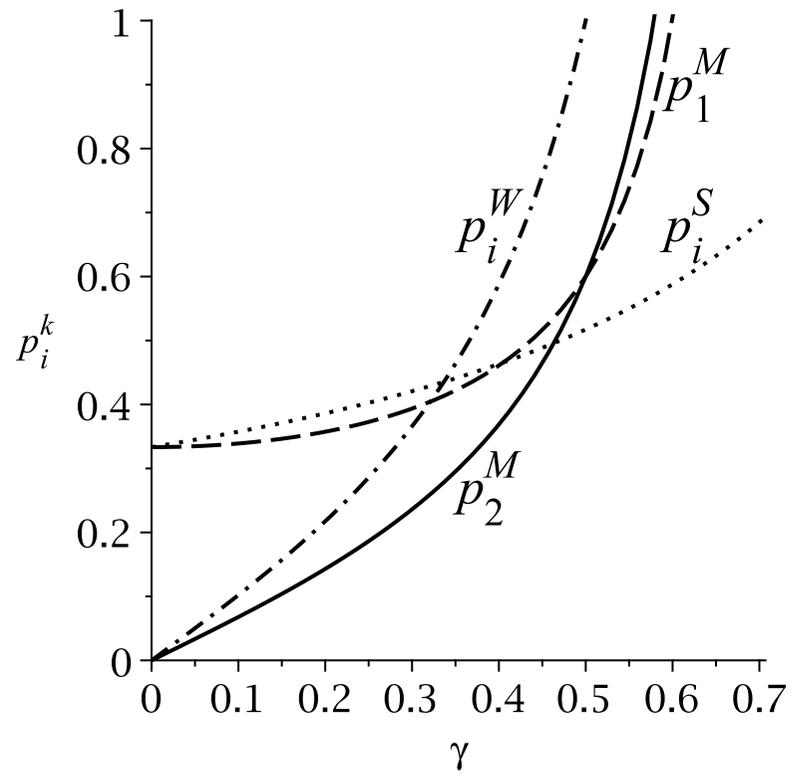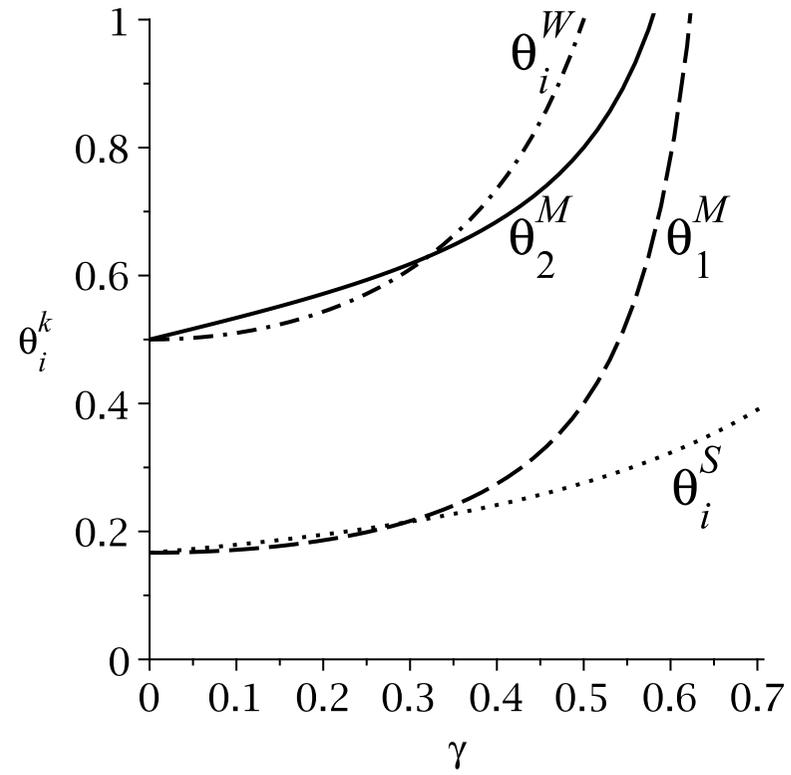Figure 1: Prices in the three market configurations ($\alpha = 1/2$, $\beta = 1$ and $c_1 = c_2 = 0$)

Figure 2: Qualities in the three market configurations ($\alpha = 1/2$, $\beta = 1$ and $c_1 = c_2 = 0$)
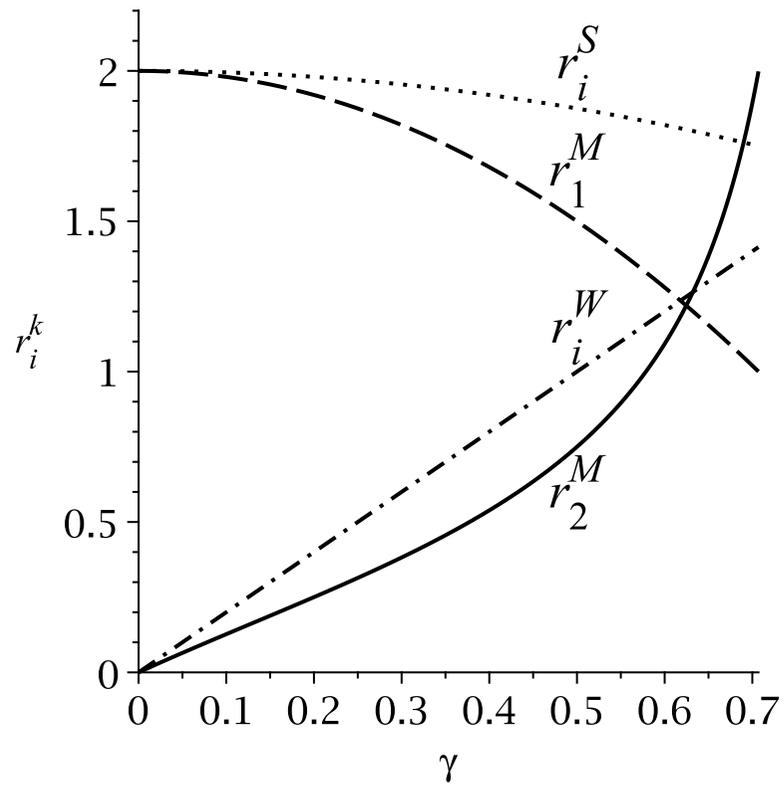
Figure 3: Price-Quality ratios in the three market configurations ($\alpha = 1/2$, $\beta = 1$ and $c_1 = c_2 = 0$)