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Abstract

This study investigates loss aversion when the reference point is a state-dependent random variable. This case describes, for example, a money manager being evaluated relative to a risky benchmark index rather than a fixed target return level. Using a state-dependent structure, prospects are more (less) attractive if they depend positively (negatively) on the reference point. In addition, the structure avoids an inherent aversion to risky prospects and yields no losses when the prospect and the reference point are the same. Related to this, the optimal reference-dependent solution equals the optimal consumption solution (no loss aversion) when the reference point is selected completely endogenously. Given that loss aversion is widespread, we conclude that the reference point generally includes an important exogenously fixed component. For example, the typical investment benchmark index is externally fixed by the investment principal for the duration of the investment mandate. We develop a choice model where adjustment costs cause stickiness relative to an initial exogenous reference point.

Keywords

Reference-dependent preferences, stochastic reference point, loss aversion, disappointment theory, regret theory.

JEL Classification

D81, C23, C91, C93.

1 Introduction

A key problem of reference-dependent choice theories is the specification of the relevant reference point. Traditionally, the reference point is interpreted as an exogenously fixed and constant value, for example, the current wealth level of the decision maker. Recent studies have examined risky choice with an endogenous and/or stochastic reference point. Shalev (2000) allows the reference point to be determined endogenously as part of the decision-maker's optimization problem. Sugden (2003) allows the reference point to be a random variable rather than a constant; see also Schmidt, Starmer, and Sugden (2008). Using a stochastic reference point is reminiscent of measuring the investment performance of a money manager relative to a risky benchmark portfolio like the S&P 500 index rather than a fixed target return. Köszegi and Rabin (2006, 2007) combine both ideas and use a reference point that is both endogenous and stochastic. This paper analyzes an alternative model of stochastic reference points. To simplify the exposition and discussion, and for the sake of comparison, we largely adhere to the assumptions and terminology of Köszegi and Rabin (2006, 2007), but our conclusions apply more generally.

The Köszegi and Rabin (2006) model basically builds on disappointment theory (see, for example, Bell 1985, Loomes and Sugden 1986, Gul 1991, Cillo and Delquié 2006). It assumes that the decision maker compares every possible outcome of a given prospect with every possible outcome of the reference point. The decision maker therefore experiences loss (disappointment) when the outcome of the prospect in a given state-of-the-world falls below the outcome of the reference point in other states. By contrast, the Sugden (2003) model builds on regret theory (Loomes and Sugden 1982, Bell 1982, 1983). The decision maker compares the prospect and the reference point only in the same state and not across states and experiences loss (regret) only if the outcome of the prospect falls below the outcome of the reference point in the same state. For many applications, the latter, regret-based or state-dependent preference structure seems more plausible than the former, disappointment-based structure. For example, for the money manager who benchmarks against the market index, the most relevant reference point for the realized portfolio value in a given period seems to be the realized value of the market index in the same period, and the value in other states-of-the-world seems less relevant. This study therefore examines loss aversion with a state-dependent reference point and the endogenous selection of the reference point. The analysis yields a number of surprising insights.

First, the disappointment-based structure implies that the decision maker is indifferent to the statistical dependency between the prospect and the reference point. A prospect that is positively correlated with the reference point is seen as equally risky as an uncorrelated or negatively correlated prospect. Intuitively, it seems that a prospect is more attractive if it depends positively on the reference point and is less attractive in case of negative dependence. For example, for the money manager who benchmarks against the market index, long positions in stocks generally will feel safer and entail smaller gains and losses than holding short positions in the same stocks, although the two positions yield a comparable *univariate* risk profile. In fact, perfectly replicating the market index creates a perfectly positive dependence with the reference point and avoids all possible losses. The state-dependent model captures this intuition, and a prospect that is positively correlated with the reference point will appear to be safer and causes smaller losses, while a negative correlation will feel riskier and yield larger losses.

Second, across-state comparison introduces an aversion to risky prospects, which will yield losses even when the prospect and the reference point are the same. In many cases, the reference point is exogenously fixed (in part or in whole). For example, the reference point may be set by an external principal, as is true for a benchmark index in an external investment mandate. Alternatively, the decision maker may adjust slowly to new information or surprise events, for example, an unexpected change in the composition of the benchmark index. In these cases, it seems natural that loss aversion influences behavior and leads to different behavior than a reference-independent model. By contrast, when the reference point is completely endogenous, we may expect that it equals the optimal solution to the reference-independent choice problem and therefore loss aversion does not influence behavior. However, this is not true for the disappointment-based model: reference-dependent behavior generally deviates from reference-independent behavior, even if the reference point is completely endogenous. By contrast, the optimal solution in the state-dependent model equals the reference-independent solution if the reference point is fully endogenous. Loss aversion influences behavior only if the reference point includes an exogenous component and the decision maker is not entirely free to select the reference-independent solution as her reference point. Our model captures this exogenous component using costly adjustment from an initial, exogenous reference point.

Like Köszegi and Rabin (2006), our analysis does not account for subjective probability weighting. Since probability weighting is known to be strong even for simple fifty-fifty gambles with a constant reference point, it seems unlikely that a model with a stochastic reference point is complete without accounting for this phenomenon. Fortunately, our arguments in favour of a state-dependent reference point structure do not critically depend on probability weighting.

The outline of this paper is as follows. Section 2 discusses the stochastic reference point model proposed by Köszegi and Rabin (2006). Section 3 introduces the state-dependent stochastic reference point model and discusses its properties. Section 4 applies the two stochastic reference point models to US investment benchmark data. Section 5 concludes. The Appendix includes our formal proofs.

2 The Stochastic Reference Point Model

Throughout the text, we will use Ω for the *state-space*, $\mathbb{P}[A]$ for the *probability* that event $A \subseteq \Omega$ occurs, and $\mathcal{X}$ is the collection of *feasible prospects* $X : \Omega \to \mathbb{R}$ (for instance, budget feasible portfolio payoffs).

Köszegi and Rabin (2006) define the *reference-dependent utility* of $X \in \mathcal{X}$ given the reference point $Y \in \mathcal{X}$ as follows:

Definition 2.1.

$$U(X|Y) = \int \int u(x|y)\, dF_Y(y)\, dF_X(x) \tag{2.1}$$

where $F_X(x) = \mathbb{P}[X \leq x]$ *and* $F_Y(y) = \mathbb{P}[Y \leq y]$ *are the distribution functions of* X *and* Y*, respectively, and*

$$u(x|y) = \eta_1\, m(x) + \eta_2\, \mu(m(x) - m(y)), \tag{2.2}$$

$m : \mathbb{R} \to \mathbb{R}$ *is a continuously differentiable, strictly increasing "consumption" utility function, and* $\mu : \mathbb{R} \to \mathbb{R}$ *is a "universal" gain-loss utility function which satisfies the following properties:*

A0. $\mu(x)$ *is continuous for all* x *and twice differentiable for* $x \neq 0$*;*

A1. $\mu(x)$ *is strictly increasing;*

A2. If $y > x > 0$*, then* $\mu(y) + \mu(-y) < \mu(x) + \mu(-x)$*;*

A3. $\mu''(x) \leq 0$ *for* $x > 0$ *and* $\mu''(x) \geq 0$ *for* $x < 0$*;*

A4. $\frac{\lim_{x\to 0} \mu'(-|x|)}{\lim_{x\to 0} \mu'(|x|)} = \lambda > 1.$

The parameters $\eta_1, \eta_2 \in \mathbb{R}^+$ give the weights between consumption utility m and gain-loss utility μ. Köszegi and Rabin (2006) assume $\eta_1 = 1$. Our analysis will also use the expected consumption utility $M(X) = \int m(x)\, dF_X$ and the consumption certainty equivalent $C(X) = m^{-1}(M(X))$. If $m(x) = x$ for all x and $\eta_1 = 0$, the piecewise-power value function of Tversky and Kahneman (1992) arises as a special case of Equation (2.1). Note that for this specification of gain-loss utility, the curvature in the domain of losses should be equal to the curvature in the domain of gains in order to obey Assumption A2, as shown by Köbberling and Wakker (2005). As discussed by Köszegi and Rabin (2007), the model allows for consumption utility to dominate gain-loss utility for large-stake prospects. Hence, the model can reconcile loss aversion for modest stakes with risk aversion for large stakes.

Definition 2.1 does not account for subjective probability weighting. Since probability weighting is known to be strong even for simple fifty-fifty gambles with a constant reference point, it seems unlikely that the model is complete without accounting for this phenomenon. To the best of our knowledge, the only model that includes probability weighting with a stochastic reference point is by Schmidt, Starmer, and Sugden (2008), who define cumulative decision weights as in Cumulative Prospect Theory of Tversky and Kahneman (1992), i.e., they apply a weighting function to the cumulative and decumulative distribution of gains and losses, respectively. However, to us it is not immediately clear how probability weighting would enter in our model. Is consumption utility affected in the same way as gain-loss utility? Are the probabilities of the evaluated prospect, F_X, affected in the same way as the probabilities of the reference point, F_Y? Since our arguments do not critically depend on probability weighting, we leave these questions for further research.

It will be useful for our analysis to consider a stronger version of assumption A3:

A3′. $\mu''(x) = 0$ for $x \neq 0$.[1]

This assumption does not allow for the piecewise-power function of Tversky and Kahneman (1992). However, it does allow for a piecewise-linear gain-loss function. Note that a piecewise-linear gain-loss utility μ does not imply piecewise-linear reference-dependent utility u, because consumption utility m is not restricted.

In case of discrete distributions with S states of nature, i.e., $\Omega = \{1, \ldots, S\}$ and $p_s =$

$\mathbb{P}[\{s\}]$, reference-dependent utility corresponds to:

$$U(X|Y) = \sum_{s=1}^{S} \sum_{s'=1}^{S} u(X(s)|Y(s'))\, p_s\, p_{s'}. \tag{2.3}$$

The model combines every possible outcome of the prospect with every possible outcome of the reference point and evaluates every combination at the product of the two marginal probabilities. The double summation implies that the decision maker considers a total of S^2 combinations of outcomes for every pair of evaluated prospect and reference point. As in disappointment theory, the decision maker experiences a loss (disappointment) when the outcome of the prospect in a given state falls below the outcome of the reference point in another state. The decision maker is therefore predicted to be indifferent to the statistical dependence between the prospect X and the reference point Y:

$$U(X|Y) = U(\tilde{X}|\tilde{Y}) \tag{2.4}$$

for any $\tilde{X}$ and $\tilde{Y}$ which have the same marginal distributions as X and Y, irrespective of the dependence structure. However, our intuition says that a prospect would appear less risky in case of positive dependence and more risky in case of negative dependence, in the same way as an investment portfolio with a positive market beta appears less risky than a negative-beta portfolio to an investor who benchmarks against a market index. Indeed, indifference to the dependence structure can lead to counterintuitive choices, as shown in the following example:

Example 2.1. *Let $\Omega = \{1, 2\}$ and $\mathbb{P}[\{1\}] = 1/2$. We define the risky prospects X and Y as* *follows:*

$$X(1) = 0, X(2) = 101$$
$$Y(1) = 0, Y(2) = 100.$$

Suppose that $m(x) = x$, $\mu(x) = x$ *if* $x \geq 0$ *and* $\mu(x) = \lambda x$, $\lambda > 1$, *if* $x < 0$, *and* $\eta_1 = \eta_2 = 1$. *The decision maker faces the exogenous stochastic reference point* Y. *Faced with this reference point, she faces a choice between the two risky prospects,* Y *and* X. *In this case,* X *strictly dominates* Y *and the preference for* X *is obvious. Indeed, the relevant values for expected reference-dependent utility are*

$$U(Y|Y) = 50 + \frac{1}{2}\frac{1}{2}(0-0) + \frac{1}{2}\frac{1}{2}(100-0) + \frac{1}{2}\frac{1}{2}\lambda\,(0-100) + \frac{1}{2}\frac{1}{2}(100-100) = \frac{100}{4}(3-\lambda)$$

$$U(X|Y) = \frac{101}{2} + \frac{1}{2}\frac{1}{2}(0-0) + \frac{1}{2}\frac{1}{2}(101-0) + \frac{1}{2}\frac{1}{2}\lambda\,(0-100) + \frac{1}{2}\frac{1}{2}(101-100) = \frac{100}{4}(3-\lambda)+1$$

and the decision maker is predicted to prefer X *to* Y. *In this case,* X *and* Y *have a perfectly positive dependence. Now assume that a perfectly negative dependence:*

$$X'(1) = 101, X'(2) = 0,$$

Equation (2.3) does not account for dependencies and hence the decision maker is still predicted to prefer X' *to* Y. *However, it seems that a loss-averter would want to avoid the situation* $(Y(2), X'(2)) = (100, 0)$ *by choosing* Y.

Indifference to dependence structure is particularly difficult to understand when one evaluates a risky prospect that is also used as the reference point – "*auto-evaluation*". In this case, a perfectly positive dependence arises and the decision maker will not experience any losses in the sense of negative deviations from the reference point. For example, an investor who benchmarks against a market index experiences no losses when she perfectly replicates the index. However, the model predicts that the joint probabilities are not relevant and the decision maker experiences losses (disappointment), even in case of auto-evaluation. This contrasts with the original interpretation of the reference point as a "neutral" prospect, according to which the decision maker experiences no gains or losses when she would selects this prospect; see Kahneman and Tversky (1979, Page 274). In general, auto-evaluating a risky prospect yields losses and implies negative gain-loss utility. By contrast, auto-evaluating a riskless prospect always avoids losses and yields zero gain-loss utility. This introduces an inherent aversion to risky prospects and implies, among other things, that auto-evaluating a risky prospect is always less favorable than auto-evaluating its consumption certainty equivalent:

Lemma 2.1. *For any $Y \in \mathcal{X}$ we have*

$$U(Y|Y) \leq \eta_1 M(Y), \text{ and} \tag{2.5}$$

$$U(Y|Y) = \eta_1 M(Y) \text{ if and only if } Y \text{ is riskless.} \tag{2.6}$$

Consequently, if Y is stochastic and $\eta_2 > 0$ then

$$U(Y|Y) < U(C(Y)|C(Y)).$$

Thus far, the reference point was exogenously given. Köszegi and Rabin (2006) develop a framework to endogenously determine the reference point. They introduce the following definitions:

Definition 2.2. *A* personal equilibrium *(PE) is a prospect $Y \in \mathcal{X}$ such that*

$$U(Y|Y) \geq U(X|Y)$$

for all $X \in \mathcal{X}$. We denote by $\mathcal{X}_{PE} \subset \mathcal{X}$ the set of personal equilibria.
A preferred personal equilibrium *(PPE) is a personal equilibrium with maximal reference-dependent utility:*

$$X \in \arg\max\{U(Z|Z) : Z \in \mathcal{X}_{PE}\}.$$

If $Y \notin \mathcal{X}_{PE}$ is taken as reference point, the decision maker will find a prospect X that is preferred to Y, and will use X as the new reference point. Under assumption A3′ on the gain-loss function, the change of reference point does not cause a preference reversal, i.e., X is preferred to Y also with respect to the new reference point (Köszegi and Rabin 2006, Proposition 1.3). Therefore, the decision maker will replace the reference point with the preferred prospect as long as a personal equilibrium has not been reached. The preferred personal equilibrium is the personal equilibrium with maximal reference-dependent utility.

The aversion to risky prospects implies that any riskfree personal equilibrium is also a preferred personal equilibrium:

Proposition 2.1. *Let $X \in \mathcal{X}_{PE}$ be deterministic. Under assumption A3', X is a PPE.*

This result demonstrates the counterintuitive implications of cross-state comparisons. It also implies that a preferred personal equilibrium need not maximize consumption utility, not even on the set of personal equilibria. Consider the following example:

Example 2.2. *We assume the same setup of Example 2.1. Consider the choice between the fifty-fifty gamble Y for 0 or 100, and a sure thing Z that pays $z \in [0, 100]$ with full certainty.*

Because consumption utility is assumed to be linear, Y is the consumption optimum if $z \leq 50$ and Z is the optimum if $z \geq 50$. The first step to implement the stochastic reference point model is to compute the relevant expected reference-dependent utilities:

$$\begin{aligned} U(Y|Y) &= \frac{100}{4}(3-\lambda) \\ U(Z|Y) &= z + \frac{1}{2}(z-0) + \frac{1}{2}\lambda(z-100) = \frac{1}{2}(3\,z + \lambda\,z - 100\,\lambda) \\ U(Y|Z) &= 50 + \frac{1}{2}\lambda\,(0-z) + \frac{1}{2}(100-z) = \frac{1}{2}(200 - \lambda\,z - z) \\ U(Z|Z) &= z + (z-z) = z. \end{aligned}$$

It follows directly that Y is a personal equilibrium ($U(Y|Y) \geq U(Z|Y)$) if $z \leq 50$ and Z is a personal equilibrium ($U(Z|Z \geq U(Y|Z)$) if $z \geq 200/(3+\lambda)$. Thus, for $z < 200/(3+\lambda)$

and $z > 50$*, there exists a unique personal equilibrium, which is the preferred personal equilibrium and is equal to the consumption optimum. However, for* $z \in [200/(3+\lambda), 50]$*, both alternatives are equilibria. Interestingly, the riskless equilibrium* Z *is then always preferred to the risky equilibrium* Y*, because* $U(Y|Y) < U(Z|Z)$ *for* $z \in [200/(3+\lambda), 50]$*. This result is surprising, because* Y *rather than* Z *is the consumption optimum for* $z \in [200/(3+\lambda), 50]$*. This result reflects bias of the model against risky alternatives. The risky personal equilibrium yields negative gain-loss utility because the decision maker is assumed to derive negative gain-loss utility from the situation where* Y *pays 0, while the reference point pays 100, a situation that has zero probability of occurring since the reference point equals* Y*.*

The purpose of this example is to demonstrate the divergence between the preferred personal equilibrium and the consumption optimum under simplifying assumptions. In a real-life choice experiment, many subjects would deviate from the consumption optimum in the example by choosing the riskless alternative even if it has the lowest expected outcome (for example, $z = 45$). One possible explanation for these choices is that the subjects do not endogenously select their reference point, but simply fix it at, for example, their normal hourly wage, introducing loss aversion. An alternative explanation is probability weighting, which generally is strong even for fifty-fifty gambles and introduces a "certainty effect". To account for this effect, we may use a rank-dependent consumption utility model as the benchmark. Using the same reasoning as in the example, the reference-dependent model would then predict a stronger aversion to the risky alternative than the consumption model.

The preferred personal equilibrium characterizes risk preferences before making an anticipated risky choice. Köszegi and Rabin (2007) also introduce the concept of choice-acclimating personal equilibrium (CPE) to describe risk preferences after the choice has been made. The CPE maximizes reference-dependent utility $U(Z|Z)$ over all risky prospects rather than over personal equilibria (as in Definition 2.2), that is, the CPE corresponds to $X \in \arg\max\{U(Z|Z) : Z \text{ from } \mathcal{X}\}$. This paper focuses on pre-choice risk preferences and the preferred personal equilibrium. However, it follows directly from Proposition 3.2 below that the post-choice CPE in our framework simply reduces to the consumption optimum, that is, $X \in \arg\max\{M(Z) : Z \text{ from } \mathcal{X}\}$.

3 The State-dependent Reference Point Model

We deviate from Köszegi and Rabin (2006) in two ways: (i) we use a more general set of admissible reference points and (ii) we replace the disappointment-based preference structure with a state-dependent, regret-based structure. The analysis deviates from Sugden (2003) model with a fixed state-dependent reference point, be allowing the reference point to be selected endogenously.

Thus far, it was assumed that all feasible prospects are admissible reference points, and vice versa. We now consider a more general specification where the set of admissible reference points is a subset of all feasible prospects, $\mathcal{Y} \subseteq X$. In our analysis, there is no need to consider non-feasible prospects $X \notin \mathcal{X}$. However, for other purposes, it may be useful to allow for, for example, an optimistic "utopia prospect" or a pessimistic "dystopia prospect" as the reference point. It seems plausible that $\mathcal{Y}$ would depend on the decision maker's initial subjective valuation of the prospects and therefore the specification of $\mathcal{Y}$ will be discussed after introducing our preference structure.

In the spirit of regret theory, we consider the following alternative, state-dependent structure:

Definition 3.1. *Let $\mathcal{Y} \subseteq \mathcal{X}$. For a risky prospect $X \in \mathcal{X}$ and an admissible reference point $Y \in \mathcal{Y}$, the state-dependent reference-dependent utility of X given Y is defined as*

$$\tilde{U}(X|Y) = \int \int u(x|y)\, d^2 H_{X,Y}(x,y). \tag{3.7}$$

where $H_{X,Y}(x,y) = \mathbb{P}[X \leq x, Y \leq y]$ is the joint cumulative distribution function of X and Y, and u is defined as in Equation (2.2).

The state-dependent model evaluates the outcome of the prospect and the reference point at their joint probabilities, rather than the product of the marginal probabilities, and thus also incorporates the statistical dependence between the prospect and the reference point. In case of a discrete probability distribution with S states of nature, this boils down to comparing the outcomes of the prospect with those of the reference point in the same state of nature and not with outcomes in other states:

$$\tilde{U}(X|Y) = \sum_{s=1}^{S} u(X(s)|Y(s))\, p_s. \tag{3.8}$$

Using a state-dependent reference point, the decision maker does not experience negative gain-loss utility (disappointment) from the fact that bad states yield worse outcomes than good states, as Equation (2.3) would predict. Rather, she derives negative gain-loss utility (regret) when the chosen prospect falls below the reference point in the same state.

If two random variables X and Y are independent, then the joint cumulative distribution function of X and Y is the product of the corresponding marginal distributions:

$$H_{X,Y}(x,y) = F_X(x)\, F_Y(y).$$

In this case, the two specifications of reference-dependent utility coincide:

$$\tilde{U}(X|Y) = \int\int u(x|y)\, dF_Y(y)\, dF_X(x) = U(X|Y). \tag{3.9}$$

However, the two models generally diverge if the prospect and the reference point are dependent. Compared to the state-dependent model, the Köszegi and Rabin (2006) model generally overestimates the true joint probabilities of gains or losses in case of positive dependence between X and Y and underestimates the joint probabilities in case of negative dependence. In fact, the decision maker may even experience illusionary gains and losses that have a zero probability of occurring. In contrast to the disappointment specification, the regret specification is not invariant with respect to the dependence structure. We formalize this observation using the concept of positively and negatively associated random variables.

Definition 3.2. *Two random variables X and Y are said to be positively associated if*

$$\mathtt{Cov}(f(X), g(Y)) \geq 0$$

for every pair of non-decreasing functions f and g such that the above covariance exists.[2] *Negative association holds if the above inequality is reversed.*

Using the state-dependent function, decision makers generally have a preference for prospects that are positively associated with the reference point and an aversion to prospects with a negative association:

Proposition 3.1. *Let $(X,Y) \in \mathcal{X} \times \mathcal{Y}$ be a pair of prospects and consider a second pair of prospects $(\tilde{X},\ \tilde{Y})$ with same marginal distributions as the first pair, i.e., $F_{\tilde{X}} \equiv F_X$ and $F_{\tilde{Y}} \equiv F_Y$, and such that $\tilde{X}$ is independent from $\tilde{Y}$. If u satisfies assumption A3′ then* *(i)* $\tilde{U}(X|Y) \geq \tilde{U}(\tilde{X}|\tilde{Y})$ *if* X *and* Y *are positively associated.*

(ii) $\tilde{U}(X|Y) \leq \tilde{U}(\tilde{X}|\tilde{Y})$ *if* X *and* Y *are negatively associated.*

The following example illustrates the implications of Proposition 3.1:

Example 3.1. *We assume the same setup of Example 2.1. Assuming a perfectly positive dependence, the relevant values of expected reference-dependent utility are*

$$\begin{aligned}\tilde{U}(Y|Y) &= 50,\\ \tilde{U}(X|Y) &= \frac{101}{2} + \frac{1}{2}(0-0) + \frac{1}{2}(101-100) = 51\end{aligned}$$

and X *is preferred to* Y*. However, assuming a perfect negative correlation, expected state-dependent reference-dependent utility for* X' *given* Y *is*

$$\tilde{U}(X'|Y) = \frac{101}{2} + \lambda\,\frac{1}{2}(0-100) + \frac{1}{2}(101-0) = \frac{100}{2}(2-\lambda) + 1$$

and the loss averter prefers Y *to* X' *in order to avoid the loss situation* $(Y(2), X'(2)) = (100, 0)$*.*

By accounting for the dependence structure, the inherent aversion to risky prospects disappears:

Proposition 3.2. $\tilde{U}(Y|Y) = \eta_1\, M(Y)$ *for all* $Y \in \mathcal{Y}$ *and therefore* $\tilde{U}(Y|Y) = \tilde{U}(c(Y)|c(Y))$*.*

Similar to Proposition 1.3 in Köszegi and Rabin (2006), but under more general conditions, if a prospect is preferred to the reference point, then the same preference relationship holds if the prospect is taken as reference point:

Proposition 3.3. *Let* $X, Y \in \mathcal{Y}$ *with* $\mathbb{P}[X \neq Y] > 0$. *If* $\tilde{U}(X|Y) \geq \tilde{U}(Y|Y)$ *then* $\tilde{U}(X|X) > \tilde{U}(Y|X)$.

This result motivates the following definitions of state-dependent personal equilibrium and state-dependent preferred personal equilibrium:

Definition 3.3. *A element* $Y \in \mathcal{Y}$ *is a* state-dependent personal equilibrium given $\mathcal{Y}$ *(SPE) if*

$$\tilde{U}(Y|Y) \geq \tilde{U}(X|Y)$$

for all $X \in \mathcal{Y}$. *We denote the set of state-dependent personal equilibria in* $\mathcal{Y}$ *by* $\mathcal{Y}_{SPE}$.
A state-dependent preferred personal equilibrium given $\mathcal{Y}$ *(SPPE) is a risky prospect* $Y \in \mathcal{Y}_{SPE}$ *such that*

$$Y \in \arg\max\{\tilde{U}(Z|Z) : Z \in \mathcal{Y}_{SPE}\}.$$

Note that for a SPE we restrict the condition $\tilde{U}(Y|Y) \geq \tilde{U}(X|Y)$ to hold only for prospects X in $\mathcal{Y}$, i.e., only for admissible reference points. Therefore, in our setting, the (S)PPE generally differs from the optimal prospect, while in Köszegi and Rabin (2007) the decision maker selects a PPE as both the reference point as well as the optimal prospect given the reference point.

Recall that the disappointment-based model and the regret-based model generally differ, even if $\mathcal{Y} = \mathcal{X}$, unless the prospect and the reference point are statistically independent. Therefore, the stochastic model and the state-dependent model generally yield different sets of personal equilibria and different preferred personal equilibria. This occurs even when all prospects are statistically independent, because the definition of personal equilibrium requires auto-evaluation – a case with perfectly positive dependence. The following example shows that not every state-dependent personal equilibrium is a personal equilibrium:

Example 3.2. *We assume the same setup of Examples 2.1 and 2.2 (thus we also assume $\mathcal{Y} = \mathcal{X}$). The state-dependent model computes the reference-dependent utilities as follows:*

$$\begin{aligned}
\tilde{U}(Y|Y) &= 50 + \frac{1}{2}(0-0) + \frac{1}{2}(100-100) = 50 \\
\tilde{U}(Z|Y) &= U(Z|Y) = z + \frac{1}{2}(z-0) + \frac{1}{2}\lambda\,(z-100) = \frac{1}{2}(3\,z + \lambda\,z - 100\,\lambda) \\
\tilde{U}(Y|Z) &= U(Y|Z) = 50 + \frac{1}{2}\lambda\,(0-z) + \frac{1}{2}(100-z) = \frac{1}{2}(200 - \lambda\,z - z) \\
\tilde{U}(Z|Z) &= U(Z|Z) = z.
\end{aligned}$$

Therefore Y is a state-dependent personal equilibrium ($\tilde{U}(Y|Y) \geq \tilde{U}(Z|Y)$) if $\tilde{U}(Z|Y) \leq 50$, or $z \leq 100\,(1+\lambda)/(3+\lambda)$. Similarly, Z is a state-dependent personal equilibrium ($\tilde{U}(Z|Z) \geq \tilde{U}(Y|Z)$) if $\tilde{U}(Y|Z) \leq z$, or $z \geq 200/(3+\lambda)$. Thus, for $z < 100\,(1+\lambda)/(3+\lambda)$ and $z > 200/(3+\lambda)$, there exists a unique state-dependent personal equilibrium, which equals the state-dependent preferred personal equilibrium and the consumption optimum. However, for $z \in [200/(3+\lambda), 100\,(1+\lambda)/(3+\lambda)]$, we have two state-dependent personal equilibria and the state-dependent preferred personal equilibrium is the consumption optimum. By contrast, Example 2.2 shows that for $z \in [50, 100\,(1+\lambda)/(3+\lambda)]$ the risky prospect Y is not a personal equilibrium. In contrast to Proposition 2.1, the example also shows that a riskfree state-dependent personal equilibrium is not necessarily a state-dependent preferred personal equilibrium. Indeed, for $z \in [200/(3+\lambda), 50]$ the riskfree prospect Z is a state-dependent personal equilibrium, but not a preferred personal equilibrium. Table 1 summarizes the comparison given in Examples 2.1, 2.2 and 3.2 between the stochastic reference point model and the state-dependent model.

Under the general assumptions about risk preferences used thus far, we can also find examples where not every personal equilibrium is a state-dependent personal equilibrium.[3] However, if we impose more structure on risk preferences, such examples are excluded, and every personal equilibrium is a state-dependent personal equilibrium:

Proposition 3.4. *Suppose that m is bounded and μ satisfies assumption A3′. Then every personal equilibrium in $\mathcal{Y}$ is a state-dependent personal equilibrium, i.e., $\mathcal{Y}_{PE} \subset \mathcal{Y}_{SPE}$.*

While comparison across states of nature generally moves the PPE away from the optimal solution to the reference-independent choice problem, the SPPE generally equals the consumption optimum:

Proposition 3.5. *Let $Y \in \mathcal{Y}$ be a state-dependent preferred personal equilibrium and let $\eta_1 > 0$.*

(i) $Y \in \arg\max\{M(Z) : Z \in \mathcal{Y}_{SPE}\}$.

(ii) Under assumption A3', $Y \in \arg\max\{M(Z) : Z \in \mathcal{Y}\}$. Moreover, any prospect in $\arg\max\{M(Z) : Z \in \mathcal{Y}\}$ is a SPPE.

Loss-aversion in our model generally does not affect choice behavior if the reference point is completely endogenous and adjusts immediately to new information or unexpected events (i.e., $\mathcal{Y} = \mathcal{X}$). The decision maker is then free to select any choice alternative and reference point, and she may select the consumption optimum for both. This combination maximizes both components of expected reference-dependent utility: (i) the consumption optimum by definition maximizes expected consumption utility and (ii) expected gain-loss utility achieves its maximal value of zero in case of auto-evaluation. Thus, the reference-dependent solution equals the consumption optimum when the reference point is completely endogenous. Given the wealth of evidence showing that loss aversion affects choice behavior, this finding suggests that the reference point generally includes an important exogenous component. In our model, this means that only a subset of the feasible prospects is considered as a candidate reference point (i.e., $\mathcal{Y} \subset \mathcal{X}$).

We now turn to the specification of the set of admissible reference points, $\mathcal{Y}$. Without claims to generality, the following specification is sufficiently flexible for our purposes:

$$\mathcal{Y} = \{X \in \mathcal{X} \,:\, c(X, X_0) \leq c_0\}. \tag{3.10}$$

In this specification, $X_0 \in \mathcal{X}$ is an external reference point that may reflect, for example, a past reference point or a past solution from a previous decision problem, in case of repeated decision making. For an investor who considers rebalancing her portfolio, the current portfolio composition, or the solution to her previous rebalancing problem, could represent an external point of reference. Alternatively, X_0 may be an external benchmark that is imposed by a principal or an external advice or a social norm. For example, for a money manager, the external reference point could be a general market index or a customized benchmark portfolio specified by a client. The function c measures the subjective adjustment cost or mental effort of deviating from X_0. It seems plausible that the costs would decrease with the experience and education of the decision maker and the available decision time and decision support tools. For a given decision maker and decision problem, the adjustment costs would seem to depend on the "economic distance" between a candidate reference point X and the initial reference point X_0. One possible specification is:

$$c(X, X_0) = \tilde{U}(X_0|X_0) - \tilde{U}(X|X_0). \tag{3.11}$$

In this case, the adjustment costs of a candidate reference point X depends on its "initial value," or the reference-dependent utility given the initial reference point X_0. The initial value captures the decision-maker's prior subjective judgement regarding the prospects. T¡he mental cost of considering a candidate reference point is higher if the initial value is lower. Loss aversion lowers the initial valuation and hence increases the adjustment costs and shrinks the admissible set. c_0 measures the maximum admissible adjustment costs. Setting $c_0 \geq \max\{c(X, X_0) : X \in \mathcal{X}\}$ yields the extreme, unrestricted case with $\mathcal{Y} = \mathcal{X}$, or the situation that was considered by Köszegi and Rabin (2006). In the extreme case of $c_0 = 0$, the decision maker would consider only alternative reference points that improve the initial valuation. Such reference points represent obvious improvement possibilities that can be detected even without changing the reference point. Still, the decision maker would avoid all reference points with less salient improvement possibilities – ones that can only be detected after first updating the reference point. If X_0 already maximizes the initial valuation (and is a unique solution), then X_0 is also a state-dependent preferred equilibrium in $\mathcal{X}$ and the reference point is in effect completely fixed, as in the Sugden (2003) model. Apart from being fixed, X_0 could also take a non-stochastic, constant value, as in the traditional interpretation of the reference point, for example, in Prospect Theory.

As discussed above, loss-aversion in our model generally does not affect choice behavior if the reference point is completely endogenous, that is, $\mathcal{Y} = \mathcal{X}$. However, more generally, loss aversion increases the mental effort required to adjust the reference point and may exclude some prospects from consideration. Loss aversion introduces a preference for solutions that have a positive correlation with the initial reference point (and involve relatively low adjustment costs) and an aversion to negative correlation. The decision maker generally deviates from the general consumption optimum in order to reduce her exposure to losses relative to her initial reference point. Prospects that are positively correlated with the initial reference point will look more attractive, because these involve smaller losses and lower adjustment costs than uncorrelated or negatively correlated prospects. This is consistent with the prediction of Köszegi and Rabin (2007) that a prior expectation to take on a risk will decrease the willingness to pay for insurance against that risk.

While loss aversion generally causes deviations from the unrestricted consumption optimum, its effect is limited to excluding certain prospects and it does not introduce new candidate solutions:

Proposition 3.6. *Let $\mathcal{Y} \subset X$ be defined as in Equation (3.10) for some $c_0 \in \mathbb{R}$, where c is the cost function given in Equation (3.11). Then under assumption A3', a state-dependent personal equilibrium given $\mathcal{Y}$ is a state-dependent personal equilibrium given $\mathcal{X}$, i.e., $\mathcal{Y}_{SPE} \subseteq \mathcal{X}_{SPE}$.*

Thus, the preferred state-dependent personal equilibrium given $\mathcal{Y}$ will always be one of the state-dependent personal equilibria of the unrestricted case.

4 Empirical application

We analyze historical returns to the one-month US Treasury bill ("bills"), the US common stock market index constructed by the Center for Research in Security Prices (CRSP) of the University of Chicago Booth School of Business ("stocks") and 50/50 mixtures of bills and bonds ("mix funds"). We consider returns with a daily, weekly, monthly and annual frequency. The sample includes the daily, weekly and monthly returns from July 1, 1963 to

April 30, 2010, and the yearly returns from 1963 to 2009, a total of 11,789 daily observations, 2,444 weekly observations, 562 monthly observations and 47 annual observations. Returns are evaluated in excess of the T-bill rate, so that the bills have an excess return of zero and are assumed to be completely risk free. The T-bill series are from Ibbotson Associates; the stock series are from Kenneth French' online data library.

As in the examples in the main text, we assume risk-neutral, linear consumption utility ($m(x) = x$) and use a piecewise-linear gain-loss utility function ($\mu(x) = x$ if $x \geq 0$ and $\mu(x) = 2\,x$ if $x < 0$). We also considered other specifications, including risk averse, logarithmic consumption utility ($m(x) = \ln(100 + x)$) and the Tversky and Kahneman (1992) value function $\mu(x) = x^{\alpha}$ if $x \geq 0$ and $\mu(x) = -\lambda\,(-x)^{\alpha}$ if $x < 0$, using the Tversky and Kahneman (1992) parameters ($\alpha = 0.88$, $\lambda = 2.25$). However, the specification of the preference parameters proved to be less important than the specification of the reference point and the choice of the return frequency.

We use the historical returns as equally likely states-of-the-world. We estimate the expected consumption utility and gain-loss utility using the sample average over all states. These averages are then used to identify the personal equilibriums and preferred personal equilibriums. Given the high average excess return to stocks, it is not surprising that the consumption optimum is to invest in stocks for every return frequency in our sample. Since the excess returns on bills is always zero, consumption utility of bills is always zero too. Stocks and mix funds by contrast have positive consumption utility on average.

To account for sampling error, we estimate the probability that stocks, bills and mix funds represent a personal equilibrium or a preferred personal equilibrium using bootstrapping. We generate 10,000 pseudo-samples through random sampling with replacement from the original sample, and compute average consumption utility and gain-loss utility in every pseudo-sample. Next, we compute the fraction of the pseudo-samples where stocks, bills or mix funds represent a personal equilibrium or a preferred personal equilibrium. The results suggest that the full-sample results are robust to sampling variation.

The first four columns of Table 2 show results for the disappointment-based model of Köszegi and Rabin (2006).

For daily and weekly returns, investing in bills is a personal equilibrium. When the reference point equals the riskless rate, investing in bills looks more attractive than investing in stocks or mix funds. Consumption utility and gain-loss utility of bills are always zero and hence average reference-dependent utility equals zero. Stocks and mix funds have positive consumption utility, but the large possible losses (disappointment) relative to the riskless rate introduce negative average gain-loss utility, and reference-dependent utility takes a negative value on average.

Investing in stocks is not a personal equilibrium for daily and weekly returns. According to the model, stocks may cause losses even to investors who use stock returns as their reference point. A prospective stock investor is assumed to be afraid that stocks would go down, while the reference point goes up, a situation that will of course never occur when stock returns are the reference point. For example, the largest weekly "loss" in the sample occurs by comparing the stock market return of minus 18.40 percent in the week of October

6-10, 2008 with the stock market return of plus 16 percent in the week of October 7-11, 1974. For this reason, bills and mix funds achieve a higher average reference-dependent utility than stocks if stock returns are the reference point. Similarly, mix funds are not a personal equilibrium, because bills look more attractive than mix funds when mix fund returns are the reference point.

For monthly returns, all three asset classes are a personal equilibrium. Thus, every asset class is optimal for investors who benchmark against the returns of that asset class. However, the reference point is endogenous and the investor selects the preferred personal equilibrium, or the personal equilibrium with the highest expected reference-dependent utility. Since bills yield a zero expected reference-dependent utility and stocks and mix funds yield negative values, bills are the preferred equilibrium.

Thus, for daily to monthly return frequencies, the preferred personal equilibrium is bills and does not equal the optimal solution to the investment problem - stocks. The preference for bills reflects the inherent aversion to risky choices that was discussed in Section 2; while bills by definition yield zero gain-loss utility when compared to the riskless rate, auto-evaluation of stocks and mix funds yields negative gain-loss utility.

For annual returns, stocks are the only personal equilibrium. Bills and mix funds are not personal equilibriums; stocks look more attractive than bills and mix funds when the riskless rate or mix fund returns are the reference point.

The last four columns of Table 2 show results for the regret-based, state-dependent model, which avoids comparing outcomes across states-of-the-world and focuses on withinstate comparison only. We first assume full endogeneity for the reference point, or $c_0 = +\infty$ and $\mathcal{Y} = \mathcal{X}$; we will consider binding adjustment costs below. The regret-based model is identical to the disappointment-based model when the prospect or the reference point is riskless; differences arise only when the prospect and the reference point are both stochastic. Hence, the two models yield identical utility levels for bills, stocks or mix funds relative to the riskless rate and bills relative to stock or mix fund returns. However, evaluating stocks or mix funds relative to stock or mix fund returns now makes stocks and mix funds look more favorable.

For daily, weekly and monthly returns, each of the three asset classes is a state-personal equilibrium. Since holding stocks or mix funds avoids possible losses (regret) relative to that asset class, gain-loss utility is zero and reference-dependent utility equals consumption utility and is positive on average - in contrast to the negative values for the Köszegi and Rabin model. The preferred personal equilibrium in this case is stocks, or the consumption optimum. For annual returns, the preference for stocks is even stronger; bills and mix funds are not even a personal equilibrium.

The above results illustrate that loss aversion does not affect optimal choice if the state-dependent reference point is fully endogenous. We now turn to the case with an exogenous initial reference point and binding adjustment costs. Table 3 summarizes the results for the extreme case with $c_0 = 0$, that is, the investor allows only adjustments of the reference point that improve the initial valuation. The initial reference point X_0 could be set at bills, stocks or mix funds. We will first discuss the results for X_0 =bills.

For daily, weekly and monthly returns, bills are a personal equilibrium over $\mathcal{X}$ and hence maximize the initial valuation. Therefore, bills are the only admissible reference point, or $\mathcal{Y} = \{\text{bills}\}$, and hence the preferred personal equilibrium. For annual returns, however, stocks and mix funds achieve a higher initial value than bills and also represent admissible reference points, that is, $\mathcal{Y} = \{\text{bills, stocks, mix funds}\}$. Thus, in effect, this is the unrestricted case from Table 2, and stocks are the preferred state-dependent personal equilibrium.

These results show how costly adjustment can cause deviations from the consumption optimum. If the reference point is fixed at a target rate-of-return, loss aversion will affect investment by making bills appear more attractive to myopic investors with a relatively short investment horizon.

We have thus far assumed X_0=bills. If we assume that the initial reference point is stocks, or X_0=stocks, the preference for stocks is even stronger than in the unrestricted case of Table 2, and stocks are the preferred equilibrium for every return frequency. If we set X_0 =mix funds, we find results that are comparable to those for X_0 =bills; mix funds are the preferred equilibrium for weekly to monthly returns, but stocks are the preferred equilibrium for annual returns.

5 Conclusion

While the typical implementation of reference-dependent choice theories exogenously fixes the reference point at a given constant, recent research has dealt with the possibility that the reference point is a random variable and that the reference point is endogenously determined as part of the decision maker's optimization problem. We add to this literature by examining loss aversion with a state-dependent reference point. The model essentially extends the Sugden (2003) model for an exogenous stochastic reference point to the case where the reference point is endogenous (in part or in whole), and it modifies the Köszegi and Rabin (2006) model by changing the underlying reference-dependent preference structure from "disappointment-based" to "regret-based" and allowing for exogenous component in the reference point.

The Köszegi and Rabin (2006) model compares every possible outcome of the prospect with every possible outcome of the reference point, as in disappointment theory. The decision maker experiences losses when the outcome of the prospect in a given state falls below the outcome of the reference point in other states. She is indifferent to the statistical dependency between the prospect and the reference point. Comparing across states also introduces an aversion to risky prospects, which yield negative gain-loss utility (disappointment), even in the case of auto-evaluation. This aversion generally moves the preferred personal equilibrium away from the decision maker's consumption optimum. For example, in our empirical application, investors are predicted to invest in riskless bills, while investing in stocks maximizes their expected consumption utility.

The state-dependent reference point model leads to different results. The decision-maker experiences negative gain-loss utility (regret) when the prospect falls below her reference point in the same state. Therefore, prospects are more attractive if they depend positively on the reference point and are less attractive in case of negative dependence. The state-dependent model is neutral in the sense that it avoids an inherent aversion to risky prospects and yields no loss when the prospect and the reference point are the same. In addition, an related to this, the model ensures that the preferred personal equilibrium equals the consumption optimum, when the reference point is fully endogenous.

In the state-dependent model, loss aversion influences behavior only if the decision maker is not free to select the consumption optimum as her reference point. Given that loss aversion is widespread, we conclude that the reference point generally includes an important exogenously fixed component or adjust slowly to new information or unexpected events. A case in point is an investment benchmark index that is externally fixed by the investment principal for the duration of the investment mandate. Our model captures this exogenous component using costly adjustment from an initial, exogenous reference point. The fixed state-dependent reference point of Sugden (2003) arises the special case with prohibitive adjustment costs, provided the decision maker cannot improve upon the initial reference point without updating the reference point.

Further research could focus on the dynamics of a stochastic reference point - how does it originate and how quickly does it adjust to new information or surprise events? Does the adjustment speed depend on, for example, problem presentation, decision time and experience? Another interesting research topic is probability weighting. Does probability weighting affect consumption utility in the same way as gain-loss utility? Does it affect the probabilities of the evaluated prospect, F_X, in the same way as the probabilities of the reference point, F_Y?

Notes

[1]Assumption A3′ implies that $(x,y) \mapsto u(u|y)$ is supermodular. A function $\phi : \mathbb{R}^2 \to \mathbb{R}$ is supermodular if for all $(x_1,y_1),(x_2,y_2) \in \mathbb{R}^2$ we have

$$\phi(\min\{x_1,x_2\},\min\{y_1,y_2\}) + \phi(\max\{x_1,x_2\},\max\{y_1,y_2\}) \geq \phi(x_1,y_1) + \phi(x_2,y_2).$$

On $\mathbb{R}^2$, supermodularity is equivalent to the property of having increasing differences, i.e., the function $\phi(\cdot,y) - \phi(\cdot,y')$ is nondecreasing for all $y \geq y'$; see **?**. Under assumption A3′, the function

$$u(x|y) - u(x|y') = \eta_2 \begin{cases} m(y') - m(y) & ,x \geq y \geq y' \\ (\lambda - 1)\, m(x) - \lambda\,(m(y) - m(y')) & ,y \geq x \geq y' \\ \lambda\,(m(y') - m(y)) & ,y \geq y' \geq x. \end{cases}$$

is nondecreasing in x for all $y \geq y'$.

[2]Note that each random variable X is positively associated with itself (Joe 1997, Lemma 2.1). Moreover, two random variables X and Y are positively (negatively) associated if and only if they are positive (negative) quadrant dependent, i.e.,

$$H_{X,Y}(x,y) \geq (\leq) F_X(x)\, F_Y(y)$$

for all $x, y \in \mathbb{R}^2$ (see Joag-Dev and Proschan 1983, Property P1).

[3]Let $\Omega = \{1,2,3\}$ and $\mathbb{P}\left[\{s\}\right] = \frac{1}{3}$ for $s = 1,\ldots,3$. We define the risky prospects X and Y as follows:

$$X(1) = 111.1, X(2) = 100, X(3) = 89$$
$$Y(1) = 110, Y(2) = 100, Y(3) = 90.$$

Suppose that $m(x) = x$, $\mu(x) = 1 - \exp(-0.1\,x)$ if $x \geq 0$ and $\mu(x) = 20\,(\exp(0.01\,x) - 1)$ if $x < 0$ (the index of loss aversion is $\lambda = 2$), and $\eta_1 = \eta_2 = 1$. Then $\mathcal{X}_{SPE} = \{X\}$ while $\mathcal{X}_{PE} = \{X,Y\}$. The example exploits the different curvatures of the value function over gains and losses. We use a piecewise-exponential function, since a piecewise-power function with different powers for gains and losses violates assumption A2, as demonstrated in Köbberling and Wakker (2005).

A Proofs

A.1 Proof of Lemma 2.1

Let $Y \in \mathcal{X}$ then

$$
\begin{aligned}
U(Y|Y) &= \int\int u(y|z)\,dF_Y(z)\,dF_Y(y) \\
&= \eta_1 \int\int m(y)\,dF_Y(z)\,dF_Y(y) + \eta_2 \int\int \mu(m(y)-m(z))\,dF_Y(z)\,dF_Y(y) \\
&= \eta_1 \int m(y)\,dF_Y(y) + \eta_2 \int\int_{z>y} \mu(m(y)-m(z))\,dF_Y(z)\,dF_Y(y) \\
&\quad +\eta_2 \int\int_{z<y} \mu(m(y)-m(z))\,dF_Y(z)\,dF_Y(y) \\
&= \eta_1 \int m(y)\,dF_Y(y) + \eta_2 \int\int_{y>z} \mu(m(z)-m(y))\,dF_Y(y)\,dF_Y(z) \\
&\quad +\eta_2 \int\int_{z>y} \mu(m(z)-m(y))\,dF_Y(z)\,dF_Y(y) \\
&= \eta_1\, M(Y) + \eta_2 \int\int_{z>y} \mu(m(y)-m(z))\,dF_Y(y)\,dF_Y(z) \\
&\quad +\eta_2 \int\int_{z>y} \mu(m(z)-m(y))\,dF_Y(z)\,dF_Y(y) \\
&= \eta_1\, M(Y) + \eta_2 \int\int_{z>y} [\mu(m(y)-m(z)) + \mu(m(z)-m(y))]\,dF_Y(z)\,dF_Y(y) \\
&= \eta_1\, M(Y) + \eta_2 \int\int_{z>y} [\mu(-(m(z)-m(y))) + \mu(m(z)-m(y))]\,dF_Y(z)\,dF_Y(y).
\end{aligned}
$$

The second term vanishes if Y is riskless. If Y is stochastic, i.e., $\mathbb{P}[Y=a] < 1$ for all $a \in \mathbb{R}$, and since m is strictly increasing, we have

$$\int\int_{z>y} [\mu(m(y)-m(z)) + \mu(m(z)-m(y))]\,dF_Y(z)\,dF_Y(y) < 0$$

by property A2. This proves the statement of the Lemma.

A.2 Proof of Proposition 2.1

Without loss of generality $\eta_2 > 0$. Let

$$GL(Z|Y) = (1/\eta_2)\,(U(Z|X) - \eta_1\, M(Z))$$

be the gain-loss utility. If $\eta_1 = 0$ the statement is clear, since $GL(Z|Z) \leq 0$ for all $Z \in \mathcal{X}$.

Let $\eta_1 > 0$. We prove the statement by contradiction. Assume that $X = x$ is not a PPE. Then it exists $Z \in \mathcal{X}_{PE}$ with

$$U(Z|Z) > U(X|X).$$

It follows:

$$\begin{aligned} U(Z|X) &= \eta_1\, M(Z) + \eta_2\, GL(Z|X) = \eta_1\, M(Z) + \eta_2\, GL(Z|Z) - \eta_2\, GL(Z|Z) + \eta_2\, GL(Z|X) \\ &= U(Z|Z) + \eta_2\,(GL(Z|X) - GL(Z|Z)) \\ &> U(X|X) + \eta_2\,(GL(Z|X) - GL(Z|Z)). \end{aligned}$$

If we prove $GL(Z|X) - GL(Z|Z) \geq 0$, then $U(Z|X) > U(X|X)$, a contradiction to $X \in \mathcal{X}_{PE}$. The following properties are satisfied:

(i) $M(Z) > M(X)$.

(ii) There exists $z' \in \mathrm{supp}(Z)$, such that $z' > x$.

We first prove these two properties:

(i) $M(Z) = (1/\eta_1)\,(U(Z|Z) - \eta_2\, GL(Z|Z)) \geq (1/\eta_1)\, U(Z|Z) > (1/\eta_1)\, U(X|X) = M(X)$.

(ii) Suppose that for all $z' \in \mathrm{supp}(Z)$ we have $z' \leq x$. Then $x \geq Z$ almost surely and therefore $M(X) = M(x) \geq M(Z)$ since M is monotone. This contradicts property (i). Thus property (ii) holds.

Property (i) implies:

$$\begin{aligned} 0 \ &\leq\ M(Z) - M(X) = \int_{\mathbb{R}} (m(z) - m(x))\, dF_Z(z) \\ &=\ \int_{z>x} (m(z) - m(x))\, dF_Z(z) + \int_{z<x} (m(z) - m(x))\, dF_Z(z) \end{aligned}$$

and thus

$$\int_{z>x} (m(z) - m(x))\, dF_Z(x) \geq - \int_{z<x} (m(z) - m(x))\, dF_Z.$$

Property (ii) implies:

$$\int_{z>z'} (m(z) - m(z'))\, dF_Z(z)\, dF_Z(z') \geq \int_{z>x} (m(z) - m(x))\, dF_Z(z)$$

Under assumption A3′ we have

$$\begin{aligned} GL(Z|Z) \ &=\ (1-\lambda) \int_{z>z'} (m(z) - m(z'))\, dF_Z(z)\, dF_Z(z') \\ &\overset{(i)}{\leq}\ (1-\lambda) \int_{z>x} (m(z) - m(x))\, dF_Z(z) \\ &=\ \int_{z>x} (m(z) - m(x))\, dF_Z(z) - \lambda \int_{z>x} (m(z) - m(x))\, dF_Z(z) \\ &\overset{(ii)}{\leq}\ \int_{z>x} (m(z) - m(x))\, dF_Z(z) + \lambda \int_{z<x} (m(z) - m(x))\, dF_Z(z) \\ &=\ GL(Z|X). \end{aligned}$$

Therefore $GL(Z|X) \geq GL(Z|Z)$ and thus $U(Z|X) \geq U(X|X)$, a contradiction to $X \in \mathcal{X}_{PE}$. This prove the statement.

A.3 Proof of Proposition 3.1

Christofides and Vaggelatou (2004) show that if X and Y are positively associated then

$$\mathbb{E}\left[\phi(X,Y)\right] \geq \mathbb{E}\left[\phi(\tilde{X},\tilde{Y})\right]$$

for every supermodular function $\phi : \mathbb{R}^2 \to \mathbb{R}$ such that the expectations exist (we say that the pair (X,Y) dominates the pair $(\tilde{X},\tilde{Y})$ by supermodular order). The inequality sign is reverted in the latter equation if X and Y are negatively associated. Under Assumption A3$'$ the function $\phi : (x,y) \mapsto u(x|y)$ is supermodular. Consequently,

$$\tilde{U}(X|Y) = \mathbb{E}\left[\phi(X,Y)\right] \geq \mathbb{E}\left[\phi(\tilde{X},\tilde{Y})\right] = \tilde{U}(\tilde{X}|\tilde{Y})$$

if X and Y are positively associated. Similarly,

$$\tilde{U}(X|Y) \leq \tilde{U}(\tilde{X}|\tilde{Y})$$

if X and Y are negatively associated.

A.4 Proof of Proposition 3.2

If $X = Y$, then $H_{X,X}(x,y) = F_X(\min\{x,y\})$. We have:

$$\int\int \mu(m(x) - m(y))\, d^2H(x,y) = \int\int \mu(m(x) - m(y))\, d^2F_X(\min\{x,y\}) = 0$$

since $d^2F_X(\min\{x,y\}) = 0$ for $x \neq y$. Therefore, the gain-loss utility is zero and this proves the statement.

A.5 Proof of Proposition 3.3

Let $X, Y \in \mathcal{Y}$, then

$$
\begin{aligned}
&\tilde{U}(Y|X) + \tilde{U}(X|Y) = \\
&= \eta_1 M(X) + \eta_1 M(Y) \\
&\quad +\eta_2 \int\int \mu(m(y) - m(x))\, d^2H(x,y) + \eta_2 \int\int \mu(m(x) - m(y))\, d^2H(x,y)
\end{aligned}
$$

Since $\tilde{U}(X|X) + \tilde{U}(Y|Y) = \eta_1 M(X) + \eta_1 M(Y)$ by (i), it is sufficient to show that

$$\int\int \mu(m(y) - m(x))\, d^2H(x,y) + \int\int \mu(m(x) - m(y))\, d^2H(x,y) < 0.$$

We have

$$
\begin{aligned}
&\int\int \mu(m(y) - m(x))\, d^2H(x,y) + \int\int \mu(m(x) - m(y))\, d^2H(x,y) \\
&= \int\int \mu(-(m(x) - m(y)))\, d^2H(x,y) + \int\int \mu(m(x) - m(y))\, d^2H(x,y)
\end{aligned}
$$

Property A2 implies that $\mu(-(m(x)-m(y))) + \mu(m(x)-m(y)) < 0$ for all $x \neq y$ (also using that m is strictly increasing). Thus, if $\mathbb{P}[X \neq Y] > 0$, then

$$\int\int \mu(-(m(x) - m(y)))\, d^2H(x,y) + \int\int \mu(m(x) - m(y))\, d^2H(x,y) < 0$$

and this proves the statement.

A.6 Proof of Proposition 3.4

Let $Y \in \mathcal{Y}_{PE}$. If Y is riskless, than the statement is obvious since $U(Z|Y) = \tilde{U}(Z|Y)$ for all $Z \in \mathcal{Y}$. Therefore we assume that Y is stochastic (and thus its cumulative distribution function is not degenerated).

Let $Z \in \mathcal{Y}$. If Z is riskless, then

$$\tilde{U}(Z|Z) \geq U(Y|Y) \geq U(Y|Y) = \tilde{U}(Z|Y),$$

and Z is not preferred to Y if Y is the reference point. Therefore, we also assume that Z is stochastic (and thus its cumulative distribution function is not degenerated).

Let Z^* be a random variable with the same marginal distribution of Z, and Y and Z^* have joint distribution $\min\{F_Y(x), F_Z(y)\}$ (it corresponds to the upper Fréchet bound; see Joe 1997). By Property (2.4), $U(Z^*|Y) = U(Z|Y)$.

Let $\phi : (x, y) \mapsto u(x|y)$. Since ϕ is continuous, bounded and supermodular, then by Tchen (1980, Corollary 2.2)

$$\tilde{U}(Z^*|Y) = \mathbb{E}\left[\phi(Z^*, Y)\right] \geq \mathbb{E}\left[\phi(Z, Y)\right] = \tilde{U}(Z|Y).$$

Therefore, if we prove that $\tilde{U}(Z^*|Y) \leq \tilde{U}(Y|Y)$ then also $\tilde{U}(Z|Y) \leq \tilde{U}(Y|Y)$, and the statement follows. For the sake of simplicity, we denote Z^* by Z.

For any function g we have

$$\int_{x_0}^{x_1} g(t)\, dt = \int_{\mathbb{R}} 1_{\{x_1 > t\}}\, g(t)\, dt - \int_{\mathbb{R}} 1_{\{x_0 > t\}}\, g(t)\, dt.$$

From this property and assumption A3$'$, for any $x, y \in \mathbb{R}$ we obtain:

$$\phi(x, y) - \phi(x_0, y) = \int_{x_0}^{x} g(t, y)\, dm(t) = \int_{\mathbb{R}} 1_{\{x > t\}}\, g(t, y)\, dm(t) - \int_{\mathbb{R}} 1_{\{x_0 > t\}}\, g(t, y)\, dm(t)$$

where $g(t, y) = \eta_1 + \eta_2\, \lambda_y(t)$,

$$\lambda_y(t) = \begin{cases} \lambda & , y > t \\ 1 & , y \leq t \end{cases},$$

and $\lambda > 1$ is defined in A4.

Let $\tilde{Y}$ and $\tilde{Z}$ be independent copies of Y and Z, i.e., $\tilde{Y}$ and $\tilde{Z}$ have the same marginal distributions of Y and Z, respectively, and are both independent from Y and Z. Using the formula for $\phi(x,y) - \phi(x_0,y)$, we have:

$$\begin{aligned}\phi(\tilde{Z},Y) - \phi(\tilde{Y},Y) &= \int_{\mathbb{R}} (1_{\{\tilde{Z}>t\}} - 1_{\{\tilde{Y}>t\}})\, g(t,Y)\, dm(t)\\ \phi(Z,Y) - \phi(Y,Y) &= \int_{R} (1_{\{Z>t\}} - 1_{\{Y>t\}})\, g(t,Y)\, dm(t).\end{aligned}$$

We take the expectations and we apply Fubini's theorem; it follows:

$$\begin{aligned}U(Z|Y) - U(Y|Y) &= \mathbb{E}\left[\phi(\tilde{Z},Y)\right] - \mathbb{E}\left[\phi(\tilde{Y},Y)\right] = \int_{\mathbb{R}} (F_Y(t) - F_Z(t))\, \mathbb{E}\left[g(t,Y)\right]\, dm(t)\\ \tilde{U}(Z|Y) - \tilde{U}(Y|Y) &= \mathbb{E}\left[\phi(Z,Y)\right] - \mathbb{E}\left[\phi(Y,Y)\right] = \int_{\mathbb{R}} \mathbb{E}\left[(1_{\{Z>t\}} - 1_{\{Y>t\}})\, g(t,Y)\right]\, dm(t).\end{aligned}$$

Using that $g(t,Y) = \eta_1 + \eta_2\, \lambda\, 1_{\{Y>t\}} + \eta_2\, 1_{\{Y\leq t\}}$ we derive the expected values of $g(t,Y)$ and $(1_{\{Z>t\}} - 1_{\{Y>t\}})\, g(t,Y)$:

$$\begin{aligned}U(Z|Y) - U(Y|Y) &= (\eta_1 + \lambda\, \eta_2) \int_{R} (F_Y(t) - F_Z(t))\, dm(t)\\ &\quad -(\lambda - 1)\eta_2 \int_{\mathbb{R}} (F_Y(t) - F_Z(t))\, F_Y(t)\, dm(t)\\ \tilde{U}(Z|Y) - \tilde{U}(Y|Y) &= (\eta_1 + \lambda\, \eta_2) \int_{R} (F_Y(t) - F_Z(t))\, dm(t)\\ &\quad -(\lambda - 1)\eta_2 \int_{\mathbb{R}} (F_Y(t) - H_{Y,Z}(t,t))\, dm(t)\end{aligned}$$

and therefore

$$\tilde{U}(Z|Y) - \tilde{U}(Y|Y) =$$

$$= U(Z|Y) - U(Y|Y) - \eta_2(\lambda - 1)\int_R [(F_Y(t) - H_{Y,Z}(t,t)) - (F_Y(t) - F_Z(t))F_Y(t)]\, dm(t).$$

The first term is negative since $Y \in \mathcal{Y}_{PE}$; the second term is also negative since $H_{Y,Z}(t,t) = \min\{F_Y(t), F_Z(t)\}$:

$$F_Y(t) - H_{Y,Z}(t,t) - (F_Y(t) - F_Z(t))F_Y(t) = \begin{cases} (F_Z(t) - F_Y(t))\, F_Y(t) & , F_Y(t) \leq F_Z(t) \\ (F_Y(t) - F_Z(t))\,(1 - F_Y(t)) & , F_Y(t) > F_Z(t) \end{cases}.$$

Thus $\tilde{U}(Z|Y) \leq \tilde{U}(Y|Y)$ and since this is true for all $Z \in \mathcal{Y}$, Y is a state-dependent personal equilibrium, i.e., $Y \in \mathcal{Y}_{SPE}$.

A.7 Proof of Proposition 3.5

(i) Follows directly from the definition of SPPE and Proposition 3.2.

(ii) Let Y be a SPPE and suppose that there exists $W \in \mathcal{Y}$ such that $M(W) > M(Y)$. Without loss of generality, we take $W \in \arg\max\{M(Z) : Z \in \mathcal{Y}\}$. Let $V \in \mathcal{Y}$, then $M(W) \geq M(V)$. Under assumption A3′, the function μ is concave, thus by Jensen's inequality we have:

$$\mathbb{E}\left[\mu(m(V) - m(W))\right] \leq \mu\left(\mathbb{E}\left[m(V) - m(W)\right]\right) = \mu(M(V) - M(W)) \leq 0.$$

Therefore,

$$\tilde{U}(V|W) = \eta_1\, m(V) + \eta_2\, \mathbb{E}\left[\mu(m(V) - m(W))\right] \leq \eta_1\, m(V) \leq \eta_1\, M(W) = \tilde{U}(W|W),$$

i.e., W is a state-dependent personal equilibrium given $\mathcal{Y}$. By (i), the SPPE has maximal consumption utility over the set of SPE's given $mathcalY$, which contradicts $M(W) > M(V)$. This also shows that $W \in \arg\max\{M(Z) : Z \in \mathcal{Y}\}$ is a SPPE given $\mathcal{Y}$.

A.8 Proof of Proposition 3.6

Assume that $Y \in \mathcal{Y}_{SPE}$ but $Y \notin \mathcal{X}_{SPE}$. The we find $X \in \mathcal{X} \setminus \mathcal{Y}$ such that

$$\tilde{U}(X|Y) > \tilde{U}(Y|Y).$$

If $\eta_2 = 0$, the statement is clear.

Let $\eta_2 > 0$ and

$$\tilde{GL}(Z|W) = \frac{1}{\eta_2} (\tilde{U}(Z|W) - \eta_1 M(Z))$$

for all $(Z, W) \in \mathcal{X} \times \mathcal{Y}$.

It follows:

$$\begin{aligned}
&\tilde{U}(X|X_0) - \tilde{U}(Y|X_0) = \\
&= \eta_1 M(X) + \eta_2 \tilde{GL}(X|X_0) - \eta_1 M(Y) - \eta_2 \tilde{GL}(Y|X_0) \\
&= \eta_1 M(X) + \underbrace{(\eta_2 \tilde{GL}(X|Y) - \tilde{GL}(X|Y))}_{=0} + \eta_2 \tilde{GL}(X|X_0) - \eta_1 M(Y) - \eta_2 \tilde{GL}(Y|X_0) \\
&= \tilde{U}(X|Y) - \tilde{GL}(X|Y)) + \eta_2 \tilde{GL}(X|X_0) - \tilde{U}(Y|Y) - \eta_2 \tilde{GL}(Y|X_0) \\
&= \tilde{U}(X|Y) - \tilde{U}(Y|Y) + \eta_2 (\tilde{GL}(X|X_0) - \tilde{GL}(X|Y)) - \tilde{GL}(Y|X_0)) \\
&\geq \eta_2 (\tilde{GL}(X|X_0) - \tilde{GL}(X|Y)) - \tilde{GL}(Y|X_0))
\end{aligned}$$

The latter inequality follows since $\tilde{U}(X|Y) > \tilde{U}(Y|Y)$.

Moreover, from Definition 3.1, it follows

$$\tilde{GL}(X|X_0) = \mathbb{E}\left[\mu(m(X) - m(X_0))\right] = \mathbb{E}\left[\mu(m(X) - m(Y) + m(Y) - m(X_0))\right].$$

Under Assumption A3′, μ is super-additive, i.e., $\mu(x+y) \geq \mu(x) + \mu(y)$ for all $x, y \in \mathbb{R}$. It follows:

$$\begin{aligned}
\tilde{GL}(X|X_0) &= \mathbb{E}\left[\mu(m(X) - m(Y) + m(Y) - m(X_0))\right] \\
&\geq \mathbb{E}\left[\mu(m(X) - m(Y)) + \mu(m(Y) - m(X_0))\right] \\
&= \mathbb{E}\left[\mu(m(X) - m(Y))\right] + \mathbb{E}\left[\mu(m(Y) - m(X_0))\right] \\
&= \tilde{GL}(X|Y) + \tilde{GL}(Y|X_0).
\end{aligned}$$

Therefore

$$\tilde{U}(X|X_0) - \tilde{U}(Y|X_0) \geq \eta_2\left(tildeGL(X|X_0) - \tilde{GL}(X|Y) - \tilde{GL}(Y|X_0)\right) \geq 0$$

and $\tilde{U}(X|X_0) \geq \tilde{U}(Y|X_0)$. This implies that $X \in \mathcal{Y}$. A contradiction to $Y \in \mathcal{Y}_{SPE}$.

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z	$\left[0, \frac{200}{3+\lambda}\right]$	$\left[\frac{200}{3+\lambda}, 50\right]$	$\left[50, \frac{100\,(1+\lambda)}{3+\lambda}\right]$	$\left[\frac{100\,(1+\lambda)}{3+\lambda}, 100\right]$
CO	Y	Y	Z	Z
PE	Y	Y, Z	Z	Z
PPE	Y	$\mathbf{Z}$	Z	Z
SPE	Y	Y, Z	Y, Z	Z
SPPE	Y	$\mathbf{Y}$	Z	Z

Table 1: The table shows consumption optimum (CO), personal equilibria (PE), preferred personal equilibria (PPE), state-dependent personal equilibria (SPE) and state-dependent preferred personal equilibria (SPPE) for a risk neutral decision maker who face the choice between a fifty-fifty gamble Y for 0 or 100, and a sure thing that pays $z \in [0, 100]$.

	Köszegi and Rabin model				State-dependent model			
	Daily	Weekly	Monthly	Yearly	Daily	Weekly	Monthly	Yearly
U(Bills|Bills)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
U(Stocks|Bills)	-0.276	-0.564	-0.653	6.379	-0.276	-0.564	-0.653	6.379
U(Mix Funds|Bills)	-0.138	-0.282	-0.326	3.190	-0.138	-0.282	-0.326	3.190
U(Bills|Stocks)	-0.359	-0.930	-2.389	-16.981	-0.359	-0.930	-2.389	-16.981
U(Stocks|Stocks)	-0.471	-1.058	-2.018	-4.171	0.021	0.091	0.434	5.840
U(Mix Funds|Stocks)	-0.387	-0.924	-2.045	-9.723	-0.169	-0.419	-0.977	-5.571
U(Bills|Mix Funds)	-0.179	-0.465	-1.194	-8.491	-0.179	-0.465	-1.194	-8.491
U(Stocks|Mix Funds)	-0.346	-0.741	-1.177	1.957	-0.128	-0.236	-0.109	6.110
U(Mix Funds|Mix Funds)	-0.235	-0.529	-1.009	-2.086	0.010	0.046	0.217	2.920
Bills are (S)PE	1.0000	1.0000	0.9097	0.1632	1.0000	1.0000	0.9156	0.1697
Stocks are (S)PE	0.0000	0.0098	0.5509	0.9500	1.0000	1.0000	1.0000	0.9992
Mix funds are (S)PE	0.2120	0.1350	0.5189	0.0819	1.0000	1.0000	0.9156	0.1689
Bills are (S)PPE	1.0000	1.0000	0.9097	0.1632	0.0104	0.0201	0.0125	0.0148
Stocks are (S)PPE	0.0000	0.0000	0.0903	0.8368	0.9896	0.9799	0.9875	0.9852
Mix funds are (S)PPE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 2: The table shows the results from applying reference-dependent utility models to daily, weekly and monthly excess returns of bills, stocks, and mix funds with 50% stocks and 50% bills from July 1, 1963, to April 31, 2010, and yearly returns for 1963-2009. The first four columns show the results for the Köszegi and Rabin model, while the last four columns show the results for our state-dependent model with a fully endogenous reference point. The first nine rows give the average of reference-dependent utility U(Bills|Bills), U(Stocks|Bills), U(Mix Fund|Bills), U(Bills|Stocks), U(Stocks|Stocks), U(Mix Fund|Stocks), U(Bills|Mix Fund), U(Stocks|Mix Fund) and U(Mix Fund|Mix Fund), assuming risk neutral consumption utility ($m(x) = x$) and a piecewise-linear value function ($\mu(x) = x$ for $x \geq 0$ and $\mu(x) = 2\,x$ for $x < 0$). Additionally, for the state-dependent model we have assumed that the set of admissible reference points corresponds to $\mathcal{Y} = \{X \in \mathcal{X} \,:\, c(X, X_0) \leq c_0\}$, where $c(X, X_0) = \tilde{U}(X_0|X_0) - \tilde{U}(X|X_0)$ and $c_0 = \infty$, thus $\mathcal{Y} = \mathcal{X} = \{$Bills, Stocks, Mix Funds$\}$ in all cases. The last six rows contain bootstrap results. We generated 10,000 pseudo-samples through random sampling with replacement from the original sample, and computed average reference-dependent utility in every pseudo-sample. Next, we computed the fraction of the pseudo-samples where bills, stocks or the mix fund represent a (S)PE or (S)PPE. The stock series are from Kenneth French' online data library; the T-bill series are from Ibbotson Associates.

	Daily	Weekly	Monthly	Yearly
X_0=Bills				
$\mathcal{Y}$ = {**Bills**}	1.000	1.000	0.915	0.169
$\mathcal{Y}$ = {Bills, Stocks}	0.000	0.000	0.000	0.000
$\mathcal{Y}$ = {Bills, Mix Funds}	0.000	0.000	0.000	0.000
$\mathcal{Y}$ = {Bills, **Stocks**, Mix Funds}	0.0000	0.000	0.085	0.831
X_0=Stocks				
$\mathcal{Y}$ = {**Stocks**}	1.000	1.000	1.000	1.000
$\mathcal{Y}$ = {Stocks, Bills}	0.000	0.000	0.000	0.000
$\mathcal{Y}$ = {Stocks, Mix Funds}	0.000	0.000	0.000	0.000
$\mathcal{Y}$ = {Stocks, Bills, Mix Funds}	0.000	0.000	0.000	0.000
X_0=Mix Fund				
$\mathcal{Y}$ = {**Mix Funds**}	1.000	1.000	0.915	0.169
$\mathcal{Y}$ = {Mix Funds, Bills}	0.000	0.000	0.000	0.000
$\mathcal{Y}$ = {Mix Funds, **Stocks**}	0.000	0.000	0.085	0.831
$\mathcal{Y}$ = {Mix Funds, Bills, Stocks}	0.000	0.000	0.000	0.000

Table 3: The table shows the results from the applying state-dependent utility model to daily, weekly and monthly excess returns of bills, stocks, and mix funds with 50% stocks and 50% bills from July 1, 1963, to April 31, 2010, and yearly returns for 1963-2009. We assumed a risk neutral consumption utility ($m(x) = x$) and a piecewise-linear value function ($\mu(x) = x$ for $x \geq 0$ and $\mu(x) = 2\,x$ for $x < 0$). The set of admissible reference points corresponds to $\mathcal{Y} = \{X \in \mathcal{X} \,:\, c(X, X_0) \leq c_0\}$, where $c(X, X_0) = \tilde{U}(X_0|X_0) - \tilde{U}(X|X_0)$ and $c_0 = 0$. The table reports bootstrap results. We generated 10,000 pseudo-samples through random sampling with replacement from the original sample, and computed average reference-dependent utility in every pseudo-sample. For every initial reference point $X_0 \in$ {Bills, Stocks, Mix Funds} we computed the fraction of the pseudo-samples where the set of admissible reference points corresponded to $\mathcal{Y}$ reported in the first column for the corresponding values of the exogenously given reference point X_0. The SPPE given $\mathcal{Y}$ is bold-faced. The stock series are from Kenneth French' online data library; the T-bill series are from Ibbotson Associates.