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Labor Market Policy Instruments and the Role of Economic Turbulence[1]

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Abstract

Times of high unemployment always inspire debates on the role of labor market policy and its optimal implementation. This paper uses a dynamic model of search unemployment and bilateral wage bargaining to characterize optimal labor market policy in a possibly turbulent environment. A firing externality, generated by the existence of a partial unemployment insurance system, distorts the pre-policy equilibrium along three margins: job creation, job acceptance, and job destruction. Optimal policy is characterized by a payroll tax, a firing tax, and a hiring subsidy. Endogenous job acceptance demands that a firing tax and a hiring subsidy have to be set equal in any case and cannot be used to correct for the possible failure of the Hosios condition. In that case the optimal policy mix has to be extended by either an output or recruitment tax/subsidy. It is further shown that the derived policy mix is robust to the introduction of economic turbulence in form of state-dependent worker transitions between skill classes. This is crucial as widely discussed intergroup redistribution schemes, like in-work benefits targeted at low-skilled workers, are rendered considerably less effective in that case. Instead of redistribution from high- to low-skilled workers or from firing firms to unemployed workers, the paper identifies a scheme involving redistribution from firing to hiring firms to be optimal.

Keywords

Search and matching, employment subsidies, economic turbulence, policy spill-over.

JEL Classification

E24, E61, J08

1 Introduction

Times of high unemployment always inspire debates on the role of labor market policy. While most scholars agree that unemployment rates in the OECD, especially of low-skilled workers, are excessively high, there is little consensus on what should be done about it, leading to a variety of policy advice. Take for example the recent debate in Germany on the controversial subject of which of two specific policy instruments should be introduced: wage subsidies (e.g. Sinn et al., 2006) versus hiring subsidies (e.g. Brown et al., 2007). While there exists much empirical work focusing on the relative effectiveness of specific instruments, the aim of this paper is to develop a theoretical characterization of optimal policy. We will employ a dynamic equilibrium model of search unemployment, rich enough to incorporate three important decision margins: job creation, job acceptance, and job destruction, in a possibly turbulent environment. Turbulence is introduced to the model, in the spirit of Ljungqvist and Sargent (1998), as state-dependent transitions of workers between skill classes, i.e. unemployed workers lose their skills in the course of time. This adds an additional channel for policy spill-over to the framework which plays an important role in case of policy targeting. Possible inefficiencies of the initial, pre-policy equilibrium come in the form of a typical search externality and a firing externality, which we will focus on in more detail. The need to finance an existing partial unemployment insurance (UI) system, not internalized by firms, gives rise to the latter externality. We introduce a set of policy instruments: payroll, output and firing taxes as well as wage, hiring and recruitment subsidies, while we do not allow for taxation of unemployed workers. The analysis shows that the optimal policy mix can be characterized by a payroll tax as a fiscal instrument set to finance the UI system and a 'firing tax equal hiring subsidy'-scheme similar to Ligthart and Heijdra (2000) and Heijdra and Ligthart (2002), representing redistribution from firing to hiring firms, to correct for the distortions. This contrasts the results of Blanchard and Tirole (2008) (henceforth BT) who in a static approach, neglecting job creation and acceptance, characterize optimal policy by redistribution from firing firms to unemployed workers, which resembles the experience rating system implemented in the United States. The derived optimal policy mix has to be expanded by either an output or a recruitment tax/subsidy in case of 'unbalanced' search externalities to correct for the failure of the Hosios (1990)-condition. An important finding is that the characterization of optimal policy is robust to the introduction of economic turbulence. This is crucial as a lot of existing policy advice is rendered considerably less effective in that case. The cross-financed wage subsidy scheme for low-skilled workers, representing redistribution from high- to low-skilled workers, as proposed by Mortensen and Pissarides (2003) (henceforth MP) is an example for such a policy advice. They assume skill classes to operate in complete juxtaposition, except for the connection via the government's budget constraint, which underestimates the adverse effect on high-skilled workers who become more picky concerning their acceptance and continuation decision as their fall back option increases with the wage subsidy for low-skilled workers. In addition the skill composition deteriorates as a result of such a targeting scheme.

This paper relates to several strands of the literature. As mentioned above there are numerous empirical studies[1] that deal with estimating the employment effects of various policy instruments. But typical inference from comparing treated and untreated individuals to evaluate 'big scale' policies faces the following problems. First, a policy has actually to be in place. Second, 'big scale' policies will induce general equilibrium effects which lead to a violation of the necessary 'stable unit treatment value assumption' (see Angrist et al., 1996). In their study on counseling, Cahuc and Barbanchon (2010) argue how micro evaluations neglecting crowding out, adverse spill-over effects on non-targeted persons, and other equilibrium effects can lead to misguided policy advice. We therefore base our analysis on a model of equilibrium search unemployment rich enough to capture those effects. Dynamic search and matching models have been widely used to evaluate employment subsidies ever since the influential paper of Mortensen and Pissarides (1994). However, the conclusions so far are mixed. While Bovenberg et al. (2000) and Cardullo and Van der Linden (2006) argue that wage subsidies can substantially reduce unemployment, Boone and van Ours (2004) and Oskamp and Snower (2008) find no such effect. What these type of studies typically have in common is that they embed the frictional labor market in complex CGE-models which makes it hard to disentangle different effects or draw conclusions concerning the optimal design of policy which is at the heart of our paper. A more theoretical treatment - probably most closely related to this paper - is provided by MP. They analytically derive optimal policy before presenting some simulation results for non-optimal policy schemes. We will extend their analysis on several dimensions. First, in their optimal policy characterization, MP solely concentrate on the distorting effects of subsidies and taxes while fiscal effects are suppressed by allowing for non-distortionary consumption taxes. Hence, the firing externality in the spirit of BT does not play a role in their setting. Second, we introduce an additional margin (job acceptance) which will alter optimal policy if search externalities are 'unbalanced'. Third and most importantly, we introduce economic turbulence and discuss its role for policy design. Another closely related paper is Michau (2009) who extends the BT-setting to a dynamic search and matching framework. He explicitly models the UI problem with risk-averse workers and finds that the welfare maximizing allocation is characterized by full insurance and output

[1] Empirical evidence on the effects of wage subsidies is summarized by Katz (1996) for the United States, Bell et al. (1999) for the United Kingdom, and Bonin et al. (2002) for Germany. Boockmann et al. (2007) provide some evidence on 'hiring subsidy'-like grants in Germany. A review on the empirical effects of employment protection is presented in Lazear (1990).

maximization. As Nash bargained wages with positive bargaining power of the workers are incompatible with full insurance, he finds that in a second best a social planner would reduce labor market tightness, implemented by a positive spread between a firing tax and a hiring subsidy. This is done to reduce wages and therefore decrease under-provision of UI. We do not consider this trade-off between insurance and output maximization here by conditioning our analysis on the existence of a partial UI system as the focus lies on role of economic turbulence in designing policy. The idea of economic turbulence is inspired by work of Ljungqvist and Sargent (1998). While this strand of the literature, including Pissarides (1992), Ljungqvist and Sargent (2004), and Den Haan et al. (2005), is concerned with the influence of skill depreciation during unemployment on the persistence of unemployment, its implication for policy design has received little attention so far.

The outline of the paper is as follows. First, a simple intragroup model is developed featuring only one skill class. The optimal policy mix, implementing the social planner's solution, is characterized, which will provide good intuition for the more complex intergroup model discussed in section 3. We extend the intragroup by an additional skill class and allow for redistribution as well as economic turbulence in form of state-dependent transitions of workers between the skill classes. After showing how the characterization of optimal policy in the intergroup model relates to the intragroup case we present some simulation results in order to highlight also the quantitative dimension of the results.

2 A simple intragroup model

The model is based on the standard dynamic Mortensen and Pissarides (1994)-framework enriched by endogenous acceptance. In this section we consider only one skill class. There are two types of rational, forward looking agents: workers and firms. Labor force L is comprised of atomistic risk neutral workers. The assumption of risk neutrality is discussed more thoroughly in section 2.2. There is a sufficiently large number of risk neutral firms that can enter the labor market instantaneously but are subject to per-period net flow costs c for posting a vacancy. For production each firm needs one worker who will inelastically supply one unit of labor if employed. The three decision margins: job creation (θ), job acceptance ($\underline{x}$), and job destruction ($\hat{\underline{x}}$) are best understood when looking at the life cycle of a job. First, firms decide to post vacancies according to a free entry condition which fixes labor market tightness θ. The search friction implies that it takes time to fill a vacancy during which a firm has to pay per-period gross posting costs C that are reduced by a recruitment subsidy R to $c = C - R$. Eventually a worker and a firm are matched according to a matching technology m. This can be interpreted as meeting for a job interview. Only then the agents will learn how well suited an applicant is for the specific job. This is modeled as drawing a job-specific productivity x from a known distribution $G(\cdot)$. If the realization of the draw is higher than the according reservation productivity, referred to as 'outside' cut-off, i.e. $x > \underline{x}$, the job is started and the firm receives a one-time hiring subsidy H. Technically, this is one of the main difference compared to MP who assume that every job is created at maximum idiosyncratic productivity, trivializing the acceptance decision because job offers are rejected with probability zero. [2] During production the firm receives net off tax output $(1-\tau)x$ and an in-work benefit or wage subsidy D that partly compensates for the wage w, stemming from a Nash bargaining game, it has to pay to the worker. A new idiosyncratic productivity shock arrives with probability π^n. If a new draw is lower than the endogenous 'inside' cut-off, i.e. $x < \hat{\underline{x}}$, the job is destroyed and the firm has to pay a separation or firing tax F. To summarize the featured instruments. Three different subsidies will be analyzed: a periodic lump-sum wage subsidy (D), a one-time hiring subsidy to the firm (H), and a recruitment subsidy (R). On the other hand we will analyze three distortionary taxes, namely: firing taxes (F), linear output taxes (τ), and linear[3] payroll taxes (t). An important assumption we make is that unemployed workers cannot be taxed. Part of the value of non-work is home production which cannot be transformed into tax revenue. This rules out non-distortionary consumption taxation. Analytically, the model can be described as follows[4].

As usual for this kind of framework an aggregate matching function $m(u, v)$, which maps the stock of unemployed (u) and the stock of vacancies (v) into the flow of new matches (m), is assumed to be homogeneous of degree one with elasticity w.r.t. u of $0 < \eta < 1$. Defining labor market tightness as $\theta \equiv \frac{v}{u}$ results in the matching probability functions (2.1) and (2.2) for firms and workers, respectively.

$$\text{prob. of a match for the firm:} \quad \frac{m(u,v)}{v} = q(\theta) \tag{2.1}$$

$$\text{prob. of a match for the worker:} \quad \frac{m(u,v)}{u} = \theta q(\theta) \tag{2.2}$$

with $q'(\cdot) < 0$, $q''(\cdot) < 0$ and $m(u, v) \leq \min(u, v)$. Further define: $q^f = q(\theta)(1 - G(\underline{x}))$ and $q^w = \theta q(\theta)(1 - G(\underline{x}))$ as the joint probabilities of matching and accepting. A worker can

[2]In an alternative interpretation this relates to Hall (2005) who also allows for less qualified persons to apply. In contrast to our analysis, he assumes that the qualification of an applicant is not completely revealed to the employer in the first meeting. This can only be resolved if the employer decides to costly evaluate the application.

[3]The linearity assumption does not drive the fundamental results but helps to keep the mathematics straightforward. Note that the lump-sum component of the wage subsidy going to the worker and the linear component t can mimic a regressive or progressive tax schedule. The implementation of the efficient allocation, as derived in section 2.2, does not require a wage subsidy.

[4]The notation is based on Pissarides (2000). A description of all used variables can be found in appendix section G.

be either employed (e) or unemployed (u), that is we abstract from transitions into and out of labor force, hence $e + u = L$. Each state is associated with a specific present value, U for being unemployed and $W(x)$ or $\hat{W}(x)$ for becoming or being employed, respectively. A firm participating in the labor market can be in two states. Either it is looking for a worker which has value V or it is employing a worker which gives $J(x)$ or $\hat{J}(x)$. In general, the hat-notation always indicates that the worker or the firm has already been in the same state before the arrival of a shock. Or put differently, 'without hat' can be referred to as the initial or 'outside' value while 'with hat' denotes the continuation or 'inside' value. Given the assumption of perfect capital markets, where r denotes the exogenous interest rate, we can write both asset equations of working as follows:

$$rW(x) = (1-t)w(x) + \pi^n \left[(1 - G(\underline{\hat{x}}))\, \hat{W}^{\hat{e}} + G(\underline{\hat{x}})U - W(x) \right] \tag{2.3}$$

$$r\hat{W}(x) = (1-t)\hat{w}(x) + \pi^n \left[(1 - G(\underline{\hat{x}}))\, \hat{W}^{\hat{e}} + G(\underline{\hat{x}})U - \hat{W}(x) \right] \tag{2.4}$$

A just recently employed worker's felicity equals after tax wage income $(1-t)w(x)$ or $(1-t)\hat{w}(x)$, respectively. When a shock arrives he loses $W(x)$ and gains U if the new productivity draw x is lower than the 'inside' cut-off $\underline{\hat{x}}$, hence with probability $G(\underline{\hat{x}})$. With probability $(1 - G(\underline{\hat{x}}))$ he gets $\hat{W}^{\hat{e}}$, which denotes the conditional expectation[5] of the value of being employed. The asset value of being unemployed is given by

$$rU = z + q^w (W^e - U) \tag{2.5}$$

where z denotes the value of non-work which is composed of unemployment compensation b and home production h in a linear way, $z = b + h$. Turning to the firms' side the asset value of a vacancy can be written as

$$rV = -c + q^f (J^e + H - V), \qquad \text{where} \qquad c = C - R \tag{2.6}$$

Two subsidies enter this relationship. In case of an accepted match the firm has to give up the value of a vacancy V but gets the expected value of a job for the firm J^e plus a hiring subsidy H. The gross flow costs of maintaining a vacancy C minus the recruitment subsidy R give the net costs c. As free entry is imposed and V is decreasing in θ, in equilibrium V is driven down to zero which will pin down θ, hence:

$$V = 0 \Rightarrow \theta \tag{2.7}$$

[5] The conditional expectation of some random variable $X(x)$ w.r.t. $\underline{\hat{x}}$ is defined as $E(X(x)|x > \underline{\hat{x}}) = X^{\hat{e}} = \int_{\underline{\hat{x}}}^{\infty} \frac{X(\tilde{x})}{1-G(\underline{\hat{x}})}\, dG(\tilde{x})$. Note the difference in notation compared to $E(X(x)|x > \underline{x}) = X^e$.

The asset values of a job are given in (2.8) and (2.9):

$$rJ(x) = (1-\tau)x - w(x) + D + \pi^n \left[(1-G(\underline{\hat{x}}))\hat{J}^{\hat{e}} - G(\underline{\hat{x}})F - J(x)\right] \tag{2.8}$$

$$r\hat{J}(x) = (1-\tau)x - \hat{w}(x) + D + \pi^n \left[(1-G(\underline{\hat{x}}))\hat{J}^{\hat{e}} - G(\underline{\hat{x}})F - \hat{J}(x)\right] \tag{2.9}$$

In the current period a firm receives after tax production $(1-\tau)x$ minus wage rate $w(x)$ or $\hat{w}(x)$ plus a wage subsidy D[6]. In case of a separation, which occurs with probability π^n and the probability of $x < \underline{\hat{x}}$, a firm has to pay a firing tax F. Observe that given the wage determination explained below a firm and a worker will always mutually agree to destroy or create a job, i.e. both sides have the same reservation productivities. Hence, the notions of a 'firing' and 'separation' tax are equivalent. The reservation productivities are pinned down by the following conditions

$$J(\underline{x}) + H = 0 \Rightarrow \underline{x} \tag{2.10}$$

$$\hat{J}(\underline{\hat{x}}) + F = 0 \Rightarrow \underline{\hat{x}} \tag{2.11}$$

The first relation states that after meeting for an interview and observing the match specific productivity x, a job will only be generated if the value of a job including the one-time hiring subsidy is non-negative. The second condition reflects that a firm will only want to continue a job if its value covers at least the firing tax. Wages are determined via Nash bargaining and are renegotiated every time a shock arrives. The Nash wages are given as solutions to the following optimization problems, where the weight ω can be interpreted as the worker's bargaining power.

$$w(x) = \text{argmax}\,(W(x) - U)^{\omega}\,(J(x) + H)^{1-\omega} \tag{2.12}$$

$$\hat{w}(x) = \text{argmax}\left(\hat{W}(x) - U\right)^{\omega}\left(\hat{J}(x) + F\right)^{1-\omega} \tag{2.13}$$

Lemma 2.1. *If $F = H$ then $w(x) = \hat{w}(x)$.*

Proof. Note that $w(x) = \hat{w}(x)$ implies $W(x) = \hat{W}(x)$ and $J(x) = \hat{J}(x)$ by construction. If $F = H$, the two problems (2.12) and (2.13) are identical. ∎

The result of this lemma is more general and can be extended to non-linear utility and non-linear wage income taxation[7]. Given our assumptions the equilibrium 'outside' and

[6]Note that with Nash bargaining it does not matter economically whether the wage subsidy is given to the worker or the firm but the interpretation of w changes. In our setting w and $(1-t)w$ are interpreted as gross and net wages received by worker already including all subsidies.

[7]Insert $v(w(x) - T(x))$ in (2.3) and $v(\hat{w}(x) - T(x))$ in (2.4), with the mild conditions $v'(\cdot) > 0$ and $w(x) - T(x) > 0$ for otherwise arbitrary functions $v(\cdot)$ and $T(\cdot)$. The proof still holds.

'inside' wage rates can be solved for explicitly[8]:

$$w(x) = (1-\omega)\frac{z}{1-t} + \omega((1-\tau)x + D + c\theta + rH) - \omega\pi^n(F-H) \tag{2.14}$$

$$\hat{w}(x) = (1-\omega)\frac{z}{1-t} + \omega((1-\tau)x + D + c\theta + rF) \tag{2.15}$$

Observe that the 'inside' and 'outside' wage distributions are directly related to the productivity distribution $G(\cdot)$ for x larger than the respective cut-off. A wage subsidy D will increase both wage schedules by the share the worker can claim in the process of bargaining, ωD. While a recruitment subsidy R, which is included in c, decreases both wages to the same extent, they respond differently to a hiring subsidy H and a firing tax F. A hiring subsidy will increase the 'outside' wage of a worker while it does not affect the 'inside' wage as the subsidy is already sunk by then. A firing tax will abate 'outside' wages as firms are more cautious about hiring workers because they eventually have to pay F. In contrast, 'inside' wages will be inflated by F because firms are more willing to hold on to workers once they are employed. The relationship of 'outside' and 'inside' wage is simply $w(x) = \hat{w} - (r + \pi^n)\omega(F - H)$. At last, in equilibrium the government's budget constraint has to hold.

$$\begin{aligned} 0 = {} & (L-u)\bar{w}t + (L-u)\bar{x}\tau + (L-u)\pi^n G(\underline{\hat{x}})F \\ & - uq^w H - \theta u R - (L-u)D - ub \end{aligned} \tag{2.16}$$

where $\bar{w}$ and $\bar{x}$ denote average wage and productivity, respectively. The first line represents tax income from the payroll tax, the output tax and the firing tax. The second line gives expenditure on hiring, recruitment, and wage subsidies as well as unemployment benefits.

2.1 Equilibrium

The equilibrium vector $\langle u, \theta, \underline{x}, \underline{\hat{x}} \rangle$ is pinned down by the four equations (2.17) to (2.20)[9]. Equilibrium is partly recursive, i.e. only (2.17) and (2.18), henceforth referred to as the JD-JC system, have to be solved simultaneously for θ and $\underline{\hat{x}}$ after inserting (2.19). The job creation (JC) curve, which is derived from the free entry condition, equates expected gain and cost of a vacancy

$$JC \quad : \quad (1-\omega)\left(\frac{(x^e - \underline{\hat{x}})(1-\tau)}{\pi^n + r} - F + H\right) - \frac{c}{q^f} = 0 \tag{2.17}$$

The first term is the expected gain of job creation for a firm, i.e. the firm's after tax share of excess output discounted by $\pi^n + r$. The gain is additionally raised or lowered

[8]See appendix section C for derivation.

[9]See appendix section C for a detailed derivation of (2.17)-(2.20).

depending on whether the hiring subsidy H exceeds the firing tax F, or vice versa. The second term reflects the expected costs of job creation, i.e. the net flow cost c times the average duration of a vacancy $1/q^f$.

$$JD \quad : \quad (1-\tau)\underline{\hat{x}} + D + \frac{\pi^n(1-\tau)}{\pi^n + r}\int_{\underline{\hat{x}}}^{\infty}(\tilde{x} - \underline{\hat{x}})\, dG(\tilde{x}) \\ - \frac{z}{1-t} + rF - \frac{\omega}{1-\omega}c\theta = 0 \tag{2.18}$$

The first line of the job destruction (JD) condition, which represents the 'inside' cut-off condition, gives the lowest acceptable joint inside value of a job, i.e. the after tax reservation product plus a wage subsidy D and the option value of keeping a worker as her productivity might change. The second line can be interpreted as the joint outside value, which increases in z and θ, as both raise the worker's outside option, and decreases in F. The analytic relationship of the 'outside' to the 'inside' productivity cut-off is novel compared to other studies that do not take endogenous job acceptance into account.

$$\underline{x} = \underline{\hat{x}} + \frac{(\pi^n + r)}{(1-\tau)}(F - H) \tag{2.19}$$

Observe that both cut-offs coincide in a policy free environment where $F = H = 0$. A hiring subsidy H will put a wedge between those cut-offs in a way that agents more easily accept than destroy a job ($\underline{x} < \underline{\hat{x}}$). A firing tax F has the opposite consequence, $\underline{x} > \underline{\hat{x}}$. Having derived all three decision variables θ, $\underline{\hat{x}}$, and $\underline{x}$, we can compute unemployment u. Just insert in the typical Beveridge curve (2.20), which is derived by setting the change in u, i.e. $\dot{u} = (L-u)\pi^n G(\underline{\hat{x}}) - uq^w$, to zero.

$$u = \frac{\pi^n G(\underline{\hat{x}})}{\pi^n G(\underline{\hat{x}}) + q^w} \cdot L \tag{2.20}$$

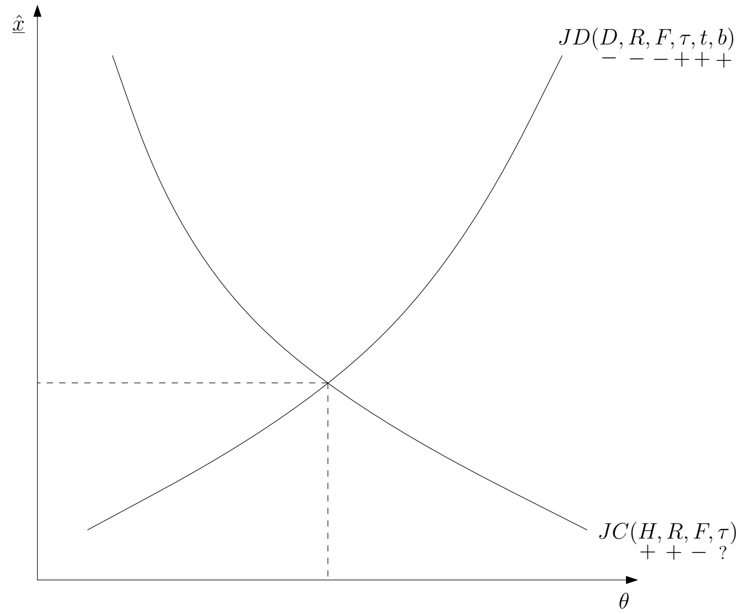
As mentioned, the recursion of the system reduces the problem to solving only two equations simultaneously. Therefore, we can conveniently analyze comparative statics in the JD-JC diagram[10], drawn in the θ-$\underline{\hat{x}}$-space (see Pissarides, 2000). The JC-curve is sloping downward because firms post fewer vacancies the higher $\underline{\hat{x}}$, as average duration of a job decreases in $\underline{\hat{x}}$. The JD-curve slopes upward because workers want to terminate jobs more easily the higher θ, as their outside options increase in labor market tightness. Hence, the curves intersect at most once, as illustrated by figure 2.1, which makes the equilibrium unique in case of existence. We will now shortly address the effects of uncompensated changes in our policy instruments[11]. A wage subsidy D has no effect on the JC-curve

[10]See appendix section F.1 for more details.
[11]See appendix section F.2 for the analytic derivation.

but shifts out the JD-curve. Hence, equilibrium labor market tightness θ will go up, the reservation productivities $\underline{x} = \hat{\underline{x}}$ will fall, leading to more job creation, more acceptance and less destruction. Therefore, unemployment will unambiguously decrease. A hiring subsidy H works quite differently. While there is no effect on the JD-curve the JC-curve will shift outward. This raises labor market tightness and consequently job creation as well as job destruction. In relation to job destruction, job acceptance is boosted, i.e. $\underline{x} < \hat{\underline{x}}$. Whether job acceptance rises or falls in absolute terms is ambiguous. Proposition 2.1 states a condition for the direction of the absolute effect.

Figure 2.1: Uncompensated comparative statics

(a) The job destruction, job creation diagram



(b) Equilibrium effects

	θ	$\hat{\underline{x}}$	$\underline{x}^*$	u
b	$-$	$+$	$=$	$+$
R	$+$	$+$	$=$	?
D	$+$	$-$	$=$	$-$
H	$+$	$+$	$<$	?
F	$-$	$-$	$>$	?
τ	$-$	?[12]	$=$	?[12]
t	$-$	$+$	$=$	$+$

* This column gives the effect on $\underline{x}$ in relation to $\hat{\underline{x}}$.

Proposition 2.1. (Hiring subsidy and job acceptance) *A hiring subsidy can lead to more or less job acceptance. Assume for simplicity that* $t = \tau = 0$. *Whenever* $\nabla^{-1}\omega c(1-\Omega) < \pi^n + r$ *the effect of* H *on* $\underline{x}$ *will be negative, leading to more job acceptance.*

Proof. Differentiating (2.19) w.r.t. H gives $\frac{\partial \underline{x}}{\partial H} = \frac{\partial \hat{\underline{x}}}{\partial H} - (\pi^n + r)$. Inserting for $\frac{\partial \hat{\underline{x}}}{\partial H}$ derived using the implicit function theorem and rearranging completes the proof[13]. ∎

A firing tax F has very similar but inverted effects compared to a hiring subsidy H. While the JC-curve moves inward to the same extent, additionally also the JD-curve shifts outward as long as $r > 0$. Hence, if $F = H$ rise simultaneously all shifts of the JC-curve cancel, while the shift in the JD-curve remains.

[12]The sufficient and necessary condition for $\hat{\underline{x}}$ to increase (assuming $F = H$ and $\omega = \eta$ for simplicity) is: $[\pi^n + r + q^w G(\underline{x})]\, \underline{x} > [q(\theta) - \pi^n]\,(r + \pi^n)\Gamma$. Hence, it is also sufficient to raise u.

[13]∇ denotes the determinant of the JD-JC system which is always positive. Ω is the derivative of the conditional expectation with respect to the cut-off. See appendix section F.1 for details.

Proposition 2.2. (The $F = H$ scheme) *Let $r > 0$. Then $F = H > 0$ leads to more job creation and acceptance, less job destruction and consequently reduced unemployment.*

Proof. This follows directly from (2.17), (2.18), and (2.19). ∎

The positive effect of this scheme, also described in Ligthart and Heijdra (2000) and Heijdra and Ligthart (2002), can be explained as follows. Looking at the life cycle of a job, a $F = H$ scheme can be compared to an interest free loan to the firm, as it gets H at the beginning of a job and eventually pays back the same amount without capital user costs. The gain is therefore reflected in the rF-term in the job destruction condition (2.18). Due to the dynamic structure of the model the alternative interpretation when considering the cross-section of firms at a specific point in time is that $F = H$ implies redistribution from the firing to the hiring firms. A recruitment subsidy affects both curves. Both shift outward leading to an increase in labor market tightness, but the JC-curve moves more strongly implying more job destruction. Compared to a hiring subsidy which is only paid if a job is created, a recruitment subsidy is received by the firm irrespective of whether a match occurs or not. The main consequence is that a hiring subsidy will partly go to the worker, while the latter subsidy is already sunk in the wage bargaining. All effects are summarized in table 2.1. These uncompensated comparative static exercises provide intuition through which channels our policy instruments work. In order to characterize optimal policy we have to develop a notion of efficiency and we have to close the government's budget constraint to restrict the analysis to policy that is implementable. This is done in the next section.

2.2 Efficiency and the optimal policy mix

Efficiency can be distorted in many ways. We will consider two possibilities: first, the typical search externality that comes from the way workers and firms are matched. Second and more at the focus our analysis, we will consider a firing externality in the spirit of BT stemming from the requirement to finance unemployment benefits which is not taken into account by the agents. We will concentrate on this fiscal externality and not on the problem of how unemployment compensation, which we take as given, should be optimally set. Michau (2009) explicitly models the insurance problem with risk-averse workers in a comparable setup and finds that the welfare optimum requires full insurance, i.e. $w = z$ and output maximization. As full insurance is incompatible with Nash bargained wages, we condition our efficiency analysis on the prior implementation of a partial insurance system. We assume that insurance is not perfect but that b is set such that the value of non-work z is close to the value of work[14]. Our model with risk-neutrality can then be

[14] This is also reflected in the calibration choices later on.

considered as a linear approximation of a more complex model that features concavity in the utility function (see Hagedorn and Manovskii (2008) for similar argumentation). The quality of the approximation naturally decreases in the difference of w and z, which as argued above, is assumed to be small. In order to analyze named inefficiencies we compute the solution to the social planner's problem of maximizing total output[15], which is given by the following three reduced equations for socially optimal job creation, job destruction, and job acceptance[16]:

$$(1-\eta)\frac{x^e-\underline{\hat{x}}}{\pi^n+r}-\frac{C}{q^f}=0 \tag{2.21}$$

$$\underline{\hat{x}}+\frac{\pi^n}{\pi^n+r}\int_{\underline{\hat{x}}}^{\infty}(\tilde{x}-\underline{\hat{x}})\,dG(\tilde{x})-h-\frac{\eta}{1-\eta}C\theta=0 \tag{2.22}$$

$$\underline{x}=\underline{\hat{x}} \tag{2.23}$$

Comparing those relations with the decentralized equilibrium equations (2.17)-(2.19) in a policy free world, i.e. $b=F=\tau=t=D=H=R=0$, reveals that they coincide if and only if $\omega=\eta$ (Hosios, 1990). From now on we will follow a Ramsey approach and assume that unemployment compensation $b>0$ is exogenously given and has to be financed with the least possible distortions using our instruments. Subtracting (2.21)-(2.23) from (2.17)-(2.19) gives the conditions that the policies in question have to fulfill to restore efficiency.

$$\frac{x^e-\underline{\hat{x}}}{\pi^n+r}\left[(1-\omega)(1-\tau)-(1-\eta)\right]+\frac{R}{q^f}=(1-\omega)(H-F) \tag{2.24}$$

$$-\tau\left(\underline{\hat{x}}+\pi^n\Gamma\right)-\frac{b+th}{1-t}+D+rF-C\theta\left[\frac{\omega}{1-\omega}-\frac{\eta}{1-\eta}\right]+\frac{\omega}{1-\omega}R\theta=0 \tag{2.25}$$

$$F=H \tag{2.26}$$

where $\Gamma\equiv\frac{1}{r+\pi^n}\int_{\underline{\hat{x}}}^{\infty}(\tilde{x}-\underline{\hat{x}})\,dG(\tilde{x})$. In addition, the government's budget constraint[17] must be met:

$$0=(L-u)\bar{w}t+(L-u)\bar{x}\tau+q^w u(F-H)-\theta uR-(L-u)D-ub \tag{2.27}$$

The important consequence from introducing a job acceptance margin is that $F=H$ has to hold even if the Hosios condition is not fulfilled[18]. In what follows we characterize two

[15]In case of risk-neutral agents the solutions to the problems of maximizing output or utilitarian welfare coincide.

[16]See appendix section D for derivation.

[17]Note that in equilibrium the number of outflows $\pi^n G(\underline{x})(L-u)$ is equal to the inflows $q^w u$. Hence, $F=H$ is budget neutral in equilibrium. One should keep in mind that the introduction of a $F=H$ scheme shifts the JD-curve inward leading to more outflow out of and less inflow into unemployment. Hence, during transition the outlay on H will exceed the revenue generated by F.

[18]Because of lemma 2.1 this finding also generalizes to a framework with risk-averse workers and is independent of whether welfare or output is maximized.

alternative implementations of the optimal allocation, one involving hiring and the other using wage subsidies. We depict the limitations to both schemes.

Let us first assume that the search externalities do not distort the equilibrium, i.e. $\omega = \eta$. Inserting (2.26) in (2.24) reveals that output taxation and recruiting cost subsidization are not required for efficiency, hence: $\tau = R = 0$. Unemployment benefits then have to be financed using the payroll tax $t = \frac{b}{\bar{w}} \frac{u}{L-u} > 0$, which is chosen to fulfill (2.27). As a compensated firing tax, $F = H$, is budget neutral, we can set F in order to fulfill (2.25), hence $F = \frac{b+th}{(1-t)r} > 0$.

Proposition 2.3. (Implementation 1a) *In case of unemployment compensation $b > 0$ and $\omega = \eta$ it is possible to implement the optimal allocation and balance the budget using a payroll tax, $t > 0$, a firing tax and a hiring subsidy, $F = H > 0$.*

Observe the difference compared to BT. In their framework the optimal policy consists of zero payroll taxes and a firing tax to finance unemployment benefits and offset the involved distortions. Here, a firing tax will distort the acceptance margin unless a firing tax is fully compensated by a hiring subsidy. As both instruments together are budget neutral a firing tax cannot be used for financing unemployment compensation. Instead of the redistribution from the firms to the workers as in BT, we require redistribution from employed to unemployed workers and from firing to hiring firms.

Now consider the case where $\omega \neq \eta$. Observe that at least one of the two policy instruments τ or R, is needed to satisfy equation (2.24). First we focus on output taxation, hence setting $R = 0$. The efficient output tax rate is then given by $\tau = 1 - \frac{1-\eta}{1-\omega}$ which is smaller than zero i.e. a subsidy if $\omega > \eta$ and positive if $\omega < \eta$. Therefore, the budget-solving payroll tax rate will be higher ($\omega > \eta$) or smaller ($\omega < \eta$) compared to the benchmark tax rate where the Hosios condition holds. Again F is set to fulfill (2.25) and therefore the implementation of the optimal allocation is complete. Note that the case $F < 0$ cannot be ruled out now. Instead of τ one could alternatively use $R = \frac{x^e - \hat{x}}{\pi^n + r}(\omega - \eta)q^f$ by the same argument.

Proposition 2.4. (Implementation 1b) *In case of unemployment compensation $b > 0$ and $\omega \neq \eta$ it is possible to implement the optimal allocation and balance budget using a payroll tax t, a firing tax and a hiring subsidy, $F = H$, and at least one of the following two instruments: output (τ) or recruitment (R) tax/subsidy.*

MP do not explicitly consider the case of $\omega \neq \eta$ but it is easy to see that their job creation curve can be moved to the optimum just by adjusting $F \neq H$ accordingly. In our case this is not possible as $F = H$ is always required to offset the distortions on the job acceptance margin. Hence, the job creation curve can only be shifted by additional instruments, such as an output or recruitment tax/subsidy.

The above implementations might require the firing tax to be of considerable magnitude. This will certainly be an issue when firms are liquidity constrained, e.g. $F \leq F_{max}$ (see BT) which will eventually prevent the implementation of the optimal allocation. This becomes even more severe in the following extension. One can assume that F only partly improves the government's budget, say by F_{tax} as a fixed part $F_{cost} = F - F_{tax}$ reflects sunk firing costs, e.g. the administrative costs of a lay-off, etc. Obviously, $F = H$ is no longer budget neutral, implying that the payroll tax t has to rise to close the budget constraint and $F = H$ have to be even higher to undo the additional distortion of the increased payroll tax. Hence, it is more likely to hit F_{max}.

Note that a wage subsidy D is not required for achieving efficiency but possibly provides an alternative implementation. For simplicity assume again that $\omega = \eta$ and set $F = H = \tau = R = 0$. The lump-sum wage subsidy D, in addition to unemployment compensation b, is financed using a payroll tax t, ergo $D = \bar{w}t - \frac{u}{L-u}b$. The job destruction curve will coincide with its social optimal counterpart if and only if $\frac{b}{1-t} + \frac{ht}{1-t} + \frac{u}{L-u}b = \bar{w}t$. For $u > 0$ we can derive a necessary condition for the replacement ratio, namely $\frac{b}{\bar{w}} < \frac{1}{4}$. The contrapositive reads:

Proposition 2.5. (Implementation 2) *In case of unemployment compensation $b > 0$ and $\omega = \eta$ it is **not** possible to implement the optimal allocation and balance budget using only a payroll tax t and a wage subsidy D, if the replacement ratio is higher than 25%.*

Proof. $\frac{b}{1-t} + \frac{ht}{1-t} + \frac{u}{L-u}b = \bar{w}t \overset{u>0}{\Longrightarrow} \frac{b}{1-t} + \frac{ht}{1-t} < \bar{w}t \Rightarrow \frac{b}{\bar{w}} < t(1-t) - t\frac{h}{\bar{w}} \Rightarrow \frac{b}{\bar{w}} < \max_{t,\frac{h}{\bar{w}} \in [0,1)\times[0,1)} \left[t(1-t) - t\frac{h}{\bar{w}}\right] = \frac{1}{4} \Leftrightarrow \frac{b}{\bar{w}} < \frac{1}{4}$ ∎

This implementation is very specific to the way we introduced those instruments (D lump-sum and t linear). A possible implementation with $D > 0$ and $t > 0$ would mimic a progressive tax schedule. As this condition is hardly met in any OECD economy anyways we shall focus on implementation 1 in what follows.

3 An intergroup model with economic turbulence

So far, we focused on intragroup redistribution. Allowing for intergroup redistribution enriches the model considerably because it enables us to evaluate more realistic policies. MP find in a numerical simulation that a wage subsidy targeted at low-skilled workers and financed by high-skilled workers works quite well in bringing down overall unemployment[19]. Besides the connection via the government's budget constraint, they assume the two skill classes to operate in complete juxtaposition. The issue that "targeting is likely to damage the quality and quantity of labor supply" (Bovenberg et al., 2000) is therefore hardly addressed. The aim of this section is to show how the optimal policy mix is altered by the presence of economic turbulence and we find that a scheme as proposed by MP might be considerably less effective in such an environment. The idea that increased economic turbulence affects labor market outcomes is related to Ljungqvist and Sargent (1998), who assume that unemployed workers lose their skills in the course of time[20] as they can not keep up to date with new production technologies[21]. In a broader interpretation, these new production techniques and requirements emerge as a result of ongoing restructuring from manufacturing to services, spread of new information technologies, internationalization of production, etc. which all lead to expeditious changes in the economic environment, and render previous ways of production obsolete. Hence, a worker who is only familiar with outdated techniques is less productive when confronted with state-of-the-art production technology.

The key differences compared to the simple intragroup model described above follow from the introduction of a second skill class with the property that the productivity distribution function of the high-skilled (h) first-order stochastically dominates the cdf of the low-skilled (l), i.e. $G_h(x) \leq G_l(x)\ \forall x$. Introducing economic turbulence is modeled as follows. High-skilled workers lose their skills conditional on job loss and during unemployment with probability π^l, which means that they can only draw from $G_l(\cdot)$ when they are matched again. Low-skilled workers, on the other hand, receive a skill upgrade during employment, reflecting 'learning-on-the-job', with probability π^h, which allows them to draw a new productivity from $G_h(\cdot)$ instead of $G_l(\cdot)$. Hence, the skill composition is endogenous. For simplicity we assume that the skill of a specific worker can be observed by firms and the government at any time. Hence, a firm can direct search towards the skill class which is more profitable for the firm[22]. An individual can be in 4 different states, employed with high or low skills and unemployed with high or low skills, where we assume that total labor

[19]For the 'European calibration' they find that a 20% wage subsidy decreases low-skilled unemployment from 16.2% to 7.6% while the unemployment rate of high-skilled workers rises from 4.5% to only 4.9%.

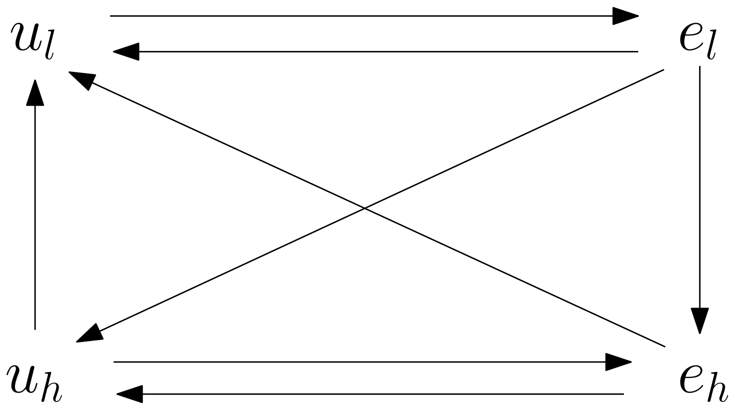
[20]Empirical evidence for skill loss upon separation or during unemployment, which is often approximated by the difference between the old wage and the re-employment wage, is widely documented. See for example Fallick (1996) for the U.S. and Burda and Mertens (2001) for Germany.

[21]This strand of the literature was initially concerned with persistence in unemployment, see e.g. Pissarides (1992), Ljungqvist and Sargent (1998, 2004), and Den Haan et al. (2005).

[22]We abstract from undirected search, as presented e.g. in Albrecht and Vroman (2002), as it would add an additional externality to our framework. Firms would not internalize the positive effect of employment on the average quality of the pool of workers from which they source, which would lead to inefficiently low job creation.

force is normalized to 1, hence: $e_l + u_l + e_h + u_h = L_l + L_h = 1$. Transitions between these states are illustrated by figure 3.1 and are formally reported in appendix section A.1. Note that we now additionally allow for exogenous, productivity unrelated, separation at a rate π^x, which does not provide additional analytic insight, but is important to quantitatively match the model to the data. Besides the productivity distributions we allow high- and

Figure 3.1: The transition flows



low-skilled workers to differ in other dimensions, like the matching technologies, as well. Differences are indicated by the subscript $j \in \{h, l\}$. All the assumptions of the intragroup model still apply unless stated otherwise. Hence, the models are nested, i.e. the intragroup model is a special case of the intergroup model with $\pi^h = \pi^l = 0$ and dropped skill indices. The asset value of unemployment for the low-skilled workers is the same as before, while high-skilled workers lose U_h in case they are not matched with probability π^l and only get U_l instead[23].

$$rU_j = z_j + q_j^w \left(W_l^e - U_l\right) + \mathbb{I}_h(j)(1 - q_h^w)\pi^l \left(U_l - U_h\right) \tag{3.1}$$

The value of working[24] differs for both skill classes as follows. While the outside option of a low-skilled worker is only U_l, the possibility of a skill loss has to be incorporated in the outside option of a high-skilled worker, hence: $\bar{U} \equiv \pi^l U_l + (1 - \pi^l)U_h$. On the other hand only a low-skilled worker can receive a skill upgrade during work.

$$\begin{aligned} rW_j(x) =& (1 - t_j)w_j(x) + \pi^x \left[\mathbb{I}_l(j)U_l + \mathbb{I}_h(j)\bar{U} - W_j(x)\right] \\ &+ \pi^n \left[(1 - G_j(\hat{\underline{x}}_j))\hat{W}_j^{\hat{e}} + G_j(\hat{\underline{x}}_j)\left(\mathbb{I}_l(j)U_l + \mathbb{I}_h(j)\bar{U}\right) - W_j(x)\right] \\ &+ \mathbb{I}_l(j)\pi^h \left[(1 - G_h(\hat{\underline{x}}_h))\hat{W}_h^{\hat{e}} + G_h(\hat{\underline{x}}_h)\bar{U} - W_l(x)\right] \end{aligned} \tag{3.2}$$

We turn to the firms' side. As the skill of the workers can be perfectly observed, firms are

[23] $\mathbb{I}$ denotes the indicator function of form $\mathbb{I}_i(j) = \begin{cases} 1 & \text{if } j = i \\ 0 & \text{if } j \neq i \end{cases}$.

[24] Note that the 'inside' asset values ($\hat{W}_h(x)$ and $\hat{W}_l(x)$) are set up analogously and are not reported in the text but only in the appendix section B for the sake of completeness.

able to discriminate and specifically post a vacancy for high- or low-skilled workers. A firm will enter the labor market that generates higher returns. We further assume that it can reassess this decision every period. Let us therefore define $V^m \equiv \max\{V_h, V_l\}$. The values of posting vacancies in the high- and the low-skill market, respectively, are given as:

$$rV_j = -c_j + q_j^f \left(J_j^e + H_j - V_j\right) + (1 - q_j^f)\left(V^m - V_j\right), \text{ with } c_j \equiv C_j - R_j \tag{3.3}$$

Employing a high-skilled worker yields a per-period return of rJ_h similar to before, while rJ_l again accounts for the possibility of a skill upgrade.

$$\begin{aligned} rJ_j(x) =&(1-\tau_j)x - w_j(x) + D_j + \pi^x\left[(V^m - F_j) - J_j(x)\right] \\ &+\pi^n\left[(1 - G_j(\underline{\hat{x}}_j))\hat{J}_j^{\hat{e}} + G_j(\underline{\hat{x}}_j)(V^m - F_j) - J_j(x)\right] \\ &+\mathbb{I}_l(j)\pi^h\left[(1 - G_h(\underline{\hat{x}}_h))\hat{J}_h^{\hat{e}} + G_h(\underline{\hat{x}}_h)(V^m - F_h) - J_l(x)\right] \end{aligned} \tag{3.4}$$

Wages are again determined by Nash bargaining and are related in the following way: $w_j(x) = \hat{w}_j(x) - r_j\omega(F_j - H_j)$, where $r_h = r + \pi^x + \pi^n$ and $r_l = r_h + \pi^h$. Wages now do not only depend on their 'own' endogenous variables and parameters but also on those of the other skill group[25]. Importantly, this dependence is asymmetric. While wages of both skill classes increase in the own outside options, i.e. $\frac{\partial w_h}{\partial U_h} > 0$ and $\frac{\partial w_l}{\partial U_l} > 0$, as before, this is not true for the 'cross terms', i.e. $\frac{\partial w_h}{\partial U_l} > 0$ and $\frac{\partial w_l}{\partial U_h} < 0$. These derivatives capture policy spill-over of a targeting scheme. Subsidizing low-skilled workers will increase U_l and lead high-skilled workers to bargain a higher wage as their fall back option, which includes that they eventually become low-skilled, increases[26]. By contrast, if high-skilled workers are subsidized, low-skilled workers will bargain a lower wage because working with low skills includes the increased option value of becoming high-skilled. We will now characterize the equilibrium.

3.1 Equilibrium

The equilibrium vector $\langle u_h, u_l, e_h, e_l, \theta_h, \theta_l, \underline{x}_h, \underline{x}_l, \underline{\hat{x}}_h, \underline{\hat{x}}_l\rangle$ is pinned down by equations (3.5)-(3.7) and the steady state flow equations (A.1). In comparison to the intragroup model, the job creation conditions hardly change

$$JC_j \quad : \quad (1-\omega)\left(\frac{(x_j^e - \underline{\hat{x}}_j)(1-\tau_j)}{r_j} - F_j + H_j\right) = \frac{c_j}{q_j^f} \tag{3.5}$$

[25] See appendix section C for an explicit derivation of all four wage schedules.

[26] Note that is effect should be even stronger if workers are risk-averse.

The job destruction conditions are now more involved. After defining $\Gamma_j \equiv \frac{1}{r_j}\int_{\underline{\hat{x}}_j}^{\infty}(\tilde{x} - \underline{\hat{x}}_j)\, dG_j(\tilde{x})$, they read

$$\begin{aligned} JD_j \quad : \quad & (1-\tau_j)\underline{\hat{x}}_j - \hat{w}_j(\underline{\hat{x}}_j) + D_j + rF_j - \mathbb{I}_l(j)\pi^h(F_h - F_l) \\ & +(1-\omega)\pi^n(1-\tau_j)\Gamma_j + \mathbb{I}_l(j)(1-\omega)\pi^h(1-\tau_h)\Gamma_h = 0 \end{aligned} \tag{3.6}$$

The relationship of the cut-off productivities, representing the job acceptance decision, is given by

$$JA_j \quad : \quad \underline{x}_j = \underline{\hat{x}}_j + \frac{r_j}{1-\tau_j}(F_j - H_j) \tag{3.7}$$

Analogously to before equilibrium is partly recursive. After inserting (3.7) in (3.5) in order to compute x_j^e, one can solve the remaining JD_j-JC_j system of 4 equations for θ_h, θ_l, $\underline{\hat{x}}_h$, and $\underline{\hat{x}}_l$. Knowing θ_j, $\underline{\hat{x}}_j$, and $\underline{x}_j$ enables us to solve for e_j and u_j using (A.1).

3.2 Efficiency and the optimal policy mix

As before we start out by computing the solution to the social planner's problem, which is documented in appendix section E. Again, efficiency in a policy-free world is guaranteed if and only if $\omega = \eta$. Hence, the Hosios (1990)-condition generalizes to the complex intergroup model. We use the same Ramsey approach as before, i.e. b_h and b_l are exogenously given and have to be financed with the least possible distortion. As the implementation of the optimum should be feasible we are bound to the following budget constraint that allows for intergroup redistribution

$$0 = GB_h + GB_l \tag{3.8}$$

$$\begin{aligned} GB_j =& e_j\left[\bar{w}_j t_j + \bar{x}_j\tau_j - D_j\right] - u_j b_j - \theta_j u_j R_j - q_j^w u_j H_j \\ & + e_j\pi^n G_j(\underline{\hat{x}}_j)F_j + e_j\pi^x F_j + \mathbb{I}_h(j)e_l\pi^h G_h(\underline{\hat{x}}_h)F_h \end{aligned} \tag{3.9}$$

Many insights from the intragroup model generalize to the extended model. First, $F_j = H_j$ is again a necessary condition for efficiency. Second, if $\omega = \eta$ we do not require output taxation τ_j or a recruiting subsidy R_j. If $\omega \neq \eta$ we need at least one of those instruments. These are important guidelines for finding an implementation of the optimal allocation for the complex intergroup model which is a non-trivial task because of several complications. First, even $b_j > 0$ and $b_{i\neq j} = 0$ requires both payroll tax rates to be non-zero. Second, a $F_j = H_j$-scheme is only budget neutral if $F_l = H_h$ as there are no taxes paid or subsidies received if a worker 'upgrades'. Consequently, whenever $F_l > H_h$ the effect on the budget is negative, if $F_l < H_h$ it is positive. If we require $F_h = F_l = H_h = H_l$ then the implementation of the efficient allocation is given by the vector $\langle t_h, t_l, F_l\rangle$ that satisfies the governments budget constraint (3.8) and pushes the JD-curves to their optimum, i.e. (C.16) minus (E.17) = 0 and (C.17) minus (E.16) = 0. If we do not require $F_l = H_h$ then we have an additional degree of freedom and can have many optimal implementations. The central insight is that the idea of implementation 1 generalizes to the complex model. Hence, the $F = H$-scheme is robust to the presence of economic turbulence.

3.3 Simulation

Although the theoretical treatment gives a lot of insight we will perform some numerical simulations[27] for two reasons. First, we provide quantitative evidence for the possible welfare gain when the optimal policy is implemented. Second, we will show that the quantitative relevance of the additional spill-over effects of targeting schemes, which arise in presence of economic turbulence, is considerable. We will do so by revisiting the effects of a cross-financed wage subsidy scheme proposed by MP who find that such a policy can considerably increase employment and output in their 'European calibration' case. For the sake of comparability we will focus on Germany as a representative European economy. The first task is to find a reasonable calibration for the model to fit German labor market characteristics. We specify the functional forms of $q_j(\cdot)$ and $G_j(\cdot)$ following MP, Den Haan et al. (2005), or Ljungqvist and Sargent (1998, 2004)

$$q_j(\theta_j) = A_j \theta_j^{-\eta} \tag{3.10}$$

and a uniform distribution on the interval $[\underline{\kappa}, \bar{\kappa}]$

$$G_j(x) = \frac{x - \underline{\kappa}_j}{\bar{\kappa}_j - \underline{\kappa}_j} \tag{3.11}$$

A period is chosen to be a month. Targeting an interest rate of 5 % p.a. results in $r = 0.0041$ or $\beta = 0.9959$. Nash bargaining is chosen to be symmetric as done by many authors[28]. Estimates for the elasticity of the matching function vary between 0.45 (Fahr and Sunde, 2001) and 0.7 (Burda and Wyplosz, 1994). For simplicity, we abstract from inefficiencies generated by search externalities. Hence, we set $\eta = 0.5$ in order to fulfill the Hosios (1990)-condition. The expectations of the two productivity distributions were chosen to be $E(X_l) = 1$ and $E(X_h) = 1.35$. As data for per worker productivity broken down into skill classes is not available, gross wage was used as a proxy by assuming that productivities and wages are sufficiently proportional. During 2002 - 2006 a white collar worker earned approximately 1.35 as much as a blue collar worker (Statistisches Bundesamt, 2007). Variances were set such that the model's wage predictions result in a wage ratio of

[27] The simulations were performed using MATLAB. The code is available on my website `http://sites.google.com/site/schusterphilip/`.

[28] See Hall and Milgrom (2008) for an additional motivation of setting $\omega = 0.5$.

approximately $1:1.35$. This implies $Var(X_l) = \frac{1^2}{12}$ and $Var(X_h) = \frac{1.3^2}{12}$. A crucial choice is the value of non-work z_j which will be set as a compromise between two different calibration strategies. As mentioned earlier we impose linearity in the value of non-work, hence $z_j = h + b_j$, which implies that the effects of db_j and dh are equal. We further assume that there is no skill specific difference in the value of home production. In line with the results of the OECD tax-benefit calculator we target replacement ratios of $\frac{b_h}{w_h} = 0.6$ and $\frac{b_l}{w_l} = 0.65$. Exploiting cross-country variation, Costain and Reiter (2008) estimate the semi-elasticity of unemployment with respect to the replacement ratio $\frac{b}{w}$, i.e. $\frac{d \ln u}{d \frac{b}{w}}$ approximately in the range of $[2, 3]$. We set a common $h = 0.25$ low enough to hit the upper bound of the Costain and Reiter (2008)-target, namely $\frac{d \ln u}{d \frac{b}{w}} \approx 3.2$[29]. This implies total replacement ratios of $\frac{z_h}{w_h} = 0.77$ and $\frac{z_l}{w_l} = 0.87$ and, in terms of productivity, $\frac{z_h}{av.prod_h} = 0.717$ and $\frac{z_l}{av.prod_l} = 0.814$. Those figures are just high enough to be in line with corresponding value of 0.71 derived by Hall and Milgrom (2008) for the U.S. using a completely different calibration approach relying on estimates of the Frisch elasticity. As one would expect this number to be even higher in Germany, because of the more generous compensation system, 0.71 seems to be a reasonable choice as a lower bound. Hence, our calibration compromises between both targets. It addresses the argument of Hagedorn and Manovskii (2008) that the value of non-work is substantially high, but at the same time produces a realistic responsiveness of unemployment to changes in benefits. In order to finance the expenditure on b_h and b_l we set $t_h = 0.065$ and $t_l = 0.05$, which reflects progression in the existing tax system.

The transition probabilities π^l and π^h are chosen in order to replicate the empirical skill distribution. We use the following targets based on the publicly available statistics provided by the German federal employment agency (Bundesagentur für Arbeit, 2008). Among the unemployed the ratio of blue to white collar workers is approximately 60 %, hence $\frac{u_l}{u} \approx 0.6$. We further target $\frac{e_l}{e} \approx 0.2$, where the low-skilled are measured as workers with no professional education and apprentices. Given an unemployment rate of $u = 0.1$, this gives a skill composition of the labor force of $\frac{L_l}{L} \approx 0.25$. We set $\pi^h = 0.01$, which implies that it takes on average 8 years and 4 months to become high-skilled, conditional on no job loss. A skill loss occurs after 1 year and 10 months on average, i.e. $\pi^l = 0.05$. Those values are in line with the choices of Den Haan et al. (2005) and Ljungqvist and Sargent (2004). These papers and MP also inspire the choice for the rate at which new shocks arrive, i.e. $\pi^n = 0.02$. As we do not interprete the average duration of a vacancy or the number of vacancies but just target the duration of unemployment we are free to choose

[29]The model delivers $\frac{d \ln u}{dh} = 2.2$, $\frac{d \frac{b_h}{w_h}}{db_h} = 0.6$ and $\frac{d \frac{b_l}{w_l}}{db_l} = 0.73$. After averaging the effects by the skill share in the group of unemployed one can retrieve the sought-after semi-elasticity as $\frac{d \ln u}{d \frac{b}{w}} = \frac{d \ln u}{dh} / \frac{d \frac{b}{w}}{db}$.

$C_h = 1.509$ and $C_l = 0.274$ in order to normalize $\theta_h = \theta_l = 1$[30]. The probability of an exogenous split $\pi^x = 0.00668$ and the scaling factors[31] $A_h = 0.563$ and $A_l = 0.148$ are set to replicate an unemployment rate of $u = 0.1$ and average duration of unemployment of 9 months (long term averages for 1998-2007, Bundesagentur für Arbeit, 2008). Table 3.1 summarizes the calibration choices and results for the decentralized economy, which serves as our benchmark.

Table 3.1: Decentralized economy, benchmark

			Parameters				
β	0.996	b_h	0.950	H_h	0.000	$\underline{\kappa}_h$	0.700
r	0.004	b_l	0.750	H_l	0.000	$\bar{\kappa}_h$	2.000
ω	0.500	C_h	1.509	F_h	0.000	$\underline{\kappa}_l$	0.500
η	0.500	C_l	0.274	F_l	0.000	$\bar{\kappa}_l$	1.500
π^x	0.007	R_h	0.000	D_h	0.000	A_h	0.563
π^n	0.020	R_l	0.000	D_l	0.000	A_l	0.148
π^h	0.010	τ_h	0.000	t_h	0.065	h	0.250
π^l	0.050	τ_l	0.000	t_l	0.050		
			Results				
type	θ_j	$\underline{x}_j$	$\hat{\underline{x}}_j$	L_j	e_j	u_j	duration
h:	1.000	1.345	1.345	0.740	0.702	0.038	3.524
l:	1.000	0.950	0.950	0.260	0.198	0.062	12.296
total:	-	-	-	1.000	0.900	0.100	9.000
type	$G_j(\underline{x}_j)$	av. prod	av. w_j	av. $\hat{w}_j$	repl.	tot. repl.	welfare
h:	0.496	1.673	1.562	1.562	0.608	0.768	1.108
l:	0.450	1.225	1.151	1.151	0.651	0.869	0.210
total:	-	1.574	1.472	1.472	-	-	1.318

Note: 'repl.' gives the replacement ratio, i.e. $\frac{b_j}{\bar{w}_j}$. 'tot. repl.' is $\frac{z_j}{\bar{w}_j}$. 'av.' denotes average. 'welfare' is per-period in steady state. Other variables as in the paper.

In contrast, table 3.2 shows the results of the social optimum. Our welfare criterion increased by 5.6 %. Unemployment is at 4 % compared to 10 %, while average duration of unemployment should optimally be 3 months instead of 9. Comparing the endogenous decision variables we observe two things. First, reservation productivities for accepting and destructing jobs are inefficiently high, especially for the low-skilled who reject almost every second offer instead of one out of four which would be optimal. Second, job creation is inefficiently low. Again, this is more servere for low-skilled workers where market tightness is about one forth of what it should be.

Implementation of the optimal allocation

[30] This normalization is more thoroughly described in Shimer (2005).

[31] The chosen ratio of A_h to A_l is admittedly a little bit arbitrary but could be fixed if average duration of unemployment is known for each skill class seperately. For the time being average duration of unemployment was set to 3 and a half months for high-skilled and a little bit more than 1 year for low-skilled.

Table 3.2: Social planner's solution

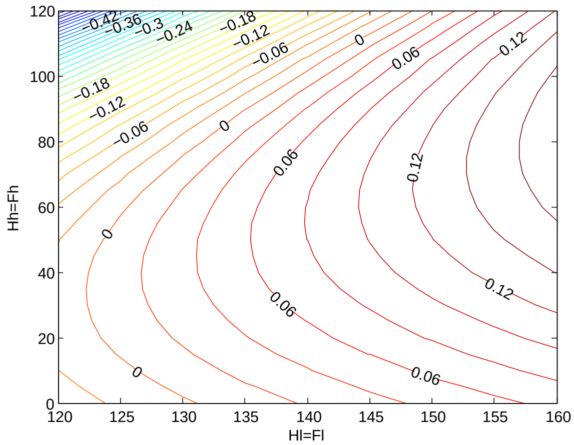
Parameters							
β	0.996	b_h	-	H_h	-	$\underline{\kappa}_h$	0.700
r	0.004	b_l	-	H_l	-	$\bar{\kappa}_h$	2.000
ω	-	C_h	1.509	F_h	-	$\underline{\kappa}_l$	0.500
η	0.500	C_l	0.274	F_l	-	$\bar{\kappa}_l$	1.500
π^x	0.007	R_h	-	D_h	-	A_h	0.563
π^n	0.020	R_l	-	D_l	-	A_l	0.148
π^h	0.010	τ_h	-	t_h	-	h	0.250
π^l	0.050	τ_l	-	t_l	-		
Results							
type	θ_j	$\underline{x}_j$	$\hat{\underline{x}}_j$	L_j	e_j	u_j	duration
h:	1.821	1.239	1.239	0.850	0.824	0.026	2.248
l:	3.677	0.738	0.738	0.150	0.137	0.014	4.630
total:	-	-	-	1.000	0.961	0.039	3.066
type	$G_j(\underline{x}_j)$	av. prod	av. w_j	av. $\hat{w}_j$	repl.	tot. repl.	welfare
h:	0.415	1.620	-	-	-	-	1.257
l:	0.238	1.119	-	-	-	-	0.136
total:	-	1.548	-	-	-	-	1.392

Note: see table 3.1.

Let us now address possible implementations of the social optimum. We have learned from the previous sections that in case of $\omega = \eta$ we do not require output taxation τ_j or a recruiting subsidy R_j. Further, given proposition 2.5 for the intragroup model and the high empirical replacement ratios an implementation relying on wage subsidies does not seem to be very promising. Hence, we try to implement the corresponding intergroup variant of the policy scheme suggested in proposition 2.3. We proceed as follows. First we set $F_j = H_j$. As $\omega = \eta$ the job creation conditions coincide with their social optimal counterparts. Given $F_j = H_j$ and b_j we can now compute the tax rates t_j that satisfy the two optimal job destruction conditions simultaneously. All the possible pairs of $F_h = H_h$ and $F_l = H_l$ that satisfy the budget constraint, i.e. set the budget surplus to 0, represent an implementation of the optimal allocation. Figure 3.2 illustrates these social optimal combinations. Moving along the optimal isoline does not only change the combination of $F_h = H_h$ and $F_l = H_l$ but also the corresponding optimal tax rates as shown by table 3.2. The higher $F_h = H_h$ the higher t_h has to be compared to t_l. The striking result is that such schemes involve tremendously high firing taxes and hiring subsidies. To get a feeling for magnitude: the lowest possible value for $F_l = H_l$ is still more than 100 times larger than the monthly wage of a low-skilled in our benchmark case. Hence, it is of interest how close we can get to the optimum if we are limited in the extend to which firing taxes and hiring subsidies can be introduced as the effects presumably work in a concave way. Consider the uniform policy case $F_j = H_j = 90$ representing a 'half-way' policy mix. Payroll taxes are adjusted to satisfy the budget constraint and keep the ratio

Figure 3.2: First best implementations in the intergroup model

(a) Budget surplus for efficient tax rates and all combinations of $H_j = F_j$



(b) Possible implementations of the optimal allocation

$H_h = F_h$	$H_l = F_l$	t_h	t_l
0	131.2	-0.062	0.738
20	123.4	-0.046	0.668
40	122.4	-0.033	0.594
60	125.6	-0.022	0.519
80	131.5	-0.014	0.443
100	139.2	-0.007	0.365
120	148.3	-0.001	0.287
140	158.3	0.004	0.208
160	169.0	0.008	0.129
180	180.3	0.012	0.050
200	192.0	0.016	-0.029

$\frac{t_h}{t_l}$ constant. Such a policy gives a welfare gain of 4.4 % instead of 5.6 %. Unemployment is reduced to 5.7 % and unemployment duration to 4.5 months. If our scheme is set to a value of $F_j = H_j = 10$, which amounts to approximately 7 months of average wage, unemployment is still reduced by 1 %-point, unemployment duration by one month, while welfare increases by 1 %. Another finding is that our instruments allow us to trade off welfare and employment. Consider - for example - again $F_h = F_l = 10$ this time as only source of tax income and instead of using hiring subsidies, tax revenue is spent on recuitment subsidies $R_h = R_l = 0.24$. Such a scheme reduces overall unemployment to 6.3 % and average duration of unemployment to 5.7 months, while it distorts job acceptance and creation and leaves welfare practically unchanged compared to the benchmark.

Cross-financed wage subsidy schemes

We argued in the theoretical part of this section how the presence of economic turbulence can create additional spill-over effects from targeted to untargeted workers. In this section we try to quantify this for a particular targeting scheme, namely a wage subsidy for low-skilled workers financed by high-skilled workers as studied by MP. Although a policy like that does not fulfill the criteria of being optimal, MP propose it as a 'better than nothing' scheme especially useful to reduce unemployment. We show that this conclusion is overthrown when economic turbulence is taken into account. To have a reference point we first replicate the MP result in our model when turbulence is switched off, i.e. $\pi^h = \pi^l$. Hence, the skill composition of the labor force is not endogenous anymore but exogenously fixed, i.e. $L_h = 0.7398$. To replicate our targets for unemployment, its duration and composition we have to recalibrate some of the remaining transition probabilities[32]. We

[32] In detail, $A_h = 2.23$, $A_l = 0.28$, $\pi^n = 0.01$, $\pi_h^x = 0.0065$, and $\pi_l^x = 0.0185$. In addition, as wages

then rerun the MP experiment by increasing D_l stepwise from 0 to 0.5. This is done in an uncompensated way and also if financed by the high-skilled workers. Table 3.3 summarizes the results.

Table 3.3: Effect of a low wage subsidy on unemployment rates

	D_l change uncompensated						D_l change compensated by t_h					
	no turbulence			turbulence			no turbulence			turbulence		
D_l	u_h	u_l	u	u_h	u_l	u	u_h	u_l	u	u_h	u_l	u
0.0	5.08	23.98	10.00	5.08	23.98	10.00	5.08	23.98	10.00	5.08	23.98	10.00
0.1	5.08	19.29	8.78	5.80	22.09	10.39	5.15	19.29	8.83	6.63	23.69	11.98
0.2	5.08	16.18	7.97	6.74	20.40	10.95	5.26	16.18	8.10	-	-	-
0.3	5.08	13.95	7.39	7.99	18.90	11.71	5.39	13.95	7.62	-	-	-
0.4	5.08	12.30	6.96	9.68	17.54	12.68	5.56	12.30	7.31	-	-	-
0.5	5.08	10.99	6.62	12.10	16.30	13.91	5.76	10.99	7.12	-	-	-

Note: Unemployment rates are computed in percent relative to L_j. '-' denotes break down of equilibrium.

As in MP a low-wage subsidy scheme seems to be very effective in reducing overall unemployment, which can be brought down to 7.02 % for $D_l = 0.66$ in the tax-compensated scenario. However, when we take economic turbulence into account, the results reverse. It is striking that even in the uncompensated case, i.e. the subsidy is given away for free, total unemployment will increase with D_l. Two effects, one boosting u_h and the other dampening the reduction in u_l come into play. In a first direct effect a rise in D_l increases the value of working as low-skilled (W_l) and consequently the value of being unemployed (U_l) with low skills. That is where the mechanism stops in the non-turbulence framework. In our case additional indirect effects start to work. As U_l increases so does the fall back option of the high-skilled workers ($\bar{U}$), which will raise the reservation productivities, inflate wages and therefore reduce vacancy creation for high-skilled workers. Consequently, u_h has to rise. In a third round, as the value of being high-skilled drops in relative terms this feeds back in a negative way to the low-skilled workers as the motive of accepting a low-wage job in order to eventually become high-skilled diminishes. A direct consequence is that the skill composition in the labor force in shifted towards low-skilled workers. This is the reason why such a scheme can lead to a break down of the equilbrium even for small values of D_l if the subsidy is financed through t_h as the high-skill tax base can collapse. In conclusion, a low-wage subsidy might be useful to increase low-skill employment, but is less effective in reducing low-skill unemployment, let alone total unemployment.

slightly differ we have to set the tax rates (keeping the relative ratio constant) to $t_h = 0.054$ and $t_l = 0.046$. Again, we choose $C_h = 0.571$ and $C_l = 0.178$ in order to normalize $\theta_h = \theta_l = 1$.

4 Conclusion

A dynamic model of equilibrium unemployment and bilateral wage bargaining is used to characterize optimal labor market policy in a possibly turbulent environment. The pre-policy equilibrium is distorted by a firing externality, created by an existing partial unemployment insurance system, along three decision margins: job creation, job acceptance, and job destruction. We apply a Ramsey approach and try to find a solution to the problem of financing exogenously fixed unemployment benefits with the least possible distortions using a rich set of policy instruments: payroll, output, and firing taxes as well as wage, hiring, and recruitment subsidies. It is shown that the optimal policy mix consists of a payroll tax to finance unemployment compensation and a firing tax that is offset by a hiring subsidy. The latter part can be interpreted as redistribution from firing to hiring firms and helps to undo the distortions created by the payroll tax system. The reason is that a 'firing tax equal hiring subsidy'-scheme, while not distorting job acceptance and job creation, leads to less job destruction as such a policy represents an interest free loan to the firm. The derived optimal policy mix deviates from the static framework results of Blanchard and Tirole (2008) who argue that benefits should be completely financed through firing taxes. This idea does not completely transfer to our dynamic set-up. In any case a firing tax has to be compensated one-for-one by a hiring subsidy to prevent distortions along the job acceptance margin. Hence, in the case of 'unbalanced' search externalities that distort job creation, the failure of the Hosios condition cannot be corrected by a spread between the firing tax and the hiring subsidy. Instead either an output or recruitment tax/subsidy have to be used in addition.

The important feature of the derived policy mix is that it is robust to the introduction of economic turbulence in the interpretation of Ljungqvist and Sargent (1998), i.e. skill loss during unemployment. This is crucial as a lot of existing policy advice is rendered considerably less effective in that case. We demonstrate this by reassessing a cross-financed wage subsidy scheme for low-skilled workers as - for example - suggested by Mortensen and Pissarides (2003). While they assume skill classes to operate in complete juxtaposition, except for the connection via the government's budget constraint, possible skill loss during unemployment implies that high-skilled workers become more picky concerning their acceptance and continuation decision as their fall back option, including subsidized low-skill employment, increases. The skill composition deteriorates as a result of such a targeting scheme and the finding of Mortensen and Pissarides (2003) that unemployment can be considerably reduced is overthrown. In conclusion, the paper argues that instead of redistribution from firing firms to unemployed workers (Blanchard and Tirole, 2008) or from high- to low-skilled workers (Mortensen and Pissarides, 2003), a scheme involving redistribution from firing to hiring firms should be preferred.

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Appendix

A Laws of motion

$$
\begin{aligned}
\dot{u}_h =& e_h \left[\pi^x + \pi^n G_h(\underline{\hat{x}}_h)\right](1-\pi^l) + e_l \pi^h G_h(\underline{\hat{x}}_h)(1-\pi^l) \\
& - u_h \left[(1-q_h^w)\pi^l + q_h^w\right] \\
\dot{e}_l =& (1-e_h-e_l-u_h)q_l^w - e_l\left[\pi^x + \pi^n G_l(\underline{\hat{x}}_l) + \pi^h\right] \\
\dot{e}_h =& e_l \pi^h (1-G_h(\underline{\hat{x}}_h)) + u_h q_h^w - e_h\left[\pi^x + \pi^n G_h(\underline{\hat{x}}_h)\right] \\
\dot{y}_h =& - y_h(\pi^x+\pi^n) + e_h\pi^n \widetilde{G}_h(\underline{\hat{x}}_h) + e_l \pi^h \widetilde{G}_h(\underline{\hat{x}}_h) \\
& + u_h \theta_h q_h(\theta_h) \widetilde{G}_h(\underline{x}_h) \\
\dot{y}_l =& - y_l(\pi^x+\pi^n+\pi^h) + e_l \pi^n \widetilde{G}_l(\underline{\hat{x}}_l) \\
& + (1-u_h-e_l-e_h)\theta_l q_l(\theta_l)\widetilde{G}_l(\underline{x}_l)
\end{aligned} \tag{A.1}
$$

Equilibrium states are derived by setting the left hand sides to zero.

B Unreported value functions and Nash bargaining

Unreported value functions:

$$
\begin{aligned}
r\hat{W}_j(x) =& (1-t_j)\hat{w}_j(x) + \pi^x \left[\mathbb{I}_l(j)U_l + \mathbb{I}_h(j)\bar{U} - \hat{W}_j(x)\right] \\
& + \pi^n \left[(1-G_j(\underline{\hat{x}}_j))\hat{W}_j^{\hat{e}} + G_j(\underline{\hat{x}}_j)\left(\mathbb{I}_l(j)U_l + \mathbb{I}_h(j)\bar{U}\right) - \hat{W}_j(x)\right] \\
& + \mathbb{I}_l(j)\pi^h\left[(1-G_h(\underline{\hat{x}}_h))\hat{W}_h^{\hat{e}} + G_h(\underline{\hat{x}}_h)\bar{U} - \hat{W}_l(x)\right]
\end{aligned} \tag{B.1}
$$

$$
\begin{aligned}
r\hat{J}_j(x) =& (1-\tau_j)x - \hat{w}_j(x) + D_j + \pi^x\left[(V^m - F_j) - \hat{J}_j(x)\right] \\
& + \pi^n\left[(1-G_j(\underline{\hat{x}}_j))\hat{J}_j^{\hat{e}} + G_j(\underline{\hat{x}}_j)(V^m - F_j) - \hat{J}_j(x)\right] \\
& + \mathbb{I}_l(j)\pi^h\left[(1-G_h(\underline{\hat{x}}_h))\hat{J}_h^{\hat{e}} + G_h(\underline{\hat{x}}_h)(V^m - F_h) - \hat{J}_l(x)\right]
\end{aligned} \tag{B.2}
$$

Nash bargaining implies:

$$
W_h - \bar{U} = \frac{\omega(1-t_h)}{1-\omega}(J_h + H_h) \text{ and } \hat{W}_h - \bar{U} = \frac{\omega(1-t_h)}{1-\omega}(\hat{J}_h + F_h) \tag{B.3}
$$

$$
W_l - U_l = \frac{\omega(1-t_l)}{1-\omega}(J_l + H_l) \text{ and } \hat{W}_l - U_l = \frac{\omega(1-t_l)}{1-\omega}(\hat{J}_l + F_l) \tag{B.4}
$$

or

$$
W_h - \bar{U} = \widetilde{\omega}_h s_h \text{ and } \hat{W}_h - \bar{U} = \tilde{\omega}_h \hat{s}_h \tag{B.5}
$$

$$
W_l - U_l = \widetilde{\omega}_l s_l \text{ and } \hat{W}_l - U_l = \tilde{\omega}_l \hat{s}_l \tag{B.6}
$$

$$J_j + H_j = \widetilde{\widetilde{\omega}}_j s_j \text{ and } \hat{J}_j + F_j = \widetilde{\widetilde{\omega}}_j \hat{s}_j \tag{B.7}$$

where

$$\widetilde{\omega}_j \equiv \frac{\omega(1-t_j)}{1-\omega t_j} \text{ and } \widetilde{\widetilde{\omega}}_j \equiv \frac{1-\omega}{1-\omega t_j} \text{ and } (1-\omega)\widetilde{\omega}_j = \omega(1-t_j)\widetilde{\widetilde{\omega}}_j$$

C Derivation of the equilibrium conditions

This section formally derives the equilibrium conditions for the intergroup model. As the model is nested, the conditions for the simple intragroup model can be found by dropping the skill index and setting $\pi^h = \pi^l = 0$. Let us first define $\tilde{r} = r + \pi^l$, $r_h = r + \pi^x + \pi^n$, and $r_l = r_h + \pi^h$. Equilibrium is determined by the free entry conditions (C.1) and the cut-off conditions (C.2).

$$V_l = V_h = 0 \quad \Rightarrow \quad V^m = 0 \quad \Rightarrow \quad J_h^e = \frac{c_h}{q_h^f} - H_h \text{ and } J_l^e = \frac{c_l}{q_l^f} - H_l \tag{C.1}$$

$$\hat{J}_j(\underline{\hat{x}}_j) + F_j = 0 \Rightarrow \underline{\hat{x}}_j \text{ and } J_j(\underline{x}_j) + H_j = 0 \Rightarrow \underline{x}_j \tag{C.2}$$

Take conditional expectation of (B.4), insert in (3.1) and eliminate J_l^e using the free entry condition (C.1) to get

$$rU_l = z_l + \frac{\omega(1-t_l)}{1-\omega} c_l \theta_l \tag{C.3}$$

Proceeding analogously for U_h results in

$$rU_h = z_h + \frac{\omega(1-t_h)}{1-\omega} c_h \theta_h - \pi^l (U_h - U_l) \tag{C.4}$$

We use C.4 and C.3 to solve for the difference in the values of unemployment

$$U_h - U_l = \frac{z_h - z_l}{\tilde{r}} + \frac{\omega}{1-\omega} \frac{(1-t_h)c_h\theta_h - (1-t_l)c_l\theta_l}{\tilde{r}} \tag{C.5}$$

Wages:

To get the wage equations proceed as follows. Multiplying (B.3) by r and rearranging gives $\omega(1-t_h)r\hat{J}_h(x) - (1-\omega)r\hat{W}_h(x) = -(1-\omega)r\bar{U} - \omega(1-t_h)rF_h$. Replace $r\hat{W}_h(x)$ and $r\hat{J}_h(x)$ by (B.1) and (B.2). Most of the remaining values cancel out after eliminating them using the FOCs from the Nash bargaining (B.5)-(B.7), and their conditional expectations. Solving for $\hat{w}_h(x)$ gives

$$\hat{w}_h(x) = \omega \left[(1-\tau_h)x + D_h + rF_h \right] + \frac{1-\omega}{1-t_h} r\bar{U} \tag{C.6}$$

Eliminating the remaining values of being unemployed, realizing that $r\bar{U} = r(1 -$

$\pi^l)\left[U_h - U_l\right] + rU_l$, results in

$$\begin{aligned}\hat{w}_h(x) &= \frac{1-\omega}{1-t_h}\left(\frac{(1-\pi^l)r}{\tilde{r}}z_h + \frac{\pi^l(1+r)}{\tilde{r}}z_l\right) + \omega\left[(1-\tau_h)x + D_h + rF_h\right] \\ &+ \omega\left[\frac{(1-\pi^l)r}{\tilde{r}}c_h\theta_h + \frac{1-t_l}{1-t_h}\frac{\pi^l(1+r)}{\tilde{r}}c_l\theta_l\right] \qquad \text{(C.7)}\end{aligned}$$

The derivation of the 'outside' wage works analogously and results in $w_h(x) = \hat{w}_h(x) - r_h\omega(F_h - H_h)$ given

$$w_h(x) = \omega\left[(1-\tau_h)x + D_h - (\pi^x + \pi^n)F_h + r_hH_h\right] + \frac{1-\omega}{1-t_h}r\bar{U} \qquad \text{(C.8)}$$

We proceed the same way to get $\hat{w}_l(x)$ and $w_l(x)$.

$$\begin{aligned}\hat{w}_l(x) =& \omega\left[(1-\tau_l)x + D_l - \pi^h(F_h - F_l) + rF_l\right] \\ &+ \frac{1-\omega}{1-t_l}\left[rU_l - \pi^h(1-\pi^l)(U_h - U_l)\right] \\ &+ \omega\pi^h(1 - G_h(\underline{\hat{x}}_h))\frac{t_h - t_l}{1-t_l}\widetilde{\widetilde{\omega}}_h\hat{s}_h^{\hat{e}}\end{aligned} \qquad \text{(C.9)}$$

Note that in case of $t_l \neq t_h$, $\widetilde{\widetilde{\omega}}_h\hat{s}_h^{\hat{e}}$ does not drop out and is replaced by $\frac{c_h}{q_h^f} + \widetilde{\widetilde{\omega}}_h\Sigma$. See below for the derivation. Eliminating the values of being unemployed gives

$$\begin{aligned}\hat{w}_l(x) &= -\frac{1-\omega}{1-t_l}\pi^h(1-\pi^l)\left[\frac{z_h - z_l}{\tilde{r}} + \frac{\omega}{1-\omega}\frac{(1-t_h)c_h\theta_h - (1-t_l)c_l\theta_l}{\tilde{r}}\right] \\ &+ \omega\left[(1-\tau_l)x + D_l - \pi^h(F_h - F_l) + rF_l + c_l\theta_l\right] + \frac{1-\omega}{1-t_l}z_l \\ &+ \omega\pi^h(1 - G_h(\underline{\hat{x}}_h))\frac{t_h - t_l}{1-t_l}\left[\frac{c_h}{q_h^f} + \widetilde{\widetilde{\omega}}_h\Sigma\right] \qquad \text{(C.10)}\end{aligned}$$

Similar to before the 'outside' wage is given by $w_l(x) = \hat{w}_l(x) - r_l\omega(F_l - H_l)$ knowing that

$$\begin{aligned}w_l(x) =& \omega\left[(1-\tau_l)x + D_l - \pi^hF_h - (\pi^x + \pi^n)F_l + r_lH_l\right] \\ &+ \frac{1-\omega}{1-t_l}\left[rU_l - \pi^h(1-\pi^l)(U_h - U_l)\right] \\ &+ \omega\pi^h(1 - G_h(\underline{\hat{x}}_h))\frac{t_h - t_l}{1-t_l}\widetilde{\widetilde{\omega}}\hat{s}_h^{\hat{e}}\end{aligned} \qquad \text{(C.11)}$$

For the derivation of Σ we start out by noting that the surplus functions are linear of form $s_h(x) = s_h^0 + s_h^1 x$, and further

$$\hat{s}_h(x) = s_h(x) + (1-\omega t_h)(F_h - H_h) = s_h^0 + s_h^1 x + (1-\omega t_h)(F_h - H_h) \qquad \text{(C.12)}$$

as will be established below. Taking conditional expectation gives

$$\begin{aligned}\hat{s}_h^{\hat{e}} &= \int_{\underline{\hat{x}}_h}^{\infty} \left[\frac{s_h^0 + s_h^1 \tilde{x} + (1-\omega t_h)(F_h - H_h)}{1 - G_h(\underline{\hat{x}}_h)} \right] dG_h(\tilde{x}) \\ &= s_h^0 + (1-\omega t_h)(F_h - H_h) + s^1 \frac{\widetilde{G}(\underline{\hat{x}}_h)}{1 - G_h(\underline{\hat{x}}_h)}\end{aligned} \tag{C.13}$$

Where the partial expectation is defined as $\widetilde{G}_j(x) = \int_x^\infty \tilde{x} dG_j(\tilde{x})$. Taking conditional expectation of $s_h(x) = s_h^0 + s_h^1 x$, eliminating s_h^0 by using (C.13) and inserting for s_h^1 establishes $\hat{s}_h^{\hat{e}} = s_h^e + \Sigma$. Combine (C.1) and (B.7) to get $s_h^e = \frac{c_h}{q_h^f} \frac{1}{\widetilde{\widetilde{\omega}}_h}$ which gives $\widetilde{\widetilde{\omega}}_h \hat{s}_h^{\hat{e}} = \frac{c_h}{q_h^f} + \widetilde{\widetilde{\omega}}_h \Sigma$, with

$$\Sigma = \underbrace{\frac{(1-\tau_h)(1-\omega t_h)}{r_h}}_{s_h^1} \left[\frac{\widetilde{G}_h(\underline{\hat{x}}_h)}{1 - G_h(\underline{\hat{x}}_h)} - \frac{\widetilde{G}_h(\underline{x}_h)}{1 - G_h(\underline{x}_h)} \right] \underbrace{+(1-\omega t_h)(F_h - H_h)}_{\hat{s}_h(x) - s_h(x)}$$

Note that that $\Sigma = 0$ if $F_h = H_h$ because it also implies $\underline{x}_h = \underline{\hat{x}}_h$ as we will prove below.

Job creation conditions:

The job creation conditions are derived as follows. Subtract (B.2) evaluated at $\underline{\hat{x}}_j$ from (3.4) and replace $\hat{J}_j(\underline{\hat{x}})$ by $-F_j$ using (C.2). Taking conditional expectation w.r.t. $\underline{x}_j$ and replacing J_j^e using (C.1) gives the **job creation curves**:

$$JC_j \quad : \quad (1-\omega) \left(\frac{(x_j^e - \underline{\hat{x}}_j)(1-\tau_j)}{r_j} - F_j + H_j \right) = \frac{c_j}{q_j^f} \tag{C.14}$$

Job destruction conditions:

First define: $\Gamma_j \equiv \frac{1}{r_j} \int_{\underline{\hat{x}}_j}^{\infty} (\tilde{x} - \underline{\hat{x}}_j)\, dG_j(\tilde{x})$. Subtract (B.2) evaluated at $\underline{\hat{x}}_j$ from themselves and eliminate $\hat{J}_j(\underline{\hat{x}})$ by $-F_j$ again using (C.2). Use the conditional expectation w.r.t. $\underline{\hat{x}}_j$ of the resulting expressions $\hat{J}_h(x)$ and $\hat{J}_l(x)$ to eliminate $\hat{J}_j^{\hat{e}}$ in (B.2). Evaluate again at $\underline{\hat{x}}_j$ and make use of (C.2) to arrive at

$$\begin{aligned}JD_j \quad : \quad & (1-\tau_j)\underline{\hat{x}}_j - \hat{w}_j(\underline{\hat{x}}_j) + D_j + rF_j - \mathbb{I}_l(j)\pi^h(F_h - F_l) \\ & +(1-\omega)\pi^n(1-\tau_j)\Gamma_j + \mathbb{I}_l(j)(1-\omega)\pi^h(1-\tau_h)\Gamma_h = 0\end{aligned} \tag{C.15}$$

Eliminating the wages and diving by $(1-\omega)$ then gives the final **job destruction curves**:

$$\begin{aligned}&(1-\tau_h)\underline{\hat{x}}_h + D_h + rF_h - \frac{1}{1-t_h}\left[\frac{(1-\pi^l)r}{\tilde{r}}z_h + \frac{\pi^l(1+r)}{\tilde{r}}z_l\right]\\ &-\frac{\omega}{1-\omega}\left[\frac{(1-\pi^l)r}{\tilde{r}}c_h\theta_h + \frac{1-t_l}{1-t_h}\frac{\pi^l(1+r)}{\tilde{r}}c_l\theta_l\right] + \pi^n(1-\tau_h)\Gamma_h = 0\end{aligned} \tag{C.16}$$

$$\begin{aligned}&(1-\tau_l)\underline{\hat{x}}_l + D_l + rF_l - \pi^h(F_h - F_l) - \frac{z_l}{1-t_l} - \frac{\omega}{1-\omega}c_l\theta_l\\ &+\frac{\pi^h(1-\pi^l)}{1-t_l}\frac{z_h - z_l}{\tilde{r}} + \pi^h(1-\pi^l)\frac{\omega}{1-\omega}\left[\frac{1-t_h}{1-t_l}\frac{c_h\theta_h}{\tilde{r}} - \frac{c_l\theta_l}{\tilde{r}}\right]\\ &-\pi^h(1-G_h(\underline{\hat{x}}_h))\frac{\omega}{1-\omega}\frac{t_h - t_l}{1-t_l}\left[\frac{c_h}{q_h^f} + \widetilde{\widetilde{\omega}}_h\Sigma\right]\\ &+\pi^n(1-\tau_l)\Gamma_l + \pi^h(1-\tau_h)\Gamma_h = 0\end{aligned} \tag{C.17}$$

Cut-off relationships:
The relation between the reservation productivities $\underline{x}_j$ and $\underline{\hat{x}}_j$ stems from a simple observation. The cut-off conditions in (C.2) in combination with (B.3) and (B.4) imply that firms and workers will always mutually agree on creating and destroying jobs. Hence, $\underline{x}$ and $\underline{\hat{x}}$ set the joint surpluses to 0. The surpluses in equilibrium are given by

$$s_h(x) = W_h(x) + J_h(x) - \bar{U} + H_h \quad \text{and} \quad s_l(x) = W_l(x) + J_l(x) - U_l + H_l \tag{C.18}$$

$$\hat{s}_h(x) = \hat{W}_h(x) + \hat{J}_h(x) - \bar{U} + F_h \quad \text{and} \quad \hat{s}_l(x) = \hat{W}_l(x) + \hat{J}_l(x) - U_l + F_l \tag{C.19}$$

Note that by lemma 2.1 both surplus functions and hence cut-offs coincide if $F = H$, a result which even holds in a more general framework with non-linear utility and non-linear payroll tax. Given our assumptions, observe that for the same x the difference between the surplus functions is given by: $s_j(x) - \hat{s}_j(x) = \frac{-t_j}{r_j}(w_j(x) - \hat{w}_j(x)) + H_j - F_j = -(1-\omega t_j)(F_j - H_j)$, which is independent of x. Hence, the surplus functions have the following linear structure

$$s_j(x) = s_j^0 + s_j^1 x - (1-\omega t_j)(F_j - H_j) \tag{C.20}$$

$$\hat{s}_j(x) = s_j^0 + s_j^1 x \tag{C.21}$$

From (3.2) and (3.4) we infer that $s_j^1 = \frac{(1-\tau_j)(1-\omega t_j)}{r_j}$. The cut-offs solving $s_j(\underline{x}) = 0$ and $s_j(\underline{\hat{x}}) = 0$ are therefore given by

$$\underline{x}_j = -\frac{s_j^0 - (1-\omega t_j)(F_j - H_j)}{s_j^1} \tag{C.22}$$

$$\underline{\hat{x}}_j = -\frac{s_j^0}{s_j^1} \tag{C.23}$$

Hence, the **relationship of the cut-offs** can be written as

$$JA_j \quad : \quad \underline{x}_j = \underline{\hat{x}}_j + \frac{r_j}{1-\tau_j}(F_j - H_j) \tag{C.24}$$

D Derivation of the social planner's optimum in the simple intragroup model

The constrained social optimum is derived by maximizing the social welfare function $\Theta(\cdot)$ subject to the matching constraints and the evolution of total production y, hence:

$$\max_{\{\underline{x},\underline{\hat{x}},\theta\}} \Theta = \max_{\{\underline{x},\underline{\hat{x}},\theta\}} \int_0^\infty e^{-rt}(y + uh - C\theta u)dt \tag{D.1}$$

subject to:

$$\dot{u} = \pi^n G(\underline{\hat{x}})(L-u) - q^w u \tag{D.2}$$

$$\dot{y} = u\theta q(\theta) \int_{\underline{x}}^\infty \tilde{x}\, dG(\tilde{x}) + (L-u)\pi^n \int_{\underline{\hat{x}}}^\infty \tilde{x}\, dG(\tilde{x}) - \pi^n y \tag{D.3}$$

We set up the present-value Hamiltonian

$$\begin{aligned}\mathcal{H} = {} & e^{-rt}(y + uh - C\theta u) + \lambda_1 \left[\pi^n G(\underline{\hat{x}})(L-u) - q^w u\right] \\ & + \lambda_2 \left[u\theta q(\theta) \int_{\underline{x}}^\infty \tilde{x}\, dG(\tilde{x}) + (L-u)\pi^n \int_{\underline{\hat{x}}}^\infty \tilde{x}\, dG(\tilde{x}) - \pi^n y\right]\end{aligned} \tag{D.4}$$

The optimality conditions, i.e. $\frac{\partial \mathcal{H}}{\partial \underline{x}} = 0$, $\frac{\partial \mathcal{H}}{\partial \underline{\hat{x}}} = 0$, $\frac{\partial \mathcal{H}}{\partial \theta} = 0$, $\frac{\partial \mathcal{H}}{\partial u} = -\dot{\lambda}_1$, $\frac{\partial \mathcal{H}}{\partial y} = -\dot{\lambda}_2$, imply (D.5) to (D.9):

$$\lambda_1 - \underline{x}\lambda_2 = 0 \tag{D.5}$$

$$\lambda_1 - \underline{\hat{x}}\lambda_2 = 0 \tag{D.6}$$

From (D.5) and (D.6) we infer that the cut-off productivities irrespective of whether one arrives at or has already been in a job coincide, i.e. $\underline{x} = \underline{\hat{x}}$. From now on we will just use $\underline{x}$. Before stating the remaining first order conditions define $\widetilde{G}(\underline{x}) \equiv \int_{\underline{x}}^\infty \tilde{x}\, dG(\tilde{x})$ and $\Gamma \equiv \frac{1}{r+\pi^n} \int_{\underline{x}}^\infty (\tilde{x} - \underline{x})\, dG(\tilde{x})$:

$$-e^{-rt}C - \lambda_1(1-\eta)q(\theta)(1 - G(\underline{x})) + \lambda_2(1-\eta)q(\theta)\widetilde{G}(\underline{x}) = 0 \tag{D.7}$$

$$-e^{-rt}(h - C\theta) - \lambda_1\left[\pi^n G(\underline{x}) + q^w\right] + \lambda_2\left[\theta q(\theta) - \pi^n\right]\widetilde{G}(\underline{x}) = -\dot{\lambda}_1 \tag{D.8}$$

$$e^{-rt} - \pi^n \lambda_2 = -\dot{\lambda}_2 \tag{D.9}$$

Eliminating λ_1 in (D.7) using (D.5) gives

$$-e^{-rt}C + \lambda_2(1-\eta)q(\theta)(r+\pi^n)\Gamma = 0 \tag{D.10}$$

which implies the following relationships for λ_1 and λ_2:

$$\lambda_1 = \frac{e^{-rt}C\underline{x}}{(1-\eta)q(\theta)(r+\pi^n)\Gamma} \quad \text{and} \quad \lambda_2 = \frac{e^{-rt}C}{(1-\eta)q(\theta)(r+\pi^n)\Gamma} \tag{D.11}$$

Differentiating (D.10) w.r.t. t and subtracting (D.10) again results in the following relations:

$$\dot{\lambda}_2 = -\lambda_2 r \quad \text{and consequently} \quad \dot{\lambda}_1 = -\lambda_1 r \tag{D.12}$$

Inserting for λ_2 and $\dot{\lambda}_2$ in (D.9) and rearranging gives the reduced optimality condition that has a similar structure compared the job creation condition:

$$(1-\eta)\frac{x^e - \underline{x}}{\pi^n + r} - \frac{C}{q^f} = 0 \tag{D.13}$$

To derive the last reduced optimality condition, i.e. the job destruction condition counterpart, we eliminate λ_1, λ_2 and $\dot{\lambda}_1$ in (D.8) and rearrange:

$$\underline{x} - h + \pi^n\Gamma - \frac{\eta}{1-\eta}C\theta = 0 \tag{D.14}$$

E Derivation of the social planner's optimum in the intergroup model

Again we maximize discounted social welfare

$$\int_0^\infty e^{-rt}\left[y_h + y_l + (u_h + u_l)h - u_h C_h \theta_h - u_l C \theta_l\right] dt \tag{E.1}$$

where $u_l = (1 - u_h - e_h - e_l)$, subject to the evolution of the employment states $\dot{u}_h$, $\dot{e}_l$, $\dot{e}_h$ and of total production $\dot{y}_h$ and $\dot{y}_l$ as given by (A.1), over the choice variables $\underline{x}_j$, $\hat{\underline{x}}_j$ and θ_j. We set up the present-value Hamiltonian

$$\begin{aligned}\mathcal{H} = e^{-rt}&\left[y_h + y_l + (1 - e_h - e_l)h - u_h C_h \theta_h - (1 - u_h - e_h - e_l)C\theta_l\right] \\ &+ \lambda_1 \dot{u}_h + \lambda_2 \dot{e}_l + \lambda_3 \dot{e}_h + \lambda_4 \dot{y}_h + \lambda_5 \dot{y}_l\end{aligned} \tag{E.2}$$

The optimality conditions $\frac{\partial \mathcal{H}}{\partial \underline{x}_j} = 0$, $\frac{\partial \mathcal{H}}{\partial \hat{\underline{x}}_j} = 0$ imply

$$\lambda_1(1-\pi^l) - \lambda_3 - \underline{x}_h\lambda_4 = 0 \quad \text{and} \quad \lambda_1(1-\pi^l) - \lambda_3 - \hat{\underline{x}}_h\lambda_4 = 0 \tag{E.3}$$

$$\lambda_2 + \underline{x}_l \lambda_5 = 0 \quad \text{and} \quad \lambda_2 + \hat{\underline{x}}_l \lambda_5 = 0 \tag{E.4}$$

Hence, reservation productivities have to coincide again, i.e. $\underline{x}_j = \hat{\underline{x}}_j$. For simplicity will will just use $\underline{x}_j$ from now on. Define $\widetilde{G}_j(\underline{x}_j) \equiv \int_{\underline{x}_j}^{\infty} \tilde{x}\, dG_j(\tilde{x})$ and $\Gamma_j \equiv \frac{1}{r_j}\int_{\underline{x}_j}^{\infty} \left(\tilde{x} - \underline{x}_j\right) dG_j(\tilde{x})$ and note their relationship $r_j\Gamma_j = \widetilde{G}_j(\underline{x}_j) - \underline{x}_j(1 - G_j(\underline{x}_j))$ which will be used frequently in what follows. Next, we set $\frac{\partial \mathcal{H}}{\partial \theta_h} = 0$ and eliminate $\lambda_1(1-\pi^l) - \lambda_3$ using (E.3) to get

$$-e^{-rt}C_h + \lambda_4 r_h \Gamma_h (1-\eta) q_h(\theta_h) = 0 \tag{E.5}$$

which solved for λ_4 implies

$$\lambda_4 = \frac{e^{-rt}C_h}{(1-\eta)q_h(\theta_h) r_h \Gamma_h} \quad \text{and} \quad \dot{\lambda}_4 = -r\lambda_4 \tag{E.6}$$

Inserting again in (E.5) gives

$$\begin{aligned} \lambda_1(1-\pi^l) - \lambda_3 &= \frac{e^{-rt}C_h \underline{x}_h}{(1-\eta)q_h(\theta_h) r_h \Gamma_h} \quad \text{and} \\ \dot{\lambda}_1(1-\pi^l) - \dot{\lambda}_3 &= -r\left(\lambda_1(1-\pi^l) - \lambda_3\right) \end{aligned} \tag{E.7}$$

Proceeding analogously for θ_l implies

$$\lambda_2 = \frac{e^{-rt}C_l}{(1-\eta)q_l(\theta_l) r_l \Gamma_l} \quad \text{and} \quad \dot{\lambda}_2 = -r\lambda_2 \tag{E.8}$$

$$\lambda_5 = \frac{e^{-rt}C_l \underline{x}_l}{(1-\eta)q_l(\theta_l) r_l \Gamma_l} \quad \text{and} \quad \dot{\lambda}_5 = -r\lambda_5 \tag{E.9}$$

The optimality condition for y_h reads $e^{-rt} - \lambda_4(\pi^x + \pi^n) = -\dot{\lambda}_4$. We eliminate λ_4 and $\dot{\lambda}_4$ to get the optimal job creation condition for high-skilled jobs

$$(1-\eta)q_h(\theta_h)\Gamma_h = C_h \quad \text{or} \quad (1-\eta)\left(\frac{x_h^e - \hat{\underline{x}}_h}{r_h}\right) = \frac{C_h}{q_h^f} \tag{E.10}$$

Similarly, transforming $\frac{\partial \mathcal{H}}{\partial y_l} = e^{-rt} - \lambda_5(\pi^x + \pi^n + \pi^h) = -\dot{\lambda}_5$ gives the optimal low-skill job creation condition

$$(1-\eta)q_l(\theta_l)\Gamma_l = C_l \quad \text{or} \quad (1-\eta)\left(\frac{x_l^e - \hat{\underline{x}}_l}{r_l}\right) = \frac{C_l}{q_l^f} \tag{E.11}$$

Combine those two conditions with our expressions for the co-states to get

$$\lambda_1(1-\pi^l) - \lambda_3 = \frac{e^{-rt}\underline{x}_h}{r_h}, \lambda_2 = \frac{e^{-rt}\underline{x}_l}{r_l}, \lambda_4 = \frac{e^{-rt}}{r_h}, \lambda_5 = \frac{e^{-rt}}{r_l} \tag{E.12}$$

Compute $\frac{\partial \mathcal{H}}{\partial e_l} = -\dot{\lambda}_2$, eliminate all known co-states and transform to get

$$\underline{x}_l - h - \frac{\eta}{1-\eta} C_l \theta_l + \pi^h \Gamma_h + \pi^n \Gamma_l + \frac{\lambda_1}{e^{-rt}}(1-\pi^l)\pi^h = 0 \tag{E.13}$$

Note that this equation implies that $\dot{\lambda}_1 = -r\lambda_1$ and consequently $\dot{\lambda}_3 = -r\lambda_3$. Next, we calculate $\frac{\partial \mathcal{H}}{\partial u_h} = -\dot{\lambda}_1 = r\lambda_1$ which gives

$$\lambda_1 \tilde{r} = -e^{-rt}\left[C_h\theta_h - C_l\theta_l\right] + e^{-rt}\theta_h q_h(\theta_h)\Gamma_h - e^{-rt}\theta_l q_l(\theta_l)\Gamma_l \tag{E.14}$$

Use the job creation conditions (E.10) and (E.11) to eliminate $q_j(\theta_j)\Gamma_j$ by $\frac{C_j}{1-\eta}$ and rearrange to arrive at

$$\frac{\lambda_1}{e^{-rt}} = \frac{\eta}{1-\eta}\left[\frac{C_h\theta_h - C_l\theta_l}{\tilde{r}}\right] \tag{E.15}$$

Insert this expression in (E.13) to derive the optimal job destruction condition for low-skilled workers

$$\hat{\underline{x}}_l - h - \frac{\eta}{1-\eta} C_l\theta_l + \pi^h(1-\pi^l)\frac{\eta}{1-\eta}\left[\frac{C_h\theta_h - C_l\theta_l}{\tilde{r}}\right] + \pi^n\Gamma_l + \pi^h\Gamma_h = 0 \tag{E.16}$$

Compute $\frac{\partial \mathcal{H}}{\partial e_h} = -\dot{\lambda}_3$ and eliminate $-\dot{\lambda}_3$ using $\lambda_3 = e^{-rt}(1-\pi^l)\frac{\eta}{1-\eta}\left[\frac{C_h\theta_h - C_l\theta_l}{\tilde{r}}\right]$. Rearranging reveals the optimal job destruction condition for high-skilled workers

$$\hat{\underline{x}}_h - h - \frac{\eta}{1-\eta}\left[\frac{(1-\pi^l)r}{\tilde{r}}C_h\theta_h + \frac{\pi^l(1+r)}{\tilde{r}}C_l\theta_l\right] + \pi^n\Gamma_h = 0 \tag{E.17}$$

Observe how $\pi^l = \pi^h = 0$ make the conditions collapse to their intragroup forms as derived in appendix section D.

F More comparative statics for the intragroup model

F.1 JD-JC diagram

Note that the determinant of the Jacobian of the JD-JC system is always positive, as $JD_\theta \equiv \frac{\partial JD}{\partial \theta} < 0$, $JD_{\hat{\underline{x}}} \equiv \frac{\partial JD}{\partial \hat{\underline{x}}} > 0$, $JC_\theta \equiv \frac{\partial JC}{\partial \theta} < 0$, and $JC_{\hat{\underline{x}}} \equiv \frac{\partial JC}{\partial \hat{\underline{x}}} < 0$, i.e. $Det(JDJC) = JD_\theta JC_{\hat{\underline{x}}} - JC_\theta JD_{\hat{\underline{x}}} \equiv \nabla > 0$. The elements of the inverse of the Jacobian of $JDJC$ system have the following signs:

$$(Jac_{JDJC})^{-1} = \nabla^{-1}\begin{pmatrix} JC_{\hat{\underline{x}}} & -JD_{\hat{\underline{x}}} \\ -JC_\theta & JD_\theta \end{pmatrix} = \begin{pmatrix} - & - \\ + & - \end{pmatrix}$$

To prove that the JD-curve slopes upward and the JC-curve is downward sloping proceed as follows. Total differentiation of the JD-curve w.r.t. $\underline{\hat{x}}$ and θ gives

$$\frac{(1-\tau)(1-G(\underline{\hat{x}}))\pi^n + (1-\tau)r}{\pi^n + r} d\underline{\hat{x}} = \frac{\omega c}{1-\omega} d\theta, \text{ hence}$$

$$\frac{d\theta}{d\underline{\hat{x}}}|_{\mathrm{JD}} > 0, \text{ the JD-curve is increasing.}$$

Before deriving the slope of the JC-curve, let us define $\frac{\partial x^e}{\partial \underline{x}} = \frac{g(\underline{x})(x^e - \underline{x})}{1-G(\underline{x})} \equiv \Omega$.

Assumption F.1. $\Omega < 1$. *This is true in any case for some distributions (e.g. uniform, normal,...) and very likely to be true for others (e.g. log-normal, with sufficiently small variance)*[33].

Again, total differentiation reveals that:

$$\left[\frac{(1-\tau)(1-\omega)}{\pi^n + r}(\Omega - 1) - \frac{c}{(q^f)^2} g(\underline{x}) q(\theta)\right] d\underline{\hat{x}} = \frac{\eta c}{q^w} d\theta,$$

$$\frac{d\theta}{d\underline{\hat{x}}}|_{\mathrm{JC}} < 0, \text{ the JC-curve is decreasing.}$$

F.2 Policy effects

If total effects are not mentioned, it means that they are ambiguous.

Wage subsidy (D)

$$\frac{d\theta}{dD}|_{\mathrm{JD}} = \frac{1-\omega}{\omega c} > 0 \quad \text{and} \quad \frac{d\theta}{dD}|_{\mathrm{JC}} = 0$$

Effect: The JD-curve shifts outward. The JC-curve does not move. $\theta \uparrow, \underline{\hat{x}} = \underline{x} \downarrow$, $u \downarrow$.

Hiring subsidy (H)

$$\frac{d\theta}{dH}|_{\mathrm{JD}} = 0 \quad \text{and} \quad \frac{d\theta}{dH}|_{\mathrm{JC}} = -\frac{q^w(1-\omega)\left[\Omega - 1\right]}{\eta c} > 0$$

Effect: The JD-curve does not move. The JC-curve shifts outward. $\theta \uparrow$, $\underline{\hat{x}} \uparrow$, $\underline{x} < \underline{\hat{x}}$. To determine the effect on the direction of $\underline{x}$ see proposition 2.1.

Recruitment subsidy (R)

$$\frac{d\theta}{dR}|_{\mathrm{JD}} = \frac{\theta}{c} > 0 \text{ and } \frac{d\theta}{dR}|_{\mathrm{JC}} = \frac{\theta}{\eta c} > 0$$

[33] It is easy to analytically show that for the uniform distribution $\Omega = 1/2, \forall \underline{\hat{x}}$. Statements about the other distributions are based on numerical simulations.

Effect: The JD- and the JC-curves shift outward. $\theta \uparrow$. As the JC-curve moves stronger we have that: $\underline{\hat{x}} = \underline{x} \uparrow$.

Firing tax (F)

$$\frac{d\theta}{dF}|_{\text{JD}} = \frac{(1-\omega)r}{\omega c} > 0 \quad \text{and} \quad \frac{d\theta}{dF}|_{\text{JC}} = \frac{q^w(1-\omega)\left[\Omega - 1\right]}{\eta c} < 0$$

Effect: The JD-curve shifts outward and the JC-curve shift inward. $\underline{\hat{x}} \downarrow, \underline{x} > \underline{\hat{x}}$. Using the implicit function theorem one can show that $\theta \downarrow$.

Output taxes (τ)

$$\frac{d\theta}{d\tau}|_{\text{JD}} = -\frac{(1-\omega)\left[(x^{\hat{e}} - \underline{\hat{x}})(1-G(\underline{\hat{x}}))\pi^n + (\pi^n + r)\underline{\hat{x}}\right]}{\omega c(\pi^n + r)} < 0$$

$$\frac{d\theta}{d\tau}|_{\text{JC}} = -\frac{q^w(1-\omega)\left[(x^e - \underline{\hat{x}})(1-\tau) - \Omega(F-H)(\pi^n + r)\right]}{(1-\tau)(\pi^n + r)\eta c}$$

This expression is smaller than 0, i.e. the JC shifts inward, whenever $F = H$. The bigger F in comparison to H, the smaller the inward shift.

Effect: The JD- and the JC-curves shift inward. $\theta \downarrow$.

Payroll taxes (t)

$$\frac{d\theta}{dt}|_{\text{JD}} = -\frac{z(1-\omega)}{(1-t)^2\omega c} < 0 \quad \text{and} \quad \frac{d\theta}{dt}|_{\text{JC}} = 0$$

Effect: The JD-curve shifts inward. The JC does not move, implying $\theta \downarrow, \underline{\hat{x}} \uparrow$.

G List of variables

Table G.1: Variable names

$j = l, h$	subscript indicating the skill type
$\hat{}$	hat notation refers to 'inside'-variables
$\bar{x}$	average productivity
$\bar{w}$	average wage
ω	bargaining weight for the worker
$\underline{\kappa}$	bound (lower) for $G(\cdot)$
$\bar{\kappa}$	bound (upper) for $G(\cdot)$
$G(x)$	cdf for productivity draws
X^e	conditional expectation of some random variable X w.r.t. $\underline{x}$
$X^{\hat{e}}$	conditional expectation of some random variable X w.r.t. $\underline{\hat{x}}$
∇	determinant of the JD-JC system
F	firing taxes
H	hiring subsidy
h	home production
μ	instantaneous value of non-work ($\mu = b + h$)
r	interest rate, $r = \frac{(1-\beta)}{\beta}$
L	labor force
θ	labor market tightness
e	mass of employed people
u	mass of unemployed people
$\widetilde{G}(x)$	partial expectation of productivity
$g(x)$	pdf for productivity draws
π^l	prob. of downgrade
π^x	prob. of exogenous separation
q^f	prob. of filling a vacancy
q^w	prob. of finding and accepting a job
$q(\theta)$	prob. of match for the firm
$\theta q(\theta)$	prob. of match for the worker
π^n	prob. of new productivity draw
π^h	prob. of upgrade
x	productivity
R	recruitment subsidy
$\underline{x}$	reservation productivity, 'outside'
$\underline{\hat{x}}$	reservation productivity, 'inside'
Θ	social welfare
$s(x)$	surplus function
y	total production
τ	output tax rate
b	unemployment compensation
C	vacancy creation costs (gross)
c	vacancy creation costs (net of subsidies, i.e. $c = C - R$)
U	value of a being unemployed
$J(x)$	value of employment for the firm
$W(x)$	value of employment for the worker
V	value of a vacancy
D	wage subsidy (lump-sum)
t	payroll tax rate
η	weight in the matching function