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July 2015 Discussion Paper no. 2015-17

School of Economics and Political Science, Department of Economics

University of St.Gallen

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| Publisher: | School of Economics and Political Science<br>Department of Economics<br>University of St.Gallen<br>Bodanstrasse 8<br>CH-9000 St. Gallen<br>Phone +41 71 224 23 25 |
| Electronic Publication: | Fax +41 71 224 31 35<br>http://www.seps.unisg.ch |

Measuring spot variance spillovers when (co)variances are time-varying –

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**Abstract**

In highly integrated markets, news spreads at a fast pace and bedevils risk monitoring and optimal asset allocation. We therefore propose global and disaggregated measures of variance transmission that allow one to assess spillovers locally in time. Key to our approach is the vector ARMA representation of the second-order dynamics of the popular BEKK model. In an empirical application to a four-dimensional system of US asset classes - equity, fixed income, foreign exchange and commodities - we illustrate the second-order transmissions at various levels of (dis)aggregation. Moreover, we demonstrate that the proposed spillover indices are informative on the value-at-risk violations of portfolios composed of the considered asset classes.

**Keywords**

Multivariate GARCH, spillover index, value-at-risk, variance spillovers, variance decomposition.

**JEL Classification**

C32, C58, F3, G1.

# 1 Introduction

The financial and banking crisis of 2008, the political controversies over the fiscal cliff in the US in 2011, and the persistent fragility of real economic activity have raised the concerns of corporate risk managers, central bankers, and policy makers over measuring and monitoring economic and financial interdependence. Accordingly, a number of dependence and association measures have been suggested over the past years, such as the systemic expected shortfall of Acharya et al. (2010), the conditional value-at-risk of Adrian and Brunnermeier (2011), and the spillover or financial connectedness measures of Diebold and Yilmaz (2009, 2012, 2014).

Among these, the spillover measures of Diebold and Yilmaz (2009, 2012, 2014) have garnered much attention, because in contrast to other measures, they allow one to track the associations between individual variables and the system as a whole at all levels, from pairwise to system-wide, in a mutually consistent way. Technically, the spillover measures are derived from the forecast-error variance decomposition of a vector autoregressive model (VAR). For an application to variance spillovers, one therefore constructs the indices from VAR models estimated on measures of realized variance, such as range-based variance estimates or other estimates obtained from high-frequency intra-day data – see, among others, Yilmaz (2013), Baruník et al. (2014), Fengler and Gisler (2015), Louzis (2015) for recent applications.

Because the index is based on the forecast-error variance decomposition of a single VAR, it produces static, i.e., average, spillover information. While undoubtedly valuable, it would be of even greater use to have more timely spillover information, especially for variance spillovers. There is ample evidence that conditional variance is time-varying, and it is natural to expect that spillovers are as well. Diebold and Yilmaz (2009, 2012) therefore suggest computing the indices from VAR models that are estimated on rolling subsamples. In this way, one obtains an impression of the time-varying patterns of spillovers, showing for instance how a certain event may have contributed and changed the dependence structure of the system. However, as with all rolling window approaches, the estimates reflect only the average information of the current estimation window. Since the subsamples must be of sufficient length to provide reasonably accurate parameter estimates, e.g., one year of daily data, the rolling window indices are probably more useful for a retrospective analysis than for the timely monitoring of spillovers. For this aim, one would need a time-$t$ conditional spillover index.

In this paper, we propose such an index. We adopt the ideas of Diebold and Yilmaz (2009, 2012) to construct variance spillover indices that are updated with time-$t$ information. To this end, we build on multivariate GARCH (MGARCH) models of the BEKK-type (Engle and Kroner, 1995) and calculate the indices from the forecast error decomposition that is derived from the vector moving average (VMA) representation of the squared and vectorized return process. This process is driven by serially uncorrelated heteroskedastic innovations and, as we show here, its conditional covariance matrix can be derived analytically. This allows us to absorb the regime dependence into the parameters of the VMA representation. The variance spillover indices that are based on this time-varying VMA representation therefore take full advantage of the time-$t$ conditional information of the prevailing variance regime. In contrast to rolling window estimates, they convey on-the-spot variance spillover information. In our empirical applications, we not only show that the time-$t$ conditional variance spillover indices allow a study of the prompt impact of major economic or political events, but also that they are informative about the likelihood of value-at-risk violations. We therefore provide a new instructive tool for the instantaneous monitoring of variance spillovers.

Aside from the value of timely spillover information, our approach differs in methodological terms from the extant literature in that we derive the spillover indices from a full-fledged model of variance-covariance dynamics. This has advantages that are more than just conceptual. First and foremost, our variance spillover indices take full advantage of the informational content embedded in covariances. It appears commonplace to expect covariance dynamics to play a decisive role in the mechanisms of variance spillovers. In Fengler and Gisler (2015), a first step toward incorporating covariances into variance spillover measures is made, but the authors follow the traditional route in applying a VAR model to vectorized realized variance-covariance matrices estimated from intra-day data. Thus, they do not build on a model that ensures positive definiteness of the dynamic variance-covariance matrices. Of course, the recent realized variance literature has proposed such models, yet at the expense of nonlinear transformations of the variance-covariance matrices, which renders the clear attribution of the shocks to specific variables intricate if not impossible – see, e.g., Bauer and Vorkink (2011) and Golosnoy et al. (2012). The most attractive property of the Diebold and Yilmaz (2009, 2012) framework would thereby be lost.

With the MGARCH model, we circumvent this difficulty, because the VMA representation of the vector collecting the squared observations and the crossproducts of the observations remains linear in a serially uncorrelated vector innovation process. Hence, our approach integrates familiar concepts of VAR modeling (impulse response functions, forecast error variance decompositions) with established tools to address key topics in empirical finance, such as the forecasting of variance-covariance matrices, the determination of the value-at-risk and portfolio optimization. In summary, our MGARCH approach to defining variance spillover indices is not only naturally adaptive to time variation of second-order moments, but also affords a structural perspective on the sources of risk and on risk transmission through time. In this way, our approach complements the traditional variance transmission and contagion literature that studies (off-diagonal) parameter significance in MGARCH models to assess spillovers – see Pericoli and Sbracia (2003) for a survey of this rich literature.

Section 2 provides a brief sketch of the BEKK model and its translation into a vectorized representation of the 'squared' multivariate GARCH process. This representation is picked up in Section 3 to define indices of variance spillover. An empirical analysis of returns of four major US asset classes (equity, fixed income, foreign exchange, commodities) is provided in Section 4. Section 5 concludes.

## 2 The Multivariate GARCH model in BEKK and vec form

In this section, we discuss the so-called BEKK representation of the conditional covariance matrix of a vector of speculative returns. We refer to Bauwens et al. (2006) for an exhaustive presentation of the various model specifications. In addition, we are explicit about the translation of the BEKK model into the linear vec representation of a multivariate GARCH model.

### 2.1 BEKK model

We consider an $N$-dimensional vector of returns (first differences of log asset prices)

$$r_t = \mu_t + \varepsilon_t = \mu_t + H_t^{1/2}\xi_t, \quad \xi_t \overset{iid}{\sim} N(0, I_N),\ t = 1, 2, 3, \ldots, T\ , \tag{1}$$

where both the conditional mean $\mu_t$ and the conditional covariance $H_t$ are assumed to be measurable with respect to a filtration $\mathcal{F}_{t-1}$. We set $\mu_t = 0$ and estimate the model on the series of centered daily returns. In (1), $H_t^{1/2}$ denotes the symmetric matrix square root of $H_t$.[1] The innovation vector $\xi_t$ is assumed to be independent and identically normally distributed (iid). The assumption of conditional normality of $\xi_t$ is commonly adopted to implement (Quasi) Maximum Likelihood estimation of the parameters, but it is not essential for the subsequent discussions.

The so-called BEKK model has the attractive feature that under mild restrictions applying to the initial conditions, the process of conditional covariances $H_t$ is positive definite by construction (Engle and Kroner, 1995). In addition, other multivariate GARCH variants are special cases of the BEKK specification, for example, the factor model of Engle et al. (1990), the orthogonal GARCH model of Alexander (2001, pp. 21–38), its generalization introduced by van der Weide (2002) and the Cholesky GARCH of Dellaportas and Pourahmadi (2012).

[1] The square root of a symmetric positive definite matrix $Z$ is defined as $Z^{1/2} = \Gamma\Lambda^{1/2}\Gamma'$, where the columns of $\Gamma$ contain the eigenvectors of $Z$, and $\Lambda^{1/2}$ is diagonal with the positive square roots of the eigenvalues on its diagonal.

In its most flexible form, the BEKK$(p,q,K)$ representation of the conditional covariance $\text{Cov}_{t-1}[\varepsilon_t] = E_{t-1}[\varepsilon_t\varepsilon_t'] = H_t$ is given by

$$H_t = CC' + \sum_{k=1}^{K}\sum_{i=1}^{q} F_{ki}'\varepsilon_{t-i}\varepsilon_{t-i}'F_{ki} + \sum_{k=1}^{K}\sum_{i=1}^{p} G_{ki}'H_{t-i}G_{ki}\ . \tag{2}$$

In (2), $C$ is a lower triangular matrix and $F_{ki}$ and $G_{ki}$ are $N \times N$ parameter matrices. In this work, we focus on the simplest case $p = q = K = 1$, which is also by far the most popular model order obtaining

$$H_t = CC' + F'\varepsilon_{t-1}\varepsilon_{t-1}'F + G'H_{t-1}G, \tag{3}$$

where we have suppressed the subscripts of the BEKK parameter matrices for notational convenience. For this model, the parameter vector is given by $\gamma = (\text{vech}(C)', \text{vec}(F)', \text{vec}(G)')'$.[2] In cases $N = 2, 3$ and $N = 4$ this amounts to 11, 24 and 42 parameters, respectively. With regard to parameter estimation, Jeantheau (2000) and Comte and Lieberman (2003) have shown, respectively, consistency and asymptotic normality of the (quasi) Maximum-Likelihood (QML) estimator $\hat{\gamma}$ under particular regularity conditions.

Encompassing all linear covariance specifications, the vec representation provides a general framework to compare the dynamic features implied by alternative covariance models, such as impulse response functions (Hafner and Herwartz, 2006). For the derivation of the BEKK implied vec form, some elementary matrices turn out to be useful, namely the elimination matrix $L_N$, the duplication matrix $D_N$ and its generalized inverse $D_N^+$.[3] Let $\eta_t = \text{vech}(\varepsilon_t\varepsilon_t')$ and $h_t = \text{vech}(H_t)$. Then, the vec representation of the BEKK model in (3) is given by

$$h_t = \upsilon + A\eta_{t-1} + Bh_{t-1}, \tag{4}$$

[2] The vec-operator stacks the columns of a matrix into a vector. For a square matrix, the vech-operator stacks the elements on and below the diagonal into a vector.

[3] Let $N^* = N(N+1)/2$. With reference to a symmetric square $N \times N$ matrix $Z$, the $N^* \times N^2$ elimination matrix $L_N$ is defined by the property $\text{vech}(Z) = L_N\text{vec}(Z)$. Conversely, the $(N^2 \times N^*)$ dimensional duplication matrix $D_N$ is defined by $\text{vec}(Z) = D_N\text{vech}(Z)$. Because $D_N'D_N$ is nonsingular, the Moore-Penrose inverse or generalized inverse of $D_N$ is $D_N^+ = (D_N'D_N)^{-1}D_N'$. See Lütkepohl (1996).

where $\upsilon = \text{vech}(CC')$, $A = D_N^+(F \otimes F)' D_N$ and $B = D_N^+(G \otimes G)' D_N$.

Now consider the $N^* = N(N+1)/2$ dimensional vector of mean zero random variables

$$u_t = \eta_t - h_t. \tag{5}$$

The process $u_t$ can be easily shown to be free of serial correlation. Hence, using the definition in (5) to replace $h_t$ in (4), $u_t$ serves as an innovation or forecast error process in VARMA type representations of $\eta_t$. However, due to the intrinsic nonlinearity of $\varepsilon_t = H_t^{1/2}\xi_t$, the vector innovation process $u_t$ is not identically distributed over the time dimension. In particular, time variation of the fourth-order moments of $\varepsilon_t$ implies that the (conditional) covariance of $u_t$ is also time-varying, i.e., $\text{Cov}_{t-1}[u_t] = E_{t-1}[u_t u_t'] = \Sigma_t$. Below we provide an explicit representation of $\Sigma_t$ in terms of $h_t$ and fourth-order moments of the iid innovations $\xi_t$.

Let $\mathcal{A} = A + B$ and denote by $L$ the lag operator such that $L\eta_t = \eta_{t-1}$. Substituting for $h_t$ in (4), one obtains

$$\begin{aligned}
\eta_t &= \upsilon + A\eta_{t-1} + B(\eta_{t-1} - u_{t-1}) + u_t & (6)\\
\Leftrightarrow (I - \mathcal{A}L)\eta_t &= \upsilon + (I - BL)u_t & (7)\\
\Leftrightarrow \eta_t &= (I - \mathcal{A}L)^{-1}\upsilon + (I - \mathcal{A}L)^{-1}(I - BL)u_t & (8)\\
&= \tilde{\upsilon} + \Phi(L)(1 - BL)u_t, & (9)\\
&= \tilde{\upsilon} + \Theta(L)u_t, & (10)
\end{aligned}$$

with $\tilde{\upsilon} = (I - \mathcal{A})^{-1}\upsilon$ and $\Phi(L) = (I - \mathcal{A}L)^{-1}$. The vector disturbance $u_t$ is serially uncorrelated and conditionally heteroskedastic, $u_t \sim (0, \Sigma_t)$. Moreover, in (9) the parameter matrices specifying the operator $\Phi(L) = (I_N - AL)^{-1} = I_N + \Phi_1 L + \Phi_2 L^2 + \Phi_3 L^3 + \Phi_4 L^4 + \Phi_5 L^5 + \ldots$ are

$$\Phi_0 = I_N,\ \Phi_i = \mathcal{A}\Phi_{i-1},\ i = 1, 2, 3, 4, \ldots\ .$$

Summarizing the autoregressive and moving average part of the vec representation, the operator $\Theta(L)$ in (10) conforms with the parameterization

$$\Theta_0 = I,\ \Theta_1 = \mathcal{A} - B = A,\ \Theta_i = \mathcal{A}\Theta_{i-1},\ i = 2, 3, \ldots.$$

Since the matrix $\text{Cov}[u_t] = \Sigma_t$ is typically not diagonal, the elements of $u_t$ are simultaneously correlated. As an implication, the coefficient matrices $\Theta_k$, $k = 0, 1, 2, \ldots$ are not suitable for describing how isolated shocks are transmitted to forecast uncertainties or impulse responses attached to particular variables in $\eta_t$. To cope with cross equation correlation, it has become a convention to extract impulse responses from suitably orthogonalized shocks. For this purpose, the model in (10) can be rephrased as

$$\begin{aligned} \eta_t &= \tilde{\upsilon} + \Theta(L)\Sigma_t^{1/2}\Sigma_t^{-1/2}u_t & (11) \\ &= \tilde{\upsilon} + \Psi_t(L)\nu_t, & (12) \end{aligned}$$

where $\nu_t = \Sigma_t^{-1/2}u_t$, $\Psi_t(L) = \Theta(L)\Sigma_t^{1/2}$. In contrast to the elements in $u_t$, the elements in $\nu_t$ are suitably orthogonalized such that it is reasonable to trace the effects of isolated shocks on forecast uncertainties and the impulse responses attached to the variables in $\eta_t$. Unlike standard impulse response patterns in homoskedastic VARs, however, the operator $\Psi(L)$ depends on $\Sigma_t$ and, hence, is time-varying. Specifically, we have

$$\Psi_t(L) = \Psi_{t0} + \Psi_{t1}L + \Psi_{t2}L^2 + \Psi_{t3}L^3 + \ldots \quad (13)$$

where $\Psi_{t0} = \Sigma_t^{1/2}$, $\Psi_{tk} = \Theta_k\Sigma_{t-k}^{1/2}$. As with the usual concepts of impulse response analysis in VAR models, the coefficients in $\Psi_{tk}$, $k = 0, 1, 2, \ldots$ describe how (unit) shocks in the elements of $\nu_t$ impact the variables in $\eta_t = \text{vech}(\varepsilon_t\varepsilon_t')$ simultaneously $(k = 0)$ and over time $(k = 1, 2, \ldots)$. For their practical implementation, it remains to derive $\text{Cov}_{t-1}[u_t] = \Sigma_t$ to which we turn next.

## 2.2 The conditional covariance of GARCH VARMA innovations

To determine $\Sigma_t = E_{t-1}[u_t u_t']$, we notice from symmetry of $H_t^{1/2}$ that

$$\eta_t = \text{vech}(\varepsilon_t\varepsilon_t') = \text{vech}(H_t^{1/2}\xi_t\xi_t'H_t^{1/2}) \; .$$

By the definition of $\Sigma_t$ and because $\text{vec}(H_t) = D_N\text{vech}(H_t) = D_N h_t$, we have

$$\Sigma_t = \text{Cov}_{t-1}\left[\text{vech}(H_t^{1/2}\xi_t\xi_t'H_t^{1/2}) - h_t\right]$$

$$
\begin{aligned}
&= E_{t-1}\left[(\text{vech}(H_t^{1/2}\xi_t\xi_t' H_t^{1/2}) - h_t)(\text{vech}(H_t^{1/2}\xi_t\xi_t' H_t^{1/2}) - h_t)'\right] \\
&= E_{t-1}\left[L_N\left(\mathcal{H}_t \text{vec}(\xi_t\xi_t') - D_N h_t\right)\left(\text{vec}(\xi_t\xi_t')'\mathcal{H}_t - D_N h_t\right) L_N'\right] \ ,
\end{aligned}
$$

where we use result 7.3 (6) in Lütkepohl (1996, p. 100) for the calculus with vectorized matrices, and define the $N^2 \times N^2$ matrix $\mathcal{H}_t = H_t^{1/2} \otimes H_t^{1/2}$. Hence, $\Sigma_t$ reads as

$$
\begin{aligned}
\Sigma_t &= E_{t-1}\left[L_N \mathcal{H}_t \text{vec}(\xi_t\xi_t')\text{vec}(\xi_t\xi_t')'\mathcal{H}_t L_N'\right] + L_N D_N h_t h_t' D_N' L_N' \\
&- E_{t-1}\left[L_N \mathcal{H}_t \text{vec}(\xi_t\xi_t') h_t' D_N' L_N'\right] - E_{t-1}\left[L_N D_N h_t \text{vec}(\xi_t\xi_t')\mathcal{H}_t L_N'\right] \\
&= L_N \mathcal{H}_t \widetilde{\Omega} \mathcal{H}_t L_N' + L_N D_N h_t h_t' D_N' L_N' - 2 L_N \mathcal{H}_t \text{vec}(I_N) h_t' D_N' L_N' \ , \qquad (14)
\end{aligned}
$$

where we use $E[\text{vec}(\xi_t\xi_t')] = \text{vec}(I_N)$ in (14). In addition, replacing $\text{vec}(\xi_t\xi_t')$ by $\xi_t \otimes \xi_t$, we get

$$
E_{t-1}[\text{vec}(\xi_t\xi_t')\text{vec}(\xi_t\xi_t')'] = E_{t-1}[(\xi_t\xi_t') \otimes (\xi_t\xi_t')'] = \widetilde{\Omega} \ . \qquad (15)
$$

Collecting fourth-order moments of $\xi_t$, the matrix $\widetilde{\Omega}$ in (15) is of dimension $N^2 \times N^2$ with typical $N \times N$ dimensional blocks $\Omega_{ij}$, $i, j = 1, 2, \ldots, N$. Let $\omega_{kl}^{(ij)}$ denote a typical element of the block $\Omega_{ij}$ of $\widetilde{\Omega}$. Specifically, along the diagonal, the block matrices $\Omega_{ii}$ have typical elements

$$
\omega_{ii}^{(ii)} = \kappa, \ \omega_{jj}^{(ii)} = 1, j \neq i \text{ and } \omega_{ij}^{(ii)} = 0, \ i \neq j. \qquad (16)
$$

The quantities $\omega_{ii}^{(ii)} = \kappa$ refer to the fourth-order moments of elements in $\xi_t$; under the Gaussian assumption, we would have $\kappa = 3$. Off-diagonal blocks $\Omega_{ij}$, $i \neq j$, are such that

$$
\omega_{ij}^{(ij)} = 1, \text{ and } \omega_{kl}^{(ij)} = 0 \text{ for } (k, l) \neq (i, j).
$$

# 3 Measuring variance spillovers

## 3.1 A time-varying forecast-error variance decomposition of squared returns

Conditional on time $t$, the $G$-step ahead forecast error for $\eta_{t+G}$ reads as

$$
\eta_{t+G} - \hat{\eta}_{t+G|t} = \Psi_{t+G,0|t}\nu_{t+G} + \Psi_{t+G-1,1|t}\nu_{t+G-1} + \ldots + \Psi_{t+1,G-1|t}\nu_{t+1} \ . \qquad (17)
$$

The parameters in the $i$th row of the matrices $\Psi_{t+G,0|t}, \Psi_{t+G-1,1|t}, \ldots, \Psi_{t+1,G-1|t}$ describe how the elements in $\nu_{t+G}, \ldots, \nu_{t+1}$ contribute to the forecast errors of variable $i$ at horizon $G$. Conditional on time $t$, the parameter matrices in (17) are

$$\Psi_{t+1,G-1|t} = \Theta_{G-1}\Sigma_{t+1}^{1/2}, \;\; \Psi_{t+2,G-2|t} = \Theta_{G-2}\widehat{\Sigma}_{t+2|t}^{1/2}, \;\; \ldots, \;\; \Psi_{t+G,0|t} = \Theta_0\widehat{\Sigma}_{t+G|t}^{1/2} .$$

While $\Sigma_{t+1}$ is measurable with respect to information available in the forecast origin $t$, future covariances $\Sigma_{t+1}, \Sigma_{t+2}, \ldots, \Sigma_{t+G}$ depend on the MGARCH covariances $H_s$, $s = t+2, t+3, \ldots, t+G$. According to the BEKK representation in (3), the recursive one-step ahead predictors of the conditional covariances are

$$\begin{aligned} \widehat{H}_{t+1|t} \;=\; H_{t+1} \;&=\; CC' + F'\varepsilon_t\varepsilon_t'F + G'H_tG \\ \text{and } \widehat{H}_{t+i|t} \;&=\; CC' + F'\widehat{H}_{t+i-1|t}'F + G'\widehat{H}_{t+i-1|t}G, \; i = 2, 3, \ldots \end{aligned} \tag{18}$$

Let $\psi_{ij}^{(t,k,G)}$ denote a typical element of the matrix $\Psi_{t+G-k,k}$, $k = 0, 1, \ldots, G-1$. In accordance with the VAR literature (see Lütkepohl, 2007, p. 63-64), the proportion of the $G$-step forecast-error variance of variable $i$, accounted for by innovations in variable $j$, is given by

$$\lambda_{t,ij}^{(G)} = \frac{\sum_{g=1}^{G}\left(\psi_{ij}^{(t,g,G)}\right)^2}{\sum_{g=1}^{G}\sum_{j=1}^{N^*}\left(\psi_{ij}^{(t,g,G)}\right)^2} , \tag{19}$$

where $N^* = N(N+1)/2$ is the number of variables, i.e., the dimension of $\text{vech}(\varepsilon_t\varepsilon_t')$.

The time-specific measures in (19) serve as a basis for the definition of the spillover statistics in the following section. For their actual implementation, we will assume $E[\xi_{i,t}^4] = 3$, $i = 1, \ldots, N$, in (16) to specify the conditional covariances $\Sigma_t$. While this may seem rather strong an assumption, from the definition of $\lambda_{t,ij}^{(G)}$ as a ratio, one may easily imagine that the approximation error implied by the normality assumption is minor despite the actual excess kurtosis of return innovations.[4]

[4]Indeed, when we replace $E[\xi_{i,t}^4] = 3$ by the empirical fourth-order moments of the leptokurtic innovations $\xi_{i,t}$, $i = 1, \ldots, N$, the mean (standard deviation) of the absolute differences between the respective indices of total variance spillovers shown in Section 4 is 0.007 (0.003).

### 3.2 Variance spillover indices

Diebold and Yilmaz (2009) motivate to use statistics of the form in (19) to define spillover indices. It is, however, important to observe that $\lambda_{t,ij}^{(G)}$ in (19) depends on the construction of underlying shocks $\nu_t$ and the determination of $\Sigma_t^{1/2}$. Both $\nu_t$ and $\Sigma_t^{1/2}$ lack invariance under rotation, or, put differently, rival definitions are observationally equivalent in $\eta_t$. Specifically, consider a counterpart of (11)

$$\eta_t = \tilde{\upsilon} + \Theta(L)\Sigma_t^{1/2} Q Q' \Sigma_t^{-1/2} u_t,\ QQ' = I,\ Q \neq I, \tag{20}$$

where $Q$ is a rotation matrix. In the literature on structural VARs, the identification of $\Sigma_t^{1/2}Q$ has attracted huge interest (Amisano and Giannini, 1997). Typically, external information, for instance, derived from economic theory, is employed to address model identification. Recently, sign restrictions have become a prominent identification approach. In this simulation based framework, those matrix candidates $\Sigma_t^{1/2}Q$ are considered to contribute to identification that imply impulse response patterns consistent with economic theory (Faust, 1998; Uhlig, 2005). Because economic theory of the contemporaneous relations among daily financial data is scarce, the decomposition set out in (11) can only be justified in the light of its economic content and the plausibility of the statistical functionals derived from the definitions in (11) or (19). When discussing the empirical implications of our model, we will justify the identifying content of $\Sigma_t^{1/2}$ in the light of the detected patterns of aggregate total variance spillovers and disaggregate asset-specific net variance spillovers.[5]

For the construction of the spillover index, note that we have $\sum_j^{N^*} \lambda_{t,ij}^{(G)} = 1$ and $\sum_{i,j}^{N^*} \lambda_{t,ij}^{(G)} = N^*$. Let $\eta_{i,t}$, $i = 1, \ldots, N^*$, denote an element of $\eta_t$. Then a measure

[5] As an alternative to the symmetric matrix square root, Diebold and Yilmaz (2014) build on the Cholesky factorization for identification. Since the Cholesky factorization is order-dependent, they justify this choice by showing that they recover very similar spillover patterns for other randomized orderings. In the light of their Fig. 5, it seems likely that the spillover index obtained by averaging all indices of the randomized orderings will be close to the index based on the symmetric matrix square root.

of total spillovers can be defined as

$$\mathcal{S}_t^{(G)} = \frac{\sum_{i,j=1,i\neq j}^{N^*} \lambda_{t,ij}^{(G)}}{N^*} \ . \tag{21}$$

In (21), $\mathcal{S}_t^{(G)}$ measures the fraction of the forecast-error variance of the variables $\eta_{i,t}$, $i = 1, \ldots, N^*$, that is attributable to shocks in all other variables $\eta_{j,t}$, $j = 1, \ldots, N^*, j \neq i$. Thus, it is an index of how the shocks spill across the system. Since it is built from information conditional on time-$t$, it is a spot measure of variance spillovers.

Besides the total index (21), many other spot variance indices are possible. Following Diebold and Yilmaz (2012), we can therefore define directional spillovers between all variables involved. The directional spillovers received by variable $i$ from all other variables $j$ are defined as

$$\mathcal{R}_{t,i}^{(G)} = \frac{\sum_{j=1,j\neq i}^{N^*} \lambda_{t,ij}^{(G)}}{N^*} \ , \tag{22}$$

whereas the directional spillovers transmitted by variable $i$ to all other variables $j$ are measured by

$$\mathcal{T}_{t,i}^{(G)} = \frac{\sum_{j=1,i\neq j}^{N^*} \lambda_{t,ji}^{(G)}}{N^*} \ . \tag{23}$$

The directional spillovers provide decompositions of the spillover index into spillovers coming from (or to) a specific source. Furthermore, it is meaningful to compute their difference

$$\mathcal{N}_{t,i}^{(G)} = \mathcal{T}_{t,i}^{(G)} - \mathcal{R}_{t,i}^{(G)} \ , \tag{24}$$

because one learns about the net contribution of variable $i$ to the entire transmission process.

Since the elements in $\eta_t = \text{vech}(\varepsilon_t \varepsilon_t')$ correspond to patterns of variation and covariation, it is of interest to further distinguish between these two groups. Let $J_{cov}$ and $I_{cov}$ be the sets of all $i, j = 1, \ldots, N^*$ that index a covariance, and define $J_{var}$ and $I_{var}$ accordingly. As suggested in Fengler and Gisler (2015), we define

$$\mathcal{R}_{t,i}^{(G,cov)} = \frac{\sum_{j\in J_{cov},j\neq i}^{N^*} \lambda_{t,ij}^{(G)}}{N^*} \ , \qquad \mathcal{T}_{t,i}^{(G,cov)} = \frac{\sum_{j\in J_{cov},j\neq i}^{N^*} \lambda_{t,ji}^{(G)}}{N^*} \ , \tag{25}$$

$$\mathcal{R}_{t,i}^{(G,var)} = \frac{\sum_{j \in J_{var}, j \neq i}^{N^*} \lambda_{t,ij}^{(G)}}{N^*}, \qquad \mathcal{T}_{t,i}^{(G,var)} = \frac{\sum_{j \in J_{var}, j \neq i}^{N^*} \lambda_{t,ji}^{(G)}}{N^*}, \tag{26}$$

which can be interpreted as the directional spillovers received by variable $i$ from all covariances $j$ (left-hand side of (25)) or transmitted by variable $i$ to all covariances $j$ (right-hand side of (25)); and likewise for the variances in (26). As discussed above, for each $i$, the differences among these indices, e.g., $\mathcal{N}_{t,i}^{(G,cov)} = \mathcal{T}_{t,i}^{(G,cov)} - \mathcal{R}_{t,i}^{(G,cov)}$, provide insights into the net spillovers between covariances and variances.

Based on this, one defines the following total covariance and total variance spillover indices. An index of total *own (co)variance spillovers*, which measures the spillovers between covariances (between variances, right-hand side), is given by

$$\mathcal{S}_t^{(G,ocov)} = \sum_{i \in I_{cov}} \frac{\sum_{j \in J_{cov}, j \neq i}^{N^*} \lambda_{t,ij}^{(G)}}{N^*}, \qquad \mathcal{S}_t^{(G,ovar)} = \sum_{i \in I_{var}} \frac{\sum_{j \in J_{var}, j \neq i}^{N^*} \lambda_{t,ij}^{(G)}}{N^*}. \tag{27}$$

Moreover, the total *cross (co)variance spillovers*, which are spillovers from covariances to variances (variances to covariances, right-hand side), is defined by

$$\mathcal{S}_t^{(G,ccov)} = \sum_{i \in I_{var}} \frac{\sum_{j \in J_{cov}, j \neq i}^{N^*} \lambda_{t,ij}^{(G)}}{N^*}, \qquad \mathcal{S}_t^{(G,cvar)} = \sum_{i \in I_{cov}} \frac{\sum_{j \in J_{var}, j \neq i}^{N^*} \lambda_{t,ij}^{(G)}}{N^*}. \tag{28}$$

It holds that $\mathcal{S}_t^{(G)} = \mathcal{S}_t^{(G,ocov)} + \mathcal{S}_t^{(G,ovar)} + \mathcal{S}_t^{(G,ccov)} + \mathcal{S}_t^{(G,cvar)}$. The indices (27) and (28) therefore shed light on the relative contribution of variance and covariance spillovers to the total index. It is also useful to study the net cross spillover index between variances and covariances given by

$$\begin{aligned} \mathcal{N}_t^{(G,cross)} &= \sum_{i \in I_{cov}} (\mathcal{T}_{t,i}^{(G,var)} - \mathcal{R}_{t,i}^{(G,var)}) \\ &= - \sum_{i \in I_{var}} (\mathcal{T}_{t,i}^{(G,cov)} - \mathcal{R}_{t,i}^{(G,cov)}) = \mathcal{S}_t^{(G,ccov)} - \mathcal{S}_t^{(G,cvar)}. \end{aligned} \tag{29}$$

This index decodes the total net exposure of all covariances vis-à-vis variance spillovers.

# 4 Empirics

## 4.1 Data

For our applications, we consider the same set of four key US asset classes as in Diebold and Yilmaz (2012): equity, fixed income, foreign exchange and commodities, yet on a larger sample. We study returns of the S&P 500 index, the 10-year treasury bond yields, the New York Board of Trade US dollar index futures, and the Dow-Jones/UBS commodity index obtained from Thomson Reuters Datastream. The sample period is from March 1, 1995, to December 31, 2014, with 5176 daily returns altogether. See Fig. 1 for an overview of the data.

## 4.2 Estimation of the BEKK model

To estimate the four-dimensional variance specification for the vector of asset returns, we use a modified version of the module 'arch_mg.src' that is part of the software *JMulti* (Lütkepohl and Krätzig, 2004, *http://www.jmulti.de/*). We verify the estimated parameters to correspond to a maximum of the log-likelihood function by multiplying each parameter estimate with 0.995 and 1.005 and checking the reductions of the log-likelihood. For inferential purposes, we use the estimates of the analytical ML and QML covariance matrices as provided in Hafner and Herwartz (2008). Given that the multivariate GARCH innovations $\xi_t$ are not multivariate Gaussian distributed, the QML covariance matrix is more reliable for diagnosing parameter significance.

Table 2 documents estimation results. Along with the coefficient estimates, we present ML and QML $t$-statistics. The BEKK parameters enter the conditional covariance $H_t$ in 'squared' form. Hence, the diagonal entries of the matrices $F$ and $G$ are in line with the common univariate GARCH estimates. In univariate applications to daily data, the news response parameter is often estimated to be about 0.05 which is close to the squared diagonal elements of $\hat{F}$. Similarly, in univariate GARCH models, the autoregressive parameter is often found to be around 0.95 which accords with the squared diagonal elements of $\hat{G}$. Apart from the autoregressive dynamics and the responses to own news, the BEKK model allows for cross equation covariance dynamics. Parameterized by the off-diagonal elements of $\hat{F}$ and $\hat{G}$, such cross equation dynamics are diagnosed significant at conventional levels. Evaluating the significance of ML (QML) $t-$ratios, we find five (one) out of twelve off-diagonal elements of $\hat{F}$ to differ from zero with 10% significance. Similarly, as regards the off-diagonal elements of $\hat{G}$, four ML and two QML diagnostics are significant at the 10% level, respectively. In addition to testing the parameters for significance, we find that a likelihood ratio test against the diagonal BEKK model strongly supports the presence of off-diagonal dynamics. Log-likelihood estimates for the unrestricted model and the diagonal BEKK model are, respectively, 69867.4 and 69825.1. Hence, in a test of the joint insignificance of the off-diagonal elements, the respective test statistic is 84.6, which is highly significant with respect to the critical values of a $\chi^2(24)$ distribution.[6]

Fig. 2 displays the estimated conditional standard deviations of the four asset classes (upper panel), and the six BEKK implied pairwise correlations (lower panel). The estimated conditional standard deviations reflect the typical features of volatility clustering. Starting with the subprime crises at the end of 2007, conditional standard deviations have accelerated over all asset classes (except foreign exchange). While conditional second-order moments of equity and fixed income indices are of similar magnitude until 2011, for the most recent part of the sample, fixed income risk turns out to be more pronounced in comparison with stock market volatility. Pairwise correlations in the lower panel of Fig. 2 show that the comovements of almost all asset classes exhibit strong time variation. With regard to the two asset classes with highest volatility on average, it turns out that the correlation between

[6]The 1% critical value of a $\chi^2(24)$ distribution is 42.98. The $\chi^2(24)$ distribution might only approximate the true distribution of the test statistic under the diagonal BEKK and violation of conditional normality. However, given the magnitude of the quasi LR statistic, it is most likely that its 'true' distribution under the restrictive model would also indicate significant off-diagonal dynamics at conventional nominal levels.

equity and fixed income markets is markedly negative (positive) in the beginning (at the end) of the sample period. The recent periods of turmoil starting in 2008 are, in particular, characterized by strongly negative correlations among foreign exchange markets and the remaining asset classes.

## 4.3 Descriptive conditional spillover analysis

### 4.3.1 The time-varying total variance spillover index

We start the analysis by considering the time evolution of the total spillover index (21) displayed in the upper panel of Fig. 3. The index is plotted along with a number of major political and economic events – see Table 1 for a compilation of the events and their dates of occurrence. This follows Diebold and Yilmaz (2009, 2012) and Yilmaz (2013); because our modeling approach allows us to compute the index at the daily frequency, however, we can exactly spot these events and analyze their impact to a degree of detail that is not feasible in rolling window applications. At the same time, we can study the long-term cyclical trends of variance spillovers in the 20 years of our sample. For the forecast horizon, we set $G = 5$, i.e., we consider about a week.

According to Fig. 3, the spillovers are very moderate between 1995 and 2001, hovering at or below 10%. Although events like the Thai Bhat devaluation (1), which is seen as the starting point of the Asian crisis, the Russian crisis (2), the first market disruptures at the beginning of the dot-com crisis, such as the April 14, 2000 NASDAQ crash (3), and the 09/11 twin-tower attacks (4) make the spillover index soar, they are rather short-lived and have no long-lasting impact on variance spillovers.

A first major period of increased variance spillovers can be detected in the forefront of the geopolitical tensions surrounding the pending US-led war in Iraq (5). At the outbreak of the war, the index spikes to unprecedented levels of 40%, after which it returns to previous levels. The most important period of increased interdependence and variance spillovers by far, however, is the crisis complex of the subprime mortgage crunch, the banking crisis, and the US recession from December 2007 to June 2009, all accompanied by extraordinary US central bank measures and by the political controversies over the impending US debt limits in 2011 and 2013.[7] Several incidents can be clearly distinguished: Freddie Mac's announcement that it would no longer take the worst subprime risks (6); the Northern Rock crisis (7); the Carlyle Capital Corporation's press release on failing to meet margin calls on one of its mortgage bond funds (8); the Lehman default (9). Between 2006 and the end of 2008, the index rises continuously from about 5% to about 25%, and remains at about these levels till the end of 2012. The overall climax of the index is reached in November 2011 with about 60%. Over this crisis, we also document the announcements of the major monetary policy measures of the Fed, later known as 'quantitative easing': the first program to purchase the direct obligations of housing-related government-sponsored enterprises announced in November 2008 (10); the expansion of the program to buy long-term Treasury securities of November 2010, (11); the operation 'twist' to influence the term structure of interest rates (13); the open-ended bond purchasing program of agency mortgage-backed securities of September 2012 (14). It is remarkable that despite their exceptional nature, none of these policy announcements has any visible, ameliorating impact on variance spillovers. It is only at the end of 2012 that the spillover index levels start to retreat. Interestingly, the political debates about the US fiscal cliffs in 2011 (12) and 2013 (15) are also hardly detectable in the graph.

As we have argued above, using the symmetric eigenvalue decomposition of the contemporaneous covariance $\Sigma_t$ for identification deserves further economic underpinnings. As a first justification of our identification scheme, consider the lower part of Fig. 3, which displays a 20-days moving average of the daily US Economic Policy Uncertainty Index (EPUI) of S. R. Baker, N. Bloom, and S. J. Davis. This index measures policy-related economic uncertainty as derived from newspaper coverage

[7] For these dates, we borrow from a time-line of events published on the website of the Federal Reserve Bank at St. Louis at https://www.stlouisfed.org/financial-crisis/full-timeline and the press releases linked to this site.

of policy-related economic uncertainty, from expiring US federal tax code provisions and from disagreement among economic forecasters.[8] For illustrational reasons, the index is scaled, such as to have the same standard deviation as the total spillover index, and it is reflected along the horizontal axis.

The similarity between both graphs is striking. The total spillover index moves almost in a one-to-one fashion with the moving average of the EPUI. While the amplitudes may differ in detail, both indices exhibit the same long-term trends as well as very similar reactions to the events singled out and discussed above. The correlation among both indices is indeed very high: 57%. Hence, the symmetric eigenvalue decomposition of $\Sigma_t$ supports the detection of an economically well-founded index of variance spillovers.

Despite the one week ahead forecasts, the graph of the spillover index in Fig. 3 is 'in-sample' since the underlying parameter estimates are obtained from the full sample. For real-time applications, it is natural to ask how much the spillover graph would change if one worked in a framework that was entirely ex ante. To explore this question, we employ rolling subsamples that comprise 1500 return observations each to estimate the BEKK model as described in Section 4.2. To economize on computation time, the windows are shifted only every 250 observations after each estimation. For given parameter estimates, the covariance paths $H_t$, $t = 1501, 1502, \ldots, T$, are determined by updating the variance-covariance dynamics with the observed time series innovations $\varepsilon_t$.

In Fig. 4, we superimpose such a fully ex ante spillover plot with the previous graph of Fig. 3. Due to parameter variations, we find moderate deviations between the two indices, in particular between 2006 and 2007 and in 2014. Overall, however, we find strong agreement between the two indices.

[8] Data and methodological details can be found on *http://www.policyuncertainty.com/index.html*.

### 4.3.2 Further decompositions of the total spillover index

What drives the total variance index? This and related questions can be answered by studying the subindices of Section 3.2. In Fig. 5, we decompose the total index into own and cross (co)variance spillovers.

Two observations are evident. First, the major features of the total spillover index are traced out by the own covariance ($\mathcal{S}_t^{(G,ocov)}$), the cross covariance ($\mathcal{S}_t^{(G,ccov)}$) and the cross variance spillover graphs ($\mathcal{S}_t^{(G,cvar)}$). All three are approximately of equal size, their fluctuations are highly correlated and their paths are very much akin to the total index. Therefore each of them reflects very similar information as the EPUI. Second, the own variance spillover index ($\mathcal{S}_t^{(G,ovar)}$) is markedly different from the other three series. This is remarkable because the own variance spillover index corresponds to what the standard variance spillover literature, which ignores covariances when computing the total index, would report as the total variance spillover index.[9] While similar observations are also made in Fengler and Gisler (2015), in our BEKK model with fully specified covariance dynamics, this discrepancy is even more eye-catching. It suggests that most of the systemic interdependence is propagated through the joint variance-covariance dynamics rather than the variance dynamics alone. This interpretation is also confirmed by comparing the reactions of the various indices to the selected events discussed in the previous section. Nevertheless, the plot also reveals that the net exposure of all covariances vis-à-vis the variances (recall that $\mathcal{N}_t^{(G,cross)} = \mathcal{S}_t^{(G,ccov)} - \mathcal{S}_t^{(G,cvar)}$) is negative on average; thus, overall, the covariances receive more spillovers from the variances than they transmit back.

As a more disaggregated decomposition, we present in Fig. 6 the asset-specific net exposures of variances $\mathcal{N}_{t,i}^{(G,var)}$ and covariances $\mathcal{N}_{t,i}^{(G,cov)}$. The top left panel reveals that generally stock markets as well as bonds are transmitters of variance spillovers. Whereas they are of about equal size in the first half of the sample, the bond net variance spillovers dominate since 2003. They are particularly strong

[9] Note, however, that the absolute scales are different, because in a spillover analysis with $N$ assets without covariances, one has $N^* = N$ instead of $N^* = N(N+1)/2$ as scaling constant.

from 2010 to 2012, in which time the stock markets even become net receivers of variance spillovers. It should be noted that this period falls into the times when the Fed adopted extraordinary monetary measures to influence the bond markets – see Table 1. It therefore appears natural that bond markets are positive net transmitters of variance spillovers. Referring to our discussions in Section 3.2, we read these characteristics as supportive evidence for the identification scheme based on the symmetric eigenvalue decomposition.

In contrast to stocks and bonds, over the entire sample, the commodity market is a net transmitter and the foreign exchange market a net receiver of variance spillovers (top right element in Fig. 6). Moreover, both net variance spillovers exhibit pronounced trends from 1995 to about 2010/2012, which reflects their increasing importance for investors as asset classes. The net receiver position of the foreign exchange market becomes particularly pronounced from 2008 onward. Because the Fed's quantitative easing programs that were effective since then, as a side effect, tended to weaken the dollar against other major currencies, the net receiver position of the foreign exchange market is again economically plausible, which supports the adopted identification scheme.

Finally, the lower panel of Fig. 6 shows the net covariance spillovers $\mathcal{N}_{t,i}^{(G,cov)}$. Overall, they fluctuate around zero, but with deviations of about two percent around zero, they are of smaller size than the variance net spillovers $\mathcal{N}_{t,i}^{(G,var)}$. This implies, interestingly, that covariance spillovers – in contrast to the variance spillovers – are generally less asymmetric among the different asset classes.

### 4.4 Informational content of time-$t$ conditional variance spillovers: the case of value-at-risk predictions

The indices of spot variance transmission introduced in Section 3 might be used for a real time (daily) assessment of potential threats to risk monitoring. In the following, we assess whether the proposed indices of risk transmission can contribute to an informationally efficient evaluation of portfolio risks.

#### 4.4.1 Value-at-risk

In financial practice and particularly in risk management, GARCH models have become a standard econometric tool to evaluate risk measures such as the value-at-risk (VaR) – see Andersen et al. (2013) for a discussion of GARCH-based approaches to quantify the VaR. For a portfolio with shares $w_t$ and conditional on time $t-1$, the VaR at level $\alpha$ is the (negative) return quantile

$$\mathrm{VaR}_t^{(\alpha)} = -q_\epsilon(\alpha)s_t \; , \tag{30}$$

where $s_t$ is the conditional standard deviation, $s_t = \sqrt{w_t' H_t w_t}$, and $q_\epsilon(\alpha)$ is the empirical $\alpha$-quantile of standardized portfolio returns $\{\epsilon_t = w_t' r_t / s_t\}_{t=1}^T$. We consider three portfolios with time-invariant composition $w_t = w$, namely (i) a portfolio assigning equal weight to all asset classes: equity, bonds, foreign exchange and commodities (denoted by ewp); (ii) a portfolio consisting only of equity (eqp); and (iii) a portfolio assigning equal weight to all asset classes except equity (noeq). In addition, we consider the minimum variance portfolios (mvp) with portfolio weights $w_t = H_t^{-1}\mathbf{1}/c$, where $c = \mathbf{1}' H_t^{-1}\mathbf{1}$ and $\mathbf{1}$ is a four-dimensional vector of ones (see, e.g., Campbell et al., 1997, Chap. 5).

#### 4.4.2 Value-at-risk diagnosis

The backtesting of VaR estimates relies on the series of binary auxiliary variables, so-called VaR hits,

$$\tilde{y}_{t,\alpha} = I(w_t' r_t \leq -\mathrm{VaR}_t^{(\alpha)}) \; . \tag{31}$$

An unconditionally valid risk assessment requires that the mean of the hit process in (31) be $\alpha$. For an informationally efficient risk assessment, it is required that conditional on time $t-1$ information, deviations of the hit process $\tilde{y}_{t,\alpha}$ from its unconditional expectation $\alpha$ be first-order unpredictable. For an assessment of the VaR estimates, we apply the dynamic quantile (DQ) test introduced in Engle and Manganelli (2004), because informational efficiency can be tested within this framework in a straightforward manner. Under the null hypothesis of the DQ test, the

VaR model is (conditionally and unconditionally) well specified. Specifically, it is tested whether the centered hits, $y_{t,\alpha} = \tilde{y}_{t,\alpha} - \alpha$, follow a martingale difference sequence conditional on information that is available in time $t-1$. Established indicators of informational inefficiency comprise the history of the VaR hit process $y_{t-i,\alpha}$, $i = 1, 2, \ldots, k$. In the present framework, it is natural to regard (lagged) indices of variance transmission as indicators of informational inefficiency of the VaR. In summary, we consider the following DQ regression model

$$y_{t,\alpha} \;=\; \beta_0 + \sum_{k=1}^{5} \beta_k y_{t-k,\alpha} + x'_{t-1}\boldsymbol{\delta} + e_t\,, \tag{32}$$

where the $Q$-dimensional vector $x_{i,t-1}$ collects predetermined measures of risk transmission as introduced in Section 3.[10] The null hypothesis of correct conditional and unconditional coverage of the model reads as $H_0 : \beta_k = 0,\ \forall\, k = 0, 1, \ldots, 5,$ and $\delta_q = 0,\ \forall\, q = 1, 2, \ldots, Q$. Because the regression is specified for centered binary variables, the significance of $\hat{\beta}_0$ indicates in a separate test that the VaR model violates the unconditional coverage criterion. We assess the martingale property for the hit processes derived from three levels of VaR coverage, namely $\alpha = 0.010, 0.025$ and $\alpha = 0.050$.

Table 3 documents some diagnostic results for the standardized portfolio returns (innovations) $\epsilon_t = w'_t r_t/(w'_t H_t w_t)$. In case of a valid specification of the dynamic covariance process, these innovations should have mean zero and unit variance and should not indicate any kind of non-modeled or remaining conditional heteroskedasticity. The documented moments of $\epsilon_t$ show that for almost all portfolios, the standard error of portfolio innovations is close to unity. As the only exception, the second-order moments of the equity portfolio have a standard error of 0.95.

[10] While DQ regressions have turned out to dominate rival VaR diagnostics in terms power against misspecified VaRs (Berkowitz et al., 2011), their implementation relies on the binary hit processes $\tilde{y}_{t,\alpha}$. As it is visible from (31) their determination comes along with a substantial loss of information. Against this background, the VaR diagnostic introduced by Gaglianone et al. (2011) promises further power improvements in comparison with the DQ-test, since it addresses directly the conditional validity of the quantile $\text{VaR}_t^{(\alpha)}$. Our empirical results, however, suggest that the DQ test based on the spillover indices is sufficiently powerful as a diagnostic checking of the VaR.

Higher order moments reveal some negative skewness of portfolio innovations, and the fourth-order moments are between four and five for all considered portfolios.

For some portfolios, we diagnose patterns of remaining heteroskedasticity in return innovations. In particular, the portfolios including equity show significant ARCH-LM diagnostics at order five, while low-order diagnostics do not indicate departures from an iid distribution. This may reflect the presence of outliers or the fact that the BEKK model is symmetric, i.e., positive and negative shocks of a given magnitude impact symmetrically on the conditional second-order moments. In summary, both in-sample and ex ante portfolio innovations indicate that the employed four-dimensional BEKK model is largely suitable for extracting second-order characteristics of portfolio returns.

The DQ diagnostics are shown in Tables 4 to 6. Subjecting the model-implied VaR estimates to the DQ tests shows that for most portfolios, the process of VaR hits does not exhibit significant serial patterns. Autoregressive models of order five mostly lack any significant explanatory content for the process of centered VaR hits. Moreover, the mean hit frequencies are in line with the nominal VaR levels.[11] The purely autoregressive DQ test regression indicates with 5% significance some misspecification of the risk model applied to in-sample portfolios that include equity components. However, since the full sample covers a period of almost 20 years, the significance of the DQ statistics might be due to violations of model stability.

While the standard DQ diagnostic indicates accuracy of the employed risk models, the indices of risk transmission carry predictive content for the dynamic patterns of centered VaR hits. For the smallest coverage level ($\alpha = 0.01$), overall, four spillover indices (the receiver variance $\mathcal{R}_t^{(G,var)}$, receiver covariance $\mathcal{R}_t^{(G,cov)}$, transmitter covariance $\mathcal{T}_t^{(G,cov)}$, and net variance $\mathcal{N}_t^{(G,var)}$) contribute significantly to the explanation of the occurrence of overly negative returns. By means of these indicators, risk model misspecifications can be diagnosed at the 5% significance level

[11]In Tables 4 to 6, we do not document specific tests of the unconditional coverage. However, all $t$-statistics of the intercept estimates $\hat{\beta}_0$ of the DQ regression indicate insignificance at conventional levels. Detailed results on testing for unconditional coverage are available upon request.

for the equal weight and equity portfolio (ex ante), the portfolio excluding equity (in-sample and ex ante) and the minimum variance portfolio (in-sample). At the coverage level of $\alpha = 0.025$, VaR dynamics for the portfolio excluding equity (in-sample) are significantly explained by particular spillover indices. For the highest VaR level ($\alpha = 0.05$), the equal weight portfolios (ex ante) indicate misspecification of the risk model.

The DQ diagnostics show that the VaR estimates may suffer from violations of informational efficiency, but the evidence is only mildly specific on the particular indices of variance transmission that are most informative for the process of VaR hits. In this context, it is worthwhile to recall that the DQ tests indicate joint significance (or insignificance) of both the autoregressive patterns and the spillover statistics. To shed light on the marginal explanatory content of the autoregressive parameters on the one hand, and the indices of variance transmission on the other hand, the lower panels of Tables 4 to 6 document statistics that are derived from the marginal degrees of explanation in the DQ regressions. To be specific, we provide $\sqrt{R^2}$ statistics for regressions such that VaR hits are either explained by autoregressive patterns or by the lagged variance spillover indices.[12] The largest statistics are printed in bold. Purely autoregressive DQ regressions which have higher explanatory content than all regressions that condition exclusively on variance indices are rare (three out of thirty). For the vast majority of the regressions, the highest degrees of explanation are obtained by conditioning on indices of variance spillover.[13]

Distinguishing the particular indices with highest explanatory content, it turns out that generally covariance spillovers (in comparison to variance spillovers) have some lead in explaining the VaR hit processes. Similarly, indices of variance trans-

[12]The $R^2$ statistics are unadjusted for the number of explanatory variables. Given the size of the sample, numerical differences between adjusted and unadjusted degrees of explanation are negligible.

[13]Relating the $TR^2$ with the critical values from $\chi^2$ distributions of either four or six degrees of freedom, we find that about one third of the $7 \times 5 \times 2 \times 3$=210 statistics (seven groups of spillover indices, five portfolios, in-sample vs ex ante, three nominal VaR levels) are significant at the 5% level.

mission have stronger explanatory content in comparison to indices of variance reception. From an overall perspective, the measure of net covariance transmission ($\mathcal{N}_t^{(G,cov)}$) appears to have the highest explanatory content. Of all employed groups of VaR hit indicators for 11 (out of 30) regression models (portfolios), this set of indicators obtains the highest marginal degree of explanation in DQ regressions.

In particular, we find that the overall index of variance spillovers ($\mathcal{S}_t^{(G)}$) has only minor content to unravel informational inefficiency of the VaR estimates. As it is visible for the case of the minimum variance portfolios, the horizon $G$ to which the spillover statistics refer has only negligible impact on the DQ diagnosis. For the alternative choices $G = 1$ and $G = 5$, the inferential outcomes and degrees of explanation are rather close to one other.

## 5 Concluding remarks

In this work, we have proposed a variance spillover index which is derived from the forecast-error variance decomposition of the squared returns of a multivariate GARCH model of the BEKK class. On this basis, our variance spillover indices are time-$t$ conditional and take full advantage of time-varying covariance information.

Empirically, we study a system of four major US asset classes. As they are highly responsive to the news innovation process, our indices allow one to simultaneously study the immediate impact of singular, surprising events at the daily frequency, the implications of lingering times of political and economic uncertainty, and long-term secular trends of market interdependence. In an application to risk management, we demonstrate that the indices are informative about the likelihood of value-at-risk violations. Hence, they are not only of interest in a descriptive-analytic sense, but also for predictive purposes in standard problems of empirical finance.

Analytically, our approach is attractive because we rely on the very general vec representation of the multivariate GARCH model. It is therefore not limited to the BEKK class: any multivariate GARCH model having a vec representation could be treated in this way. As a potential shortcoming, the curse of dimensionality could be cited as a problem known to afflict such models. Using covariance targeting strategies and suitable parameter restrictions, one might shift the limits further than we do. We leave this for future research.

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## Tables and figures

| Event | Date | Description |
|---|---|---|
| (1) | Jul 02, 1997 | Thai Bhat devaluation, start of the 1997 Asian crisis |
| (2) | Aug 17, 1998 | Russia defaults on domestic debt, start of Russian crisis |
| (3) | Apr 14, 2000 | NASDAQ crash, dot-com crisis till March 2003 |
| (4) | Sep 11, 2001 | 09/11 attacks |
| (5) | Mar 20, 2003 | US-led war in Iraq |
| (6) | Feb 27, 2007 | Freddie Mac refuses to take worst subprime risks |
| (7) | Sep 14, 2007 | Northern Rock crisis |
| (8) | Mar 05, 2008 | Carlyle Capital Corp. fails to meet margin calls on a mortgage bond fund |
| (9) | Sep 15, 2008 | Lehman Brothers default |
| (10) | Nov 25, 2008 | Quantitative easing 1: Fed buys mortgage-backed securities |
| (11) | Nov 03, 2010 | Quantitative easing 2: Fed buys long-term Treasury bonds |
| (12) | Aug 01, 2011 | Fiscal cliff 2011: House passes 2011 debt ceiling bill |
| (13) | Sep 21, 2011 | Fed announces operation Twist |
| (14) | Sep 13, 2012 | Quantitative easing 3: open-ended bond purchasing program of agency mortgage-backed securities |
| (15) | Jul 22, 2013 | Fiscal cliff 2013: House passes 2013 Continuing Appropriations Act |

Table 1: Calendar of political and economic events highlighted in Figs. 3, 4, and 5.

| $\hat{C}$ | | | | $\hat{F}$ | | | | $\hat{G}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.16E[-03] | - | - | - | .2685 | .0325 | −.0115 | .0045 | .9562 | −.0031 | .0048 | −.0008 |
| (7.73) | | | | (13.4) | (1.40) | (−1.94) | (0.40) | (144.8) | (−0.42) | (2.78) | (−0.25) |
| −5.28E[-04] | 1.04E[-03] | - | - | .0063 | .2332 | .0034 | .0004 | .0011 | .9699 | −.0013 | .0006 |
| (−1.93) | (3.84) | | | (0.81) | (9.82) | (0.86) | (0.05) | (0.52) | (147.2) | (−1.31) | (0.36) |
| −1.13E[-04] | 2.31E[-04] | 2.78E[-04] | - | .0519 | −.0230 | .1406 | −.0309 | −.0050 | −.0023 | .9856 | .0067 |
| (−1.25) | (2.08) | (3.52) | | (1.63) | (−0.56) | (13.26) | (−1.51) | (−0.69) | (−0.22) | (394.0) | (1.89) |
| −1.70E[-04] | −1.70E[-05] | 1.05E[-04] | 5.29E[-04] | −.0067 | −.0040 | −.0083 | .1408 | .0057 | −.0018 | −.0012 | .9884 |
| (−0.94) | (−0.07) | (0.58) | (3.16) | (−0.35) | (−0.12) | (−0.93) | (10.7) | (1.18) | (−0.27) | (−0.66) | (353.0) |

Table 2: The table documents QML parameter estimates of the unrestricted BEKK model. Entries in parentheses and square brackets are the ML and QML $t$-ratios determined by means of the analytical derivatives as provided in Hafner and Herwartz (2008). See also the module '*arch_mg.src*' implemented in *JMulTi* (Lütkepohl and Krätzig, 2004).

| | In-sample BEKK estimates | | | | Ex ante BEKK estimates | | | |
|---|---|---|---|---|---|---|---|---|
| | ewp | eq | noeq | mvp | ewp | eq | noeq | mvp |
| $G$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| std($\epsilon$) | 0.996 | 0.993 | 0.993 | 1.011 | 0.994 | 0.950 | 1.009 | 0.989 |
| $\overline{\epsilon^3}$ | -0.214 | -0.457 | -0.018 | -0.202 | -0.185 | -0.403 | 0.004 | -0.114 |
| $\overline{\epsilon^4}$ | 4.217 | 4.692 | 4.045 | 4.294 | 4.273 | 4.112 | 4.613 | 4.005 |
| LM(1) | 0.213 | 0.112 | 0.283 | 0.015 | 0.276 | 0.245 | 0.255 | 0.040 |
| LM(5) | 0.001 | 0.002 | 0.274 | 0.089 | 0.007 | 0.012 | 0.289 | 0.129 |

Table 3: The table shows the descriptive statistics of the standardized portfolio returns (standard deviation, third and fourth-order moment, ARCH-LM tests of order 1 and 5). Four portfolio compositions are considered: equal weight (ewp), only equity (eqp), equal weight without equity (noeq), minimum variance portfolio (mvp). Second-order characteristics of portfolio returns are evaluated by means of full sample information (left-hand side), or using ex ante BEKK parameters determined by means of rolling sample windows.

| | In-sample BEKK estimates | | | | | Ex ante BEKK estimates | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | ewp | eq | noeq | mvp | | ewp | eq | noeq | mvp | |
| $G$ | 1 | 1 | 1 | 1 | 5 | 1 | 1 | 1 | 1 | 5 |
| | Dynamic Quantile (DQ) diagnostics ($p$-values times 100) | | | | | | | | | |
| AR | 0.018 | 0.614 | 37.38 | 7.812 | | 10.43 | 5.857 | 15.95 | 49.32 | |
| $\mathcal{R}_t^{(G,var)}$ | 0.122 | 2.461 | 32.25 | 3.253 | 3.272 | 28.50 | 10.40 | 12.17 | 47.70 | 45.70 |
| $\mathcal{R}_t^{(G,cov)}$ | 0.007 | 0.199 | 9.082 | 9.593 | 9.855 | 14.90 | 1.869 | 11.75 | 48.39 | 47.29 |
| $\mathcal{T}_t^{(G,var)}$ | 0.013 | 1.233 | 4.958 | 7.746 | 8.091 | 8.704 | 21.93 | 8.489 | 41.33 | 50.91 |
| $\mathcal{T}_t^{(G,cov)}$ | 0.006 | 0.361 | 1.296 | 6.647 | 6.451 | 2.738 | 5.658 | 2.614 | 50.26 | 57.79 |
| $\mathcal{N}_t^{(G,var)}$ | 0.034 | 0.828 | 15.21 | 3.570 | 3.986 | 6.588 | 14.76 | 7.640 | 39.00 | 47.12 |
| $\mathcal{N}_t^{(G,cov)}$ | 0.029 | 1.075 | 1.691 | 2.484 | 3.374 | 18.75 | 2.893 | 10.89 | 36.97 | 35.51 |
| $\mathcal{S}_t^{(G)}$ | 0.040 | 0.806 | 42.10 | 12.39 | 12.40 | 16.06 | 8.564 | 18.87 | 60.28 | 60.99 |
| | Marginal $\sqrt{R^2}$ (times 100) | | | | | | | | | |
| AR | **7.276** | **5.996** | 3.578 | 4.781 | | 5.507 | 5.927 | 5.002 | 3.900 | |
| $\mathcal{R}_t^{(G,var)}$ | 2.298 | 1.980 | 3.243 | 4.565 | 4.560 | 2.503 | 3.104 | 4.170 | 3.718 | 3.827 |
| $\mathcal{R}_t^{(G,cov)}$ | 5.267 | 5.067 | 4.991 | 4.222 | 4.190 | 4.409 | **5.978** | 4.971 | 4.450 | 4.518 |
| $\mathcal{T}_t^{(G,var)}$ | 4.159 | 2.969 | 4.868 | 3.725 | 3.682 | 4.193 | 2.068 | 4.289 | 4.011 | 3.583 |
| $\mathcal{T}_t^{(G,cov)}$ | 5.383 | 4.718 | **6.148** | 4.580 | 4.601 | **6.004** | 5.033 | **6.137** | 4.384 | 4.058 |
| $\mathcal{N}_t^{(G,var)}$ | 3.619 | 3.275 | 4.017 | 4.463 | 4.371 | 4.682 | 2.713 | 4.510 | 4.113 | 3.767 |
| $\mathcal{N}_t^{(G,cov)}$ | 4.705 | 3.989 | 6.015 | **5.381** | **5.152** | 4.334 | 5.642 | 4.910 | **4.962** | **5.050** |
| $\mathcal{S}_t^{(G)}$ | 0.107 | 1.224 | 1.224 | 0.032 | 0.019 | 0.387 | 1.538 | 1.130 | 0.439 | 0.123 |

Table 4: Inferential ($p$-values) and descriptive (marginal $R^2$) statistics from DQ regressions for VaR estimates with nominal coverage $\alpha = 1\%$. All DQ regressions include autoregressive dynamics up to order 5. Additional joint misspecification indicators included in enhanced DQ regressions are listed rowwise. The degrees of freedom for testing the null hypothesis of a conditionally valid VaR model are 6 for the purely autoregressive design, and 10 and 12, respectively, for enhanced DQ regressions comprising either (four) variance or (six) covariance spillovers. The lower panel documents the degrees of explanation for the autoregressive DQ model ('AR') and marginal degrees of explanation for the group of spillover indices achieved within enhanced DQ regressions. Entries in bold face indicate the group of DQ misspecification indicators with highest marginal explanatory content. $G$ is the horizon of variance spillovers. DQ regressions for in-sample and ex ante analysis comprise 5176 and 3676 observations, respectively. For further notes, see Table 3.

| | In-sample BEKK estimates | | | | | Ex ante BEKK estimates | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | ewp | eq | noeq | mvp | | ewp | eq | noeq | mvp | |
| $G$ | 1 | 1 | 1 | 1 | 5 | 1 | 1 | 1 | 1 | 5 |
| | Dynamic Quantile diagnostics ($p$-values times 100) | | | | | | | | | |
| AR | 1.435 | 3.757 | 68.00 | 32.72 | | 40.81 | 8.927 | 92.31 | 54.48 | |
| $\mathcal{R}_t^{(G,var)}$ | 3.730 | 15.25 | 36.29 | 9.118 | 9.409 | 31.78 | 17.11 | 48.15 | 39.35 | 32.46 |
| $\mathcal{R}_t^{(G,cov)}$ | 0.109 | 8.998 | 8.126 | 5.139 | 5.561 | 14.62 | 16.91 | 80.39 | 71.23 | 62.20 |
| $\mathcal{T}_t^{(G,var)}$ | 0.174 | 2.131 | 1.232 | 17.94 | 18.74 | 36.50 | 23.21 | 13.43 | 44.22 | 58.79 |
| $\mathcal{T}_t^{(G,cov)}$ | 0.085 | 4.937 | 0.175 | 17.19 | 15.91 | 16.65 | 20.83 | 8.368 | 55.19 | 28.98 |
| $\mathcal{N}_t^{(G,var)}$ | 0.510 | 5.157 | 3.710 | 11.27 | 11.41 | 15.37 | 10.18 | 12.24 | 57.78 | 73.02 |
| $\mathcal{N}_t^{(G,cov)}$ | 0.630 | 14.24 | 0.248 | 5.277 | 7.987 | 18.02 | 8.112 | 19.33 | 40.39 | 39.01 |
| $\mathcal{S}_t^{(G)}$ | 2.323 | 6.363 | 72.64 | 41.03 | 41.05 | 42.74 | 8.836 | 94.45 | 64.76 | 65.80 |
| | Marginal $\sqrt{R^2}$ (times 100) | | | | | | | | | |
| AR | 5.568 | **5.089** | 2.782 | 3.683 | | 4.083 | **5.488** | 2.318 | 3.700 | |
| $\mathcal{R}_t^{(G,var)}$ | 2.499 | 1.374 | 3.526 | 4.641 | 4.615 | 3.942 | 3.265 | 4.477 | 3.872 | 4.183 |
| $\mathcal{R}_t^{(G,cov)}$ | 5.740 | 3.406 | 5.323 | 5.593 | **5.541** | **5.504** | 4.192 | 3.921 | 3.259 | 3.682 |
| $\mathcal{T}_t^{(G,var)}$ | 4.855 | 3.850 | 5.894 | 3.957 | 3.910 | 3.417 | 2.614 | 5.901 | 3.693 | 3.060 |
| $\mathcal{T}_t^{(G,cov)}$ | **5.849** | 3.828 | **7.145** | 4.606 | 4.675 | 5.158 | 3.577 | **6.794** | 3.973 | **5.003** |
| $\mathcal{N}_t^{(G,var)}$ | 4.333 | 3.014 | 5.294 | 4.438 | 4.424 | 4.797 | 4.024 | 5.970 | 3.099 | 2.298 |
| $\mathcal{N}_t^{(G,cov)}$ | 4.977 | 2.801 | 6.916 | **5.595** | 5.293 | 5.347 | 5.006 | 6.093 | **4.490** | 4.552 |
| $\mathcal{S}_t^{(G)}$ | 0.544 | 0.267 | 0.868 | 0.735 | 0.735 | 1.228 | 2.393 | 0.860 | 0.543 | 0.256 |

Table 5: Inferential ($p$-values) and descriptive (marginal $R^2$) statistics of the dynamic quantile regressions for the hit processes of VaR estimates with nominal coverage $\alpha$ =2.5%. For further notes, see Table 4.

| | In-sample BEKK estimates | | | | | Ex ante BEKK estimates | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | ewp | eq | noeq | mvp | | ewp | eq | noeq | mvp | |
| $G$ | 1 | 1 | 1 | 1 | 5 | 1 | 1 | 1 | 1 | 5 |
| | Dynamic Quantile diagnostics ($p$-values times 100) | | | | | | | | | |
| AR | 0.026 | 0.006 | 90.28 | 31.17 | | 19.24 | 15.19 | 63.07 | 70.23 | |
| $\mathcal{R}_t^{(G,var)}$ | 0.048 | 0.037 | 80.29 | 13.75 | 13.90 | 0.467 | 8.438 | 11.30 | 22.76 | 12.26 |
| $\mathcal{R}_t^{(G,cov)}$ | 0.000 | 0.058 | 55.38 | 6.678 | 7.310 | 0.031 | 16.23 | 56.03 | 63.28 | 48.50 |
| $\mathcal{T}_t^{(G,var)}$ | 0.004 | 0.011 | 14.86 | 24.27 | 25.64 | 10.32 | 40.70 | 13.33 | 29.00 | 33.94 |
| $\mathcal{T}_t^{(G,cov)}$ | 0.001 | 0.075 | 8.177 | 7.936 | 6.249 | 1.336 | 19.16 | 25.70 | 47.30 | 26.63 |
| $\mathcal{N}_t^{(G,var)}$ | 0.003 | 0.045 | 34.14 | 21.00 | 21.89 | 2.443 | 23.21 | 8.391 | 37.76 | 58.45 |
| $\mathcal{N}_t^{(G,cov)}$ | 0.001 | 0.222 | 30.75 | 3.732 | 5.611 | 0.005 | 19.57 | 12.83 | 35.79 | 37.04 |
| $\mathcal{S}_t^{(G)}$ | 0.049 | 0.015 | 93.31 | 41.40 | 41.33 | 10.46 | 19.40 | 62.10 | 75.37 | 76.68 |
| | Marginal $\sqrt{R^2}$ (times 100) | | | | | | | | | |
| AR | **7.056** | **7.493** | 2.053 | 3.714 | | 4.801 | **5.041** | 3.373 | 3.209 | |
| $\mathcal{R}_t^{(G,var)}$ | 3.576 | 2.483 | 2.888 | 4.205 | 4.194 | 6.697 | 4.692 | 5.494 | 5.207 | **5.775** |
| $\mathcal{R}_t^{(G,cov)}$ | 6.776 | 3.267 | 4.162 | 5.304 | 5.240 | 8.459 | 4.633 | 4.102 | 4.226 | 4.770 |
| $\mathcal{T}_t^{(G,var)}$ | 4.996 | 3.502 | 4.969 | 3.524 | 3.460 | 4.179 | 2.048 | 5.539 | 4.871 | 4.697 |
| $\mathcal{T}_t^{(G,cov)}$ | 6.186 | 2.948 | **5.850** | 5.166 | 5.337 | 6.339 | 4.221 | 5.528 | 4.767 | 5.585 |
| $\mathcal{N}_t^{(G,var)}$ | 5.269 | 2.302 | 4.287 | 3.731 | 3.682 | 5.569 | 3.470 | 5.933 | 4.563 | 3.734 |
| $\mathcal{N}_t^{(G,cov)}$ | 6.556 | 1.967 | 4.886 | **5.704** | **5.438** | **9.280** | 4.486 | **6.092** | **5.264** | 5.201 |
| $\mathcal{S}_t^{(G)}$ | 0.607 | 0.074 | 0.780 | 0.322 | 0.348 | 2.474 | 1.568 | 1.832 | 1.124 | 0.964 |

Table 6: Inferential ($p$-values) and descriptive (marginal $R^2$) of the dynamic quantile regressions for the hit processes of VaR estimates with nominal coverage $\alpha$ =5.0%. For further notes, see Table 4.

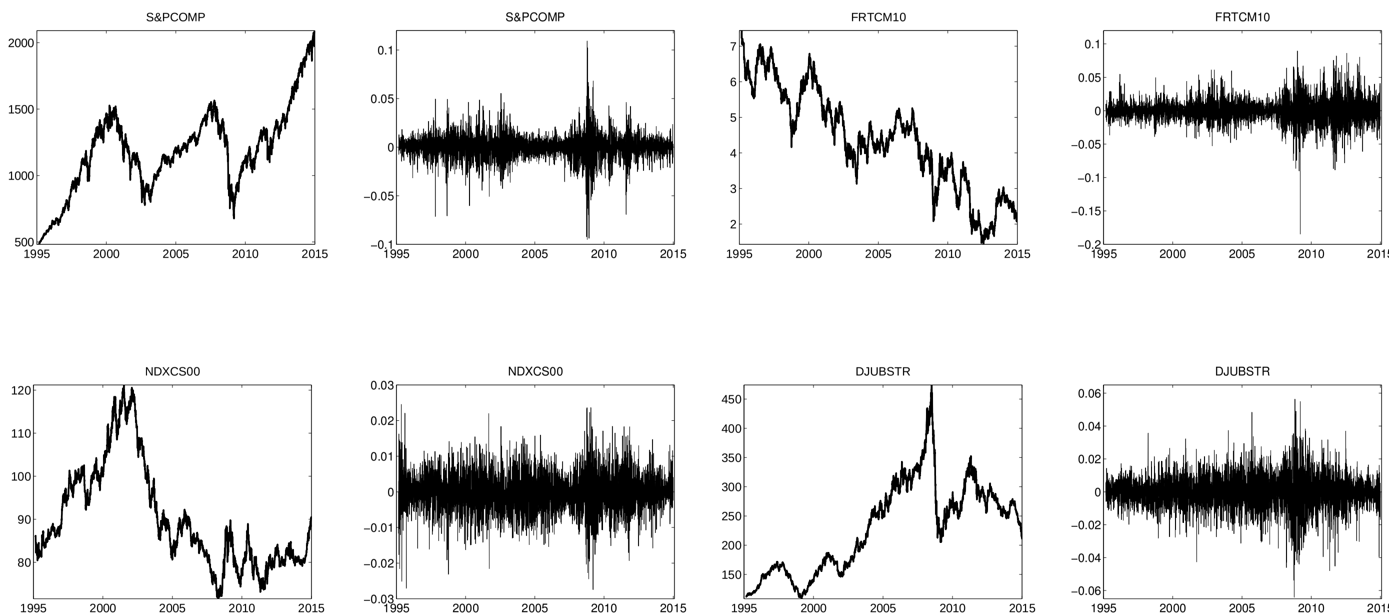
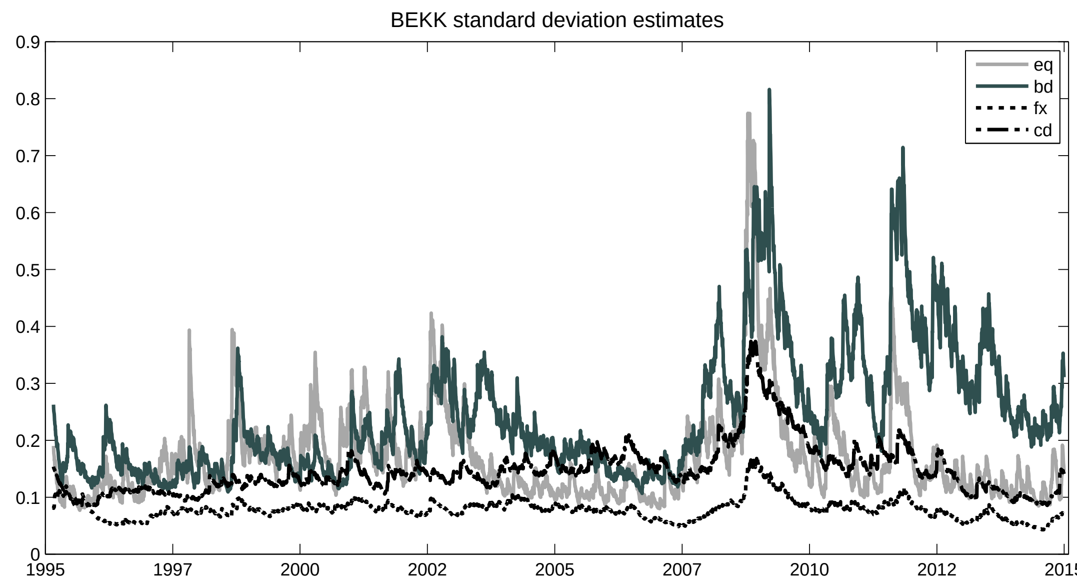
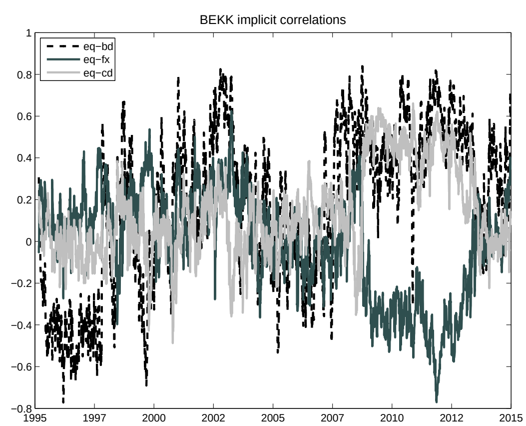


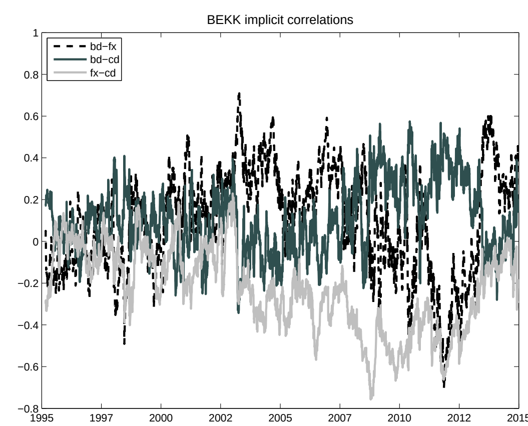

Figure 1: Price levels and log returns of the S&P 500 index (top left panel), the 10-year treasury bond yields (top right panel), the New York Board of Trade US dollar index futures (lower left panel) and the Dow-Jones/UBS commodity index (lower right panel). Sample from March 1, 1995, to December 31, 2014.




(a)




(b)




(c)

Figure 2: Estimated BEKK implicit correlations of the four asset classes: equity (eq), bonds (bd), foreign exchange (fx), and commodities (cd). Sample from March 1, 1995, to December 31, 2014.

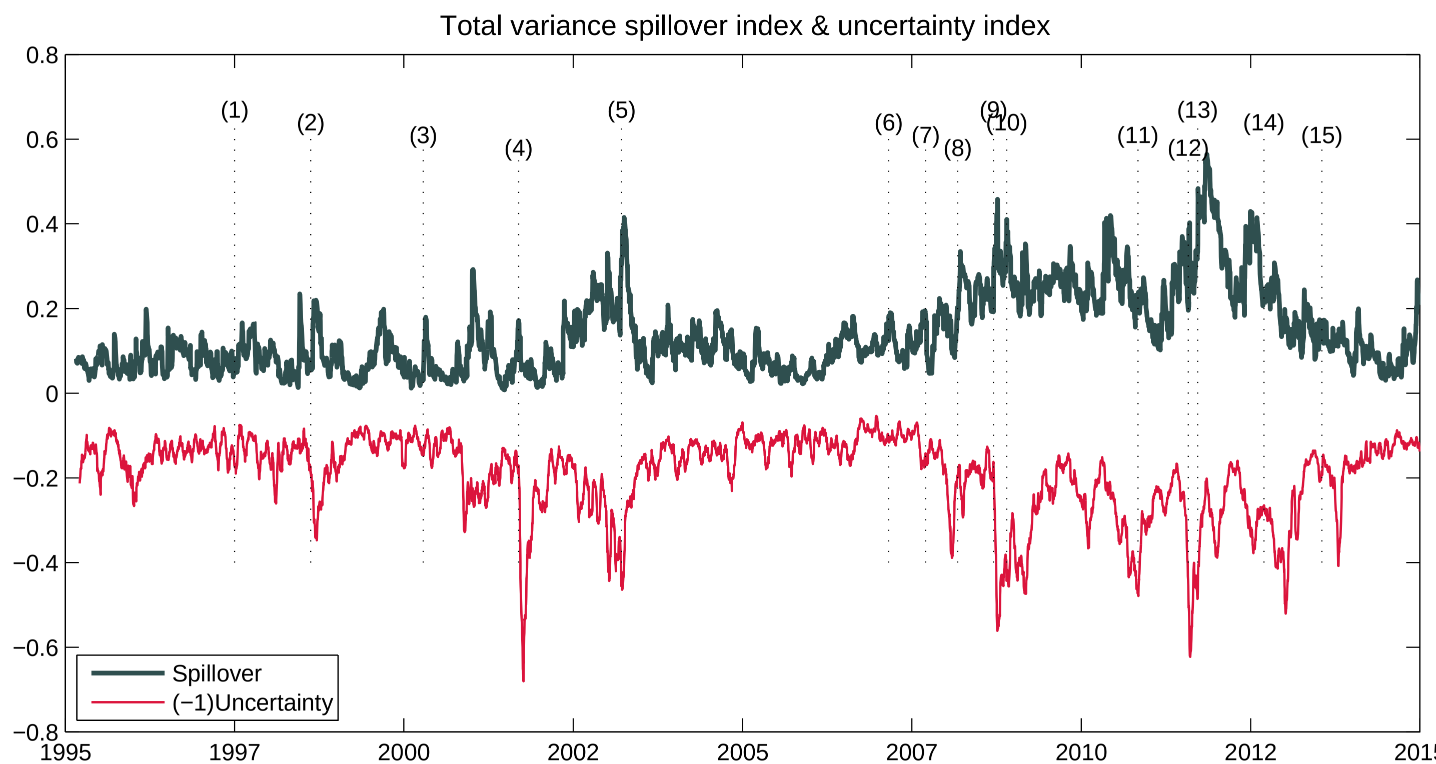



Figure 3: Total variance spillover index $\mathcal{S}_t^{(5)}$ and the 20-days moving average of the daily US Economic Policy Uncertainty Index (EPUI) of S. R. Baker, N. Bloom, and S. J. Davis. The EPUI is scaled (has the same standard deviation as the total variance spillover index) and reflected along the horizontal axis. See Table 1 for a listing of the events tagged in the plot.

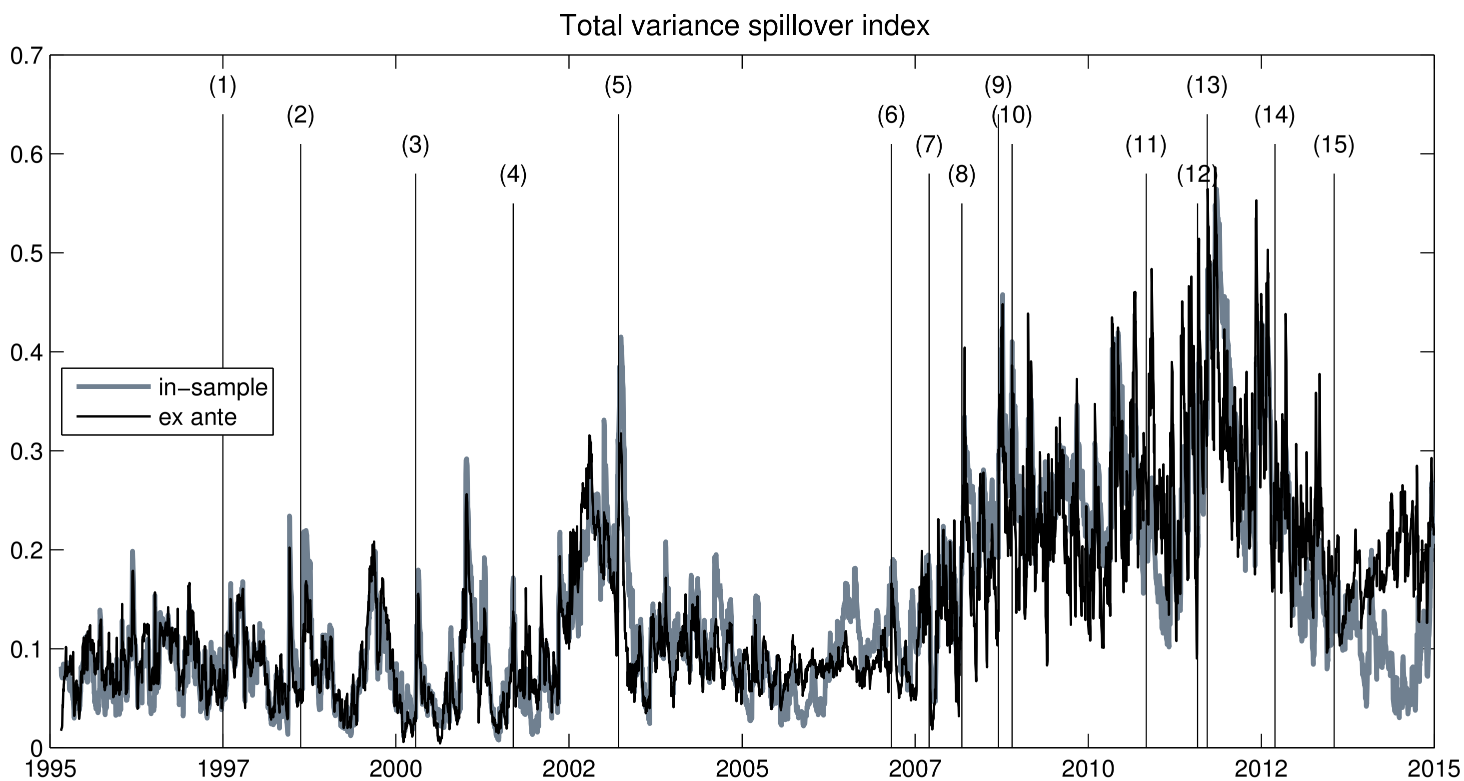



Figure 4: Total variance spillover index $\mathcal{S}_t^{(5)}$ of Figure 3 (light grey) and a fully ex ante total variance spillover index based on rolling window estimates (black line). See Table 1 for a listing of the events tagged in the plot.

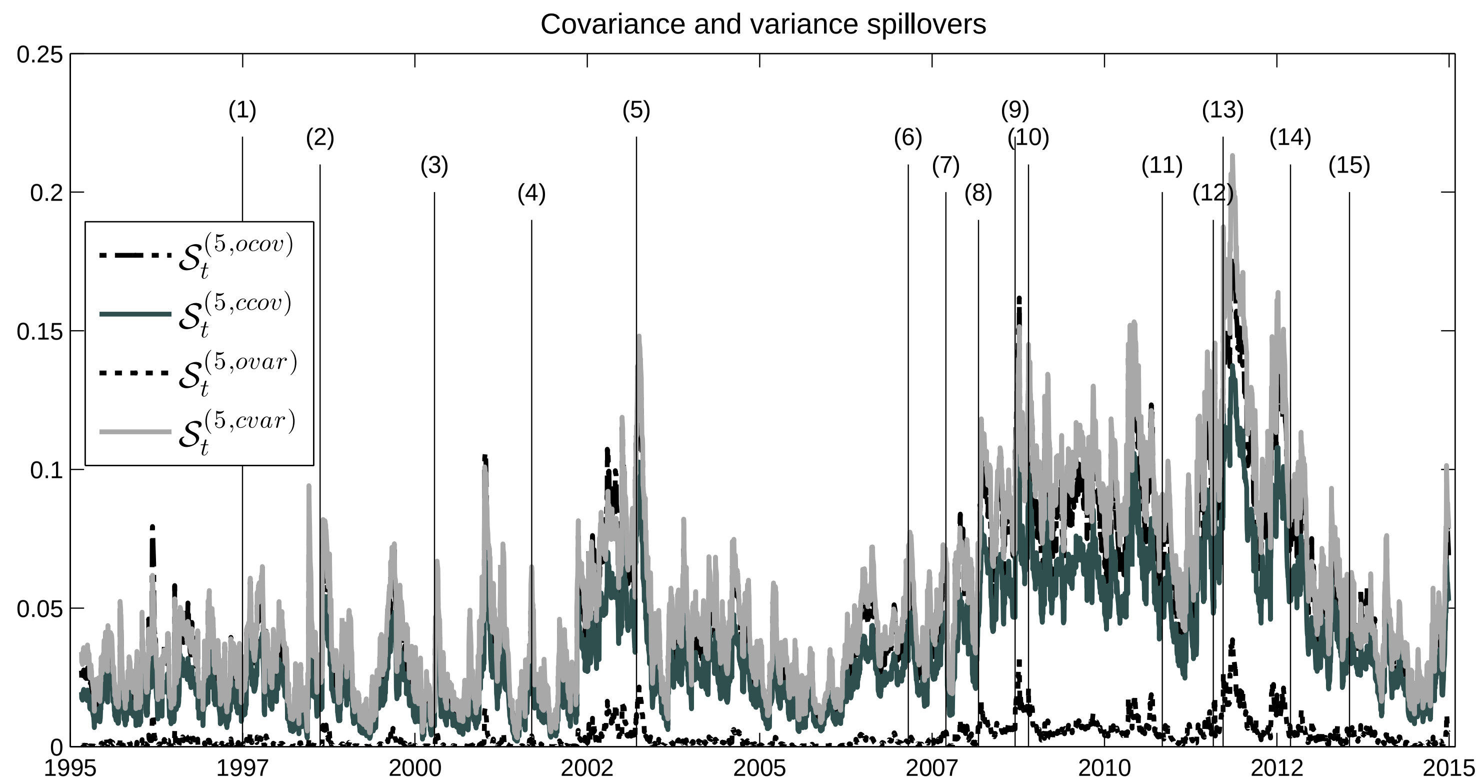



Figure 5: Own and cross variance and covariance spillovers. See Table 1 for a listing of the events tagged in the plot.

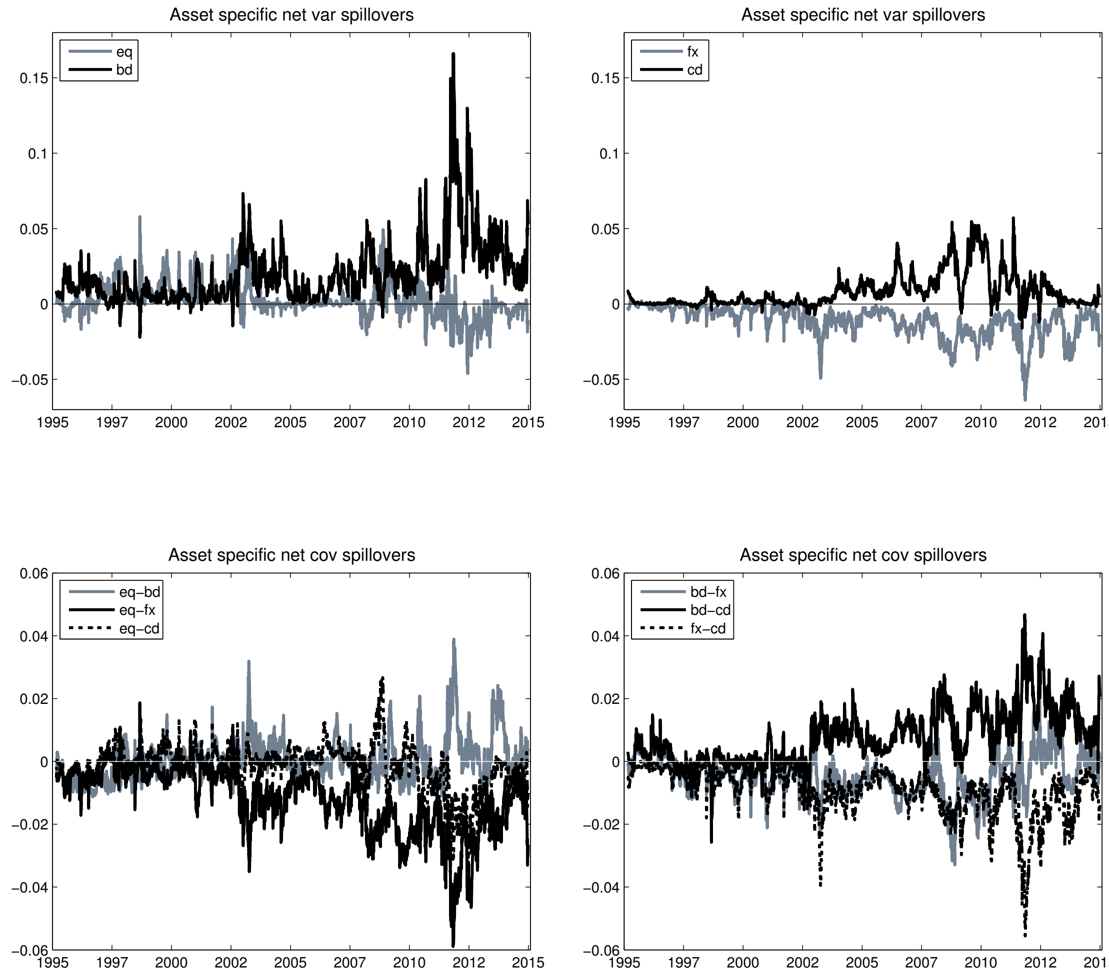



Figure 6: Net variance $\mathcal{N}_{t,i}^{(5,var)}$ (top panel) and net covariance $\mathcal{N}_{t,i}^{(5,cov)}$ (lower panel) spillovers aggregated per asset class: equity (eq), bonds (bd), foreign exchange (fx), and commodities (cd). Sample from March 1, 1995, to December 31, 2014.