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Income Expectations, Limited Liquidity, and Anomalies in Intertemporal Choice[1]

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Abstract

Intertemporal choices are a ubiquitous part of our economic lives. Decisions about education, savings and health, all involve tradeoffs between costs and benefits materializing at different points in time. Yet, a large body of experimental evidence questions the descriptive validity of the economic benchmark model (exponential discounted utility) by documenting a series of behavioral patterns allegedly violating its key predictions. Observed discount rates typically lie far beyond market interest rates, tend to decline in time horizon and in outcome magnitude, and seem to be larger for gains than for losses. Hyperbolic preference models resolve these issues only partly: These models accommodate excessive short-run discounting, but fail to predict both outcome dependence and sign dependence. This paper demonstrates that all these "anomalies" are rationalizable without introducing exotic preferences. Instead, an interplay between liquidity constraints and income expectations is able to produce these stylized facts, even if economic agents are fully rational and have a pure rate of time preference close to the market interest rate. Liquidity-constrained agents who dislike fluctuations in the consumption path, but expect their income to rise in the near future prefer to allocate newly available cash inflows at earlier dates than their pure rate of time preference suggests. The assumptions underlying this mechanism are likely to hold for typical participants in laboratory and field experiments. Beyond that, our approach also provides an explanation for a number of phenomena which have remained largely unexplained so far, such as reasons for observed discount rates to increase in time delay, the heterogeneity of discount rates across different commodities and regions, and the co-occurrence of stationarity and dynamic inconsistency. The mechanism is easily distinguished from hyperbolic preferences and optimistic outlook, and its key predictions are retained under bounded rationality and partial asset integration.

Keywords

Time Preferences, Intertemporal Choice, Hyperbolic Discounting, Magnitude Effect, Sign Effect, Stationarity, Time Inconsistency, Expectations, Liquidity Constraints.

JEL Classification

D03, D84, D91.

1 Introduction

Many of the most important choices we make involve alternatives with consequences materializing at different points in time. Prominent examples are how much to save for future consumption, when to pay off debts, or in which training to invest. Understanding what drives these choices is paramount for predicting individual behavior and market outcomes. In particular, the design of incentive mechanisms, information programs or optimal defaults preventing individuals from undersaving or taking actions which are detrimental to their own health needs to be based on a sound knowledge of where and how to intervene.

This paper contributes to a better understanding of intertemporal choice behavior. It is motivated by a number of puzzling findings apparently inconsistent with exponential discounted utility, the canonical model of intertemporal choice. Most solutions proposed to address these issues suffer from fundamental shortcomings. They typically focus on one single "anomaly" only and fail to predict the vast number of equally important stylized facts. Hyperbolic preference models (Ainslie, 1975; Herrnstein, 1981; Mazur, 1987; Laibson, 1997; Harris and Laibson, 2001), for example, capture excessive short-run discounting. These models, however, do not provide an explanation for why subjects' behavior differs from standard predictions in many other ways.

Empirical evidence documents the following results. First, aggregate behavior departs in systematic ways from exponential discounting (Loewenstein and Thaler, 1989; Loewenstein and Prelec, 1992; Frederick, Loewenstein, and O'Donoghue, 2002). Quite robust "anomalies" are that discount rates lie far beyond market interest rates, decline in time horizon and in outcome magnitude, and are larger for gains compared to losses. A coherent explanation encompassing all these findings is still lacking.

Second, quantitative results, such as estimated discount rates or the size of the effects, vary tremendously across and within studies (Frederick, Loewenstein, and O'Donoghue, 2002).[1] This seems puzzling as many experiments reporting quite distinct findings are based on similar designs and are conducted among similar cohorts, i.e. subjects that are relatively homogenous with respect to their education, wealth and age. Related to this fact, important behavioral differences are found between studies involving monetary and primary outcomes and between studies conducted in different geographic regions. Discount rates, for instance, differ between wealth and health outcomes (Chapman, 1996) as well as between money and effort outcomes (Augenblick, Niederle, and Sprenger, 2014). While some studies (e.g. Reuben, Sapienza, and Zingales (2010)) find strong correlations across monetary and primary outcomes, others do not (e.g. Chapman (1996)). Regarding regional differences, Tanaka and Munro (2014) document that farmers living in less favorable agro-climatic zones discount future rewards more steeply than those living in more favorable zones. Taken together, these results indicate that particular characteristics of the outcomes under consideration and environmental factors are important drivers of heterogeneity in time discounting.

[1] In their review article, Frederick, Loewenstein, and O'Donoghue (2002) report discount rates ranging from -6 percent per annum to infinity.

Third, longitudinal studies show that behavior is not as temporally stable as existing preference models suggest (Airoldi, Read, and Frederick, 2012). This indicates that intertemporal choices may be influenced by other factors than intrinsic preferences. Indeed, in a recent experimental study, Halevy (2015) shows that dynamic inconsistencies cannot be fully understood by static preference reversals. His data suggests that a considerable fraction of subjects change behavior over the passage of time in a way not predicted by models imposing particular restrictions on discount functions, such as hyperbolic preference models.

Finally, many studies find a substantial fraction of subjects exhibiting discount rates increasing in time horizon (Read, Frederick, Orsel, and Rahman, 2005; Sayman and Öncüler, 2007; Airoldi, Read, and Frederick, 2012; Abdellaoui, Attema, and Bleichrodt, 2010; Epper, Fehr-Duda, and Bruhin, 2011; Abdellaoui, Bleichrodt, and l'Haridon, 2013).[2] So far, it is not clear whether this behavior is due to errors, traits or other reasons.

The present paper addresses the question to what extent these behavioral patterns can be rationalized within the bounds of standard economic theory. We explore the following intuition: When rational economic agents attempt to sustain a smooth consumption path, but are prevented from doing so because they are borrowing constrained and only hold very limited liquid assets, their income expectations can have important effects on intertemporal choices. Opting for new alternatives materializing at dates when consumption is expected to be relatively low permits constrained agents to reach consumption paths which better measure up to their goals (i.e. their preferences).

That considerations such as limited access to liquidity can play an important role in time discounting is supported by empirical evidence. In a study conducted among rural households in developing countries, Holden, Shiferaw, and Wik (1998) find that liquidity-constrained households show much higher discount rates than households not facing such constraints. Pender (1996) measures discount rates in rural India, and argues along similar lines: Exorbitantly high real interest rates charged by moneylenders generate binding credit constraints, which, again, explain individuals' excess impatience. More recently, Dean and Sautmann (2014) report evidence consistent with the idea that constraints affect measures of time preferences. Using data from a panel study in Mali, they find that the marginal rate of substitution is not very informative about underlying time preferences, by means that partial credit constraints can play a more important role in determining actual choices. A previous version of the present paper (Epper, 2010) documents related results for two samples of Swiss university students: Students who state to be cash constrained discount future rewards much steeper than those who do not. Similar results are found for a representative sample of the German-speaking Swiss population (Epper, Fehr-Duda, and Schubert, 2011).

The idea that liquidity constraints provide a powerful approach for explaining empirical regularities is not new. Theoretical work by Deaton (1991) shows that such restrictions can explain many important findings in consumer behavior not captured by competing models. Pender (1996) and Dean and Sautmann (2014) draw similar conclusions for the kind of in-

[2] Studies reporting similar results are Frederick (1999); Rubinstein (2003); Read, Airoldi, and Loewe (2005) and Attema, Bleichrodt, Rohde, and Wakker (2010).

tertemporal choices we are interested in here. The novelty of our research is to apply this idea to unravel "anomalies" and other puzzling findings in intertemporal choice behavior. We argue that new alternatives are not evaluated in full isolation, as it is usually assumed in empirical studies, but that there are situations where subjective income expectations are reflected in individuals' intertemporal choice behavior. Indeed, there is direct evidence that narrow bracketing (Rabin and Weizsäcker, 2009) fails in the kind of situations we consider (Epper, 2010; Dean and Sautmann, 2014). Epper (2010), Section 3, for example, shows that subjects (partially) integrate experimental income with their out-of-lab consumption plans, even if stakes are comparatively small.

We present the following insights. First, we show that all allegedly anomalous patterns naturally arise for a liquidity-constrained, relatively impatient agent with positive rational expectations. In fact, all the "anomalies" are predicted to be closely intertwined with each other. Consequently, our model prognosticates strong interaction effects across different violations of exponential discounted utility. Behavior that appears as apparently hyperbolic from a static point of view, is not necessarily time inconsistent from a dynamic point of view. Hence, our results contradict some of the conclusions drawn in the hyperbolic discounting literature. Further, heterogeneity in the agents' constraints and subjective expectations can provide an explanation for why subjects with fairly homogenous characteristics may substantially differ in their choice behavior.

Second, our approach is first to provide a rationale for so far unexplained behavioral patterns found in empirical data. As time passes, the agent's life and job situation might change, or she may be confronted with an altered economic environment. Her access to liquidity and her consumption plans are therefore likely to vary over time. As a result, our model gives a justification for the apparent dynamic instability (or time variance) of choice behavior. Furthermore, if the agent expects her income to substantially decline in the not so distant future, but is unable to accumulate sufficient liquid assets to smooth away the upcoming low-consumption periods, she will exhibits increasing discount rates. As a consequence, the distribution of exponential, hyperbolic and counter-hyperbolic types in the population may be largely governed by the liquidity constraints subjects face and the expectations they hold.

Third, we relax the assumption of perfect rationality to obtain a possibly more plausible model of intertemporal choice. We pursue two routes: In the first extension, we discuss the possibility that agents only partially integrate new consumption opportunities with their existing plans. There are two knife-edge cases: (i) The model motivated above, i.e. the case where all new incomes are perfectly integrated with existing plans *(perfect asset integration)*, and (ii) a model where all plans outside of the scope of the current decision situation are completely ignored *(narrow bracketing)*. In the former case, choices are typically more informative about expectations than preferences, while, in the latter case, they are perfectly informative about time preferences. Empirical evidence suggests that reality lies somewhere in between the two extremes (Epper, 2010).

In the second extension, we explore the possibility that agents hold overoptimistic beliefs about the future, but are not (necessarily) liquidity constrained. This alternative explanation for a link between subjective expectations and time discounting is motivated by the finding that subjects often hold optimistically biased beliefs when it comes to assessing future life events, such as future income flows (Weinstein, 1980, 1987; Dominitz, 1998; Armor and Taylor, 2002). Similar to agents with hyperbolic preferences and contrary to the above rational model, their behavior will be dynamically inconsistent, however. Recent research also indicates that the same mechanism can explain why people suffer from self-control problems. Nordgren, van Harreveld, and van der Pligt (2009) find that people often overestimate their capacity for impulse control, leading them to overexpose themselves to temptations.[3]

There are ways to distinguish between competing explanations of anomalously-appearing behavior, i.e. hyperbolic preferences, optimistically biased beliefs, or liquidity constraints together with positive income expectations. And the fact that different motives can generate rather similar behavior is important for policy makers. For example, some policy measures suggested in the literature might be problematic (if not harmful) for rational agents who undersave due to a temporal shortage of liquidity. We discuss the implications of this finding.

The present paper is related to some recent contributions analyzing the role of constraints and expectations in intertemporal choice. Dean and Sautmann (2014) theoretically and empirically analyze the effect of credit constraints on measured time preferences.[4] Their theoretical analysis differs in several important ways from ours: First, while Dean and Sautmann (2014) also seek to understand the effect of credit constraints on time discounting, they do not attempt to rationalize anomalies in intertemporal choice within the standard model. In fact, they ex-ante assume that preferences are quasi-hyperbolic. Our approach differs in that we obtain hyperbolic discounting as a result, given that certain conditions about the agent's access to liquidity and her expectations hold. Second, Dean and Sautmann (2014)'s primary interest lies on income and expenditure shocks, while we are concerned with *anticipated* changes in future income.[5] Indeed, our most central results are the outcome of an interplay between liquidity constraints and the agent's income expectations. What motivates our approach is direct empirical evidence reporting strong correlations between expected income and discount rates. Epper (2010), Section 3, for example shows that students' out-of-lab income expectations are predictive for discount rates they reveal in the laboratory. Finally, Dean and Sautmann (2014) and the present approach differ in their modeling approaches. Dean and Sautmann (2014) follow Harris and Laibson (2001), and they model constraints by letting the interest rate to depend negatively on savings, i.e. they presume that borrowing gets more expensive the more credit the agent holds. In contrast, we follow ideas discussed in the buffer-stock literature (Deaton, 1991), assume the interest rate to be exogenously given, and require the agent's wealth to be (weakly) positive. In our case, the liquidity constraint

[3] There might also be rational reasons for problems of self-control. Subjects are required to exert willpower in order to resist temptation. If they (temporarily) only possess limited cognitive resources, they may not be able to exhibit the self-control they need to.

[4] Similar points are raised by Pender (1996), Cubitt and Read (2007), and Meier and Sprenger (2010).

[5] For experimental evidence on the effect of income shocks on time discounting see Haushofer, Schunk, and Fehr (2013).

becomes binding as the liquid assets held by the agent are relatively small.

Ambrus, Asgeirsdottir, Noor, and Sandor (2014) provide experimental evidence for the hypothesis that income expectations influence elicited discount rates. They find that subjects with constant expected income show less present bias than subjects with non-constant income expectations. Together with the results presented in Epper (2010), this is possibly the most direct evidence we can find for expectations driving discounting behavior. Note that Ambrus, Asgeirsdottir, Noor, and Sandor (2014) do not attempt to provide a unifying explanation for all the behavioral patterns we seek to explain. The method they propose to compensate for the confounding effect of income expectations on discount rates can be very useful for measuring time preferences and testing the theory introduced here.

Along similar lines, Gerber and Rohde (2015) propose a method to elicit discount rates when baseline consumption is changing over time. Noor (2009) provides a related analysis. Both papers conclude that preference reversals may be produced by changes in baseline consumption. However, they remain silent about the origin of non-constant consumption plans. The present paper fills this gap. We seek to uncover the underlying mechanism generating the "anomalies" reported in the literature, while these other studies mainly focus on describing one particular finding and providing tools for experimentalists to control for potential bias in time preferences.

Carvalho, Meier, and Wang (forthcoming) report evidence in line with the mechanism outlined in the present paper. They examine intertemporal choices of low-income U.S. households before and after payday. Their subjects face considerable shortage of liquidity, and this shortage is more pronounced before than after receiving the wage payment. Moreover, they find that subjects exhibit significant present bias before payday, but not shortly afterwards. This prediction is perfectly consistent with our model: Liquidity constraints together with positive income expectations (i.e. the impending wage payment) produce excessive short-run discounting.

The remainder of this paper is structured as follows. Section 2 introduces the theoretical model. Section 3 derives predictions under the assumption of perfect rationality and compares these predictions to existing empirical evidence. Section 4 discusses some relevant extensions and implications. In particular, it covers the case of bounded rationality, biased beliefs, and possibilities to discriminate between competing mechanisms. The case of stochastic consumption plans is deferred to the appendix. Section 5 concludes.

2 Model

This section introduces our basic model. We first focus on behavior of an agent with rational expectations who is limited with respect to her borrowing opportunities and only holds few liquid assets. The particular situation we are interested in, is how this rational agent evaluates new alternatives by integrating them into her existing consumption plans. We remain completely within standard economic theory and follow standard preference assumptions (see e.g. Fishburn and Rubinstein (1982) or Manzini and Mariotti (2009)). An alternative explanation for the link between subjective income expectations and discounting behavior is given in Section 4.

The agent we consider is characterized by the following preferences. First, she has constant impatience, i.e. for any two points in time with fixed temporal distance, she has the same propensity to exchange sooner for later consumption, ceteris paribus. Under this assumption, the agent attributes weight $d(t) = \delta^t$, with $\delta \in (0, 1]$, to future consumption utility, where d is the discount function and $\eta = -\ln(\delta)$ her (constant) pure rate of time preference. Strotz (1955) shows that such preferences are a necessity for dynamically consistent behavior.[6] We further assume that the agent is relatively impatient i.e. that her rate of time preference η exceeds the real interest rate r. Only if this condition is satisfied, the agent has a need for additional liquidity sooner rather than later and, hence, will demand credit or prefer to dissave from the liquid assets she holds (see Deaton (1991)). Second, the agent has an aversion towards consumption fluctuations, i.e. she favors less variable consumption paths over more variable consumption paths, ceteris paribus. Her preference over consumption quantities is captured by an instantaneous utility function u satisfying standard regularity conditions (Inada, 1963).[7] Comparatively more concave utility functions imply a stronger preference for consumption smoothing.[8]

The agent has a consumption plan. This plan consists of a sequence of expected future consumption spendings. To form a plan, the agent uses her current information about future income. We assume rational expectations, i.e., on average, predicted income $\hat{y}_t$ coincides with realized income y_t at any given point in time t, such that $\mathbb{E}[\hat{y}_t - y_t] = 0$.[9] Three types of income expectations are distinguished: An agent who does not expect her income to change within the relevant time horizon is said to hold *constant income expectations*. Similarly, an agent who expects her income to substantially rise within the relevant time horizon holds *positive income expectations*. The opposite is the case for *negative income expectations*.

Borrowing and saving permit the agent to transfer income back and forth in time. Likewise, an agent may dissave from her (liquid) assets during periods with low income, but save during periods with high income. This allows her to sustain a smooth consumption path even if income is subject to variation. Consequently, if access to liquidity is not restricted, expected short-term income changes should have little effect on actual consumption spendings.[10] It is

[6] As we will see later, however, such preferences are not sufficient to establish dynamic consistency.

[7] Viz. the utility function fulfills the following conditions: (1) $u(0) = 0$, (2) continuous differentiability, (3) $u' > 0$ and $u'' < 0$ (concavity), (4) $u'(0) = \infty$ and $u'(\infty) = 0$.

[8] Comparatively more concave by means of an Arrow-Pratt-type measure $-u''/u'$. Note that utility embodies the same preference property as in atemporal settings, where a comparatively more concave utility function expresses a stronger aversion towards variability of outcomes at one single point in time.

[9] More precisely, the actual realization of consumption then deviates from expectations by some symmetric, independent random error, i.e. an error term which has an an expected value of zero, is i.i.d. and uncorrelated with the information the agent holds at the relevant point in time.

[10] For a patient agent with $\eta \leq r$ similar predictions arise even if there are limited borrowing opportunities (see Deaton (1991) for an elaborate discussion). For $\eta = r$, for example, Schechtman (1976) and Bewley (1977) show that, under i.i.d. or stationary stochastic income processes, consumption converges to the mean of income even if liquidity constraints are present, leading the agent to reach a perfectly smooth consumption path. Similarly, if

only expected lifetime income, but not its fluctuations, which governs current consumption in this case (Modigliani and Brumberg, 1954; Friedman, 1957).

Predictions may differ fundamentally if the agent is limited with respect to her possibilities to access sources of liquidity. In what follows, we analyze behavior of a relatively impatient agent who is neither permitted to borrow, nor is endowed with many liquid assets. Nonetheless, as liquidity constraints only hamper borrowing, but not saving, the agent can still overcome expected future low-income periods. Her limited borrowing opportunities and her limited assets, however, prevent her from smoothing away any enduring low-income period preceding a substantial growth in future income. As a result, consumption behavior responds asymmetrically to expected short-term income changes: The agent's expectations about changes in future income are informative about behavior only if income expectations are positive, but not if they are negative or constant.[11]

That liquidity constraints play an important role in many situations is motivated by the following facts. First, there exists support that a substantial fraction of the population in both, rich and poor countries, is affected by such limitations (Zeldes (1989), Deaton (1991)). Estimates from the Panel Study of Income Dynamics (PSID), for example, suggest that about 20% of U.S. households are credit constrained (Hall and Mishkin, 1982; Jappelli, 1990).[12] Probably most interestingly, the vast majority of empirical evidence on anomalies in intertemporal choice is based on experiments conducted among students or residents of developing countries, i.e. subjects who tend to be exceptionally exposed to such constraints and are also more likely to select into studies which promise monetary rewards. Second, there is direct empirical support for such constraints affecting time discounting and consumption behavior. Holden, Shiferaw, and Wik (1998), for instance, find that liquidity-constrained households in developing countries exhibit much higher discount rates than households not facing such constraints. Related findings are reported in Pender (1996); Epper (2010); Epper, Fehr-Duda, and Schubert (2011); Dean and Sautmann (2014); Carvalho, Meier, and Wang (forthcoming) (see the literature review in Section 1). Others (Altonji and Siow, 1987; Shea, 1995; Drakos, 2002) find that (aggregate) consumption moves asymmetrically with predictable changes in income, a pattern explainable by liquidity constraints, but not myopia. Finally, a certain degree of impatience is required such that an agent demands additional liquidity. If her preferences instead do not comply with these requirements, liquidity constraints will not bind. However, for our theoretical argument it is sufficient that the pure rate of time preference lies slightly above the real interest rate, and there is good reason to believe that this is in fact the case for the majority of subjects. Epper, Fehr-Duda, and Bruhin (2011), for instance, report average discount rates of 21.3% for the group of subjects with constant discount rates.

$\eta < r$, the agent will save indefinitely. As Deaton (1991) says it: *"[...] saving, not borrowing, is [such an agent] main concern"* (p.1225, text in brackets added).

[11] Contrarily, if a poor agent is neither permitted to borrow nor save, responses should be symmetric. Her consumption plan then reflects her income expectations.

[12] Epper, Fehr-Duda, and Schubert (2011) conduct experiments with Swiss university students and a representative sample of the Swiss German-speaking population. Self-reports indicate that 44.8% and 9.7% of the university students and the general population were cash-constrained during the time of the survey, respectively.

Andreoni and Sprenger (2012a) find comparable results.

We now derive our formal model. It builds on standard exponential discounted utility theory (Samuelson, 1937; Fishburn and Rubinstein, 1982). We do, however, introduce liquidity constraints. The basic assumptions we make closely follow those in Deaton (1991).[13] The agent chooses a consumption allocation which maximizes her total expected discounted consumption utility such that

$$\max_{(c_0,\dots,c_T)} \mathbb{E}\left[\sum_{\tau=0}^{T} \delta^{\tau} u(c_\tau) \middle| I_0\right] , \tag{1}$$

where $\mathbb{E}\left[\cdot|I_0\right]$ is her expectation conditional on the information I available at the decision date 0, c_τ is her *planned consumption* in period τ, and T is the planning horizon. In period $\tau+1$, the agents liquid assets are $w_{\tau+1} = (1+r)(w_\tau + y_\tau - c_\tau)$, where r is the real interest rate and y_τ her exogenous (net) labor income earned in period τ. The borrowing restriction takes the form $w_\tau \geq 0$. The agent is only allowed to consume out of her "cash-on-hand" $w_\tau + y_\tau$, i.e. consumption c_τ is bounded from above at $c_\tau \leq w_\tau + y_\tau$ and from below at zero. We shall further assume that w_τ is relatively small. The nonnegative, but small liquid assets the agent maintains are only intended to insure herself against transitory and small negative income shocks. If not stated otherwise, it will therefore be convenient to presume that the agent can sustain a constant consumption path before and after a substantial rise in income. This is possible because small income fluctuations are easily smoothed away by consuming out of the small buffer-stock the agent holds.[14]

Our particular interest lies in how new, i.e. previously unanticipated, alternatives are integrated into the agent's existing (ex-ante) consumption plan. Thereby, we restrict our attention to singular and certain cash in- or out-flows (for short: outcomes) of which the agent learns at time $\tau = 0$. An alternative is a dated outcome x_t, where x is the outcome amount and t is the outcome date.[15] Information about liquidity constraints is contained in the consumption plan. As limited access to liquidity makes it impossible to transfer future income to earlier dates, positive income expectations will be reflected in improving baseline consumption plans. Since saving is still allowed, however, this will not be the case for negative income expectations.[16] Consequently, liquidity constraints limit the agent's possibilities to rearrange her ex-ante consumption plan during the evaluation of new alternatives.

Figure 1 illustrates how expected income is transformed into an ex-post consumption plan

[13] See also Schechtman (1976) and Bewley (1977) who impose similar borrowing constraints.

[14] Small fluctuations of (disposable) income are more likely to occur than substantial changes in labor income. A precautionary agent may therefore anticipate such small fluctuation and build up a (more or less small) buffer-stock.

[15] In the typical intertemporal choice experiment subjects have to choose between sooner smaller and later larger outcomes. Commonly, it is impossible for the subject to anticipate these outcomes in advance as she does only have limited information about what is going to happen in the study, what size the payoffs will have and when they will be paid out.

[16] An agent with substantial negative income expectations must have sufficient opportunities to save income during high-income periods, however. Only this allows her to smooth away the anticipated low-consumption period.

in the presence and absence of liquidity constraints, respectively. A detailed description can be found in the figure's annotations.

Suppose an agent is indifferent between two new alternatives, a present outcome z_0 and a future outcome x_t, such that $z_0 \sim x_t$.[17] Assume that she fully integrates z_0 and x_t into her ex-ante consumption plan $(c_0, ..., c_T)$. The indifference can then be expressed as

$$u(c_0 + z_0) + \mathbb{E}\left[\delta^\tau u(c_t)|I_0\right] = u(c_0) + \mathbb{E}\left[\delta^t u(c_t + x_t|I_0)\right] \ . \tag{2}$$

Note that all time periods other than $\tau = 0$ and $\tau = t$ appear on both sides of the equation, and, hence, they cancel out. Put differently, consumption in the other periods is unaffected by the new alternatives z_0 and x_t. Rearranging the above terms, we get

$$u(c_0 + z_0) - u(c_0) = \mathbb{E}\left[\delta^t \Big(u(c_t + x_t) - u(c_t)\Big)\Big|I_0\right] \ , \tag{3}$$

where the left-hand side of the equation denotes the additional utility from increasing consumption by z today, and the right-hand side of the equation denotes the additional utility from increasing consumption by x at t. If not stated otherwise, we set $c_0 = u(c_0) \overset{!}{=} 0$, and denote the change in baseline consumption relative to current consumption by $g_t = c_t - c_0$. This is motivated by the fact that typical choice data does not permit identification of the absolute level of consumption.

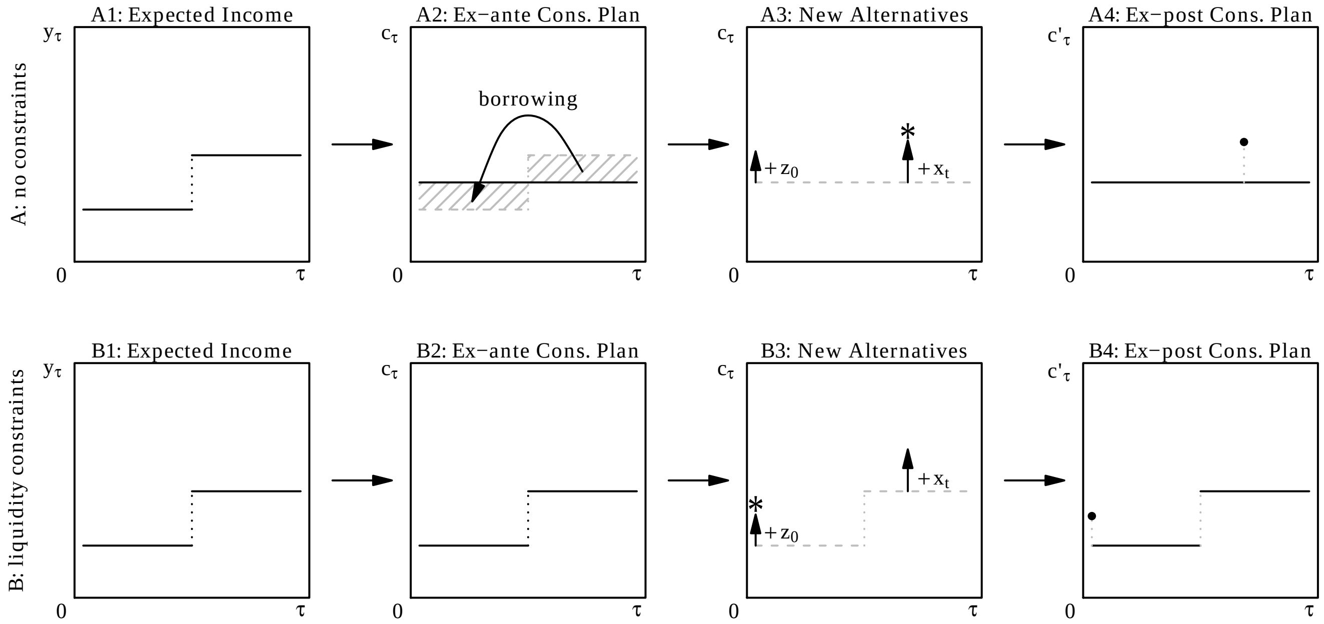
Under these assumptions, the agent evaluates any newly available dated outcome x_t relative to her expected change in consumption compared to today. The dated outcome x_t is evaluated as

$$U_0(x_t) = \mathbb{E}\left[\delta^t \Big(u(g_t + x_t) - u(g_t)\Big)\Big|I_0\right] \ . \tag{4}$$

The availability of new alternatives opens up new possibilities for the agent to, at least partly, overcome her limited borrowing capabilities and her deficient liquid assets. A liquidity-constrained agent with the above preferences and positive income expectations can make the huge gap between low and high baseline consumption periods smaller by allocating new outcomes at earlier dates where baseline consumption is expected to be low compared to dates where it is expected to be high. The intuition for this result is straightforward. Due to concavity of the instantaneous utility function, marginal utility of consumption is larger at periods where planned consumption is small compared to when it is large. This result directly follows from Equation 4. Note that the relatively low quantity of cash-on-hand together with the limited access to credit and the agent's relative impatience implies that, if there is no uncertainty, she does not have an incentive to spread consumption over the remaining low-income periods. Rather, she will consume the entire amount at the point in time it materializes (see Deaton (1991)).

[17] For a similar analysis under hyperbolic time preferences and constant baseline consumption see Loewenstein and Prelec (1992). Here, we only consider binary choices. However, the model is easily extended to the case where the agent chooses from a larger set of alternatives.

Figure 1: Illustration - Income Expectations and Consumption Plans



For illustrative purposes, we assume that the agent possesses a very small amount of liquid assets. The agent's preferences can be characterized by a constant pure rate of time preference slightly above the real interest rate and a concave instantaneous utility function. We explore the case, where the agent expects her income to rise within the relevant time horizon. **Row A** depicts the situation where the agent has full access to the credit market. **Panel A1** shows her positive income expectations. In **Panel A2**, the agent borrows against future labor income, and, hence, increase here short-run consumption in exchange to longer-run consumption. The resulting consumption plan is smooth (solid black line). **Panel A3** introduces the choice between a smaller-sooner payment z_0 and a larger-later payment x_t. The agent opts for the payment at t (marked by $*$). **Panel A4** plots the resulting consumption path, which is constant, except of a peak where the payment x_t occurs.
Row B depicts behavior in the presence of liquidity constraints. **Panel B1** shows the positive income expectations. As borrowing is not possible, these income expectations are reflected in the ex-ante consumption plan **(Panel B2)**. Being confronted with the same two payments as in the upper row, the agent opts for z_0 **(Panel B3)**. Intuitively, this is the case, as marginal utility from increasing consumption at $\tau = 0$ is larger than marginal utility from increasing consumption at $\tau = t$. Choosing the smaller-sooner payment is a direct consequence of a preference for consumption smoothing. In other words, the choice of the new alternative can be seen as a way to (partly) overcome the limited access to the capital market. **Panel B4** plots the ex-post consumption plan, with a peak in the present, and a pattern reflecting the positive income expectations afterwards.

3 Predictions

In what follows, we present the model predictions and put them into contrast with existing empirical evidence. To do this, we first obtain the present equivalent $\hat{z}_0$ making the agent indifferent to the dated outcome x_t, such that $\hat{z}_0 \sim x_t$, using the model we introduced in the previous section. Then, we calculate the predicted (per annum) discount rate $\hat{\eta}$ from the equation $\hat{z}_0 = e^{-\hat{\eta}t}x_t$:

$$\hat{\eta} = -\frac{1}{t} \ln \left(\frac{\hat{z}_0}{x_t} \right) . \quad (5)$$

By choosing this procedure, we closely follow the empirical literature, which typically reports discount rates inferred from smaller-sooner vs. larger-later tradeoffs. This permits us to directly compare predictions with actual observations.

We make a number of assumptions which facilitate our analysis. First, if not stated otherwise, we assume isoelastic utility. We discuss the effect of alternative utility specifications at some later point. Second, for expositional simplicity, we also assume that there is no uncertainty about the agent's (ex-ante) consumption plan, i.e. that $(c_0, ..., c_T)$ is *a priori* known. This allows us to cope without the expectation operator in the following subsections. As discussed in Appendix A.1, this assumption is not too much of a limitation, however, as any uncertain consumption plan can be replaced by a 'certainty equivalent' consumption plan leaving qualitative results intact.

We are primarily interested in comparing behavior of a liquidity-constrained, relatively impatient agent holding positive income expectations with an agent who does either not face such constraints, because she is relatively wealthy, can borrow, is sufficiently patient, or has constant income expectations. Remember that according to our rational model, income expectations only contain information about the agent's behavior if she is limited with respect to her liquidity.

The next subsections are devoted to illustrate the dependency of predicted discount rates on time horizon, outcome magnitude and outcome sign. Further predictions are sketched out in a special subsection.

3.1 Time Horizon

We first analyze the impact of delay on predicted discount rates. Our model's predictions are a direct result of the basic intuition provided earlier. Liquidity-constrained agents with positive income expectations allocate new cash inflows at dates where baseline consumption is expected to be relatively low. In such situations, low-consumption periods precede high-consumption periods. Liquidity-constrained agents then show a comparatively stronger preference for more immediate payoffs than agents who do not face such constraints or expect to maintain a constant baseline consumption path, holding all other things fixed. This effect, however, diminishes as the delay grows larger, resulting in predicted discount rates declining hyperbolically in time horizon and converging towards constant long-run rates. Intuitively, high baseline consumption periods gain comparatively more weight in the agent's total welfare, overcompensating the utility generated by consuming in the relatively short-lasting low baseline consumption periods. Hence, for a liquidity-constrained, relatively impatient agent positive income expectations generate a markup on otherwise constant discount rates. The size of this markup is predominantly driven by how large the agent expects her income to rise within the relevant time horizon.

To see this, consider the following formal derivation. According to our model, the predicted present equivalent $\hat{z}_0$ making the agent indifferent to the dated outcome x_t is equal to

$$\begin{aligned} \hat{z}_0 &= u^{(-1)}\left(\mathbb{E}\left[\delta^t\left(u(c_t+x_t)-u(c_t)\right)\Big|I_0\right]+u(c_0)\right)-c_0 \\ &= u^{(-1)}\left(\delta^t\left(u(g_t+x_t)-u(g_t)\right)\right)\,, \end{aligned} \tag{6}$$

where the second line takes into account the above assumptions that the consumption plan is *a priori* known and that $c_0 = u(c_0) \stackrel{!}{=} 0$. Remember that the ex-ante consumption plan already contains the agent's information about liquidity constraints. For our results to hold, it is important that the expected rise in consumption occurs somewhere in between the point in time the decision is made and the point in time the dated outcome materializes.

Calculating the discount rate inferred from our model by inserting the predicted present equivalent in Equation 6 into Formula 5, differentiating with respect to t, and rearranging, gives

$$\frac{\partial\hat{\eta}}{\partial t} = -\frac{1}{t}\left[\underbrace{-\frac{1}{t}\ln\left(\frac{\hat{z}_0}{x_t}\right)}_{\hat{\eta}} - \underbrace{(-\ln(\delta))}_{\eta}\underbrace{\frac{u(\hat{z}_0)}{\hat{z}_0 u'(\hat{z}_0)}}_{1/\varepsilon}\right]\,. \tag{7}$$

The interpretation of the term in square brackets is straightforward. It is the difference between the agent's *behavior*, i.e. the predicted discount rate $\hat{\eta}$, and her *'deep' preferences*, i.e. the product of the pure rate of time preference η and the reciprocal of the elasticity of the utility function $1/\varepsilon$. The following comparative statics illustrates the basic mechanism.

For $u'' < 0$ and $g_t > 0$, the fraction $\hat{z}_0/x_t$ decreases as g_t increases, ceteris paribus. This is the case since larger g_ts lead to stronger discounting of prospective consumption utility, i.e. due to concavity of u, the utility difference $u(g_t + x_t) - u(g_t)$ becomes smaller the larger the expected change in consumption g_t gets. As δ^t is a constant with $\delta^t \in (0, 1]$, and $u^{(-1)}$ is a strictly monotone function, the predicted discount rate $\hat{\eta}$ increases as consumption expectations g_t grow larger. u being isoelastic means that $1/\varepsilon$ is constant. Given all this, the difference in square brackets is positive and $\partial\hat{\eta}/\partial t < 0$. In plain terms, predicted discount rates decline in time horizon, with higher consumption expectations leading to a more pronounced difference between behavior and underlying 'deep' preferences.

A number of additional findings directly result from Equation 7. First, anticipated baseline consumption changes do not take effect under utility linear in consumption, simply because

g_t cancels out in $\hat{\eta}$ and $1/\varepsilon = 1$. Similarly, no effect is predicted for $g_t = 0$ if u is isoelastic. As a consequence, both concave utility and an improving consumption plan are necessary to produce the effect we are interested in. Second, in the long run, predicted discount rates are not distinguishable from constant rates even if $g_t > 0$, since $\lim_{t \to \infty} \partial\hat{\eta}/\partial t = 0$. Third, contrary to hyperbolic discounting models, decreasing discount rates are not a consequence of the agent's 'deep' preferences, but they are the result of her income expectations and the borrowing limitations preventing her from smoothing away the preceding low-consumption periods. In our model, the size of the effect therefore depends on anticipated consumption changes g_t, but no such dependency is predicted for hyperbolic preference models. Fourth, it directly follows from Equation 7 that there is an interaction between decreasing discount rates and the magnitude effect. Holding g_t constant, predicted discount rates decline less strongly in time horizon the larger x becomes, ceteris paribus. A more elaborate analysis of this effect follows in the next subsection.

As a short note, consider the case where the elasticity of utility is non-constant. There are two cases for which this applies: Either the utility function does not have a power or logarithmic form, or the absolute level of baseline consumption c_0 is an argument of the isoelastic utility function (see first line in Equation 6). An example for the first case is exponential utility. In the second case, c_0 takes the role of an 'anti-index' of concavity of u (for a discussion see Wakker (2008)). Both, exponential utility and taking into account the absolute level of consumption can, in principle, generate discount rates decreasing in time horizon as well as the other "anomalies". However, within plausible parameter ranges, the effects are too small to produce meaningful effects in line with empirical findings. Moreover, the direct empirical evidence linking liquidity constraints and income expectations to elicited discount rates clearly speaks against the conjecture that non-constant elasticities are the key driver of observed violations of exponential discounted utility.

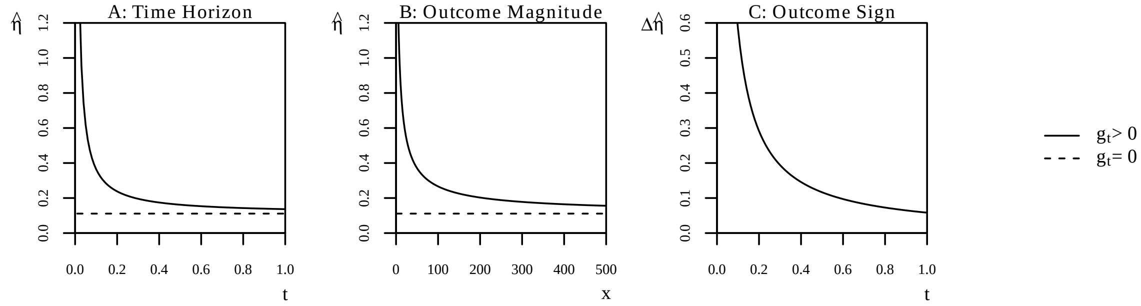
Panel A of Figure 2 illustrates our findings. It plots predicted discount rates for both, an agent with an improving consumption plan (solid curve) and an agent with a constant consumption plan (dashed line). As can be seen, positive income expectations induce decreasing discount rates for the liquidity-constrained agent. Short-run discount rates appear much larger than long-run discount rates. Despite the agent having a constant pure rate of time preference, her expectations drive a significant wedge between her time preferences and her discounting behavior. Very similar results can be obtained for losses, i.e. consumption reductions rather than consumption increases.[18] However, due to concavity of the utility function, predicted discount rates for losses converge faster towards constant rates.[19] We come back to this result later.

The predictions our model makes accord well with the empirical evidence on intertemporal choice behavior. For monetary rewards, Thaler (1981) observes annualized median

[18] This either requires total consumption to be nonnegative, or, alternatively, an appropriate normalization of the utility function.

[19] Due to concavity, subtracting x units from c_t leads to a utility loss which is in absolute terms larger than the utility gain when adding x units to c_t. In utility terms, losses then loom larger than gains.

Figure 2: Model Predictions



The three panels depict the model's key predictions. **Panel A:** For constant consumption plans, $g_t = 0$, the predicted discount rate $\hat{\eta}$ is constant in time delay t (dashed horizontal line). Improving consumption plans $g_t > 0$ produce a markup on these discount rates, ceteris paribus. The predicted discount rates decline in time horizon (solid hyperbolic curve). **Panel B:** For constant consumption plans, $g_t = 0$, the predicted discount rate does not depend on outcome magnitude x holding t fixed (dashed horizontal line). Improving consumption plans $g_t > 0$ lead to excessive discounting for small stakes (solid hyperbolic curve). This effect diminishes as stakes grow larger. **Panel C:** This figure plots the difference between discount rates predicted for gains and discount rates predicted for losses, i.e. $\Delta\hat{\eta} = \hat{\eta}^{+} - \hat{\eta}^{-}$ against time delay t. No gain-loss asymmetry is predicted for $g_t = 0$. For $g_t > 0$, however, the asymmetry between gains and losses is most pronounced for short delays t, but it diminishes when increasing the delay t. Similar results obtain when plotting this difference in domain-specific discount rates against outcome magnitude x. *Remarks:* Graphical illustrations assume that the agent expects an increase of consumption between the point in time the decision is made and the point in time the outcome materializes. The following parameters were used to construct the graphs: $\eta = 0.1$, $u(z) = z^{0.9}$, $g_t = 2$; Panel A: $x = 40$; Panel B: $t = 1/12$; Panel C: $x = 1$.

discount rates of several dozen to hundred percent. Discount rates decline sharply in time horizon. Similar findings are reported by Benzion, Rapoport, and Yagil (1989) and many others (see e.g. Redelmeier and Heller (1993); Chapman and Elstein (1995); Chapman (1996); Pender (1996); Frederick, Loewenstein, and O'Donoghue (2002); Epper, Fehr-Duda, and Bruhin (2011); Halevy (2015)).[20] Empirical evidence also supports our long-run predictions. Pender (1996), for example, finds that discount rates far away from the present cannot be distinguished from constant ones. Similar results are reported for negatively signed outcomes (Benzion, Rapoport, and Yagil, 1989; Chapman, 1996). Most studies find much lower discount rates in this domain, a finding corresponding well with our predictions.[21]

[20] Some studies do not find support for discount rates decreasing in time horizon. Examples are Andreoni and Sprenger (2012a) and Augenblick, Niederle, and Sprenger (2014). Augenblick, Niederle, and Sprenger (2014) argue that decreasing discount rates might be a consequence of not controlling for equal transaction costs across different points in time or not making all payments equally reliable. Arguably, these are important factors which can produce or amplify decreasing discount rates. However, Epper, Fehr-Duda, and Bruhin (2011) control on equal transaction cost and put special care on making payments reliable. They still find a majority of subjects exhibiting decreasing discount rates. We suspect that the differences between these conflicting results might be found in sample differences (richer vs. poorer, i.e. in the mechanism described in the present paper), or the different elicitation methods used by these studies (the two studies cited in the beginning of this footnote use convex time budgets while most other studies use binary choices or lists of such choices). The results reported in Carvalho, Meier, and Wang (forthcoming) speak in favor of the former explanation.

[21] It should be noted, however, that implementing losses in experiments is a difficult task, as it is usually not possible to force participants to pay money to the experimenter. The results in this domain might therefore crucially depend on the framing chosen.

Two important properties of intertemporal preferences have been widely discussed in the literature. The first property, *stationarity*, says that preferences should not revert when moving two dated outcomes along the time axis, while keeping the delay between the two outcomes fixed. Stationarity is a static property. It involves comparison of two binary choices viewed from the very same point in time. Formally, stationarity is defined as follows:

Definition 1 (Stationarity) *If* $x_t \sim z_{t+\lambda}$*, then* $x_s \sim z_{s+\lambda}$*,* $\forall x, z$*, and* $0 \leq t < s$*,* $\lambda > 0$*.*

According to this property, preferences are fully determined by the temporal distance λ between the two dated outcomes. Put differently, the delay of these two outcomes relative to the point in time the decision is made should not affect preferences. It has been shown that stationarity is necessary for exponential discounted utility (Strotz, 1955; Fishburn and Rubinstein, 1982), and that preference reversals of the form $x_t \succ z_{t+\lambda}$ and $x_s \prec z_{s+\lambda}$ are the critical criterion giving rise to hyperbolic preference models (Prelec, 2004; Bleichrodt, Rohde, and Wakker, 2009).

The second property, *dynamic consistency*, says that preferences should not revert when reevaluating choices at some later point in time. As its name says, dynamic consistency is a dynamic property. It involves comparison of the very same binary choice viewed from different points in time. Formally, dynamic inconsistency is defined as follows:

Definition 2 (Dynamic inconsistency) *If* $x_s \sim z_{s+\lambda}$*, then* $x_{s-\theta} \sim^{\theta} z_{s+\lambda-\theta}$*,* $\forall x, z$*, and* $\lambda > 0$*,* $0 < \theta \leq s$*,*

where $\sim^{\theta}$ denotes preference at the later point in time, θ time units after the original assessment. According to this property, preferences should be fully determined at one single point in time. Put differently, the point in time from which the two dated outcomes are evaluated should not affect preferences.

The large part of the hyperbolic preference literature makes no distinction between violations of stationarity and violations of dynamic consistency (see e.g. Thaler (1981) or Augenblick, Niederle, and Sprenger (2014)). Indeed, it is often concluded that static preference reversals fully inform about dynamic preference reversals. This view is perfectly valid if and only if the agent's 'deep' hyperbolic preferences completely describe her choice behavior, i.e. if preferences are hyperbolic and stable over the passage in time, and no factors other than the agent's pure preferences influence choice. Under these assumptions, only the temporal distance relative to the point in time the decision is made will matter, and we typically expect that $x_t \succ z_{t+\lambda}$, $x_{t-\theta} \succ^{\theta} z_{t+\lambda-\theta}$, but $x_s \prec z_{s+\lambda}$. Row A of Figure 3 illustrates this choice pattern. In the absence of any changes in the environment, hyperbolic preferences predict that non-stationarities and dynamically inconsistencies are one and the same.

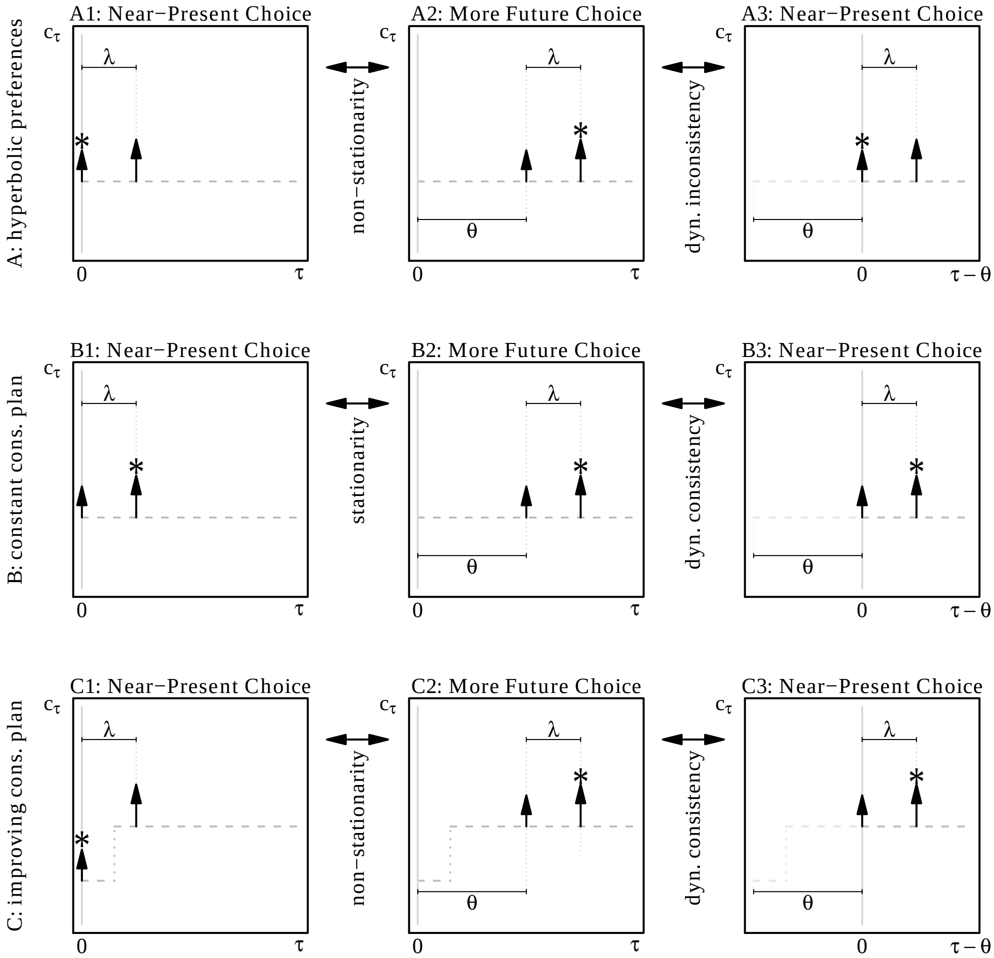
Empirical research indicates that static preference reversals cannot fully explain dynamic preference reversals. Airoldi, Read, and Frederick (2012) conduct a longitudinal study, which allows both, tests of stationarity and tests of dynamic consistency. Their results suggest that other factors than pure preferences play a role, and, indicate that violations of stationarity should be uncoupled from violations of dynamic consistency. Halevy (2015)'s experimental design permits a separate test of violations of the two properties. He finds that subjects who violate stationarity do not necessarily violate dynamic consistency, and vice versa.

Our model provides an explanation for these findings: Still assuming that 'deep' preferences are characterized by a constant pure rate of time preference and concave instantaneous utility, our model makes the following predictions. First, if a constant consumption plan can be retained, we predict neither static nor dynamic preference reversals (see row B of Figure 3). In this case, both, stationarity and dynamic consistency hold. Similar predictions sustain in the case of perfect access to the capital market. Second, if the consumption plan is improving within the relevant time horizon, we are able to predict static, but not dynamic preference reversals (see row C of Figure 3). Intuitively, anticipated changes of baseline consumption are tightly linked to the *calendar date* at which they occur. Assuming that no new information regarding income or constraints becomes available over the passage in time, the consumption plan remains the same. In contrast, it is the *temporal delay* with regard to the point in time the decision is made which generates the non-stationarities and dynamic inconsistencies in hyperbolic preference models. Row C and A visualize the predictions and show that they differ in dramatic ways between our model and hyperbolic preferences. Finally, if the consumption plan is constant over the relevant time horizon, but new information about substantial and systematic changes in future income or liquidity constraints becomes available over the course in time, the model is able to predict dynamic preference reversals (not depicted in Figure 3). Our model can therefore predict all possible combinations of static and dynamic preference reversals, and it remains an empirical question whether this mechanism is able to fully explain the results documented in Halevy (2015).

Our approach can also provide an explanation for why many studies find a substantial fraction of subjects exhibiting discount rates increasing in time horizon (see for instance Read, Frederick, Orsel, and Rahman (2005); Sayman and Öncüler (2007); Airoldi, Read, and Frederick (2012); Abdellaoui, Attema, and Bleichrodt (2010); Epper, Fehr-Duda, and Bruhin (2011); Abdellaoui, Bleichrodt, and l'Haridon (2013)). If an agent expects her income to decline substantially in the not so distant future, but is unable to accumulate sufficient liquid assets to overcome the anticipated low-consumption periods, she will exhibit increasing discount rates. The story is the same as above, but here the agent is limited with respect to putting aside present income for future consumption. The effects are again asymmetric, but this time reversed. Opting for new alternatives materializing during future low-consumption periods may help her to partially overcome these limitations. As a result, predicted discount rates lie below her 'deep' time preference. To the best of our knowledge, our approach is the first providing a possible explanation for this economically relevant, but puzzling finding. We suspect that relatively poor agents with a marginal propensity to consume out of their "cash-on-hand" close to one and substantial negative expectations are most likely to exhibit such behavior.

Decreasing discount rates motivated the development of hyperbolic preference models. Understanding why discount rates decrease or increase in time horizon and which mech-

Figure 3: Illustration - Stationarity and Dynamic Consistency



The figure depicts static and dynamic preference reversals under three different setups. *Stationarity* posits that revealed preferences do not revert between column 1 and column 2. *Dynamic consistency* posits that revealed preferences do not revert between column 2 and the column 3.

Row A shows predictions for a hyperbolic preferences model with narrow bracketing. The agent exhibits excessive short-run discounting and prefers the smaller-sooner payment over the larger-later payment when confronted with the choice between two near-present outcomes (indicated by $*$ in **Panel A1**). She prefers the larger-later payment, however, when moving the two outcomes into the future while keeping their temporal distance constant ($*$ in **Panel A2**). Her preference reverts when reconsidering this choice at some later point in time (**Panel A3**), and she again favors the smaller-sooner over the larger-later payment. It is only the delay relative to the point in time the decision is made which matters in this model. As a consequence, hyperbolic preferences predict that stationarity and dynamic consistency are violated simultaneously.

Row B illustrates predictions for standard preferences where the agent can retain a smooth consumption plan. The agent prefers the larger-later payment ($*$ in **Panel B1**). The same is the case when moving the two payments into the future while keeping the temporal distance between the two fixed ($*$ in **Panel B2**). **Panel B3** depicts the situation where the agent is reconsidering the choice she made before (see Panel B2). Again, behavior does not change in this setup. The predictions for an exponential preferences model with narrow bracketing are equivalent.

Row C shows predictions for standard preferences where the agent has an improving consumption plan. The agent picks the smaller-sooner payment ($*$ in **Panel C1**) which permits her to, at least partially, compensate for the relatively low consumption periods in the near present. Confronted with the choice between the same two payments in the farther future (**Panel C2**), she opts for the larger-later option (as in Panel B2). The agent appears to violate stationarity. She won't revert her choice when reconsidering it at some later point in time **(Panel C3)**, however, as her consumption plan remains stable. This result requires that new information which became available over the passage of time does not systematically change the agent's plan. Our approach therefore untangles the link between stationarity and dynamic consistency postulated by the hyperbolic discounting literature.

anism is at play is important for modeling consumption-savings decisions, asset allocation over time and many other relevant economic behaviors.

3.2 Outcome Magnitude

Our second prediction concerns the magnitude effect, i.e. the empirical regularity that smaller outcomes are discounted more heavily than larger ones. This effect is a direct consequence of the agent's temporal allocation of new cash inflows at dates where marginal utility is greatest. As changes in baseline consumption have a higher impact on smaller outcomes compared to larger outcomes, ceteris paribus, predicted discount rates decline in outcome magnitude. The effect is largely governed by the marginal rate of intertemporal substitution between future and present consumption, and it is more pronounced the more future baseline consumption is expected to grow. Negligible or distant positive changes in baseline consumption are unlikely to result in significant magnitude-dependency, however.

To see this, consider the derivative of predicted discount rates $\hat{\eta}$ with respect to the outcome magnitude x:

$$\frac{\partial \hat{\eta}}{\partial x} = -\frac{1}{t}\left[\underbrace{\frac{\delta^t u'(g_t + x_t)}{u'(\hat{z}_0)}}_{\text{MRS}_{t,0}}\frac{1}{\hat{z}_0} - \frac{1}{x_t}\right]. \tag{8}$$

Remember that the predicted present equivalent is equal to $\hat{z}_0 = u^{(-1)}\big(\delta^t(u(g_t + x_t) - u(g_t))\big)$. The size of the magnitude effect largely depends on how much the agent expects her consumption to rise during the time horizon under consideration. The first term in square brackets, the product of the marginal rate of intertemporal substitution between future and present consumption $\text{MRS}_{t,0}$ and the reciprocal of the present equivalent $1/\hat{z}_0$, rises as g_t becomes larger.

The following comparative static proves the magnitude effect and its dependence on g_t. Consider two dated outcomes with $\bar{x}_t > \underline{x}_t > 0$. $u' > 0$ implies that $\hat{z}_0(\bar{x}_t) > \hat{z}_0(\underline{x}_t)$, ceteris paribus. From $u'' < 0$ it follows that $\left(u^{(-1)}\right)'' > 0$ and $\hat{z}_0'' > 0$. As a result, the discount fraction $\hat{z}_0(\bar{x}_t)/\bar{x}_t$ is larger than the discount fraction $\hat{z}_0(\underline{x}_t)/\underline{x}_t$. Since $\hat{\eta}_{\bar{x}_t} < \hat{\eta}_{\underline{x}_t}$, predicted discount rates decrease as x_t grows larger. Following the same argument, it is easily shown that the effect is more pronounced as g_t increases, holding x_t and all other things fixed.

Some additional results emerge from Equation 8. First, anticipated changes in baseline consumption do not take effect if u is a linear function of its argument, i.e. if $u' = 1$. For this case, no magnitude effect is predicted. Similar results are obtained for $g_t = 0$ and u isoelastic. Then, the first term in square brackets reduces to $1/x_t$, which is the lower bound of the product in the first term in the square bracket given that $g_t \geq 0$ and $u'' < 0$. Second, predicted discount rates converge to rates constant in outcome magnitude as x_t tends towards infinity, holding all other things fixed, as $\lim_{x_t \to \infty} \partial\hat{\eta}/\partial x_t = 0$. Similarly, the magnitude effect diminishes as the time horizon grows in direction of infinity, since $\lim_{t\to\infty} \partial\hat{\eta}/\partial x_t = 0$. This result, once again, indicates that the two effects, discount rates decreasing in time horizon and the magnitude effect, are closely intertwined.

Panel B of Figure 2 illustrates these findings. For a liquidity-constrained, relatively impatient agent with positive income expectations, predicted discount rates are substantially larger for small outcomes as compared to large outcomes, but they decline as the stake size increases (solid curve). This is not the case for a liquidity-constrained agent with constant income expectations (dotted line). Such an agent is not sensitive to changes in outcome magnitude, but reveals predicted discount rates constant in stake size. The same predictions emerge for an agent who is not liquidity-constrained, irrespective of her income expectations. We also predict a magnitude effect for losses, i.e. consumption reductions. Again, due to concavity of the utility function the effect is predicted to be less pronounced in this domain. Details follow in the next subsection.

Our predictions dovetail nicely with the empirical evidence provided by numerous studies (Thaler, 1981; Loewenstein, 1987; Benzion, Rapoport, and Yagil, 1989; Holcomb and Nelson, 1992; Raineri and Rachlin, 1993; Shelley, 1993; Green, Fristoe, and Myerson, 1994; Green, Myerson, Lichtman, Rosen, and Fry, 1996; Kirby and Marakovic, 1995; Kirby, 1997; Kirby, Petry, and Bickel, 1999; Halevy, 2015). For rewards, Thaler (1981) not only finds that median discount rates decrease in outcome magnitude, but also that *"subjects' actions are closer to the normative model, the larger are the stakes"* (p.206). Similar results, although much less pronounced, are also found for losses (Thaler, 1981; Benzion, Rapoport, and Yagil, 1989; Shelley, 1993; Chapman, 1996).

Despite its prevalence and its relevance for understanding choices over small and large stakes, most discounting models, such as hyperbolic preference models, fail to predict a magnitude effect. Some exceptions exist. Loewenstein and Prelec (1992) propose a descriptive reference-dependent model which imposes specific assumptions on the value function to accommodate the magnitude effect. Noor (2011) assumes a magnitude-dependent discount factor and derives hyperbolic preferences as a special case. Epper and Fehr-Duda (2014) demonstrate that the presence of a per-period risk to get less than a promised reward can produce magnitude-dependence. Walther (2010) argues that various transaction costs or the propensity to think twice when expected gains or losses are relatively large can generate a magnitude effect. Holden (2014) presents a theory of mental zooming which accommodates both the magnitude effect and hyperbolic discounting. The magnitude effect and its interaction with delay is relevant as it can provide a better understanding of purchase decisions of durable goods. When different alternatives are compared, the relatively small, recurring costs often get too little weight compared to the relatively large, up-front purchasing costs. Importantly, in such situations, magnitude dependency is hardly separable from time horizon effects.

3.3 Outcome Sign

So far, we found that subjective income expectations can induce asymmetric discounting behavior with respect to differently signed outcomes. The sign effect, i.e. the regularity that gains are discounted more heavily than losses, is predicted for typical agents in our model. The intuition is the following. Due to concavity of the instantaneous utility function, sacrificing consumption always hurts more than increasing consumption by the same absolute quantity yields pleasure. It follows from the previously examined effects that the difference between domain-specific discount rates diminishes as the time horizon or the absolute outcome magnitude becomes larger. As a result, behavior is predicted to be more symmetric the larger these dated outcome's two arguments become, ceteris paribus.

The formal derivation straightforwardly follows from the above results. The sign effect emerges directly from applying Jensen's inequality to the gain-loss difference of predicted discount rates $\Delta\hat{\eta} = \hat{\eta}^{+} - \hat{\eta}^{-}$. Again, under standard assumptions, it holds that $\lim_{t\to\infty} \partial \left[\Delta\hat{\eta}\right] / \partial t = 0$ and $\lim_{x\to\infty} \partial \left[\Delta\hat{\eta}\right] / \partial x_t = 0$.

Panel C of Figure 2 shows our predictions by plotting the difference of the domain-specific discount rates against time horizon. Liquidity-constrained agents with positive expectations show asymmetric discounting behavior (solid curve). The asymmetry between gains and losses is largest for short time horizons, but diminishes the farther in the future the outcome materializes. Symmetric behavior is predicted for an agent with constant expectations (dotted line) as well as for a agent not facing liquidity constraints. Similarly, the sign effect vanishes as stakes grow larger.

Empirical studies such as Yates and Watts (1975), Thaler (1981), Loewenstein (1987), Loewenstein (1988), Benzion, Rapoport, and Yagil (1989), MacKeigan, Larson, Draugalis, and Bootman (1993), and Chapman (1996), all find that gains are discounted more heavily than losses. Evidence also seems to suggest that this effect diminishes as the time horizon and the outcome magnitude grows larger (see e.g. the results in Thaler (1981)).

Most competing discounting models do not predict sign-dependency. A notable exception is Loewenstein and Prelec (1992). In their reference-dependent model this anomaly arises by ex-ante assuming different elasticities of the value function for gains and losses. Hyperbolic preferences alone, however, do not permit any conclusion regarding the sign effect. A better understanding of these asymmetries, however, is crucial for interpreting choices where both, costs and benefits come into play, such as investment decisions or the purchase of durable goods.

3.4 Other Predictions

Our model makes a series of additional predictions beside the well-known stylized facts we discussed so far. Some of these predictions have, to the best of our knowledge, not yet been tested.

The formal derivation of the above "anomalies" has shown that they are inextricably linked with each other. This is the case, as they all originate from the agent's allocation of new cash inflows at dates where baseline consumption is comparatively low. Put differently, they are all the result of one and the same mechanism. We therefore expect the extent of departures from exponential discounting with respect to time horizon, outcome magnitude and outcome sign to be strongly and positively correlated across subjects. We are not aware of any direct test of this prediction.

A few remarks are in order. First, note that we only predict such interactions for relatively short time horizons and modest stake sizes. If these conditions are not met, interactions might be rather hard to detect. This prediction has important implications for some applications. For example, exponential discounting might be a good approximation when evaluating strategies mitigating global warming, even when only very limited financial resources are available and significant technological progress can be expected in the future.

Second, that one common mechanism drives distortions in discount rates can be used to falsify our approach and it can be used to test it against other models of intertemporal choice. Hyperbolic preference models, for example, attribute decreasing discount rates to 'deep' preferences, but fail to predict magnitude- and sign-dependency. The more complete model introduced by Loewenstein and Prelec (1992) attempts to capture every single anomaly by imposing separate assumptions on the discount function and the value function. These models therefore do not predict interactions between "anomalies".

Third, we predict the "anomalies" to be most pronounced for agents with improving consumption plans, or - more specific - for those agents facing binding liquidity constraints and having positive income expectations. Earlier in this article, we reviewed some literature which is in line with this prediction. The dependence of discount rates on these factors can be used to test our model against models relying on narrow bracketing. Moreover, as a preference for consumption smoothing is crucial for predicting the "anomalies", we also expect a correlation between utility curvature and the degree of departures from exponential discounting.

As agents differ with respect to their preferences, their access to liquidity and their income expectations, the above mechanism might also provide a potential explanation for the huge variation within and between studies (Frederick, Loewenstein, and O'Donoghue, 2002). While most other models assume that behavioral heterogeneity is solely driven by interpersonal differences in 'deep' preferences, our analysis reveals the importance of environmental factors and expectations for understanding the vast behavioral differences. Beside the direct evidence on the link between liquidity constraints, income expectations and time discounting listed in the first section of this paper, this could also explain why poorer subjects typically reveal higher discount rates (see e.g. Tanaka, Camerer, and Nguyen (2010)). Further, it also illustrates how problematic it can be to use discount rates stemming from experiments with typical subjects to predict behavior of a more general or different population, or to even rely on such results when formulating policy recommendations. Students, for example, are likely to exhibit stronger departures from exponential discounting than middle-aged employees, as the latter had more opportunities to accumulate sufficient wealth in the past and, hence, are less likely to have difficulties accessing fresh capital (see e.g. Epper, Fehr-Duda, and Schubert (2011) for related evidence).

The very same mechanism can potentially also help to understand a number of related findings. First, there are apparent differences in discount rates across regions. Tanaka and Munro (2014), for example, recognize that *"[r]isk attitudes and discount rates are not merely reflection of personal preferences but represent economic and other conditions of the individuals and households"* (p.152). They find that farmers in more favorable agro-climatic zones discount future outcomes less heavily than others. A plausible explanation for this finding is that these farmers also had more chances to accumulate wealth in the past, and, hence, that they less likely face liquidity constraints than farmers operating in harsher environments. It would be interesting to know whether farmer's expectations indeed correlate with their discount rates, as predicted by our model. While it is evident that market frictions, underdeveloped capital markets or the lack of (micro)credit might explain differences of discount rates across (developing and developed) countries, we are not aware of any elaborate study examining intertemporal choices among a wide range of regions. There are many important questions yet to be answered. For example, do people end up in poverty because of the institutional deficiencies they face or do they select into poverty because they are notoriously impatient? Our model suggests that the former mechanism might play a prevalent role, but also that this mechanism can potentially not be fully discerned from 'deep' impatience (we need this assumption to make the constraint binding). The existing empirical evidence, however, does not permit any causal conclusions.

Second, a similar argument can explain why we find differences in time discounting across different commodities, such as between monetary and primary outcomes. Chapman (1996), for example, reports the typical "anomalies" for both, choices involving money and choices involving health. However, she finds no evidence for correlation of discount rates across the two domains. Also, average discount rates for positive health outcomes typically exceed those for money outcomes (see e.g. Chapman (1996)). Similarly, Augenblick, Niederle, and Sprenger (2014) find only very limited evidence of decreasing discount rates for money, but rather pronounced present bias in an effort task. The key for understanding these behavioral differences between monetary and primary rewards might lie in the characteristics of the particular commodity under consideration: Money is fungible and it is comparatively easily transferred back and forth in time, given that the agent has unrestricted access to the capital market. This is not the case for health and real effort (at least in the Augenblick, Niederle, and Sprenger (2014) setting). While it is clearly not possible to buy or borrow quantities of health in exchange to money or future units of health, it also seems very unlikely that people in the Augenblick, Niederle, and Sprenger (2014) real effort experiments hire others to do their work. Choices involving primary outcomes might therefore look very similar to choices involving monetary outcomes in the presence of liquidity constraints. That is, it might be the different characteristics of commodities (e.g. the existence of markets or possibilities to intertemporally reallocate), and not 'deep' preferences, which are fundamental for an understanding of the behavioral differences we observe. The results presented by Carvalho, Meier, and Wang (forthcoming) are in in line with this reasoning: The low-income households they survey exhibit present bias in a real effort task, irrespective of whether they are asked before or after the paycheck arrived. However, they only show present-biased behavior for monetary outcomes prior to the wage payment, but not afterwards.

4 Discussion

We now present two model extensions and a series of key implications. So far, we demonstrated that many well-documented findings reported in the intertemporal choice literature can be generated by fully rational agents taking into account their limited access to liquidity. In the next two sections we weaken the rationality assumption in two ways, and obtain (possibly more plausible) conditions leading to qualitatively similar conclusions. In the first extension (Section 4.1), we relax the assumption of full asset integration. In the second extension (Section 4.2), we relax the assumption of rational expectations. An additional extension is deferred to Appendix A.1. It demonstrates that our predictions remain intact if the agent is uncertain about her consumption plan. Later, in Section 4.3 we outline some important implications of two extreme variants of our model in comparison to hyperbolic preferences.

4.1 Extension I: Partial Asset Integration

It is straightforward to extend our model to accommodate partial asset integration, and our predictions remain qualitatively robust when this is done. Compared to the case of full asset integration, the effects will be less pronounced, however. For the extreme case of narrow bracketing, we predict no departure from exponential discounting.

Consider the following (extended) valuation function:

$$U_0(x_t) = \mathbb{E}\left[\delta^t \Big(u(\kappa g_t + x_t) - u(\kappa g_t)\Big)\Big| I_0\right] , \tag{9}$$

where $\kappa \in [0, 1]$ denotes the (asset) integration parameter. This model contains two notable edge cases. First, for $\kappa = 1$ we have the case where the agent performs full asset integration. This is the rational model discussed before. Second, for $\kappa = 0$ we have the case of narrow bracketing (Rabin and Weizsäcker, 2009). This model is equivalent to the exponential discounted utility model serving as a benchmark in most empirical studies. It essentially says that consumption opportunities outside of the current choice situations (e.g. the laboratory) are completely ignored by the subject.

From Equation 9 it is straightforward to see that our predictions remain intact for $\kappa > 0$. However, putting less weight on changes in baseline consumption, i.e. a smaller κ, also means that the "anomalies" will be less pronounced, ceteris paribus. This is the case since the term representing the anticipated changes, κg_t, becomes comparatively smaller as κ decreases.[22]

[22] In Appendix A.1 we show that uncertainty in the consumption plan can have similar effects.

It directly follows from Equation 9 that observed choices will be typically more informative about expectations than 'deep' preferences if $\kappa > 0$, while they will be perfectly informative about 'deep' preferences if $\kappa = 0$.

Epper (2010), Section 3, presents some evidence for the model introduced in this paper, and shows that not all the anomalous behavior can be explained by the rational mechanism discussed before. Partial asset integration is one possible explanation for this finding. It is noteworthy, however, that the results in Epper (2010), Epper, Fehr-Duda, and Schubert (2011), Ambrus, Asgeirsdottir, Noor, and Sandor (2014) and Dean and Sautmann (2014) are not in line with narrow bracketing, as this would imply that liquidity constraints or income expectations are not correlated with choices in the experiments or surveys.

4.2 Extension II: Bounded Rationality

There is a second mechanism through which subjective income expectations can translate into the above patterns of discounting behavior, and this mechanism does not necessarily require liquidity constraints nor does it require the agent to be relatively impatient. Instead, this mechanism replaces the rational expectation assumption by optimistically biased expectations about the future. More precisely, an agent prone to optimistic bias expects her consumption opportunities in the future to be better than they actually are. Such biased beliefs can be easily accommodated by our model, and there is sound empirical evidence that they are indeed prevalent in situations alike to the ones we are interested here. Again, our key predictions remain intact when such considerations play a role, two notable exceptions being dynamic preference reversals and possibly the dependence of behavior on scarce liquidity.

Empirical evidence reports that subjects are often too optimistic when it comes to evaluating future life events (Weinstein, 1980, 1987; Armor and Taylor, 2002). They typically overestimate their future earnings (Dominitz, 1998) and their ability to resist future temptations (Nordgren, van Harreveld, and van der Pligt, 2009). For our model, this means that future income would be systematically overestimated such that $\mathbb{E}[\hat{y}_t - y_t] > 0$ for any point in time t, where $\hat{y}$ is the income predicted by the agent and y is the realized income. In the most extreme case, an optimistically biased agent would completely ignore any possibility to intertemporally transfer income back and forth in time by saving, dissaving or borrowing, and, as a consequence, she would use new cash inflows for instantaneous consumption. If this is the case, income y is equivalent to consumption c. Now set $c_0 = 0$, as baseline consumption is completely irrelevant for this agent. $g_t > 0$ in our original model (see Equation 4) can then be interpreted as the optimistic bias. As this optimistic bias will be always strictly positive, the agent's beliefs about the future imply that marginal utility derived from consumption today is always larger than the marginal utility derived from consumption at some future point in time, irrespective of her pure rate of time preference. Even if there is no (rationally) anticipated increase in baseline consumption over the relevant time horizon, the key anomalies would still be predicted. However, there are some exceptions and differences to the above rational model which allow us to distinguish the two mechanisms. We discuss them in detail below.

The model in which the agent ignores any outside consumption plan is likely too extreme. A more plausible model could be one which is close to the rational model above, but still permits some degree of overoptimism. One way to integrate this intuition in our model is to postulate that g_t does not fully measure rational changes in baseline consumption, but is instead a convex combination of rational anticipations and optimism. For example, let $g_t = \lambda h_t + (1 - \lambda) b_t$, where h_t measures the rational change in baseline consumption during the relevant time period and b_t measures optimism expressed in monetary terms. $\lambda \in [0, 1]$ denotes the weight the agent attributes to rational changes of consumption, and, hence, it can be interpreted as a rationality index.[23] According to this extended model, optimistic bias can have the opposite effect than partial asset integration in that it can amplify the "anomalies" if $b_t > h_t$, while, again, (partially) linking non-stationarities to dynamic inconsistencies. One reason why such an extended model could be useful is recent empirical evidence showing that schooling makes subjects' expectations more rational (Shenoy, 2014). We therefore expect λ to increase with schooling, and, consequently, dynamic inconsistencies to be more prevalent among subjects with lower education.

4.3 Key Implications

We now explore possibilities to distinguish between different mechanisms generating anomalies in intertemporal choice. We focus on three models: First, our *rational model* in which departures from exponential discounting are generated by an interplay between liquidity constraints and (rational) income expectations. Second, an *optimism model* in which these departures are generated solely by the agent's overly positive outlook for the future. This model corresponds to the specification in Section 4.2 where $\lambda = 0$. The typical agent in this model therefore abstracts from integration of new alternatives into existing consumption plans. And, third, a pure *hyperbolic preference model* which attributes decreasing discount rates to 'deep' time preferences, i.e. a hyperbolic discount function.

These three models make a series of distinct predictions and they have very different consequences. The different predictions permit researchers to discriminate among the models, i.e. they enable an understanding of whether preferences, beliefs or constraints drive distortions in discount rates. The different implications are relevant for developing policy measures which counteract potentially problematic behavior. Note that the boundedly rational model we introduced in the last section can make predictions somewhere in between the rational model and the optimism model. For example, it can predict that both, stationarity and dynamic consistency are violated, despite the agent taking into account his constraints. The boundedly rational model is easier to distinguish from hyperbolic preferences than the pure optimism model as it relaxes the bracketing assumption inherent in the latter two.

Table 1 provides an overview of our key findings. At a first glance, predictions for the

[23] The two parameters λ and b_t are obviously hard to recover from data, and in typical settings only jointly identified.

Table 1: Different Models, Different Predictions

	Criterion	**Rational Model**	**Optimism Model**	**Hyperbolic Preference Model**
(0)	**Main driver of discounted utility violations**	constraints (+ rational expectations)	(biased) beliefs	'deep' preferences
(1)	**Anomalies predicted under typical assumptions**	all[a][b] & interactions	all[a][c] & interactions	excessive short-run discounting[d]
(2)	**Can predict increasing discount rates**	yes[e]	yes[f]	no[g]
(3)	**Static (revealed) preference reversals**	not necessarily	yes	yes
(4)	**Dynamic (revealed) preference reversals**	no[h]	yes[i]	yes[i]
(5)	**Preference for commitment**	no	depending on awareness	depending on awareness
(6)	**Bracketing**	full asset integration	(more or less) narrow	narrow
(7)	**Discount rates correlate with liquidity constraints**	yes	possibly[j]	no
(8)	**Discount rates correlate with income expectations**	yes[k]	possibly[j]	no
(9)	**New information about future income affects discount rates**	yes	possibly[l]	no
(10)	**Causality**	constraints generate "anomalies"	optimism generates "anomalies"	hyperbolic preferences lead agents into constraints
(11)	**Effective policies**	none[m]	compulsory savings plan, defaults, information	compulsory savings plan, defaults, (commitment)

[a] See Section 3 for details.

[b] Required assumptions are liquidity constraints, positive income expectations, concavity of instantaneous utility and relative impatience.

[c] Required assumptions are optimistic bias and concave instantaneous utility.

[d] Requires that Pratt–Arrow convexity of the logarithm of the discount function is strictly positive (Prelec, 2004).

[e] If the agent expects a substantial decline of future income, but no accumulation of wealth is possible.

[f] Requires pessimistic beliefs.

[g] Most hyperbolic discount models fail to accommodate such patterns. But see Bleichrodt, Rohde, and Wakker (2009) for a discussion and discount functions which allows for increasing discount rates.

[h] Assuming that new information becoming available over the passage in time is integrated in a Bayesian way.

[i] Presuming naiveté or the absence of commitment devices.

[j] Might affect the agent's optimism/pessimism.

[k] Only if liquidity constraints are present.

[l] New information might influence the agent's optimism/pessimism.

[m] See main text. Easier access to liquidity (e.g. via microcredit) might help rational agents to find a way out of poverty.

optimism model and the hyperbolic preference model look rather similar, while predictions for the rational model substantially depart from these two models. However, as shown below, there are ways to discriminate between all three models, and this is relevant as they have very different implications.

The following results emerge.[24] First, both models consistent with the representation introduced earlier, i.e. the rational model and the optimism model, predict the entire set of “anomalies” as well as their interaction. In contrast, hyperbolic preference models fail to predict the magnitude effect and the sign effect as well as the other patterns outlined in Section 3.4. To accommodate these two effects, such models would require further restrictions on the value function. Loewenstein and Prelec (1992) achieve this by assuming the value function to be more elastic for outcomes that are larger in absolute magnitude (generating the magnitude effect) and more elastic for losses than for gains (generating the sign effect). Again, such an approach would not predict correlations across anomalies.

Second, traditional hyperbolic preference models cannot reconcile discount rates increasing in time horizon. Bleichrodt, Rohde, and Wakker (2009) propose two discount functions which resolve this limitation. Models incorporating such discount functions are useful for describing behavior. However, they remain silent about the underlying mechanism generating increasing discount rates. Our approach fills this gap and makes sharp and testable predictions regarding whether or not such distortions should be observed.

Third, fourth and fifth, the models make distinct predictions when it comes to violations of stationarity and dynamic consistency, as well as when it comes to the relationship between these two properties. As we have seen before, the rational model uncouples non-stationarities from dynamic inconsistencies and it is able to generate various combinations of static and dynamic preference reversals. Contrary to this, these two kinds of preference reversals are one and the same in the optimism model and the hyperbolic preference model. However, there is one notable exception. If the agent is aware of her dynamically inconsistent 'deep' preferences or her consistently overoptimistic beliefs she might prevent dynamic preference reversals by commitment. A possible commitment strategy for sophisticated agents is to put wealth into illiquid assets in order to inhibit future selves from consuming “too much”. While sophisticated agents gained some attention in the theoretical literature (see e.g. O'Donoghue and Rabin (1999)), there appears to be only very limited demand for commitment in both, real-world and experimental settings (Laibson, 2015).[25] Further, note that dynamic inconsistencies also pose a problem for welfare evaluations. Irrespective of whether it is what the agent wants (i.e. her preferences) or what she expects for the future (i.e. her beliefs), there will always be a point in time from which past choices appear to be suboptimal. For rational agents no such problems occur and there is one unique way to judge whether some action is beneficial or detrimental to overall welfare.

Sixth, seventh, eight and ninth, the models make different predictions when the correla-

[24] Listed in order of appearance in Table 1.

[25] Laibson (2015) also points at another issue: In many cases the cost of commitment are just too high, such that commitment does not pay off.

tion of discount rates with variables outside of the actual choice situation and the response to newly available information is concerned. The hyperbolic preference model we consider predicts no such correlations. However, the typical optimistic agent's mood might be (more or less strongly) influenced by changes in her environment or the availability of new information, which might again affect her discounting behavior. For rational agents, we predict that liquidity constraints have an effect on intertemporal choices, and, if this is the case, that income expectations should be reflected in consumption plans and discounting behavior. As new information about systematic changes in future income becomes available, the rational agent should respond by integrating this information into her existing consumption plan.

Tenth, distinct causal mechanisms are at work in the three models. Positive attitudes drive anomalously-looking behavior in the optimism model. Possibly more interestingly, the rational model and the hyperbolic preference model make reverse causal predictions with regard to the link between decreasing discount rates and liquidity constraints. While notorious overconsumption due to hyperbolic preferences might expose agents to further constraints and lead them into poverty, the rational model argues that it is these limitations which eventually produce hyperbolically-appearing discounting behavior. It is an important empirical question which mechanism is more relevant in explaining the co-occurrence of present bias and liquidity constraints.

Eleventh, policy measures attempting to prevent negative consequences of excessive short-run discounting can have diverse effects on agents' welfare, depending on which mechanism eventually drives behavior. Consider, for example, a policy maker observing that a considerable fraction of the population is reluctant to save for retirement. While compulsory savings plans forcing agents to pay a given fraction of their labor income into a savings account might work well for agents with hyperbolic preferences or optimistic beliefs, such plans may have adverse effects for rational agents. Compelling rational agents to allocate parts of their income to illiquid assets can potentially direct them into an even more difficult situation of poverty, from which it gets even harder to escape. Alternative policy measures can be less problematic, as they only affect irrational agents, but not rational ones. Incentive schemes, such as tax deductions on income transferred to a savings account, can help those with hyperbolic preferences or optimistic beliefs to save more while they have no effect on rational agents. However, such policies won't help the latter agents to get access to the liquidity they urgently need. Defaults, which leave the agent with the option to opt out of a savings plan, might have similar effects. Still, as opting out typically comes at a cost, they might put rational agents worse off compared to irrational agents not making use of this possibility. Such defaults are therefore not optimal when costs such as transaction costs or asymmetric cost of account withdrawal come into play. Policies such as commitment devices will not help most agents, but only those who are aware of their notoriously dynamically inconsistent behavior. Information is possibly the least problematic policy instrument. It can help agents with systematically biased beliefs to make better choices, but will improve neither choices of agents with hyperbolic preferences nor those of rational agents. That similar discounting behavior can be produced by both, rational and irrational agents is key, and a better understanding of the actual mechanism driving the apparent violations of exponential discounted utility can help developing appropriate policy measures.

The role of the institutional environment can arguably not be neglected. Access to micro-credit, for example, might assist agents to find out of (transitory) liquidity constraints which were caused by severe wealth shocks in the past. Facilitated access to liquidity might therefore be crucial for rational agents to find their way out of poverty. To make another example, people are often reluctant to purchase energy-efficient durables. The temporal shortage of liquidity can be one reason for why they are not willing to consider the long-run cost savings of more efficient alternatives (see e.g. Epper, Fehr-Duda, and Schubert (2011)). If this is the case, providing further information, e.g. in the form of energy labels, will not help to encourage energy-efficient investments. Nevertheless, alternative ways of financing these investments could be a solution.

This brings us back to the question which of the three competing mechanisms is the most plausible. In the last section, we reviewed evidence in favor of the rational model. We do not conjecture that this model explains all the evidence, but suspect that a boundedly rational model with λ closer to one than zero is possibly best in line with the empirical findings (see Section 4.2). While there is evidence for optimistic bias, the nature of static and dynamic preference reversals clearly speak in favor of the hypothesis that some considerations captured by the rational model are relevant.

If departures from exponential discounting are predominantly driven by optimism, then one might argue that it does not matter whether we model intertemporal choices as the outcome of beliefs (as in the optimism model) or as the outcome of preferences (as in the hyperbolic preference model). However, our results demonstrate that hyperbolic preferences are only suited well as a reduced form when the focus lies on excessive short-run discounting, but variation in outcome magnitude and outcome sign can be ruled out. It seems that we first need further empirical tests of the above mechanisms before drawing a final conclusion.

5 Conclusion

We introduce a novel and unifying rationalization of "anomalies" in intertemporal choice. We find that subjective expectations can have substantial impact on agents' discounting behavior and that they can drive systematic departures from exponential discounting beyond those predicted by other approaches. In the first part of the paper we discuss the case of a fully rational agent with standard preferences, and we show that all the apparently puzzling behavioral patterns naturally occur as a result of an interplay between liquidity constraints and positive income expectations. We review empirical evidence in line with this mechanism, and point at novel and testable predictions this model makes.

Our approach differs from previous attempts to accommodate "anomalies" in intertemporal choice. In behavioral economics, for example, actual behavior is typically contrasted with standard economic preferences, while other potentially important factors affecting choice behavior are left aside. We argue that the agent's constraints and expectations cannot be neglected when one wants to understand individual decision making and its variation across different domains. Indeed, the fact that some commodities are not tradable might be key for interpreting differences between intertemporal choices involving monetary and primary outcomes. Moreover, there are many behavioral patterns which are hard to reconcile with stable hyperbolic preferences. One example is people's reluctance to buy health insurance when they are in a good state of health, but their willingness to spend a very large part of their wealth for health care once they got diagnosed a life-threatening disease. It seems to be more plausible that this change in behavior is driven by a change in constraints than by an instability of 'deep' time preferences.

The fact that anomalously-appearing behavior is not necessarily driven by irrational types, but might well be caused by fully rational agents considering their constraints poses some challenges for applied economics. It suggests that constant discount rates are not the proper criterion for identifying rational types, and that it might be problematic to attribute all departures from exponential discounting to exotic 'deep' preferences.

In the second part of the paper, we relax the rationality assumption and show that our key predictions remain intact for the cases of bounded rationality and optimistically biased expectations. There are ways to distinguish hyperbolic preferences from optimistic bias and our rational mechanism, and this is important as all three models have very distinct economic implications. We hope to see some direct empirical tests of the above described mechanism in the future.

A Appendix

A.1 Extension III: Stochastic Consumption Plans

In the main text, we assume that future consumption is *a priori* known. As we show in this appendix, our qualitative predictions remain robust if one allows for uncertainty in the consumption plan. This extension potentially leads a series of new comparative static results which are beyond the scope of the model we discussed so far.

There are two reasons for a stochastic component in consumption plans. First, future income is typically subject to uncertainty. For example, a student in search for a job cannot be sure about the wage offers she will receive in the future. Similarly, many employment contracts contain a variable wage component, such as tips, bonuses or other performance-dependent payments, making it impossible to perfectly predict future labor income. Further, an employee might face a chance of getting promoted and, hence, might expect a positive likelihood to move into a higher wage bracket, or she might consider the possibility of getting unemployed.

Second, unanticipated and substantial changes in wealth could alter the agent's access to liquidity. Examples are positive wealth shocks, such as inheritance, or negative wealth shocks, such as a large and unexpected loss of (uninsured) assets. Both, the likelihood of income shocks and the likelihood of wealth shocks should be reflected in the agent's consumption plan, and, hence, the agent's discount rates (see e.g. Haushofer, Schunk, and Fehr (2013) for some related empirical evidence).

There are different ways to incorporate uncertainty in our model. The possibly simplest approach is to recognize that for any uncertain (ex-ante) consumption plan there exists a certainty-equivalent consumption plan. Formally, suppose that for any point in time t, there is a stochastic consumption level $\tilde{g}_t = g_t + \varepsilon$ with $\mathbb{E}[\varepsilon \mid g_t] = 0$ and ε being independently and identically distributed noise. The agent is indifferent between the degenerate consumption level, i.e. the certainty equivalent q_t, and the stochastic consumption level, i.e. the random variable $\tilde{g}_t$, such that $q_t \sim \tilde{g}_t$. To make one further assumption explicit, we presume that both, risk aversion and consumption smoothing are entailed in the instantaneous utility function u. We do this for simplicity, and to demonstrate the effect the introduction of risk and risk aversion has on intertemporal choices.

We now consider the situation where a newly available and certain alternative x_t is evaluated against the ex-ante consumption plan.[26] Note that under the above assumptions, the following equivalency holds:

$$U_0(x_t) = \delta^t \Big(u(q_t + x_t) - u(q_t) \Big) = \mathbb{E}\left[\delta^t \Big(u(g_t + x_t) - u(g_t) \Big) \Big| I_0 \right] . \qquad (10)$$

[26] We do not discuss the case where x_t is uncertain, as this has been done before. Epper and Fehr-Duda (2014) and Epper and Fehr-Duda (forthcoming) analyze this situation theoretically, and Epper, Fehr-Duda, and Bruhin (2011) and Andreoni and Sprenger (2012b) provide empirical results. A more complicated, but potentially promising future direction is where both x_t and c_t are stochastic and one is a hedge against the other.

A risk averse agent is willing to accept a certainty-equivalent consumption plan which lies below the expectation of the stochastic consumption plan. That is, for any point in time t, we have $q_t \leq \mathbb{E}[\tilde{g}_t]$. q_t gets smaller, the comparatively more risk averse the agent is, i.e. the more the agent dislikes spread, ceteris paribus. Similarly, holding risk preferences fixed, increasing risk in the sense of Rothschild and Stiglitz (1970) has similar effects, i.e. q_t shrinks as $\tilde{g}_t$ gets more risky, ceteris paribus.

The following key results naturally arise from Equation 10 and Jensen's inequality. First, holding all other things fixed, comparatively more risk averse agents will exhibit less pronounced changes in the certainty-equivalent consumption plan than others. This is the case as the evaluation of new alternatives involves consumption changes relative to the point in time the decision is made. Only today is certain, but future consumption is always subject to some risk. Risk aversion, however, lets q_t diminish, such that the effect on discount rates and the above discussed "anomalies" gets relatively less pronounced.

Second, increases in risk, by means of adding more noise to g_t, have similar effect. More uncertainty in the consumption plan means less pronounced declines in discount rates, a less pronounced magnitude effect and a less pronounced sign effect, ceteris paribus. These results contrast the findings in Epper and Fehr-Duda (2014) who examine the risk of ending up with an outcome lower than the one promised (survival risk) and the risk attached to receiving the outcome itself (prospect risk). Survival risk together with Allais common ratio violations can generate both, hyperbolic-like discounting, a magnitude effect as well as many other important interaction effects between time and risk. However, this form of risk can generate and even amplify these effects, while the risk in the consumption plan discussed here has the opposite, i.e. a compressing, effect.

The effect of anticipated changes in consumption are therefore very different from the expectation of potential income or wealth shocks. In this paper, we are primarily interested in the former. Accumulation of sufficient wealth, i.e. precautionary savings, can help the agent to overcome both, anticipated low-income periods and unanticipated fluctuations in income. Borrowing money against future labor income or the purchase of insurance are ways to resolve such issues, when no sufficiently large buffer-stock exists.

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