Universität St.Gallen

Banks and Sovereigns: A Model of Mutual Contagion

Alexander Gruber, Michael Kogler

August 2016 Discussion Paper no. 2016-14

Banks and Sovereigns: A Model of Mutual Contagion[1]

Alexander Gruber, Michael Kogler

Author's address: Alexander Gruber
University of St.Gallen (FGN-HSG)
Varnbüelstrasse 19
CH-9000 St.Gallen
Phone +41 78 854 3242
Email   alexander.gruber@unisg.ch

Michael Kogler
University of St.Gallen (FGN-HSG)
Varnbüelstrasse 19
CH-9000 St.Gallen
Phone   +41 71 224 2156
Fax   +41 71 224 2887
Email   michael.kogler@unisg.ch

Abstract

The recent crisis has revealed that bank and sovereign risks are inherently intertwined. This paper develops a model of the bank-sovereign nexus to identify the main spillovers and to study the implications of guarantees and capital regulation. We show how banks' asset risk may trigger a sovereign default through taxation and deposit insurance. The latter can be contagious because of its cost or stabilizing by avoiding liquidation losses. Since sovereign risks receive preferential regulatory treatment, banks purchase government bonds. This creates the opportunity for adverse feedback loops such that a sovereign default is the very reason for bank failure.

Keywords

Sovereign Debt Crisis, Financial Risk, Contagion, Deposit Insurance

JEL Classification

G11, G21, G28, H63

1 Introduction

The recent financial crisis has emphatically demonstrated that bank and sovereign risks are inherently and inevitably intertwined. A crisis of the banking industry may trigger disastrous consequences for the economy as a whole and induce the governments to intervene. In fact, rescue packages for distressed banks and other systemically important financial institutions took center stage in many countries in the recent past. Given that the size of the banking sector often corresponds to a multiple of GDP, the sovereign exposure to such financial risks was exorbitant in many cases. This is especially true for the euro area where the fiscal responsibility for such interventions still lies within national borders although banks have long expanded beyond. As a result, public debt levels successively increased: According to Laeven and Valencia (2012), the public debt-to-GDP ratio increased by almost 20 percentage points in the euro area between 2008 and 2011; this increase was particularly sharp in Ireland (72pp), Greece (45pp), and Spain (31pp). The economic viability of the GIIPS - Greece, Ireland, Italy, Portugal and Spain - and their ability to repay their outstanding debt was suddenly at stake. The abrupt awareness of countries' vulnerability and possible sovereign defaults drove apart the bond spreads in the euro area. Banks, however, had built up a large sovereign exposure as documented by the 2014 EBA stress test: Belgian banks, for example, held euro area sovereign bonds worth 16 percent of total assets. Italian (14 percent) and German, Portuguese, and Spanish banks (between 10 and 13 percent) showed a similar exposure (ESRB, 2015). The preferential treatment of sovereign bonds in the Basel accords was certainly conducive to this trend. Especially in the GIIPS countries, bond holdings are characterized by a significant home bias: Domestic bonds represented 85 percent of banks' (Euro area) sovereign exposure in Italy, and 87 percent in Ireland, 93 percent in Spain and Portugal, and 98 percent in Greece (ESRB, 2015). As the creditworthiness of certain governments decreased, banks were forced to reappraise some of these positions and - as for Greece's debt haircut - to take real losses. A vicious spiral with negative spillovers from banks to sovereigns and *vice versa* emerged. An even more disastrous credit crunch and further contagion between euro area member states could so far only be averted by massive policy interventions and bailouts.

On closer inspection, the crisis thus revived our awareness for the inherent fragility of banks, which fund themselves with unparalleledly low levels of equity, and their unique

interconnectedness with other banks, sovereigns, and market participants. These characteristics set banks apart from ordinary companies and provide the basis for their systemic relevance. Although influential strands of literature consider this fragility a necessary disciplining device, it proved to be a source of financial instability associated with severe negative consequences such as bank runs and contagion. A more critical assessment has thus been advocated by Pfleiderer (2014), Admati and Pfleiderer (2010) and Admati and Hellwig (2013), who question the upside of this 'self-imposed' fragility and draw attention to the negative consequences for bank governance, financial stability, and welfare.

This paper contributes to the emerging literature on the bank-sovereign nexus in several ways: First of all, it develops a comprehensive theoretical framework that highlights the interplay of bank and sovereign risks by combining a fully-fledged model of banks, which are invested in risky assets, with a classical version of sovereign debt fragility with multiple equilibria. This allows us to capture the key mechanisms of contagion between banks and sovereigns, namely, government guarantees, taxation, and sovereign bond holdings. Importantly, the focus on risks which emerge from the bank's asset side captures a stylized fact of the recent crisis. After all, the latter originated in the sub-prime mortgage market. Existing literature on the bank-sovereign nexus has primarily dealt with contagion issues coming from the public sector. Furthermore, the paper explores the consequences of government guarantees for depositors on sovereign risk and domestic welfare, which sets it apart from other contributions that focus on the implications of *ex ante* bailouts à la Acharya et al. (2014). The welfare and risk effects crucially depend on avoiding the cost of a disorderly bank liquidation and on the possibility to shift the bailout cost onto foreign bondholders. Notably, we find that the provision of deposit insurance can either trigger or prevent a sovereign default. Finally, we investigate the implications of tighter capital requirements for bank and sovereign risks in a setting in which government bonds receive preferential treatment in the sense that they do not need to be backed by equity (as in Basel III): This setup provides strong incentives for banks to invest in such assets, which makes them sensitive to the fiscal state. In turn, this creates the possibility for adverse feedback loops in which banks may be weakened or even fail because the government defaults. Interestingly, relatively low levels of fiscal fragility may actually improve financial stability since higher bond returns provide a buffer which improves banks' robustness to poor loan performance. This relationship reverses,

however, when a certain level of fiscal fragility is exceeded. Stricter capital requirements are likely to enhance the resilience of sovereigns and banks in our set-up although the analysis also points at potential countervailing effects.

The remainder of this paper is organized as follows: Section 2 first reviews the related literature, and section 3 then introduces the model. Subsequently, section 4 characterizes potential equilibria and examines the consequences of providing government guarantees on sovereign risk and domestic welfare. Section 5 discusses a variant with capital regulation and section 6 eventually concludes.

2 Literature

This paper particularly relates to the literature on financial and sovereign debt fragility as well as to recent contributions on the interaction of bank and sovereign risks: Financial fragility is often modeled by a combination of risky bank assets and small equity. A tractable approach that exemplifies this key feature is a stochastic loan return as in Dermine (1986) and Boyd et al. (2009): Bad realizations of borrowers' returns translate into loan losses, which, if large enough, may wipe out a bank's equity. On the liquidity side, Diamond and Dybvig (1983) investigate the role of excess maturity transformation for banks' inherent susceptibility to runs. They show that a 'good' equilibrium with optimal risk sharing between depositors with different liquidity needs may give way to a 'bad' one, in which all depositors panic and withdraw their deposits.[1] Bank risks, however, must not be examined in isolation. Instead, they are intimately linked through at least two mechanisms: interbank lending and fire sales. Following Diamond and Dybvig (1983), Allen and Gale (2000) develop a network model of interbank lending. Although the latter is beneficial *per se* and allows for optimal risk sharing in order to withstand independent liquidity shocks, it may lead to contagion in case of correlated shocks. Depending on the network structure and the liquidation value of the bank's assets, the crisis of a single institution may then spread over to other banks and become systemic. Likewise, Shleifer and Vishny (1992, 2011) identify the contagious effect of fire sales: They argue that banks which face substantial liquidity withdrawals might be forced to quickly liquidate parts of their assets at a dislocated price. That, in turn, may cause a further deterioration of

---

[1]Another branch of the literature, for example, Diamond and Rajan (2000, 2005) emphasizes the importance of financial fragility as a commitment device in the presence of a hold-up problem.

other banks' balance sheets, which subsequently forces them to sell their assets as well; either because they violate regulatory standards or because depositors start to withdraw. Furthermore, Diamond and Rajan (2011) relate fire sales to the freeze of credit markets. They show that the prospect of future fire sales alone suffices to depress the current asset prices and to cause a 'seller's strike' *ex ante*. Eventually, Greenwood et al. (2015) develop a model of contagion through fire sales and focus on each bank's exposure and contribution to system-wide deleveraging.

Sovereign debt fragility on the other hand arises because a government's ability or willingness to repay its debt may depend on the interest rate, which, in turn, hinges on investors' expectations about future debt repayment. This gives rise to multiple equilibria and self-fulfilling debt crises: If investors are pessimistic about debt repayment, they require a high interest rate, which increases the debt burden and weakens fiscal stability thus justifying their pessimism. In a seminal contribution, Calvo (1988) shows that such a mechanism can be generated by the possibility of debt repudiation, which may lead to multiple equilibria. In our paper, we subsequently rely on a textbook version of this model by Romer (2001), who essentially replaces debt repudiation by a stochastic tax revenue. Detragiache (1996) shows that some of these equilibria materialize as a liquidity crisis while Cole and Kehoe (2000) focus on a so-called crisis zone where sovereign risk depends on market participants' expectations and study its fundamental determinants as well as optimal debt policy. The empirical relevance of multiple equilibria in the context of sovereign debt is documented, for example, in Reinhart and Rogoff (2011) or De Grauwe and Ji (2013).

Recent events have raised the need for a more integrated view on financial and sovereign debt fragility thereby laying the ground for topical research on the bank-sovereign nexus, to which this paper contributes. On the theoretical side, Bolton and Jeanne (2011), for example, stress the role of sovereign bonds as a collateral in interbank lending. Sovereign risk compromises this function and hampers a bank's lending capacity. An extension to a two-country model shows that banks tend to diversify their bond holdings and that this diversification, although beneficial *ex ante*, may trigger financial contagion *ex post*. In a similar spirit, Gennaioli et al. (2014) relate the strength of financial institutions to cross-country capital flows and the governments' decision to default. The authors conclude that better financial institutions increase capital inflows to a country and reduce

6

the attractiveness of government default. Cooper and Nikolov (2013) connect the model of sovereign debt fragility by Calvo (1988) with the model of bank fragility by Diamond and Dybvig (1983) and focus on two channels of mutual contagion: banks' sovereign bond holdings and explicit or implicit government guarantees. They find that a sudden drop in confidence in the sovereign's creditworthiness may abruptly shift the economy to a pessimistic equilibrium associated with costly bank runs. They also study the role of deposit insurance, which may prevent runs but also exacerbate a looming fiscal crisis. Using a global games approach, Leonello (2015) shows that government guarantees connect banks' withdrawal and governments' roll-over risks as the actions of depositors and bondholders become strategic complements. Guarantees may trigger a feedback loops between a banking and sovereign debt crisis. Motivated by the Irish example, Acharya et al. (2014) study the impact of bank bailouts on sovereign risk. A bailout alleviates the under-provision of financial services due to debt overhang but also provokes distortive taxation of the non-financial sector. The latter can be avoided by a sovereign default, which, however, further weakens the solvency of banks. The intimate linkages between financial and sovereign risk are also documented by empirical evidence: Acharya et al. (2014), for example, show that the recent crisis and the corresponding bailouts caused a risk transfer to the government while Battistini et al. (2013) point out the significant home bias of European banks' sovereign bond portfolios and its negative consequences. Similarly, Mody and Sandri (2012) provide evidence for the strong impact of the banking sector's performance on risk premia on euro area sovereign bonds. Furthermore, they highlight that problems in the banking sector exert particularly negative effects in countries with low growth prospects and high initial debt burdens. Bénassy-Quéré and Roussellet (2014) provide quantitative simulations to show how implicit government guarantees for systemic banks undermine fiscal sustainability measured by the gap between the tax rate necessary for a sustainable debt ratio and the current tax rate. They find that such guarantees tend to increase the tax gap but there is considerable heterogeneity across EU countries and depending on how the bailout cost is measured. Reinhart and Rogoff (2011) eventually demonstrate that these insights hold for a long-run perspective as well. Using data from nearly two centuries, they find that sovereign debt crises have been frequently preceded by banking crises in the past.

3   The Model

This section outlines the baseline model: The main source of risk in the economy is bank loans (e.g., mortgages, consumer or commercial loans, asset-backed securities) characterized by a stochastic return: Bad realizations, which may, for example, reflect a large share of non-performing loans or write-offs on asset-backed securities, may cause substantial losses that quickly wipe out a bank's small equity - a feature that captures the asset risk dimension of financial fragility. Bank risk may then spread to the sovereign through two channels - government guarantees and taxation - and may trigger a sovereign default, which, in turn, may exert adverse feedback effects due to the sovereign bond holdings of banks. Sovereign debt fragility is therefore the second source of risk in the economy. It arises due to the interaction of investors' expectations about sovereign risk and the required return on government bonds. Hence, multiple equilibria, which differ in the extent and mechanisms of bank-sovereign contagion, may emerge.

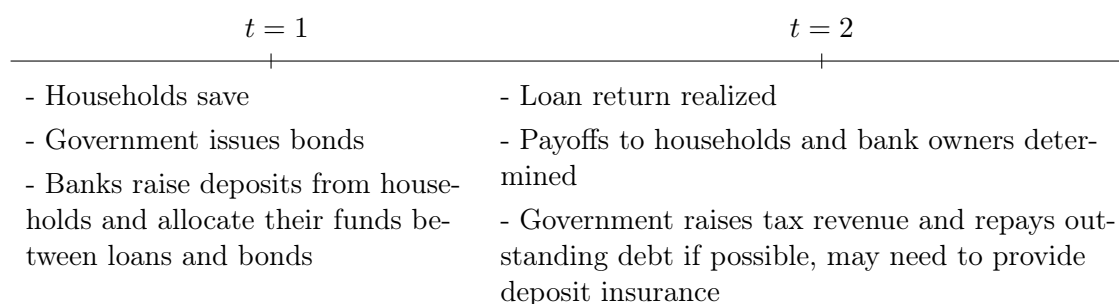| $t = 1$	$t = 2$
- Households save	- Loan return realized
- Government issues bonds	- Payoffs to households and bank owners determined
- Banks raise deposits from households and allocate their funds between loans and bonds	- Government raises tax revenue and repays outstanding debt if possible, may need to provide deposit insurance

Figure 1: *Time Line*

There are two periods and the model economy is populated by three types of agents: First, there exists a continuum of measure one of identical *banks*. Each bank is funded by exogenous equity E and raises an amount D of deposits from households, which are protected by a deposit insurance scheme. A bank can invest into two types of assets: (i) bank loans which yield a stochastic return and (ii) sovereign bonds. Consistent with our focus on systemic crises, loan returns are correlated across banks. Bank owners are risk-neutral and protected by limited liability; they receive the bank's final-period equity and consume at date 2. Second, risk-averse, identical *households* derive utility from consumption at both dates. They earn labor income modeled as a deterministic endowment at both dates; income is larger at date 1 than at date 2: $W_1 > W_2$. In order to smooth consumption, they deposit savings D with the bank. Third, the *government*

8

assumes two roles: It issues sovereign bonds B to cover exogenous initial expenditures or to roll-over legacy debt; these are purchased by banks and risk-neutral international investors. In order to repay its outstanding obligations, the government raises tax revenue at date 2. Moreover, the government also provides deposit insurance, which is tax-funded and - if actually provided - equivalent to rescue package for distressed banks.

3.1 Banks

The main characteristic of banks in our model is that they operate a risky technology, a feature shared with Dermine (1986) and Boyd et al. (2009), and are endowed with little equity. These models essentially build on a lender-borrower framework à la Jaffee and Modigliani (1969), complemented with the risk of bank failure due to correlated loan returns and an oligopolistic loan market à la Cournot. Motivated by our focus on the bank-sovereign nexus, we replace the risk-free asset in Boyd et al. (2009) by sovereign bonds, the risk-return profile of which endogenously emerges, and include equity to have a richer capital structure. To keep the analysis tractable, however, we rely on perfectly competitive banks and omit an explicit model of borrowers. These twists generate a framework in which the bank assumes an active role and allow us to derive novel insights about the mechanisms of contagion as well as the impact of government guarantees.

The bank funds itself by exogenous equity E and deposits D raised from households. Since deposits are insured, they earn the (gross) risk-free interest rate normalized to one. The bank allocates these funds among two assets: First, an amount L is invested in loans that are characterized by assumption 1.

ASSUMPTION 1 *Loans yield a stochastic (gross) return A per unit; $A \in [0, \bar{A}]$ is distributed according to some continuous twice-differentiable distribution function $F(A)$ with $E(A) = \int_0^{\bar{A}} A dF(A) > 1$. Conditional on bank failure, depositors can recover at most a liquidation value vA with $v \leq 1$.*

Hence, loans are risky and may trigger bank failure in case they perform poorly. They can be interpreted as credit to small businesses that invest in risky projects. Assumption 1 implies that the liquidation of bank loans is costly; $v < 1$ may, for instance, represent a bank run scenario in which a shock triggers an immediate, disorderly liquidation of the bank. Assets may then have to be sold at a dislocated price. Alternatively, suppose

that loan collection requires specific skills as in Diamond and Rajan (2000). If the bank fails, its owners receive a zero payoff and depositors cannot force them to use the bank's capabilities on their behalf such that they lose a fraction of each loan's value. Second, the bank can purchase an amount G of sovereign bonds with a binary payoff $\widetilde{R}$ which equals $R \geq 1$ (per unit) if the government is solvent (with probability $1 - p$) and zero otherwise (with probability p). The bank observes the return on sovereign bonds R as well as the sovereign default probability p, both of which it takes as given. Since bank owners are protected by limited liability and only consider the upside of their bank's investments, the bank maximizes its expected equity value $E[\max\{\pi, 0\}]$ by solving:

PROGRAM 1 *The bank chooses loans L, sovereign bonds G, and deposits D to maximize its expected equity value*

$$\max_{L,G,D} \int_{A^*}^{\bar{A}} AL + \widetilde{R}G - D \, dF(A) \tag{1}$$

subject to a funding constraint

$$L + G = E + D \tag{2}$$

A^* *is the minimum realization for which the bank succeeds (failure threshold):*

$$A^* = \max\left\{\frac{D - \tilde{R}G}{L}, 0\right\} \tag{3}$$

The bank fails as soon whenever the stochastic loan return falls short of A^*; this threshold crucially depends on sovereign bond repayment. The latter is not exogenous as it depends on the realized loan return. Hence, the bank forms expectations about the bond repayment conditional on its own performance: If bank and sovereign risks were independent, the bank would simply earn an *ex ante* return on bonds equal to $[1 - F(A^*)](1 - p)R$, that is, the probability that it succeeds times the expected return on sovereign bonds. Since loans are the main source of risk in the economy, however, bank and sovereign risks are interconnected. Consequently, the bank determines the asset allocation using the probability of bond repayment conditional on its own survival, $1 - p_C$, instead of the 'true' repayment probability, $1 - p$, such that its expected return on sovereign bonds equals $[1 - F(A^*)](1 - p_C)R$. From Bayes' theorem, the conditional default probability

$p_C = Prob(Bonds\ not\ repaid|Bank\ survives)$ is:

$$p_C = \frac{Prob(Bonds\ not\ repaid,\ bank\ survives)}{Prob(Bank\ survives)} = \frac{\max\{\int_{A^*}^{\hat{A}} dF(A), 0\}}{1 - F(A^*)} = \frac{\max\{p - F(A^*), 0\}}{1 - F(A^*)}$$

$\hat{A} = F^{-1}(p)$ denotes the realization of the stochastic loan return consistent with the sovereign default probability p. The integral captures all realizations of the stochastic loan return for which the bank survives and the government defaults (i.e., $A^* \leq A < \hat{A}$). Figure 2 illustrates how bank risk depends on the repayment of sovereign bonds for two given values of $\hat{A}$.
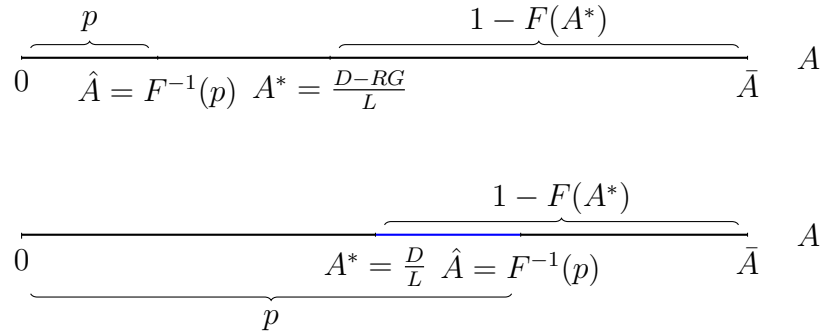

Figure 2: *Bank Risk and Bond Repayment*

If sovereign is lower than bank risk such that bonds are always repaid as long as the bank succeeds (i.e., if $p \leq F(A^*)$ and $\widetilde{R} = R$ at $A = A^*$) as shown in the upper part, the conditional default probability, p_C, is zero because for no realization banks survive and the government defaults. Thus, the bank essentially considers them risk-free and earns an *ex ante* bond return $[1 - F(A^*)]R$. If, in contrast, sovereign risk is higher than bank risk (i.e., if $p > F(A^*)$ and $\widetilde{R} = 0$ at $A = A^*$) as shown in the lower part, there is a possibility that the bank survives but bonds are not repaid (shown in blue). The conditional default probability equals $p_C = \frac{p - F(A^*)}{1 - F(A^*)}$, and banks earn an *ex ante* bond return of $[1 - F(A^*)](1 - p_C)R = (1 - p)R$. The failure threshold is *ceteris paribus* higher due to losses on bonds. Combining the two cases, the expected bond return from the bank's perspective is $[1 - \max\{F(A^*), p\}]R$. The optimization problem is:

$$\max_{L,D} \int_{A^*}^{\bar{A}} ALdF(A) + [1 - \max\{F(A^*), p\}]R(D + E - L) - [1 - F(A^*)]D \qquad (4)$$

The first two terms capture the expected returns on loans and bonds, respectively, the third term represents the expected repayment to depositors. Moreover, one can determine

which of the two cases explained above materializes (i.e., whether $F(A^*) \geq p$ or $F(A^*) < p$) based on the sovereign default probability p: The bank's failure thresholds depending on bond repayment - $A^*_{|\tilde{R}=R}$ and $A^*_{|\tilde{R}=0}$ - are illustrated in figure 3. As noted above, a sovereign default immediately shifts up this threshold and weakens the bank's capacity to withstand a poor loan performance. As long as bonds are repaid (i.e., if the government honors its debt for $A = A^*$), holding them reduces the bank's exposure to loan risk, and it can withstand worse realizations of A. Hence, bank risk is higher whenever a bank holds many loans and only few bonds such that the failure threshold (lower curve) increases in loans. In this scenario, a bank features the highest possible risk level, which corresponds to $A^* = \frac{D}{D+E}$, in case it does not hold any sovereign bonds but only loans (i.e., $L = D + E$ and $G = 0$). As soon as the bonds are not repaid, however, banks are more vulnerable because large bond holdings merely translate into losses such that the failure threshold (upper curve) decreases in loans. In this case, $A^* = \frac{D}{D+E}$ exactly denotes the minimum feasible level of bank risk instead, which materializes if the bank is exclusively invested in loans. One can thus define a critical probability of sovereign default:

$$\bar{p} = F\left(\frac{D}{D+E}\right)$$

Since bank and sovereign risks are interconnected, bond repayment is endogenous and related to banks' risk profile: First, if $p \leq F(A^*)$, bond repayment at $A = A^*$ requires that $p < \bar{p}$. Otherwise, p would exceed the *highest* possible level of bank risk in this case, $\bar{p}$, and contradict the initial assumption that bonds are repaid if the bank succeeds, $p \leq F(A^*)$. Second, if $p > F(A^*)$, no repayment at $A = A^*$ requires $p \geq \bar{p}$ as p would otherwise lie below the *lowest* possible level of bank risk, $\bar{p}$, again violating the initial assumption. Consequently, the bank's failure threshold can be defined in terms of the sovereign default probability p:

$$A^* = \begin{cases} \max\left\{\frac{D-R(D+E-L)}{L}, F^{-1}(p)\right\}, & \text{if } p \leq \bar{p} \\ \min\left\{\frac{D}{L}, F^{-1}(p)\right\}, & \text{if } p > \bar{p} \end{cases} \tag{5}$$

Intuitively, $p \leq \bar{p}$ implies that the bank is at least as vulnerable as the government and that the latter can withstand a worse realization of A. Hence, the bank still receives the bond repayment at the failure threshold (i.e., $\tilde{R} = R$). The reverse is true for $p > \bar{p}$.
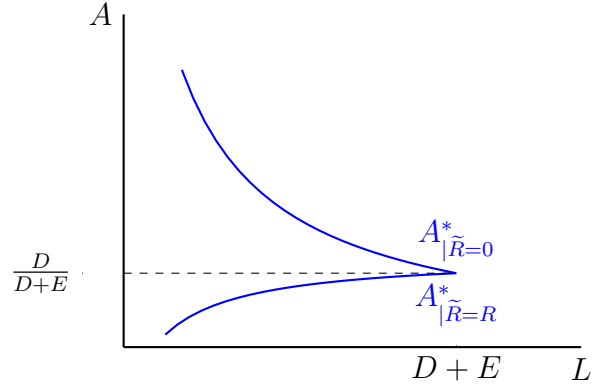
Figure 3: *Bank's Failure Threshold*

From figure 3, one may also conclude that whenever, for a given bond return, the sovereign default threshold $\hat{A} = F^{-1}(p)$ is in the area below (above) the lower (upper) of these two curves, bank failure is more (less) likely than a sovereign default as the government withstands a worse realization of the loan return. Moreover, if it lies in the area between the two curves, the bank will survive as long as bonds are repaid (since $A^*_{|\widetilde{R}=R} < A$) but will fail as soon as they are not (since $A^*_{|\widetilde{R}=0} > A$). Hence, the bank fails as soon as the government does not repay the bonds. Such a case captures the idea of an adverse feedback as a sovereign default immediately pushes banks into bankruptcy.

One can solve the optimization problem (4) separately for each using to derive optimal bank size and asset allocation. Since the objective function increases in deposits, the deposit demand D is perfectly elastic at the risk-free interest rate.[2] As a result, the bank is willing to accept any amount of deposits such that its size $D + E$ is predetermined by equity endowment and household savings. Similarly, expected bank profits are a linear or convex function of loans, which implies that no interior maximum $L \in (0, D + E)$ exists. This feature essentially reduces the problem to a binary comparison of expected profits from exclusively investing in either loans ($L = D + E$) or sovereign bonds ($L = 0$).[3] A bank chooses the former as long as $\pi(D + E) \geq \pi(0)$. The results are summarized in:

LEMMA 1 *The bank's deposit demand D is perfectly elastic. The cutoff R' is given by*

$$R' = \frac{1}{1-p} \left[E(A) + \int_0^{\frac{D}{D+E}} F(A)dA - \frac{pD}{D+E} \right] \tag{6}$$

---

[2] This feature that keeps the analysis tractable arises due to perfect competition and no convex costs. In Boyd et al. (2009), for example, Cournot competition ensures an interior solution.

[3] This is consistent with the finding of Rochet (1992) that in the presence of deposit insurance without actuarially fair premia, value-maximizing banks have a convex objective function and fully specialize in one risky asset.

It decreases in equity E if $p < \bar{p}$ and increases if $p > \bar{p}$. The bank provides an amount

$$L = \begin{cases} D + E, & \text{if } R \leq R' \\ 0, & \text{if } R > R' \end{cases} \tag{7}$$

of loans and invests $G = D + E - L$ in sovereign bonds. Its failure threshold is:

$$A^* = \begin{cases} \frac{D}{D+E}, & \text{if } R \leq R' \\ F^{-1}(p), & \text{if } R > R' \end{cases} \tag{8}$$

Proof: See Appendix A.1.

The bank invests in the asset that promises a higher expected return as illustrated in figure 4: The shaded area represents allocations for which the bank exclusively holds loans, and R' defines the critical bond return such that it is indifferent between the two assets. As soon as the bond return exceeds R', the bank purchases sovereign bonds only.
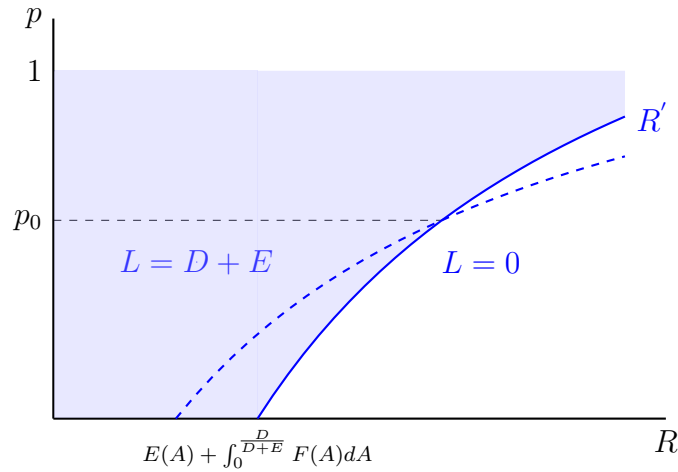
Figure 4: *Bank's Asset Allocation*

The cut-off critically depends on the two assets' risk-return profile: If the likelihood of a sovereign default p increases, banks are only willing to buy bonds when they are compensated by a higher return. In the absence of limited liability, R' would simply be pinned down by the equalization of expected returns, $E(A) = (1 - p)R$, represented by the dashed line in figure 4. Limited liability distorts this choice, which is captured by the

14

second and third term of expression (6), such that the bank tends to invest in a riskier allocation. More precisely, the choice at the extensive margin is distorted in favor of loans if bonds are relatively safe, $p < p_0 \equiv \frac{D+E}{D} \int_0^{\frac{D}{D+E}} F(A)dA$, and vice versa.

3.2 Households

Households consume C_t and earn labor income W_t at both dates, where $W_1 > W_2$. There is no discounting. Since their income at date 1 is higher and households smooth consumption, they save and deposit their savings D with the bank. Their date 2-consumption is subject to taxation with tax rate t such that consumption spending is $(1+t)C_2$. Households consider deposits safe and deposit insurance credible.[4] Deposits earn the (gross) risk-free rate normalized to one such that the optimization problem is:

$$\max_D u \underbrace{(W_1 - D)}_{C_1} + Eu \underbrace{\left(\frac{W_2 + D}{1+t} \right)}_{C_2} \tag{9}$$

We rely on the logarithmic utility function $u(C_t) = log(C_t)$ to keep the analysis tractable as households' decisions are independent of the tax rate: Income and substitution effect just offset each other, and the choice is independent of the uncertain date 2-tax rate.

LEMMA 2 *Due to logarithmic utility, savings amount to*

$$D = \frac{W_1 - W_2}{2} \tag{10}$$

and do not depend on the tax rate. Households' consumption is $C_1 = (W_1 + W_2)/2$ and $C_2 = (W_1 + W_2)/2(1+t)$.

Proof: Follows from the first-order condition of (9) using the log utility function. *Q.E.D.*

3.3 Government

The government's role is essentially shaped by three key characteristics: debt, taxes, and default. First of all, the government issues an exogenous amount B of sovereign bonds at date 1 either to roll over legacy debt or to cover initial expenditures. These bonds

---

[4]The government has the fiscal capacity (Assumption 3).

promise a gross return R and are sold to domestic banks and to risk-neutral international investors. The former's demand equals G, the latter's is perfectly elastic as long as they earn an expected bond return that equals the risk-free (gross) interest rate:

$$(1-p)R = 1 \tag{11}$$

This key condition ensures 'fair' pricing of sovereign bonds. The presence of foreign investors is crucial in this regard as the bank's asset allocation is distorted by limited liability and risk-averse households refrain from buying risky bonds in general.[5] We therefore impose the following assumption on the bond volume B:

ASSUMPTION 2 $B > \frac{W_1 - W_2}{2} + E$

This ensures that the amount of available government bonds is large enough to meet the demand of domestic banks even if they invest all their funds in sovereign bonds (i.e., if $G = D + E$) and that a fraction of bonds is held by foreign investors. The share of these securities held by domestic banks is therefore defined as:

$$\omega = \frac{G}{B}$$

Moreover, the government raises taxes from households and bank owners at date 2 in order to (i) repay its debt and (ii) to fund the deposit insurance scheme if necessary. In principle, the tax is designed as a consumption tax[6] t but it is subsequently expressed in terms of the equivalent income tax $\tau \in [0, 1]$, which is more intuitive.[7] The tax rate τ guarantees a balanced budget. It is, however, constrained by an upper bound $\bar{\tau} \leq 1$. $\bar{\tau} = 1$ seems to be a natural maximum for the tax capacity although institutional limitations, tax evasion and other frictions may in fact justify a smaller ceiling. This idea is related to Cooper and Nikolov (2013) although, in their model, the ceiling is stochastic and the very source of sovereign risk. The tax ceiling $\bar{\tau}$ satisfies two conditions:

ASSUMPTION 3 $(i)\ \bar{\tau} \geq \frac{2B}{W_1 + W_2}, \qquad (ii)\ W_2 > E$

---

[5] To be indifferent between deposits which are considered safe due to deposit insurance, they would require an additional risk premium.

[6] This is to keep the analysis tractable. A classical income tax would make deposits sensitive to the tax rate (see section 3.2) and require households to correctly anticipate the tax policy depending on bank and sovereign risks.

[7] Recall the relationship between τ and the consumption tax t, i.e., $1 - \tau = \frac{1}{1+t}$.

Whereas the former guarantees - in conjunction with assumption 2 - that deposit insurance is feasible and credible even for a complete loss on loans,[8] the latter ensures that $\bar{\tau} \leq 1$ is indeed possible. Yet, the government may eventually default even though it manages to successfully bail-out depositors. This occurs if it fails to raise sufficient tax revenue to cover all cost, namely, deposit insurance and outstanding debt. Importantly, default entails a full haircut on sovereign bonds, which is a common assumption in related models such as Cooper and Nikolov (2013). Deposit insurance, in contrast, is still provided if necessary.

The very reason of a sovereign default is therefore the government's two-way exposure to the risky banks loans: After all, loan performance (i.e., the realization of the stochastic return A) influences (i) date 2-consumption of bank owners and the tax base as well as (ii) the cost of providing deposit insurance in case of bank failure. Hence, a sovereign default eventually occurs due to weak fundamentals rather than strategic considerations like in Calvo (1988). One can derive a precise sovereign default threshold $\hat{A}$: The government repays its debt if the realized loan return exceeds $\hat{A}$ and defaults otherwise. This threshold determines the default probability p:

$$p = F(\hat{A}) \tag{12}$$

First of all, suppose that the bank survives because its loan portfolio performs well (i.e., $A \geq A^*$). Deposit insurance is not needed in such a scenario and the government's date 2 expenditures entirely consist of the debt repayment. Taxes are levied on consumption spending of both households, $W_2 + D$, and bank owners, $AL + RG - D$, such that the tax rate follows from the balanced budget condition:

$$BR = \tau[AL + R(D + E - L) + W_2] \tag{13}$$

As soon as the level of τ implied by this condition exceeds the ceiling $\bar{\tau}$, the government defaults because it would need to impose an unfeasibly high tax rate to collect sufficient revenue. The reason for that is the low tax base due to insufficient dividend income of

---

[8]In an extreme case in which loans completely fail and sovereign bonds are not repaid, the cost of deposit insurance is D. Substituting for B in the first inequality using assumption 2; maximum date 2 tax revenue $\bar{\tau}(D + W_2)$ exceeds the cost.

bank owners. Substituting for τ in expression (13) yields the sovereign default threshold:

$$\hat{A}_{|A \geq A^*} = \max \left\{ \frac{D - R(D + E - L)}{L} + \frac{BR - \bar{\tau}(D + W_2)}{\bar{\tau} L}, 0 \right\} \tag{14}$$

Second, suppose that banks fail due to poor loan performance (i.e., $A < A^*$). The government incurs costs of deposit insurance which equal guaranteed deposits net of the residual value of bank assets:

$$DC(A) = D - AL - R(D + E - L) \tag{15}$$

Added to the bond repayment, they constitute the second part of the government's date 2 expenditures. The balanced budget condition pins down the tax rate:

$$BR + DC(A) = \tau(D + W_2) \tag{16}$$

Again, the government defaults whenever $\tau \geq \bar{\tau}$. But as opposed to (13) above, the constellation is now relatively worse since the tax base is lower and depositors have to be bailed out. Combining (15) and (16) yields the sovereign default threshold:

$$\hat{A}_{|A < A^*} = \max \left\{ \frac{D - R(D + E - L)}{L} + \frac{BR - \bar{\tau}(D + W_2)}{L}, 0 \right\} \tag{17}$$

The government's default threshold $\hat{A}$ as well as the bank failure threshold A^*, which follows from (5), are illustrated by the blue and red lines in figure 5, respectively. This reveals the existence of three possible outcomes: First, the government tends to be relatively more stable than banks (i.e., $\hat{A} < A^*$) whenever the interest rate on its debt imposed by foreign bondholders is relatively low and falls short of the cutoff R_0. This corresponds to classical bank-sovereign contagion as a poor loan performance causes bank failure, which may eventually trigger a sovereign default because of deposit insurance cost. Second, the government is less stable than banks (i.e., $\hat{A} > A^*$) in case the bond return is relatively large and exceeds the cutoff R_1. This represents an outcome where the debt burden is so large that the government may even default in the absence of bank failure; low bank dividends and tax revenue are sufficient to trigger a sovereign default. Third, bank and sovereign risks coincide in an interim region, $R_0 < R < R_1$: Banks would

survive for the loan return $A = \hat{A}$ but fail as soon as they incur losses on their sovereign bond holdings. This captures an adverse feedback that arises because a mediocre loan performance triggers a sovereign default, which, in turn, puts banks in jeopardy.
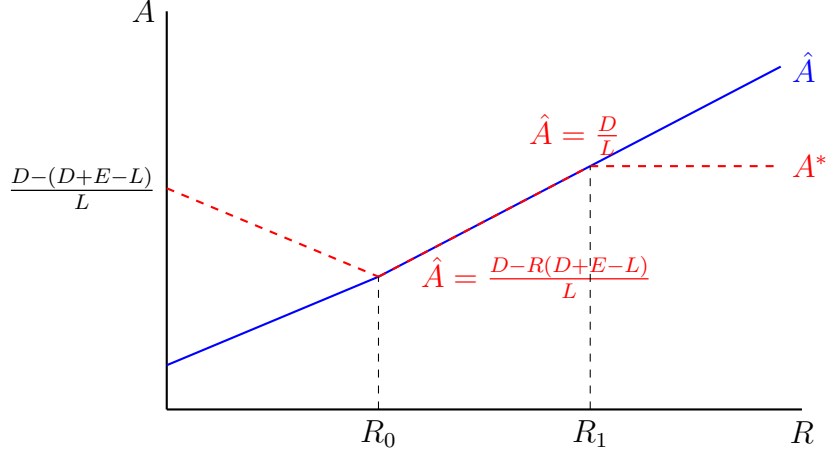

Figure 5: *Sovereign Default Threshold*

Note that these three cases exactly correspond to those in figure 3 but are now characterized in terms of the bond return. It can be shown that the default threshold $\hat{A}$ is described by (17) for $R \leq R_0$ and by (14) for $R > R_0$. Graphically, the corresponding curves intersect at the cutoff $R = R_0$; the sovereign default threshold represented by the solid blue line has a kink but is continuous. Both cutoffs follow from $\hat{A} = A^*$, that is, equalizing (17) and $A^* = \frac{D-R(D+E-L)}{L}$ as well as (14) and $A^* = \frac{D}{L}$. Using $G = \omega B$ yields:

$$R_0 = \frac{\bar{\tau}(D + W_2)}{B} \tag{18}$$

$$R_1 = \frac{\bar{\tau}(D + W_2)}{(1 - \omega\bar{\tau})B} \tag{19}$$

Note that $R_0 > 1$ is due to the first part of assumption 3. Obviously, these two cutoffs coincide whenever banks do not hold any sovereign bonds (i.e., $\omega = 0$): In such a case, the third outcome, which entails an adverse feedback, vanishes as banks are not exposed to sovereign risk at all. The latter scenario is, in contrast, more likely to be an equilibrium outcome if banks hold a large share of sovereign bonds (i.e., ω and R_2 are large). Consequently, one may rewrite the government's default threshold as

$$\hat{A} = \begin{cases} \frac{D-R(D+E-L)}{L} + \max\left\{\frac{BR-\bar{\tau}(D+W_2)}{L}, \frac{BR-\bar{\tau}(D+W_2)}{\bar{\tau}L}\right\}, & \text{if } R \leq R_2 \\ \bar{A}, & \text{if } R > R_2 \end{cases} \tag{20}$$

19

where $R_2 = \frac{\bar{\tau}[\bar{A}L+W_2]}{(1-\omega\bar{\tau})B}$ denotes the bond return above which the government defaults with certainty (i.e., it defaults even if the maximum loan return $\bar{A}$ is realized). The first term in curly brackets is relevant if $R \leq R_0$ while the second expression is applicable for $R > R_0$. Obviously, in both cases, the sovereign default threshold positively depends on the debt burden BR and the size of the commitment to deposit insurance D, but negatively on the tax capacity $\bar{\tau}$ and the bank's assets L and $G = D + E - L$, which effectively reduces the costs of providing deposit insurance by raising the bank's liquidation value.

3.4   Market Clearing

At date 1, the markets for deposits and government bonds clear:

$$W_1 - C_1 = D, \quad B = G + (1-\omega)B$$

The deposit supply consists of households' labor income that is not consumed, the bond supply is exogenous; banks invest an amount G in bonds, and the remainder is purchased by international investors as long as the return is fair. Aggregating these constraints using the balance sheet identity $L + G = D + E$ yields the date 1 aggregate budget constraint

$$C_1 + L + B = W_1 + E + (1-\omega)B \tag{21}$$

which implies that consumption, investment (lending), and government expenditures are funded by the domestic endowment of households and bank owners and the capital inflow from international investors. At date 2, consumption of households and bankers equal:

$$C_2 = (1-\tau)(D+W_2), \quad C_2^B = (1-\tau)\max\{AL + G\widetilde{R} - D, 0\}$$

Substituting $\tau[D + W_2 + \max\{AL + G\widetilde{R} - D, 0\}] = B\widetilde{R} + \max\{D - AL - G\widetilde{R}, 0\}$ from the government's budget constraint yields the aggregate budget constraint at date 2:

$$C_2 + C_2^B = W_2 + AL - (1-\omega)B\widetilde{R} \tag{22}$$

Consumption depends on the realization of the loan return A and on the bond repayment. In the absence of a sovereign default (i.e., $\widetilde{R} = R$), there is a capital outflow as bonds

20

are repaid to foreign investors. In case of a sovereign default, there is no such outflow. Combining (21) and (22) and noting that $E(\widetilde{R}) = (1-p)R = 1$ in equilibrium implies that expected consumption is financed by labor income, equity endowment, and the surplus earned on loans such that the model is closed:

$$C_1 + E(C_2) + E(C_2^B) = W_1 + W_2 + E + [E(A) - 1]L \tag{23}$$

4 Equilibrium Analysis

4.1 Equilibrium Allocation

Combining the optimal decisions of banks and households as well as the government's policy establishes the following proposition:

PROPOSITION 1 *The equilibrium allocation $\{A^*, \hat{A}, D, G, L, p, R, R'\}$ is characterized by conditions (2), (6) - (8), (10) - (12), and (20). From (20), the sovereign default threshold for $L = D + E$ is*

$$\hat{A}_{|L=D+E} = \begin{cases} \frac{D}{D+E} + \max\left\{\frac{BR - \bar{\tau}(D+W_2)}{D+E}, \frac{BR - \bar{\tau}(D+W_2)}{\bar{\tau}(D+E)}\right\}, & \text{if } R \leq R_2 \\ \bar{A}, & \text{if } R > R_2 \end{cases} \tag{24}$$

with $R_2 = \frac{\bar{\tau}[\bar{A}(D+E)+W_2]}{B}$. Two types of equilibria may exist:

- *The 'good' equilibrium with $p_g < 1$ and $R_g < R_2$ exists if (i) $\exists R \in [1, R_2)$ such that $F[\hat{A}_{|L=D+E}(R)] \leq 1 - \frac{1}{R}$ and (ii) $F[\hat{A}_{|L=D+E}(R_g)] \leq \frac{D+E}{D}\left[E(A) + \int_0^{\frac{D}{D+E}} F(A)dA - 1\right]$.*

- *The 'bad' equilibrium with $p_b = 1$ and $R_b \to \infty$ always exists.*

In each equilibrium, banks exclusively hold loans such that $L = D + E$, $A^* = \frac{D}{D+E}$, and $\hat{A} = \hat{A}_{|L=D+E}$.

Proof: See Appendix A.1.

Multiple equilibria arise because investors' expectations about a sovereign default determine their required bond return, which, in turn, influences the government's debt-servicing cost and its default probability. Such dynamics may turn into a self-fulfilling

prophecy, which eventually results in one of the equilibria outlined in proposition 1. This is a standard occurrence in many models of public debt crises as exemplified in Romer (2001) and Cooper and Nikolov (2013). In our model, however, the uncertainty about the government's ability to repay originates from risky banks that either affect expenditures or tax revenue rather than a shock to the sovereign's fiscal position itself.
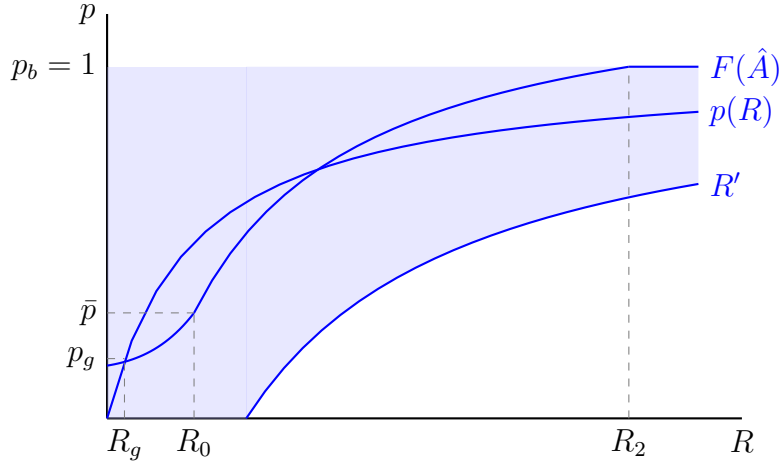

Figure 6: *Multiple Equilibria*

Figure 6 illustrates a combination with two equilibria given a bell-shaped density function:[9] The 'good' equilibrium is characterized by a low bond return R_g and a moderate default probability p_g. The 'bad' equilibrium features an infinitely high bond return for which the government defaults with certainty (i.e., $R_2 \to \infty$ and $p_b = 1$). The stability of these equilibria is consistent with the debt crisis model of Romer (2001). An additional equilibrium with intermediate bond return and default probability may exist conditional on the 'good' equilibrium. It is, however, unstable under plausible dynamics. Both equilibria are located in the shaded area above R', where banks prefer to hold loans such that only international investors purchase sovereign bonds. Graphically, an equilibrium is determined by the intersection of bond pricing and default curve $p(R)$ and $F(\hat{A})$. The latter is a simple transformation of the default threshold.

The result that a bank unconstrained by any regulatory requirements never purchases fairly priced, domestic sovereign bonds is one of the key insights of the baseline model and requires some comments: Sovereign bonds would need to yield a relatively high return $(R > R')$ in order to be more attractive than loans (i.e., to yield a higher expected

---

[9]Depending on the shape of the distribution function, more than two stable equilibria might exist (see Cooper and Nikolov (2013)) for a graphical illustration). The additional equilibria share features of the 'good' type.

return taking into account all effects of limited liability). Since foreign investors price government bonds fairly, however, such a high return would only be consistent if sovereign risk is comparatively high as well. As soon as banks only hold sovereign bonds, though, there is no risk in the economy, and the sovereign default probability equals zero implying a low bond return.[10] An equilibrium with sufficiently attractive bond returns for banks is, therefore, inconsistent with fair bond pricing.

Yet, we observe considerable sovereign bond holdings of the banking sector and a significant home bias in reality. This can be explained by several factors: First, deviations from fair pricing of bonds may offer attractive returns, at least temporarily. Central bank stimuli and other demand-side effects currently serve as important examples. Second, capital[11] and liquidity requirements may limit the lending capacity and force the bank to (partly) invest in alternative assets that might be associated with lower returns. Importantly, the Basel accords generally consider sovereign bonds as safe such that their risk weight is zero; they are also eligible for the new liquidity requirements. Third, sovereign bonds play an important role as a collateral for interbank borrowing and repo transactions, which provides another rationale.[12]

Intuitively, a country is likely to end up in the *'good' equilibrium* whenever it is fiscally sound, that is, its public debt level B is low or the tax capacity $\bar{\tau}$ high. Optimistic expectations about the sovereign's creditworthiness then translate into low debt-servicing cost. As in both equilibria, the bank exclusively holds loans, defaults with probability $\bar{p}$ and - depending on whether $R_g < R_0$ as illustrated in figure 6 or not - may be more or less stable than the government. In particular, the adverse feedback outcome discussed above is ruled out in the baseline allocation. The 'good' equilibrium exists as long as bond returns R exist, for which (i) the default lies below the bond pricing curve and the (ii) the bank prefers loans to bonds as $R \leq R'$. This holds true if the country is fiscally sound such that its default probability implied by the threshold $\hat{A}$ is small for low bond returns. Sovereign risk may even vanish in the 'good' equilibrium if the amount of outstanding bonds and deposit insurance obligations is lower than the potential tax income at date 2 even for a complete loss on loans, that is, if $B + D \leq \bar{\tau}(D + W_2)$ such that the default threshold is $\hat{A} = 0$ for $R = 1$ and sovereign bonds are indeed risk-free.

---

[10]Nevertheless, the 'bad' equilibrium may still exist but banks then exclusively hold loans.

[11]Capital regulation is explored in section 5.

[12]See Bolton and Jeanne (2011) for a model of interbank borrowing with risky sovereign bonds.

The *'bad' equilibrium*, in contrast, materializes when a self-fulfilling spiral of pessimism translates into an excessively high bond return such that the government defaults with certainty. Given that bonds are never repaid in this scenario, however, no investor is willing to purchase them in the first place. Since the bank is exclusively invested in loans anyway, it is always more stable than the government (i.e., $A^* < \hat{A} = \bar{A}$).[13] The 'bad' equilibrium is particularly relevant as soon as the 'good' does not exist: This may occur, for instance, in case of a highly indebted country with an insufficient tax capacity. Hence, its actual default probability exceeds the default probability implied by fair bond pricing for all finite values of R. Graphically, this means that default and bond pricing curve never intersect and only coincide in the limit.

4.2 Bank and Sovereign Risks

Since banks exclusively hold loans in equilibrium, they are not exposed to sovereign risk and thus insensitive to fiscal fundamentals. They fail whenever loans perform so poorly that their date 2 equity is wiped out and deposits are not covered anymore. Hence, the failure threshold equals the leverage ratio:

$$A^* = \frac{D}{D + E}$$

Obviously, bank risk increases in deposits, $\frac{\partial A^*}{\partial D} > 0$, and decreases in equity, $\frac{\partial A^*}{\partial E} < 0$. The latter provides a buffer to absorb loan losses and unambiguously lowers bank risk. Sovereign risk, in contrast, crucially depends on banks' loan performance and capital structure. Recall that there are two different cases how an equilibrium may emerge: First, banks may be more vulnerable than the government ($A^* \geq \hat{A}$). This is the case whenever the latter is fiscally sound such that it pays low interest rates in equilibrium, $R_g < R_0$, as illustrated in figure 6. Contagion then runs from the banking sector to the government and is driven by the cost of deposit insurance or rescue packages as it recently happened in Ireland and Spain. Second, banks may be more stable than the government ($A^* < \hat{A}$). This always occurs in the 'bad' equilibrium and can also be a property of the 'good' one if the debt servicing costs are rather high such that $R_g > R_0$. The tax potential of households is quite small in this scenario compared to the public debt level

---

[13]The government can still collect sufficient revenue to provide deposit insurance due to assumption 3.

B. Bank-sovereign contagion thus occurs because loans do not perform well enough such that bank dividends and the tax base are low. As a result, the government cannot raise sufficient revenue to repay its outstanding debt. This may, to some extent, capture the case of highly indebted countries like Italy and Greece, in which the tax base has often been small due to tax evasion and lax fiscal authorities. In general, the government defaults as soon as the necessary tax rate to cover date 2 expenditures (debt repayment BR and possibly deposit insurance cost DC) is no longer feasible. Its default threshold $\hat{A}$ follows directly from condition (24). A sovereign default involves a full haircut on sovereign bonds whereas deposit insurance is still provided. The sensitivities of default probability $p = F(\hat{A})$ and bond return R can be summarized as follows:

COROLLARY 1 *In the 'good' equilibrium, the sensitivities of the sovereign default probability are as follows: $\frac{\partial p}{\partial \bar{\tau}} < 0$, $\frac{\partial p}{\partial B} > 0$, and $\frac{\partial p}{\partial E} < 0$; these imply $\frac{\partial R}{\partial \bar{\tau}} < 0$, $\frac{\partial R}{\partial B} > 0$, and $\frac{\partial R}{\partial E} < 0$. Sovereign risk decreases in the tax capacity and bank equity but increases in the public debt burden.*

Proof: See Appendix A.1.

A higher tax capacity, a lower public debt burden, and a banking sector funded by more equity reduce sovereign risk in the 'good' equilibrium and thus depress its debt-servicing costs. This result is not surprising: A sound fiscal policy and a well-capitalized banking sector are widely considered to improve a country's fiscal stability. This is due to the fact that - in equilibrium - default probability and bond return need to be consistent with each other: If public debt B increases, for instance, the default probability rises as well. The bond return adjusts upwards until it is consistent with the higher sovereign risk.

4.3   Deposit Insurance and Sovereign Risk

Interestingly, one can make use of this model to show why government guarantees may either preserve fiscal stability by preventing costly bank failures or jeopardize it by putting the government itself into distress. The latter was, for instance, the case in Ireland and, to a lesser extent, in Spain. For that purpose, we compare sovereign risk in the baseline model in which deposit insurance is provided whenever necessary with sovereign risk in a hypothetical scenario in which the government deviates from its commitment and does not rescue distressed banks. The latter describes the relevant alternative to rescue

packages frequently employed in the current crisis. Yet, deposits were considered safe because the explicit or implicit government guarantees in place were credible. Hence, focusing on an *ex post* deviation from the guarantee better captures the alternative than an allocation without deposit insurance at all. While the sovereign default threshold in the baseline model is given by (20), the default threshold without deposit insurance, $\hat{A}_N$, is subsequently determined by

$$BR = \bar{\tau}[\ \underbrace{D + W_2}_{Households'\ Inc.}\ + \underbrace{\hat{A}_N(D + E) - D]}_{Bankers'\ Inc.} \tag{25}$$

or

$$BR = \bar{\tau}[\underbrace{W_2 + v\hat{A}_N(D + E)}_{Households'\ Inc.}] \tag{26}$$

depending on whether banks are solvent at $A = \hat{A}_N$. These conditions follow from the government's date 2 budget constraint. The solvency of banks matters because the assets' liquidation value is smaller than one ($v < 1$) in the absence of deposit insurance as described for assumption 1. This may be rationalized by a bank run that requires an immediate and costly liquidation of the assets. The default threshold for a government that deviates from its initial commitment follows from (25) and (26):

$$\hat{A}_N = \begin{cases} \frac{BR - \bar{\tau}W_2}{v\bar{\tau}(D+E)} & \text{if } R < \max\{R_0 - \frac{\bar{\tau}(1-v)D}{B}, 1\} \\ \frac{D}{D+E} & \text{if } \max\{R_0 - \frac{\bar{\tau}(1-v)D}{B}, 1\} \leq R < R_0 \\ \hat{A}, & \text{if } R \geq R_0 \end{cases} \tag{27}$$

The discontinuity is entirely due to $v < 1$, which causes a further erosion of the tax base as soon as banks fail. In fact, the liquidation costs associated with bank failure are the very reason for a sovereign default if $R \in \left[R_0 - \frac{\bar{\tau}(1-v)D}{B}, R_0\right]$. For $R \geq R_0$, the thresholds with and without deposit insurance just coincide as the government defaults because of insufficient tax revenue and is more vulnerable than banks anyway. Accordingly, a government that deviates defaults with probability $p_N = F(\hat{A}_N)$. Note that sovereign bonds are not fairly priced *ex post* in such a scenario.

Interestingly, it is not *a priori* clear how the provision of deposit insurance influences sovereign risk. This is because there are two countervailing effects. The choice to refrain from a bailout spares important expenses but also triggers considerable liquidation costs

captured by $v < 1$, which reduces the tax base. The magnitude of the latter is thus crucial for the impact of deposit insurance on sovereign risk.

PROPOSITION 2 *Deposit insurance lowers sovereign risk (i.e., $p < p_N$) if the loan liquidation value v is sufficiently small:*

$$v < \min \left\{ \frac{BR - \bar{\tau}W_2}{\bar{\tau}[BR - \bar{\tau}W_2 + (1 - \bar{\tau})D]}, 1 \right\} \equiv v_m(R) \qquad (28)$$

In the 'good' equilibrium both cases are possible; in the 'bad' equilibrium $p = p_N$ holds irrespective of the liquidation value v.

Proof: See Appendix A.1.

Proposition 2 is illustrated in figure 7: In the shaded area, the liquidation value of bank loans is small ($v \leq v_m$) such that providing deposit insurance indeed prevents a massive erosion of the tax base because liquidation costs would cause a significant drop in household income and tax revenue otherwise. Government guarantees therefore reduce sovereign risk ($p < p_N$). If the liquidation value is large, in contrast, this effect is outweighed by the cost of deposit insurance that undermines fiscal stability ($p > p_N$). For $R \leq R_0$, the commitment towards depositors thus tends to reduce sovereign risk as long as liquidation costs are sizable or the debt-servicing cost is high. If $R > R_0$, however, the government defaults irrespective of bank failure and sovereign risk is independent of deposit insurance. Intuitively, the fiscal state is so weak that the extra cost of deposit insurance is small compared to the debt burden.
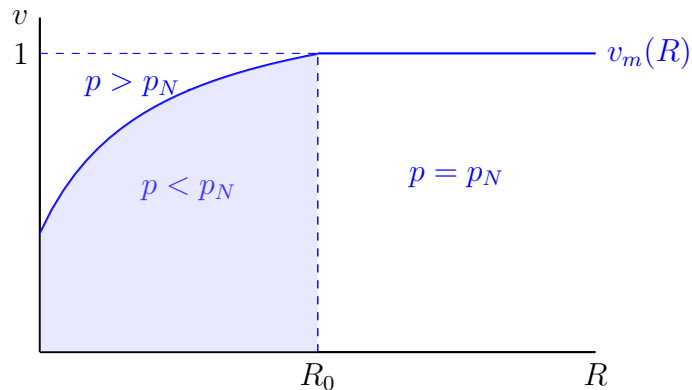

Figure 7: *Deposit Insurance and Sovereign Risk*

Figure 8 shows two examples that illustrate the impact of deposit insurance on the sovereign default. τ and τ_N are the tax rates necessary to cover all outstanding obli-

gations as a function of the loan performance A depending on whether the government honors its guarantees. The tax rate decreases in A because higher loan returns increase the available resources or reduce the cost of deposit insurance. Recall that the sovereign default threshold, $\hat{A}$, is determined by the intersection of τ and the maximum feasible tax rate $\bar{\tau}$. The left panel illustrates a scenario in which deposit insurance triggers a sovereign default: If the realized bond return is between $\hat{A}_N$ and $\hat{A}$ (in the blue-shaded area), fulfilling the commitment towards depositors requires a tax rate τ that is infeasible such that the government defaults. Providing no deposit insurance, in contrast, allows for a tax rate τ_N below the ceiling $\bar{\tau}$. Such a scenario may occur if the liquidation value of bank loans is large ($v > v_m$ in figure 7). In the right panel, in contrast, providing deposit insurance prevents a sovereign default if loan return A is between $\hat{A}$ and $\hat{A}_N$. This is due to a small liquidation value ($v < v_m$ and $R < R_0$), which leads to a massive erosion of tax base and revenue in the absence of deposit insurance.
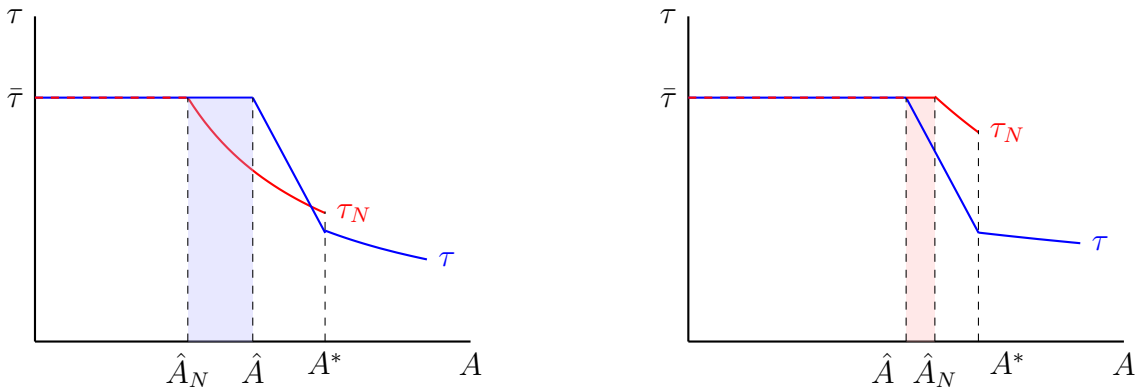

Figure 8: *Default Mechanisms and Deposit Insurance*

4.4 Deposit Insurance and Welfare

Another closely associated issue about government guarantees and rescue packages is whether they are welfare-improving: We focus on the question of whether it is efficient to provide deposit insurance in case the bank fails or whether a deviation from the initial commitment can raise domestic welfare.[14] Since households' savings and their date 1 consumption are independent of an *ex post* decision on whether to satisfy the commitment or not, it is sufficient to look at date 2 domestic welfare, which consists of households'

---

[14]We again focus on an *ex post* deviation instead of a scenario without deposit insurance at all as it is consistent with the fact that investors and depositors often indeed expected governments to rescue distressed banks.

and bankers' utility derived from consumption:

$$V_2 = u(C_2^H) + C_2^B$$

In principle, we compare two different welfare profiles at date 2, namely, domestic welfare with and without deposit insurance. For that purpose, however, it suffices to compare the consumption levels.

4.4.1 Consumption Profile

We first characterize aggregate consumption at date 2, which consists of households' and bankers' consumption $C_2 = C_2^H + C_2^B$: Due to non-linearities associated with default and policy interventions, C_2 is a non-continuous function of the stochastic loan return A.

If the government provides deposit insurance and bails out distressed banks, aggregate consumption equals $C_2 = [1 - \tau(A)][D + W_2 + \max\{A(D + E) - D, 0\}]$. Recall that bankers consume only as long as $A > A^*$. After substituting for the tax rate τ using the government's budget constraints (13) and (16), one obtains:

$$C_2 = \begin{cases} W_2 + A(D + E) - BR & \text{if } A \geq \hat{A} \\ W_2 + A(D + E) & \text{if } A < \hat{A} \end{cases} \tag{29}$$

Hence, aggregate consumption equals total income net of public debt; the discrete jump at the sovereign default threshold $\hat{A}$ results from the full haircut on public debt. The latter reduces the tax burden as well as the tax rate at date 2 thereby raising domestic consumption. The tax rate[15] may, however, still be positive if a deposit insurance scheme needs to be funded. This consumption profile generally emerges in both equilibria. In the 'bad' equilibrium, however, the government defaults first, which implies that there is no public debt that needs to be repaid at date 2. Aggregate consumption is captured by the lower part of (29) since $\hat{A} = \bar{A}$.

Whenever a government does not provide deposit insurance, its potential expenditures at date 2 only come to BR. Compared to the scenario above, consumption differs in two fundamental ways: First, the default threshold changes to $\hat{A}_N$ given by (27); second, liquidation costs reduce the value of the bank's assets to v as soon as the bank fails (i.e., if

---

[15]Assumption 3 ensures that it never exceeds $\bar{\tau}$.

$A < A^*$). While the former affects consumption indirectly because of taxation, the latter reduces income and consumption directly. After substituting for the tax rate τ_N using the government's budget constraints (13) and (16), the following consumption profile arises:

$$C_2^N = \begin{cases} W_2 + [1 - \mathbb{1}_{A<A^*}(1-v)]A(D+E) - BR & \text{if } A \geq \hat{A}_N \\ W_2 + [1 - \mathbb{1}_{A<A^*}(1-v)]A(D+E) & \text{if } A < \hat{A}_N \end{cases} \tag{30}$$

The term in square brackets equals one if the bank succeeds and v otherwise such that loans are worth only $vA(D+E)$ in case the bank fails in a disorderly way.

4.4.2 Welfare Implications

Deposit insurance therefore affects consumption and welfare (i) by preventing a costly liquidation of the bank's assets and (ii) through its effect on the sovereign default threshold. While the former always increases consumption, the effect of the latter is ambiguous and strongly depends on how guarantees affect sovereign risk (see proposition 2). A binary comparison of the two consumption profiles C_2 and C_2^N yields the following corollary:

COROLLARY 2 *If in equilibrium (i) $R \geq R_0$ or (ii) $R < R_0$ and $v \geq v_m(R)$, deposit insurance can always increase domestic welfare. If (iii) $R < R_0$ and $v < v_m(R)$, the welfare effect depends on the realization of A: It can be positive for $A \notin [\hat{A}, \hat{A}_N]$ but is negative for $A \in [\hat{A}, \hat{A}_N]$.*

Proof: This follows from the comparison of the consumption profiles (29) and (30) using the default threshold $\hat{A}_N$ given by (27). The positive effect in (i) and (ii) is due to $\hat{A} \geq \hat{A}_N$. To show (iii), one compares (29) and (30): The result is $C_2 > C_2^N$ for $A < \hat{A}$ and $A \in (\hat{A}_N, A^*)$ due to $v < 1$ and $C_2 < C_2^N$ for $A \in [\hat{A}, \hat{A}_N]$. The latter requires $BR > (1-v)A(D+E)$ which follows from the last inequality after substituting for consumption. If satisfied for the maximum value $A = \hat{A}_N$, this relation is obviously true for all $A \in [\hat{A}, \hat{A}_N]$: $\hat{A}_N$ is at most $\frac{D}{D+E}$ such that $BR > (1-v)D$. This is ensured by assumption 2, which requires $B > D$. Q.E.D.

One can relate these cases to the three regions in figure 7: In the first case, which corresponds to the region $p = p_N$, the provision of deposit insurance does not affect sovereign risk such that $\hat{A}$ and $\hat{A}_N$ coincide. It is still welfare-improving if a bank failure

would imply positive liquidation costs (i.e., if $v < 1$). If depositors can recover the full liquidation value of the bank's assets ($v = 1$), however, deposit insurance is essentially a zero-sum game because the costs increase the tax burden one-to-one without affecting consumption. In the second case, which is highlighted by region $p > p_N$, providing deposit insurance increases domestic consumption as it (i) prevents costly liquidation and (ii) raises sovereign risk thus shifting the cost of deposit insurance onto foreign bondholders. In the third case, as indicated by the region $p < p_N$, however, these effects have opposite signs: While preventing costly liquidation is still welfare-improving, providing deposit insurance makes a sovereign default less likely. Hence, there are fewer opportunities to remove the debt burden. The second, negative effect dominates whenever present, that is, if loan performance is such that government guarantees indeed prevent a sovereign default, $A \in [\hat{A}, \hat{A}_N]$. This is shown by the shaded area in the right panel of figure 8. The intuition is that loans are performing quite poorly. The positive effect of preventing additional liquidation costs, which are proportional to the loans' realized value, is thus dominated by the negative effect of not shifting the public debt burden to foreign investors. If A is outside this region, however, a sovereign default is independent of deposit insurance and only the positive effect of avoiding costly liquidation exists.

The welfare implications of government guarantees crucially depend also on the type of equilibrium. In the 'bad' one, where sovereign default occurs with certainty and $R > R_0$, the first of the three cases matters. Fulfilling the commitment to depositors is welfare-improving only in the presence of liquidation costs and a zero-sum game, where deposit insurance is essentially paid by the households themselves through higher taxes, otherwise. In the 'good' equilibrium, however, deposit insurance may become decisive for welfare. Moreover, the welfare properties of deposit insurance may have implications for the credibility of deposit insurance: In the first two cases, rescuing distressed banks is always optimal *ex post* such that a benevolent government will indeed rescue a failing bank. Deposit insurance is then both time-consistent and credible. As a side effect, it could be argued that the disciplining role of depositors through the threat of bank runs - as claimed in Diamond and Rajan (2000), for example - can therefore not be rationalized under such circumstances. In the third case, however, the government might have an incentive to deviate from its initial commitment depending on the performance of bank loans. Deposit insurance could be time-inconsistent in such a scenario but is still provided

due to legal obligations. Agents may otherwise anticipate that the commitment might not be fulfilled and revise their expectations. Households, for instance, might demand a risk-adjusted deposit interest rate while investors would impose a different bond return due to implications of deposit insurance for sovereign default.

Eventually, the finding of a potentially welfare-improving sovereign default requires some comments. Clearly, the possibility to remove the debt (and tax) burden by defaulting on bonds that are exclusively held by foreign investors in equilibrium raises domestic consumption and welfare. However, a default in our model only occurs due to bad fundamentals, namely, if the government cannot collect sufficient revenue to cover all its date 2 expenditures. This sets it apart from contributions, which model default as a strategic decision. In our model, defaulting on bonds would thus always be optimal *ex post* regardless of the fiscal capacity such that only the 'bad' equilibrium would prevail. The result that sovereign default is welfare-improving should, however, be interpreted with some caution for several reasons. First, a static framework does not capture negative future effects such as damaged reputation and limited access to the international capital market. Second, a sovereign default may entail high macroeconomic and political costs, for example, employment losses in the public sector, political instability or social unrest. This could be easily added to the model either as reduced-form social costs or - following Cooper and Nikolov (2013) - as lower date 2 labor income W_2.[16] Third, a considerable fraction of sovereign bonds is often held by domestic investors such as banks, pension funds, and insurance companies. The domestic welfare gain of defaulting on these bonds is likely to be smaller in reality. Fourth, a default implies a full haircut on bondholders while the residual remains with the government.

5 Capital Regulation

The bank's asset allocation has been unconstrained in the model so far. In reality, however, banks face numerous regulatory restrictions, in particular, capital requirements. A key aspect for this analysis is that they limit the bank's lending capacity but do not constrain sovereign bond holdings due to positive risk weights for the former and zero risk weights for the latter. Consequently, capital regulation is one factor that explains why

---

[16]This would also alter households' savings choice and make deposits sensitive to sovereign risk.

banks purchase fairly priced sovereign bonds in equilibrium. Such bond holdings provide a richer characterization of the bank-sovereign nexus: While the two main channels of bank-sovereign contagion - government guarantees and taxation - persist in such an allocation, a scenario with adverse feedback loops may occur as well. This happens if banks fail or are considerably more vulnerable because of a sovereign default. Besides such a case, banks can also become sensitive to fiscal fundamentals like public debt or tax capacity because the bond return, which reflects the risk of a sovereign default, becomes a critical determinant of bank risk.

5.1 Banks

Due to capital requirements, banks need to finance a fraction of their loans by equity whereas sovereign bonds have a risk weight of zero and do not require any equity funding. Their asset allocation is subject to the regulatory constraint

$$L \leq \mu E, \tag{31}$$

where μ denotes the equity multiplier.[17] Given a minimum capital requirement of 8% as in Basel II, loans must not be larger than 12.5 times its equity. Consequently, the bank chooses deposits and asset allocation in order to maximize expected profits

$$\max_{L,D} \int_{A^*}^{\bar{A}} AL dF(A) + [1 - \max\{F(A^*), p\}]R(D + E - L) - [1 - F(A^*)]D \tag{32}$$

subject to the regulatory constraint (31). Using a similar logic as in the baseline model, one can derive the corresponding failure threshold A^* based on its general definition (3): As long as sovereign bonds are repaid, they provide a buffer to absorb loan losses. The bank's failure threshold therefore increases in loans and is at most $\frac{D - R(D + E - \mu E)}{\mu E}$. The latter represents the case in which the bank provides the maximum amount of loans possible: $L = \mu E$. This scenario is captured by the lower, upward-sloping curve in figure 9. If they are not repaid, however, holding bonds immediately reduces a bank's capacity to absorb loan losses. The failure threshold then decreases in loans, which at the margin yield A^* while bonds yield zero. It is at least $\frac{D}{\mu E}$ such that the bank is most stable if it provides the maximum amount of loans, $L = \mu E$, and holds as few bonds as possible. The

---

[17]If the capital requirement is k, the multiplier equals $\mu = 1/k$.

upper, downward-sloping curve in figure 9 illustrates this case. Consequently, if bonds are repaid if the bank fails (i.e., if $\widetilde{R} = R$ at $A = A^*$), the government is necessarily more robust than the least stable bank such that $\hat{A} = F^{-1}(p) < \frac{D-R(D+E-\mu E)}{\mu E}$ or, equivalently, $p < F\left(\frac{D-R(D+E-\mu E)}{\mu E}\right) \equiv p_1$. Otherwise, it would default at $A = A^*$ and sovereign bonds would not be repaid. Similarly, if bonds are not repaid when the bank fails (i.e., if $\widetilde{R} = 0$ at $A = A^*$), the government is necessarily less robust than the most stable bank such that $\hat{A} = F^{-1}(p) > \frac{D}{\mu E}$ or, equivalently, $p > F\left(\frac{D}{\mu E}\right) \equiv p_2$. In contrast to the baseline model, the cutoffs p_1 and p_2 differ because capital requirements prevent an all-loans bank. Whenever sovereign risk is in this interim region, $p \in [p_1, p_2]$, the bank survives if bonds are repaid but fails otherwise such that a sovereign default is the very reason for its failure.
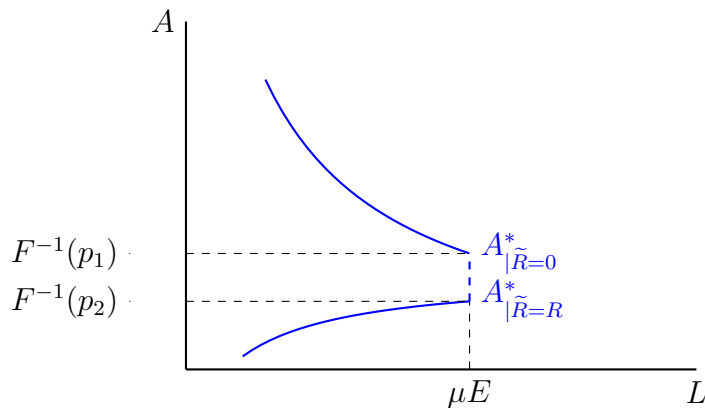

Figure 9: *Bank's Failure Threshold*

One can summarize the bank's failure threshold as a function of sovereign risk:

$$A^* = \begin{cases} \max\left\{\frac{D-R(D+E-L)}{L}, F^{-1}(p)\right\}, & \text{if } p < p_1 \\ F^{-1}(p), & \text{if } p \in [p_1, p_2] \\ \min\left\{\frac{D}{L}, F^{-1}(p)\right\}, & \text{if } p > p_2 \end{cases} \tag{33}$$

We solve for the bank's optimal asset allocation using this definition of the default threshold. Again, the bank's optimization problem is convex or linear in L and D such that a corner solution emerges. Hence, the demand for deposit is indeterminate and perfectly elastic at the risk-free interest rate; any amount of deposits supplied by households is accepted. Regarding the asset allocation, the bank chooses between two options: It either provides as much loans as possible and invests the remainder in sovereign bonds, $L = \mu E$

and $G = D + E - \mu E$; or it only purchases sovereign bonds, $L = 0$ and $G = D + E$. As in the baseline model, the bank chooses the former if this allocation promises higher expected profits, that is, if $\pi(\mu E) \geq \pi(0)$. The results are summarized as follows:

LEMMA 3 *The bank's deposit demand D is perfectly elastic. Define the cutoff:*

$$R' = \begin{cases} \frac{1}{1-p}\left[E(A) + \int_0^{\frac{D-R(D+E-\mu E)}{\mu E}} F(A)dA - \frac{p[D-R(D+E-\mu E)]}{\mu E} \right], & if\ p < p_1 \\[3mm] \frac{1}{1-p}\left[E(A) + \int_0^{F^{-1}(p)} F(A)dA - pF^{-1}(p) \right], & if\ p \in [p_1, p_2] \\[3mm] \frac{1}{1-p}\left[E(A) + \int_0^{\frac{D}{\mu E}} F(A)dA - \frac{pD}{\mu E} \right], & if\ p > p_2 \end{cases} \tag{34}$$

R' decreases in the capital requirement if $p < p_1$, is unchanged if $p \in [p_1, p_2]$, and increases if $p > p_2$. The bank's loan volume equals

$$L = \begin{cases} \mu E, & if\ R \leq R' \\[2mm] 0, & if\ R > R' \end{cases} \tag{35}$$

and its sovereign bond holdings are $G = D + E - L$. The bank's failure threshold is either

$$A^* = \begin{cases} \frac{D-R(D+E-\mu E)}{\mu E}, & if\ p < p_1 \\[2mm] F^{-1}(p), & if\ p \in [p_1, p_2] \\[2mm] \frac{D}{\mu E}, & if\ p > p_2 \end{cases} \tag{36}$$

if $R \leq R'$ or $A^* = F^{-1}(p)$ if $R > R'$.

Proof: See Appendix A.1.

The bank's portfolio consists of both loans and sovereign bonds as long as the bond return is small enough such that expected bank profits from investing in a combined portfolio of loans and bonds are higher than in case of a bonds-only portfolio. Note that - although defined in a piecewise manner - the cutoff return R' exhibits no discrete jumps and is increasing in p. Tighter regulation induces banks to favor the safer portfolio.

5.2 Equilibrium

The choices of the households and the government are similar to those in the baseline model. Focusing on the case of banks with the combined portfolio (see discussion in Appendix A.1), these results and lemma 3 establish:

PROPOSITION 3 *The equilibrium allocation $\{A^*, \hat{A}, D, G, L, p, R, R'\}$ is characterized by conditions (2), (10) - (12), (20), and (34) - (36). Using (20), define the sovereign default threshold for $L = \mu E$:*

$$\hat{A}_{|L=\mu E} = \begin{cases} \frac{D - R(D + E - \mu E)}{\mu E} + \max\left\{\frac{BR - \bar{\tau}(D + W_2)}{\mu E}, \frac{BR - \bar{\tau}(D + W_2)}{\bar{\tau}\mu E}\right\}, & \text{if } R \leq R_2 \\ \bar{A}, & \text{if } R > R_2 \end{cases} \quad (37)$$

R_2 equals $\frac{\bar{\tau}(W_2 + \bar{A}\mu E)}{(1 - \omega\bar{\tau})B}$. Two types of equilibria may exist:

- *The 'good' equilibrium with $p_g < 1$ and $R_g < R_2$ exists if $\exists R \in [1, R_2)$ such that $F[\hat{A}_{|L=\mu E}(R)] < 1 - \frac{1}{R}$.*

- *The 'bad' equilibrium with $p_b = 1$ and $R \to \infty$ always exists.*

In each type of equilibrium, banks hold a combination of loans and sovereign bonds: They provide the maximum amount of loans $L = \mu E$ and invest the remainder in sovereign bonds, $G = D + E - \mu E$. The bank failure threshold is

$$A^* = \begin{cases} \frac{D - R(D + E - \mu E)}{\mu E}, & \text{if } R \leq R_0 \\ \min\left\{\frac{D - R(D + E - \mu E)}{\mu E} + \frac{BR - \bar{\tau}(D + W_2)}{\bar{\tau}\mu E}, \frac{D}{\mu E}\right\}, & \text{if } R > R_0 \end{cases}$$

and the government's default threshold equals $\hat{A} = \hat{A}_{|L=\mu E}$.

Proof: See Appendix A.1.

The preferential treatment of sovereign bonds, which are subject to zero risk weights, is one reason why banks invest in fairly priced bonds. Without regulation, they do not purchase any bonds at all because fair pricing makes them less attractive than loans. Hence, banks are sensitive to sovereign risk through bond return and repayment, and a scenario with adverse feedback loops is possible.

5.3 Comparative Statics

Three cases with fundamentally different channels of bank-sovereign contagion are possible: First, the banks are less stable than the government (i.e., $A^* > \hat{A}$) whenever the bond return is low. Bank-sovereign contagion may thus occur because of government guarantees. Second, they are at least as stable as the government (i.e., $A^* < \hat{A}$), which is the case if the bond return exceeds the cutoff R_0. Hence, the tax capacity is small compared to the level of public debt such that quite a large taxable income from bank owners (i.e., a high realization of A) is necessary to raise sufficient revenue. In reality, this describes highly indebted countries with a small tax base. Contagion thus occurs even without bank failure in the first place simply due to a small dividend income of bank owners that leads to an erosion of the tax base. Third, an adverse feedback may occur in the latter scenario whenever the bank fails because sovereign bonds are not repaid.

5.3.1 Sovereign Risk

The sovereign default threshold is given by (37), which means that the default probability $p = F(\hat{A})$ and the bond return R react to changes in the fiscal and regulatory environment as follows:

COROLLARY 3 *In the 'good' equilibrium, the sovereign default probability satisfies $\frac{\partial p}{\partial \bar{\tau}} < 0$ and $\frac{\partial p}{\partial B} > 0$, which implies $\frac{\partial R}{\partial \bar{\tau}} < 0$ and $\frac{\partial R}{\partial B} > 0$. The sensitivities of p and R to the regulatory multiplier μ are positive as long as $R \leq R_0$ but can be of either sign for $R > R_0$.*

Proof: See Appendix A.1.

A fiscally sound country characterized by low public debt and a high tax capacity generally features a reduced probability of sovereign default, which translates into relatively small bond returns. A more fragile country, however, is more likely to default and thus borrows at higher interest rates.

The impact of capital requirements on sovereign risk is more subtle: If a government is less likely to default than banks (i.e., if $\hat{A} \leq A^*$ or, equivalently, $R \leq R_0$), tighter capital requirements reduce sovereign risk, as banks are less exposed to loan risk and absorb a larger amount of potential losses. This, in turn, lowers the cost of deposit insurance

that might ultimately trigger its own default. Hence, tighter capital regulation clearly improves fiscal stability. If the government is more vulnerable in equilibrium (i.e., if $\hat{A} > A^*$ or, equivalently, $R > R_0$), however, this effect can be ambiguous. Recall that a sovereign default occurs in this case because the tax base is so small that the government cannot raise sufficient revenue. The impact of regulation on bank profits is therefore critical: Tighter capital requirements effectively force banks to reallocate assets from loans to sovereign bonds. This may increase the realized bank profit as long as the return of bonds exceeds that of loans at the sovereign default threshold (i.e., as long as $R > \hat{A}$) such that tighter regulation, due to its impact on taxable date 2 income, still reduces sovereign risk. If the sovereign default threshold $\hat{A}$ is very high, however, bank profits and the tax base increase if banks are allowed to hold more loans. These still perform very well at the default threshold and yield a larger (taxable) payoff than sovereign bonds (i.e., $\hat{A} > R$). This is the reason why a tighter regulatory stance might even weaken fiscal stability in these countries characterized by high sovereign risk and high interest rates in equilibrium. For capital regulation to increase sovereign risk, the default threshold and the bond return need to be very high, which is a rather unlikely scenario for the 'good' equilibrium, however.[18] Under such circumstances, it appears more likely that the 'good' equilibrium ceases to exist such that only the 'bad' equilibrium remains. Apart from this special case, tighter capital requirements reduce sovereign risk. This also implies that the government is more stable and benefits from lower interest rates in an allocation where banks hold both loans and bonds than in the unconstrained equilibrium.

5.3.2 Bank Risk

Bank's sovereign bond holdings link bank and sovereign risks through bond return and repayment, which creates the possibility of an adverse feedback. Recall that the failure threshold of banks considerably varies depending on the equilibrium bond return

$$
A^* = \begin{cases} \frac{D - R(D + E - \mu E)}{\mu E}, & \text{if } R \leq R_0 \\ \frac{D - R(D + E - \mu E)}{\mu E} + \frac{BR - \bar{\tau}(D + W_2)}{\bar{\tau} \mu E}, & \text{if } R \in (R_0, R_1) \\ \frac{D}{\mu E}, & \text{if } R \geq R_1 \end{cases} \tag{38}
$$

---

[18]This is only feasible if the maximum loan return $\bar{A}$ is large enough such that $R_2 > \frac{\bar{\tau} W_2}{B - \bar{\tau}(D + E)}$.

38

where the cutoffs R_0 and R_1 follow from (18) and (19). Recall figure 5: As long as the equilibrium bond return is small ($R \leq R_0$) such that the government is more stable than banks, bonds provide a cushion to absorb loan losses. If, however, the bond return is large ($R \geq R_1$) due to a substantial debt burden, in which case the government is less stable than the bank, bond holdings directly translate into losses at the bank failure threshold A^*; this uses up equity and weakens banks' resilience. In an intermediate case $R \in (R_0, R_1)$, the bank's failure is conditional upon non-repayment of bonds: It would survive for an even worse loan performance if bonds were repaid but it cannot absorb losses on both assets. As a result, the sovereign default is the very reason for bank failure and both thresholds (A^* and $\hat{A}$) just coincide. This case reflects the negative feedback loop as a poor loan performance itself does not push banks into bankruptcy but triggers a sovereign default, which immediately leads to bank failure. Using the bond return's sensitivities from corollary 3, differentiating (38) yields:

COROLLARY 4 *The sensitivities of bank risk, $F(A^*)$, differ between three cases:*

- *If $R \leq R_0$, bank risk increases in the tax capacity, $\frac{\partial A^*}{\partial \bar{\tau}} > 0$, and decreases in public debt, $\frac{\partial A^*}{\partial B} < 0$, whereas capital requirements have an ambiguous effect, $\frac{\partial A^*}{\partial \mu}$.*

- *If $R \in (R_0, R_1)$, bank risk decreases in the tax capacity, $\frac{\partial A^*}{\partial \bar{\tau}} < 0$, and increases in public debt, $\frac{\partial A^*}{\partial B} > 0$, whereas the effect of capital requirements can be of either sign, $\frac{\partial A^*}{\partial \mu}$.*

- *If $R \geq R_1$, bank risk does not directly depend on fiscal fundamentals, $\frac{\partial A^*}{\partial \bar{\tau}} = \frac{\partial A^*}{\partial B} = 0$, and decreases in the regulatory multiplier $\frac{\partial A^*}{\partial \mu} < 0$.*

Proof: See Appendix A.1.

Bank risk exhibits significant differences depending on the equilibrium bond return. In the first case, $R \leq R_0$, bonds are always repaid as long as the bank survives. Hence, they reduce banks' exposure to loan risk and generate profits that may serve as a buffer to absorb loan losses. The bond return plays a prominent role as it reduces bank risk by generating higher profits and links it to fiscal fundamentals. Since a higher public debt level, B, and a lower tax capacity, $\bar{\tau}$, raise the bond return as shown in corollary 3, they even enhance banks' resilience. Therefore, slightly increasing sovereign risk in a

country that is still fiscally sound (i.e., pays a relatively low bond return in equilibrium) may reduce bank risk because higher bond returns raise profits and (final-period) equity. The impact of the regulatory multiplier μ is, in principle, ambiguous in such a scenario because of two countervailing effects: On the one hand, tighter regulation directly lowers the bank's exposure to risky loans thereby making it more resilient. On the other hand, it reduces sovereign risk as shown in corollary 3 such that bond returns fall; the latter, in turn, reduces bank profits and thus its capacity to withstand a poor loan performance. However, it is likely that the positive direct effect prevails. The effects associated with the bond return are of course expected to be less pronounced in reality where banks hold diversified portfolios of sovereign bonds and may, in fact, substitute foreign for domestic bonds if the latter become riskier. However, banks' sovereign exposures are also characterized by a significant home bias that is empirically well documented.

In the second case, $R \in (R_0, R_1)$, a bank failure is triggered by the non-repayment of government bonds such that sovereign and bank risk are similar and their default thresholds coincide. In other words, banks fail because of an adverse feedback. As a result, increases in sovereign risk imply higher bank risk. Weaker fiscal fundamentals (B and $\bar{\tau}$), therefore, increase bank risk, whereas the effects of tighter capital regulation are generally ambiguous but likely positive as discussed in section 5.3.1.

In the third case, $R \geq R_1$, banks are more stable than the government, and do not immediately fail if bonds are not repaid. Put differently, sovereign risk is so high that the government defaults even though bank loans perform well. This case also captures bank risk in the 'bad' equilibrium, in which the country experiences a sovereign debt crisis and defaults irrespective of the banking sector's loan performance. Bank risk then only depends on bank characteristics and is disconnected from fiscal fundamentals in the sense that they have no direct impact on failure threshold and probability. Although there is no scope for any immediate feedback like in the second case, the sovereign default weakens banks' resilience. Interestingly, relaxing capital requirements (i.e., raising the regulatory multiplier) reduces bank risk in this scenario: The intuition is that banks hold more loans, which are worth A^* at the margin, and fewer sovereign bonds, which are worth zero.

The interaction between bank and sovereign risks is a key feature of this model. In fiscally sound countries that pay a low bond return, there is a tension between bank and sovereign risks because holding bonds only yields small profits due to low returns such that banks'

loss-absorbing capacity is limited. This means that bank risk is *ceteris paribus* higher in a risk-free country ($R = 1$) than in a still solid country with a positive default probability (i.e., if R is close to R_0). As soon as sovereign risk and the corresponding bond return increase above a cutoff, the relation between bank and sovereign risks is reversed because bank failure crucially depends on bond repayment. Improving fiscal fundamentals then makes bond repayment more and a bank failure less likely. If bond return and debt burden are so large that a sovereign default occurs even though bank loans perform relatively well, for instance, in a 'bad' equilibrium, the bank can absorb the bond losses and its own risk only depends on leverage and loan performance. Hence, bank risk does not directly depend on fiscal fundamentals but is higher as a result of a sovereign default.

6   Conclusion

We present a model of the bank-sovereign nexus, which has been a prominent feature in the ongoing European banking and debt crisis. In order to study contagion between banks and sovereigns and to analyze the impact of government guarantees for stability and welfare, the model uniquely combines financial fragility with sovereign debt fragility in the form of multiple equilibria and self-fulfilling debt crises. Unlike in most other papers on that topic, risks originate in the banking sector, more precisely, from the banks' asset side. A poor loan performance directly hits the bank but may also cause a sovereign default. This is because the fate of the two is tied together by deposit insurance cost and taxation. Importantly, the provision of deposit insurance can either trigger or prevent a sovereign default. The outcome depends crucially on the liquidation value of the bank's assets: A government safeguards its own stability whenever its intervention prevents high costs of a disorderly bank liquidation but may jeopardize it otherwise. The provision of deposit insurance lifts domestic consumption levels as well as welfare as it avoids significant bankruptcy costs and may effectively shift the debt burden onto foreign bondholders. This is a key difference to other contributions on sovereign debt crises, in which default is the result of a strategic decision.

Banks in this model only purchase fairly priced bonds due to their preferential regulatory treatment, that is, because no equity has to be held against them. Bond holdings make them sensitive to the fiscal state and cause the unhealthy symbiosis between the banking

sector and the sovereign. Therefore, it is possible that banks only fail because of sovereign default. Adverse feedback loops of that sort were the source of major problems in recent years. The model is able to rationalize both the Irish case, in which banks stood at the heart of the problem, and, to a lesser extent, the Greek scenario, in which a sudden loss of confidence in the government was the decisive trigger. The interplay between the risks of banks and sovereigns reveals a number of interesting interdependencies. We find, for instance, that financial and fiscal stability may not always work in the same direction in the sense that higher bond returns, which are the result of weaker fiscal fundamentals, provide a buffer to absorb loan losses to the bank and thus stabilize the latter. While this is true for fiscally sound countries, the effect reverses for banks located in unstable regions. Stricter capital requirements tend to enhance the resilience of both banks and sovereigns, but also raise awareness of potentially countervailing effects. The novel findings of this analysis clarify the fundamental mechanisms of contagion between governments and banks and outline possible consequences of the policy options at hand. They also rationalize important implications of deposit insurance, potential welfare benefits of sovereign default, as well as consequences of tighter capital requirements.

Bank-sovereign contagion and adverse feedback effects are at the core of our analysis. Motivated by recent crises in Ireland and Spain, the focus of our paper is clearly on risks originating in the banking sector. In further research, one may also include other sources of risk such as political or macroeconomic shocks. Moreover, our analysis provides one explanation for why banks hold fairly priced domestic sovereign bonds, namely, that they receive preferential treatment in the current regulatory framework. Further motivations for bond holdings, such as their role as a collateral in the interbank market, and their effect on the relation between bank stability and sovereign risks may be explored as well. Our paper studies systemic crises with correlated risks. An extension could analyze which mechanisms of the bank-sovereign nexus prevail and how deposit insurance affect risk and welfare in case of imperfect correlation.

References

Acharya, V. V., I. Drechsler, and P. Schnabl (2014). A Phyrric Victory? - Bank Bailouts and Sovereign Credit Risk. *Journal of Finance 69(9)*, 2689–2739.

Admati, A. R. and M. Hellwig (2013). Does Debt Discipline Bankers? An Academic Myth about Bank Indebtedness. Rock Center for Corporate Governance at Stanford University Working Paper No. 132.

Admati, A. R. and P. Pfleiderer (2010). Increased-Liability Equity: A Proposal to Improve Capital Regulation of Large Financial Institutions. Stanford Graduate School of Business Research Paper No. 2043.

Allen, F. and D. Gale (2000). Financial Contagion. *Journal of Political Economy 108(11)*, 1–33.

Battistini, N., M. Pagano, and S. Simonelli (2013). Systemic Risk and Home Bias in the Euro Area. *Economic Papers of the European Commission 494*, 1–45.

Bénassy-Quéré, A. and G. Roussellet (2014). Fiscal Sustainability in the Presence of Systemic Banks: The Case of EU Countries. *International Tax and Public Finance 21(3)*, 436–467.

Bolton, P. and O. Jeanne (2011). Sovereign Default Risk and Bank Fragility in Financially Integrated Economies. *IMF Economic Review 59(2)*, 162–194.

Boyd, J. H., G. De Nicolo, and A. M. Jalal (2009). Bank Competition, Risk and Asset Allocations. IMF Working Paper No. 09/143.

Calvo, G. (1988). Servicing the Public Debt: The Role of Expectations. *American Economic Review 78(4)*, 647–661.

Cole, H. L. and T. J. Kehoe (2000). Self-Fulfilling Debt Crises. *Review of Economic Studies 67(1)*, 91–116.

Cooper, R. and K. Nikolov (2013). Government Debt and Banking Fragility: The Spreading of Strategic Uncertainty. NBER Working Paper No. 19278.

De Grauwe, P. and Y. Ji (2013). Self-fulfilling Crises in the Eurozone: An Empirical Test. *Journal of International Money and Finance 34*, 15–36.

Dermine, J. (1986). Deposit Rates, Credit Rates and Bank Capital. The Klein-Monti Model Revisited. *Journal of Banking and Finance 10*, 99–114.

Detragiache, E. (1996). Rational Liquidity Crises in the Sovereign Debt Market: In Search of a Theory. *IMF Staff Papers 43(3)*, 545–570.

Diamond, D. W. and P. H. Dybvig (1983). Bank Runs, Deposit Insurance, and Liquidity. *Journal of Political Economy 91(3)*, 401–419.

Diamond, D. W. and R. G. Rajan (2000). A Theory of Bank Capital. *Journal of*

Finance 55(6), 2431–2465.

Diamond, D. W. and R. G. Rajan (2005). Liquidity Shortages and Banking Crises. *Journal of Finance 60(2)*, 615–47.

Diamond, D. W. and R. G. Rajan (2011). Fear of Fire Sales and the Credit Freeze. *Quarterly Journal of Economics 126(2)*, 557–591.

European Systemic Risk Board (2015). ESRB Report on the Regulatory Treatment of Sovereign Exposures. March 2015.

Gennaioli, N., A. Martin, and S. Rossi (2014). Sovereign Default, Domestic Banks and Financial Institutions. *Journal of Finance 69(2)*, 819–866.

Greenwood, R., A. Landier, and D. Thesmar (2015). Vulnerable Banks. *Journal of Financial Economics 118*, 471–485.

Jaffee, D. M. and F. Modigliani (1969). A Theory and Test of Credit Rationing. *American Economic Review 59(5)*, 850–872.

Laeven, L. and F. Valencia (2012). Systemic Banking Crises Database: An Update. IMF Working Paper, No. 12/163.

Leonello, A. (2015). Government Guarantees and the Two-way Feedback Between Banking and Sovereign Debt Crises. Working Paper.

Mody, A. and D. Sandri (2012). The Eurozone Crisis: How Banks and Sovereigns Came to Be Joined at the Hip. *Economic Policy 70*, 199–230.

Pfleiderer, P. (2014). Chameleon Models: The Misuse of Theoretical Models in Financial Economics. Working Paper.

Reinhart, C. M. and K. Rogoff (2011). From Financial Crash to Debt Crisis. *American Economic Review 101(5)*, 1676–1706.

Rochet, J.-C. (1992). Capital Requirements and the Behaviour of Commercial Banks. *European Economic Review 36(5)*, 1137–1178.

Romer, D. (2001). *Advanced Macroeconomics*. New York: McGraw-Hill.

Shleifer, A. and R. W. Vishny (1992). Liquidation Values and Debt Capacity: A Market Equilibrium Approach. *Journal of Finance 47(4)*, 1343–1366.

Shleifer, A. and R. W. Vishny (2011). Fire Sales in Finance and Macroeconomics. *Journal of Economic Perspectives 25(1)*, 29–47.

A Appendix

A.1 Proofs and Derivations

Proof of Lemma 1: We distinguish between two cases: Suppose first $p \leq \bar{p}$ such that bonds are repaid for the realization of A at which the bank fails and $\max\{F(A^*), p\} = F(A^*)$. Integrating the term $\int_{A^*}^{\bar{A}} dF(A)$ yields the expected bank profit:

$$\pi = \left[\bar{A} - F(A^*)A^* - \int_{A^*}^{\bar{A}} F(A)dA \right] L + [1 - F(A^*)][R(D + E - L) - D]$$

where by (5), its failure threshold is $A^* = \max\left\{ \frac{D - R(D+E-L)}{L}, F^{-1}(p) \right\}$.[19] The first-order condition w.r.t. D is nonnegative due to $R \geq 1$:

$$\frac{\partial \pi}{\partial D} = [1 - F(A^*)](R - 1) \geq 0$$

Hence, the bank always raises the maximum amount of deposits households are willing to supply at the risk-free interest rate. The first-order condition w.r.t. L is

$$\frac{\partial \pi}{\partial L} = \bar{A} - F(A^*)A^* - \int_{A^*}^{\bar{A}} F(A)dA - [1 - F(A^*)]R$$

The objective function is linear or convex in L as the second-order condition is nonnegative

$$\frac{\partial^2 \pi}{\partial L^2} = f(A^*)(R - A^*)\frac{\partial A^*}{\partial L} \geq 0$$

because of $\frac{\partial A^*}{\partial L} = \frac{R(D+E)-D}{L^2} > 0$ if $A^* > F^{-1}(p)$ and $\frac{\partial A^*}{\partial L} = 0$ if $A^* = F^{-1}(p)$.[20] Note that (5) implies $R \geq A^*$. Therefore, the optimal choice is determined by the corner solution $L = \{D + E, 0\}$. The bank chooses $L = D + E$ and $G = 0$ if $\pi(D + E) \geq \pi(0)$:

$$\left[\bar{A} - \frac{\bar{p}D}{D + E} - \int_{D/(D+E)}^{\bar{A}} F(A)dA \right](D + E) - (1 - \bar{p})D \geq (1 - p)[R(D + E) - D]$$

---

[19]The threshold is $[D - R(D + E - L)]/L$ if $L \geq [R(D + E) - D]/[R - F^{-1}(p)]$ and $F^{-1}(p)$ else.

[20]Note that the objective function π is linear for $L < [R(D + E) - D]/[R - F^{-1}(p)]$ and convex for larger L; there is no discrete jump of π at $L = [R(D + E) - D]/[R - F^{-1}(p)]$.

which uses $A^* = \frac{D}{D+E}$ if $L = D + E$ and $A^* = F^{-1}(p)$ if $L = 0$. Otherwise, the bank purchases sovereign bonds only ($L = 0$ and $G = D + E$). Rearranging yields

$$\bar{A} - \int_{D/(D+E)}^{\bar{A}} F(A)dA \geq (1-p)R + \frac{pD}{D+E} \tag{39}$$

where the l.h.s. equals $E(A) + \int_0^{D/(D+E)} F(A)dA$. This gives the maximum bond return R'.

Second, suppose instead that $p > \bar{p}$. Bonds are then not repaid for the realization of A at which the bank fails and $\max\{F(A^*), p\} = p$. Hence, the bank's expected profit is:

$$\pi = \left[\bar{A} - F(A^*)A^* - \int_{A^*}^{\bar{A}} F(A)dA\right]L + (1-p)R(D+E-L) - [1 - F(A^*)]D$$

The default threshold is $A^* = \min\left\{\frac{D}{L}, F^{-1}(p)\right\}$ by (5).[21] The first-order condition w.r.t. D is

$$\frac{\partial \pi}{\partial D} = (1-p)R - [1 - F(A^*)]$$

In equilibrium, bonds are priced such that this condition is nonnegative and the bank raises the maximum amount of deposits supplied by households. The first-order condition w.r.t. L is

$$\frac{\partial \pi}{\partial L} = \bar{A} - F(A^*)A^* - \int_{A^*}^{\bar{A}} F(A)dA - (1-p)R$$

The objective function is linear or convex in L as the second-order condition is again nonnegative

$$\frac{\partial^2 \pi}{\partial L^2} = -f(A^*)A^* \frac{\partial A^*}{\partial L} \geq 0$$

because of $\frac{\partial A^*}{\partial L} = -\frac{D}{L^2} < 0$ if $A^* > F^{-1}(p)$ and $\frac{\partial A^*}{\partial L} = 0$ if $A^* = F^{-1}(p)$.[22] The bank chooses $L = D + E$ and $G = 0$ if $\pi(D+E) \geq \pi(0)$:

$$\left[\bar{A} - \frac{\bar{p}D}{D+E} - \int_{D/(D+E)}^{\bar{A}} F(A)dA\right](D+E) - (1-\bar{p})D \geq (1-p)[R(D+E) - D]$$

Rearranging yields the cutoff R'. Finally, one obtains the sensitivity of R' by totally

---

[21]More precisely, the default threshold is D/L if $L \geq D/F^{-1}(p)$ and $F^{-1}(p)$ else.
[22]The objective function is linear for $L < D/F^{-1}(p)$ and convex for larger L; there is no discrete jump of π at $L = D/F^{-1}(p)$.

differentiating (39):

$$\frac{\partial R'}{\partial E} = \frac{D}{(D+E)^2} \frac{p - \bar{p}}{1 - p}$$

such that $\frac{\partial R'}{\partial E} < 0$ if $p < \bar{p}$ and $\frac{\partial R'}{\partial E} > 0$ if $p > \bar{p}$. $Q.E.D.$

Proof of Proposition 1: The 'bad' equilibrium always exists, as for $R \to \infty$, $\hat{A} = \bar{A}$, and $p = 1$. The bond return is determined by (11) which means that a default with certainty implies that $R \to \infty$ is indeed justified. Since $R' \to \infty$, the bank prefers holding loans only, $L = D + E$. To prove the existence of the 'good' equilibrium, we proceed in two steps: First, suppose $L = D + E$ such that $\hat{A} = \hat{A}_{|L=D+E}$. Given that a bond return $R \in [1, R_2)$ with $F[\hat{A}_{|L=D+E}(R^*)] \leq 1 - \frac{1}{R}$ exists, the continuity[23] of both $F(A)$ and $\hat{A}_{|L=D+E}$, which implies that $F[\hat{A}_{|L=D+E}(R)]$ is non-decreasing in R, together with $F[\hat{A}_{|L=D+E}(1)] \geq 0 = 1 - \frac{1}{1}$ ensure the existence of the 'good' equilibrium with $R_g \leq R$. Graphically, bond pricing and default curve intersect. Hence, one can identify an equilibrium candidate; for it to be a true equilibrium, one needs to show that $R_g \leq R'$ is satisfied such that the bank is indeed willing to hold loans only: Substituting $R_g = \frac{1}{1-p}$ from the bond pricing condition into (6) implies:

$$p \leq \frac{D+E}{D} \left[E(A) + \int_0^{\frac{D}{D+E}} F(A) dA - 1 \right] \tag{40}$$

This means that the default probability implied by R_g, $p_g = F[\hat{A}_{|L=D+E}(R_g)]$, needs to satisfy the above condition. If condition (40) is violated for all potential values of R^*, implying that the bank would prefer to hold sovereign bonds only, the candidate identified above is no equilibrium. Therefore, only the 'bad' equilibrium exists in this case. In general, a bank holding sovereign bonds only cannot be an equilibrium outcome: The government's default probability is then either zero or one and, by (11), the bond return is either one or infinity. These values, in turn, are smaller than the cutoff R'; the bank would then prefer loans over bonds. Since $L = D + E$ in each type of equilibrium, $A^* = \frac{D}{D+E}$ and $\hat{A} = \hat{A}_{|L=D+E}$ immediately follow. $Q.E.D$

Proof of Corollary 1: The systems (11) and (24) jointly determine $\hat{A}$ and R. Since

---

[23]Note that $\hat{A}_{|L=D+E}(R)$ has two kinks at $R = R_0$ and $R = R_2$ but no jumps.

$R < R'$ in the 'good' equilibrium, the system is

$$J^1 = \hat{A} - \frac{D}{D+E} - \max\left\{\frac{BR - \tau(D+W_2)}{D+E}, \frac{BR - \tau(D+W_2)}{\bar{\tau}(D+E)}\right\} = 0$$

$$J^2 = [1 - F(\hat{A})]R - 1 = 0$$

Provided that in equilibrium $R \leq R_0$, the Jacobian matrix is

$$J = \begin{bmatrix} 1 & -\frac{B}{D+E} \\ -f(\hat{A})R & 1 - F(\hat{A}) \end{bmatrix}$$

Denote the Jacobian determinant by ∇:

$$\nabla = 1 - F(\hat{A}) - \frac{f(\hat{A})BR}{D+E} > 0 \tag{41}$$

The sign is derived using a specific property of the equilibrium, which follows from the intersection of the default threshold and the bond pricing equation: In the 'good' equilibrium, the bond pricing curve is steeper than the default threshold (i.e., $1/R^2 > f(\hat{A})d\hat{A}/dR$). This property is necessary for the existence of the equilibrium since, for $R = 1$, $F[\hat{A}(1)] \geq 0 = p(1)$.[24] substituting $1 - F(\hat{A}) = 1/R$, which holds in equilibrium, into (41) implies that $\nabla > 0$ in the 'good' equilibrium. Cramer's rule yields:

$$\frac{\partial R}{\partial B} = \frac{1}{\nabla}\frac{f(\hat{A})R^2}{D+E} > 0, \quad \frac{\partial \hat{A}}{\partial B} = \frac{1}{\nabla}\frac{(1-F(\hat{A}))R}{D+E} > 0$$

$$\frac{\partial R}{\partial \bar{\tau}} = -\frac{1}{\nabla}\frac{f(\hat{A})(D+W_2)R}{D+E} < 0, \quad \frac{\partial \hat{A}}{\partial \bar{\tau}} = -\frac{1}{\nabla}\frac{(1-F(\hat{A}))(D+W_2)}{D+E} < 0$$

$$\frac{\partial R}{\partial E} = -\frac{1}{\nabla}\frac{f(\hat{A})R(BR - \bar{\tau}W_2 + (1-\bar{\tau})D)}{(D+E)^2} < 0, \quad \frac{\partial \hat{A}}{\partial E} = -\frac{1}{\nabla}\frac{(1-F(\hat{A}))(BR - \bar{\tau}W_2 + (1-\bar{\tau})D)}{(D+E)^2} < 0$$

If in equilibrium $R > R_0$, the Jacobian matrix is

$$J = \begin{bmatrix} 1 & -\frac{B}{\bar{\tau}(D+E)} \\ -f(\hat{A})R & 1 - F(\hat{A}) \end{bmatrix}$$

---

[24]For a graphical exposition, refer to figure 6.

The Jacobian determinant is

$$\nabla = 1 - F(\hat{A}) - \frac{f(\hat{A})BR}{\bar{\tau}(D+E)} > 0$$

The sign of ∇ is derived using the same approach as above; thus, the signs of the sensitivities do not differ from the case $R \leq R_0$. Applying Cramer's rule yields:

$$\frac{\partial R}{\partial B} = \frac{1}{\nabla}\frac{f(\hat{A})R^2}{\bar{\tau}(D+E)} > 0, \quad \frac{\partial \hat{A}}{\partial B} = \frac{1}{\nabla}\frac{(1-F(\hat{A}))R}{\bar{\tau}(D+E)} > 0$$

$$\frac{\partial R}{\partial \bar{\tau}} = -\frac{1}{\nabla}\frac{f(\hat{A})(D+W_2)R}{\bar{\tau}(D+E)} < 0, \quad \frac{\partial \hat{A}}{\partial \bar{\tau}} = -\frac{1}{\nabla}\frac{(1-F(\hat{A}))(D+W_2)}{\bar{\tau}(D+E)} < 0$$

$$\frac{\partial R}{\partial E} = -\frac{1}{\nabla}\frac{f(\hat{A})R(BR - \bar{\tau}W_2 + (1-\bar{\tau})D)}{\bar{\tau}(D+E)^2} < 0, \quad \frac{\partial \hat{A}}{\partial E} = -\frac{1}{\nabla}\frac{(1-F(\hat{A}))(BR - \bar{\tau}W_2 + (1-\bar{\tau})D)}{\bar{\tau}(D+E)^2} < 0$$

The signs of the sensitivities in corollary 1 then follow from $p = F(\hat{A})$. $Q.E.D.$

Proof of Proposition 2: Given (27), one needs to distinguish between three intervals of the equilibrium bond return R: First, if $R \geq R_0$, the default thresholds $\hat{A}$ and $\hat{A}_N$ coincide such that $p = p_N$ irrespective of v. Second, if $R \in [R_0 - \frac{(1-v)\bar{\tau}D}{B}, R_0)$, $\hat{A}_N = \frac{D}{D+E} = A^*$ whereas $\hat{A} < A^*$ for all $R < R_0$, which implies that $\hat{A}_N > \hat{A}$ and $p_N > p$. Consequently, $\hat{A}_N \geq \hat{A}$ for all $R \geq R_0 - \frac{(1-v)\bar{\tau}D}{B}$. Rearranging this condition yields

$$v \leq \frac{BR - \bar{\tau}W_2}{\bar{\tau}D} \tag{42}$$

Third, if $R < R_0 - \frac{(1-v)\bar{\tau}D}{B}$, $\hat{A}_N = \frac{BR - \bar{\tau}W_2}{v\bar{\tau}(D+E)}$ and $\hat{A} = \frac{D}{D+E} + \frac{BR - \bar{\tau}(D+W_2)}{D+E}$. Solving $\hat{A}_N \geq \hat{A}$ for v yields

$$v \leq \frac{BR - \bar{\tau}W_2}{\bar{\tau}[BR - \bar{\tau}W_2 + (1-\bar{\tau})D]} \tag{43}$$

For $p \geq p_N$, the equilibrium allocation (i.e., the combination of R and v) needs to satisfy either (42) or (43). Since all combinations that fulfill (43) are also consistent with (42) and since $v \leq 1$, condition (28) characterizes all allocations for which deposit insurance does not increase the government's default threshold and vulnerability. $Q.E.D.$

Proof of Lemma 3: This proof is similar to the proof of lemma 1: First, it is shown that the bank's objective function is increasing in or independent of D and either linear or convex in L such that the bank is willing to accept any amount of deposits and its

optimal asset allocation is a corner solution (i.e., the bank either provides no loans at all or the maximum amount possible μE). Second, we characterize the asset allocation depending on sovereign bond characteristics (i.e., p and R) by comparing expected profits for the two corner solutions. First, suppose $p < p_1$, that is, the government is still solvent for the realization of A at which the bank fails. The bank's expected profit is:

$$\pi = \left[\bar{A} - F(A^*)A^* - \int_{A^*}^{\bar{A}} F(A)dA \right] L + [1 - F(A^*)][R(D + E - L) - D]$$

where by (34), its default threshold equals $A^* = \max\left\{ \frac{D - R(D+E-L)}{L}, F^{-1}(p) \right\}$.[25] The first-order condition w.r.t. L is nonnegative due to $R \geq 1$:

$$\frac{\partial \pi}{\partial D} = [1 - F(A^*)](R - 1) \geq 0$$

The first-order condition w.r.t. L is

$$\frac{\partial \pi}{\partial L} = \bar{A} - F(A^*)A^* - \int_{A^*}^{\bar{A}} F(A)dA - [1 - F(A^*)]R$$

The objective function is linear or convex as the second-order condition is nonnegative

$$\frac{\partial^2 \pi}{\partial L^2} = f(A^*)(R - A^*)\frac{\partial A^*}{\partial L} \geq 0$$

as $\frac{\partial A^*}{\partial L} \geq 0$ and $R \geq A^*$. Therefore, expected profit is maximized either if $L = \mu E$ or $L = 0$. The bank chooses $L = \mu E$ as long as it yields a larger expected profit, $\pi(\mu E) \geq \pi(0)$:

$$\left[\bar{A} - p_1 F^{-1}(p_1) - \int_{F^{-1}(p_1)}^{\bar{A}} F(A)dA \right] \mu E + (1 - p_1)[R(D + E - \mu E) - D] \geq (1 - p)[R(D + E) - D]$$

This inequality uses $A^* = F^{-1}(p_1) = \frac{D - R(D+E-\mu E)}{\mu E}$ if $L = \mu E$ and $A^* = F^{-1}(p)$ if $L = 0$. Using these definitions and rearranging yields the first part of (36).

---

[25]More precisely, the default threshold is again $[D - R(D+E-L)]/L$ if $L \geq [R(D+E) - D]/[R - F^{-1}(p)]$ and $F^{-1}(p)$ else.

Second, suppose that $p \in [p_1, p_2]$, the expected bank profit is

$$\pi = \left[\bar{A} - pF^{-1}(p) - \int_{F^{-1}(p)}^{\bar{A}} F(A)dA \right] L + (1-p)[R(D+E-L)-D]$$

where $A^* = F^{-1}(p)$ irrespective of D and L. Obviously, the objective function is linear. While it is non-decreasing in D such that the bank accepts any amount of deposits, the asset allocation is determined by comparing the corner solutions: The bank chooses $L = \mu E$ as long as

$$\left[\bar{A} - pF^{-1}(p) - \int_{F^{-1}(p)}^{\bar{A}} F(A)dA \right] \mu E + (1-p)[R(D+E-\mu E)-D] \geq (1-p)[R(D+E)-D]$$

and $L = 0$ otherwise. Rearranging yields the second part of (36).

Eventually, suppose $p > p_2$. The government defaults for the realization of A at which the bank fails. Hence, the bank's expected profit is:

$$\pi = \left[\bar{A} - F(A^*)A^* - \int_{A^*}^{\bar{A}} F(A)dA \right] L + (1-p)R(D+E-L) - [1-F(A^*)]D$$

The default threshold equals $A^* = \min \left\{ \frac{D}{L}, F^{-1}(p) \right\}$.[26] The first-order condition w.r.t. D is

$$\frac{\partial \pi}{\partial D} = (1-p)R - [1 - F(A^*)]$$

In equilibrium, bonds are priced such that this condition is nonnegative and the bank raises any amount of deposits supplied by households. The first-order condition w.r.t. L is

$$\frac{d\pi}{dL} = \bar{A} - F(A^*)A^* - \int_{A^*}^{\bar{A}} F(A)dA - (1-p)R$$

The objective function is again linear or convex in L due to

$$\frac{\partial^2 \pi}{\partial L^2} = -f(A^*)A^* \frac{\partial A^*}{\partial L} \geq 0$$

---

[26]More precisely, it is D/L if $L \geq D/F^{-1}(p)$ and $F^{-1}(p)$ else.

as $\frac{\partial A^*}{\partial L} \leq 0$. The bank chooses $L = \mu E$ if

$$\left[\bar{A} - \frac{p_2 D}{\mu E} - \int_{D/\mu E}^{\bar{A}} F(A) dA \right] \mu E + (1-p)R(D+E-\mu E) - (1-p_2)D \geq (1-p)[R(D+E)-D]$$

This inequality uses $A^* = F^{-1}(p_2) = \frac{D}{\mu E}$ if $L = \mu E$ and $A^* = F^{-1}(p)$ if $L = 0$. Applying these definitions and rearranging yields the third part of (36). $Q.E.D.$

Proof of Proposition 3: In the extension, we focus on the scenario where the bond return implied by fair bond pricing (11) never exceeds the cutoff R' given by (34) such that the bank always holds $L = \mu E$ and $G = D + E - \mu E$ if bonds are fairly priced. This requires

$$E(A) + \int_0^{\frac{D}{\mu E}} F(A) dA - \frac{D}{\mu E} \geq 1 \tag{44}$$

If this relation is satisfied, $R \leq R'$ for all $p \in [0,1]$. Since any equilibrium requires fairly priced bonds, only the asset allocation $L = \mu E$ and $G = D + E - \mu E$ is consistent with equilibrium. The bond return is determined by (11). A default with certainty therefore implies that $R \to \infty$ is indeed justified. Since $R' \to \infty$, the bank prefers to hold a combination of loans and bonds with $L = \mu E$ and $G = D + E - \mu E$. The 'good' equilibrium exists whenever there exists a bond return $R \in (1, R_2)$ such that $F[\hat{A}_{|L=\mu E}(R)] < 1 - \frac{1}{R}$: Since $F[\hat{A}_{|L=\mu E}(1)] \geq 0 = 1 - \frac{1}{1}$, the continuity[27] of $F(A)$ and $\hat{A}_{|L=\mu E}(R)$, which also means that $F[\hat{A}_{|L=\mu E}(R)]$ is increasing in R, ensures that the 'good' equilibrium with $R_g \leq R$ exists. Note that the equilibrium asset allocation $L = \mu E$ and $G = D + E - \mu E$ is ensured by the additional condition (44) which implies $\hat{A} = \hat{A}_{|L=\mu E}$ and that the existence of R is sufficient for the existence of the equilibrium. $Q.E.D$

Proof of Corollary 3: The system (11) and (37) jointly determines $\hat{A}$ and R. Since $R < R'$ in the 'good' equilibrium, the system is

$$J^1 = \hat{A} - \frac{D - R(D+E-\mu E)}{\mu E} - \max\left\{ \frac{BR - \tau(D+W_2)}{\mu E}, \frac{BR - \tau(D+W_2)}{\bar{\tau}\mu E} \right\} = 0$$

$$J^2 = [1 - F(\hat{A})]R - 1 = 0$$

---

[27]Note that $\hat{A}_{|L=\mu E}(R)$ has two kinks at $R = R_0$ and $R = R_2$ but no jumps.

Provided that in equilibrium $R \leq R_0$, the Jacobian matrix is

$$
J = \begin{bmatrix} 1 & -\frac{(1-\omega)B}{\mu E} \\ \\ -f(\hat{A})R & 1 - F(\hat{A}) \end{bmatrix}
$$

The Jacobian determinant is

$$
\nabla = 1 - F(\hat{A}) - \frac{f(\hat{A})(1-\omega)BR}{\mu E} > 0
$$

The sign of the Jacobian determinant again follows from the property of the 'good' equilibrium that the bond pricing curve is steeper than the default threshold (i.e., $1/R^2 > f(\hat{A})d\hat{A}/dR$). Substituting $1 - F(\hat{A}) = 1/R$ implies $\nabla > 0$ such that Cramer's rule yields:

$$
\frac{\partial R}{\partial B} = \frac{1}{\nabla}\frac{f(\hat{A})R^2}{\mu E} > 0, \quad \frac{\partial \hat{A}}{\partial B} = \frac{1}{\nabla}\frac{(1-F(\hat{A}))R}{\mu E} > 0
$$
$$
\frac{\partial R}{\partial \bar{\tau}} = -\frac{1}{\nabla}\frac{f(\hat{A})R(D+W_2)}{\mu E} < 0, \quad \frac{\partial \hat{A}}{\partial \bar{\tau}} = -\frac{1}{\nabla}\frac{(1-F(\hat{A}))(D+W_2)}{\mu E} < 0
$$
$$
\frac{\partial R}{\partial \mu} = \frac{1}{\nabla}\frac{f(\hat{A})R[R(D+E) - D - (BR - \bar{\tau}(W_2+D))]}{\mu^2 E} > 0
$$
$$
\frac{\partial \hat{A}}{\partial \mu} = \frac{1}{\nabla}\frac{(1-F(\hat{A}))[R(D+E) - D - (BR - \bar{\tau}(W_2+D))]}{\mu^2 E} > 0
$$

If in equilibrium $R > R_0$, the Jacobian matrix is

$$
J = \begin{bmatrix} 1 & -\frac{(1-\omega\bar{\tau})B}{\bar{\tau}\mu E} \\ \\ -f(\hat{A})R & 1 - F(\hat{A}) \end{bmatrix}
$$

By the same argument as above, it can be shown that the Jacobian determinant is positive:

$$
\nabla = 1 - F(\hat{A}) - \frac{f(\hat{A})(1-\omega\bar{\tau})BR}{\bar{\tau}\mu E} > 0
$$

Using Cramer's rule yields:

$$
\frac{\partial R}{\partial B} = \frac{1}{\nabla}\frac{f(\hat{A})R^2}{\bar{\tau}\mu E} > 0, \quad \frac{\partial \hat{A}}{\partial B} = \frac{1}{\nabla}\frac{(1-F(\hat{A}))R}{\bar{\tau}\mu E} > 0
$$
$$
\frac{\partial R}{\partial \bar{\tau}} = -\frac{1}{\nabla}\frac{f(\hat{A})R(D+W_2)}{\bar{\tau}\mu E} < 0, \quad \frac{\partial \hat{A}}{\partial \bar{\tau}} = -\frac{1}{\nabla}\frac{(1-F(\hat{A}))(D+W_2)}{\bar{\tau}\mu E} < 0
$$

$$\frac{\partial R}{\partial \mu} = \frac{1}{\nabla} \frac{f(\hat{A})R[\bar{\tau}W_2 - R(B - \bar{\tau}(D+E))]}{\bar{\tau}\mu^2 E}, \quad \frac{\partial \hat{A}}{\partial \mu} = \frac{1}{\nabla} \frac{(1 - F(\hat{A}))[\bar{\tau}W_2 - R(B - \bar{\tau}(D+E))]}{\bar{\tau}\mu^2 E}$$

The sensitivities $\frac{\partial R}{\partial \mu}$ and $\frac{\partial \hat{A}}{\partial \mu}$ are positive as long as $\bar{\tau}[R(D+E) + W_2] > BR$ or, equivalently, $R < \frac{\bar{\tau}W_2}{B - \bar{\tau}(D+E)}$.[28] Rearranging yields $\hat{A} < R$, that is, as long as the sovereign default threshold is smaller than the equilibrium bond return, sovereign risk increases in the regulatory multiplier. As soon as $\hat{A} > R$, sovereign risk decreases in the multiplier. The signs of the sensitivities in corollary 3 then follow from $p = F(\hat{A})$. Q.E.D.

Proof of Corollary 4: If $R \leq R_0$ in equilibrium, $A^* = \frac{D - R(D+E-\mu E)}{\mu E}$ with partial derivatives:

$$\frac{\partial A^*}{\partial B} = -\frac{D + E - \mu E}{\mu E} \frac{\partial R}{\partial B} < 0$$

$$\frac{\partial A^*}{\partial \bar{\tau}} = -\frac{D + E - \mu E}{\mu E} \frac{\partial R}{\partial \bar{\tau}} > 0$$

$$\frac{\partial A^*}{\partial \mu} = \frac{R(D+E) - D}{\mu^2 E} - \frac{D + E - \mu E}{\mu E} \frac{\partial R}{\partial \mu}$$

The signs of $\frac{\partial A^*}{\partial B}$ and $\frac{\partial A^*}{\partial \bar{\tau}}$ follow directly from the sensitivities of R summarized in corollary 3. The sign of $\frac{\partial A^*}{\partial \mu}$ is unclear given that R increases in μ. If $R \in (R_0, R_1)$, the bank and sovereign default threshold coincide such that $\frac{\partial A^*}{\partial B} > 0$ and $\frac{\partial A^*}{\partial \bar{\tau}} < 0$ and $\frac{\partial A^*}{\partial \mu}$ is ambiguous. If $R \geq R_1$, $A^* = \frac{D}{\mu E}$ is independent of the bond return, which implies that $\frac{\partial A^*}{\partial B} = \frac{\partial A^*}{\partial \bar{\tau}} = 0$ and $\frac{\partial A^*}{\partial \mu} < 0$. Q.E.D.

---

[28]Note that $R_0 < \frac{\bar{\tau}W_2}{B - \bar{\tau}(D+E)}$ such that a positive sign of these sensitivities is, in principle, feasible if the equilibrium bond return exceeds R_0.