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Optimal Bank Regulation, the Real Sector, and the State of the Economy¹

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Abstract

Concerns about the procyclicality of bank regulation have motivated recent reforms that include countercyclical measures. This paper analyzes how optimal capital requirements, which balance a trade-off between financial stability and investment of the real sector, adjust during a downturn. Adding an endogenous loan market reveals equilibrium effects that strongly influence the adjustment and allows studying the implications of real shocks. The results suggest a nuanced adjustment depending on the shock: In a capital crunch, capital requirements are relaxed to prevent a sharp decline in investment. If productivity decreases, they are tightened as preserving financial stability entails a smaller cost.

Keywords

Capital Regulation, Credit Markets, Banking Crisis, Business Cycle

JEL Classification

G21, G28

1 Introduction

The interplay of the banking system in general and capital regulation in particular with the business cycle has figured prominently in the context of banking reform and is one of the main aspects in several policy reports.¹ The fundamental problem is well known: During a downturn, many banks experience negative funding shocks as, for example, more frequent loan losses weaken their capitalization while it is particularly challenging to raise new equity such that regulatory constraints become binding. At the same time, traditional, risk-sensitive capital requirements tighten as risk weights increase to account for the generally higher loan risk. In order to meet the regulatory requirements, banks thus deleverage and cut lending, which may even lead to a credit crunch. This clearly procyclical behavior aggravates the downturn with a potentially adverse feedback on financial stability. Yet, bank loans are riskier in bad times such that a larger capital buffer is necessary to prevent a banking crisis. In addition, the investment prospects in the real sector are often rather poor, and a smaller loan supply as a result of binding regulatory constraints may thus turn out to be less problematic because fewer investments would be realized even if funding was available. The conflicting goals of ensuring bank safety and preventing a further decline of investment and aggregate demand to some extent reflect the tension between micro- and macroprudential regulation. With Basel III, regulators try to mitigate the procyclicality of capital requirements through a countercyclical and a capital conservation buffer.² As a result, regulation tends to be tougher in good times when the risk of unsustainable lending booms and asset price bubbles is high and more relaxed in bad times when recapitalization is difficult.

We study this trade-off and provide a normative analysis of how capital requirements should adjust to different economic shocks. The paper presents a model of the capital structure where equity provides a buffer against loan losses and thus lowers the risk of a banking crisis. As an innovation, we explicitly model the real sector consisting of bank-dependent entrepreneurs thereby endogenizing the loan market. This approach reveals important equilibrium effects that influence the optimal regulatory adjustment and highlights how bank regulation depends on real sector shocks and characteristics. At the core of this paper is an extensive comparative statics analysis with two scenarios: (i) a

¹For example, Brunnermeier et al. (2009), Turner Review (2009), and FSB (2009).

²See sections III and IV in BCBS (2010).

shortage of bank capital (henceforth: capital crunch) that limits banks' lending capacity and (ii) a lower productivity of entrepreneurs that reduces loan demand and the value of investment. The optimal capital requirements, which balance the trade-off between the stability of banks and the ability of entrepreneurs to finance profitable investments, relate to the state of the economy through the lending rate, which provides a *de facto* substitute for equity, and the welfare-maximizing level of bank risk. Their adjustment fundamentally differs between the two scenarios: Capital requirements should be relaxed in a capital crunch to prevent a contraction of lending but they should be stricter if productivity declines and the lending rate and the value of investment are low. Compared to alternative systems like risk-sensitive or flat capital requirements, optimal regulation allows for a more flexible adjustment of the economy in a downturn and thus mitigates its adverse welfare consequences.

The analysis builds on the literature on the real effects of capital regulation and, more generally, of funding shocks³: Since the introduction of the Basel accords, their real and especially their procyclical effects have been extensively studied.⁴ As a first benchmark, the Modigliani-Miller theorem, however, implies that capital requirements do not have any pronounced real effects as they can be fulfilled with outside equity which should not raise the cost of capital. Such arguments have recently been emphasized, for instance, by Admati et al. (2011); quantitative simulations by Miles et al. (2012) imply only minor long-run effects on customers' borrowing costs even if capital requirements strongly increase. Nevertheless, equity can be scarce and expensive⁵ especially during bad times such that capital requirements have the potential to affect lending and investment. Blum and Hellwig (1995) highlight two key frictions that create such real effects: First, banks do not recapitalize by issuing new equity and deleverage instead, second, firms cannot fully substitute bank loans with other funds. They show that whenever capital requirements are binding, equilibrium output and prices become more sensitive to aggregate demand shocks thereby amplifying macroeconomic fluctuations. Furthermore, Heid (2007) shows that banks may hold voluntary buffers in excess of capital charges. These buffers mitigate

³A seminal theoretical contribution is Holmström and Tirole (1997) who study the (heterogeneous) effects of shocks to the supply of different types of capital.

⁴For an overview about links between capital requirements and the real economy, see, Goodhart and Taylor (2006).

⁵For example due to tax benefits of debt finance or asymmetric information cost of equity and signaling considerations as emphasized by Myers and Majluf (1984).

but do not offset the procyclical effects of capital requirements. Further theoretical contributions on the procyclicality of capital regulation include, among others, Estrella (2004), Zhu (2008), and Covas and Fujita (2010). On the empirical side, early evidence of how binding regulatory constraints affect lending is provided by Peek and Rosengren (1995a) who study the New England capital crunch in the early 1990s when capital requirements were actively enforced. They find that assets of banks subject to formal enforcement actions shrink significantly faster than those of banks without and that loans to bank-dependent borrowers are most strongly affected. Using a sample of French firms, Fraise et al. (2013) find that a one percentage point increase in bank capital requirements lowers credit by eight and firm borrowing by four percent. Hence, firms can only partly compensate the smaller loan supply. In a similar spirit, Aiyar et al. (2014) present evidence for the UK and stress the role of loans from foreign banks as substitutes. The procyclicality of capital requirements, in particular of Basel II, is documented, for example, by Kashyap and Stein (2004) and Gordy and Howells (2006) for American, Repullo et al. (2010) for Spanish, and Andersen (2011) for Norwegian banks.

This paper contributes to the literature on the optimal adjustment of bank regulation to macroeconomic shocks: Kashyap and Stein (2004)⁶ show that if the shadow value of bank capital varies over the cycle, optimal capital regulation should be countercyclical. More precisely, they argue for a family of risk curves, which map the risk of each asset into a capital charge, where each curve is associated with a specific shadow value. This preserves the sensitivity of capital requirements across asset categories with different risks but allows for an adjustment over the cycle. In a dynamic equilibrium model with time-varying loan risk, Repullo and Suarez (2013) characterize the welfare properties of different regulatory systems and conclude that optimal capital regulation is procyclical. However, the cyclical variation is less pronounced compared to Basel II for sufficiently large values of the social cost of bank failure. Several contributions analyze capital regulation in models with agency problems: Dewatripont and Tirole (2012) show that capital requirements allocate the control rights of bank shareholders and debtholders as to ensure managerial effort and prevent gambling for resurrection. Regulation should neutralize macroeconomic shocks that would otherwise distort incentives; this is achieved by countercyclical capital buffers or capital insurance. Repullo (2013) stresses the role of

⁶The underlying model can be found in the 2003 working paper version.

costly bank capital in a risk-shifting model: He studies the trade-off between mitigating risk shifting and preserving the lending capacity of banks. Given a shortage of bank capital, its shadow value increases and optimal capital requirements are relaxed. If they remained unchanged, banks would be safer but aggregate investment would sharply drop. Focusing on credit cycles, Gersbach and Rochet (2012) argue that countercyclical capital regulation implemented, for example, as an upper bound on short-term debt corrects the misallocation of credit between good and bad states of nature thereby dampening fluctuations. Several options how cyclically-varying capital regulation can be implemented have been suggested, in particular, direct and indirect smoothing rules for capital requirements [e.g., Gordy and Howells (2006), Brunnermeier et al. (2009), Repullo et al. (2010)] and the build-up of countercyclical buffers [e.g., FSB (2009), BCBS (2010)], which are envisaged by Basel III. Alternative proposals include dynamic provisioning, contingent convertibles and capital insurance [e.g., Kashyap et al. (2008)], and regulatory discretion. Yet, it is too early to present evidence about the consequences of such countercyclical measures but Jiménez et al. (2015) evaluate a comparable policy introduced in Spain already in 2000: dynamic provisions. These provisions are built up from retained earnings during a boom to cover loan losses in bad times when equity is scarce, and they are, in fact, similar to countercyclical capital buffers. They find that dynamic provisions significantly mitigate the fall of bank lending and firm borrowing during the financial crisis. In the good times during the early 2000s, banks that had to build up larger provisions reduced their loan supply but firms could easily substitute by borrowing from less affected banks. The main contribution of this paper is a normative analysis of how capital requirements adjust to economic shocks especially during a downturn. A full-fledged model of the real sector and the loan market identifies equilibrium effects associated with the lending rate that together with changes in the desired risk level determine the regulatory adjustment. In addition, this extension allows analyzing whether and how optimal regulation responds to real shocks (e.g., productivity), which have not been studied so far despite their importance in macroeconomics. The model is most closely related to Repullo (2013) and Repullo and Suarez (2013): Compared to the former, we explicitly model the real sector with an endogenous loan market; the role of bank capital as a buffer requires positively correlated defaults but is more conventional than the incentive mechanism otherwise. This contribution differs from Repullo and Suarez (2013), who do include the real sector

in their dynamic framework, by its focus on variations in financial factors and productivity instead of loan risk and by providing an analytical solution.

The remainder of this paper is organized as follows: Section 2 outlines the model. Section 3 characterizes the equilibrium and analyzes optimal capital requirements and its adjustment. It also provides a numerical example. Finally, section 4 concludes.

2 The Model

We develop a static, partial equilibrium model of an economy populated by four types of risk-neutral agents: entrepreneurs representing the real sector and banks, investors (bank shareholders), and depositors representing the financial sector. Banks attract deposits and equity from depositors and investors and lend to entrepreneurs, who can invest in profitable but risky projects. Whenever such a project fails, the entrepreneur defaults and the bank incurs a loan loss. The risk characteristics crucially depend on whether the economy experiences a recession, which is revealed after projects were initiated: Usually, only idiosyncratic risk matters such that each bank can diversify its loan portfolio. In a recession, however, systemic risk materializes and the defaults of entrepreneurs are positively correlated. As a result, banks may fail whenever too many borrowers simultaneously default and their equity cannot fully absorb all losses. This leads to a banking crisis, the costs of which are not completely internalized and thus provide a rationale for regulation. The timing is as follows: (i) banks attract capital from depositors and investors and lend to entrepreneurs who invest, (ii) it is revealed whether project risks are independent (normal state) or positively correlated (recession), and (iii) projects mature and the contracts are settled.

The following friction makes sure that capital requirements have the potential to affect the real economy:

ASSUMPTION 1 *Entrepreneurs can finance their projects with bank loans only.*

Hence, entrepreneurs are bank-dependent and do not raise funds directly from investors or depositors. Evidence of Fraisse et al. (2013) and Jiménez et al. (2015) supports this assumption especially during bad times. In a broader context, this of course concerns only some firms like, for example, small businesses, whereas others can access the capital

market. Another friction - whether banks raise new equity or deleverage to satisfy capital requirements - endogenously emerges depending on the scarcity of bank capital.

2.1 Entrepreneurs

The real sector consists of a continuum of measure one of penniless entrepreneurs. Each of them can undertake a risky investment project characterized by:

ASSUMPTION 2 *The unit-size project yields a binary return*

$$\tilde{R} = \begin{cases} R, & 1 - p_0 \\ \alpha, & p_0 \end{cases} \quad (1)$$

with $R > 1 > \alpha$. The net present value is positive: $\mu \equiv (1 - p_0)R + p_0\alpha - 1 > 0$.

Subsequently, we interpret the return R as the entrepreneur's productivity and α as the liquidation value. If the project fails, the latter is appropriated by the lender and $1 - \alpha$ equals the loss given default. Failure and success probability, p_0 and $1 - p_0$, are *ex ante* probabilities that consist of an idiosyncratic and a systemic component; the latter gives rise to correlated defaults.

The loan demand is modeled as in Repullo and Martinez-Miera (2010): Entrepreneurs face heterogeneous opportunity costs, $u \sim U[0, 1]$. They may represent, for instance, forgone labor income or the value of leisure. Only an entrepreneur whose opportunity cost is smaller than the expected net return on investment borrows and invests:

$$u \leq (1 - p)(R - r_L) \equiv \hat{u}(r_L) \quad (2)$$

\hat{u} defines the marginal entrepreneur who is just indifferent between investing and choosing the outside option. Since opportunity costs are uniformly distributed, \hat{u} also equals the fraction of investing entrepreneurs (i.e., with opportunity costs below the threshold) and thus the loan demand. It decreases in the lending rate r_L and the *ex ante* project risk p_0 and increases in productivity R . The surplus of an active entrepreneur equals $(1 - p_0)(R - r_L) - u$; the aggregate surplus of the real sector is $\pi^E = \int_0^{\hat{u}} (1 - p_0)(R - r_L) - u du$. Since they undertake a single investment, the correlation of projects matters little for individual entrepreneurs. However, it is crucial for banks as they fail whenever too

many entrepreneurs simultaneously default. We suggest an intuitive and tractable model of project correlation across entrepreneurs: The economy may experience either normal conditions or a recession, which is revealed after the projects are initiated and determines to what extent failures are independent or correlated:

ASSUMPTION 3 *In normal times (probability $1 - \theta$), projects are independent and each of them succeeds with probability $1 - p$ and fails with probability p . In a recession (probability θ), a fraction x of projects immediately fails. $x \in [0, 1]$ is distributed according to some continuous, differentiable distribution function $F(x)$. The remaining projects continue and succeed with probability $1 - p$ and fail with probability p*

Hence, x captures the systemic and p the idiosyncratic risk component. Projects are generally independent but a recession is associated with an adverse shock to a stochastic fraction of projects, which thus immediately fail. This represents a macroeconomic shock that has the potential to affect all entrepreneurs at the same time like, for example, a fall in aggregate demand or - in a small, open economy - a sudden appreciation of the currency. Figure 1 illustrates the possible outcomes: In normal times, a project succeeds and yields the payoff R with probability $1 - p$ and fails and yields the payoff α with probability p . In a recession, a fraction x of all projects fails due to the shock, whereas the unaffected projects independently succeed or fail according to idiosyncratic risk. The stochastic variable x measures the severity of a recession, high realizations imply a severe recession.

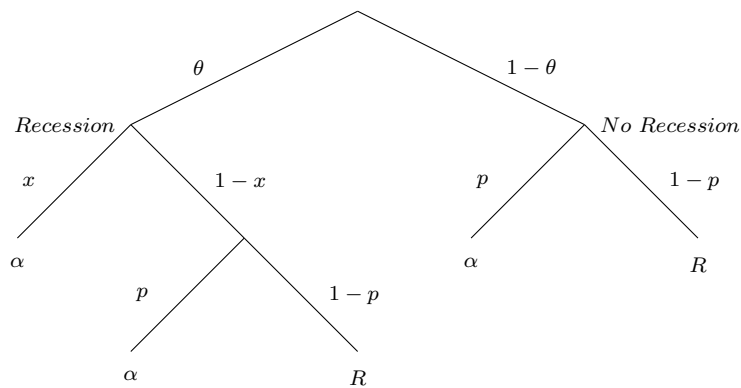


Figure 1: *Probability Tree*

Eventually, table 1 summarize a project's success and failure probabilities *ex ante* as well as in a recession and in normal times using $x_0 \equiv E(x) = \int_0^1 x dF(x)$. Intuitively, a recession revises the failure probability up compared to the idiosyncratic probability p .

	Success	Failure
Ex ante	$1 - p_0 = (1 - p)(1 - \theta x_0)$	$p_0 = p + \theta x_0(1 - p)$
Recession	$(1 - p)(1 - x)$	$p + x(1 - p)$
No Recession	$1 - p$	p

Table 1: *Probabilities*

2.2 Banks

There is a continuum of measure one of banks that lend to entrepreneurs; more specifically, they provide unit-size loans to a mass L of active entrepreneurs. Each bank can raise funds from two sources: deposits (share $1 - k$) and bank capital (share k). Bank owners are protected by limited liability. Deposits are elastically supplied at the risk-free (gross) interest rate normalized to one but depositors require a compensation for bearing the bank's failure risk giving rise to a risk-adjusted deposit rate $r \geq 1$. One might alternatively interpret r as the risk-free rate plus an actuarially fair deposit insurance premium. Bank capital is provided by investors (i.e., outside shareholders) who require an expected (gross) return on equity $\gamma \geq 1$.

Bank risk crucially depends on whether the economy experiences a recession or not: In normal times, loans are uncorrelated because a deterministic fraction $1 - p$ is repaid and a fraction p fails. Hence, the portfolio is diversified and the bank is safe. A recession, in contrast, leads to the failure of a stochastic fraction $p + x(1 - p)$ of loans: a share x due to the macroeconomic shock and a share $(1 - x)p$ due to project-specific risk. The bank thus receives the full repayment r_L from a fraction $(1 - x)(1 - p)$ of borrowers and the liquidation value α from a fraction $p + (1 - x)p$. It succeeds as long as enough loans are repaid such that losses are small. This requires the share of entrepreneurs who immediately fail to be smaller than the failure threshold \hat{x} given by equalizing the bank's assets and liabilities:

$$(1 - \hat{x})(1 - p)r_L + [p + \hat{x}(1 - p)]\alpha = r(1 - k) \quad (3)$$

Hence, the liabilities, $r(1 - k)L$, are just covered by the assets consisting of repaid and liquidated loans. In other words, the bank's end-of-period equity is zero. Obviously, the failure threshold increases in the deposit rate and in the idiosyncratic project risk and decreases in the capital ratio, the lending rate, and the liquidation value. Importantly,

the deposit rate is endogenous because depositors require a risk-adjusted interest rate. As soon as the recession is more severe such that a more borrowers default, the loss wipes out the bank's equity and its liabilities are not covered. Bank failures are correlated because banks are identical and defaults occur due to an economy-wide shock, which gives rise to a systemic banking crisis if $x > \hat{x}$. Consequently, the economy can be in three different states: In normal times, banks diversify their portfolios such that they are safe. In a (mild) recession, there are more frequent defaults in the real sector because of the macroeconomic shock. However, banks can absorb these additional losses. Whenever the shock is more severe, the recession transforms into a systemic banking crisis. The *ex ante* probabilities of the three outcomes are $1 - \theta$, $\theta F(\hat{x})$, and $\theta[1 - F(\hat{x})]$ respectively. Importantly, bank risk originates from defaults in the real economy that can be attributed to a recession. Whether these defaults trigger a banking crisis or not, in contrast, depends on bank characteristics that determine the failure threshold \hat{x} . We add the assumption that a banking crisis is costly for society:

ASSUMPTION 4 *A banking crisis entails a social cost c per unit of loans.*

This reduced-form approach captures, for example, the costs of bank runs and contagion, the loss of lender-borrower relationships or disruptions to the payment systems.⁷ The failure of banks to internalize these costs is the very reason why a market equilibrium is inefficient and thus provides a rationale for regulation. This is a common motivation in the literature applied, for instance, by Kashyap and Stein (2004), Repullo (2013), and Repullo and Suarez (2013). An alternative interpretation would be a utility loss of depositors, who lose money during a banking crisis. The deposit market can only compensate income losses (by offering a risk-adjusted interest rate) but not the additional utility loss, which is thus uninsurable and represents an externality of banks' leverage choice. Such an approach essentially mimics risk aversion and has been applied in a model of optimal unemployment insurance by Blanchard and Tirole (2008).

Eventually, bank's expected surplus equals:

$$\begin{aligned} \pi^B = & \theta \int_0^{\hat{x}} (1-x)(1-p)r_L + [p+x(1-p)]\alpha - r(1-k)dF(x)L \\ & + (1-\theta)[(1-p)r_L + p\alpha - r(1-k)]L - \gamma kL \end{aligned} \quad (4)$$

⁷Note that wealth losses of depositors (or the cost of providing deposit insurance) are fully internalized as deposits are correctly priced.

It consists of the profit in normal times (with probability $1 - \theta$) and the expected profit in a recession (with probability θ) net of the required return on equity. In both states, the profit equals gross interest income from repaid and the liquidation value of failed loans minus deposit repayment. Since bank owners are protected by limited liability, the payoff is zero in case of failure (i.e., if $x > \hat{x}$). To maximize its surplus, the bank determines the capital ratio k and the loan supply L .

2.3 Depositors and Investors

The supply side is modeled as in Repullo (2013) with an elastic deposit and a fixed bank capital supply: First, risk-neutral depositors elastically supply deposits as long as they yield an expected return equal to the (gross) risk-free interest rate that is normalized to one. Hence, there is market discipline as the interest rate compensates depositors for bearing the risk of bank failure such that:

$$E \left[\min \left\{ r, \frac{(1-p)(1-x)r_L + [p + x(1-p)]\alpha}{1-k} \right\} \right] = 1 \quad (5)$$

One may interpret this condition as a participation constraint of depositors: Whenever a bank succeeds, depositors earn an interest rate r . In case of failure, however, each depositor inherits a share $1/(1-k)L$ of bank assets $[(1-p)(1-x)r_L + (p + x(1-p))\alpha]L$. Consequently, they earn the interest rate in normal times or in a mild recession [which occur with probability $1 - \theta + \theta F(\hat{x})$] and inherit the assets in a banking crisis:

$$[1 - \theta + \theta F(\hat{x})]r + \theta \int_{\hat{x}}^1 \frac{(1-p)(1-x)r_L + [p + x(1-p)]\alpha}{1-k} dF(x) = 1 \quad (6)$$

Since they are paid a risk-adjusted interest rate, depositors' expected surplus is zero. As long as the deposit rate satisfies the participation constraint, they are willing to supply any quantity.

Second, investors supply a fixed amount K of bank capital and require an expected return on equity γ which is at least one:

$$K(\gamma) = \begin{cases} K, & \text{if } \gamma \geq 1 \\ 0, & \text{if } \gamma < 1 \end{cases} \quad (7)$$

Hence, the expected surplus of investors is $\pi^I = (\gamma - 1)K(\gamma) \geq 0$. A fixed supply of bank capital is typical for models with funding shocks such as Holmström and Tirole (1997) as this formulation allows capturing such shocks by comparative statics. An alternative is an exogenous excess return on equity such as in Repullo and Suarez (2013).

2.4 Markets

In this economy, three markets exist - a market for loans, deposits, and bank capital. The loan market clears as soon as $L = \hat{u}$ such that the loan supply equals the fraction of entrepreneurs who invest. This pins down the lending rate r_L . Given the perfectly elastic supply, the deposit market is in equilibrium whenever banks promise a deposit rate that satisfies the participation constraint of depositors (5). Eventually, the market for bank capital is in equilibrium if $K(\gamma) = kL$ thereby determining the return on equity. However, this market may not clear if bank capital is abundant in supply such that $K > kL > 0$ even if the required returns on equity and deposits are the same ($\gamma = 1$).

2.5 State of the Economy

The state of the economy characterizes the economic conditions. We examine the optimal adjustment of capital requirements to a financial and a real shock and focus on (i) the availability of bank capital⁸ given by the fixed supply K and (ii) entrepreneurs' productivity R . The supply of bank capital immediately affects banks. As soon as they face binding capital requirements and borrowers are bank-dependent, a shortage of bank capital - a capital crunch - may force them to cut lending, which has real effects because it constrains entrepreneurial investment. The empirical relevance of capital crunches is documented, for example, by Bernanke and Lown (1991) and Peek and Rosengren (1995b). Such a scenario is also at the core of Repullo's (2013) analysis. It represents a financial shock that can be the result of swings in investors' moods and optimism.

The project return R , in contrast, captures technology or productivity shocks that feature prominently in macroeconomics and also characterizes entrepreneurs' investment prospects. It is a crucial determinant of their investment decisions and thus influences

⁸In our static setting where banks raise new equity, the interpretation of changes in the supply of bank capital appears suitable. In a dynamic model, a broader interpretation would also include shocks to the current capitalization, for example, due to loan losses.

the loan demand. Although banks are not directly, optimal regulation can be adjusted for two reasons: equilibrium effects associated with changes in the lending rate that influence bank risk and changes in the value of projects that tilt the underlying trade-off between financial stability and investment.

3 Equilibrium Analysis

This section characterizes two allocations: the market equilibrium and the social optimum where all costs of a banking crisis are internalized. The latter is the reason for market failure in the sense that banks are inadequately capitalized and may provide too large an amount of loans. Subsequently, we show how the optimal allocation can be decentralized using capital requirements and study their adjustment to economic shocks.

An outcome of key interest is bank risk or, more precisely, the likelihood of a banking crisis: It is jointly determined by the identity that equalizes assets and liabilities (3) and the participation constraint of depositors (5) that pin down the failure threshold and the deposit rate respectively:

LEMMA 1 *Banks fail in a recession if a fraction $x > \hat{x}$ of entrepreneurs immediately default. This threshold is characterized by*

$$1 - k - \alpha - (r_L - \alpha)(1 - p)H(\hat{x}) = 0 \quad (8)$$

where

$$H(\hat{x}) = (1 - \theta)(1 - \hat{x}) + \theta \int_{\hat{x}}^1 F(x) dx \quad (9)$$

is a decreasing function of \hat{x} with $H'(\hat{x}) = -[1 - \theta + \theta F(\hat{x})] < 0$, $H(0) = 1 - \theta x_0$ and $H(1) = 0$. The failure threshold increases in the capital ratio, the lending rate, and the liquidation value.

Proof: See Appendix A.1.

Condition (8) relates bank risk to the capital structure and the lending rate. Well-capitalized banks that earn a high lending rate are particularly safe. Moreover, a bank can be risk-free whenever it succeeds in repaying deposits even if all borrowers simultaneously default [i.e., the threshold equals $\hat{x} = 1$ such that $H(\hat{x}) = 0$]. This requires a capital

ratio of at least $1 - \alpha$, which suffices to cover the loss given default. In the extreme case $\alpha = 0$, this corresponds to an all-equity financed bank.

3.1 Market Equilibrium

Each bank determines its capital ratio k and loan supply L as well as the interest rate offered to depositors r in order to maximize the expected surplus π^B which is given in (26) subject to depositors' participation constraint (6). By substituting the latter into the objective function to eliminate r , one obtains the consolidated problem:

$$\pi^B = \max_{k,L} [(1 - p_0)r_L + p_0\alpha - (1 - k) - \gamma k] L \quad (10)$$

The bank's optimal choices are summarized in

LEMMA 2 *The bank's capital ratio is indeterminate, $k \in [0, 1]$, if $\gamma = 1$ and zero, $k = 0$, if $\gamma > 0$. The loan supply is elastic at the lending rate*

$$r_L = \frac{1 - p_0\alpha}{1 - p_0} \quad (11)$$

such that banks earn a zero expected surplus: $\pi^B = [(1 - p_0)r_L + p_0\alpha - 1]L = 0$.

Proof: Substituting the participation constraint of depositors (6) into the objective function of the bank (26) yields the consolidated problem (10). The indeterminacy of the capital structure follows from the first-order condition $\partial\pi^B/\partial k = 1 - \gamma \leq 0$; $\partial\pi^B/\partial L = (1 - p_0)r_L + p_0\alpha - (1 - k) - \gamma k = 0$ implies that banks provide loans until they earn a zero expected surplus; substituting either $\gamma = 1$ or $k = 0$ gives (11). *Q.E.D.*

The loan supply is elastic because of the linear technology and the elastic supply of deposits. Hence, the lending rate exactly compensates banks for bearing the project risk leading to zero expected profits. In other words, the bank itself (i.e., its inside shareholders) does not earn a rent. In line with Modigliani-Miller, the capital structure is indeterminate: Since equity has no advantage over debt because the latter is correctly priced and wealth losses of depositors are internalized, the bank is indifferent as long as both types of capital have the same costs. Whenever bank capital is more expensive (i.e., $\gamma > 1$), the capital ratio is zero. Hence, only a required return on equity of one is

consistent with equilibrium such that at least some banks have a positive capital ratio. The irrelevance of the capital structure is of course a strong result: It would disappear in the presence of guarantees or tax distortions, which imply a strict preference for debt, or bank borrowing frictions such as limited pledgeability, which require a minimum capital ratio.

Entrepreneurs decide about investment: A project is undertaken if the expected profit exceeds the idiosyncratic opportunity cost u . Derived from the extensive margin, loan demand equals the fraction of entrepreneurs with sufficiently low opportunity costs, $\hat{u}(r_L)$, defined in (2). Together with loan market equilibrium, $L = \hat{u}(r_L)$, this yields:

LEMMA 3 *Equilibrium lending and investment is:*

$$L = (1 - p_0)R + p_0\alpha - 1 = \mu \quad (12)$$

It increases in the project return and is insensitive to the bank capital supply.

Proof: Investment immediately follows from the market clearing condition, $L = \hat{u}(r_L)$, by substituting (2) and (11) for loan demand and lending rate. *Q.E.D.*

Investment equals the project's expected net return. This follows from entrepreneurs' investment choice at the extensive margin combined with the risk-adjusted lending rate (11). The latter guarantees that entrepreneurs earn the project's expected net present value, $(1 - p_0)(R - r_L) = \mu$. Since opportunity costs are uniformly distributed, investing is attractive for exactly a fraction $\hat{u} = \mu$ of entrepreneurs.

3.2 The Regulator's Problem

In the market equilibrium, the social cost of a banking crisis, $C = \theta[1 - F(\hat{x})]cL$, is not internalized. Welfare W consists of the expected surplus of entrepreneurs, banks, and investors net of the social cost, $W = \pi^E + \pi^B + \pi^I - C$. Substituting $K(\gamma) = kL$ as well as eliminating the deposit rate in (6) and (8) yields:

$$W = \int_0^{\hat{u}} (1 - p_0)(R - r_L) - u du + [(1 - p_0)r_L + p_0\alpha - 1 - \theta(1 - F(\hat{x}))c] L \quad (13)$$

The first term is expected surplus of the real sector; the second term captures the surplus of the financial sector (i.e., banks and investors) net of all costs associated with a banking crisis. The regulator determines lending and investment, the marginal entrepreneur, the capital ratio of banks, and the lending rate in order to maximize welfare, which leads to full internalization. Note that the lending rate does affect welfare because of its effect on bank risk, which matters whenever failure entails a cost. In principle, the lending rate should thus be as high as possible to minimize bank risk but it is restricted: The marginal entrepreneur, \hat{u} , needs to earn a zero surplus in order to invest. This adds a participation constraint of entrepreneurs. Inserting $L = \hat{u}$, which holds in equilibrium, in (13) the optimization problem of a welfare-maximizing regulator is:

PROGRAM 1 *The regulator determines lending and investment L , banks' capital ratio k , and the lending rate r_L to maximize social welfare*

$$\max_{k, L, r_L} [\mu - \theta(1 - F(\hat{x}))c]L - \frac{L^2}{2} \quad (14)$$

subject to the participation constraint of entrepreneurs, $(1 - p_0)(R - r_L) = L$, and the capital availability constraint, $K \geq kL$.

In contrast to Modigliani-Miller's irrelevance theorem, the capital structure has welfare consequences as a higher capital ratio reduces the risk of a costly banking crisis: The capital ratio is chosen according to the first-order condition

$$\theta f(\hat{x})c \underbrace{\frac{1}{(r_L - \alpha)(1 - p)[1 - \theta + \theta F(\hat{x})]}}_{\frac{\partial \hat{x}}{\partial k}} = \lambda_1 \quad (15)$$

where λ_1 is the Lagrange multiplier of the capital availability constraint. The left-hand side captures the marginal gains of a higher capital ratio, namely, the lower risk of a banking crisis. The marginal cost equals the shadow value of bank capital captured by the multiplier. By the Envelope theorem, the latter measures the welfare contribution of bank capital $\partial W / \partial K = \lambda_1$. As long as equity does not cover the loss given default, $k < 1 - \alpha$ (see lemma 1), we have $\partial \hat{x} / \partial k > 0$ such that the shadow value of bank capital is positive and additional equity increases welfare.

Lending and investment are determined according to the first-order condition

$$\mu - L - \theta[1 - F(\hat{x})]c - \lambda_1 k - \lambda_2 = 0 \quad (16)$$

where λ_2 denotes the Lagrange multiplier of the participation constraint. Intuitively, the marginal welfare gains from lending (i.e., the expected return of financing an additional project) equal the marginal costs consisting of the opportunity and social failure cost. Investment also tightens both the participation and the capital availability constraint thereby raising bank risk because lending rate or capital ratio decrease.

3.3 Equilibrium Allocation

Based on the first-order conditions and constraints of program 1, one can characterize the optimal allocation:

PROPOSITION 1 *The failure threshold \hat{x}^* and bank lending L^* are jointly determined by the system:*

$$J^1(L^*, \hat{x}^*) = \mu - L^* - \theta[1 - F(\hat{x}^*)]c - \frac{\theta f(\hat{x}^*)c}{1 - \theta + \theta F(\hat{x}^*)} \left[\frac{1 - \alpha + \frac{H(\hat{x}^*)L^*}{1 - \theta x_0}}{\left(R - \frac{L^*}{1 - p_0} - \alpha\right)(1 - p)} - H(\hat{x}^*) \right] = 0 \quad (17)$$

$$J^2(L^*, \hat{x}^*) = K - \left[1 - \alpha - \left(R - \frac{L^*}{1 - p_0} - \alpha\right)(1 - p)H(\hat{x}^*) \right] L^* = 0 \quad (18)$$

Lending $L^ \leq \mu$ increases in the supply of bank capital, $\partial L^*/\partial K \geq 0$, and productivity, $\partial L^*/\partial R > 0$. The failure threshold \hat{x}^* increases in the supply of bank capital, $\partial \hat{x}^*/\partial K \geq 0$, but may increase or decrease in productivity. The optimal allocation requires the capital ratio:*

$$k^* = 1 - \alpha - \left(R - \frac{L^*}{1 - p_0} - \alpha\right)(1 - p)H(\hat{x}^*) \quad (19)$$

Proof: See Appendix A.1.

Compared to the market equilibrium with $L = \mu$, condition (16) implies that lending and investment are usually smaller and the lending rate is higher. The reason is that internalizing the social costs of a banking crisis requires the use of scarce equity. The optimal capital structure given by (19) ensures that the balance sheet of each bank is

consistent with the socially optimal failure threshold \hat{x}^* . This is the reason why the capital structure is not irrelevant à la Modigliani-Miller. Instead, the required capital ratio mechanically follows from the definition of the failure threshold (8).

Essentially, the optimal allocation trades off the benefit of lower bank failure risk against smaller lending and investment. This trade-off emerges as long as bank capital, which is necessary to improve the stability and resilience of banks, is scarce. A larger supply thus relaxes the capital availability constraint such that more investments are financed without increasing risk and bank risk decreases by improving their capitalization. An increase in entrepreneurs' productivity, in contrast, makes investment more valuable thereby tilting the trade-off more in favor of investment. In order to mobilize additional funds, the failure threshold of banks likely falls such that risk increases. This outcome materializes as long as capital requirements are tight and the risk of failure is low.

The supply of bank capital can be large enough to make banks risk-free such that they succeed in a recession so severe that all loans fail ($x = 1$) and only the liquidation value is recovered:

COROLLARY 1 *If the supply of bank capital exceeds $K \geq (1 - \alpha)\mu \equiv K_0$, the capital ratio is $k^* = 1 - \alpha$ such that banks are risk-free, $\hat{x}^* = 1$, and lending and investment are similar to the market equilibrium, $L^* = \mu$.*

Proof: A risk-free bank (i.e., $\hat{x} = 1$) requires $k \geq 1 - \alpha$ (see lemma 1). In this case, $\partial\hat{x}/\partial k = \partial\hat{x}/\partial r_L = 0$ such that $\lambda_1 = \lambda_2 = 0$. Hence, $L = \mu$ follows from (17) and the participation constraint of depositors requires $r_L = (1 - p_0\alpha)/(1 - p_0)$. This outcome is only feasible if $K \geq K_0$ according to the capital availability constraint. *Q.E.D.*

Although lending and investment are similar to the unregulated market equilibrium, the latter is not necessarily efficient because the capital structure is indeterminate: On average, banks' capitalization may be sufficient but in the absence of regulation, some banks may have too small a capital ratio, $k < 1 - \alpha$, and are risky. As soon as bank capital is abundant in supply ($K \geq K_0$), the trade-off between financial stability and real investment disappears and a high capital ratio does not entail any costs for the real economy. This case is consistent with the key argument of Admati et al. (2011).

3.4 Optimal Capital Requirements

Capital requirements allow decentralizing the social optimum in a market economy. Essentially, the regulator needs to ensure that all banks have the optimal capital structure:

COROLLARY 2 *The optimal allocation can be implemented by minimum capital requirements*

$$k \geq 1 - \alpha - (r_L - \alpha)(1 - p)H(\hat{x}^*) \equiv k^*(r_L, \hat{x}^*) \quad (20)$$

that increase in the failure threshold and decrease in the lending rate and the liquidation value. Capital requirements bind if $K \leq K_0$.

Proof: See Appendix A.1.

The capital requirements are a function of the failure threshold \hat{x}^* (henceforth: target risk), the lending rate r_L and the liquidation value α : A lower target risk naturally requires more equity, whereas a higher lending rate and liquidation value allow for a lower capital ratio without increasing bank risk. They increase a bank's income, $(1 - x)(1 - p)r_L + (p + (1 - x)p)\alpha$, thereby providing an additional buffer such that the same failure risk materializes even with a smaller capital ratio. Consequently, a high lending rate or liquidation value are substitutes for capital on the 'risk front'. A similar substitution effect is found by Repullo and Suarez (2013) for the capital structure of banks that hold voluntary buffers in order to avoid binding regulatory constraints in the future. Therefore, the state of the economy - entrepreneurs' productivity and the supply of bank capital - influences capital requirements through two main channels: (i) target risk and (ii) equilibrium lending rate. As soon as the supply of bank capital is large enough to make all banks risk-free without constraining investment (i.e., $K \geq K_0$), however, capital requirements equal the loss given default $k = 1 - \alpha$ and are insensitive to economic shocks.

Implementing the optimal allocation (proposition 1) with capital requirements is straightforward: The optimal capital structure varies with the lending rate and thus ensures by construction that the failure threshold is indeed \hat{x}^* . In addition, it achieves the optimal lending and investment scale L^* : Banks maximize their expected surplus π^B subject to the regulatory constraint $k \geq k^*(r_L, \hat{x}^*)$. The first-order condition $(1 - p_0)r_L + p_0\alpha - 1 - (\gamma - 1)k = 0$, the loan demand of entrepreneurs $\hat{u} = (1 - p_0)(R - r_L)$, and market clearing

$L = \hat{u}$ imply that loans

$$L = \mu - (\gamma - 1)k \quad (21)$$

decrease in the capital ratio k and in the required return on equity γ .⁹ Together with market clearing for bank capital, $K = kL$, this condition determines equilibrium lending and return on equity: Since capital requirements are binding, market clearing coincides with the capital availability constraint in the regulator's program. Therefore, the lending and investment scale is optimal, $L = L^*$.¹⁰ The return on equity endogenously adjusts: As long as bank capital is scarce, $K < K_0$, such that $k^* < 1 - \alpha$, the market for bank capital clears thereby determining loans $L = L^* < \mu$. From condition (21), equity earns an excess return compared to deposits $\gamma > 1$. If $K > K_0$, capital requirements make banks risk-free such that the externality vanishes and maximum lending is optimal $L^* = \mu$. Accordingly, equity earns the same return as deposits, $\gamma = 1$.¹¹

3.4.1 Capital Requirements and the State of the Economy

This section studies how the optimal capital requirements described in corollary 2 are adjusted in two different scenarios: a capital crunch, that is, a contraction of the bank capital supply K , and an adverse shock to entrepreneurs' productivity R . The capital crunch captures a key concern in the procyclicality debate¹², while productivity shocks play a central role in macroeconomics as one of the driving forces of the business cycle. Capital requirements is linked to the state of the economy through two endogenous channels, target risk and the equilibrium lending rate, which are directly or indirectly affected by the shock. Comparative statics reveal the optimal adjustment:

PROPOSITION 2 *Optimal capital requirements $k^*(r_L, \hat{x}^*)$ increase in the supply of bank capital, $\frac{\partial k(r_L, \hat{x}^*)}{\partial K} > 0$, and decrease in entrepreneurs' productivity, $\frac{\partial k(r_L, \hat{x}^*)}{\partial R} < 0$. Whenever the supply of bank capital is abundant, $K \geq K_0$, capital requirements are independent of the state of the economy.*

Proof: See Appendix A.1.

⁹If equity earns no excess return over debt (i.e., if $\gamma = 1$), lending is independent of the capital structure and similar to the market equilibrium.

¹⁰Since the demand monotonically increases in L and the supply is fixed, the solution is $L = L^*$.

¹¹Capital requirements are binding if $\gamma > 1$ but can be slack if $\gamma = 1$; in this case, banks may choose a capital ratio higher than $1 - \alpha$ but they never lend more than μ .

¹²See, for instance, Repullo (2013).

If more equity is available, banks can increase the loan supply and raise the capital ratio to reduce risk. Proposition 1 shows that a combination of both is optimal, which drives the response of the capital structure: First, a larger loan supply *ceteris paribus* reduces the equilibrium lending rate. This lowers the bank's interest income such that it can absorb fewer loan losses. Only a higher capital ratio can preserve target risk. Second, it is optimal to lower bank risk, which requires an even higher capital ratio. Intuitively, the capital availability constraint is relaxed such that improving banks' crisis resilience is less costly in terms of forgone investment. Both effects clearly imply tighter capital requirements. Conversely, the optimal response to a capital crunch is to relax them: The contraction of the loan supply generates a positive equilibrium effect as the lending rate increases, which allows reducing the capital ratio without affecting bank risk. In addition, tolerating a higher risk level is optimal as the decline of investment would otherwise be too strong. Thus, capital requirements are procyclical in the sense that they are tightened (relaxed) if more (less) bank capital is available. This result is qualitatively similar to Repullo (2013) and supports recent reforms with the aim of mitigating the procyclical effects of financial regulation.

A positive technology shock or, more generally, attractive investment prospects increase the value of investment projects such that even entrepreneurs with high opportunity costs find it profitable to invest and loan demand increases. Such a real shock indirectly affects capital requirements: First, the higher loan demand leads to an increase in lending rate and interest income. This allows for a lower capital ratio without undermining financial stability. Second, tolerating higher a risk level is usually optimal especially if capital requirements are tight. This implies a further decrease in the capital ratio.¹³ The main purpose of relaxing capital requirements whenever investment opportunities improve is to allow banks to accommodate the higher loan demand such that they can fund more projects despite a fixed capital supply. Bank regulation should not hamper entrepreneurial activity and investment when they are most promising. If investment prospects worsen, in contrast, capital requirements should be tightened to account for the declining lending rate and to exploit the low project value and loan demand in order to maintain or even reduce bank risk. In other words, preserving or even improving

¹³Even if an increase in risk is not optimal, the effect of a higher lending rate prevails, and the capital ratio unambiguously decreases. This somewhat ambiguous risk impact is precisely due to two counteracting effects: the higher lending rate versus the lower capital ratio.

financial stability only entails a relatively small welfare cost. This finding can be related to the 'cleansing effect' of recessions emphasized, among others, by Caballero and Hammour (1994): Regulation should thus support such an effect by cutting funds for low productivity investments with a small net surplus and aim at improving financial stability instead. Hence, capital requirements are countercyclical in the sense that they decrease (increase) if productivity improves (declines). This type of adjustment is more in line with risk-sensitive systems like Basel II; however, the tighter regulatory stance in a downturn is motivated by lower interest rates and project value instead of generally riskier loans.

The analysis offers three main insights: First, optimal regulation is related to the state of the economy through target risk and the equilibrium lending rate. Thus, it is also sensitive to developments in the real sector. Second, the cyclical adjustment fundamentally differs depending on the type of economic shock: Capital requirements are clearly countercyclical in case of productivity shocks but procyclical with regard to fluctuations in the supply of bank capital. In part, this difference arises because of an opposite equilibrium effect: A contraction of the bank capital supply leads to a higher lending rate as banks reduce their loan supply. This, in turn, allows for a lower capital ratio without undermining banks' crisis resilience. A productivity decline, in contrast, lowers loan demand such that the lending rate falls, which mechanically requires a higher capital ratio to avoid an increase in risk. In particular, the regulator may adjust target risk: In case of a shortage of bank capital, a higher risk should be tolerated, whereas the response to a declining productivity is often to reduce risk. Intuitively, the latter can be achieved at a lower cost because of the unattractive investment prospects. Third, whenever a downturn involves both declining productivity and a shortage of bank capital at the same time, the optimal adjustment depends on the relative magnitude of the two effects.

3.4.2 Comparison

This section compares optimal regulation to two alternative systems: flat and risk-sensitive capital requirements. Defining a constant ratio of bank capital to total assets, the former are similar in kind to the leverage ratio envisaged in Basel III. The latter define a minimum ratio of capital to risk-weighted assets and essentially target a particular risk level such as the target one-year solvency probability of 99.9% in Basel II (corresponding

to a failure probability of 0.1% in our framework); risk-sensitive capital requirements remain an essential part of Basel III.

In case banks are subject to flat capital requirements, $k = \bar{k}$, their failure threshold is determined by $1 - \bar{k} - \alpha - (r_L - \alpha)(1 - p)H(\hat{x}) = 0$. Thus, bank risk decreases in the capital requirement, the lending rate, and the liquidation value. Taking into account loan demand and market clearing, lending and the return on equity are jointly determined by the bank's first-order condition and the equilibrium in the bank capital market

$$L = \mu - (\gamma - 1)\bar{k}, \quad K = \bar{k}L \quad (22)$$

In particular, lending and investment are simply a multiple of the bank capital supply: $L = K/\bar{k}$. Whenever the supply of bank capital increases, banks increase loans by a factor $1/\bar{k}$. In contrast, a higher loan demand due to more productive entrepreneurs only increases the lending rate and the return on equity thereby offsetting any quantity effect. In the risk-sensitive system, the failure threshold is a fixed policy parameter, $\hat{x} = \hat{x}'$. Intuitively, the regulator sets capital requirements consistent with a particular probability of a banking crisis. Condition (8) implies:

$$k(r_L, \hat{x}') = 1 - \alpha - (r_L - \alpha)(1 - p)H(\hat{x}') \quad (23)$$

The only difference to optimal capital requirements is the exogenous target risk.¹⁴ Again, lending and interest rates follow from the bank's first-order condition combined with loan market clearing

$$\begin{aligned} L = \mu - (\gamma - 1) [1 - \alpha - (r_L - \alpha)(1 - p)H(\hat{x}')] &= 0, \\ K = [1 - \alpha - (1 - p)(r_L - \alpha)H(\hat{x}')] L & \end{aligned} \quad (24)$$

with $r_L = R - L/(1 - p_0)$. Differentiating market clearing using r_L from the second condition implies that lending increases in both the bank capital supply and entrepreneurs' productivity. Capital requirements passively adjust to offset changes in the lending rate and to keep failure risk constant: Since the former decreases if banks can access more capital such that the loan supply increases, capital requirements are tightened to avoid

¹⁴In case the latter was appropriately chosen and adjusted (i.e., if $\hat{x}' = \hat{x}^*$), the allocation would coincide with the optimal allocation.

higher risk. In case of a higher productivity, in contrast, the loan demand and lending rate increase and the higher revenue from repaid loans provides an additional buffer implying a lower capital ratio. The cyclical adjustment is thus qualitatively similar to that of optimal capital requirements but it is entirely driven by changes in the equilibrium lending rate, whereas a welfare-maximizing regulator also adjusts the target risk. Hence, one may expect that the adjustment is less pronounced.

Capital Requirements			
	Optimal	Flat	Risk-Sensitive
Capital Crunch ($K \downarrow$)			
Capital Ratio	\downarrow	\leftrightarrow	\downarrow
Bank Risk	\uparrow	\downarrow	\leftrightarrow
Lending	\downarrow	\downarrow	\downarrow
Adverse Productivity Shock ($R \downarrow$)			
Capital Ratio	\uparrow	\leftrightarrow	\uparrow
Bank Risk	\downarrow	\uparrow	\leftrightarrow
Lending	\downarrow	\leftrightarrow	\downarrow

Table 2: *Adjustment in a Downturn*

Table 2 summarizes how the economy adjusts to (i) a capital crunch and (ii) a decline in productivity depending on the regulatory system. Both optimal and risk-sensitive capital requirements are relaxed if the bank capital supply falls and tightened if productivity deteriorates. However, the extent of their responses differs: Generally, an economy with optimal regulation adjusts at both margins - risk and lending - to a shock, whereas in an economy with risk-sensitive system, only lending adjusts. In a capital crunch, optimal capital requirements are relaxed in order to prevent a massive contraction of the loan supply thereby tolerating a higher risk, whereas risk-sensitive capital requirements target a fixed risk level and are only relaxed because of a higher lending rate. Flat capital requirements are, by definition, independent of the state of the economy. A shortage of bank capital directly lowers the loan supply, which is a multiple of bank capital; through a higher equilibrium lending rate, this even makes banks safer. In this system, an adverse productivity shock only increases bank risk as lower loan demand drives down the lending rate but the loan volume is completely insensitive due to the inelastic loan supply.

The flexible adjustment of optimal regulation ensures that the allocation maximizes welfare even after an adverse shock. However, such a policy may be difficult to implement

because of strong informational requirements: Whereas no additional information is required under flat capital requirements, regulators in a risk-sensitive system need new information at the bank level. In particular, they need to observe new interest rates and how they influence risk through a bank's interest income and expenses in order to correctly adjust the capital ratio. On top of that, optimal regulation also involves an adjustment of the intended risk level. This requires more aggregate information, for example, about investment opportunities in the real economy and the availability of bank capital. These strong informational requirements may explain why regulatory systems may deviate from the benchmark in reality. If the information set of regulators is constrained and acquiring relevant information is costly, risk-sensitive regulation with a passive adjustment only is a second-best option as its implications are to a large extent qualitatively similar.

3.4.3 Numerical Example

In this section, we compute the equilibrium of the model and provide a numerical example in order to illustrate the adjustment to adverse financial and real shocks. The purpose of this example is purely illustrative. The baseline calibration is $R = 1.5$, $p_0 = 0.15$, $\alpha = 0.55$, $\theta = 0.3$, $c = 0.5$, $K = 0.03$.¹⁵ Furthermore, the macroeconomic shock x is uniformly distributed on the unit interval. The expected net return μ , which equals investment in the unregulated market equilibrium, is 0.3537. Hence, bank capital is clearly scarce as $K = 0.03 < 0.1609 = (1 - \alpha)\mu$. The optimal allocation is characterized by lending of 0.2516, a capital ratio of 11.92%, and a (gross) lending rate of 1.204. Banks fail in a recession as soon as more than 43.23% of their borrowers default which corresponds to an *ex ante* failure probability of 11.35%; social welfare (aggregate surplus) equals 0.044. We simulate two scenarios depending on whether optimal, flat or risk-sensitive capital requirement are in place: (i) a capital crunch with the supply of bank capital K decreasing by one third to 0.02 and (ii) an adverse productivity shock that lowers the project return by 6.67 percent to 1.4 such that the expected net return μ decreases to 0.2725. The values of the baseline allocation are used to fix the flat capital requirements ($\bar{k} = 11.92\%$) and target risk in the risk-sensitive system ($\hat{x}' = 0.4342$). Contrary to optimal regulation, either the capital ratio or target risk remain constant.

¹⁵Following Repullo and Martinez-Miera (2010), we set the liquidation value α similar to the Basel II IRB approach, which suggests a loss given default (i.e., $1 - \alpha$) of 0.45 for senior claims on corporates (foundation approach); see par. 273 in BCBS (2004).

	Capital Requirements		
	Optimal	Flat	Risk-Sensitive
Capital Crunch (K -33.3%)			
Capital Requirement (in pp)	-3.88	-	-2.43
Bank Risk (in pp)	+1.36	-1.56	-
Lending (in %)	-1.15	-33.31	-16.26
Welfare (in %)	-4.09	-14.32	-6.36
Adverse Productivity Shock (R -6.67%)			
Capital Requirement (in pp)	+5.08	-	+2.48
Bank Risk (in pp)	-1.65	+2.22	-
Lending (in %)	-29.85	-	-17.21
Welfare (in %)	-45.45	-55.00	-47.27

Table 3: *Numerical Example*

In a capital crunch, optimal capital requirements are relaxed from 11.92 to 8.04 percent: This significantly mitigates the decline of lending (-1.15%), which will fall by one third if capital requirements are not adjusted or still by more than 16 percent if only a passive adjustment as a result of the higher lending rate occurred. However, it moderately raises the probability of a banking crisis from 11.4 to 12.7 percent. The shortage of bank capital only causes a small welfare loss under optimal and risk-sensitive capital regulation; flat capital requirements perform significantly worse. The former is remarkable because the adjustment of risk and lending clearly differ between the optimal and risk-sensitive system. Since an adverse productivity shock reduces loan demand and makes investment less valuable, it is optimal to exploit this and to strongly increase capital requirement from almost 12 to 17 percent in order to reduce the probability of a banking crisis by 1.65 percentage points. Both lower loan demand and tighter regulation cause a strong decline of lending by almost 30 percent. In the risk-sensitive system, capital requirements are only passively adjusted to account for the lower lending rate and lending falls by roughly one quarter. The adjustment under flat capital requirements markedly differs: The resource allocation remains unchanged but as the lower loan demand reduces the gross lending rate from 1.2 to 1.1, financial stability is weakened with banks' failure probability increasing to 13.57 percent. In general, the welfare loss defined in terms of aggregate surplus¹⁶ is higher than during a capital crunch partly because, in addition to distortions of risk or

¹⁶This is the reason why the relative welfare losses are so large; if welfare is defined in terms of (gross) output, they are substantially smaller.

lending, all entrepreneurs are less productive. Again, the difference between optimal and risk-sensitive capital requirements is small, whereas flat capital requirements exacerbate the welfare loss.

3.5 Entrepreneurial Moral Hazard

Borrowing and lending is characterized by many frictions, for instance, borrowers who are protected by limited liability and cannot be costlessly monitored by their lenders may have an incentive to allocate funds to riskier investments or to deliberately reduce effort. Adverse selection and moral hazard make the lending rate a critical determinant of loan risk as shown by Stiglitz and Weiss (1981). Subsequently, we add such a credit friction to the model but stick to a formulation with private benefits in the spirit of Holmström and Tirole (1997) instead of different project returns¹⁷. This extension also demonstrates how moral hazard in the real economy can influence the regulation of banks.

Suppose that the effort of an entrepreneur is critical for the project's success. More specifically, the entrepreneur may exert effort such that the project succeeds with an *ex ante* probability $1 - p_0$ or exert no effort ('shirking') such that the success probability falls to $1 - p'_0$ with $p'_0 = p_0 + \Delta p_0$ and $\Delta p_0 > 0$.¹⁸ Shirking yields private benefits b for the entrepreneur but makes the project unprofitable $(1 - p'_0)R + p'_0\alpha - 1 < 0$. Importantly, the bank does not observe effort, which gives rise to entrepreneurial moral hazard. As a result, the lending contract needs to be incentive-compatible as to guarantee effort:

$$(1 - p_0)(R - r_L) \geq (1 - p'_0)(R - r_L) + b \quad \Rightarrow \quad R - r_L \geq \frac{b}{\Delta p_0} \equiv \beta \quad (25)$$

Essentially, moral hazard limits the lending rate by an upper bound $R - \beta$. The parameter β is a measure of corporate governance; higher values point to a more severe agency problem. It ultimately depends on the institutional quality in a country and thus on factors like, for example, investor protection and transparency. A regulator does not observe effort either and thus needs to choose an incentive-compatible allocation. Subsequently, we focus on a binding incentive compatibility constraint, $r_L = R - \beta$; the agency problem

¹⁷This allows separating project return and corporate governance such that the cyclical adjustment is better comparable to the baseline model.

¹⁸Shirking increases the idiosyncratic project risk by Δp such that $\Delta p_0 = (1 - \theta x_0)\Delta p$ and $p'_0 = p_0 + \Delta p_0 = p_0 + \Delta p + \theta x_0[1 - (p_0 + \Delta p)]$.

is thus severe enough and the baseline allocation would not be incentive-compatible.¹⁹ Hence, the participation constraint is slack and the regulator's choices are instead subject to the binding incentive compatibility constraint:

PROGRAM 2 *The regulator maximizes welfare by choosing the capital ratio k , lending L , and the lending rate r_L*

$$\max_{k,L,r_L} [\mu - \theta(1 - F(\hat{x}))c]L - \frac{L^2}{2} \quad (26)$$

subject to the incentive compatibility constraint of entrepreneurs, $r_L = R - \beta$, and the capital availability constraint, $K \geq kL$.

Solving this program yields the second-best allocation:

LEMMA 4 *Bank lending and failure threshold are determined by the system*

$$J^1(L, \hat{x}') = \mu - L' - \theta[1 - F(\hat{x}')c] - \frac{\theta f(\hat{x}')c}{1 - \theta + \theta F(\hat{x}')} \left[\frac{1 - \alpha}{(1 - p)(R - \beta - \alpha)} - H(\hat{x}') \right] = 0 \quad (27)$$

$$J^2(L, \hat{x}') = K - [1 - \alpha - (R - \beta - \alpha)(1 - p)H(\hat{x}')]L = 0 \quad (28)$$

Lending increases in the supply of bank capital K and entrepreneurs' productivity R but decreases in the corporate governance parameter β . The failure threshold increases in the supply of bank capital but the sensitivity with respect to productivity and corporate governance can be of either sign.

Proof: See Appendix A.1.

Apart from the fixed interest rate, the first-order condition for loans and the capital availability constraint are similar to the baseline model. The equilibrium allocation involves capital regulation:

PROPOSITION 3 *Banks are subject to optimal capital requirements*

$$k'(r_L, \hat{x}') = 1 - \alpha - (r_L - \alpha)(1 - p)H(\hat{x}') \quad (29)$$

¹⁹However, moral hazard must not be too severe, $\beta \leq \mu$, as banks would otherwise earn negative expected profits or their shareholders a negative return. This follows from $\pi^B = [(1 - p)r_L + p\alpha - 1 - (\gamma - 1)k]L = [\mu - \beta - (\gamma - 1)k]L \geq 0$, using $r_L = R - \beta$. $\beta > \mu$ would lead to full credit rationing. Whenever $K \geq K_0$, the lending rate is small such that the IC is slack; $r_L = (1 - p\alpha)/(1 - p) < R - \beta$. Hence, entrepreneurial moral hazard may only arise if scarce bank capital is required to internalize the social cost of a banking crisis.

with $r_L = R - \beta$ that increase in the supply of bank capital, $\partial k'(r_L, \hat{x}')/\partial K > 0$, and the corporate governance parameter, $\partial k'(r_L, \hat{x}')/\partial \beta > 0$, and decrease in entrepreneurs' productivity, $\partial k'(r_L, \hat{x}')/\partial R < 0$.

Proof: See Appendix A.1.

This proposition has two main implications: First, moral hazard leads to an artificially low lending rate as higher rates would destroy incentives, an effect reinforced by a poor corporate governance. Since banks earn a smaller interest income and are *ceteris paribus* riskier, the optimal capital ratio increases such that poor governance of firms and entrepreneurial projects requires tighter bank regulation. In the presence of credit frictions, optimal regulation thus depends on the institutional quality. Second, the adjustment of capital requirements to economic shocks is qualitatively similar to the baseline model without moral hazard. However, the mechanisms differ: Since the lending rate is fixed, some equilibrium effects that are relevant in the standard model disappear. Capital requirements are relaxed in capital crunch only because tolerating a higher bank risk is optimal and not due to a declining lending rate. The tightening in case of declining productivity, in contrast, still results from the two effects described above: The lending rate decreases to preserve the incentive of entrepreneurs, which requires a higher capital ratio to keep bank risk constant. In addition, the regulator may adjust the latter in either way but even if higher risk is tolerated, the first, positive effect prevails.

4 Conclusion

This paper provides a normative analysis of the cyclical adjustment of capital requirements. Regulation aims at internalizing all costs of a systemic banking crisis, which originates from a recession in the real economy with correlated defaults lead to that lead to bank failure. As long as bank capital is scarce, optimal regulation balances the trade-off between financial stability and the investment capacity of the real sector. Optimal capital requirements adjust to financial and real shocks because of two channels: target risk and the equilibrium lending rate. The latter, which, contrary to related models like Kashyap and Stein (2004) and Repullo (2013), is endogenized by a full-fledged model of the real sector, plays an important role as a *de facto* substitute for bank capital on the

'risk front' and thus strongly influences the optimal regulatory adjustment. The main finding is that there are striking differences depending on the driving force of a downturn: If the supply of bank capital falls, banks face difficulties to raise equity such that they may reduce the loan supply. Therefore, capital requirements should be less strict: On the one hand, a smaller loan supply raises the lending rate, which has a stabilizing effect by making banks safer due a higher interest income. On the other hand, tolerating a higher risk of a banking crisis is optimal in order to prevent a sharp fall of lending and investment. In a downturn primarily characterized by poor investment prospects and a low loan demand of entrepreneurs, capital requirements should be tighter: Declining lending rates undermine a bank's resilience such that the risk level is only preserved at a higher capital ratio. More importantly, the number of attractive investments is small; such circumstances even allow a regulator to further improve financial stability by tighter capital requirements at a relatively small welfare cost. Consequently, regulation should be tough whenever bank capital is easily available and only few investments are promising but can be relaxed whenever equity is scarce and investment prospects are good. Contrary to flat or risk-sensitive capital requirements, optimal regulation allows the economy to adjust to shocks at two different margins - lending and risk - whereas one of them is *de facto* fixed in the two alternative systems, which exacerbates the welfare losses in a downturn. The main findings also result in the presence of entrepreneurial moral hazard, which makes optimal regulation sensitive to corporate governance and institutional quality as well. Eventually, the supply of bank capital can be large enough to make banks safe without harming investment. In this case, capital requirements are insensitive to economic shocks. How do these findings compare with the countercyclical buffer envisaged in Basel III? It is of course difficult to interpret such a buffer, which is essentially a dynamic concept as the buffer is accumulated during periods of excess credit growth and effectively relaxed in a downturn, in the context of our static framework. One implication of our results is that it makes a difference whether credit growth is mainly supply- or demand-driven as capital requirements should indeed be tightened in the first but relaxed in the second case to accommodate the higher demand. Hence, the regulator should identify the precise source of credit growth and activate the buffer primarily in case attractive funding conditions of banks let their loan supply growing rapidly. A possibility would be to look at different loan categories and, for example, to compare the growth of corporate and SME loans to overall

credit growth. If the former grow more than proportionately, this may point to attractive investment prospects and strong loan demand, which do not justify tighter capital buffers. On the aggregate level, the regulator may also take into account the growth of investment in plants and equipments. In line with the focus on the real sector, our analysis of course mainly concerns corporate and business loans that finance productive real investments; credit growth driven by a strong demand for mortgages, however, does not necessarily imply that relaxing capital requirements is optimal because it may rather reflect a real estate bubble instead of more productive and valuable investments.

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A Appendix

A.1 Proofs

Proof of Lemma 1 The central condition follows from combining (3) and (6). Using integration by parts, depositors' participation constraint can be written as:

$$\theta \left[\alpha - [(1-p)(1-\hat{x})r_L + (p+\hat{x}(1-p))\alpha]F(\hat{x}) + (r_L - \alpha)(1-p) \int_{\hat{x}}^1 F(x)dx \right] + [1 - \theta + \theta F(\hat{x})]r(1-k) = 1-k \quad (\text{A.1})$$

The definition of the failure threshold implies $(1-p)(1-\hat{x})r_L + [p+\hat{x}(1-p)]\alpha = r(1-k)$, which can be used to eliminate $r(1-k)$ such that

$$\theta \left[\alpha + (r_L - \alpha)(1-p) \int_{\hat{x}}^1 F(x)dx \right] + (1-\theta)[(1-p)(1-\hat{x})r_L + (p+\hat{x}(1-p))\alpha] = 1-k \quad (\text{A.2})$$

which can be rearranged to get (8). Differentiating this condition yields the sensitivities

$$\frac{\partial \hat{x}}{\partial k} = \frac{1}{[1 - \theta + \theta F(\hat{x})](1-p)(r_L - \alpha)} > 0 \quad (\text{A.3})$$

$$\frac{\partial \hat{x}}{\partial r_L} = \frac{H(\hat{x})}{[1 - \theta + \theta F(\hat{x})](r_L - \alpha)} > 0 \quad (\text{A.4})$$

$$\frac{\partial \hat{x}}{\partial \alpha} = \frac{1 - (1-p)H(\hat{x})}{[1 - \theta + \theta F(\hat{x})](1-p)(r_L - \alpha)} > 0 \quad (\text{A.5})$$

where the latter uses $H(\hat{x}) \leq 1 - \theta x_0$. *Q.E.D.*

Proof of Proposition 1 Program 1 can be stated as a Lagrangian

$$\mathcal{L}(k, L, \lambda_1, \lambda_2) = [\mu - \theta(1 - F(\hat{x}))c]L - \frac{L^2}{2} + \lambda_1[K - kL] + \lambda_2[(1 - p_0)(R - L) - L] \quad (\text{A.6})$$

which uses $L = \hat{u}$ in equilibrium. The corresponding first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial k} = \frac{\theta f(\hat{x})cL}{(1-p)(r_L - \alpha)[1 - \theta + \theta F(\hat{x})]} - \lambda_1 L = 0 \quad (\text{A.7})$$

$$\frac{\partial \mathcal{L}}{\partial L} = \mu - (1 - F(\hat{x}))c - L - \lambda_1 k - \lambda_2 = 0 \quad (\text{A.8})$$

$$\frac{\partial \mathcal{L}}{\partial r_L} = \frac{\theta f(\hat{x})H(\hat{x})cL}{(r_L - \alpha)[1 - \theta + \theta F(\hat{x})]} - \lambda_2(1 - p_0) = 0 \quad (\text{A.9})$$

as well as participation and capital availability constraint. The first condition is equivalent to equation (15) and implies a positive capital ratio. Thus, one can rearrange (8) to express the capital ratio as a function of the risk level: $k = 1 - \alpha - (1 - p)(r_L - \alpha)H(\hat{x})$. It is more convenient and intuitive to substitute this expression for k in the above conditions and to treat \hat{x} as the unknown.

The problem can be reduced to the system (17) - (18) with two equations and two unknowns: L and \hat{x} . Substituting for the Lagrange multipliers, the capital ratio, and the lending rate in $\partial\mathcal{L}/\partial L = 0$ yields equation $J_L^1(\hat{x}, L) = 0$. The second equation, $J_L^2(\hat{x}, L) = 0$, is the capital availability constraint, in which we substitute for k using $r_L = R - \frac{L}{1-p}$. The system has the solutions $\hat{x} = 1$ and $L = \mu$ as soon as $K \geq K_0$, which allows for $k = 1 - \alpha$. Totally differentiating yields the Jacobian

$$J = \begin{bmatrix} J_L^1 & J_{\hat{x}}^1 \\ J_L^2 & J_{\hat{x}}^2 \end{bmatrix} \quad (\text{A.10})$$

where $J_i^j = \frac{\partial J^j}{\partial i}$ with $i = \{L, \hat{x}\}$ and $j = \{1, 2\}$ and

$$J_L^1 = - \left[1 + \frac{\theta f(\hat{x})c[1 - \alpha + (1 - p)(R - \alpha)H(\hat{x})]}{[1 - \theta + \theta F(\hat{x})](1 - p_0)(1 - p)(r_L - \alpha)^2} \right] < 0 \quad (\text{A.11})$$

$$J_{\hat{x}}^1 = \frac{\theta f(\hat{x})cL}{(r_L - \alpha)(1 - p_0)} + \frac{\theta c [\theta f(\hat{x})^2 - (1 - \theta + \theta F(\hat{x}))f'(\hat{x})] \left[\frac{k}{1-p} + \frac{H(\hat{x})L}{1-p_0} \right]}{[1 - \theta + \theta F(\hat{x})]^2(r_L - \alpha)} \geq 0 \quad (\text{A.12})$$

$$J_L^2 = - \left[k + \frac{(1 - p)H(\hat{x})L}{1 - p_0} \right] < 0 \quad (\text{A.13})$$

$$J_{\hat{x}}^2 = -(1 - p)(r_L - \alpha)[1 - \theta + \theta F(\hat{x})]L < 0 \quad (\text{A.14})$$

The Jacobian determinant is positive:

$$\nabla = J_L^1 J_{\hat{x}}^2 - J_L^2 J_{\hat{x}}^1 > 0 \quad (\text{A.15})$$

The comparative statics are obtained using Cramer's rule. A larger supply of bank capital increases lending and the failure threshold:

$$\frac{\partial L}{\partial K} = \frac{J_{\hat{x}}^1}{\nabla} \geq 0, \quad \frac{\partial \hat{x}}{\partial K} = -\frac{J_L^1}{\nabla} > 0 \quad (\text{A.16})$$

A higher productivity of entrepreneurs increases lending and but the response of the

failure threshold is ambiguous:

$$\frac{\partial L}{\partial R} = \frac{-J_R^1 J_x^2 + J_R^2 J_x^1}{\nabla} > 0 \quad (\text{A.17})$$

$$\frac{\partial \hat{x}}{\partial R} = \frac{-J_L^1 J_R^2 + J_L^2 J_R^1}{\nabla} = -\frac{k J_R^1 - \frac{\theta f(\hat{x})c}{1-\theta+\theta F(\hat{x})} \frac{(1-p)H(\hat{x})^2 L}{(1-p_0)(r_L-\alpha)}}{\nabla} \quad (\text{A.18})$$

where J_R^j for $j = \{1, 2\}$ are given by

$$J_R^1 = 1 - p_0 + \frac{\theta f(\hat{x})c}{1-\theta+\theta F(\hat{x})} \frac{1-\alpha + \frac{H(\hat{x})L}{1-\theta x_0}}{(1-p)(r_L-\alpha)^2} > 0, \quad J_R^2 = (1-p)H(\hat{x})L > 0 \quad (\text{A.19})$$

Note that $\partial \hat{x} / \partial R$ is usually negative unless the equilibrium capital ratio is very low and, at the same time, the social cost of bank failure is very high. *Q.E.D.*

Proof of Corollary 2 Substituting the capital ratio (20) into the definition of the failure threshold (8) shows that bank risk is, by construction, optimal: $\hat{x} = \hat{x}^*$. Each bank chooses loans and capital structure as to maximize its expected profit π^B defined in (26) subject to the regulatory constraint $k \geq k^*$. The Lagrangian is

$$\mathcal{L}(k, L, \eta) = [(1-p_0)r_L + p_0\alpha - (1-k) - \gamma k]L + \eta[k - k^*] \quad (\text{A.20})$$

where η is the Lagrange multiplier of the regulatory constraint. The corresponding first-order conditions are:

$$(1-p)r_L + p\alpha - 1 - (\gamma-1)k = 0, \quad -(\gamma-1) + \eta = 0, \quad \eta(k - k^*) = 0 \quad (\text{A.21})$$

Substituting $r_L = R - L/(1-p)$ using loan market clearing and the definition of the marginal entrepreneur, yields bank lending:

$$L = \mu - (\gamma-1)k \quad (\text{A.22})$$

If $K < K_0$, capital requirements are $k^* < 1 - \alpha$. First, we show that the regulatory constraint binds: Suppose banks chose a higher capital ratio $k > k^*$, dividing market clearing, $K = kL$, by k would yield $K/k = L = \mu - (\gamma-1)k$. The left-hand side falls short of optimal lending such that $L < L^* = K/k^*$, which is smaller than μ according

to proposition 1. From above, we have $\gamma > 1$ and $\eta > 0$, which is incompatible with complementary slackness. Consequently, capital requirements are binding: $k = k^*$. By substituting $k^* = K/L^*$ into the market clearing condition, $K = k^*L$, one observes that bank provide the optimal amount of loans, $L = L^*$.

Whenever $K \geq K_0$, capital requirements are $k^* = 1 - \alpha$. Banks choose $k \in [1 - \alpha, K/\mu]$ such that market clearing, $K \geq k[\mu - (\gamma - 1)k]$, implies $\gamma = 1$ (which implies $\eta = 0$ and allows for a possibly non-binding regulatory constraint) and banks choose optimal lending $L = \mu$. An even higher capital ratio, $k > K/\mu$ would lead to an inefficiently small amount of loans but is ruled out by complementary slackness. *Q.E.D.*

Proof of Proposition 2 Using the definition of capital requirements, (20), the sensitivity with respect to the bank capital supply K is

$$\frac{\partial k^*(r_L, \hat{x}^*)}{\partial K} = -(1-p)H(\hat{x})\frac{\partial r_L}{\partial K} + (r_L - \alpha)(1-p)[1 - \theta + \theta F(\hat{x}^*)]\frac{\partial \hat{x}^*}{\partial K} > 0 \quad (\text{A.23})$$

where $\frac{\partial r_L}{\partial K} = -\frac{1}{1-p}\frac{\partial L^*}{\partial K} < 0$; the positive sign follows from $\frac{\partial L}{\partial K} > 0$ and $\frac{\partial \hat{x}^*}{\partial K} > 0$ given by proposition 2. Similarly, one can derive the sensitivity with respect to productivity R

$$\frac{\partial k^*(r_L, \hat{x}^*)}{\partial R} = -(1-p)H(\hat{x})\frac{\partial r_L}{\partial R} + (r_L - \alpha)(1-p)[1 - \theta + \theta F(\hat{x}^*)]\frac{\partial \hat{x}^*}{\partial R} \quad (\text{A.24})$$

where $\frac{\partial r_L}{\partial R} = 1 - \frac{1}{1-p_0}\frac{\partial L^*}{\partial R} = \frac{(1-p_0)\nabla + J_R^1 J_{\hat{x}}^2 - J_R^2 J_{\hat{x}}^1}{(1-p_0)\nabla} = \frac{(1-p_0)kJ_{\hat{x}}^1 - \frac{\theta f(\hat{x})c}{1-\theta+\theta F(\hat{x})}\frac{H(\hat{x})}{r_L-\alpha}J_{\hat{x}}^2}{(1-p_0)\nabla} > 0$. However, the sign of $\frac{\partial \hat{x}^*}{\partial R}$ is ambiguous. Using the sensitivities from the proof of proposition 2, one can show that optimal capital requirements decrease in productivity:

$$\begin{aligned} \frac{\partial k^*}{\partial R} &= -\frac{1-p}{\nabla} \left[\frac{(1-p_0)kJ_{\hat{x}}^1 - \frac{\theta f(\hat{x})c}{1-\theta+\theta F(\hat{x})}\frac{H(\hat{x})}{r_L-\alpha}J_{\hat{x}}^2}{1-p_0} + (r_L - \alpha)[1 - \theta + \theta F(\hat{x})]kJ_R^1 \right] \\ &\quad - \frac{\theta f(\hat{x})c(1-p)^2 H(\hat{x})^2 L}{(1-p_0)\nabla} = -\frac{(1-p)k[H(\hat{x})J_{\hat{x}}^1 + (r_L - \alpha)(1 - \theta + \theta F(\hat{x}))J_R^1]}{\nabla} < 0 \end{aligned} \quad (\text{A.25})$$

Alternatively, one can derive this result using the capital availability constraint: Since the supply is fixed and lending increases in R , the capital ratio necessarily falls. Once $K \geq K_0$, the optimal capital requirements $k = 1 - \alpha$ are clearly independent of both entrepreneurs' productivity and the bank capital supply. *Q.E.D.*

Proof of Lemma 4 If the incentive compatibility constraint binds such that $r_L = R - \beta$, the equilibrium conditions are (27), the first-order condition w.r.t L substituting

for the Lagrange multiplier $\lambda = \frac{cf(\hat{x})}{1-F(\hat{x})} \frac{\partial \hat{x}}{\partial k}$, and (28), the capital availability constraint substituting for k and r_L . Differentiating (27) - (28) yields the Jacobian with:

$$J_L^1 = -1 < 0, \quad J_{\hat{x}}^1 = \frac{\theta c [\theta f(\hat{x})^2 - f'(\hat{x})(1 - \theta + \theta F(\hat{x}))] k}{(r_L - \alpha)(1 - p)(1 - \theta + \theta F(\hat{x}))^2} \geq 0 \quad (\text{A.26})$$

$$J_L^2 = -k < 0, \quad J_{\hat{x}}^2 = -(r_L - \alpha)(1 - p)(1 - \theta + \theta F(\hat{x}))L < 0 \quad (\text{A.27})$$

The Jacobian determinant is positive: $\nabla = J_L^1 J_{\hat{x}}^2 - J_L^2 J_{\hat{x}}^1 > 0$. Applying Cramer's rule, we find that larger supply of bank capital increases lending and the failure threshold

$$\frac{\partial L}{\partial K} = \frac{J_{\hat{x}}^1}{\nabla} > 0, \quad \frac{\partial \hat{x}}{\partial K} = -\frac{J_L^1}{\nabla} > 0 \quad (\text{A.28})$$

and that higher productivity increases lending but has an ambiguous effect on the failure threshold

$$\frac{\partial L}{\partial R} = \frac{-J_R^1 J_{\hat{x}}^2 + J_R^2 J_{\hat{x}}^1}{\nabla} > 0, \quad \frac{\partial \hat{x}}{\partial R} = \frac{J_R^2 - k J_R^1}{\nabla} \quad (\text{A.29})$$

Note that J_R^i for $i = \{1, 2\}$ are

$$J_R^1 = 1 - p_0 + \frac{\theta f(\hat{x})c}{1 - \theta + \theta F(\hat{x})} \frac{1 - \alpha}{(r_L - \alpha)^2(1 - p)} > 0, \quad J_R^2 = (1 - p)H(\hat{x})L > 0 \quad (\text{A.30})$$

Poor corporate governance of entrepreneurs lowers lending but has an ambiguous impact on the failure threshold

$$\frac{\partial L}{\partial \beta} = \frac{-J_{\beta}^1 J_{\hat{x}}^2 + J_{\beta}^2 J_{\hat{x}}^1}{\nabla} < 0, \quad \frac{\partial \hat{x}}{\partial \beta} = \frac{J_{\beta}^2 - k J_{\beta}^1}{\nabla} \quad (\text{A.31})$$

with J_{β}^j for $j = \{1, 2\}$.

$$J_{\beta}^1 = -\frac{\theta f(\hat{x})c}{1 - \theta + (\hat{x})} \frac{1 - \alpha}{(r_L - \alpha)^2(1 - p)} < 0, \quad J_{\beta}^2 = -(1 - p)H(\hat{x})L < 0 \quad (\text{A.32})$$

Q.E.D.

Proof of Proposition 3 The positive response of capital requirements $k' = 1 - \alpha - (R - \beta - \alpha)H(\hat{x})$ to a larger supply of bank capital is due to a higher failure threshold

$\partial\hat{x}'/\partial K > 0$. The sensitivity with respect to productivity follows from

$$\begin{aligned}\frac{\partial k'}{\partial R} &= -(1-p)H(\hat{x}) + (r_L - \alpha)(1-p)[1 - \theta + \theta F(\hat{x})] \frac{\partial \hat{x}}{\partial R} \\ &= -\frac{(1-p)k [H(\hat{x})J_{\hat{x}}^1 + (r_L - \alpha)[1 - \theta + \theta F(\hat{x})]J_R^1]}{\nabla} < 0\end{aligned}\tag{A.33}$$

and is negative. Similarly, the sensitivity with respect to the corporate governance parameter is

$$\begin{aligned}\frac{\partial k'}{\partial \beta} &= (1-p)H(\hat{x}) + (r_L - \alpha)(1-p)[1 - \theta + \theta F(\hat{x})] \frac{\partial \hat{x}}{\partial \beta} \\ &= \frac{(1-p)k [H(\hat{x})J_{\hat{x}}^1 - (r_L - \alpha)[1 - \theta + \theta F(\hat{x})]J_{\beta}^1]}{\nabla} > 0\end{aligned}\tag{A.34}$$

and positive. These two effects are also implied by the response of bank lending in the presence of a fixed bank capital supply. *Q.E.D.*