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Abstract

In econometrics so-called ordered choice models are popular when interest is in the estimation of the probabilities of particular values of categorical outcome variables with an inherent ordering, conditional on covariates. In this paper we develop a new machine learning estimator based on the random forest algorithm for such models without imposing any distributional assumptions. The proposed Ordered Forest estimator provides a flexible estimation method of the conditional choice probabilities that can naturally deal with nonlinearities in the data, while taking the ordering information explicitly into account. In addition to common machine learning estimators, it enables the estimation of marginal effects as well as conducting inference thereof and thus providing the same output as classical econometric estimators based on ordered logit or probit models. An extensive simulation study examines the finite sample properties of the Ordered Forest and reveals its good predictive performance, particularly in settings with multicollinearity among the predictors and nonlinear functional forms. An empirical application further illustrates the estimation of the marginal effects and their standard errors and demonstrates the advantages of the flexible estimation compared to a parametric benchmark model.

Keywords

Ordered choice models, random forests, probabilities, marginal effects, machine learning

JEL Classification

C14, C25, C40

1 Introduction

Many empirical models deal with categorical dependent variables which have an inherent ordering. In such cases the outcome variable is measured on an ordered scale such as level of education defined by primary, secondary and tertiary education or income coded into low, middle and high income level. Further examples include survey outcomes on self-assessed health status (bad, good, very good) or political opinions (do not agree, agree, strongly agree) as well as various ratings and valuations. Moreover, even sports outcomes resulting in loss, draw and win are part of such modelling framework (e.g. Goller, Knaus, Lechner, and Okasa, 2018). So far, the ordered probit or ordered logit model represent workhorse models in such cases. More generally, standard econometric modelling in this setup is based on a smooth, increasing link function which is typically a cumulative distribution function. Its argument is a linear index that models the observable covariates, in order to obtain enough structure for deriving the expectations of the choice probabilities conditional on covariates. This hinges on unknown coefficients (and possibly other parameters) that need to be estimated (e.g. Wooldridge, 2010). If the link function is chosen to be the cdf of the logistic or normal distribution, the ordered logit or the ordered probit model results, respectively (Cameron and Trivedi, 2005). Generally, in econometrics, the interest is not only in the predicted choice probabilities conditional on the covariates, but also in a relation how these probabilities vary with changing values of a specific covariate, while holding the others constant. In the framework of discrete choice models (i.e. Greene and Hensher, 2010), the respective quantities of interest are mean marginal effects (effects averaged over the sample) or marginal effects at mean (effects evaluated at the means of covariates). The main advantage of these models is the ease of estimation, usually done by maximum likelihood. However, the major disadvantage are the strong parametric assumptions which are imposed for convenience rather than derived from any substantive knowledge about the application. Unfortunately, the desired marginal effects are sensitive to these assumptions. Although there is a large literature on how to generalize these assumptions in case of binary choice models (Matzkin, 1992; Ichimura, 1993; Klein and Spady, 1993), or multinomial (unordered) choice models (Lee, 1995; Fox, 2007), limited work has been done for ordered choice models (Lewbel, 2000; Klein and Sherman, 2002; also see Stewart, 2005 for an overview).

In this paper, we exploit recent advances in the machine learning literature to build an estimator for predicting choice probabilities as well as marginal effects together with inference procedures when the outcome variable has an ordered categorical nature, while preserving to some extent also the computational ease. The proposed *Ordered Forest* estimator improves on the classical ordered choice models such as ordered logit and ordered probit models by allowing *ex-ante* flexible functional forms as well as allowing for a large covariate space. The latter is a feature of many machine learning methods, but is typically absent from standard econometrics. Additionally, the *Ordered Forest* advances also machine learning methods with the estimation of marginal effects and the inference thereof, a feature of many parametric models, but generally missing in the machine learning literature. Hence, the contribution is twofold. First, with respect to the literature on parametric estimation of the ordered choice models, the *Ordered Forest* represents a flexible estimator without any parametric assumptions, while providing essentially the same information as an ordered parametric model. Second, with respect to the machine learning literature, the *Ordered Forest* achieves more precise estimation of ordered choice probabilities, while adding estimation of marginal effects as well as statistical inference thereof.

The proposed estimator is based on the classical random forest algorithm and makes use of linear combinations of cumulative predictions of respective ordered categories, conditional on covariates. Such predictions obtained by random forest have been shown to be asymptotically normal (Wager and Athey, 2018). Thus, linear combinations of such predictions share this property as well and hence allow for conducting statistical inference. Beyond obtaining these theoretical guaranties, we also investigate the predictive performance of the estimator by comparing it to classical and other competing methods via Monte Carlo simulation study as well as in real datasets. Furthermore, an empirical example demonstrates the estimation of the marginal effects and the associated inference procedure. Moreover, a free software implementation of the *Ordered Forest* estimator has been developed in GAUSS and is available online and on *ResearchGate*. Additionally, an R-package will be submitted to the CRAN repository as well.

This paper is organized as follows. Section 2 discusses the related literature concerning machine learning methods for the estimation of ordered choice models. Section 3 reviews the random forest algorithm and its theoretical properties. In Section 4 the *Ordered Forest* estimator is introduced including the estimation of the conditional choice probabilities, marginal effects and the inference procedure. The Monte Carlo simulation is presented in Section 5. Section 6 shows an empirical application. Section 7 concludes. Further details regarding estimation methods, the simulation study and the empirical application are provided in Appendices A, B and C, respectively.

2 Literature

In econometrics, the ordered probit and ordered logit models are widely used when there are ordered response variables (McCullagh, 1980). These models build on the latent regression model assuming an underlying continuous outcome Y_i^* as a linear function of regressors X_i with unknown coefficients β, while assuming that the latent error term u_i follows the standard normal or the logistic distribution. Furthermore, the ordered discrete outcome Y_i represents categories that cover a certain range of the latent continuous Y_i^* and is determined by unknown threshold parameters α_m. Formally, in the case of the ordered logit the latent model is defined as:

$$Y_i^* = X_i'\beta + u_i, \qquad (u_i \mid X_i) \sim \mathit{Logistic}(0, \pi^2/3) \tag{2.1}$$

with threshold parameters $\alpha_1 < \alpha_2 < ... < \alpha_M$ such that:

$$Y_i = m \quad \text{if} \quad \alpha_{m-1} < Y_i^* \leq \alpha_m \quad \text{for} \quad m = 1, ..., M, \tag{2.2}$$

where the coefficients and the thresholds are commonly estimated via maximum likelihood with the delta method or bootstrapping used for inference. The above latent model is also often motivated by the quantity of interest, i.e. the conditional choice probabilities which are given by:

$$P[Y_i = m \mid X_i = x] = \Lambda\big(\alpha_m - X_i'\beta\big) - \Lambda\big(\alpha_{m-1} - X_i'\beta\big), \tag{2.3}$$

where the link function $\Lambda(\cdot)$ is the logistic cdf mapping the real line onto the unit interval. Thus, the estimated probabilities are bounded between 0 and 1. The marginal effects are further given as partial derivative of the probabilities in 2.3:

$$\frac{\partial P[Y_i = m \mid X_i = x]}{\partial x^k} = \Big[\lambda\big(\alpha_{m-1} - X_i'\beta\big) - \lambda\big(\alpha_m - X_i'\beta\big)\Big]\beta_k, \tag{2.4}$$

where x^k is the k-th element of X_i and β_k is the corresponding coefficient, while $\lambda(\cdot)$ being the logistic pdf.

Although such models are relatively easy to estimate, they impose strong parametric assumptions which hinder the flexibility of these models. Apart from the assumptions about the distribution of the error term, further functional form assumptions are being imposed. As is clear from (2.1), the coefficients β are constant across the outcome classes which is often labelled as the parallel regression assumption (Williams, 2016). This inflexibility affects both the estimation of the choice probabilities as well as the estimation of marginal effects. For these reasons, generalizations of these models have been proposed in the literature in order to relax some of the assumptions. An example of such models is the generalized ordered logit model (McCullagh and Nelder, 1989), where the parallel regression assumption is abandoned. Hence, $M-1$ models are being estimated simultaneously and the coefficients are free to vary across all M outcome classes. Boes and Winkelmann (2006) provide an excellent overview of several other generalized parametric models. However, all of these models retain some of the distributional assumptions which limit their modelling flexibility.

Besides the standard econometric literature on parametric specifications of ordered choice models (for an overview see Agresti, 2002), a new strand of literature devoted to relaxing the parametric assumptions by using novel machine learning methods is emerging. Particularly, the tree-based methods have gained considerable attention. Although the classical CART algorithms introduced by Breiman (1984) are very powerful in both regression as well as in classification (see Loh, 2011 for a review), there is a need for adjustment when predicting ordered response. In the case of regression, the *discrete* nature of the outcome is not being taken into account and in the case of classification, the *ordered* nature of the outcome is not being taken into account. For these reasons, a strand of the literature focused particularly on adjustments towards ordered classification rather than regression which excludes the estimation of the conditional probabilities as is the case in the parametric ordered choice models. For example, Kramer, Widmer, Pfahringer, and De Groeve (2000) propose a simple procedure based on the Structural CART algorithm (see Kramer, 1996) constructing a distance-sensitive classification learner. Particularly, the learner is based on a regression tree and applies specific processing rules to force the outcome into one of the ordinal classes. In this fashion, the continuous predicted values in each leaf are rounded to the nearest ordinal class or similarly, the tree is forced to predict class values in each node, and thus the leaf values result in valid classes by choosing either rounded mean, median, or mode. Another approach suggested in the literature is to modify the splitting criterion directly. In particular, the usage of alternative impurity measures as opposed to the Gini coefficient in case of classification trees have been suggested, namely the generalized Gini criterion (Breiman, 1984) or the ordinal impurity function (Piccarreta, 2008). Both of these measures put higher penalty on misclassification the more distant the predicted category is from the true one. It follows that the above methods focus on estimating ordered classes rather than estimating ordered class probabilities, as is the focus of this paper.

The above ideas, however, have not been much used in practice. The reason might be the well-known drawbacks of single trees which suffer from unstable splits and a lack of smoothness (Hastie, Tibshirani, and Friedman, 2009). This is due to the path-dependent nature of trees which makes it difficult to find the 'best' tree. A natural extension of the CART algorithms is the random forest first introduced by Breiman (2001). The method comprises of bagged trees, whereas in the tree-growing step, only a random subset of covariates is considered for the next split. This has been shown to help to decorrelate the trees and improve the prediction accuracy (Hastie et al., 2009). Hence, random forest appears to be a better choice also for nonparametric estimation of conditional probabilities of ordered responses thanks to a better predictive performance and a lower variance in comparison to the standard CART algorithms.

However, the random forest algorithm as well as CART is primarily suitable for either regression or classification exercises. As such, appropriate modifications of the standard random forest algorithm are desired in order to predict conditional probabilities of discrete outcomes while taking the ordering nature into account. Hothorn, Hornik, and Zeileis (2006b) propose a random forest algorithm building on their conditional inference framework for recursive partitioning which can also deal with ordered outcomes. The difference to standard regression forests lies in a different splitting criterion using a test statistic where the conditional distribution at each split is based on permutation tests (for details see Strasser and Weber (1999) and Hothorn et al. (2006b)). Their proposed ordinal forest regression assummes an underlying latent continuous response Y_i^* as is the case in standard ordered choice models. Hothorn et al. (2006b) define a score vector $s(m) \in \mathbb{R}^M$, with $m = 1, ..., M$ observed ordered classes. This scores reflect the distances between the classes. The authors suggest to set the scores as midpoints of the intervals of Y_i^* which define the classes. As the underlying Y_i^* is unobserved, such a suggestion results in $s(m) = m$ and ordinal forest regression collapses to a standard forest regression as pointed out by Janitza, Tutz, and Boulesteix (2016). However, although the tree building step coincides, the prediction step differs as the estimates are the choice probabilities calculated as the proportions of the respective outcome classes falling into the same leaf instead of averages of the outcomes. Then in the forest, the conditional choice probabilities $\hat{P}[Y_i = m \mid X_i = x]$ are estimated by taking the averages of the choice probabilities produced by each tree, i.e. the same aggregation scheme as in a regression forest. Janitza et al. (2016) perform also a simulation study to test the robustness of the suggested score values by setting $s(m) = m^2$, but do not find any significant differences to simple $s(m) = m$. In this case, the implicit assumption is that the class widths, i.e. the adjacent intervals of the continuous outcome variable Y_i^* determining the descrete outcome Y_i are of the same length. This, however, does not have to hold in general and these intervals might not follow any particular pattern. In order to address this issue, Hornung (2019a) proposes an ordinal forest method, which optimizes these interval widths by maximizing the out-of-bag (OOB) prediction performance of the forests. However, on the contrary to the approach of Hothorn et al. (2006b), the forest algorithm used is based on the forest as developed by Breiman (2001), while the primary target is to predict the ordinal class and the choice probabilities are obtained as relative frequencies of trees predicting the particular class. This approach could be regarded as semiparametric as it uses the nonparametric structure of the trees and assumes a particular parametric distribution (standard normal) within the optimization procedure. Hornung (2019a) shows better prediction performance of such ordinal forests which optimize the class widths of Y_i^* in comparison to the conditional forests. Without the optimization step, the author denotes such forest as the naive ordinal forest. A more detailed description of the conditional as well as the ordinal forest is provided in Appendix A.2 and A.3, respectively.

While both of the discussed approaches take the ordering information of the outcomes into account, they focus mainly on prediction and variable importance without considering estimation of the marginal effects or the associated inference for the effects which are a fundamental part of the classical econometric ordered choice models. In addition, although both of these methods demonstrate good predictive performance, none of them provides theoretical guarantees with regards to the distribution of these predictions. Further, it is worth to mention that in practice both methods suffer from considerable computational costs. In case of the conditional forest, the additional permutation tests that need to be performed to evaluate the test statistic at each split result in a considerably longer computation time. For the ordinal forest, the additional optimization step for the class widths requires a prior estimation of a large number of forests (1000 by default) which also leads to a substantially longer computation time (see Tables 22 and 23 in Appendix B.3 for further details).

There is also a strand of literature which is concerned with the estimation of ordered outcome models in high-dimensional settings based on regularization methods. Examples of this approach include penalized ordered outcome models by Wurm, Rathouz, and Hanlon (2017) who make use of a standard ordered logit/probit regression while introducing an elastic net penalization term. Harrell (2015) describes a cumulative logit model with a ridge type of penalty. Archer, Hou, Zhou, Ferber, Layne, and Gentry (2014) implement the GMIFS (generalized monotone incremental forward stagewise) algorithm for penalized ordered outcome models which is similar to the Lasso type penalty. However, although the penalized models can deal with high dimensions, when the true model is relatively 'sparse', they are not *per se* nonparametric unless a large number of polynomials and interactions of available covariates is generated prior to estimation. As such, these models are closer to global nonparametric approaches, whereas the tree-based methods such as random forests can be regarded as local nonparametric methods and do not require any specific pre-processing of the data. Even though the penalized approaches also address the ordinality of the outcome variable, due to the above mentioned conceptual differences the remainder of this paper focuses on the forest-based methods.

3 Random Forests

Random forests as introduced by Breiman (2001) became quickly a very popular prediction method thanks to its good prediction accuracy, while being relatively simple to tune. Further advantages of random forests as a nonparametric technique are the high degree of flexibility and ability to deal with large number of predictors, while coping better with the curse of dimensionality problem in comparison to classical nonparametric methods such as kernel or local linear regression (see for example Racine (2008)). Random forests are based on bootstrap aggregation, i.e. the so-called bagging of single regression (or classification) trees where the covariates considered for each next split within a tree are selected at random. More precisely, the random forest algorithm draws a bootstrap sample Z_i^* of size N from the available training data for $b = 1, ..., B$ bootstrap replications. For each bootstrapped sample, a random-forest tree $\hat{T}_b$ is grown by recursive partitioning until the minimum leaf size is reached. At each of the splits, m out of p covariates chosen at random are considered. After all B trees are grown in this fashion, the regression random forest estimate of the conditional mean $E[Y_i \mid X_i = x]$ is the ensemble of the trees:

$$\hat{RF}^B(x) = \frac{1}{B}\sum_{b=1}^{B}\hat{T}_b(x) \quad \text{with} \quad \hat{T}_b(x) = \frac{1}{\mid \{i : X_i \in L_b(x)\} \mid}\sum_{\{i:X_i \in L_b(x)\}} Y_i, \tag{3.1}$$

where $L_b(x)$ denotes a leaf containing x. Single trees, if grown sufficiently deep, have a low bias, but fairly high variance. By averaging over many single trees with randomly choosing the set of observations and split covariates, the variance of the estimator is being reduced substantially. First, the variance reduction is achieved through bagging. The higher the number of bootstrap replications, the lower the variance. Second, the variance is further reduced through the random selection of covariates. The lower is the number of considered covariates for a split, the more is the correlation between the trees reduced and consequently, the bigger is the variance reduction of the average (Hastie et al., 2009).

Another attractive feature of random forests is the weighted average representation of the final estimate of the conditional mean $E[Y_i \mid X_i = x]$. As such we can rewrite the random forest prediction as follows:

$$\hat{RF}^B(x) = \sum_{i=1}^{N} \hat{w}_i(x) Y_i, \tag{3.2}$$

where the weights are defined as:

$$\hat{w}_{b,i}(x) = \frac{\mathbf{1}(\{X_i \in L_b(x)\})}{\mid L_b(x) \mid} \quad \text{with} \quad \hat{w}_i(x) = \frac{1}{B}\sum_{b=1}^{B} \hat{w}_{b,i}(x). \tag{3.3}$$

As such the forest weights $\hat{w}_i(x)$ are again an average over all single tree weights. These tree weights capture if the training example X_i falls into the leaf $L_b(x)$ scaled by the size of that leaf. Notice, that the weights are locally adaptive. Intuitively, random forests resemble the classical nonparametric kernel regression with an adaptive, data-driven bandwidth and with limited curse of dimensionality. Additionally, one can show that in the regression case, the random forest estimate as defined in (3.1) is equivalent to the weighting estimate defined in (3.2). This weighting perspective of random forests has been firstly suggested by Hothorn, Lausen, Benner, and Radespiel-Tröger (2004) and Meinshausen (2006) in the scope of survival and quantile regression, respectively. Recently, Athey, Tibshirani, and Wager (2019) point out the usefulness of the random forest weights in various estimation tasks. In this spirit, we will later on in Section 4.3 use the forest induced weights explicitly for inference as has been recently suggested by Lechner (2019).

Despite the huge popularity of random forests, little is known about their statistical properties, which prevents valid inference. For this reason, there have been some efforts towards establishing asymptotic properties of random forests (Meinshausen, 2006; Biau, 2012; Scornet, Biau, and Vert, 2015; Mentch and Hooker, 2016). However, a major step towards formally valid inference has been done in a recent work by Wager (2014) and Wager and Athey (2018) who prove consistency and asymptotic normality of random forest predictions, under some modifications of the standard random forest algorithm. These modifications concern both the tree-building procedure as well as the tree-aggregation scheme. First, the tree aggregation is now done using subsampling without replacement instead of bootstrapping. Second, the tree building procedure introduces the major and crucial condition of so-called honesty as first suggested by Athey and Imbens (2016). A tree is honest, if it does not use the same responses for both, placing splits and estimating the within-leaf effects. This can be achieved by double-sample trees, which split the random subsample of training data Z_i into two disjoint sets of the same size, while the one is used for placing splits and the other one for estimating the effects. Furthermore, for the consistency it is essential that the size of the leaves L of the trees becomes small relative to the sample size as N gets large[1]. This is achieved by introducing some randomness in choosing the splitting variables. Particularly, each covariate receives a minimum amount of positive chance of a split. Such constructed tree is then said to be a random-split tree. Additionally, the trees are required to be α-regular, meaning that after each split, both of the child nodes contain at least a fraction α of the training data (specifically, $\alpha \leq 0.2$ is required). Lastly, trees have to be symmetric in a sense that the order of the training data is independent of the predictor output. Overall, apart from subsampling and honesty the above conditions are not particularly binding and do not fundamentally deviate from the standard random forest. Then, after assumming some additional regularity conditions[2] such as *i.i.d.* sampling and an appropriate scaling of the subsample size s_N the random forest predictions can be shown to be (pointwise) asymptotically Gaussian and unbiased.

[1] Wager and Athey (2018) point out that the leaves need to be relatively small in all dimensions of the covariate space. This implies that the high-dimensional settings are not considered and hence the theoretical asymptotic results might not hold in such settings.

[2] For a detailed description of the conditions as well as of the proof, see Wager and Athey (2018).

4 Ordered Forest Estimator

The general idea of the *Ordered Forest* estimator is to provide a flexible alternative for estimation of ordered choice models that can deal with a large-dimensional covariate space. As such, the main goal is the estimation of conditional ordered choice probabilities, i.e. $P[Y_i = m \mid X_i = x]$ as well as marginal effects, i.e. the changes in the estimated probabilities in association with changes in covariates. Correspondingly, the variability of the estimated effects is of interest and therefore a method for conducting statistical inference is provided as well. The latter two features go beyond the traditional machine learning estimators which focus solely on the prediction exercise, and complement the prediction with the same econometric output as the traditional parametric estimators.

4.1 Conditional Choice Probabilities

The main idea of the estimation of the ordered choice probabilities by a random forest algorithm lies in the estimation of cumulative, i.e. nested probabilities based on binary indicators. As such, for an *i.i.d* random sample of size $N(i = 1, ..., N)$, consider an ordered outcome variable $Y_i \in \{1, ..., M\}$ with ordered classes m. Then the binary indicators are given as $Y_{m,i} = \mathbf{1}(Y_i \leq m)$ for outcome classes $m = 1, ..., M-1$. First, the ordered model is transformed into multiple overlapping binary models which are estimated by random forests yielding the predictions for the cumulative probabilities, i.e $\hat{Y}_{m,i} = \hat{P}[Y_{m,i} = 1 \mid X_i = x]$. Second, the estimated cumulative probabilities are differenced to isolate the respective class probabilities $P_{m,i} = P[Y_i = m \mid X_i = x]$. Hence the estimate for the conditional probability of the m-th ordered class is given by subtracting two adjacent cumulative probabilities as $\hat{P}_{m,i} = \hat{Y}_{m,i} - \hat{Y}_{m-1,i}$. Formally, the proposed estimation procedure can be described as follows:

1. Create $M - 1$ binary indicator variables such as

$$Y_{m,i} = \mathbf{1}(Y_i \leq m) \qquad \text{for} \qquad m = 1, ..., M-1. \tag{4.1}$$

2. Estimate regression random forest for each of the $M - 1$ indicators.
3. Obtain predictions $\hat{Y}_{m,i} = \hat{P}[Y_{m,i} = 1 \mid X_i = x] = \sum_{i=1}^{N} \hat{w}_{m,i}(x) Y_{m,i}$.
4. Compute probabilities for each class

$$\hat{P}_{1,i} = \hat{Y}_{1,i} \tag{4.2}$$

$$\hat{P}_{m,i} = \hat{Y}_{m,i} - \hat{Y}_{m-1,i} \qquad \text{for} \qquad m = 2, ..., M \tag{4.3}$$

$$\hat{P}_{M,i} = \hat{Y}_{M,i} = 1 \tag{4.4}$$

$$\hat{P}_{m,i} = 0 \qquad \text{if} \qquad \hat{P}_{m,i} < 0 \tag{4.5}$$

$$\hat{P}_{m,i} = \frac{\hat{P}_{m,i}}{\sum_{m=1}^{M} \hat{P}_{m,i}} \qquad \text{for} \qquad m = 1, ..., M, \tag{4.6}$$

where equation (4.2) defines the probability of the lowest value of the ordered outcome variable. This follows directly from the random forest estimation as the created indicator variable $Y_{1,i}$ describes the very lowest value of the ordered outcome classes and as such, no modification of its predicted value is necessary to obtain a valid probability prediction. Equation (4.3) makes use of the cumulative (nested) probability feature. As such, the predicted values of two subsequent binary indicator variables $Y_{m,i}$ are subtracted from each other to isolate the probability of the higher order class. Equation (4.4) is given by construction as follows from the indicator function (4.1) that all values of Y_i fullfil the condition for $m = M$ and from the fact that cumulative probabilities must add up to 1. Line (4.5) ensures that the computed probabilities from (4.3) do not become negative. This might occasionally happen especially if the respective outcome classes comprise of very few observations. This issue is well-known also from the generalized ordered logit model where the parallel regression assumption is relaxed (see McCullagh and Nelder (1989), p. 155). However, even though it is possible in theory, growing honest trees seems to largely prevent this from happening in practice. Lastly, in case if negative predictions should occur and thus being set to zero, (4.6) defines a normalization step to ensure that all class probabilities sum up to 1. Notice, that such an approach requires estimation of $M - 1$ forests in the training data, which might appear to be computationally expensive. However, given that most empirical problems involve a rather limited number of outcome classes (usually not exceeding 10 distinct classes) and the relatively fast estimation of standard regression forest[3] without any additional permutation test nor optimization steps needed as is the case for the conditional or the ordinal forests, respectively, the here proposed procedure shall be computationally advantageous (see Tables 22 and 23 in Appendix B.3).

4.2 Marginal Effects

After estimating the conditional choice probabilities, it is of interest to investigate how the estimated probabilities are associated with covariates, i.e. how the changes in the covariates translate into changes in the probabilities. Typical measures for such relationships in standard nonlinear econometrics are the marginal, or, partial effects. Thus, for nonlinear models, including ordered choice models, two fundamental measures are of common interest, mean marginal effects and marginal effects at the mean of the covariates[4]. These quantities are feasible also in the case of the *Ordered Forest* estimator. Due to the character of the ordered choice model, the marginal effects on all probabilities of different values of the ordered outcome classes are estimated, i.e. $P[Y_i = m \mid X_i = x]$. In the following, let us define the marginal effect for an element x^k of X_i as follows:

$$ME_i^{k,m}(x) = \frac{\partial P[Y_i = m \mid X_i^k = x^k, X_i^{-k} = x^{-k}]}{\partial x^k}, \tag{4.7}$$

with X_i^k and X_i^{-k} denoting the elements of X_i with and without the k-th element, respectively[5]. Next, let us define the marginal effect for categorical variables as a discrete change in the following way:

$$ME_i^{k,m}(x) = P[Y_i = m \mid X_i^k = \lceil x^k \rceil, X_i^{-k} = x^{-k}] - P[Y_i = m \mid X_i^k = \lfloor x^k \rfloor, X_i^{-k} = x^{-k}], \tag{4.8}$$

where $\lceil \cdot \rceil$ and $\lfloor \cdot \rfloor$ denote rounding up and down to the nearest integer value, respectively. Notice, that in the case of a binary variable this leads to the respective probabilities being evaluated at $\lceil x^k \rceil = 1$ and $\lfloor x^k \rfloor = 0$ as is usual for ordered choice models. From the above definitions of marginal effects, we obtain the desired quantity of interest, i.e. the marginal effect at mean by evaluating $ME_i^{k,m}(x)$ at the population mean of X_i, for which the sample mean is a natural proxy. The mean marginal effect is obtained by taking sample averages of $ME_i^{k,m}(x)$, i.e. $\frac{1}{N}\sum_{i=1}^{N} ME_i^{k,m}(x)$.

[3] The computational speed of the regression forests depends on many tuning parameters, of which the number of bootstrap replications, i.e. grown trees is the most decisive one.
[4] One can evaluate the marginal effect at any arbitrarily chosen value. The default option is usually the mean or the median.
[5] As a matter of notation, capitals denote random variables, whereas small letters refer to the particular realizations of the random variable.

Having formally defined the desired marginal effects, the next issue is the estimation of these effects. For the case of binary and categorical covariates X^k, this appears straightforward as the estimated *Ordered Forest* model provides predicted values for all probabilities at all values x^k. As such, the estimate $\hat{ME}_i^{k,m}(x)$ of marginal effects defined in equation (4.8) remains as a difference of the two conditional probabilities estimated by the *Ordered Forest*. However, it is less obvious for continuous variables, where derivatives are needed. As the estimates of the choice probabilities are averaged leaf means, the marginal effect is not explicit and not differentiable. In the nonparametric literature Stoker (1996) and Powell and Stoker (1996), among others, are directly concerned with estimating average derivatives. However, these methods lack convenience of estimation and have thus not been widely adopted by empirical researchers (the issues range from estimation difficulty, possibly non-standard distribution of the estimator, to ambiguous choices of nuisance parameters). Therefore, we approximate the derivative by a discrete analogue based on the definition of a derivative as follows:

$$\hat{ME}_i^{k,m}(x) = \frac{\hat{P}[Y_i = m \mid X_i^k = x^{kU}, X_i^{-k} = x^{-k}] - \hat{P}[Y_i = m \mid X_i^k = x^{kL}, X_i^{-k} = x^{-k}]}{x^{kU} - x^{kL}} \tag{4.9}$$

$$= \frac{\hat{P}_{m,i}(x^{kU}) - \hat{P}_{m,i}(x^{kL})}{x^{kU} - x^{kL}}, \tag{4.10}$$

where x^{kU}, x^{kL} are (arbitrarily) chosen to be larger (x^{kU}) and smaller (x^{kL}) than x^k by 0.1 standard deviation of x^k, while ensuring that the support of x^k is respected. Hence, the approximation targets the marginal change in the value of the covariate X_i^k. Notice, that such an estimation of marginal effects is much more demanding exercise than solely predicting the choice probabilities. Therefore, it is expected that considerably more subsampling iterations are needed for a good performance.

4.3 Inference

The asymptotic results of Wager and Athey (2018) regarding the consistency and normality of random forest predictions hold also when dealing with binary outcomes. Then, the estimate is the conditional probability, as is the case for the *Ordered Forest* algorithm, namely $P[Y_{m,i} = 1 \mid X_i = x]$. As such, valid statistical inference can be done in respect to the probability estimate, too. This is of importance as the *Ordered Forest* estimator relies heavily on such estimates. Particularly, the *Ordered Forest* makes use of linear combinations of the probability estimates made by the random forest for both the conditional probabilities as well as for the marginal effects. Hence, the final *Ordered Forest* estimates for the probabilities and the marginal effects, based on a forest algorithm respecting the conditions discussed in Section 3, inherit the consistency and normality properties.

The here proposed method for conducting approximate inference of the estimated marginal effects utilizes the weight-based representation of random forest predictions and adapts the weight-based inference proposed by Lechner (2019) for the case of the *Ordered Forest* estimator (see also Lechner (2002) and Imbens and Abadie (2006) for related approaches). The main condition for conducting weight-based inference is to ensure that the weights and the outcomes are independent. This is achieved through sample splitting where one half of the sample is used to build the forest, and thus to determine the weights, and the other half to estimate the effects using the respective outcomes. Notice that this condition goes beyond honesty as defined in Wager and Athey (2018) as this requires not only estimating honest trees but estimating honest forest as a whole. This comes, however, at the expense of the efficiency of the estimator as less data are effectively used. Nevertheless, the simulation evidence in Lechner (2019) suggests that this efficiency loss is small, if present at all[6].

Since the *Ordered Forest* estimator is based on differences of random forest predictions for adjacent outcome categories, also the covariance term enters the variance formula of the final estimator[7] as opposed to the modified causal forests developed in Lechner (2019). Further, the estimation of marginal effects is based on differences of single *Ordered Forest* predictions which also needs to be taken into account[8]. Let us first rewrite the marginal effects in terms of weighted means of the outcomes as follows:

$$
\begin{aligned}
&\hat{ME}_i^{k,m}(x) = \frac{\hat{P}_{m,i}(x^{kU}) - \hat{P}_{m,i}(x^{kL})}{x^{kU} - x^{kL}} \\
&= \frac{1}{x^{kU} - x^{kL}} \cdot \left(\left[\sum_{i=1}^{N} \hat{w}_{i,m}(x^{kU}) Y_{i,m} - \sum_{i=1}^{N} \hat{w}_{i,m-1}(x^{kU}) Y_{i,m-1} \right] - \left[\sum_{i=1}^{N} \hat{w}_{i,m}(x^{kL}) Y_{i,m} - \sum_{i=1}^{N} \hat{w}_{i,m-1}(x^{kL}) Y_{i,m-1} \right] \right) \\
&= \frac{1}{x^{kU} - x^{kL}} \cdot \left(\left[\sum_{i=1}^{N} \hat{w}_{i,m}(x^{kU}) Y_{i,m} - \sum_{i=1}^{N} \hat{w}_{i,m}(x^{kL}) Y_{i,m} \right] - \left[\sum_{i=1}^{N} \hat{w}_{i,m-1}(x^{kU}) Y_{i,m-1} - \sum_{i=1}^{N} \hat{w}_{i,m-1}(x^{kL}) Y_{i,m-1} \right] \right) \\
&= \frac{1}{x^{kU} - x^{kL}} \cdot \left(\sum_{i=1}^{N} \tilde{w}_{i,m}(x^{kU} x^{kL}) Y_{i,m} - \sum_{i=1}^{N} \tilde{w}_{i,m-1}(x^{kU} x^{kL}) Y_{i,m-1} \right),
\end{aligned}
$$

where $\tilde{w}_{i,m}(x^{kU} x^{kL}) = \hat{w}_{i,m}(x^{kU}) - \hat{w}_{i,m}(x^{kL})$, and $\tilde{w}_{i,m-1}(x^{kU} x^{kL}) = \hat{w}_{i,m-1}(x^{kU}) - \hat{w}_{i,m-1}(x^{kL})$ are the new weights defining the marginal effect. As such the quantity of interest for inference becomes the variance of the above expression given as:

$$
\begin{aligned}
Var\left(\hat{ME}_i^{k,m}(x) \right) &= Var\left(\frac{1}{x^{kU} - x^{kL}} \cdot \left(\sum_{i=1}^{N} \tilde{w}_{i,m}(x^{kU} x^{kL}) Y_{i,m} - \sum_{i=1}^{N} \tilde{w}_{i,m-1}(x^{kU} x^{kL}) Y_{i,m-1} \right) \right) \\
&= Var\left(\frac{\sum_{i=1}^{N} \tilde{w}_{i,m}(x^{kU} x^{kL}) Y_{i,m}}{x^{kU} - x^{kL}} \right) + Var\left(\frac{\sum_{i=1}^{N} \tilde{w}_{i,m-1}(x^{kU} x^{kL}) Y_{i,m-1}}{x^{kU} - x^{kL}} \right) \\
&\quad - 2 \cdot Cov\left(\frac{\sum_{i=1}^{N} \tilde{w}_{i,m}(x^{kU} x^{kL}) Y_{i,m}}{x^{kU} - x^{kL}} ; \frac{\sum_{i=1}^{N} \tilde{w}_{i,m-1}(x^{kU} x^{kL}) Y_{i,m-1}}{x^{kU} - x^{kL}} \right),
\end{aligned}
$$

which suggests the following estimator for the variance[9]:

$$
\begin{aligned}
&\hat{Var}\left(\hat{ME}_i^{k,m}(x) \right) = \frac{N}{N-1} \cdot \frac{1}{(x^{kU} - x^{kL})^2} \cdot \\
&\cdot \left(\sum_{i=1}^{N} \left(\tilde{w}_{i,m}(x^{kU} x^{kL}) Y_{i,m} - \frac{1}{N} \sum_{i=1}^{N} \tilde{w}_{i,m}(x^{kU} x^{kL}) Y_{i,m} \right)^2 + \sum_{i=1}^{N} \left(\tilde{w}_{i,m-1}(x^{kU} x^{kL}) Y_{i,m-1} - \frac{1}{N} \sum_{i=1}^{N} \tilde{w}_{i,m-1}(x^{kU} x^{kL}) Y_{i,m-1} \right)^2 \right. \\
&\left. - 2 \cdot \sum_{i=1}^{N} \left(\tilde{w}_{i,m}(x^{kU} x^{kL}) Y_{i,m} - \frac{1}{N} \sum_{i=1}^{N} \tilde{w}_{i,m}(x^{kU} x^{kL}) Y_{i,m} \right) \cdot \left(\tilde{w}_{i,m-1}(x^{kU} x^{kL}) Y_{i,m-1} - \frac{1}{N} \sum_{i=1}^{N} \tilde{w}_{i,m-1}(x^{kU} x^{kL}) Y_{i,m-1} \right) \right),
\end{aligned}
$$

where for the marginal effects at the mean of the covariates the weights $\tilde{w}_{i,m}(x^{kU} x^{kL})$ and the scaling factor $1/(x^{kU} - x^{kL})^2$ are evaluated at the respective sample means, whereas for the mean marginal effects the average of the weights $\frac{1}{N} \sum_{i=1}^{N} \tilde{w}_{i,m}(x^{kU} x^{kL})$ and of the scaling factor $1/(\frac{1}{N} \sum_{i=1}^{N} (x^{kU} - x^{kL}))^2$ is used. Notice also the fact that the scaling factor drops out in the case of categorical covariates. Ac-

[6] The so-called cross-fitting to avoid the efficiency loss as suggested by Chernozhukov, Chetverikov, Demirer, Duflo, Hansen, Newey, and Robins (2018) does not appear to be applicable here as the independence of the weights and the outcomes would not be ensured.

[7] One could avoid the covariance term with an additional sample split, which might, however, further lead to a decreased efficiency of the estimator.

[8] Notice, that for outcome classes $m = 1$ and $m = M$, the variance formula simplifies substantially.

[9] Here, we estimate the variance with sample counterparts. An alternative approach, as in Lechner (2019), would be to first apply the law of total variance and, second, estimate the conditional moments by nonparametric methods. However, due to the presence of the covariance term the conditioning set contains 2 variables which causes the convergence rate to decrease and hence such variance estimation might even result in less precise estimates, depending on the sample size.

cording to the simulation study in Lechner (2019) the weight-based inference in case of modified causal forests tends to be rather conservative for the individual effects and rather accurate for aggregate effects. The results from the here conducted empirical applications resemble this pattern where inference for the marginal effects at the mean of the covariates is more conservative in comparison to inference for the mean marginal effects throughout all datasets (see Appendix C.3).

5 Monte Carlo Simulation

In order to investigate the finite sample properties of the proposed *Ordered Forest* estimator, we perform a Monte Carlo simulation study comparing competing estimators for ordered choice models based on the random forest algorithm. As a parametric benchmark, we take the ordered logistic regression. The considered models are specifically the following: (i) ordered logit (McCullagh, 1980), (ii) naive ordinal forest (Hornung, 2019a), (iii) ordinal forest (Hornung, 2019a), (iv) conditional forest (Hothorn et al., 2006b), and (v) *Ordered Forest* (Lechner and Okasa, 2019). Within the simulation study the *Ordered Forest* estimator is analyzed more closely to study the finite sample properties of the estimator depending on the particular forest building schemes and the way the ordering information is being taken into account. Regarding the former we study the *Ordered Forest* based on the standard random forest as in Breiman (2001), i.e. with boostrapping and *without* honesty as well as based on the modified random forest as in Wager and Athey (2018), i.e. with subsampling and *with* honesty. Regarding the latter we study an alternative approach for estimating the conditional choice probabilities which could be labelled as a 'multinomial' forest. In that case, the ordering information is not being taken into account and the probabilities of each category are estimated directly. The details of this approach are provided in Appendix A.1. Given this, the *Ordered Forest* estimator should perform better than the multinomial forest in terms of the prediction accuracy thanks to the incorporation of additional information from the ordering of the outcome classes.

Table 1: General Settings of the Simulation

Monte Carlo	
observations in training set	200 (800)
observations in testing set	10000
replications	100
covariates with effect	15
trees in a forest	1000
randomly chosen covariates	$\sqrt{p}$
minimum leaf size[10]	5

General settings regarding the sample size, the number of replications, as well as forest-specific tuning parameters for the Monte Carlo simulation are depicted in Table 1. Furthermore, a detailed description of the software implementation of the respective estimators as well as the software specific tuning parameters are discussed in Appendix B.3.

[10]Due to the conceptual differences of the conditional forests, an alternative stopping rule ensuring growing deep trees is chosen. See details in Appendix B.3.

5.1 Data Generating Process

In terms of the data generating process, we built upon an ordered logit model as defined in (2.1) and (2.2). As such we simulate the underlying continuous latent variable Y_i^* as a linear function of regressors X_i, while drawing the error term u_i from the logistic distribution. Then, the continuous outcome Y_i^* is discretized into an ordered categorical outcome Y_i based on the threshold parameters α_m. The thresholds are determined beforehand according to fixed threshold quantiles α_m^q of a large sample of $N = 1000000$ observations of the latent Y_i^* from the very same DGP to reflect the realized outcome distribution and then used afterwards in the simulations as a part of the deterministic component. Furthermore, the intercept term is fixed to zero, i.e. $\beta_0 = 0$ and thus the thresholds are relative to this value of the intercept. As a result, such DGP captures the probability of the latent variable Y_i^* falling into a particular class given the location defined by the deterministic component of the model together with its stochastic component (Carsey and Harden, 2013).

In simulations of the data generating process, different numbers of possible discrete classes are considered, particularly $M = \{3, 6, 9\}$ which corresponds to the simulation set-up used in Janitza et al. (2016) and Hornung (2019a). Further, both equal class widths, i.e. equally spaced threshold parameters α_m, as well as randomly spaced thresholds, while still preserving the monotonicity of the discrete outcome Y_i, are considered. For the latter, the threshold quantiles are drawn from the uniform distribution, i.e. $\alpha_m^q \sim U(0, 1)$ and ordered afterwards. For the former, the threshold quantiles are equally spaced between 0 and 1 depending on the number of classes. The β coefficients are specified as having fixed coefficient size, namely $\beta_1, ..., \beta_5 = 1$, $\beta_6, ..., \beta_{10} = 0.75$ and $\beta_{11}, ..., \beta_{15} = 0.5$ as is also the case in Janitza et al. (2016) and Hornung (2019a). Moreover, an option for nonlinear effects is introduced, too. As such, the coefficients of covariates are no longer linear, but are given by a sine function $sin(2X_i)$ as for example in Lin, Li, and Sun (2014), which is hard to model as opposed to other nonlinearities such as polynomials or interactions. The set of covariates X_i is drawn from the multivariate normal distribution with zero mean and a pre-specified variance-covariance matrix Σ, i.e. $X_i \sim \mathcal{N}(0, \Sigma)$, where Σ is specified either as an identity matrix and as such implying zero correlation between regressors, or it is specified to have a specific correlation structure between regressors[11] as follows:

$$\rho_{i,j} = \begin{cases} 1 & \text{for} \quad i = j \\ 0.8 & \text{for} \quad i \neq j; i, j \in \{1, 3, 5, 7, 9, 11, 13, 15\} \\ 0 & \text{otherwise} \ , \end{cases}$$

which is inspired by the correlation structure from the simulations in Janitza et al. (2016) and Hornung (2019a). Further, an option to include additional variables with zero effect is implemented as well. As such, another 15 covariates are added to the covariate space with $\beta_{16} = ... = \beta_{30} = 0$ from which 10 are again drawn from the normal distribution with zero mean and unit variance, i.e. $X_{i,0}^c \sim \mathcal{N}(0, 1)$ and 5 are dummies drawn from the binomial distribution, i.e. $X_{i,0}^d \sim \mathcal{B}(0.5)$. As the performance of the *Ordered Forest* estimator in high-dimensional settings is of particular interest, due to yet not fully understood theoretical properties in such settings, we include an option for additionally enlarging the covariate space with 1000 zero effect covariates $X_{i,0} \sim \mathcal{N}(0, 1)$, effectively creating a setting with $p >> N$. In the high-dimensional case the ordered logit is excluded from the simulations for obvious reasons. Overall, considering all the possible combinations for specifying the DGP, we end up with 72 different DGPs[12].

[11] Note that with a too high multicollinearity, the ordered logit model breaks down. With restricting the level of multi-collinearity, the logit model can be still reasonably compared to the other competing methods.

[12] For the low-dimensional setting we have $n = 4$ options for the DGP settings, out of which we can choose from none to all of

For all of them we simulate a training dataset of size $N = 200$ and a testing dataset of size $N = 10000$ for evaluating the prediction performance of the considered methods, where the large testing set enables us to reduce the prediction noise and corresponds to the setup used in Janitza et al. (2016) and Hornung (2019a) as well. Further, we focus more closely on the simulation designs corresponding to the least and the most complex DGPs for which we simulate also a training set of size $N = 800$. The former DGP (labelled as simple DGP henceforth) corresponds exactly to an ordered logit model as in (2.1) with equal class widths, uncorrelated covariates with linear effects and without any additional zero effect variables. The latter DGP (labelled as complex DGP henceforth) features random class widths, correlated covariates with nonlinear effects and additional zero effect variables. For each replication, we estimate the model on the training set and evaluate the predictions on the testing set, for all tested methods.

5.2 Evaluation Measures

In order to properly evaluate the prediction performance we use two measures of accuracy, namely the mean squared error (MSE) and the ranked probability score (RPS). The former evaluates the error of the estimated conditional choice probabilities as a squared difference from the true values of the conditional choice probabilities. Given our simulation design, we know these true values, which are given as in equation (2.3). Hence, we can define the Monte Carlo average MSE as:

$$AMSE = \frac{1}{R}\sum_{j=1}^{R}\frac{1}{N}\sum_{i=1}^{N}\frac{1}{M}\sum_{m=1}^{M}\Big(P[Y_{i,j} = m \mid X_{i,j} = x] - \hat{P}[Y_{i,j} = m \mid X_{i,j} = x]\Big)^2,$$

where j refers to the j-th simulation replication, while R being the total number of replications. The second measure, the RPS as developed by Epstein (1969) is arguably the preffered measure for the evaluation of probability forecasts for ordered outcomes as it takes the ordering information into account (Constantinou and Fenton, 2012). The Monte Carlo average RPS can be defined as follows:

$$ARPS = \frac{1}{R}\sum_{j=1}^{R}\frac{1}{N}\sum_{i=1}^{N}\frac{1}{M-1}\sum_{m=1}^{M}\Big(P[Y_{i,j} \leq m \mid X_{i,j} = x] - \hat{P}[Y_{i,j} \leq m \mid X_{i,j} = x]\Big)^2,$$

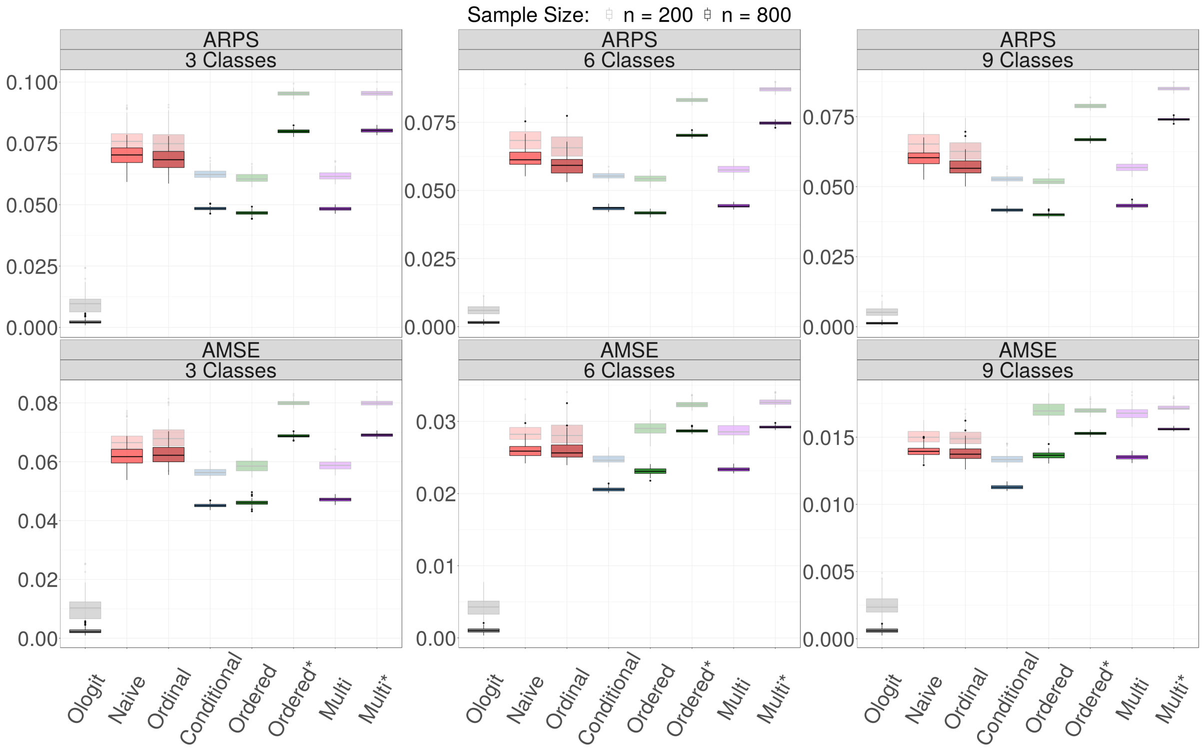
where on the contrary to the MSE, the difference between the cumulative choice probabilities is measured. The RPS can be seen as a generalization of the Brier Score (Brier, 1950) for multiple, ordered outcomes. As such, it measures the discrepancy between the predicted cumulative distribution function and the true one. The estimated cdf can be computed based on the predicted probabilities for each ordered class m of observation i, whereas the true cdf is based on the true probabilities. Note that in the case of empirical data, as opposed to the simulation data, the true cdf boils down to a step function going from 0 to 1 at the true class value of the ordered outcome Y_i for the particular observation i. As such, the more the predicted probabilities are concentrated around the true value, the lower the ARPS and hence the better the prediction. Nevertheless, although the ordering information is taken into account, the relative distance between the classes is not reflected (Janitza et al., 2016).

them, whereby the ordering does not matter, we end up with 16 possible combinations as given by the formula $\sum_{r=0}^{n}\binom{n}{r}$, each for 3 possible numbers of outcome classes resulting in 48 different DGPs. For the high-dimensional setting we have $n = 3$ options as the additional noise variables are always considered. This for all 3 distinct numbers of outcome classes yields 24 different DGPs.

5.3 Simulation Results

For the sake of brevity, here we focus mainly on the simulation results obtained for the simple and for the complex DGP, while the results for all 72 DGPs are provided in Appendix B.2. Figures 1 and 2 summarize the results for the low-dimensional setting for the simple and the complex DGP, respectively. Similarly, Figures 3 and 4 present the results for the simple and the complex DGP for the high-dimensional setting. The upper panels of the figures show the ARPS, the preferred accuracy measure, whereas the lower panels show the AMSE as a complementary measure. Within the figures the transparent boxplots in the background show the results for the smaller sample size along with the bold boxplots in the foreground showing the results for the bigger sample size. From left to right the figures present the results for 3, 6 and 9 outcome classes, respectively. The figures compare the prediction accuracy of the ordered logit, naive forest, ordinal forest, conditional forest, *Ordered Forest* and the multinomial forest, where the asterisk (*) denotes the honest version of the last two forests considered. Further tables with more detailed results and statistical tests for mean differences in the prediction errors are listed in Appendix B.1.

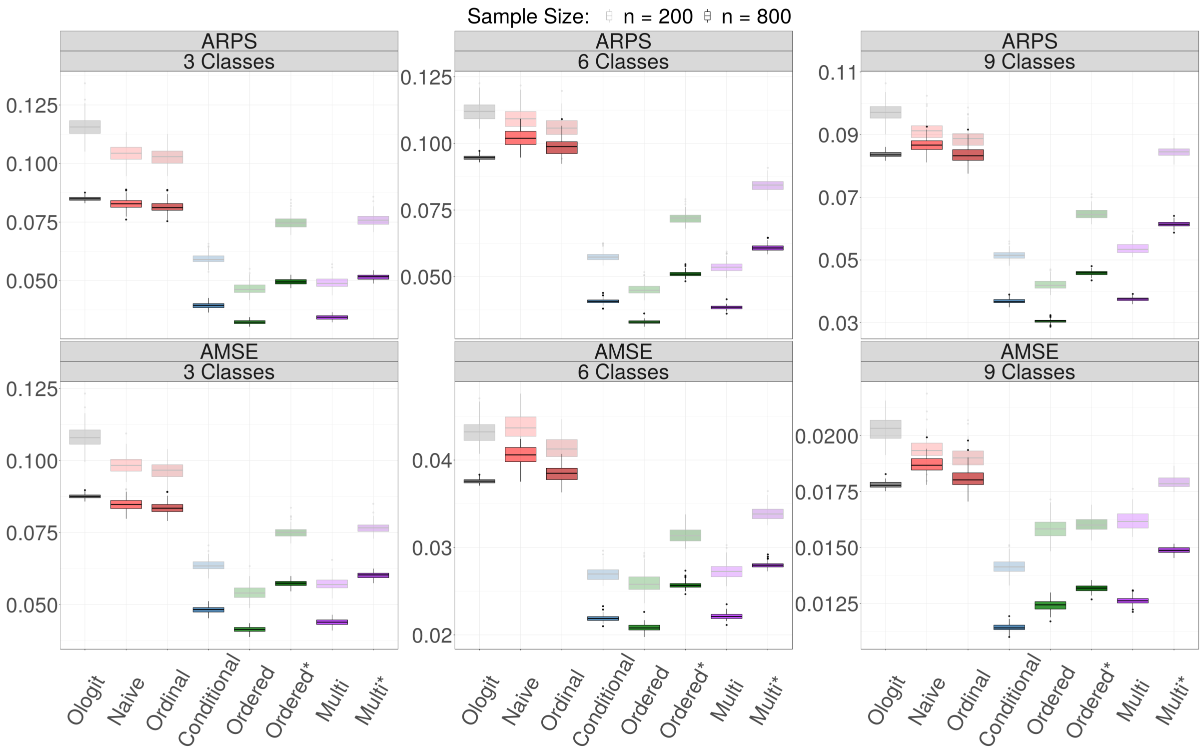
Figure 1: Simulation Results: Simple DGP & Low Dimension



Note: Figure summarizes the prediction accuracy results based on 100 simulation replications. The upper panel contains the ARPS and the lower panel contains the AMSE. The boxplots show the median and the interquartile range of the respective measure. The transparent boxplots denote the results for the small sample size, while the bold boxplots denote the results for the big sample size. From left to right the results for 3, 6, and 9 outcome classes are displayed.

In the low-dimensional setting with the simple DGP it is expected that the ordered logistic regression should perform best in terms of both the AMSE as well as the ARPS. Indeed, we do observe this results in Figure 1 as the ordered logit model performs unanimously best out of the considered models, reaching almost zero prediction error. Among the flexible forest-based estimators, the proposed *Ordered Forest* belongs to those better performing methods in terms of both accuracy measures. The honest versions of the forests lack behind what points at the efficiency loss due to the additional sample splitting. Overall, the ranking of the estimators stays stable with regards to the number of outcome categories. Additional pattern common to all estimators is the lower prediction error and increased precision with growing sample size.

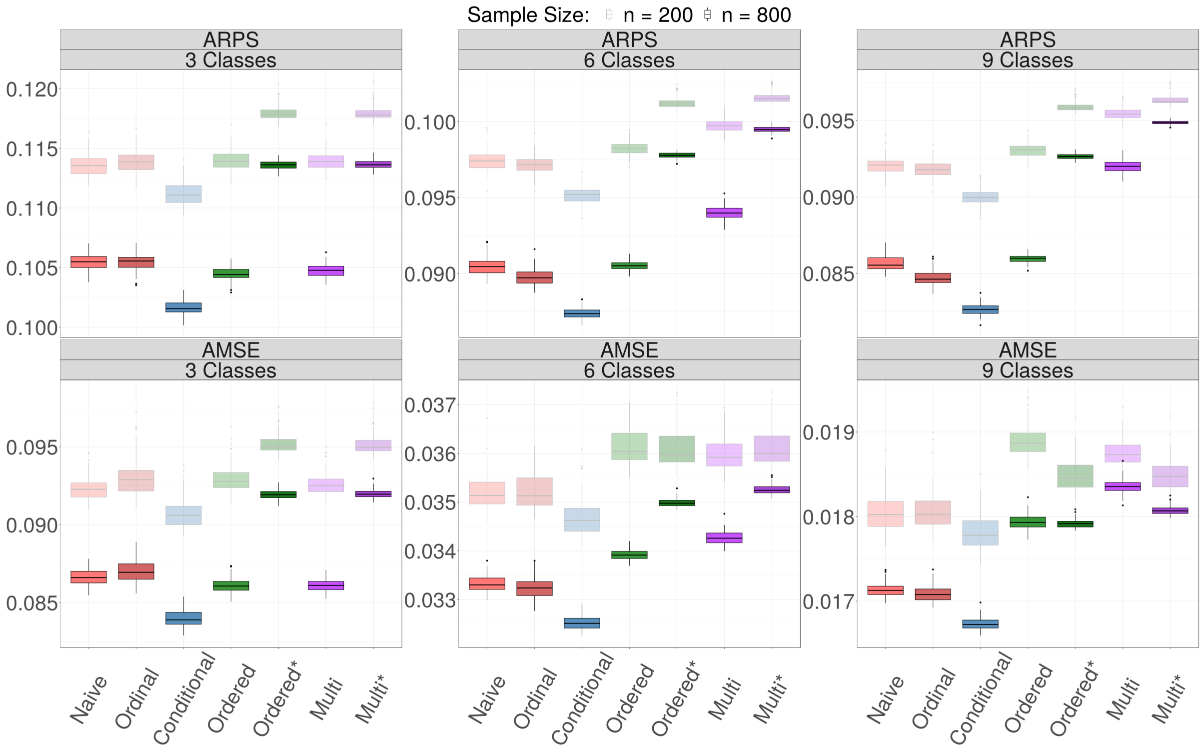
Figure 2: Simulation Results: Complex DGP & Low Dimension



Note: Figure summarizes the prediction accuracy results based on 100 simulation replications. The upper panel contains the ARPS and the lower panel contains the AMSE. The boxplots show the median and the interquartile range of the respective measure. The transparent boxplots denote the results for the small sample size, while the bold boxplots denote the results for the big sample size. From left to right the results for 3, 6, and 9 outcome classes are displayed.

In the case of the complex DGP, the performance of the flexible forest-based estimators is expected to be better in comparison to the parametric ordered logit. This can be seen in Figure 2 as the ordered logit lacks behind the majority of the flexible methods in both accuracy measures. The somewhat higher prediction errors of the naive and the ordinal forest compared to the other forest-based methods might be due to their different primary target which are the ordered classes instead of the ordered probabilities as is the case for the other methods. In this respect the conditional forest exhibits considerably good prediction performance. The *Ordered Forest* outperforms the competing forest-based estimators in terms of the ARPS throughout all outcome class scenarios and also in terms of the AMSE in two scenarios, being outperformed only by the conditional forest in case of 9 outcome classes. Interestingly, the multinomial forest performs very well across all scenarios. However, it is consistently worse than the *Ordered Forest* with bigger discrepancy between the two the more outcome classes are considered. This points to the value of the ordering information and the ability of the *Ordered Forest* to utilize it in the estimation. With regards to the sample size, we observe the same pattern as in Figure 1.

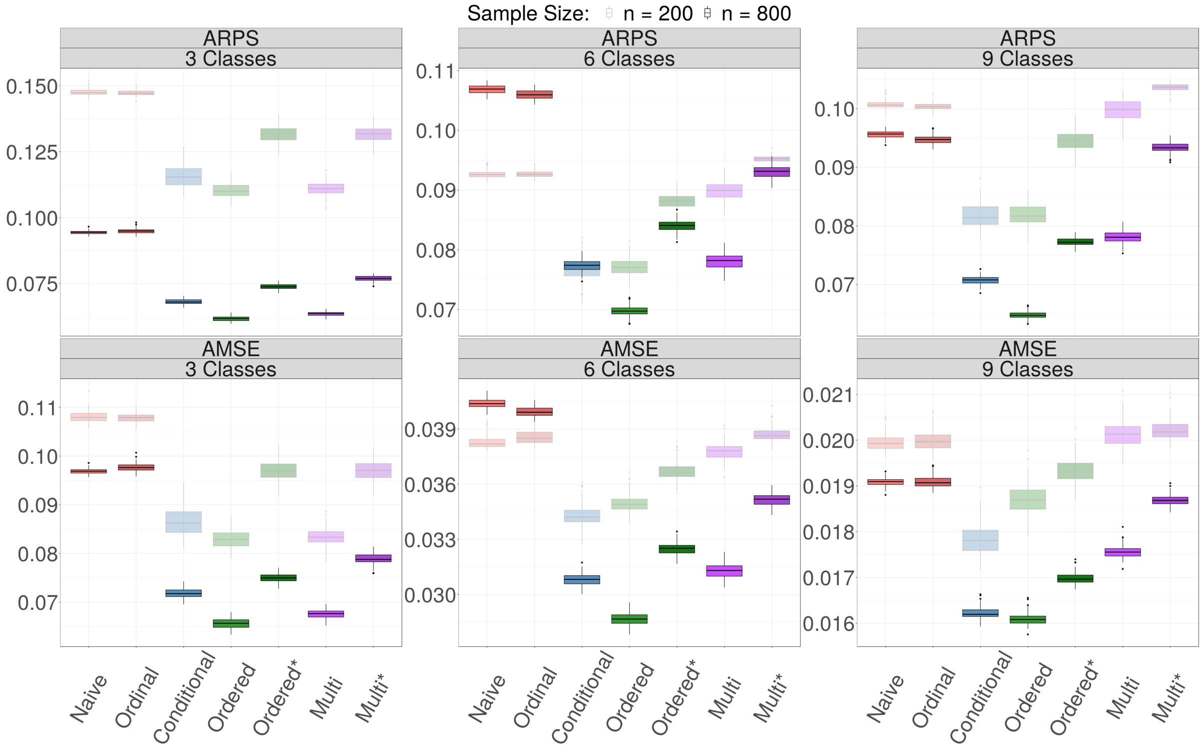
Figure 3: Simulation Results: Simple DGP & High Dimension



Note: Figure summarizes the prediction accuracy results based on 100 simulation replications. The upper panel contains the ARPS and the lower panel contains the AMSE. The boxplots show the median and the interquartile range of the respective measure. The transparent boxplots denote the results for the small sample size, while the bold boxplots denote the results for the big sample size. From left to right the results for 3, 6, and 9 outcome classes are displayed.

Considering the high-dimensional setting for the case of the simple DGP, we see in Figure 3 that the *Ordered Forest* slightly lacks behind the other methods, except the scenarios with 3 outcome classes. In comparison, the conditional forest performs best in terms of the ARPS as well as in terms of the AMSE. Also the naive and the ordinal forest exhibit better performance compared to the previous simulation designs. However, it should be noted that the overall differences in the magnitude of the prediction errors are much lower within this simulation design as compared to the previous designs. Further, taking a closer look at the ARPS results of the multinomial forest we clearly see that in the simple ordered design the ignorance of the ordering information really harms the predictive performance of the estimator the more outcome classes are considered. Additionally, it is interesting to see that the performance gain due to a bigger sample size seems to be much less for the honest version of the forests in the high-dimensional setting as opposed to the low-dimensional setting.

Figure 4: Simulation Results: Complex DGP & High Dimension



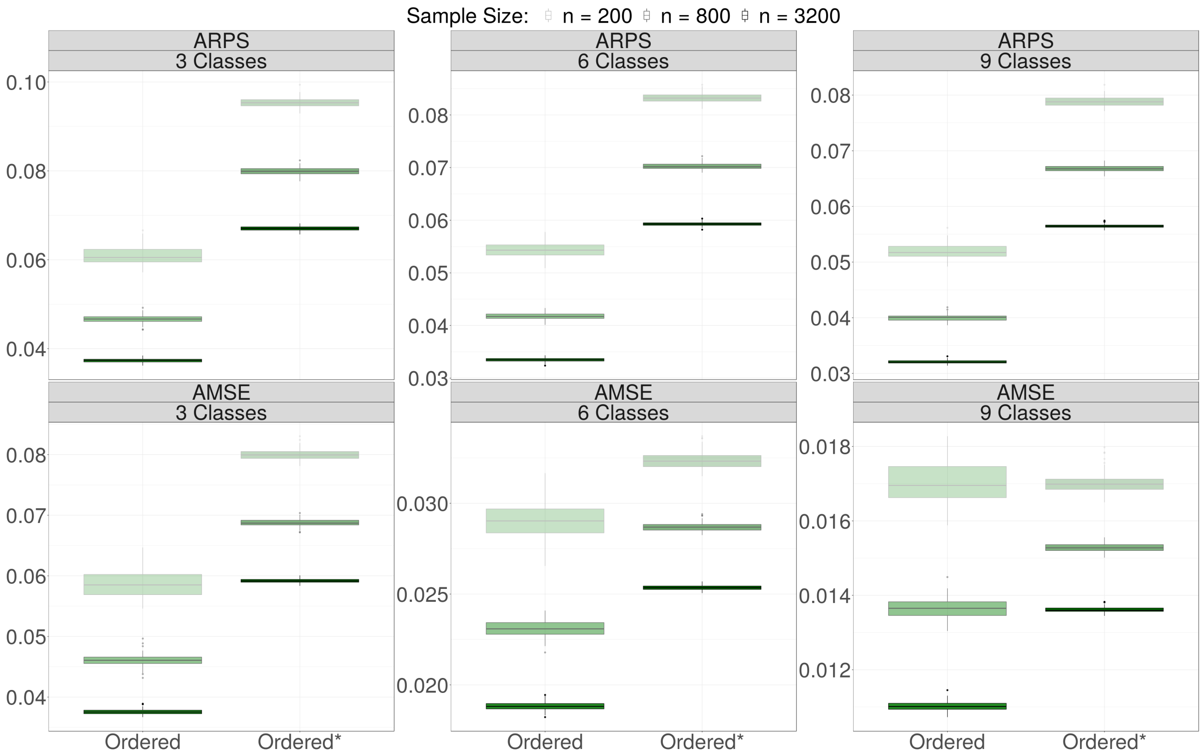
Note: Figure summarizes the prediction accuracy results based on 100 simulation replications. The upper panel contains the ARPS and the lower panel contains the AMSE. The boxplots show the median and the interquartile range of the respective measure. The transparent boxplots denote the results for the small sample size, while the bold boxplots denote the results for the big sample size. From left to right the results for 3, 6, and 9 outcome classes are displayed.

Lastly, the case of the complex DGP in the high-dimensional setting as in Figure 4 shows some interesting patterns. In general, all of the methods exhibit good predictive performance as the loss in the prediction accuracy due to the high-dimensional covariate space is small. Additionally, although dealing with the most complex design, no substantial loss in the prediction accuracy can be observed in comparison to the less complex designs. This fact demonstrates the ability of the random forest algorithm as such to effectively cope with highly nonlinear functional forms even in high dimensions. Further, it seems that the role of the sample size is of particular importance in this complex design. On the contrary to the previous designs, where the prediction accuracy increases almost by a constant amount for all estimators and thus does not change their relative ranking, here it does not hold anymore. First, some estimators seem to learn faster than others, i.e. to have a faster rate of convergence. As such in the small sample size the *Ordered Forest* has in some settings higher values of the ARPS as well as the AMSE than the conditional forest, however manages to outperform the conditional forest in the bigger training sample. This becomes most apparent in the case of 9 outcome classes. Here, the median of the ARPS is almost the same for the two methods based on the small training sample, but significantly lower for the *Ordered Forest* based on the larger training sample[13]. Second, for some estimators the prediction accuracy even worsens with the bigger training sample, which might hint on possible convergence issues. Overall, the *Ordered Forest* achieves the lowest ARPS as well as AMSE within this design, closely followed by the conditional and the multinomial forest.

[13]See Appendix B.1 for the detailed results of the statistical tests conducted.

In addition to the four main simulation designs discussed above, we also inspect all 72 different DGPs (see Appendix B.2) to analyze the performance and the sensitivity of the respective estimators to the particular features of the simulated DGPs. Let us first consider the low-dimensional case. Here, the first observation we make is the robustness of the ordered logit to small deviations from the simple DGP. As such, the prediction performance of the ordered logit does not worsen much if either noise variables, randomly spaced thresholds, or a limited multicollinearity is introduced within the DGP at a time. However, the prediction performance further worsens if these features are introduced combined. Nevertheless, the ordered logit predictions deteriorate substantially in all DGPs which include nonlinear effects, both separately as well as combined with other features. On the contrary, all forest-based estimators do well in these DGPs and clearly outperform the ordered logit. This points to the ability of random forests to naturally deal with nonlinear functional forms. Among the forest-based estimators the *Ordered Forest* outperforms the other methods particularly if the nonlinear effects are accompanied by multicollinearity of regressors as such as well as together with additional noise variables or randomly spaced thresholds (see DGPs 9, 12, 15 in Table 9, DGPs 25, 28, 31 in Table 10, and DGPs 41, 44, 47 in Table 11). Overall, the *Ordered Forest* and the conditional forest turn out to be more robust to different DGPs in terms of the prediction performance than the naive and the ordinal forest. The above results are homogeneous in respect to the number of outcome classes which corresponds also to the finding in the simulation study of Hornung (2019a). Thus, the number of outcome classes does not influence the relative performance of the considered estimators.

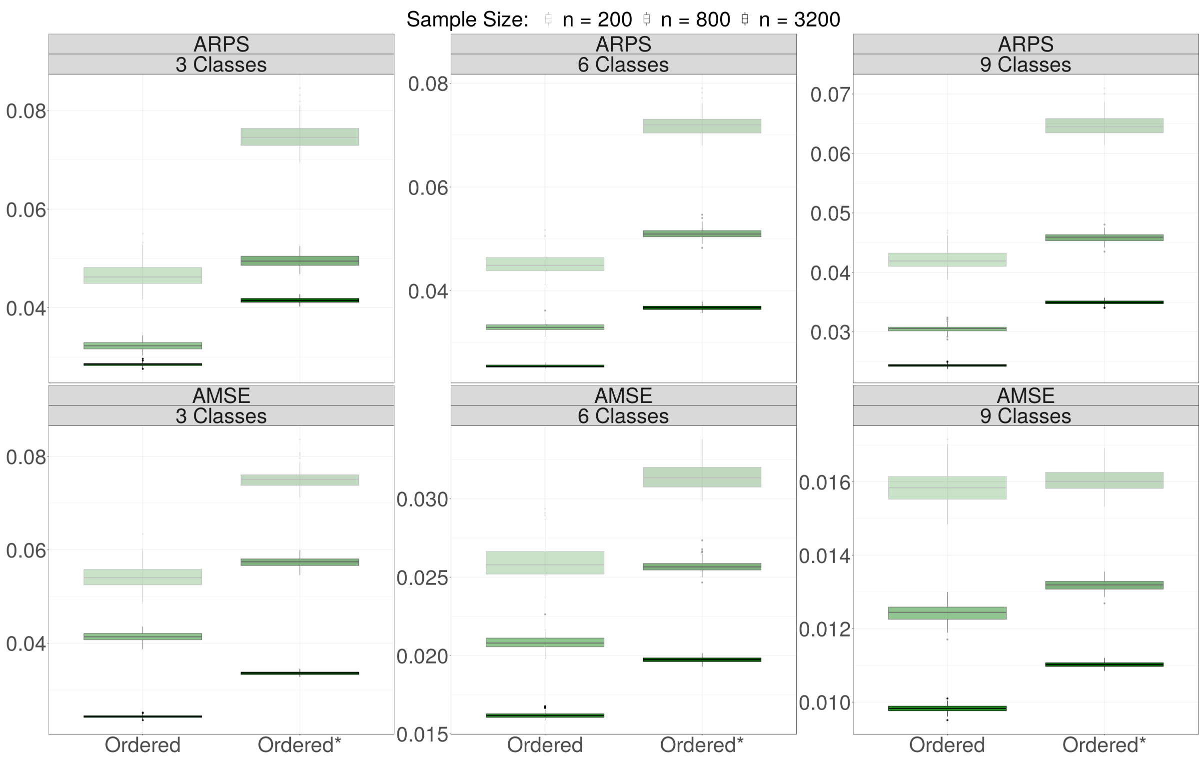
Figure 5: Ordered Forest Simulation Results: Simple DGP & Low Dimension



Note: Figure summarizes the prediction accuracy results based on 100 simulation replications. The upper panel contains the ARPS and the lower panel contains the AMSE. The boxplots show the median and the interquartile range of the respective measure. The transparent boxplots denote the results for the small sample size, the semi-transparent ones denote the medium sample size, while the bold boxplots denote the results for the big sample size. From left to right the results for 3, 6, and 9 outcome classes are displayed.

Next, considering the high-dimensional case, we again observe a good prediction accuracy of the forest-based methods when dealing with nonlinear effects as such. When these are combined with randomly spaced thresholds the naive and the ordinal forest achieve better accuracy than the other methods. However, if the randomly spaced thresholds are introduced without additional nonlinearities the conditional forest outperforms the other methods. Further, the *Ordered Forest* exhibits again good performance when dealing with multicollinearity and outperforms the other estimators in this respect as well as combined with the randomly spaced thresholds (see DGPs 51, 55 in Table 12, DGPs 59, 63 in Table 13, and DGPs 67, 71 in Table 14). For the case of multicollinearity combined with nonlinearities, both the *Ordered Forest* and the conditional forest achieve good prediction accuracy and one cannot discriminate between the two methods. Possibly, a bigger sample size would be needed in order to do so, as we have seen in the case of the complex DGP above. Lastly, similarly to the low-dimensional case, we do not observe any substantial differences in the relative prediction performance in respect to the number of outcome classes. In general, both in the low-dimensional as well as in the high-dimensional case, the honest version of the *Ordered Forest* achieves consistently lower prediction accuracy. It seems that in small samples the increase in variance due to honesty prevails the reduction in bias of the estimator.

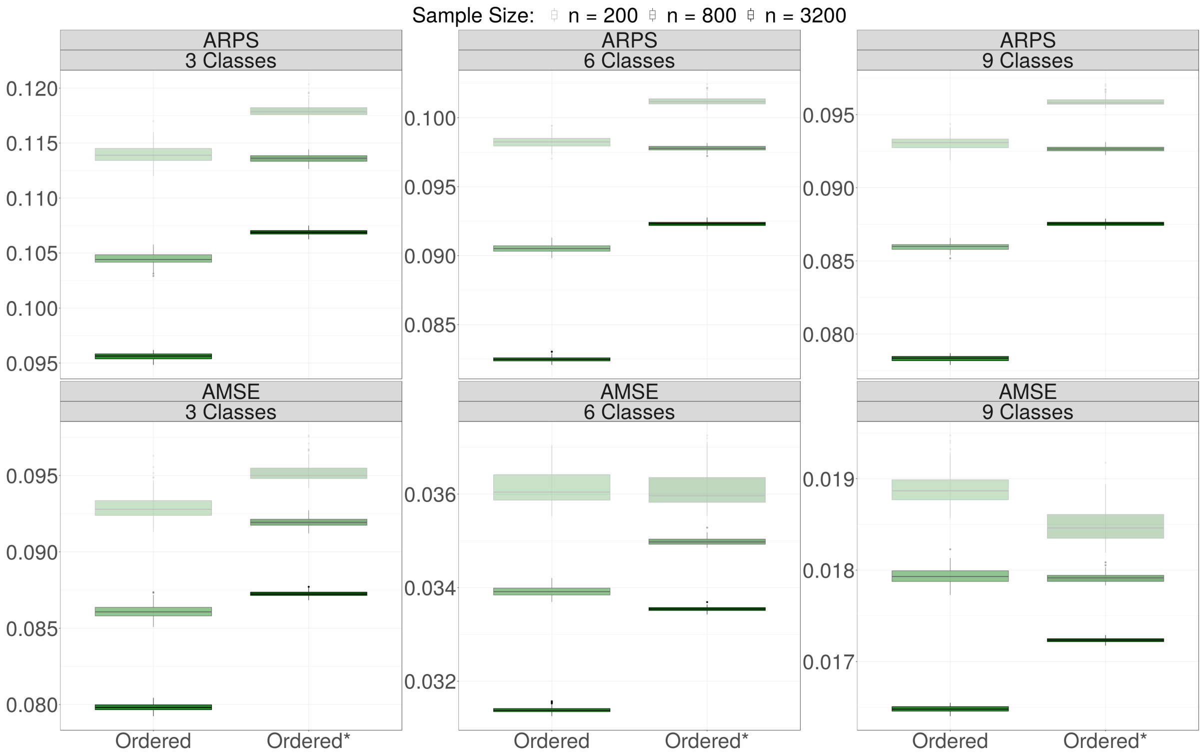
Figure 6: Ordered Forest Simulation Results: Complex DGP & Low Dimension



Note: Figure summarizes the prediction accuracy results based on 100 simulation replications. The upper panel contains the ARPS and the lower panel contains the AMSE. The boxplots show the median and the interquartile range of the respective measure. The transparent boxplots denote the results for the small sample size, the semi-transparent ones denote the medium sample size, while the bold boxplots denote the results for the big sample size. From left to right the results for 3, 6, and 9 outcome classes are displayed.

In order to further investigate the impact of the honesty feature in bigger samples as well as the convergence of the *Ordered Forest*, we quadruple the size of the training set once again and repeat the main simulation for the *Ordered Forest* and its honest version with $N = 3200$ observations. As in the main analysis Figures 5 and 6 show the simulation results for the simple and the complex DGP in the low-dimensional case, while Figures 7 and 8 present the results of the two DGPs in the high-dimensional setting. Within the tables the transparent boxplots denote the smallest sample size ($N = 200$), the semi-transparent boxplots show the medium sample size ($N = 800$), while the bold boxplots indicate the results for the biggest sample size ($N = 3200$). Similarly to the above, the upper panels of the figures show the ARPS and the lower panels show the AMSE, whereas the results for 3, 6, and 9 outcome classes are displayed fom left to right. More detailed results are included in Table 8 in Appendix B.1.

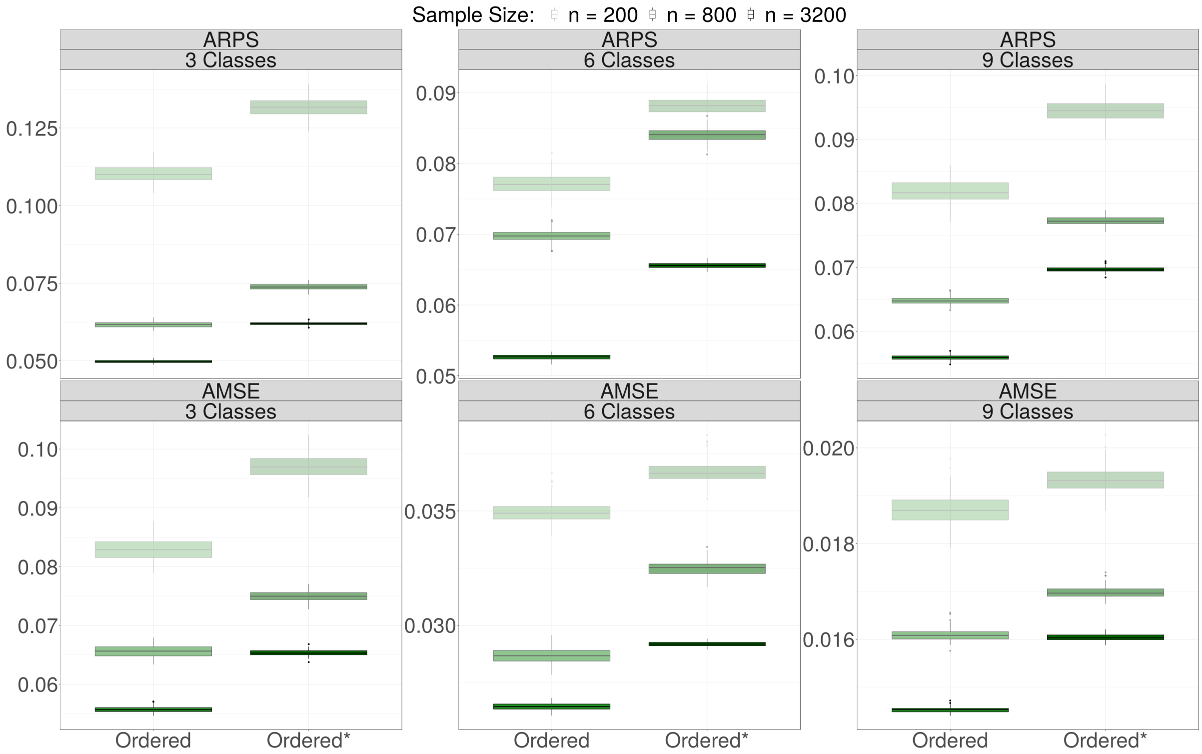
Figure 7: Ordered Forest Simulation Results: Simple DGP & High Dimension



Note: Figure summarizes the prediction accuracy results based on 100 simulation replications. The upper panel contains the ARPS and the lower panel contains the AMSE. The boxplots show the median and the interquartile range of the respective measure. The transparent boxplots denote the results for the small sample size, the semi-transparent ones denote the medium sample size, while the bold boxplots denote the results for the big sample size. From left to right the results for 3, 6, and 9 outcome classes are displayed.

The first observation we make is the convergence of both versions of the estimator in all considered scenarios (obviously based on three data points only). With growing sample size the prediction errors get lower and the precision increases. However, the rate of convergence seems to be slower than the parametric $\sqrt{N}$ rate. Clearly, this is the price to pay for the additional flexibility of the estimator. Interestingly, the convergence in MSE appears to be slightly slower for the high-dimensional case in comparison to the low-dimensional case, pointing to the theoretically limited scope of the curse of dimensionality of the forests. Generally, we also see somewhat higher prediction errors in the high-dimensional compared to the low-dimensional settings. With regards to the honesty feature, we observe the same pattern as in the smaller sample sizes, namely slightly lower prediction accuracy for the honest version of the *Ordered Forest* in all four simulation designs. The loss in the prediction accuracy due to honesty appears to be roughly constant across the considered scenarios, while in some cases, such as the complex DGP in low dimension (Figure 6), the difference in the prediction error gets smaller with bigger sample size. However, in other cases, such as the simple DGP in high dimension (Figure 7), this difference gets larger. Hence, even in the biggest sample the additional variance dominates the bias reduction. However, for a prediction exercise honesty is an optional choice, while if inference is of interest, honesty becomes binding.

Figure 8: Ordered Forest Simulation Results: Complex DGP & High Dimension



Note: Figure summarizes the prediction accuracy results based on 100 simulation replications. The upper panel contains the ARPS and the lower panel contains the AMSE. The boxplots show the median and the interquartile range of the respective measure. The transparent boxplots denote the results for the small sample size, the semi-transparent ones denote the medium sample size, while the bold boxplots denote the results for the big sample size. From left to right the results for 3, 6, and 9 outcome classes are displayed.

Finally, it should be noted that there is no 'one fits all' estimator and the choice of the particular method should be carefully done and guided by particular aspects of the estimation problem at hand. Nevertheless, the conducted simulation study provides an evidence for a good predictive performance of the new *Ordered Forest* estimator in the estimations of various ordered choice models.

6 Empirical Applications

In this section we explore the performance of the *Ordered Forest* estimator based on real datasets[14] previously used in Janitza et al. (2016) and Hornung (2019a). First, we compare our estimator in terms of the prediction accuracy to all the estimators used in the above Monte Carlo simulation. Second, we compare the *Ordered Forest* estimator also in terms of estimating marginal effects to the parametric ordered logit model. We do not consider the other forest based estimators here as these do not provide marginal effects estimation. Table 2 summarizes the datasets and the descriptive statistics are provided in Appendix C.1.

[14]The here proposed algorithm has been already applied and is in use for predicting match outcomes in football, see Goller et al. (2018) and SEW Soccer Analytics for details.

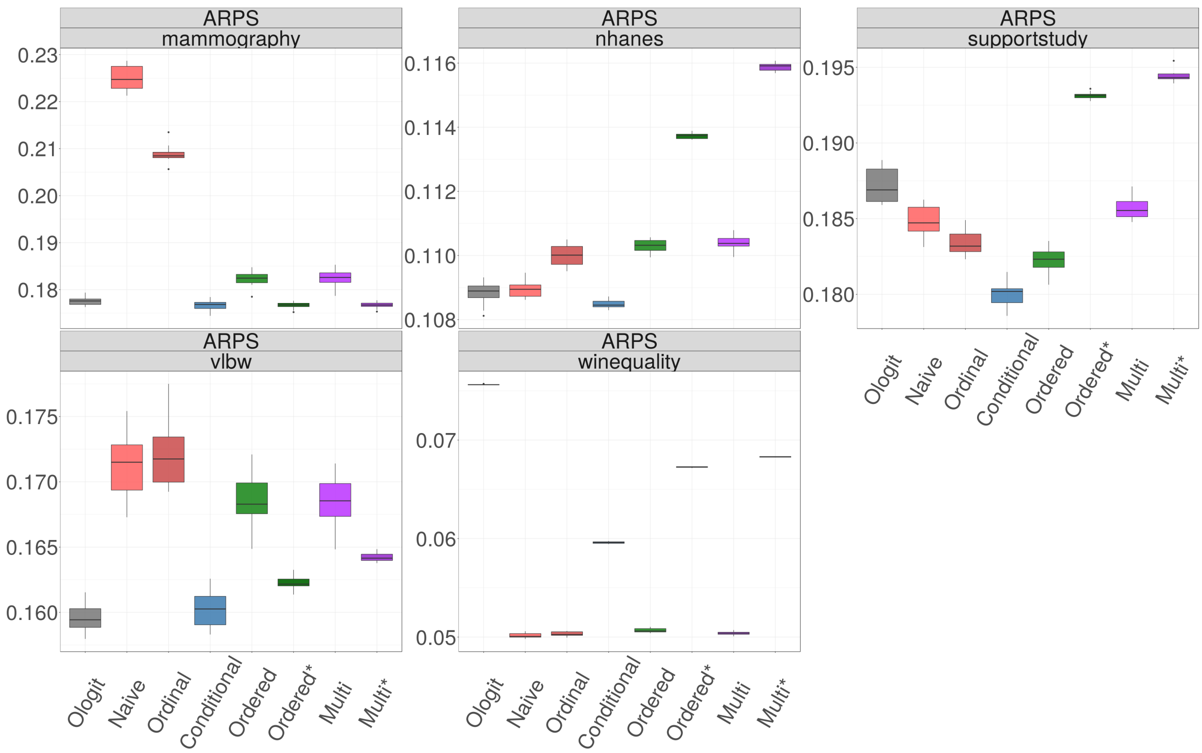
Table 2: Description of the Datasets

Datasets Summary						
Dataset	Sample Size	Outcome	Class Range			Covariates
Wine Quality	4893	Quality Score	1 (moderate)	-	6 (high)	11
Mammography	412	Visits History	1 (never)	-	3 (over year)	5
Nhanes	1914	Health Status	1 (excellent)	-	5 (poor)	26
Vlbw	218	Physical Condition	1 (threatening)	-	9 (optimal)	10
Support Study	798	Disability Degree	1 (none)	-	5 (fatal)	15

6.1 Prediction Accuracy

Similarly to Hornung (2019a) we evaluate the prediction accuracy based on a repeated cross-validation in order to reduce the dependency of the results on the particular training and test sample splits. As such we perform a 10-fold cross-validation on each dataset, i.e. we randomly split the dataset in 10 equally sized folds and use 9 folds for training the model and 1 fold for validation. This process is repeated such that each fold serves as a validation set exactly once. Lastly, we repeat this whole procedure 10 times and report average accuracy measures. The results of the cross-validation exercise for the ARPS as well as the AMSE are summarized in Figures 9 and 10, respectively. Similarly as for the simulation results Appendix C.2 contains more detailed statistics.

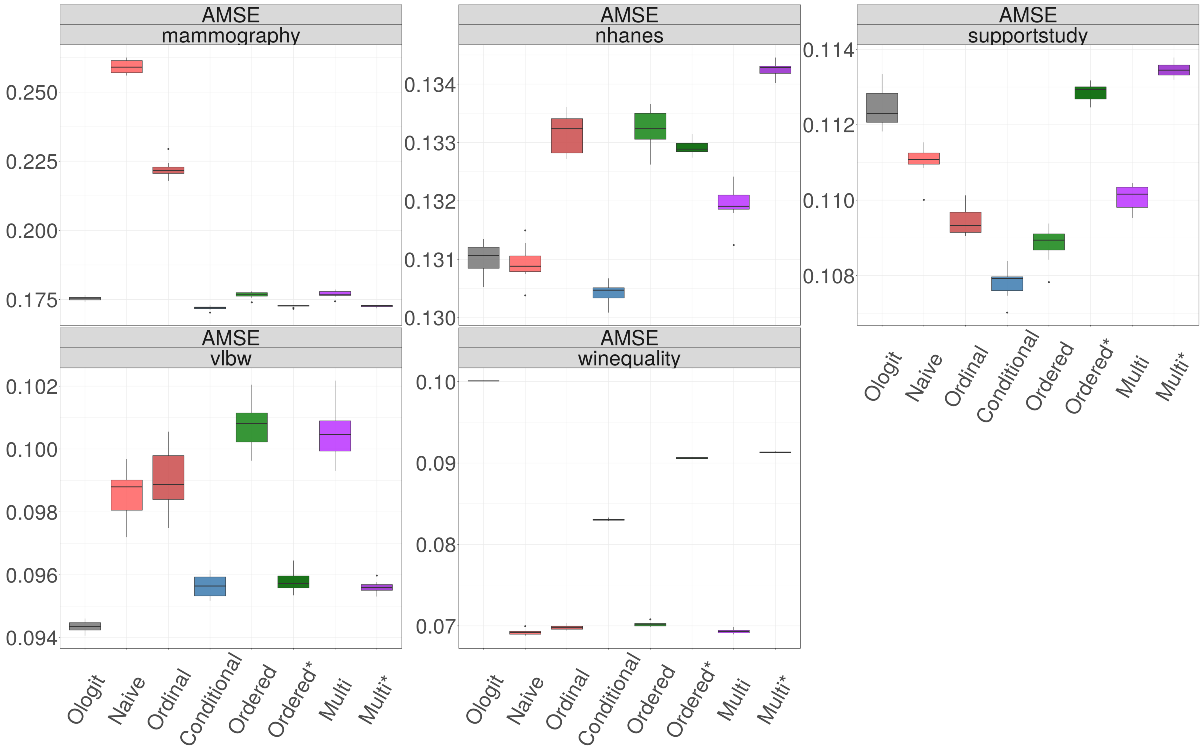
Figure 9: Cross-Validation: ARPS



Note: Figure summarizes the prediction accuracy results in terms of the ARPS based on 10 repetitions of 10-fold cross-validation for respective datasets. The boxplots show the median and the interquartile range of the respective measure.

The main difference in evaluating the prediction accuracy in comparison to the simulation study is the fact that we do not observe the outcome probabilities, but only the realized outcomes. This affects the computation of the accuracy measures as mentioned in Section 5.2 and it can be expected that the prediction errors are somewhat higher in comparison to the simulation data, which is also the case here. Overall, the results imply a substantial heterogeneity in the prediction accuracy across the considered datasets. On the one hand, the parametric ordered logit does well in small samples (*vlbw*) whereas the forest-based methods are somewhat lacking behind. This is not surprising as a lower precision in small samples is the price to pay for the additional flexibility. On the other hand, in the largest sample (*winequality*) the ordered logit is clearly the worst performing method and all forest-based methods perform substantially better. With respect to the *Ordered Forest* estimator we observe relatively high prediction accuracy for three datasets (*mammography*, *supportstudy*, *winequality*) and relatively low prediction accuracy for two datasets (*nhanes*, *vlbw*) in comparison to the competing methods. The good performance in the *winequality* and the *supportstudy* dataset is expected due to the large samples available. In case of the *mammography* dataset, even when smaller in sample size, the *Ordered Forest* maintains the good prediction performance, with its honest version doing even better. The worse performance for the *vlbw* dataset might be due to the small sample size. However, the honest version of the *Ordered Forest* performs rather well. The relatively poor performance in the case of the *nhanes* dataset comes rather at surprise as the sample size is large. Nevertheless, here the differences among all estimators are very small in magnitude, in fact the smallest among the considered datasets.

Figure 10: Cross-Validation: AMSE



Note: Figure summarizes the prediction accuracy results in terms of the AMSE based on 10 repetitions of 10-fold cross-validation for respective datasets. The boxplots show the median and the interquartile range of the respective measure.

6.2 Marginal Effects

In order to analyze the relationship between the covariates and the predicted choice probabilities we estimate the marginal effects for the *Ordered Forest* and compare these to the marginal effects estimated by the ordered logit. We estimate both common measures for marginal effects, i.e. the mean marginal effects as well as the marginal effects at covariate means. The main difference between the ordered logit and the *Ordered Forest* is the fact that the *Ordered Forest* does not use any parametric link function in the estimation of the marginal effects and as such does not impose any functional form on these estimates. As a result, the *Ordered Forest* does neither fix the sign of the marginal effects estimates nor revert it exactly once within the class range as is the case for the ordered logit (the so-called 'single crossing' feature, see i.e. Boes and Winkelmann (2006) or Greene and Hensher (2010)) but rather estimates these in a data-driven manner. Nevertheless, the *Ordered Forest*, same as the ordered logit, still ensures that the marginal effects across the class range sum up to zero (being more likely to be in the *highest* class must imply being less likely to be in the *lowest* class). As such the *Ordered Forest* not only enables a more flexible estimation of the choice probabilities but also of the marginal effects. Table 3 compares the mean marginal effects estimated by the *Ordered Forest* and the ordered logit for the *winequality* dataset, whereas Appendix C.3 contains the mean marginal effects and the marginal effects at mean for all the other datasets. The *winequality* dataset is particularly suitable for such comparison as it contains only continuous covariates which are natural for evaluation of the marginal effects and has also a sufficiently large sample size with well represented outcome classes. Table 3 contains the estimated effects for each outcome class of each covariate together with the associated standard errors, t-values, p-values as well as conventional significance levels for both the *Ordered Forest* as well as the ordered logit.

Table 3: Mean Marginal Effects: Wine Quality Dataset

Dataset		**Ordered Forest**					**Ordered Logit**				
Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
alcohol	1	0.0000	0.0001	0.4559	0.6484		-0.0017	0.0005	-3.5943	0.0003	***
	2	-0.0023	0.0021	-1.0701	0.2846		-0.0125	0.0023	-5.4096	0.0000	***
	3	-0.0582	0.0055	-10.5215	0.0000	***	-0.0612	0.0105	-5.8009	0.0000	***
	4	0.0189	0.0063	2.9994	0.0027	***	0.0163	0.0031	5.2566	0.0000	***
	5	0.0341	0.0059	5.7334	0.0000	***	0.0450	0.0077	5.8232	0.0000	***
	6	0.0075	0.0076	0.9865	0.3239		0.0141	0.0026	5.4111	0.0000	***
chlorides	1	0.0011	0.0058	0.1924	0.8474		0.0023	0.0055	0.4147	0.6784	
	2	0.2323	0.0653	3.5565	0.0004	***	0.0166	0.0398	0.4167	0.6769	
	3	1.1824	0.3295	3.5882	0.0003	***	0.0811	0.1945	0.4169	0.6768	
	4	0.0187	0.5868	0.0319	0.9745		-0.0216	0.0518	-0.4175	0.6763	
	5	-1.3851	0.4807	-2.8816	0.0040	***	-0.0596	0.1431	-0.4166	0.6770	
	6	-0.0495	0.5009	-0.0988	0.9213		-0.0187	0.0449	-0.4166	0.6769	
citric	1	0.0007	0.0003	2.7474	0.0060	***	-0.0004	0.0010	-0.4410	0.6592	
acid	2	-0.0210	0.0077	-2.7142	0.0066	***	-0.0031	0.0070	-0.4429	0.6578	
	3	-0.0766	0.0376	-2.0340	0.0420	**	-0.0151	0.0340	-0.4429	0.6578	
	4	0.0541	0.0393	1.3771	0.1685		0.0040	0.0091	0.4420	0.6585	
	5	0.0444	0.0307	1.4465	0.1480		0.0111	0.0250	0.4432	0.6576	
	6	-0.0017	0.0186	-0.0936	0.9255		0.0035	0.0079	0.4428	0.6579	
density	1	0.0182	0.0363	0.5027	0.6152		1.7954	0.4504	3.9861	0.0001	***
	2	0.8623	0.3202	2.6927	0.0071	***	13.0220	1.9652	6.6264	0.0000	***
	3	9.7099	1.8904	5.1363	0.0000	***	63.5943	8.4039	7.5672	0.0000	***
	4	-1.4862	2.7180	-0.5468	0.5845		-16.9605	2.5147	-6.7447	0.0000	***
	5	-7.8099	2.5273	-3.0902	0.0020	***	-46.7731	6.2827	-7.4447	0.0000	***
	6	-1.2944	2.4755	-0.5229	0.6010		-14.6781	2.1808	-6.7305	0.0000	***

Continued on next page

Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
fixed	1	0.0003	0.0002	1.6239	0.1044		-0.0009	0.0003	-2.8953	0.0038	***
acidity	2	0.0044	0.0014	3.0448	0.0023	***	-0.0063	0.0018	-3.5518	0.0004	***
	3	0.0015	0.0040	0.3669	0.7137		-0.0306	0.0083	-3.6996	0.0002	***
	4	-0.0081	0.0043	-1.8787	0.0603	*	0.0082	0.0022	3.6578	0.0003	***
	5	0.0027	0.0035	0.7782	0.4364		0.0225	0.0062	3.6587	0.0003	***
	6	-0.0007	0.0034	-0.2204	0.8256		0.0071	0.0020	3.5701	0.0004	***
free	1	-0.0001	0.0001	-0.8283	0.4075		-0.0000	0.0000	-3.4689	0.0005	***
sulfur	2	-0.0026	0.0005	-4.9064	0.0000	***	-0.0003	0.0001	-4.9288	0.0000	***
dioxide	3	-0.0004	0.0005	-0.8601	0.3897		-0.0017	0.0003	-5.2877	0.0000	***
	4	0.0019	0.0004	4.2409	0.0000	***	0.0004	0.0001	4.9370	0.0000	***
	5	0.0010	0.0003	3.5888	0.0003	***	0.0012	0.0002	5.2571	0.0000	***
	6	0.0002	0.0002	1.0017	0.3165		0.0004	0.0001	4.9788	0.0000	***
pH	1	0.0006	0.0012	0.4736	0.6358		-0.0079	0.0021	-3.8654	0.0001	***
	2	0.0042	0.0056	0.7548	0.4504		-0.0575	0.0093	-6.1905	0.0000	***
	3	-0.0403	0.0269	-1.4993	0.1338		-0.2810	0.0402	-6.9903	0.0000	***
	4	-0.0848	0.0380	-2.2337	0.0255	**	0.0749	0.0116	6.4570	0.0000	***
	5	0.1156	0.0322	3.5904	0.0003	***	0.2067	0.0303	6.8269	0.0000	***
	6	0.0047	0.0261	0.1785	0.8584		0.0649	0.0103	6.2841	0.0000	***
residual	1	0.0000	0.0000	0.0873	0.9304		-0.0009	0.0002	-4.2498	0.0000	***
sugar	2	-0.0028	0.0015	-1.7960	0.0725	*	-0.0065	0.0008	-8.2530	0.0000	***
	3	-0.0111	0.0040	-2.7989	0.0051	***	-0.0319	0.0031	-10.2244	0.0000	***
	4	0.0046	0.0043	1.0783	0.2809		0.0085	0.0010	8.4523	0.0000	***
	5	0.0080	0.0033	2.4287	0.0152	**	0.0234	0.0024	9.9347	0.0000	***
	6	0.0012	0.0029	0.4226	0.6726		0.0074	0.0009	8.3949	0.0000	***
sulphates	1	-0.0003	0.0003	-0.9104	0.3626		-0.0072	0.0019	-3.8794	0.0001	***
	2	-0.0012	0.0069	-0.1688	0.8660		-0.0522	0.0083	-6.2919	0.0000	***
	3	-0.0570	0.0293	-1.9436	0.0519	*	-0.2548	0.0362	-7.0366	0.0000	***
	4	0.0066	0.0293	0.2263	0.8209		0.0680	0.0109	6.2475	0.0000	***
	5	0.0807	0.0357	2.2631	0.0236	**	0.1874	0.0269	6.9790	0.0000	***
	6	-0.0288	0.0471	-0.6123	0.5403		0.0588	0.0092	6.3993	0.0000	***
total	1	-0.0000	0.0000	-0.5841	0.5592		0.0000	0.0000	0.8906	0.3732	
sulfur	2	-0.0000	0.0000	-1.0456	0.2957		0.0000	0.0000	0.9069	0.3645	
dioxide	3	0.0002	0.0001	2.0326	0.0421	**	0.0001	0.0001	0.9101	0.3628	
	4	-0.0001	0.0001	-0.8627	0.3883		-0.0000	0.0000	-0.9086	0.3636	
	5	-0.0001	0.0001	-0.9828	0.3257		-0.0001	0.0001	-0.9095	0.3631	
	6	0.0000	0.0001	0.4234	0.6720		-0.0000	0.0000	-0.9078	0.3640	
volatile	1	0.0010	0.0008	1.2894	0.1973		0.0198	0.0044	4.4792	0.0000	***
acidity	2	0.0532	0.0129	4.1351	0.0000	***	0.1434	0.0127	11.3030	0.0000	***
	3	0.6921	0.0805	8.5974	0.0000	***	0.7002	0.0420	16.6696	0.0000	***
	4	-0.3955	0.0889	-4.4495	0.0000	***	-0.1867	0.0159	-11.7703	0.0000	***
	5	-0.3015	0.0826	-3.6505	0.0003	***	-0.5150	0.0335	-15.3757	0.0000	***
	6	-0.0493	0.0752	-0.6557	0.5120		-0.1616	0.0147	-11.0298	0.0000	***

Significance levels correspond to: $***. < 0.01$, $** . < 0.05$, $*. < 0.1$.

Notes: Table shows the comparison of the mean marginal effects between the *Ordered Forest* and the ordered logit. The effects are estimated for all classes, together with the corresponding standard errors, t-values and p-values. The standard errors for the *Ordered Forest* are estimated using the weight-based inference and the standard errors for the ordered logit are obtained via the delta method.

In general, we see similar patterns in terms of the effect sizes and effect direction for both the *Ordered Forest* and the ordered logit. However, we do observe more variability in terms of the effect direction in case of the *Ordered Forest* as we would also expect given the flexibility argument discussed above. In terms of uncertainty of the effects the weight-based inference seems to be more conservative than the delta method used in the ordered logit. Nevertheless, we also detect very precise effects which are not discovered by the ordered logit.

In particular, inspecting variables such as *alcohol*, *volatile acidity*, *free sulfir dioxide* or *residual sugar*, both methods estimate comparable effects both in terms of the effect size as well as effect direction with the difference of the *Ordered Forest* achieving a lower precision for the effects of the outer classes. This might be in general due to lack of data for these outer classes (for class 1 only 20 data points are available). Contrarily, in case of the variable *density*, even though the effect direction and the precision are in line with the above, the ordered logit seems to overshoot the effect sizes quite substantially, most probably due to the parametric extrapolation in comparison to the *Ordered Forest* which does not allow any extrapolation by definition as the estimation of the effects respects the support of the covariates and thus the *Ordered Forest* estimates the effects of an order of magnitude lower. Further, for variables such as *chlorides* and *citric acid* the *Ordered Forest* identifies relevant effects which are not discovered by the ordered logit. Particularly for *chlorides*, also the effect sizes are a bit larger and the turnaround of the effect direction is shifted to a higher class. As such the effect stays positive for the first four classes whereas the ordered logit forces the switch already after the third class. In addition, we observe also variables such as *total sulfur dioxide* for which both of the methods find no evidence for relevant effects, at most evidence for a zero effect. This might be due to collinearity issues with variable *free sulfur dioxide* which takes up the whole effect of *sulfur dioxide*. Lastly, for some of the variables, namely *fixed acidity* and *pH* the estimated effects differ substantially between the *Ordered Forest* and the ordered logit. Overall, however, the main advantage of the estimation of the marginal effects by the *Ordered Forest* stems from a more flexible, data-driven approximation of possible nonlinearities in the functional form.

7 Conclusion

In this paper, we develop and apply a new machine learning estimator of the econometric ordered choice models based on the random forest algorithm. The *Ordered Forest* estimator is a flexible alternative to parametric ordered choice models such as the ordered logit or ordered probit which does not rely on any distributional assumptions and provides essentially the same output as the parametric models, including the estimation of the marginal effects as well as the associated inference. The proposed estimator utilizes the flexibility of random forests and can thus naturally deal with nonlinearities in the data and with a large-dimensional covariate space, while taking the ordering information of the categorical outcome variable into account. Hence, the estimator flexibly estimates the conditional choice probabilities without restrictive assumptions about the distribution of the error term, or other assumptions such as the single index and constant threshold assumptions as is the case for the parametric ordered choice models (see Boes and Winkelmann (2006) for a discussion of these assumptions). Further, the estimator allows also the estimation of the marginal effects, i.e. how the estimated conditional choice probabilities vary with changes in covariates. The weighted representation of these effects enables the weight-based inference as suggested by Lechner (2019). The fact that the estimator comprises of linear combinations of random forest predictions ensures the theoretical guarantees of Wager and Athey (2018). Additionally, a free software implementation of the *Ordered Forest* estimator is available in GAUSS and also an R-package will be submitted to the CRAN repository to enable the usage of the method by applied researchers.

The performance of the *Ordered Forest* estimator is studied and compared to other competing estimators in an extensive Monte Carlo simulation as well as using real datasets. The simulation results suggest good performance of the estimator in finite samples, including also high-dimensional settings. The advantages of the machine learning estimation compared to a parametric method become apparent when dealing with multicollinearity and highly nonlinear functional forms. In such cases all of the considered forest-based estimators perform better than the ordered logit in terms of the prediction accuracy. Among the forest-based estimators the *Ordered Forest* proposed in this paper performs well throughout all simulated DGPs and outperforms the competing methods in the most complex simulation designs. The empirical evidence using real datasets supports the findings from the Monte Carlo simulation. Additionally, the estimation of the marginal effects as well as the inference procedure seems to work well in the empirical examples.

Despite the attractive properties of the *Ordered Forest* estimator, many interesting questions are left open. Particularly, a further extension of the Monte Carlo simulation to study the sensitivity of the *Ordered Forest* in respect to tuning parameters of the underlying random forest as well as in respect to different simulation designs would be of interest. Similarly, the performance of the estimator with and without honesty for larger sample sizes should be further investigated. Also, the optimal choice of the size of the window for evaluating the marginal effects would be worth to explore. Additionally, a theoretical framework for the used weight-based inference would be valuable, in particular the specific conditions needed for the weights to yield a consistent estimator of standard errors. Lastly, it would be of great interest to see more real data applications of the *Ordered Forest* estimator, especially for large samples.

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A Other Machine Learning Estimators

A.1 Multinomial Forest

Considering the *Ordered Forest* estimator a possible modification for models with categorical outcome variable *without* an inherent ordering appears to be straightforward. Instead of estimating cumulative probabilities and afterwards isolating the respective class probabilities, we can estimate the class probabilities $P_{m,i} = P[Y_i = m \mid X_i = x]$ directly. As such the binary outcomes are now constructed to indicate the particular outcome classes separately. Then the random forest predictions for each class yield the conditional choice probabilities which need to be afterwards normalized to sum up to 1. Formally, consider (un)ordered categorical outcome variable $Y_i \in \{1, ..., M\}$ with classes m and sample size $N(i = 1, ..., N)$. Then, the estimation procedure can be described as follows:

1. Create M binary indicator variables such as

$$Y_{m,i} = \mathbf{1}(Y_i = m) \qquad \text{for} \qquad m = 1, ..., M. \tag{A.1}$$

2. Estimate regression random forest for each of the M indicators.
3. Obtain predictions $\hat{Y}_{m,i} = \hat{P}[Y_{m,i} = 1 \mid X_i = x] = \sum_{i=1}^{N} \hat{w}_{m,i}(x) Y_{m,i}$.
4. Compute probabilities for each class

$$\hat{P}_{m,i} = \hat{Y}_{m,i} \qquad \text{for} \qquad m = 1, ..., M \tag{A.2}$$

$$\hat{P}_{m,i} = \frac{\hat{P}_{m,i}}{\sum_{m=1}^{M} \hat{P}_{m,i}} \qquad \text{for} \qquad m = 1, ..., M, \tag{A.3}$$

where the equation (A.2) defines the probabilities of all M classes and subsequent equation (A.3) ensures that the probabilities sum up to 1 as this might not be the case otherwise. Similarly to the *Ordered Forest* estimator, also the multinomial forest is a linear combination of the respective forest predictions and as such also inherits the theoretical properties stemming from random forest estimation as described in Section 3.

A.2 Conditional Forest

The conditional forest as discussed in Section 2 is grown with the so-called conditional inference trees. The main idea is to provide an unbiased way of recursive splitting of the trees using a test statistic based on permutation tests (Strasser and Weber, 1999). To describe the estimation procedure, consider an ordered categorical outcome $Y_i \in (1, ..., M)$ with ordered classes m and an sample size $N(i = 1, ..., N)$. Further, define binary case weights $w_i \in \{0, 1\}$ which determine if the observation is part of the current leaf. Then, the algorithm developed by Hothorn et al. (2006b) can be described as follows:

1. Test the global null hypothesis of independence between any of the P covariates and the outcome, for the particular case weights, given a bootstrap sample Z_b. Afterwards, select the p-th covariate $X_{i,p}$ with the strongest association with the outcome Y_i, or stop if the null hypothesis cannot be

rejected. The association is measured by a linear statistic T given as:

$$T_p(Z_b, w) = \sum_{i=1}^{N} w_i g_p(X_{i,p}) h(Y_i), \tag{A.4}$$

where $g_p(\cdot)$ and $h(\cdot)$ are specific transformation functions.

2. Split the covariate sample space $\mathcal{X}_p$ into two disjoint sets $\mathcal{I}$ and $\mathcal{J}$ with adapted case weights $w_i \mathbf{1}(X_{i,p} \in \mathcal{I})$ and $w_i \mathbf{1}(X_{i,p} \in \mathcal{J})$ determining the observations falling into the subset $\mathcal{I}$ and $\mathcal{J}$, respectively. Then, the split is chosen by evaluating a two-sample statistic as a special case of A.4:

$$T_p^{\mathcal{I}}(Z_b, w) = \sum_{i=1}^{N} w_i \mathbf{1}(X_{i,p} \in \mathcal{I}) h(Y_i) \tag{A.5}$$

for all possible subsets $\mathcal{I}$ of the covariate sample space $\mathcal{X}_p$.

3. Repeat steps 1 and 2 recursively with modified case weights.

Hence, the above algorithm distinguishes between variable selection (step 1) and splitting rule (step 2), while both relying on the variations of the test statistic $T_p(Z_b, w)$. In practice, however, the distribution of this statistic under the null hypothesis is unknown and depends on the joint distribution of Y_i and $X_{i,p}$. For this reason, the permutation tests are applied to abstract from the dependency by fixing the covariates and conditioning on all possible permutations of the outcomes. Then, the conditional mean and covariance of the test statistic can be derived and the asymptotic distribution can be approximated by Monte Carlo procedures, while Strasser and Weber (1999) proved its normality. Finally, variables and splits are chosen according to the lowest p-value of the test statistic $T_p(Z_b, w)$ and $T_p^{\mathcal{I}}(Z_b, w)$, respectively.

Besides the permutation tests, the choice of the tranformation functions $g_p(\cdot)$ and $h(\cdot)$ is important and depends on the type of the variables. For continuous outcome and covariates, identity transformation is suggested. For the case of an ordinal regression which is of interest here, the transformation function is given through the score function $s(m)$. If the underlying latent Y_i^* is unobserved, it is suggested that $s(m) = m$ and thus $h(Y_i) = Y_i$. Hence, in the tree building the ordered outcome is treated as a continuous one (Janitza et al., 2016). Then, however, the leaf predictions are the choice probabilities computed as proportions of the outcome classes falling within the leaf, instead of fitting a within leaf constant. The final conditional forest predictions for the choice probabilities are the averaged conditional tree probability predictions. Such obtained choice probabilities are analyzed in the Monte Carlo study in Section 5.

A.3 Ordinal Forest

In the following, the algorithm for the ordinal forest as developed by Hornung (2019a) is described. To begin with, consider an ordered categorical outcome $Y_i \in (1, ..., M)$ with ordered classes m and sample size $N (i = 1, ..., N)$. Then, for a set of optimization forests $b = 1, ..., B_{sets}$:

1. Draw $M - 1$ uniformly distributed variables $D_{b,m} \sim U(0, 1)$ and sort them according to their values. Further, set $D_{b,1} = 0$ and $D_{b,M+1} = 1$.

2. Define a score set $S_{b,m} = \{S_{b,1}, ..., S_{b,M}\}$ with scores constructed as $S_{b,m} = \Phi^{-1}\big(\frac{D_{b,m} + D_{b,m+1}}{2}\big)$ for $m = 1, ..., M$, where $\Phi(\cdot)$ is the cdf of the standard normal.

3. Create a new continuous outcome $Z_{b,i} = (Z_{b,1}, ..., Z_{b,N})$ by replacing each class value m of the original ordered categorical Y_i by the m-th value of the score set $S_{b,m}$ for all $m = 1, ..., M$.

4. Use $Z_{b,i}$ as dependent variable and estimate a regression forest $RF_{S_{b,m}}$ with B_{prior} trees.

5. Obtain the out-of-bag (OOB) predictions for the continuous $Z_{b,i}$ and transform them into predictions for Y_i as follows: $\hat{Y}_{b,i} = m$ if $\hat{Z}_{b,i} \in \left]\Phi^{-1}(D_{b,m}, \Phi^{-1}(D_{b,m+1})\right]$ for all $i = 1, ..., N$.

6. Compute a performance measure for the given forest $\hat{RF}_{S_{b,m}}$ based on some performance function of type $f(Y_i, \hat{Y}_{b,i})$.

After estimating B_{sets} of optimization forests, take S_{best} of these which achieved the best performance according to the performance function. Then, construct the final set of uniformly distributed variables $D_1, ..., D_{M+1}$ as an average of those from S_{best} for $m = 1, ..., M+1$. Finally, form the optimized score set $S_m = \{S_1, ..., S_M\}$ with scores constructed as $S_m = \Phi^{-1}\left(\frac{D_m + D_{m+1}}{2}\right)$ for $m = 1, ..., M$. The continuous outcome $Z_i = (Z_1, ..., Z_N)$ is then similarly as in the optimization procedure constructed by replacing each m value of the original outcome Y_i by the m-th value of the optimized score set S_m for all $m = 1, ..., M$. Finally, estimate the regression forest RF_{final} using Z_i as the dependent variable. On one hand, the class prediction of such an ordinal forest is one of the M ordered classes which has been predicted the most by the respective trees of the forest. On the other hand, the probability prediction is obtained as a relative frequency of trees predicting the particular class. Such predicted choice probabilities are analyzed in the conducted Monte Carlo study in Section 5. Further, the so-called naive forest corresponds to the ordinal forest with omitting the above described optimization procedure.

B Simulation Study

B.1 Main Simulation Results

In the following Tables 4, 5, 6, 7 and 8 are summarized the simulation results presented in Section 5.3 in the main text. Each table specifies the particular simulation design as follows: the column *Class* indicates the number of outcome classes, *Dim.* specifies the dimension, *DGP* characterizes the data generating process as defined in the main text and *Statistic* contains summary statistics of the simulation results. In particular, the mean of the respective accuracy measure and its standard deviation. Furthermore, rows *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the non-parametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods. The alternative hypothesis is that the mean of the *Ordered Forest* is less than the mean of the other method to test if the *Ordered Forest* achieves significantly lower prediction error than the other considered methods.

B.1.1 ARPS: Sample Size = 200

Table 4: Simulation results: Accuracy Measure = ARPS & Sample Size = 200

Simulation Design				Comparison of Methods							
Class	Dim.	DGP	Statistic	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
3	Low	Simple	mean	0.0097	0.0765	0.0755	0.0625	0.0609	0.0954	0.0619	0.0954
			st.dev.	0.0042	0.0056	0.0055	0.0018	0.0020	0.0011	0.0019	0.0012
			t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0002	0.0000
			wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
3	Low	Complex	mean	0.1156	0.1044	0.1028	0.0593	0.0466	0.0748	0.0491	0.0760
			st.dev.	0.0047	0.0039	0.0038	0.0023	0.0026	0.0028	0.0024	0.0027
			t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
3	High	Simple	mean		0.1135	0.1139	0.1112	0.1140	0.1180	0.1139	0.1179
			st.dev.		0.0009	0.0010	0.0009	0.0008	0.0006	0.0008	0.0006
			t-test		1.0000	0.7676	1.0000		0.0000	0.7268	0.0000
			wilcox-test		0.9999	0.8438	1.0000		0.0000	0.7191	0.0000
3	High	Complex	mean		0.1476	0.1474	0.1156	0.1102	0.1316	0.1110	0.1317
			st.dev.		0.0013	0.0010	0.0041	0.0029	0.0031	0.0029	0.0031
			t-test		0.0000	0.0000	0.0000		0.0000	0.0287	0.0000
			wilcox-test		0.0000	0.0000	0.0000		0.0000	0.0272	0.0000
6	Low	Simple	mean	0.0062	0.0687	0.0665	0.0554	0.0544	0.0833	0.0577	0.0872
			st.dev.	0.0020	0.0048	0.0050	0.0012	0.0014	0.0009	0.0016	0.0010
			t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
6	Low	Complex	mean	0.1122	0.1093	0.1058	0.0574	0.0452	0.0719	0.0536	0.0842
			st.dev.	0.0040	0.0045	0.0044	0.0017	0.0020	0.0022	0.0021	0.0024
			t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
6	High	Simple	mean		0.0974	0.0972	0.0951	0.0983	0.1012	0.0998	0.1016
			st.dev.		0.0006	0.0006	0.0006	0.0005	0.0004	0.0005	0.0004
			t-test		1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
			wilcox-test		1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
6	High	Complex	mean		0.0927	0.0927	0.0766	0.0772	0.0882	0.0898	0.0952
			st.dev.		0.0006	0.0005	0.0020	0.0016	0.0014	0.0018	0.0006
			t-test		0.0000	0.0000	0.9878		0.0000	0.0000	0.0000
			wilcox-test		0.0000	0.0000	0.9887		0.0000	0.0000	0.0000
9	Low	Simple	mean	0.0054	0.0653	0.0629	0.0528	0.0519	0.0789	0.0569	0.0850
			st.dev.	0.0018	0.0042	0.0042	0.0012	0.0014	0.0009	0.0017	0.0009
			t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
9	Low	Complex	mean	0.0973	0.0912	0.0887	0.0515	0.0421	0.0647	0.0537	0.0845
			st.dev.	0.0031	0.0033	0.0032	0.0015	0.0016	0.0019	0.0018	0.0017
			t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
9	High	Simple	mean		0.0921	0.0918	0.0900	0.0931	0.0959	0.0955	0.0964
			st.dev.		0.0006	0.0006	0.0006	0.0005	0.0003	0.0004	0.0003
			t-test		1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
			wilcox-test		1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
9	High	Complex	mean		0.1007	0.1004	0.0817	0.0819	0.0945	0.0997	0.1036
			st.dev.		0.0007	0.0007	0.0020	0.0017	0.0015	0.0019	0.0006
			t-test		0.0000	0.0000	0.7875		0.0000	0.0000	0.0000
			wilcox-test		0.0000	0.0000	0.8473		0.0000	0.0000	0.0000

Notes: Table reports the average measures of the RPS based on 100 simulation replications for the sample size of 200 observations. The first column denotes the number of outcome classes. Columns 2 and 3 specify the dimension and the DGP, respectively. The fourth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.1.2 AMSE: Sample Size = 200

Table 5: Simulation results: Accuracy Measure = AMSE & Sample Size = 200

Simulation Design				Comparison of Methods							
Class	Dim.	DGP	Statistic	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
3	Low	Simple	mean	0.0103	0.0669	0.0682	0.0565	0.0587	0.0800	0.0587	0.0800
			st.dev.	0.0044	0.0041	0.0044	0.0015	0.0022	0.0009	0.0016	0.0010
			t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.3900	0.0000
			wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.2614	0.0000
3	Low	Complex	mean	0.1081	0.0985	0.0965	0.0637	0.0543	0.0752	0.0572	0.0768
			st.dev.	0.0039	0.0034	0.0029	0.0020	0.0026	0.0021	0.0022	0.0019
			t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
3	High	Simple	mean		0.0923	0.0931	0.0908	0.0930	0.0952	0.0926	0.0952
			st.dev.		0.0008	0.0013	0.0009	0.0009	0.0007	0.0007	0.0007
			t-test		1.0000	0.2408	1.0000		0.0000	0.9980	0.0000
			wilcox-test		1.0000	0.5433	1.0000		0.0000	0.9977	0.0000
3	High	Complex	mean		0.1081	0.1079	0.0863	0.0828	0.0970	0.0834	0.0971
			st.dev.		0.0012	0.0009	0.0028	0.0019	0.0021	0.0020	0.0021
			t-test		0.0000	0.0000	0.0000		0.0000	0.0264	0.0000
			wilcox-test		0.0000	0.0000	0.0000		0.0000	0.0364	0.0000
6	Low	Simple	mean	0.0043	0.0284	0.0283	0.0248	0.0291	0.0324	0.0287	0.0327
			st.dev.	0.0014	0.0012	0.0018	0.0007	0.0010	0.0005	0.0008	0.0005
			t-test	1.0000	1.0000	0.9998	1.0000		0.0000	0.9958	0.0000
			wilcox-test	1.0000	1.0000	1.0000	1.0000		0.0000	0.9953	0.0000
6	Low	Complex	mean	0.0433	0.0438	0.0413	0.0270	0.0260	0.0314	0.0274	0.0339
			st.dev.	0.0014	0.0017	0.0014	0.0008	0.0011	0.0009	0.0010	0.0008
			t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
6	High	Simple	mean		0.0352	0.0352	0.0347	0.0361	0.0361	0.0360	0.0361
			st.dev.		0.0003	0.0004	0.0004	0.0004	0.0004	0.0003	0.0004
			t-test		1.0000	1.0000	1.0000		0.8112	0.9994	0.6394
			wilcox-test		1.0000	1.0000	1.0000		0.8788	0.9989	0.6579
6	High	Complex	mean		0.0383	0.0386	0.0343	0.0350	0.0367	0.0378	0.0387
			st.dev.		0.0003	0.0004	0.0006	0.0005	0.0005	0.0005	0.0004
			t-test		0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
			wilcox-test		0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
9	Low	Simple	mean	0.0025	0.0150	0.0149	0.0134	0.0170	0.0170	0.0168	0.0172
			st.dev.	0.0008	0.0005	0.0007	0.0004	0.0006	0.0003	0.0005	0.0002
			t-test	1.0000	1.0000	1.0000	1.0000		0.5492	0.9993	0.0040
			wilcox-test	1.0000	1.0000	1.0000	1.0000		0.3269	0.9985	0.0003
9	Low	Complex	mean	0.0203	0.0194	0.0190	0.0142	0.0159	0.0161	0.0162	0.0179
			st.dev.	0.0006	0.0006	0.0005	0.0003	0.0005	0.0003	0.0004	0.0003
			t-test	0.0000	0.0000	0.0000	1.0000		0.0006	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0000	1.0000		0.0004	0.0000	0.0000
9	High	Simple	mean		0.0180	0.0181	0.0178	0.0189	0.0185	0.0188	0.0185
			st.dev.		0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
			t-test		1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
			wilcox-test		1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
9	High	Complex	mean		0.0200	0.0200	0.0178	0.0187	0.0193	0.0201	0.0202
			st.dev.		0.0002	0.0002	0.0003	0.0003	0.0003	0.0003	0.0002
			t-test		0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
			wilcox-test		0.0000	0.0000	1.0000		0.0000	0.0000	0.0000

Notes: Table reports the average measures of the MSE based on 100 simulation replications for the sample size of 200 observations. The first column denotes the number of outcome classes. Columns 2 and 3 specify the dimension and the DGP, respectively. The fourth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.1.3 ARPS: Sample Size = 800

Table 6: Simulation results: Accuracy Measure = ARPS & Sample Size = 800

Simulation Design				**Comparison of Methods**							
Class	Dim.	DGP	Statistic	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
3	Low	Simple	mean	0.0023	0.0701	0.0685	0.0484	0.0466	0.0799	0.0483	0.0803
			st.dev.	0.0009	0.0043	0.0045	0.0007	0.0009	0.0008	0.0008	0.0008
			t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
3	Low	Complex	mean	0.0849	0.0828	0.0813	0.0394	0.0323	0.0495	0.0344	0.0516
			st.dev.	0.0009	0.0024	0.0026	0.0012	0.0009	0.0013	0.0010	0.0012
			t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
3	High	Simple	mean		0.1055	0.1055	0.1017	0.1044	0.1136	0.1047	0.1136
			st.dev.		0.0007	0.0007	0.0006	0.0005	0.0004	0.0005	0.0003
			t-test		0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
			wilcox-test		0.0000	0.0000	1.0000		0.0000	0.0001	0.0000
3	High	Complex	mean		0.0944	0.0949	0.0681	0.0616	0.0738	0.0635	0.0770
			st.dev.		0.0007	0.0010	0.0010	0.0010	0.0010	0.0009	0.0011
			t-test		0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test		0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
6	Low	Simple	mean	0.0015	0.0619	0.0595	0.0435	0.0417	0.0702	0.0443	0.0748
			st.dev.	0.0005	0.0037	0.0039	0.0006	0.0007	0.0007	0.0006	0.0006
			t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
6	Low	Complex	mean	0.0947	0.1020	0.0986	0.0408	0.0330	0.0510	0.0384	0.0608
			st.dev.	0.0009	0.0031	0.0031	0.0009	0.0007	0.0010	0.0008	0.0012
			t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
6	High	Simple	mean		0.0905	0.0898	0.0874	0.0905	0.0978	0.0940	0.0995
			st.dev.		0.0006	0.0005	0.0004	0.0003	0.0002	0.0004	0.0002
			t-test		0.6597	1.0000	1.0000		0.0000	0.0000	0.0000
			wilcox-test		0.8939	1.0000	1.0000		0.0000	0.0000	0.0000
6	High	Complex	mean		0.1069	0.1060	0.0774	0.0698	0.0840	0.0781	0.0931
			st.dev.		0.0007	0.0007	0.0010	0.0009	0.0010	0.0013	0.0011
			t-test		0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test		0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
9	Low	Simple	mean	0.0013	0.0603	0.0570	0.0417	0.0400	0.0668	0.0432	0.0741
			st.dev.	0.0004	0.0032	0.0035	0.0006	0.0006	0.0006	0.0007	0.0006
			t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
9	Low	Complex	mean	0.0837	0.0867	0.0836	0.0368	0.0305	0.0459	0.0375	0.0614
			st.dev.	0.0009	0.0023	0.0027	0.0008	0.0006	0.0008	0.0006	0.0009
			t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
9	High	Simple	mean		0.0857	0.0847	0.0826	0.0860	0.0927	0.0920	0.0949
			st.dev.		0.0005	0.0005	0.0004	0.0003	0.0002	0.0004	0.0001
			t-test		1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
			wilcox-test		1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
9	High	Complex	mean		0.0956	0.0947	0.0708	0.0648	0.0773	0.0781	0.0933
			st.dev.		0.0006	0.0007	0.0007	0.0006	0.0007	0.0011	0.0009
			t-test		0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test		0.0000	0.0000	0.0000		0.0000	0.0000	0.0000

Notes: Table reports the average measures of the RPS based on 100 simulation replications for the sample size of 800 observations. The first column denotes the number of outcome classes. Columns 2 and 3 specify the dimension and the DGP, respectively. The fourth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.1.4 AMSE: Sample Size = 800

Table 7: Simulation results: Accuracy Measure = AMSE & Sample Size = 800

Simulation Design				**Comparison of Methods**							
Class	Dim.	DGP	Statistic	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
3	Low	Simple	mean	0.0025	0.0618	0.0624	0.0451	0.0461	0.0688	0.0472	0.0691
			st.dev.	0.0009	0.0032	0.0036	0.0006	0.0010	0.0006	0.0007	0.0006
			t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
			wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
3	Low	Complex	mean	0.0875	0.0848	0.0834	0.0482	0.0414	0.0574	0.0439	0.0602
			st.dev.	0.0008	0.0020	0.0020	0.0011	0.0010	0.0011	0.0011	0.0010
			t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
3	High	Simple	mean		0.0866	0.0870	0.0840	0.0861	0.0920	0.0861	0.0920
			st.dev.		0.0005	0.0007	0.0005	0.0005	0.0003	0.0004	0.0003
			t-test		0.0000	0.0000	1.0000		0.0000	0.5234	0.0000
			wilcox-test		0.0000	0.0000	1.0000		0.0000	0.4713	0.0000
3	High	Complex	mean		0.0969	0.0977	0.0717	0.0656	0.0749	0.0675	0.0789
			st.dev.		0.0006	0.0008	0.0009	0.0010	0.0009	0.0009	0.0011
			t-test		0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test		0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
6	Low	Simple	mean	0.0010	0.0260	0.0260	0.0206	0.0231	0.0287	0.0234	0.0292
			st.dev.	0.0003	0.0010	0.0014	0.0003	0.0005	0.0002	0.0003	0.0002
			t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
			wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
6	Low	Complex	mean	0.0376	0.0406	0.0384	0.0219	0.0208	0.0257	0.0221	0.0280
			st.dev.	0.0003	0.0010	0.0009	0.0004	0.0004	0.0004	0.0004	0.0003
			t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
6	High	Simple	mean		0.0333	0.0332	0.0325	0.0339	0.0350	0.0343	0.0353
			st.dev.		0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001
			t-test		1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
			wilcox-test		1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
6	High	Complex	mean		0.0404	0.0399	0.0308	0.0287	0.0325	0.0313	0.0352
			st.dev.		0.0002	0.0002	0.0003	0.0003	0.0004	0.0004	0.0003
			t-test		0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test		0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
9	Low	Simple	mean	0.0006	0.0140	0.0138	0.0113	0.0136	0.0153	0.0135	0.0156
			st.dev.	0.0002	0.0004	0.0006	0.0002	0.0003	0.0001	0.0002	0.0001
			t-test	1.0000	0.0000	0.0121	1.0000		0.0000	1.0000	0.0000
			wilcox-test	1.0000	0.0000	0.0241	1.0000		0.0000	1.0000	0.0000
9	Low	Complex	mean	0.0178	0.0187	0.0181	0.0114	0.0124	0.0132	0.0126	0.0149
			st.dev.	0.0001	0.0004	0.0005	0.0002	0.0003	0.0002	0.0002	0.0001
			t-test	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
9	High	Simple	mean		0.0171	0.0171	0.0167	0.0179	0.0179	0.0184	0.0181
			st.dev.		0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
			t-test		1.0000	1.0000	1.0000		0.9803	0.0000	0.0000
			wilcox-test		1.0000	1.0000	1.0000		0.9670	0.0000	0.0000
9	High	Complex	mean		0.0191	0.0191	0.0162	0.0161	0.0170	0.0176	0.0187
			st.dev.		0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
			t-test		0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
			wilcox-test		0.0000	0.0000	0.0000		0.0000	0.0000	0.0000

Notes: Table reports the average measures of the MSE based on 100 simulation replications for the sample size of 800 observations. The first column denotes the number of outcome classes. Columns 2 and 3 specify the dimension and the DGP, respectively. The fourth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.1.5 ARPS & AMSE: Sample Size = 3200

Table 8: Simulation results: Accuracy Measure = ARPS/AMSE & Sample Size = 3200

Simulation Design				ARPS		AMSE	
Class	Dim.	DGP	Statistic	Ordered	Ordered*	Ordered	Ordered*
3	Low	Simple	mean	0.0373	0.0670	0.0376	0.0591
			st.dev.	0.0004	0.0005	0.0005	0.0004
			t-test		0.0000		0.0000
			wilcox-test		0.0000		0.0000
3	Low	Complex	mean	0.0285	0.0415	0.0243	0.0336
			st.dev.	0.0004	0.0005	0.0003	0.0004
			t-test		0.0000		0.0000
			wilcox-test		0.0000		0.0000
3	High	Simple	mean	0.0956	0.1069	0.0798	0.0872
			st.dev.	0.0003	0.0002	0.0002	0.0002
			t-test		0.0000		0.0000
			wilcox-test		0.0000		0.0000
3	High	Complex	mean	0.0498	0.0620	0.0557	0.0653
			st.dev.	0.0004	0.0005	0.0005	0.0005
			t-test		0.0000		0.0000
			wilcox-test		0.0000		0.0000
6	Low	Simple	mean	0.0335	0.0593	0.0188	0.0253
			st.dev.	0.0004	0.0004	0.0002	0.0001
			t-test		0.0000		0.0000
			wilcox-test		0.0000		0.0000
6	Low	Complex	mean	0.0255	0.0367	0.0162	0.0197
			st.dev.	0.0003	0.0004	0.0002	0.0002
			t-test		0.0000		0.0000
			wilcox-test		0.0000		0.0000
6	High	Simple	mean	0.0825	0.0923	0.0314	0.0335
			st.dev.	0.0002	0.0002	0.0001	0.0000
			t-test		0.0000		0.0000
			wilcox-test		0.0000		0.0000
6	High	Complex	mean	0.0526	0.0656	0.0264	0.0292
			st.dev.	0.0004	0.0004	0.0002	0.0001
			t-test		0.0000		0.0000
			wilcox-test		0.0000		0.0000
9	Low	Simple	mean	0.0321	0.0565	0.0110	0.0136
			st.dev.	0.0003	0.0003	0.0001	0.0001
			t-test		0.0000		0.0000
			wilcox-test		0.0000		0.0000
9	Low	Complex	mean	0.0244	0.0350	0.0098	0.0110
			st.dev.	0.0002	0.0003	0.0001	0.0001
			t-test		0.0000		0.0000
			wilcox-test		0.0000		0.0000
9	High	Simple	mean	0.0783	0.0875	0.0165	0.0172
			st.dev.	0.0002	0.0002	0.0000	0.0000
			t-test		0.0000		0.0000
			wilcox-test		0.0000		0.0000
9	High	Complex	mean	0.0559	0.0697	0.0145	0.0160
			st.dev.	0.0004	0.0004	0.0001	0.0001
			t-test		0.0000		0.0000
			wilcox-test		0.0000		0.0000

Notes: Table reports the average measures of the RPS and MSE based on 100 simulation replications for the sample size of 3200 observations. The first column denotes the number of outcome classes. Columns 2 and 3 specify the dimension and the DGP, respectively. The fourth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and the honest version of the *Ordered Forest*.

B.2 Complete Simulation Results

Tables 9 to 20 below summarize the simulation results for all 72 different DGPs, complementing the main results presented in Section 5.3. Each table specifies the particular simulation design as follows: the first column *DGP* provides the identifier for the data generating process. Columns 2 to 5 specify the particular characteristics of the respective DGP, namely if the DGP features additional noise variables (*noise*), 15 in the low-dimensional case and 1000 in the high-dimensional case, nonlinear effects (*nonlin*), multicollinearity among covariates (*multi*), and randomly spaced thresholds (*random*). The sixth column *Statistic* contains summary statistics of the simulation results. In particular, the mean of the respective accuracy measure (*mean*) and its standard deviation (*st.dev.*). Furthermore, rows *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods. The alternative hypothesis is that the mean of the *Ordered Forest* is less than the mean of the other method to test if the *Ordered Forest* achieves significantly lower prediction error than the other considered methods.

B.2.1 ARPS: Low Dimension with 3 Classes

Table 9: Simulation Results: Accuracy Measure = ARPS & Low Dimension with 3 Classes

Simulation Design					Comparison of Methods								
DGP	noise	nonlin	multi	random	Statistic	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
1	✗	✗	✗	✗	mean	0.0097	0.0765	0.0755	0.0625	0.0609	0.0954	0.0619	0.0954
					st.dev.	0.0042	0.0056	0.0055	0.0018	0.0020	0.0011	0.0019	0.0012
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0002	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
2	✓	✗	✗	✗	mean	0.0216	0.0840	0.0832	0.0738	0.0754	0.1041	0.0763	0.1041
					st.dev.	0.0054	0.0046	0.0048	0.0015	0.0016	0.0013	0.0016	0.0013
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0001	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0001	0.0000
3	✗	✓	✗	✗	mean	0.0904	0.0715	0.0726	0.0688	0.0681	0.0824	0.0672	0.0824
					st.dev.	0.0045	0.0031	0.0033	0.0021	0.0022	0.0013	0.0020	0.0013
					t-test	0.0000	0.0000	0.0000	0.0132		0.0000	0.9988	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0070		0.0000	0.9976	0.0000
4	✗	✗	✓	✗	mean	0.0097	0.1236	0.1194	0.0316	0.0297	0.0449	0.0297	0.0493
					st.dev.	0.0031	0.0079	0.0079	0.0015	0.0015	0.0013	0.0014	0.0013
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.4099	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.3721	0.0000
5	✗	✗	✗	✓	mean	0.0104	0.0730	0.0698	0.0611	0.0594	0.0942	0.0607	0.0948
					st.dev.	0.0035	0.0072	0.0066	0.0017	0.0020	0.0015	0.0021	0.0016
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
6	✓	✓	✗	✗	mean	0.1052	0.0772	0.0781	0.0763	0.0768	0.0863	0.0759	0.0862
					st.dev.	0.0066	0.0025	0.0030	0.0021	0.0020	0.0011	0.0019	0.0011
					t-test	0.0000	0.1612	0.0004	0.9717		0.0000	0.9998	0.0000
					wilcox-test	0.0000	0.1979	0.0004	0.9589		0.0000	0.9996	0.0000
7	✓	✗	✓	✗	mean	0.0221	0.1349	0.1321	0.0344	0.0335	0.0502	0.0353	0.0569
					st.dev.	0.0064	0.0060	0.0057	0.0013	0.0011	0.0021	0.0013	0.0022
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
8	✓	✗	✗	✓	mean	0.0196	0.0750	0.0753	0.0669	0.0694	0.0938	0.0699	0.0940
					st.dev.	0.0056	0.0036	0.0040	0.0017	0.0019	0.0010	0.0018	0.0010
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0412	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0339	0.0000
9	✗	✓	✓	✗	mean	0.1116	0.1204	0.1170	0.0486	0.0401	0.0706	0.0422	0.0722
					st.dev.	0.0030	0.0077	0.0075	0.0022	0.0021	0.0025	0.0021	0.0024
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
10	✗	✓	✗	✓	mean	0.0905	0.0703	0.0693	0.0673	0.0668	0.0808	0.0672	0.0809
					st.dev.	0.0047	0.0042	0.0042	0.0023	0.0023	0.0013	0.0023	0.0013
					t-test	0.0000	0.0000	0.0000	0.0650		0.0000	0.0923	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0862		0.0000	0.0921	0.0000
11	✗	✗	✓	✓	mean	0.0111	0.1299	0.1284	0.0312	0.0295	0.0428	0.0298	0.0463
					st.dev.	0.0042	0.0115	0.0121	0.0017	0.0017	0.0014	0.0016	0.0015
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0868	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0901	0.0000
12	✓	✓	✓	✗	mean	0.1297	0.1232	0.1209	0.0639	0.0483	0.0809	0.0512	0.0819
					st.dev.	0.0051	0.0058	0.0055	0.0024	0.0025	0.0028	0.0023	0.0027
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
13	✓	✓	✗	✓	mean	0.0915	0.0682	0.0697	0.0675	0.0689	0.0764	0.0677	0.0764
					st.dev.	0.0063	0.0022	0.0024	0.0020	0.0020	0.0011	0.0019	0.0011
					t-test	0.0000	0.9877	0.0036	1.0000		0.0000	1.0000	0.0000
					wilcox-test	0.0000	0.9813	0.0032	1.0000		0.0000	1.0000	0.0000
14	✓	✗	✓	✓	mean	0.0235	0.1219	0.1194	0.0319	0.0312	0.0468	0.0324	0.0524
					st.dev.	0.0068	0.0052	0.0050	0.0015	0.0014	0.0020	0.0014	0.0021
					t-test	1.0000	0.0000	0.0000	0.0012		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0008		0.0000	0.0000	0.0000
15	✗	✓	✓	✓	mean	0.1118	0.1222	0.1204	0.0482	0.0396	0.0688	0.0411	0.0712
					st.dev.	0.0042	0.0087	0.0092	0.0024	0.0025	0.0026	0.0024	0.0026
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
16	✓	✓	✓	✓	mean	0.1156	0.1044	0.1028	0.0593	0.0466	0.0748	0.0491	0.0760
					st.dev.	0.0047	0.0039	0.0038	0.0023	0.0026	0.0028	0.0024	0.0027
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000

Notes: Table reports the average measures of the RPS based on 100 simulation replications for the sample size of 200 observations with 3 outcome classes. Columns 1 to 5 specify the DGP identifier and its features, namely 15 additional noise variables (*noise*), nonlinear effects (*nonlin*), multicollinearity among covariates (*multi*), and randomly spaced thresholds (*random*). The sixth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.2.2 ARPS: Low Dimension with 6 Classes

Table 10: Simulation Results: Accuracy Measure = ARPS & Low Dimension with 6 Classes

Simulation Design						Comparison of Methods							
DGP	noise	nonlin	multi	random	Statistic	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
17	✗	✗	✗	✗	mean	0.0062	0.0687	0.0665	0.0554	0.0544	0.0833	0.0577	0.0872
					st.dev.	0.0020	0.0048	0.0050	0.0012	0.0014	0.0009	0.0016	0.0010
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
18	✓	✗	✗	✗	mean	0.0129	0.0726	0.0708	0.0645	0.0669	0.0901	0.0709	0.0932
					st.dev.	0.0034	0.0026	0.0028	0.0013	0.0012	0.0007	0.0013	0.0007
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
19	✗	✓	✗	✗	mean	0.0749	0.0610	0.0608	0.0585	0.0593	0.0707	0.0597	0.0725
					st.dev.	0.0022	0.0030	0.0027	0.0016	0.0018	0.0010	0.0020	0.0010
					t-test	0.0000	0.0000	0.0000	0.9996		0.0000	0.0947	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.9995		0.0000	0.0966	0.0000
20	✗	✗	✓	✗	mean	0.0059	0.1111	0.1071	0.0285	0.0273	0.0407	0.0292	0.0539
					st.dev.	0.0016	0.0050	0.0061	0.0010	0.0009	0.0011	0.0010	0.0015
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
21	✗	✗	✗	✓	mean	0.0062	0.0670	0.0648	0.0544	0.0537	0.0816	0.0569	0.0853
					st.dev.	0.0022	0.0044	0.0044	0.0013	0.0014	0.0009	0.0015	0.0009
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
22	✓	✓	✗	✗	mean	0.0853	0.0650	0.0651	0.0644	0.0664	0.0735	0.0675	0.0748
					st.dev.	0.0049	0.0022	0.0022	0.0016	0.0014	0.0008	0.0014	0.0006
					t-test	0.0000	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
23	✓	✗	✓	✗	mean	0.0106	0.1177	0.1145	0.0313	0.0307	0.0462	0.0377	0.0640
					st.dev.	0.0028	0.0038	0.0049	0.0010	0.0008	0.0014	0.0011	0.0018
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
24	✓	✗	✗	✓	mean	0.0148	0.0745	0.0722	0.0655	0.0677	0.0919	0.0718	0.0946
					st.dev.	0.0040	0.0032	0.0029	0.0012	0.0013	0.0009	0.0014	0.0008
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
25	✗	✓	✓	✗	mean	0.0952	0.0995	0.0961	0.0439	0.0372	0.0630	0.0418	0.0747
					st.dev.	0.0020	0.0041	0.0043	0.0016	0.0016	0.0017	0.0016	0.0020
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
26	✗	✓	✗	✓	mean	0.0733	0.0590	0.0594	0.0573	0.0582	0.0691	0.0586	0.0707
					st.dev.	0.0024	0.0021	0.0020	0.0015	0.0015	0.0010	0.0015	0.0009
					t-test	0.0000	0.0017	0.0000	1.0000		0.0000	0.0660	0.0000
					wilcox-test	0.0000	0.0041	0.0000	1.0000		0.0000	0.0809	0.0000
27	✗	✗	✓	✓	mean	0.0053	0.1069	0.1046	0.0278	0.0266	0.0401	0.0286	0.0533
					st.dev.	0.0014	0.0048	0.0056	0.0010	0.0009	0.0011	0.0009	0.0015
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
28	✓	✓	✓	✗	mean	0.1090	0.1022	0.1001	0.0564	0.0447	0.0709	0.0527	0.0843
					st.dev.	0.0041	0.0031	0.0030	0.0015	0.0018	0.0020	0.0018	0.0024
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
29	✓	✓	✗	✓	mean	0.0881	0.0666	0.0662	0.0658	0.0676	0.0751	0.0697	0.0764
					st.dev.	0.0051	0.0024	0.0022	0.0016	0.0015	0.0008	0.0015	0.0006
					t-test	0.0000	0.9997	1.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
30	✓	✗	✓	✓	mean	0.0118	0.1214	0.1161	0.0317	0.0309	0.0469	0.0378	0.0642
					st.dev.	0.0032	0.0046	0.0055	0.0009	0.0008	0.0014	0.0012	0.0019
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
31	✗	✓	✓	✓	mean	0.0931	0.0956	0.0925	0.0434	0.0368	0.0619	0.0414	0.0731
					st.dev.	0.0019	0.0044	0.0045	0.0015	0.0014	0.0016	0.0014	0.0020
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
32	✓	✓	✓	✓	mean	0.1122	0.1093	0.1058	0.0574	0.0452	0.0719	0.0536	0.0842
					st.dev.	0.0040	0.0045	0.0044	0.0017	0.0020	0.0022	0.0021	0.0024
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000

Notes: Table reports the average measures of the RPS based on 100 simulation replications for the sample size of 200 observations with 6 outcome classes. Columns 1 to 5 specify the DGP identifier and its features, namely 15 additional noise variables (*noise*), nonlinear effects (*nonlin*), multicollinearity among covariates (*multi*), and randomly spaced thresholds (*random*). The sixth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.2.3 ARPS: Low Dimension with 9 Classes

Table 11: Simulation Results: Accuracy Measure = ARPS & Low Dimension with 9 Classes

Simulation Design					Comparison of Methods								
DGP	noise	nonlin	multi	random	Statistic	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
33	✗	✗	✗	✗	mean	0.0054	0.0653	0.0629	0.0528	0.0519	0.0789	0.0569	0.0850
					st.dev.	0.0018	0.0042	0.0042	0.0012	0.0014	0.0009	0.0017	0.0009
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
34	✓	✗	✗	✗	mean	0.0112	0.0693	0.0672	0.0609	0.0638	0.0855	0.0704	0.0901
					st.dev.	0.0028	0.0027	0.0027	0.0012	0.0012	0.0007	0.0013	0.0006
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
35	✗	✓	✗	✗	mean	0.0706	0.0573	0.0572	0.0555	0.0567	0.0669	0.0590	0.0698
					st.dev.	0.0023	0.0026	0.0027	0.0014	0.0015	0.0009	0.0016	0.0007
					t-test	0.0000	0.0220	0.0445	1.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0788	0.2389	1.0000		0.0000	0.0000	0.0000
36	✗	✗	✓	✗	mean	0.0052	0.1057	0.1047	0.0277	0.0263	0.0396	0.0303	0.0601
					st.dev.	0.0014	0.0046	0.0056	0.0009	0.0009	0.0010	0.0010	0.0014
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
37	✗	✗	✗	✓	mean	0.0054	0.0627	0.0608	0.0518	0.0511	0.0774	0.0558	0.0835
					st.dev.	0.0019	0.0036	0.0035	0.0012	0.0014	0.0009	0.0016	0.0010
					t-test	1.0000	0.0000	0.0000	0.0001		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0002		0.0000	0.0000	0.0000
38	✓	✓	✗	✗	mean	0.0806	0.0607	0.0608	0.0606	0.0629	0.0695	0.0661	0.0715
					st.dev.	0.0036	0.0016	0.0018	0.0013	0.0012	0.0008	0.0014	0.0007
					t-test	0.0000	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
39	✓	✗	✓	✗	mean	0.0086	0.1122	0.1102	0.0301	0.0295	0.0443	0.0408	0.0710
					st.dev.	0.0017	0.0036	0.0041	0.0009	0.0008	0.0012	0.0011	0.0017
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
40	✓	✗	✗	✓	mean	0.0106	0.0663	0.0646	0.0586	0.0615	0.0820	0.0679	0.0866
					st.dev.	0.0028	0.0026	0.0026	0.0011	0.0012	0.0008	0.0012	0.0007
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
41	✗	✓	✓	✗	mean	0.0897	0.0929	0.0897	0.0417	0.0356	0.0596	0.0424	0.0776
					st.dev.	0.0017	0.0037	0.0038	0.0014	0.0013	0.0015	0.0014	0.0018
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
42	✗	✓	✗	✓	mean	0.0701	0.0565	0.0564	0.0545	0.0556	0.0657	0.0579	0.0685
					st.dev.	0.0025	0.0024	0.0024	0.0015	0.0014	0.0008	0.0016	0.0007
					t-test	0.0000	0.0006	0.0010	1.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0028	0.0066	1.0000		0.0000	0.0000	0.0000
43	✗	✗	✓	✓	mean	0.0051	0.1034	0.1025	0.0273	0.0258	0.0394	0.0298	0.0593
					st.dev.	0.0015	0.0040	0.0045	0.0008	0.0007	0.0010	0.0009	0.0014
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
44	✓	✓	✓	✗	mean	0.1018	0.0956	0.0933	0.0534	0.0432	0.0673	0.0550	0.0873
					st.dev.	0.0035	0.0031	0.0031	0.0013	0.0016	0.0017	0.0019	0.0021
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
45	✓	✓	✗	✓	mean	0.0763	0.0587	0.0588	0.0582	0.0605	0.0664	0.0638	0.0684
					st.dev.	0.0040	0.0019	0.0018	0.0014	0.0012	0.0007	0.0011	0.0006
					t-test	0.0000	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
46	✓	✗	✓	✓	mean	0.0084	0.1079	0.1066	0.0292	0.0286	0.0432	0.0391	0.0699
					st.dev.	0.0021	0.0034	0.0040	0.0008	0.0007	0.0012	0.0012	0.0017
					t-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
47	✗	✓	✓	✓	mean	0.0881	0.0915	0.0887	0.0411	0.0352	0.0588	0.0414	0.0765
					st.dev.	0.0017	0.0039	0.0041	0.0014	0.0012	0.0014	0.0014	0.0016
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
48	✓	✓	✓	✓	mean	0.0973	0.0912	0.0887	0.0515	0.0421	0.0647	0.0537	0.0845
					st.dev.	0.0031	0.0033	0.0032	0.0015	0.0016	0.0019	0.0018	0.0017
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000

Notes: Table reports the average measures of the RPS based on 100 simulation replications for the sample size of 200 observations with 9 outcome classes. Columns 1 to 5 specify the DGP identifier and its features, namely 15 additional noise variables (*noise*), nonlinear effects (*nonlin*), multicollinearity among covariates (*multi*), and randomly spaced thresholds (*random*). The sixth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.2.4 ARPS: High Dimension with 3 Classes

Table 12: Simulation Results: Accuracy Measure = ARPS & High Dimension with 3 Classes

Simulation Design						Comparison of Methods						
DGP	noise	nonlin	multi	random	Statistic	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
49	✓	✗	✗	✗	mean	0.1135	0.1139	0.1112	0.1140	0.1180	0.1139	0.1179
					st.dev.	0.0009	0.0010	0.0009	0.0008	0.0006	0.0008	0.0006
					t-test	1.0000	0.7676	1.0000		0.0000	0.7268	0.0000
					wilcox-test	0.9999	0.8438	1.0000		0.0000	0.7191	0.0000
50	✓	✓	✗	✗	mean	0.0896	0.0899	0.0901	0.0903	0.0907	0.0901	0.0907
					st.dev.	0.0008	0.0010	0.0008	0.0007	0.0007	0.0007	0.0006
					t-test	1.0000	0.9997	0.9840		0.0002	0.9973	0.0004
					wilcox-test	1.0000	1.0000	0.9929		0.0000	0.9989	0.0000
51	✓	✗	✓	✗	mean	0.1534	0.1529	0.0827	0.0766	0.1082	0.0867	0.1134
					st.dev.	0.0011	0.0012	0.0024	0.0025	0.0029	0.0024	0.0026
					t-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
52	✓	✗	✗	✓	mean	0.1253	0.1252	0.1224	0.1248	0.1296	0.1250	0.1296
					st.dev.	0.0013	0.0013	0.0010	0.0009	0.0007	0.0009	0.0007
					t-test	0.0011	0.0115	1.0000		0.0000	0.1664	0.0000
					wilcox-test	0.0013	0.0140	1.0000		0.0000	0.1515	0.0000
53	✓	✓	✓	✗	mean	0.1299	0.1300	0.1048	0.1016	0.1200	0.1021	0.1202
					st.dev.	0.0011	0.0012	0.0034	0.0027	0.0026	0.0027	0.0025
					t-test	0.0000	0.0000	0.0000		0.0000	0.0674	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0494	0.0000
54	✓	✓	✗	✓	mean	0.0997	0.0996	0.0999	0.0998	0.1004	0.0997	0.1004
					st.dev.	0.0012	0.0013	0.0012	0.0012	0.0011	0.0011	0.0012
					t-test	0.5772	0.8438	0.3065		0.0000	0.6432	0.0000
					wilcox-test	0.6792	0.9705	0.2427		0.0000	0.7183	0.0000
55	✓	✗	✓	✓	mean	0.1678	0.1667	0.0862	0.0836	0.1167	0.0906	0.1195
					st.dev.	0.0015	0.0013	0.0026	0.0030	0.0029	0.0029	0.0029
					t-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
56	✓	✓	✓	✓	mean	0.1476	0.1474	0.1156	0.1102	0.1316	0.1110	0.1317
					st.dev.	0.0013	0.0010	0.0041	0.0029	0.0031	0.0029	0.0031
					t-test	0.0000	0.0000	0.0000		0.0000	0.0287	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0272	0.0000

Notes: Table reports the average measures of the RPS based on 100 simulation replications for the sample size of 200 observations with 3 outcome classes. Columns 1 to 5 specify the DGP identifier and its features, namely 1000 additional noise variables (*noise*), nonlinear effects (*nonlin*), multicollinearity among covariates (*multi*), and randomly spaced thresholds (*random*). The sixth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.2.5 ARPS: High Dimension with 6 Classes

Table 13: Simulation Results: Accuracy Measure = ARPS & High Dimension with 6 Classes

Simulation Design						Comparison of Methods						
DGP	noise	nonlin	multi	random	Statistic	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
57	✓	✗	✗	✗	mean	0.0974	0.0972	0.0951	0.0983	0.1012	0.0998	0.1016
					st.dev.	0.0006	0.0006	0.0006	0.0005	0.0004	0.0005	0.0004
					t-test	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
58	✓	✓	✗	✗	mean	0.0762	0.0762	0.0765	0.0773	0.0772	0.0776	0.0773
					st.dev.	0.0006	0.0006	0.0006	0.0005	0.0005	0.0004	0.0004
					t-test	1.0000	1.0000	1.0000		0.9803	0.0000	0.7833
					wilcox-test	1.0000	1.0000	1.0000		0.9838	0.0000	0.7449
59	✓	✗	✓	✗	mean	0.1336	0.1327	0.0747	0.0675	0.0968	0.0912	0.1152
					st.dev.	0.0008	0.0010	0.0013	0.0016	0.0015	0.0016	0.0017
					t-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
60	✓	✗	✗	✓	mean	0.0845	0.0845	0.0826	0.0857	0.0880	0.0872	0.0883
					st.dev.	0.0005	0.0005	0.0006	0.0004	0.0003	0.0004	0.0003
					t-test	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
61	✓	✓	✓	✗	mean	0.1091	0.1088	0.0891	0.0885	0.1026	0.1010	0.1105
					st.dev.	0.0009	0.0008	0.0025	0.0021	0.0018	0.0023	0.0010
					t-test	0.0000	0.0000	0.0547		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0626		0.0000	0.0000	0.0000
62	✓	✓	✗	✓	mean	0.0658	0.0659	0.0660	0.0669	0.0665	0.0672	0.0666
					st.dev.	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005
					t-test	1.0000	1.0000	1.0000		1.0000	0.0006	0.9998
					wilcox-test	1.0000	1.0000	1.0000		1.0000	0.0000	1.0000
63	✓	✗	✓	✓	mean	0.1167	0.1163	0.0682	0.0606	0.0872	0.0820	0.1052
					st.dev.	0.0007	0.0008	0.0014	0.0016	0.0015	0.0018	0.0015
					t-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
64	✓	✓	✓	✓	mean	0.0927	0.0927	0.0766	0.0772	0.0882	0.0898	0.0952
					st.dev.	0.0006	0.0005	0.0020	0.0016	0.0014	0.0018	0.0006
					t-test	0.0000	0.0000	0.9878		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.9887		0.0000	0.0000	0.0000

Notes: Table reports the average measures of the RPS based on 100 simulation replications for the sample size of 200 observations with 6 outcome classes. Columns 1 to 5 specify the DGP identifier and its features, namely 1000 additional noise variables (*noise*), nonlinear effects (*nonlin*), multicollinearity among covariates (*multi*), and randomly spaced thresholds (*random*). The sixth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.2.6 ARPS: High Dimension with 9 Classes

Table 14: Simulation Results: Accuracy Measure = ARPS & High Dimension with 9 Classes

Simulation Design					Comparison of Methods							
DGP	noise	nonlin	multi	random	Statistic	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
65	✓	✗	✗	✗	mean	0.0921	0.0918	0.0900	0.0931	0.0959	0.0955	0.0964
					st.dev.	0.0006	0.0006	0.0006	0.0005	0.0003	0.0004	0.0003
					t-test	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
66	✓	✓	✗	✗	mean	0.0721	0.0720	0.0724	0.0732	0.0730	0.0739	0.0731
					st.dev.	0.0006	0.0005	0.0006	0.0005	0.0004	0.0004	0.0004
					t-test	1.0000	1.0000	1.0000		0.9959	0.0000	0.8717
					wilcox-test	1.0000	1.0000	1.0000		0.9991	0.0000	0.9308
67	✓	✗	✓	✗	mean	0.1268	0.1260	0.0713	0.0648	0.0926	0.0979	0.1175
					st.dev.	0.0008	0.0009	0.0013	0.0013	0.0014	0.0017	0.0015
					t-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
68	✓	✗	✗	✓	mean	0.0904	0.0902	0.0884	0.0915	0.0941	0.0937	0.0946
					st.dev.	0.0006	0.0006	0.0005	0.0005	0.0003	0.0004	0.0003
					t-test	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	1.0000	1.0000		0.0000	0.0000	0.0000
69	✓	✓	✓	✗	mean	0.1031	0.1028	0.0838	0.0838	0.0967	0.1024	0.1061
					st.dev.	0.0007	0.0007	0.0021	0.0017	0.0016	0.0016	0.0005
					t-test	0.0000	0.0000	0.4695		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.5044		0.0000	0.0000	0.0000
70	✓	✓	✗	✓	mean	0.0706	0.0707	0.0710	0.0718	0.0716	0.0724	0.0717
					st.dev.	0.0007	0.0007	0.0006	0.0006	0.0005	0.0005	0.0006
					t-test	1.0000	1.0000	1.0000		0.9903	0.0000	0.8186
					wilcox-test	1.0000	1.0000	1.0000		0.9983	0.0000	0.8723
71	✓	✗	✓	✓	mean	0.1246	0.1238	0.0704	0.0636	0.0911	0.0966	0.1153
					st.dev.	0.0007	0.0008	0.0014	0.0013	0.0014	0.0016	0.0018
					t-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
72	✓	✓	✓	✓	mean	0.1007	0.1004	0.0817	0.0819	0.0945	0.0997	0.1036
					st.dev.	0.0007	0.0007	0.0020	0.0017	0.0015	0.0019	0.0006
					t-test	0.0000	0.0000	0.7875		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.8473		0.0000	0.0000	0.0000

Notes: Table reports the average measures of the RPS based on 100 simulation replications for the sample size of 200 observations with 9 outcome classes. Columns 1 to 5 specify the DGP identifier and its features, namely 1000 additional noise variables (*noise*), nonlinear effects (*nonlin*), multicollinearity among covariates (*multi*), and randomly spaced thresholds (*random*). The sixth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.2.7 AMSE: Low Dimension with 3 Classes

Table 15: Simulation Results: Accuracy Measure = AMSE & Low Dimension with 3 Classes

Simulation Design					**Comparison of Methods**								
DGP	noise	nonlin	multi	random	Statistic	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
1	✗	✗	✗	✗	mean	0.0103	0.0669	0.0682	0.0565	0.0587	0.0800	0.0587	0.0800
					st.dev.	0.0044	0.0041	0.0044	0.0015	0.0022	0.0009	0.0016	0.0010
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.3900	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.2614	0.0000
2	✓	✗	✗	✗	mean	0.0227	0.0723	0.0727	0.0648	0.0682	0.0859	0.0684	0.0859
					st.dev.	0.0056	0.0034	0.0038	0.0013	0.0015	0.0010	0.0014	0.0010
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.1609	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.1287	0.0000
3	✗	✓	✗	✗	mean	0.0700	0.0576	0.0609	0.0552	0.0586	0.0644	0.0565	0.0644
					st.dev.	0.0032	0.0024	0.0032	0.0016	0.0021	0.0011	0.0016	0.0011
					t-test	0.0000	0.9980	0.0000	1.0000		0.0000	1.0000	0.0000
					wilcox-test	0.0000	0.9954	0.0000	1.0000		0.0000	1.0000	0.0000
4	✗	✗	✓	✗	mean	0.0124	0.1217	0.1166	0.0378	0.0370	0.0500	0.0367	0.0554
					st.dev.	0.0040	0.0068	0.0068	0.0017	0.0018	0.0014	0.0017	0.0016
					t-test	1.0000	0.0000	0.0000	0.0005		0.0000	0.8458	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0003		0.0000	0.8530	0.0000
5	✗	✗	✗	✓	mean	0.0096	0.0594	0.0567	0.0495	0.0511	0.0726	0.0517	0.0732
					st.dev.	0.0032	0.0057	0.0047	0.0015	0.0018	0.0011	0.0017	0.0011
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0044	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0024	0.0000
6	✓	✓	✗	✗	mean	0.0809	0.0612	0.0636	0.0604	0.0638	0.0671	0.0617	0.0670
					st.dev.	0.0048	0.0019	0.0030	0.0016	0.0017	0.0010	0.0015	0.0010
					t-test	0.0000	1.0000	0.7436	1.0000		0.0000	1.0000	0.0000
					wilcox-test	0.0000	1.0000	0.9265	1.0000		0.0000	1.0000	0.0000
7	✓	✗	✓	✗	mean	0.0283	0.1297	0.1262	0.0411	0.0407	0.0548	0.0427	0.0634
					st.dev.	0.0083	0.0052	0.0049	0.0015	0.0015	0.0020	0.0017	0.0022
					t-test	1.0000	0.0000	0.0000	0.0734		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0494		0.0000	0.0000	0.0000
8	✓	✗	✗	✓	mean	0.0230	0.0722	0.0746	0.0660	0.0705	0.0855	0.0701	0.0857
					st.dev.	0.0065	0.0028	0.0038	0.0014	0.0018	0.0008	0.0015	0.0008
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.9630	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.9578	0.0000
9	✗	✓	✓	✗	mean	0.0968	0.1066	0.1012	0.0493	0.0443	0.0660	0.0465	0.0680
					st.dev.	0.0024	0.0070	0.0060	0.0018	0.0021	0.0018	0.0019	0.0016
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
10	✗	✓	✗	✓	mean	0.0667	0.0538	0.0533	0.0507	0.0530	0.0599	0.0529	0.0600
					st.dev.	0.0034	0.0033	0.0031	0.0017	0.0019	0.0010	0.0018	0.0010
					t-test	0.0000	0.0119	0.1801	1.0000		0.0000	0.6401	0.0000
					wilcox-test	0.0000	0.0332	0.2314	1.0000		0.0000	0.7041	0.0000
11	✗	✗	✓	✓	mean	0.0132	0.1201	0.1172	0.0328	0.0326	0.0427	0.0327	0.0472
					st.dev.	0.0050	0.0111	0.0113	0.0017	0.0018	0.0013	0.0018	0.0016
					t-test	1.0000	0.0000	0.0000	0.1763		0.0000	0.3026	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.1287		0.0000	0.3376	0.0000
12	✓	✓	✓	✗	mean	0.1104	0.1064	0.1027	0.0616	0.0506	0.0737	0.0540	0.0751
					st.dev.	0.0039	0.0051	0.0043	0.0020	0.0024	0.0022	0.0021	0.0019
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
13	✓	✓	✗	✓	mean	0.0765	0.0595	0.0630	0.0587	0.0632	0.0646	0.0605	0.0646
					st.dev.	0.0050	0.0016	0.0029	0.0016	0.0019	0.0011	0.0016	0.0011
					t-test	0.0000	1.0000	0.7290	1.0000		0.0000	1.0000	0.0000
					wilcox-test	0.0000	1.0000	0.8231	1.0000		0.0000	1.0000	0.0000
14	✓	✗	✓	✓	mean	0.0311	0.1273	0.1244	0.0413	0.0408	0.0553	0.0420	0.0626
					st.dev.	0.0090	0.0044	0.0043	0.0019	0.0019	0.0021	0.0018	0.0023
					t-test	1.0000	0.0000	0.0000	0.0584		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.0428		0.0000	0.0000	0.0000
15	✗	✓	✓	✓	mean	0.0878	0.1016	0.0962	0.0420	0.0374	0.0566	0.0387	0.0593
					st.dev.	0.0031	0.0081	0.0072	0.0019	0.0022	0.0018	0.0020	0.0018
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
16	✓	✓	✓	✓	mean	0.1081	0.0985	0.0965	0.0637	0.0543	0.0752	0.0572	0.0768
					st.dev.	0.0039	0.0034	0.0029	0.0020	0.0026	0.0021	0.0022	0.0019
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000

Notes: Table reports the average measures of the MSE based on 100 simulation replications for the sample size of 200 observations with 3 outcome classes. Columns 1 to 5 specify the DGP identifier and its features, namely 15 additional noise variables (*noise*), nonlinear effects (*nonlin*), multicollinearity among covariates (*multi*), and randomly spaced thresholds (*random*). The sixth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.2.8 AMSE: Low Dimension with 6 Classes

Table 16: Simulation Results: Accuracy Measure = AMSE & Low Dimension with 6 Classes

Simulation Design					Comparison of Methods								
DGP	noise	nonlin	multi	random	Statistic	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
17	✗	✗	✗	✗	mean	0.0043	0.0284	0.0283	0.0248	0.0291	0.0324	0.0287	0.0327
					st.dev.	0.0014	0.0012	0.0018	0.0007	0.0010	0.0005	0.0008	0.0005
					t-test	1.0000	1.0000	0.9998	1.0000		0.0000	0.9958	0.0000
					wilcox-test	1.0000	1.0000	1.0000	1.0000		0.0000	0.9953	0.0000
18	✓	✗	✗	✗	mean	0.0083	0.0294	0.0292	0.0272	0.0311	0.0337	0.0310	0.0341
					st.dev.	0.0021	0.0007	0.0010	0.0005	0.0006	0.0003	0.0005	0.0003
					t-test	1.0000	1.0000	1.0000	1.0000		0.0000	0.9791	0.0000
					wilcox-test	1.0000	1.0000	1.0000	1.0000		0.0000	0.9805	0.0000
19	✗	✓	✗	✗	mean	0.0245	0.0216	0.0222	0.0207	0.0257	0.0237	0.0240	0.0237
					st.dev.	0.0007	0.0009	0.0009	0.0006	0.0009	0.0004	0.0008	0.0004
					t-test	1.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	1.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
20	✗	✗	✓	✗	mean	0.0065	0.0600	0.0568	0.0238	0.0259	0.0299	0.0263	0.0356
					st.dev.	0.0017	0.0019	0.0023	0.0008	0.0009	0.0007	0.0008	0.0007
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0006	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0002	0.0000
21	✗	✗	✗	✓	mean	0.0043	0.0283	0.0282	0.0248	0.0291	0.0324	0.0289	0.0327
					st.dev.	0.0014	0.0013	0.0015	0.0007	0.0009	0.0005	0.0007	0.0005
					t-test	1.0000	1.0000	1.0000	1.0000		0.0000	0.9689	0.0000
					wilcox-test	1.0000	1.0000	1.0000	1.0000		0.0000	0.9661	0.0000
22	✓	✓	✗	✗	mean	0.0273	0.0223	0.0228	0.0220	0.0263	0.0242	0.0249	0.0243
					st.dev.	0.0015	0.0007	0.0008	0.0005	0.0007	0.0004	0.0006	0.0003
					t-test	0.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	0.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
23	✓	✗	✓	✗	mean	0.0114	0.0607	0.0580	0.0258	0.0266	0.0319	0.0305	0.0396
					st.dev.	0.0030	0.0014	0.0017	0.0007	0.0007	0.0008	0.0008	0.0007
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
24	✓	✗	✗	✓	mean	0.0088	0.0306	0.0296	0.0274	0.0306	0.0346	0.0310	0.0350
					st.dev.	0.0023	0.0010	0.0011	0.0005	0.0006	0.0004	0.0006	0.0004
					t-test	1.0000	0.6721	1.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.3992	1.0000	1.0000		0.0000	0.0000	0.0000
25	✗	✓	✓	✗	mean	0.0374	0.0396	0.0377	0.0234	0.0256	0.0292	0.0254	0.0315
					st.dev.	0.0007	0.0016	0.0013	0.0008	0.0011	0.0008	0.0009	0.0007
					t-test	0.0000	0.0000	0.0000	1.0000		0.0000	0.9637	0.0000
					wilcox-test	0.0000	0.0000	0.0000	1.0000		0.0000	0.9567	0.0000
26	✗	✓	✗	✓	mean	0.0245	0.0215	0.0224	0.0207	0.0256	0.0236	0.0241	0.0237
					st.dev.	0.0008	0.0007	0.0009	0.0006	0.0008	0.0004	0.0007	0.0005
					t-test	1.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	1.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
27	✗	✗	✓	✓	mean	0.0060	0.0587	0.0560	0.0236	0.0254	0.0297	0.0262	0.0355
					st.dev.	0.0015	0.0017	0.0019	0.0007	0.0009	0.0007	0.0009	0.0007
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
28	✓	✓	✓	✗	mean	0.0416	0.0396	0.0384	0.0271	0.0272	0.0312	0.0280	0.0338
					st.dev.	0.0014	0.0012	0.0009	0.0006	0.0009	0.0008	0.0008	0.0006
					t-test	0.0000	0.0000	0.0000	0.8880		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.8212		0.0000	0.0000	0.0000
29	✓	✓	✗	✓	mean	0.0292	0.0239	0.0240	0.0231	0.0268	0.0255	0.0261	0.0256
					st.dev.	0.0016	0.0009	0.0008	0.0005	0.0007	0.0003	0.0005	0.0003
					t-test	0.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	0.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
30	✓	✗	✓	✓	mean	0.0115	0.0618	0.0580	0.0242	0.0246	0.0306	0.0285	0.0375
					st.dev.	0.0029	0.0018	0.0020	0.0006	0.0006	0.0007	0.0008	0.0006
					t-test	1.0000	0.0000	0.0000	0.9999		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	0.9999		0.0000	0.0000	0.0000
31	✗	✓	✓	✓	mean	0.0378	0.0394	0.0375	0.0236	0.0256	0.0295	0.0256	0.0317
					st.dev.	0.0008	0.0014	0.0013	0.0007	0.0009	0.0007	0.0008	0.0006
					t-test	0.0000	0.0000	0.0000	1.0000		0.0000	0.6494	0.0000
					wilcox-test	0.0000	0.0000	0.0000	1.0000		0.0000	0.6416	0.0000
32	✓	✓	✓	✓	mean	0.0433	0.0438	0.0413	0.0270	0.0260	0.0314	0.0274	0.0339
					st.dev.	0.0014	0.0017	0.0014	0.0008	0.0011	0.0009	0.0010	0.0008
					t-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000

Notes: Table reports the average measures of the MSE based on 100 simulation replications for the sample size of 200 observations with 6 outcome classes. Columns 1 to 5 specify the DGP identifier and its features, namely 15 additional noise variables (*noise*), nonlinear effects (*nonlin*), multicollinearity among covariates (*multi*), and randomly spaced thresholds (*random*). The sixth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.2.9 AMSE: Low Dimension with 9 Classes

Table 17: Simulation Results: Accuracy Measure = AMSE & Low Dimension with 9 Classes

Simulation Design					**Comparison of Methods**								
DGP	noise	nonlin	multi	random	Statistic	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
33	✗	✗	✗	✗	mean	0.0025	0.0150	0.0149	0.0134	0.0170	0.0170	0.0168	0.0172
					st.dev.	0.0008	0.0005	0.0007	0.0004	0.0006	0.0003	0.0005	0.0002
					t-test	1.0000	1.0000	1.0000	1.0000		0.5492	0.9993	0.0040
					wilcox-test	1.0000	1.0000	1.0000	1.0000		0.3269	0.9985	0.0003
34	✓	✗	✗	✗	mean	0.0046	0.0155	0.0154	0.0144	0.0176	0.0175	0.0175	0.0178
					st.dev.	0.0011	0.0004	0.0004	0.0003	0.0004	0.0002	0.0003	0.0002
					t-test	1.0000	1.0000	1.0000	1.0000		0.9697	0.9696	0.0011
					wilcox-test	1.0000	1.0000	1.0000	1.0000		0.9359	0.9544	0.0003
35	✗	✓	✗	✗	mean	0.0123	0.0110	0.0114	0.0107	0.0147	0.0121	0.0137	0.0121
					st.dev.	0.0004	0.0004	0.0005	0.0004	0.0006	0.0003	0.0005	0.0003
					t-test	1.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	1.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
36	✗	✗	✓	✗	mean	0.0044	0.0333	0.0321	0.0148	0.0168	0.0185	0.0175	0.0222
					st.dev.	0.0010	0.0008	0.0009	0.0004	0.0005	0.0005	0.0005	0.0003
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
37	✗	✗	✗	✓	mean	0.0026	0.0152	0.0154	0.0136	0.0173	0.0172	0.0170	0.0175
					st.dev.	0.0009	0.0005	0.0006	0.0003	0.0006	0.0002	0.0004	0.0002
					t-test	1.0000	1.0000	1.0000	1.0000		0.8591	1.0000	0.0034
					wilcox-test	1.0000	1.0000	1.0000	1.0000		0.7952	1.0000	0.0032
38	✓	✓	✗	✗	mean	0.0136	0.0112	0.0115	0.0111	0.0144	0.0122	0.0137	0.0122
					st.dev.	0.0006	0.0003	0.0004	0.0003	0.0004	0.0002	0.0003	0.0002
					t-test	1.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	1.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
39	✓	✗	✓	✗	mean	0.0066	0.0335	0.0323	0.0159	0.0167	0.0192	0.0200	0.0242
					st.dev.	0.0012	0.0006	0.0007	0.0005	0.0005	0.0006	0.0005	0.0004
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
40	✓	✗	✗	✓	mean	0.0046	0.0152	0.0152	0.0142	0.0175	0.0172	0.0173	0.0174
					st.dev.	0.0011	0.0004	0.0005	0.0003	0.0004	0.0002	0.0003	0.0002
					t-test	1.0000	1.0000	1.0000	1.0000		1.0000	0.9998	0.9655
					wilcox-test	1.0000	1.0000	1.0000	1.0000		1.0000	0.9999	0.9525
41	✗	✓	✓	✗	mean	0.0190	0.0198	0.0192	0.0127	0.0158	0.0156	0.0152	0.0170
					st.dev.	0.0004	0.0007	0.0007	0.0004	0.0006	0.0004	0.0005	0.0003
					t-test	0.0000	0.0000	0.0000	1.0000		0.9652	1.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	1.0000		0.9503	1.0000	0.0000
42	✗	✓	✗	✓	mean	0.0125	0.0112	0.0117	0.0108	0.0147	0.0122	0.0138	0.0122
					st.dev.	0.0004	0.0004	0.0007	0.0003	0.0005	0.0002	0.0005	0.0002
					t-test	1.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	1.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
43	✗	✗	✓	✓	mean	0.0043	0.0335	0.0324	0.0149	0.0167	0.0187	0.0176	0.0225
					st.dev.	0.0013	0.0007	0.0009	0.0005	0.0005	0.0005	0.0005	0.0004
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
44	✓	✓	✓	✗	mean	0.0208	0.0198	0.0193	0.0143	0.0159	0.0164	0.0162	0.0181
					st.dev.	0.0007	0.0006	0.0005	0.0003	0.0006	0.0004	0.0005	0.0003
					t-test	0.0000	0.0000	0.0000	1.0000		0.0000	0.0001	0.0000
					wilcox-test	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
45	✓	✓	✗	✓	mean	0.0130	0.0110	0.0113	0.0108	0.0142	0.0118	0.0134	0.0118
					st.dev.	0.0006	0.0003	0.0003	0.0003	0.0004	0.0003	0.0003	0.0002
					t-test	1.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	1.0000	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
46	✓	✗	✓	✓	mean	0.0070	0.0335	0.0325	0.0166	0.0173	0.0200	0.0204	0.0250
					st.dev.	0.0016	0.0005	0.0007	0.0004	0.0005	0.0005	0.0005	0.0004
					t-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	1.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
47	✗	✓	✓	✓	mean	0.0192	0.0203	0.0198	0.0130	0.0159	0.0158	0.0153	0.0173
					st.dev.	0.0004	0.0007	0.0007	0.0004	0.0006	0.0004	0.0005	0.0003
					t-test	0.0000	0.0000	0.0000	1.0000		0.6681	1.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	1.0000		0.5336	1.0000	0.0000
48	✓	✓	✓	✓	mean	0.0203	0.0194	0.0190	0.0142	0.0159	0.0161	0.0162	0.0179
					st.dev.	0.0006	0.0006	0.0005	0.0003	0.0005	0.0003	0.0004	0.0003
					t-test	0.0000	0.0000	0.0000	1.0000		0.0006	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000	1.0000		0.0004	0.0000	0.0000

Notes: Table reports the average measures of the MSE based on 100 simulation replications for the sample size of 200 observations with 9 outcome classes. Columns 1 to 5 specify the DGP identifier and its features, namely 15 additional noise variables (*noise*), nonlinear effects (*nonlin*), multicollinearity among covariates (*multi*), and randomly spaced thresholds (*random*). The sixth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.2.10 AMSE: High Dimension with 3 Classes

Table 18: Simulation Results: Accuracy Measure = AMSE & High Dimension with 3 Classes

Simulation Design					Comparison of Methods							
DGP	noise	nonlin	multi	random	Statistic	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
49	✓	✗	✗	✗	mean	0.0923	0.0931	0.0908	0.0930	0.0952	0.0926	0.0952
					st.dev.	0.0008	0.0013	0.0009	0.0009	0.0007	0.0007	0.0007
					t-test	1.0000	0.2408	1.0000		0.0000	0.9980	0.0000
					wilcox-test	1.0000	0.5433	1.0000		0.0000	0.9977	0.0000
50	✓	✓	✗	✗	mean	0.0692	0.0698	0.0696	0.0702	0.0699	0.0696	0.0699
					st.dev.	0.0009	0.0013	0.0010	0.0009	0.0009	0.0008	0.0008
					t-test	1.0000	0.9907	0.9999		0.9649	1.0000	0.9852
					wilcox-test	1.0000	1.0000	1.0000		0.9887	1.0000	0.9944
51	✓	✗	✓	✗	mean	0.1385	0.1379	0.0864	0.0752	0.1008	0.0881	0.1087
					st.dev.	0.0009	0.0010	0.0019	0.0021	0.0023	0.0019	0.0018
					t-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
52	✓	✗	✗	✓	mean	0.0906	0.0904	0.0884	0.0902	0.0931	0.0902	0.0931
					st.dev.	0.0011	0.0013	0.0008	0.0008	0.0007	0.0008	0.0007
					t-test	0.0006	0.0794	1.0000		0.0000	0.3296	0.0000
					wilcox-test	0.0010	0.1853	1.0000		0.0000	0.2606	0.0000
53	✓	✓	✓	✗	mean	0.1079	0.1083	0.0910	0.0888	0.1010	0.0892	0.1013
					st.dev.	0.0009	0.0011	0.0025	0.0020	0.0019	0.0019	0.0019
					t-test	0.0000	0.0000	0.0000		0.0000	0.0936	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0745	0.0000
54	✓	✓	✗	✓	mean	0.0706	0.0703	0.0703	0.0705	0.0706	0.0704	0.0706
					st.dev.	0.0010	0.0011	0.0010	0.0009	0.0009	0.0008	0.0009
					t-test	0.1479	0.9409	0.8941		0.1495	0.7655	0.1796
					wilcox-test	0.1712	0.9972	0.9496		0.0718	0.8048	0.1178
55	✓	✗	✓	✓	mean	0.1291	0.1276	0.0725	0.0678	0.0914	0.0758	0.0954
					st.dev.	0.0016	0.0010	0.0020	0.0021	0.0021	0.0020	0.0020
					t-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
56	✓	✓	✓	✓	mean	0.1081	0.1079	0.0863	0.0828	0.0970	0.0834	0.0971
					st.dev.	0.0012	0.0009	0.0028	0.0019	0.0021	0.0020	0.0021
					t-test	0.0000	0.0000	0.0000		0.0000	0.0264	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0364	0.0000

Notes: Table reports the average measures of the MSE based on 100 simulation replications for the sample size of 200 observations with 3 outcome classes. Columns 1 to 5 specify the DGP identifier and its features, namely 1000 additional noise variables (*noise*), nonlinear effects (*nonlin*), multicollinearity among covariates (*multi*), and randomly spaced thresholds (*random*). The sixth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.2.11 AMSE: High Dimension with 6 Classes

Table 19: Simulation Results: Accuracy Measure = AMSE & High Dimension with 6 Classes

Simulation Design					Comparison of Methods							
DGP	noise	nonlin	multi	random	Statistic	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
57	✓	✗	✗	✗	mean	0.0352	0.0352	0.0347	0.0361	0.0361	0.0360	0.0361
					st.dev.	0.0003	0.0004	0.0004	0.0004	0.0004	0.0003	0.0004
					t-test	1.0000	1.0000	1.0000		0.8112	0.9994	0.6394
					wilcox-test	1.0000	1.0000	1.0000		0.8788	0.9989	0.6579
58	✓	✓	✗	✗	mean	0.0246	0.0246	0.0246	0.0257	0.0248	0.0252	0.0248
					st.dev.	0.0003	0.0003	0.0003	0.0004	0.0003	0.0002	0.0003
					t-test	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
59	✓	✗	✓	✗	mean	0.0622	0.0617	0.0459	0.0383	0.0494	0.0479	0.0553
					st.dev.	0.0003	0.0003	0.0005	0.0007	0.0007	0.0006	0.0006
					t-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
60	✓	✗	✗	✓	mean	0.0339	0.0341	0.0335	0.0350	0.0347	0.0348	0.0348
					st.dev.	0.0003	0.0004	0.0004	0.0004	0.0004	0.0003	0.0004
					t-test	1.0000	1.0000	1.0000		1.0000	0.9993	0.9999
					wilcox-test	1.0000	1.0000	1.0000		1.0000	0.9995	1.0000
61	✓	✓	✓	✗	mean	0.0397	0.0397	0.0351	0.0358	0.0383	0.0380	0.0399
					st.dev.	0.0004	0.0004	0.0007	0.0006	0.0005	0.0006	0.0004
					t-test	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
62	✓	✓	✗	✓	mean	0.0229	0.0231	0.0229	0.0241	0.0231	0.0235	0.0231
					st.dev.	0.0004	0.0005	0.0005	0.0005	0.0005	0.0004	0.0005
					t-test	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
63	✓	✗	✓	✓	mean	0.0628	0.0629	0.0481	0.0405	0.0512	0.0506	0.0583
					st.dev.	0.0003	0.0004	0.0005	0.0008	0.0007	0.0006	0.0005
					t-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
64	✓	✓	✓	✓	mean	0.0383	0.0386	0.0343	0.0350	0.0367	0.0378	0.0387
					st.dev.	0.0003	0.0004	0.0006	0.0005	0.0005	0.0005	0.0004
					t-test	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000

Notes: Table reports the average measures of the MSE based on 100 simulation replications for the sample size of 200 observations with 6 outcome classes. Columns 1 to 5 specify the DGP identifier and its features, namely 1000 additional noise variables (*noise*), nonlinear effects (*nonlin*), multicollinearity among covariates (*multi*), and randomly spaced thresholds (*random*). The sixth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.2.12 AMSE: High Dimension with 9 Classes

Table 20: Simulation Results: Accuracy Measure = AMSE & High Dimension with 9 Classes

Simulation Design						Comparison of Methods						
DGP	noise	nonlin	multi	random	Statistic	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
65	✓	✗	✗	✗	mean	0.0180	0.0181	0.0178	0.0189	0.0185	0.0188	0.0185
					st.dev.	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
					t-test	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
66	✓	✓	✗	✗	mean	0.0123	0.0123	0.0123	0.0133	0.0124	0.0129	0.0124
					st.dev.	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
					t-test	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
67	✓	✗	✓	✗	mean	0.0339	0.0337	0.0263	0.0224	0.0281	0.0284	0.0316
					st.dev.	0.0002	0.0002	0.0003	0.0005	0.0004	0.0003	0.0003
					t-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
68	✓	✗	✗	✓	mean	0.0181	0.0181	0.0179	0.0190	0.0186	0.0188	0.0186
					st.dev.	0.0002	0.0002	0.0003	0.0003	0.0003	0.0002	0.0003
					t-test	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
69	✓	✓	✓	✗	mean	0.0198	0.0199	0.0178	0.0187	0.0193	0.0201	0.0201
					st.dev.	0.0002	0.0002	0.0003	0.0003	0.0003	0.0002	0.0002
					t-test	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
70	✓	✓	✗	✓	mean	0.0124	0.0124	0.0124	0.0133	0.0125	0.0130	0.0125
					st.dev.	0.0002	0.0002	0.0002	0.0003	0.0002	0.0002	0.0002
					t-test	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
					wilcox-test	1.0000	1.0000	1.0000		1.0000	1.0000	1.0000
71	✓	✗	✓	✓	mean	0.0338	0.0337	0.0262	0.0225	0.0281	0.0285	0.0315
					st.dev.	0.0002	0.0002	0.0004	0.0005	0.0005	0.0003	0.0004
					t-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	0.0000		0.0000	0.0000	0.0000
72	✓	✓	✓	✓	mean	0.0200	0.0200	0.0178	0.0187	0.0193	0.0201	0.0202
					st.dev.	0.0002	0.0002	0.0003	0.0003	0.0003	0.0003	0.0002
					t-test	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000
					wilcox-test	0.0000	0.0000	1.0000		0.0000	0.0000	0.0000

Notes: Table reports the average measures of the MSE based on 100 simulation replications for the sample size of 200 observations with 9 outcome classes. Columns 1 to 5 specify the DGP identifier and its features, namely 1000 additional noise variables (*noise*), nonlinear effects (*nonlin*), multicollinearity among covariates (*multi*), and randomly spaced thresholds (*random*). The sixth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

B.3 Software Implementation

The Monte Carlo study has been conducted using the R statistical software (R Core Team, 2018) in version 3.5.2 (Eggshell Igloo) and the respective packages implementing the estimators used. With regards to the forest-based estimators the main tuning parameters, namely the number of trees, the number of randomly chosen covariates and the minimum leaf size have been specified according to the values in Table 1.

Table 21: Overview of Software Packages and Tuning Parameters

Software Implementation and Tuning Parameters								
method	Ologit	Naive	Ordinal	Conditional	Ordered	Ordered*	Multi	Multi*
package	rms	ordinalForest	ordinalForest	party	ranger	grf	ranger	grf
function	lrm	ordfor	ordfor	cforest	ranger	regression_forest	ranger	regression_forest
max. iterations	25	-	-	-	-	-	-	-
trees	-	1000	1000	1000	1000	1000	1000	1000
random subset	-	$\sqrt{p}$	$\sqrt{p}$	$\sqrt{p}$	$\sqrt{p}$	$\sqrt{p}$	$\sqrt{p}$	$\sqrt{p}$
leaf size	-	5	5	0	5	5	5	5
B_{sets}	-	1000	-	-	-	-	-	-
B_{prior}	-	100	-	-	-	-	-	-
performance	-	equal	-	-	-	-	-	-
S_{best}	-	10	-	-	-	-	-	-

In terms of the particular R packages used the ordered logistic regression has been implemented using the rms package (version 5.1-3) written by Harrell (2019). The respective lrm function for fitting the ordered logit has been used with the default parameters, except setting the maximum number of iterations, maxit=25 as for some of the DGPs the ordered logit has experienced convergence issues. Next, the naive and the ordinal forest have been applied based on the ordinalForest package in version 2.3 (Hornung, 2019b) with the ordfor function. As described in Appendix A.3 the ordinal forest introduces additional tuning parameters for which we use the default parameters as suggested in the package manual. Further, the conditional forest has been estimated with the package party in version 1.3-1 (Hothorn, Bühlmann, Dudoit, Molinaro, and Van Der Laan, 2006a; Strobl, Boulesteix, Zeileis, and Hothorn, 2007; Strobl, Boulesteix, Kneib, Augustin, and Zeileis, 2008). Regarding the choice of the tuning parameters, we rely on the default parameters of the cforest function. A particularity of the conditional forest is, due to the conceptual differences to standard regression forest in terms of the splitting criterion, the choice of the stopping rule. This is controlled by the significance level α (see Appendix A.2 for details). However, in order to grow deep trees we follow the suggestion in the package manual to set mincriterion$= 0$, which has been also used in the simulation study conducted in Janitza et al. (2016). Lastly, the *Ordered Forest* as well as the multinomial forest algorithms are implemented using the package ranger in version 0.11.1 (Wright and Ziegler, 2017) with the default hyperparameters. The honest versions of the above two estimators rely on the grf package in version 0.10.2 (Tibshirani, Athey, Wager, Friedberg, Miner, and Wright, 2018) with the default hyperparameters as well. A detailed overview of packages with the corresponding tuning parameters is provided in Table 21.

Furthermore, Tables 22 and 23 compare the absolute and relative computation time of the respective methods. For comparison purposes, we measure the computation time for the four main DGPs presented in Section 5.3, namely the simple DGP in the low- and high-dimensional case as well as the complex DGP in the low- and high-dimensional case, for both the small sample size ($N = 200$) and the big sample size ($N = 800$) for all considered number of outcome classes. We estimate the model based on the training set and predict the class probabilities for a test set of size $N = 10000$ as in the main simulation. We repeat this procedure 10 times and report the average computation time. The tuning parameters and the software implementations are chosen as defined in Tables 1 and 21, respectively. All simulations are computed on a 64-Bit Windows machine with 4 cores (1.80GHz) and 16GB RAM storage.

Table 22: Absolute Computation Time in Seconds

Simulation Design				**Comparison of Methods**							
Class	Dim.	DGP	Size	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
3	Low	Simple	200	0.01	1.22	10.33	46.61	0.62	1.24	0.91	1.86
3	Low	Simple	800	0.02	1.58	40.83	150.84	1.03	1.96	1.61	2.98
3	Low	Complex	200	0.02	1.19	11.93	47.43	0.63	1.26	0.98	1.92
3	Low	Complex	800	0.03	1.71	52.45	150.59	1.08	1.94	1.73	3.06
3	High	Simple	200		3.50	61.89	64.28	4.05	5.08	6.06	7.27
3	High	Simple	800		13.91	332.60	175.76	7.19	7.10	12.19	11.02
3	High	Complex	200		3.46	60.25	59.98	4.02	4.96	6.02	7.10
3	High	Complex	800		13.83	325.65	173.63	6.83	6.61	11.50	10.66
6	Low	Simple	200	0.02	1.88	12.79	46.80	1.47	3.00	1.74	3.52
6	Low	Simple	800	0.03	2.28	48.98	151.58	2.45	4.75	3.10	5.82
6	Low	Complex	200	0.03	1.85	14.75	46.97	1.56	3.12	1.85	3.66
6	Low	Complex	800	0.04	2.54	64.44	151.84	2.68	4.82	3.30	6.02
6	High	Simple	200		4.21	69.80	64.14	10.24	11.74	12.01	13.63
6	High	Simple	800		15.86	386.02	176.27	19.34	17.43	26.24	19.97
6	High	Complex	200		4.11	70.51	60.85	9.98	11.52	11.95	13.61
6	High	Complex	800		15.85	371.69	174.17	18.11	17.18	24.43	19.52
9	Low	Simple	200	0.03	2.32	20.53	46.70	2.27	4.71	2.44	5.03
9	Low	Simple	800	0.04	2.69	57.22	145.21	3.82	7.29	4.61	7.99
9	Low	Complex	200	0.03	2.29	22.86	47.36	2.40	4.83	2.65	5.28
9	Low	Complex	800	0.05	3.07	79.15	151.36	4.27	7.75	5.81	8.68
9	High	Simple	200		4.85	80.76	63.25	16.05	17.84	17.69	19.56
9	High	Simple	800		16.91	413.74	169.91	31.34	26.91	38.95	27.38
9	High	Complex	200		4.62	78.86	57.68	15.79	17.78	17.57	19.59
9	High	Complex	800		18.10	437.04	175.07	31.12	27.33	37.59	28.16

Notes: Table reports the average absolute computation time in seconds based on 10 simulation replications of training and prediction. The first column denotes the number of outcome classes. Columns 2 and 3 specify the dimension and the DGP, respectively. The fourth column contains the number of observations in the training set. The prediction set consists of 10 000 observations.

The results reveal the expected pattern for the *Ordered Forest*. The more outcome classes the longer the computation time as by definition of the algorithm more forests have to be estimated. Furthermore, we also observe a longer computation time if the number of observation and/or the number of considered splitting covariates increases which is also an expected behaviour. However, the computation time is not sensitive to the particular DGP which it should not be either. The latter two patterns are true for all considered methods. In comparison to the other forest-based methods, the computational advantage of the *Ordered Forest* becomes apparent. The *Ordered Forest* outperforms the ordinal and the conditional forest in all cases. In some cases the *Ordered Forest* is even more than 100 times faster and even in the closest cases it is more than 3 times faster than the two. In absolute terms this translates to computation time of around 1 second for the *Ordered Forest* and around 50 seconds for the ordinal and around 150 seconds for the conditional forest in the most extreme case. Contrarily, in the closest case, the computation time for the *Ordered Forest* is around 15 seconds, while for the ordinal forest this is around 80 seconds and around 60 seconds for the conditional forest. This points to the additional computation burden of the ordinal and the conditional forest due to the optimization procedure and the permutation tests, respectively. The only exception is the naive forest which does not include the optimization step. Furthermore, we observe a slightly longer computation time for the multinomial forest in comparison to the *Ordered Forest*, which is due to one extra forest being estimated. The honest versions of the two forests take a bit longer in general, but this seems to reverse once bigger samples are considered (in terms of both number of observations as well as number of considered covariates).

Table 23: Relative Computation Time

Simulation Design				**Comparison of Methods**							
Class	Dim.	DGP	Size	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
3	Low	Simple	200	0.02	1.98	16.76	75.66	1	2.02	1.48	3.02
3	Low	Simple	800	0.02	1.53	39.68	146.59	1	1.91	1.56	2.90
3	Low	Complex	200	0.03	1.87	18.79	74.70	1	1.99	1.55	3.03
3	Low	Complex	800	0.03	1.59	48.79	140.09	1	1.81	1.61	2.84
3	High	Simple	200		0.86	15.27	15.86	1	1.25	1.50	1.79
3	High	Simple	800		1.94	46.28	24.46	1	0.99	1.70	1.53
3	High	Complex	200		0.86	14.99	14.92	1	1.23	1.50	1.77
3	High	Complex	800		2.02	47.68	25.42	1	0.97	1.68	1.56
6	Low	Simple	200	0.02	1.28	8.73	31.95	1	2.05	1.19	2.40
6	Low	Simple	800	0.01	0.93	19.95	61.74	1	1.94	1.26	2.37
6	Low	Complex	200	0.02	1.18	9.45	30.09	1	2.00	1.19	2.34
6	Low	Complex	800	0.02	0.94	24.02	56.59	1	1.80	1.23	2.24
6	High	Simple	200		0.41	6.81	6.26	1	1.15	1.17	1.33
6	High	Simple	800		0.82	19.96	9.11	1	0.90	1.36	1.03
6	High	Complex	200		0.41	7.07	6.10	1	1.16	1.20	1.36
6	High	Complex	800		0.88	20.52	9.62	1	0.95	1.35	1.08
9	Low	Simple	200	0.01	1.02	9.03	20.54	1	2.07	1.07	2.21
9	Low	Simple	800	0.01	0.70	14.98	38.01	1	1.91	1.21	2.09
9	Low	Complex	200	0.01	0.95	9.51	19.69	1	2.01	1.10	2.19
9	Low	Complex	800	0.01	0.72	18.55	35.48	1	1.82	1.36	2.03
9	High	Simple	200		0.30	5.03	3.94	1	1.11	1.10	1.22
9	High	Simple	800		0.54	13.20	5.42	1	0.86	1.24	0.87
9	High	Complex	200		0.29	5.00	3.65	1	1.13	1.11	1.24
9	High	Complex	800		0.58	14.04	5.63	1	0.88	1.21	0.90

Notes: Table reports the average relative computation time with regards to the *Ordered Forest* estimator based on 10 simulation replications of training and prediction. The first column denotes the number of outcome classes. Columns 2 and 3 specify the dimension and the DGP, respectively. The fourth column contains the number of observations in the training set. The prediction set consists of 10 000 observations.

Generally, the sensitivity with regards to the computation time appears to be very different for the considered methods. For the *Ordered Forest* as well as the multinomial forest, including their honest versions, the most important aspect is clearly the number of outcome classes. For the naive and the ordinal forest the number of observations seems to be most decisive and for the conditional forest paradoxically the size of the prediction set is most relevant. Overall, the above result support the theoretical argument of the *Ordered Forest* being computationally advantageous in comparison to the ordinal and the conditional forest.

C Empirical Applications

In this section we present more detailed and supplementary results regarding the empirical evidence (Section 6) discussed in the main text. In the following the descriptive statistics for the considered datasets and the results for the prediction accuracy as well as for the marginal effects are summarized.

C.1 Descriptive Statistics

Table 24: Descriptive Statistics: mammography dataset

Mammography Dataset						
variable	type	mean	sd	median	min	max
SYMPT*	Categorical	2.97	0.95	3.00	1.00	4.00
PB	Numeric	7.56	2.10	7.00	5.00	17.00
HIST*	Categorical	1.11	0.31	1.00	1.00	2.00
BSE*	Categorical	1.87	0.34	2.00	1.00	2.00
DECT*	Categorical	2.66	0.56	3.00	1.00	3.00
y*	Categorical	1.61	0.77	1.00	1.00	3.00

Table 25: Descriptive Statistics: nhanes dataset

Nhanes Dataset						
variable	type	mean	sd	median	min	max
sex*	Categorical	1.51	0.50	2.00	1.00	2.00
race*	Categorical	2.87	1.00	3.00	1.00	5.00
country_of_birth*	Categorical	1.34	0.79	1.00	1.00	4.00
education*	Categorical	3.37	1.24	3.00	1.00	5.00
marital_status*	Categorical	2.31	1.74	1.00	1.00	6.00
waistcircum	Numeric	100.37	16.37	99.40	61.60	176.70
Cholesterol	Numeric	196.89	41.59	193.00	97.00	432.00
WBCcount	Numeric	7.30	2.88	6.90	1.60	83.20
AcuteIllness*	Categorical	1.25	0.43	1.00	1.00	2.00
depression*	Categorical	1.39	0.76	1.00	1.00	4.00
ToothCond*	Categorical	3.05	1.24	3.00	1.00	5.00
sleepTrouble*	Categorical	2.28	1.28	2.00	1.00	5.00
wakeUp*	Categorical	2.41	1.30	2.00	1.00	5.00
cig*	Categorical	1.51	0.50	2.00	1.00	2.00
diabetes*	Categorical	1.14	0.34	1.00	1.00	2.00
asthma*	Categorical	1.15	0.36	1.00	1.00	2.00
heartFailure*	Categorical	1.03	0.16	1.00	1.00	2.00
stroke*	Categorical	1.03	0.18	1.00	1.00	2.00
chronicBronchitis*	Categorical	1.07	0.26	1.00	1.00	2.00
alcohol	Numeric	3.93	20.18	2.00	0.00	365.00
heavyDrinker*	Categorical	1.17	0.37	1.00	1.00	2.00
medicalPlaceToGo*	Categorical	1.92	0.67	2.00	1.00	5.00
BPsys	Numeric	124.44	18.62	122.00	78.00	230.00
BPdias	Numeric	71.18	11.84	72.00	10.00	114.00
age	Numeric	49.96	16.68	50.00	20.00	80.00
BMI	Numeric	29.33	6.66	28.32	14.20	73.43
y*	Categorical	2.77	1.00	3.00	1.00	5.00

Table 26: Descriptive Statistics: supportstudy dataset

Supportstudy Dataset						
variable	type	mean	sd	median	min	max
age	Numeric	62.80	16.27	65.29	20.30	100.13
sex*	Categorical	1.54	0.50	2.00	1.00	2.00
dzgroup*	Categorical	3.23	2.48	2.00	1.00	8.00
num.co	Numeric	1.90	1.34	2.00	0.00	7.00
scoma	Numeric	12.45	25.29	0.00	0.00	100.00
charges	Numeric	59307.91	86620.70	28416.50	1635.75	740010.00
avtisst	Numeric	23.53	13.60	20.00	1.67	64.00
race*	Categorical	1.36	0.88	1.00	1.00	5.00
meanbp	Numeric	84.52	27.64	77.00	0.00	180.00
wblc	Numeric	12.62	9.31	10.50	0.05	100.00
hrt	Numeric	98.59	32.93	102.50	0.00	300.00
resp	Numeric	23.60	9.54	24.00	0.00	64.00
temp	Numeric	37.08	1.25	36.70	32.50	41.20
crea	Numeric	1.80	1.74	1.20	0.30	11.80
sod	Numeric	137.64	6.34	137.00	118.00	175.00
y*	Categorical	2.90	1.81	2.00	1.00	5.00

Table 27: Descriptive Statistics: vlbw dataset

Vlbw Dataset						
variable	type	mean	sd	median	min	max
race*	Categorical	1.57	0.50	2.00	1.00	2.00
bwt	Numeric	1094.89	260.44	1140.00	430.00	1500.00
inout*	Categorical	1.03	0.16	1.00	1.00	2.00
twn*	Categorical	1.24	0.43	1.00	1.00	2.00
lol	Numeric	7.73	19.47	3.00	0.00	192.00
magsulf*	Categorical	1.18	0.39	1.00	1.00	2.00
meth*	Categorical	1.44	0.50	1.00	1.00	2.00
toc*	Categorical	1.24	0.43	1.00	1.00	2.00
delivery*	Categorical	1.41	0.49	1.00	1.00	2.00
sex*	Categorical	1.50	0.50	1.00	1.00	2.00
y*	Categorical	5.09	2.58	6.00	1.00	9.00

Table 28: Descriptive Statistics: winequality dataset

Winequality Dataset						
variable	type	mean	sd	median	min	max
fixed.acidity	Numeric	6.85	0.84	6.80	3.80	14.20
volatile.acidity	Numeric	0.28	0.10	0.26	0.08	1.10
citric.acid	Numeric	0.33	0.12	0.32	0.00	1.66
residual.sugar	Numeric	6.39	5.07	5.20	0.60	65.80
chlorides	Numeric	0.05	0.02	0.04	0.01	0.35
free.sulfur.dioxide	Numeric	35.31	17.01	34.00	2.00	289.00
total.sulfur.dioxide	Numeric	138.38	42.51	134.00	9.00	440.00
density	Numeric	0.99	0.00	0.99	0.99	1.04
pH	Numeric	3.19	0.15	3.18	2.72	3.82
sulphates	Numeric	0.49	0.11	0.47	0.22	1.08
alcohol	Numeric	10.51	1.23	10.40	8.00	14.20
y*	Categorical	3.87	0.88	4.00	1.00	6.00

C.2 Prediction Accuracy

Tables 29 and 30 summarize in detail the results of the prediction accuracy exercise using real datasets for the ARPS and the AMSE, respectively. The first column *Data* specifies the dataset, the second column *Class* defines the number of outcome classes of the dependent variable and the third column *Size* indicates the number of observations. Similarly to the simulation results, the column *Statistic* contains summary statistics and statistical tests results for the equality of means between the results of the *Ordered Forest* and all the other methods.

Table 29: Empirical Results: Accuracy Measure = ARPS

Dataset Summary				**Comparison of Methods**							
Data	Class	Size	Statistic	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
mammography	3	412	mean	0.1776	0.2251	0.2089	0.1767	0.1823	0.1766	0.1826	0.1767
			st.dev.	0.0010	0.0027	0.0021	0.0013	0.0018	0.0008	0.0019	0.0007
			t-test	1.0000	0.0000	0.0000	1.0000		1.0000	0.3999	1.0000
			wilcox-test	1.0000	0.0000	0.0000	1.0000		1.0000	0.3153	1.0000
nhanes	5	1914	mean	0.1088	0.1089	0.1100	0.1085	0.1103	0.1137	0.1104	0.1159
			st.dev.	0.0004	0.0003	0.0004	0.0001	0.0002	0.0001	0.0002	0.0001
			t-test	1.0000	1.0000	0.9839	1.0000		0.0000	0.2106	0.0000
			wilcox-test	1.0000	1.0000	0.9738	1.0000		0.0000	0.2179	0.0000
supportstudy	5	798	mean	0.1872	0.1849	0.1834	0.1800	0.1823	0.1931	0.1857	0.1944
			st.dev.	0.0011	0.0010	0.0009	0.0008	0.0008	0.0003	0.0007	0.0004
			t-test	0.0000	0.0000	0.0052	1.0000		0.0000	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0073	1.0000		0.0000	0.0000	0.0000
vlbw	9	218	mean	0.1595	0.1713	0.1724	0.1603	0.1686	0.1623	0.1685	0.1642
			st.dev.	0.0011	0.0026	0.0030	0.0014	0.0021	0.0005	0.0020	0.0003
			t-test	1.0000	0.0100	0.0023	1.0000		1.0000	0.5143	1.0000
			wilcox-test	1.0000	0.0116	0.0010	1.0000		1.0000	0.5733	1.0000
winequality	6	4893	mean	0.0756	0.0501	0.0503	0.0596	0.0507	0.0673	0.0504	0.0683
			st.dev.	0.0000	0.0003	0.0002	0.0001	0.0002	0.0001	0.0002	0.0000
			t-test	0.0000	1.0000	0.9992	0.0000		0.0000	0.9971	0.0000
			wilcox-test	0.0000	0.9999	0.9986	0.0000		0.0000	0.9966	0.0000

Notes: Table reports the average measures of the RPS based on 10 repetitions of 10-fold cross-validation. The fourth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

Table 30: Empirical Results: Accuracy Measure = AMSE

Dataset Summary				**Comparison of Methods**							
Data	Class	Size	Statistic	Ologit	Naive	Ordinal	Cond.	Ordered	Ordered*	Multi	Multi*
mammography	3	412	mean	0.1754	0.2593	0.2222	0.1720	0.1766	0.1726	0.1770	0.1726
			st.dev.	0.0007	0.0025	0.0031	0.0008	0.0012	0.0004	0.0013	0.0004
			t-test	0.9923	0.0000	0.0000	1.0000		1.0000	0.2467	1.0000
			wilcox-test	0.9943	0.0000	0.0000	1.0000		1.0000	0.2179	1.0000
nhanes	5	1914	mean	0.1310	0.1309	0.1332	0.1304	0.1332	0.1329	0.1319	0.1343
			st.dev.	0.0003	0.0003	0.0003	0.0002	0.0003	0.0001	0.0003	0.0001
			t-test	1.0000	1.0000	0.7067	1.0000		0.9936	1.0000	0.0000
			wilcox-test	1.0000	1.0000	0.6579	1.0000		0.9955	1.0000	0.0000
supportstudy	5	798	mean	0.1124	0.1110	0.1094	0.1078	0.1088	0.1129	0.1101	0.1135
			st.dev.	0.0005	0.0004	0.0004	0.0004	0.0004	0.0002	0.0003	0.0002
			t-test	0.0000	0.0000	0.0020	1.0000		0.0000	0.0000	0.0000
			wilcox-test	0.0000	0.0000	0.0008	0.9999		0.0000	0.0000	0.0000
vlbw	9	218	mean	0.0944	0.0986	0.0990	0.0956	0.1008	0.0958	0.1006	0.0956
			st.dev.	0.0002	0.0008	0.0009	0.0004	0.0008	0.0003	0.0009	0.0002
			t-test	1.0000	1.0000	0.9999	1.0000		1.0000	0.7224	1.0000
			wilcox-test	1.0000	1.0000	0.9999	1.0000		1.0000	0.7821	1.0000
winequality	6	4893	mean	0.1001	0.0692	0.0698	0.0831	0.0702	0.0906	0.0693	0.0913
			st.dev.	0.0000	0.0003	0.0003	0.0001	0.0003	0.0001	0.0003	0.0001
			t-test	0.0000	1.0000	0.9960	0.0000		0.0000	1.0000	0.0000
			wilcox-test	0.0000	1.0000	0.9974	0.0000		0.0000	1.0000	0.0000

Notes: Table reports the average measures of the MSE based on 10 repetitions of 10-fold cross-validation. The fourth column *Statistic* shows the mean and the standard deviation of the accuracy measure for all methods. Additionally, *t-test* and *wilcox-test* contain the p-values of the parametric t-test as well as the nonparametric Wilcoxon test for the equality of means between the results of the *Ordered Forest* and all the other methods.

C.3 Marginal Effects

In what follows, the results for the marginal effects at mean and the mean marginal effects are presented for all considered datasets. Similarly as in the main text, the effects are computed for each outcome class of the dependent variable both for the *Ordered Forest* as well as for the ordered logit.

C.3.1 Data: mammography

Table 31: Marginal Effects at Mean: Mammography Dataset

Dataset		**Ordered Forest**					**Ordered Logit**				
Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
BSE	1	-0.2965	0.2199	-1.3481	0.1776		-0.2558	0.0912	-2.8056	0.0050	***
	2	0.1844	0.1131	1.6307	0.1030		0.1277	0.0486	2.6257	0.0086	***
	3	0.1121	0.1918	0.5845	0.5589		0.1280	0.0456	2.8076	0.0050	***
DECT	1	-0.1908	0.1932	-0.9874	0.3234		-0.0475	0.0529	-0.8978	0.3693	
	2	0.2441	0.1311	1.8623	0.0626	*	0.0237	0.0267	0.8897	0.3737	
	3	-0.0533	0.1835	-0.2906	0.7714		0.0238	0.0264	0.8997	0.3683	
HIST	1	-0.0854	0.1950	-0.4378	0.6615		-0.1829	0.0712	-2.5690	0.0102	**
	2	0.0602	0.1914	0.3143	0.7533		0.0914	0.0374	2.4459	0.0144	**
	3	0.0252	0.2662	0.0948	0.9245		0.0916	0.0359	2.5507	0.0108	**
PB	1	0.0180	0.1451	0.1243	0.9011		0.0345	0.0133	2.6056	0.0092	***
	2	-0.0990	0.1182	-0.8376	0.4023		-0.0172	0.0070	-2.4779	0.0132	**
	3	0.0810	0.1683	0.4812	0.6304		-0.0173	0.0067	-2.5860	0.0097	***
SYMPT	1	-0.2633	0.2303	-1.1433	0.2529		-0.1381	0.0300	-4.6085	0.0000	***
	2	0.3143	0.1141	2.7541	0.0059	***	0.0690	0.0173	3.9801	0.0001	***
	3	-0.0510	0.1742	-0.2928	0.7697		0.0691	0.0153	4.5242	0.0000	***
Significance levels correspond to: $* * *. < 0.01$, $* * . < 0.05$, $*. < 0.1$.											

Notes: Table shows the comparison of the marginal effects at the covariates means between the *Ordered Forest* and the ordered logit. The effects are estimated for all classes, together with the corresponding standard errors, t-values and p-values. The standard errors for the *Ordered Forest* are estimated using the weight-based inference and the standard errors for the ordered logit are obtained via the delta method.

Table 32: Mean Marginal Effects: Mammography Dataset

Dataset		**Ordered Forest**					**Ordered Logit**				
Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
BSE	1	-0.1523	0.1013	-1.5032	0.1328		-0.2227	0.0776	-2.8699	0.0041	***
	2	0.0757	0.0657	1.1511	0.2497		0.0836	0.0296	2.8229	0.0048	***
	3	0.0766	0.1064	0.7200	0.4715		0.1391	0.0507	2.7458	0.0060	***
DECT	1	-0.0825	0.0784	-1.0528	0.2924		-0.0414	0.0460	-0.8995	0.3684	
	2	0.1176	0.0534	2.2018	0.0277	**	0.0155	0.0173	0.8957	0.3704	
	3	-0.0351	0.1047	-0.3352	0.7375		0.0258	0.0288	0.8969	0.3698	
HIST	1	-0.0872	0.1180	-0.7394	0.4597		-0.1593	0.0609	-2.6154	0.0089	***
	2	0.0832	0.1063	0.7831	0.4335		0.0598	0.0242	2.4662	0.0137	**
	3	0.0040	0.1505	0.0263	0.9790		0.0995	0.0384	2.5914	0.0096	***
PB	1	0.0204	0.0695	0.2941	0.7687		0.0301	0.0113	2.6714	0.0076	***
	2	-0.0382	0.0589	-0.6483	0.5168		-0.0113	0.0044	-2.5935	0.0095	***
	3	0.0177	0.0745	0.2380	0.8119		-0.0188	0.0072	-2.5936	0.0095	***
SYMPT	1	-0.2237	0.1255	-1.7816	0.0748	*	-0.1202	0.0241	-4.9972	0.0000	***
	2	0.2472	0.0513	4.8180	0.0000	***	0.0451	0.0097	4.6455	0.0000	***
	3	-0.0236	0.0939	-0.2510	0.8018		0.0751	0.0168	4.4752	0.0000	***
Significance levels correspond to: $* * *. < 0.01$, $* * . < 0.05$, $*. < 0.1$.											

Notes: Table shows the comparison of the mean marginal effects between the *Ordered Forest* and the ordered logit. The effects are estimated for all classes, together with the corresponding standard errors, t-values and p-values. The standard errors for the *Ordered Forest* are estimated using the weight-based inference and the standard errors for the ordered logit are obtained via the delta method.

C.3.2 Data: nhanes

Table 33: Marginal Effects at Mean: Nhanes Dataset

Dataset		Ordered Forest					Ordered Logit				
Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
Acute	1	-0.0018	0.0034	-0.5200	0.6031		-0.0130	0.0052	-2.5150	0.0119	**
Illness	2	-0.0061	0.0065	-0.9416	0.3464		-0.0417	0.0171	-2.4379	0.0148	**
	3	0.0048	0.0075	0.6423	0.5206		0.0198	0.0074	2.6867	0.0072	***
	4	0.0011	0.0069	0.1615	0.8717		0.0303	0.0131	2.3121	0.0208	**
	5	0.0019	0.0115	0.1689	0.8659		0.0047	0.0021	2.1963	0.0281	**
age	1	-0.0002	0.0004	-0.5008	0.6165		-0.0004	0.0002	-2.0922	0.0364	**
	2	-0.0003	0.0010	-0.3376	0.7356		-0.0012	0.0006	-2.1036	0.0354	**
	3	0.0003	0.0009	0.2956	0.7675		0.0007	0.0003	2.0716	0.0383	**
	4	0.0002	0.0005	0.4970	0.6192		0.0009	0.0004	2.1011	0.0356	**
	5	0.0000	0.0001	0.0683	0.9456		0.0001	0.0001	2.0462	0.0407	**
alcohol	1	0.0009	0.0018	0.4777	0.6329		0.0001	0.0001	0.4906	0.6237	
	2	-0.0002	0.0046	-0.0344	0.9726		0.0002	0.0004	0.4906	0.6237	
	3	-0.0007	0.0044	-0.1676	0.8669		-0.0001	0.0002	-0.4902	0.6240	
	4	0.0001	0.0005	0.1580	0.8745		-0.0001	0.0003	-0.4906	0.6237	
	5	-0.0001	0.0004	-0.1176	0.9064		-0.0000	0.0000	-0.4901	0.6241	
asthma	1	-0.0033	0.0016	-1.9998	0.0455	**	-0.0178	0.0059	-3.0366	0.0024	***
	2	-0.0090	0.0087	-1.0383	0.2991		-0.0592	0.0208	-2.8487	0.0044	***
	3	-0.0170	0.0171	-0.9941	0.3202		0.0246	0.0067	3.6693	0.0002	***
	4	0.0275	0.0173	1.5951	0.1107		0.0452	0.0176	2.5661	0.0103	**
	5	0.0017	0.0048	0.3599	0.7189		0.0072	0.0030	2.3756	0.0175	**
BMI	1	-0.0019	0.0021	-0.9234	0.3558		-0.0022	0.0009	-2.3115	0.0208	**
	2	-0.0075	0.0058	-1.2857	0.1986		-0.0067	0.0029	-2.3256	0.0200	**
	3	0.0083	0.0055	1.5210	0.1283		0.0036	0.0016	2.2842	0.0224	**
	4	0.0010	0.0008	1.2231	0.2213		0.0046	0.0020	2.3203	0.0203	**
	5	0.0001	0.0009	0.0774	0.9383		0.0007	0.0003	2.2570	0.0240	**
BPdias	1	0.0002	0.0004	0.5164	0.6056		0.0003	0.0002	1.4160	0.1568	
	2	0.0003	0.0019	0.1860	0.8525		0.0010	0.0007	1.4177	0.1563	
	3	-0.0005	0.0019	-0.2693	0.7877		-0.0005	0.0004	-1.4083	0.1591	
	4	-0.0000	0.0001	-0.3651	0.7151		-0.0007	0.0005	-1.4175	0.1563	
	5	-0.0000	0.0001	-0.1235	0.9017		-0.0001	0.0001	-1.3992	0.1618	
BPsys	1	-0.0003	0.0001	-2.1923	0.0284	**	-0.0003	0.0002	-1.8747	0.0608	*
	2	-0.0004	0.0007	-0.5487	0.5832		-0.0009	0.0005	-1.8820	0.0598	*
	3	0.0007	0.0006	1.1788	0.2385		0.0005	0.0003	1.8567	0.0634	*
	4	-0.0000	0.0003	-0.1614	0.8718		0.0006	0.0003	1.8814	0.0599	*
	5	-0.0000	0.0002	-0.1657	0.8684		0.0001	0.0001	1.8484	0.0645	*
Cholesterol	1	-0.0001	0.0001	-1.0401	0.2983		-0.0001	0.0001	-1.8212	0.0686	*
	2	-0.0003	0.0006	-0.5020	0.6157		-0.0003	0.0002	-1.8276	0.0676	*
	3	0.0004	0.0005	0.7864	0.4317		0.0002	0.0001	1.8068	0.0708	*
	4	0.0000	0.0003	0.0349	0.9722		0.0002	0.0001	1.8253	0.0680	*
	5	0.0000	0.0000	0.0122	0.9903		0.0000	0.0000	1.7956	0.0726	*
chronic	1	-0.0016	0.0008	-2.0669	0.0387	**	-0.0155	0.0078	-1.9844	0.0472	**
Bronchitis	2	-0.0255	0.0147	-1.7405	0.0818	*	-0.0519	0.0282	-1.8362	0.0663	*
	3	0.0101	0.0213	0.4727	0.6364		0.0212	0.0082	2.5786	0.0099	***
	4	0.0119	0.0227	0.5235	0.6006		0.0398	0.0242	1.6470	0.0995	*
	5	0.0052	0.0088	0.5871	0.5571		0.0063	0.0041	1.5517	0.1207	
cig	1	-0.0080	0.0023	-3.5146	0.0004	***	-0.0006	0.0052	-0.1226	0.9024	
	2	-0.0078	0.0092	-0.8403	0.4007		-0.0020	0.0160	-0.1226	0.9024	
	3	0.0160	0.0078	2.0405	0.0413	**	0.0010	0.0085	0.1226	0.9024	
	4	-0.0003	0.0040	-0.0838	0.9333		0.0014	0.0110	0.1227	0.9024	

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Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
	5	0.0001	0.0005	0.2873	0.7739		0.0002	0.0017	0.1226	0.9024	
country of birth	1	0.0004	0.0014	0.2836	0.7767		-0.0058	0.0023	-2.4975	0.0125	**
	2	-0.0383	0.0101	-3.7986	0.0001	***	-0.0177	0.0070	-2.5150	0.0119	**
	3	0.0240	0.0095	2.5330	0.0113	**	0.0094	0.0038	2.4553	0.0141	**
	4	0.0145	0.0073	1.9856	0.0471	**	0.0122	0.0049	2.5144	0.0119	**
	5	-0.0007	0.0026	-0.2677	0.7889		0.0018	0.0008	2.4303	0.0151	**
depression	1	-0.0162	0.0068	-2.3879	0.0169	**	-0.0259	0.0039	-6.5902	0.0000	***
	2	-0.0485	0.0256	-1.8961	0.0579	*	-0.0794	0.0112	-7.0622	0.0000	***
	3	-0.0323	0.0460	-0.7014	0.4831		0.0422	0.0069	6.1140	0.0000	***
	4	0.0879	0.0510	1.7219	0.0851	*	0.0549	0.0080	6.8425	0.0000	***
	5	0.0091	0.0080	1.1391	0.2547		0.0083	0.0015	5.6826	0.0000	***
diabetes	1	-0.0065	0.0014	-4.6666	0.0000	***	-0.0360	0.0053	-6.8105	0.0000	***
	2	-0.0924	0.0237	-3.8952	0.0001	***	-0.1306	0.0198	-6.6046	0.0000	***
	3	0.0168	0.0344	0.4892	0.6247		0.0304	0.0074	4.0835	0.0000	***
	4	0.0737	0.0306	2.4114	0.0159	**	0.1160	0.0228	5.0847	0.0000	***
	5	0.0083	0.0180	0.4598	0.6457		0.0202	0.0049	4.1153	0.0000	***
education	1	0.0052	0.0020	2.5685	0.0102	**	0.0212	0.0025	8.3573	0.0000	***
	2	0.0869	0.0424	2.0509	0.0403	**	0.0649	0.0072	9.0756	0.0000	***
	3	-0.0685	0.0333	-2.0552	0.0399	**	-0.0345	0.0048	-7.2061	0.0000	***
	4	-0.0231	0.0183	-1.2651	0.2058		-0.0448	0.0050	-8.8831	0.0000	***
	5	-0.0004	0.0012	-0.3236	0.7463		-0.0068	0.0010	-6.5114	0.0000	***
heart Failure	1	0.0000	0.0000	0.0000	1.0000		-0.0288	0.0091	-3.1696	0.0015	***
	2	-0.0168	0.0073	-2.3092	0.0209	**	-0.1068	0.0395	-2.7007	0.0069	***
	3	-0.0266	0.0607	-0.4388	0.6608		0.0228	0.0094	2.4225	0.0154	**
	4	0.0369	0.0481	0.7683	0.4423		0.0960	0.0466	2.0618	0.0392	**
	5	0.0065	0.0719	0.0903	0.9280		0.0167	0.0095	1.7645	0.0777	*
heavy Drinker	1	-0.0057	0.0024	-2.3815	0.0172	**	-0.0127	0.0061	-2.0982	0.0359	**
	2	-0.0221	0.0092	-2.4012	0.0163	**	-0.0413	0.0207	-2.0010	0.0454	**
	3	0.0224	0.0088	2.5528	0.0107	**	0.0189	0.0081	2.3399	0.0193	**
	4	0.0032	0.0060	0.5317	0.5949		0.0305	0.0163	1.8718	0.0612	*
	5	0.0023	0.0100	0.2296	0.8184		0.0047	0.0027	1.7838	0.0745	*
marital status	1	-0.0007	0.0007	-0.9425	0.3459		-0.0025	0.0015	-1.6584	0.0972	*
	2	-0.0016	0.0041	-0.3818	0.7026		-0.0076	0.0045	-1.6631	0.0963	*
	3	0.0027	0.0037	0.7438	0.4570		0.0040	0.0024	1.6471	0.0995	*
	4	-0.0005	0.0026	-0.1987	0.8425		0.0052	0.0031	1.6618	0.0965	*
	5	0.0000	0.0002	0.1985	0.8426		0.0008	0.0005	1.6375	0.1015	
medical Place To Go	1	-0.0004	0.0019	-0.2220	0.8243		0.0003	0.0036	0.0728	0.9420	
	2	0.0049	0.0089	0.5515	0.5813		0.0008	0.0111	0.0728	0.9420	
	3	-0.0033	0.0073	-0.4562	0.6483		-0.0004	0.0059	-0.0728	0.9420	
	4	-0.0008	0.0037	-0.2087	0.8347		-0.0006	0.0077	-0.0728	0.9420	
	5	-0.0004	0.0025	-0.1591	0.8736		-0.0001	0.0012	-0.0728	0.9420	
race	1	0.0043	0.0014	3.0319	0.0024	***	0.0050	0.0026	1.9649	0.0494	**
	2	0.0172	0.0094	1.8371	0.0662	*	0.0154	0.0078	1.9785	0.0479	**
	3	-0.0196	0.0086	-2.2729	0.0230	**	-0.0082	0.0042	-1.9422	0.0521	*
	4	-0.0025	0.0059	-0.4262	0.6700		-0.0106	0.0054	-1.9803	0.0477	**
	5	0.0005	0.0025	0.2158	0.8292		-0.0016	0.0008	-1.9426	0.0521	*
sex	1	0.0075	0.0059	1.2739	0.2027		0.0096	0.0059	1.6327	0.1025	
	2	-0.0153	0.0098	-1.5559	0.1197		0.0294	0.0179	1.6455	0.0999	*
	3	0.0082	0.0089	0.9128	0.3613		-0.0156	0.0096	-1.6286	0.1034	
	4	-0.0002	0.0019	-0.1288	0.8975		-0.0203	0.0124	-1.6424	0.1005	
	5	-0.0001	0.0006	-0.1969	0.8439		-0.0031	0.0019	-1.6152	0.1063	
sleep Trouble	1	-0.0037	0.0017	-2.2193	0.0265	**	-0.0021	0.0024	-0.8571	0.3914	
	2	-0.0141	0.0125	-1.1235	0.2612		-0.0063	0.0073	-0.8575	0.3912	
	3	0.0109	0.0107	1.0153	0.3100		0.0033	0.0039	0.8556	0.3922	
	4	0.0069	0.0122	0.5625	0.5737		0.0043	0.0051	0.8574	0.3912	

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Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
	5	0.0000	0.0004	0.1169	0.9070		0.0007	0.0008	0.8529	0.3937	
stroke	1	0.0000	0.0000	0.0000	1.0000		-0.0312	0.0075	-4.1883	0.0000	***
	2	-0.0333	0.0094	-3.5432	0.0004	***	-0.1175	0.0326	-3.6003	0.0003	***
	3	0.0145	0.0189	0.7676	0.4427		0.0209	0.0111	1.8806	0.0600	*
	4	-0.0045	0.0228	-0.1951	0.8453		0.1086	0.0405	2.6834	0.0073	***
	5	0.0232	0.0344	0.6746	0.4999		0.0193	0.0086	2.2298	0.0258	**
Tooth	1	-0.0052	0.0021	-2.4413	0.0146	**	-0.0230	0.0025	-9.0333	0.0000	***
Cond	2	-0.1008	0.0238	-4.2327	0.0000	***	-0.0705	0.0071	-9.9072	0.0000	***
	3	0.0193	0.0488	0.3946	0.6932		0.0374	0.0050	7.5175	0.0000	***
	4	0.0862	0.0551	1.5647	0.1176		0.0487	0.0050	9.8287	0.0000	***
	5	0.0004	0.0024	0.1735	0.8623		0.0074	0.0011	6.7816	0.0000	***
waist	1	-0.0006	0.0004	-1.4128	0.1577		-0.0006	0.0004	-1.3994	0.1617	
circum	2	0.0009	0.0018	0.5026	0.6152		-0.0017	0.0012	-1.4015	0.1611	
	3	-0.0003	0.0017	-0.1823	0.8553		0.0009	0.0007	1.3900	0.1645	
	4	0.0000	0.0006	0.0112	0.9911		0.0012	0.0009	1.4026	0.1607	
	5	0.0000	0.0000	0.0000	1.0000		0.0002	0.0001	1.3859	0.1658	
wakeUp	1	-0.0018	0.0018	-1.0045	0.3152		-0.0033	0.0023	-1.4339	0.1516	
	2	-0.0088	0.0125	-0.7028	0.4822		-0.0102	0.0071	-1.4402	0.1498	
	3	0.0107	0.0116	0.9237	0.3557		0.0054	0.0038	1.4273	0.1535	
	4	-0.0002	0.0023	-0.0822	0.9345		0.0071	0.0049	1.4390	0.1501	
	5	0.0001	0.0004	0.2059	0.8369		0.0011	0.0007	1.4280	0.1533	
WBCcount	1	0.0010	0.0022	0.4391	0.6606		0.0009	0.0009	0.9893	0.3225	
	2	-0.0089	0.0080	-1.1100	0.2670		0.0027	0.0027	0.9900	0.3222	
	3	0.0038	0.0077	0.4984	0.6182		-0.0014	0.0015	-0.9864	0.3239	
	4	0.0041	0.0048	0.8583	0.3907		-0.0019	0.0019	-0.9900	0.3222	
	5	-0.0000	0.0029	-0.0022	0.9983		-0.0003	0.0003	-0.9849	0.3247	

Significance levels correspond to: $***. < 0.01$, $**. < 0.05$, $*. < 0.1$.

Notes: Table shows the comparison of the marginal effects at the covariates means between the *Ordered Forest* and the ordered logit. The effects are estimated for all classes, together with the corresponding standard errors, t-values and p-values. The standard errors for the *Ordered Forest* are estimated using the weight-based inference and the standard errors for the ordered logit are obtained via the delta method.

Table 34: Mean Marginal Effects: Nhanes Dataset

Dataset		Ordered Forest					Ordered Logit				
Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
Acute Illness	1	-0.0035	0.0020	-1.7500	0.0801	*	-0.0192	0.0076	-2.5131	0.0120	**
	2	-0.0036	0.0036	-0.9918	0.3213		-0.0253	0.0108	-2.3431	0.0191	**
	3	0.0012	0.0047	0.2525	0.8007		0.0119	0.0046	2.6025	0.0093	***
	4	0.0022	0.0062	0.3585	0.7199		0.0237	0.0101	2.3507	0.0187	**
	5	0.0037	0.0109	0.3407	0.7333		0.0090	0.0039	2.2896	0.0220	**
age	1	-0.0007	0.0002	-3.2796	0.0010	***	-0.0006	0.0003	-2.0982	0.0359	**
	2	0.0002	0.0004	0.5748	0.5654		-0.0007	0.0003	-2.1099	0.0349	**
	3	-0.0001	0.0004	-0.3504	0.7261		0.0004	0.0002	2.1012	0.0356	**
	4	0.0006	0.0002	2.4133	0.0158	**	0.0007	0.0003	2.1053	0.0353	**
	5	0.0000	0.0001	0.3204	0.7486		0.0003	0.0001	2.0708	0.0384	**
alcohol	1	0.0011	0.0009	1.1316	0.2578		0.0001	0.0002	0.4907	0.6236	
	2	0.0020	0.0021	0.9193	0.3579		0.0001	0.0002	0.4906	0.6237	
	3	0.0010	0.0020	0.4786	0.6322		-0.0001	0.0001	-0.4906	0.6237	
	4	-0.0033	0.0017	-1.9123	0.0558	*	-0.0001	0.0002	-0.4907	0.6237	
	5	-0.0007	0.0010	-0.6934	0.4881		-0.0000	0.0001	-0.4903	0.6239	
asthma	1	-0.0025	0.0011	-2.2316	0.0256	**	-0.0266	0.0089	-3.0008	0.0027	***
	2	-0.0022	0.0039	-0.5566	0.5778		-0.0368	0.0137	-2.6758	0.0075	***
	3	-0.0050	0.0090	-0.5638	0.5729		0.0153	0.0045	3.4041	0.0007	***
	4	0.0063	0.0075	0.8415	0.4001		0.0344	0.0129	2.6660	0.0077	***
	5	0.0034	0.0051	0.6653	0.5059		0.0135	0.0054	2.5135	0.0120	**
BMI	1	-0.0016	0.0006	-2.5492	0.0108	**	-0.0032	0.0014	-2.3182	0.0204	**
	2	-0.0030	0.0010	-3.1102	0.0019	***	-0.0039	0.0017	-2.3278	0.0199	**
	3	0.0030	0.0009	3.1571	0.0016	***	0.0021	0.0009	2.2983	0.0215	**
	4	0.0012	0.0006	1.9253	0.0542	*	0.0037	0.0016	2.3333	0.0196	**
	5	0.0003	0.0006	0.4878	0.6257		0.0014	0.0006	2.2857	0.0223	**
BPdias	1	-0.0002	0.0002	-1.2828	0.1996		0.0005	0.0003	1.4176	0.1563	
	2	0.0008	0.0004	2.0067	0.0448	**	0.0006	0.0004	1.4184	0.1561	
	3	-0.0002	0.0004	-0.6035	0.5462		-0.0003	0.0002	-1.4178	0.1562	
	4	-0.0003	0.0003	-1.0130	0.3111		-0.0005	0.0004	-1.4173	0.1564	
	5	-0.0000	0.0002	-0.0291	0.9768		-0.0002	0.0001	-1.4087	0.1589	
BPsys	1	-0.0001	0.0002	-0.6735	0.5006		-0.0004	0.0002	-1.8801	0.0601	*
	2	-0.0002	0.0002	-0.7668	0.4432		-0.0005	0.0003	-1.8825	0.0598	*
	3	0.0001	0.0002	0.6724	0.5013		0.0003	0.0002	1.8691	0.0616	*
	4	0.0002	0.0001	1.1666	0.2434		0.0005	0.0003	1.8840	0.0596	*
	5	-0.0000	0.0001	-0.0913	0.9273		0.0002	0.0001	1.8662	0.0620	*
Cholesterol	1	-0.0001	0.0000	-1.9336	0.0532	*	-0.0002	0.0001	-1.8253	0.0680	*
	2	0.0000	0.0001	0.3347	0.7379		-0.0002	0.0001	-1.8281	0.0675	*
	3	0.0001	0.0001	0.7192	0.4720		0.0001	0.0001	1.8160	0.0694	*
	4	-0.0000	0.0001	-0.0785	0.9374		0.0002	0.0001	1.8302	0.0672	*
	5	-0.0000	0.0000	-0.0415	0.9669		0.0001	0.0000	1.8098	0.0703	*
chronic Bronchitis	1	-0.0013	0.0009	-1.4752	0.1402		-0.0231	0.0119	-1.9495	0.0512	*
	2	-0.0124	0.0068	-1.8302	0.0672	*	-0.0324	0.0189	-1.7128	0.0867	*
	3	-0.0067	0.0106	-0.6315	0.5277		0.0133	0.0059	2.2572	0.0240	**
	4	0.0155	0.0093	1.6701	0.0949	*	0.0303	0.0177	1.7123	0.0868	*
	5	0.0049	0.0079	0.6221	0.5339		0.0119	0.0073	1.6294	0.1032	
cig	1	-0.0042	0.0016	-2.6060	0.0092	***	-0.0009	0.0076	-0.1227	0.9024	
	2	-0.0015	0.0032	-0.4697	0.6386		-0.0011	0.0093	-0.1225	0.9025	
	3	0.0054	0.0028	1.9314	0.0534	*	0.0006	0.0049	0.1225	0.9025	
	4	0.0000	0.0022	0.0198	0.9842		0.0011	0.0088	0.1226	0.9024	
	5	0.0003	0.0010	0.2799	0.7795		0.0004	0.0033	0.1227	0.9023	
country of birth	1	-0.0004	0.0007	-0.5361	0.5919		-0.0084	0.0034	-2.5107	0.0120	**
	2	-0.0197	0.0046	-4.3165	0.0000	***	-0.0103	0.0041	-2.5124	0.0120	**

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Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
	3	0.0117	0.0046	2.5288	0.0114	**	0.0054	0.0022	2.4810	0.0131	**
	4	0.0090	0.0038	2.3771	0.0175	**	0.0097	0.0038	2.5190	0.0118	**
	5	-0.0006	0.0024	-0.2627	0.7928		0.0037	0.0015	2.4750	0.0133	**
depression	1	-0.0163	0.0053	-3.0802	0.0021	***	-0.0379	0.0056	-6.7712	0.0000	***
	2	-0.0557	0.0158	-3.5277	0.0004	***	-0.0463	0.0066	-7.0602	0.0000	***
	3	-0.0171	0.0240	-0.7116	0.4767		0.0244	0.0039	6.1918	0.0000	***
	4	0.0768	0.0259	2.9620	0.0031	***	0.0435	0.0060	7.2491	0.0000	***
	5	0.0122	0.0076	1.6171	0.1059		0.0164	0.0026	6.2019	0.0000	***
diabetes	1	-0.0045	0.0008	-5.6471	0.0000	***	-0.0534	0.0075	-7.1385	0.0000	***
	2	-0.0719	0.0128	-5.6178	0.0000	***	-0.0914	0.0163	-5.5945	0.0000	***
	3	-0.0039	0.0204	-0.1924	0.8475		0.0235	0.0035	6.8145	0.0000	***
	4	0.0704	0.0152	4.6159	0.0000	***	0.0867	0.0162	5.3684	0.0000	***
	5	0.0100	0.0162	0.6167	0.5374		0.0346	0.0073	4.7343	0.0000	***
education	1	0.0034	0.0021	1.6707	0.0948	*	0.0310	0.0036	8.6650	0.0000	***
	2	0.0683	0.0196	3.4769	0.0005	***	0.0378	0.0039	9.5840	0.0000	***
	3	-0.0460	0.0147	-3.1215	0.0018	***	-0.0199	0.0023	-8.6028	0.0000	***
	4	-0.0254	0.0119	-2.1431	0.0321	**	-0.0355	0.0039	-9.2023	0.0000	***
	5	-0.0003	0.0013	-0.2455	0.8061		-0.0134	0.0018	-7.3984	0.0000	***
heart Failure	1	-0.0001	0.0000	-6.1333	0.0000	***	-0.0445	0.0147	-3.0275	0.0025	***
	2	-0.0094	0.0084	-1.1165	0.2642		-0.0723	0.0312	-2.3205	0.0203	**
	3	-0.0140	0.0327	-0.4276	0.6689		0.0189	0.0027	6.9672	0.0000	***
	4	0.0262	0.0498	0.5267	0.5984		0.0681	0.0300	2.2698	0.0232	**
	5	-0.0027	0.0702	-0.0389	0.9690		0.0298	0.0153	1.9417	0.0522	*
heavy Drinker	1	-0.0011	0.0012	-0.9120	0.3618		-0.0188	0.0090	-2.0846	0.0371	**
	2	-0.0117	0.0045	-2.5987	0.0094	***	-0.0254	0.0133	-1.9027	0.0571	*
	3	0.0049	0.0051	0.9683	0.3329		0.0116	0.0053	2.1921	0.0284	**
	4	0.0051	0.0055	0.9330	0.3508		0.0236	0.0123	1.9163	0.0553	*
	5	0.0027	0.0096	0.2867	0.7744		0.0091	0.0049	1.8575	0.0632	*
marital status	1	-0.0006	0.0008	-0.7842	0.4329		-0.0036	0.0022	-1.6598	0.0969	*
	2	-0.0009	0.0014	-0.6838	0.4941		-0.0044	0.0026	-1.6667	0.0956	*
	3	0.0011	0.0013	0.8657	0.3867		0.0023	0.0014	1.6587	0.0972	*
	4	0.0004	0.0013	0.3151	0.7527		0.0041	0.0025	1.6651	0.0959	*
	5	0.0000	0.0005	0.0106	0.9916		0.0016	0.0009	1.6484	0.0993	*
medical Place To Go	1	-0.0001	0.0014	-0.0854	0.9319		0.0004	0.0053	0.0728	0.9420	
	2	0.0037	0.0030	1.2483	0.2119		0.0005	0.0065	0.0728	0.9420	
	3	0.0017	0.0031	0.5579	0.5769		-0.0002	0.0034	-0.0728	0.9420	
	4	-0.0042	0.0020	-2.0705	0.0384	**	-0.0004	0.0061	-0.0728	0.9420	
	5	-0.0011	0.0024	-0.4592	0.6461		-0.0002	0.0023	-0.0728	0.9420	
race	1	0.0027	0.0011	2.5402	0.0111	**	0.0073	0.0037	1.9699	0.0488	**
	2	0.0132	0.0040	3.3347	0.0009	***	0.0090	0.0045	1.9840	0.0473	**
	3	-0.0110	0.0042	-2.6334	0.0085	***	-0.0047	0.0024	-1.9569	0.0504	*
	4	-0.0054	0.0032	-1.7011	0.0889	*	-0.0084	0.0042	-1.9846	0.0472	**
	5	0.0005	0.0027	0.2021	0.8398		-0.0032	0.0016	-1.9595	0.0501	*
sex	1	0.0051	0.0030	1.6862	0.0918	*	0.0140	0.0086	1.6395	0.1011	
	2	-0.0051	0.0030	-1.6929	0.0905	*	0.0171	0.0104	1.6491	0.0991	*
	3	0.0019	0.0023	0.8190	0.4128		-0.0089	0.0054	-1.6515	0.0986	*
	4	-0.0010	0.0019	-0.5180	0.6044		-0.0162	0.0099	-1.6388	0.1013	
	5	-0.0009	0.0019	-0.4967	0.6194		-0.0061	0.0037	-1.6323	0.1026	
sleep Trouble	1	-0.0017	0.0016	-1.0947	0.2736		-0.0030	0.0035	-0.8574	0.3912	
	2	-0.0018	0.0041	-0.4444	0.6568		-0.0037	0.0043	-0.8583	0.3907	
	3	-0.0037	0.0056	-0.6554	0.5122		0.0019	0.0022	0.8589	0.3904	
	4	0.0072	0.0072	1.0120	0.3115		0.0034	0.0040	0.8576	0.3911	
	5	-0.0000	0.0006	-0.0213	0.9830		0.0013	0.0015	0.8546	0.3928	
stroke	1	-0.0000	0.0000	-4.6343	0.0000	***	-0.0484	0.0120	-4.0425	0.0001	***
	2	-0.0199	0.0085	-2.3486	0.0188	**	-0.0810	0.0266	-3.0507	0.0023	***
	3	0.0087	0.0077	1.1393	0.2546		0.0190	0.0031	6.0505	0.0000	***

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Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
	4	-0.0154	0.0211	-0.7296	0.4656		0.0767	0.0259	2.9564	0.0031	***
	5	0.0265	0.0279	0.9523	0.3409		0.0337	0.0135	2.4900	0.0128	**
Tooth	1	-0.0045	0.0015	-2.9013	0.0037	***	-0.0336	0.0035	-9.5125	0.0000	***
Cond	2	-0.0609	0.0145	-4.2040	0.0000	***	-0.0411	0.0039	-10.5726	0.0000	***
	3	-0.0303	0.0271	-1.1157	0.2645		0.0216	0.0023	9.3228	0.0000	***
	4	0.0946	0.0327	2.8960	0.0038	***	0.0386	0.0038	10.1793	0.0000	***
	5	0.0010	0.0023	0.4389	0.6608		0.0145	0.0019	7.8454	0.0000	***
waist	1	-0.0006	0.0002	-3.1634	0.0016	***	-0.0008	0.0006	-1.4016	0.1610	
circum	2	-0.0013	0.0004	-3.3726	0.0007	***	-0.0010	0.0007	-1.4040	0.1603	
	3	0.0012	0.0004	3.3524	0.0008	***	0.0005	0.0004	1.4047	0.1601	
	4	0.0006	0.0003	2.3878	0.0169	**	0.0009	0.0007	1.4007	0.1613	
	5	0.0001	0.0002	0.4812	0.6304		0.0004	0.0003	1.3952	0.1630	
wakeUp	1	-0.0020	0.0016	-1.2417	0.2143		-0.0049	0.0034	-1.4365	0.1509	
	2	-0.0025	0.0035	-0.6984	0.4849		-0.0059	0.0041	-1.4398	0.1499	
	3	0.0028	0.0032	0.8858	0.3757		0.0031	0.0022	1.4280	0.1533	
	4	0.0016	0.0018	0.8958	0.3703		0.0056	0.0039	1.4407	0.1497	
	5	0.0000	0.0006	0.0277	0.9779		0.0021	0.0015	1.4353	0.1512	
WBCcount	1	-0.0011	0.0009	-1.1499	0.2502		0.0013	0.0013	0.9901	0.3221	
	2	-0.0023	0.0017	-1.3251	0.1851		0.0016	0.0016	0.9899	0.3222	
	3	0.0027	0.0015	1.7432	0.0813	*	-0.0008	0.0008	-0.9880	0.3231	
	4	0.0004	0.0011	0.3634	0.7163		-0.0015	0.0015	-0.9904	0.3220	
	5	0.0003	0.0011	0.2339	0.8151		-0.0006	0.0006	-0.9879	0.3232	

Significance levels correspond to: $***. < 0.01$, $**. < 0.05$, $*. < 0.1$.

Notes: Table shows the comparison of the mean marginal effects between the *Ordered Forest* and the ordered logit. The effects are estimated for all classes, together with the corresponding standard errors, t-values and p-values. The standard errors for the *Ordered Forest* are estimated using the weight-based inference and the standard errors for the ordered logit are obtained via the delta method.

C.3.3 Data: supportstudy

Table 35: Marginal Effects at Mean: Support Study Dataset

Dataset		Ordered Forest					Ordered Logit				
Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
age	1	-0.0008	0.0022	-0.3806	0.7035		-0.0025	0.0011	-2.3050	0.0212	**
	2	0.0013	0.0017	0.8107	0.4175		-0.0003	0.0002	-2.0671	0.0387	**
	3	-0.0013	0.0019	-0.6882	0.4913		0.0001	0.0001	1.4735	0.1406	
	4	0.0009	0.0018	0.5167	0.6054		0.0000	0.0000	1.6236	0.1045	
	5	-0.0001	0.0018	-0.0799	0.9363		0.0027	0.0012	2.3101	0.0209	**
avtisst	1	-0.0036	0.0020	-1.8184	0.0690	*	-0.0172	0.0017	-9.8795	0.0000	***
	2	0.0003	0.0017	0.1867	0.8519		-0.0022	0.0006	-3.8999	0.0001	***
	3	0.0023	0.0016	1.4074	0.1593		0.0006	0.0003	1.9432	0.0520	*
	4	-0.0004	0.0010	-0.4627	0.6436		0.0002	0.0001	2.2665	0.0234	**
	5	0.0015	0.0014	1.0804	0.2800		0.0187	0.0020	9.4419	0.0000	***
charges	1	-0.0000	0.0000	-1.3547	0.1755		0.0000	0.0000	1.8483	0.0646	*
	2	0.0000	0.0000	1.2867	0.1982		0.0000	0.0000	1.6418	0.1006	
	3	0.0000	0.0000	0.6363	0.5246		-0.0000	0.0000	-1.4242	0.1544	
	4	0.0000	0.0000	0.7488	0.4540		-0.0000	0.0000	-1.4696	0.1417	
	5	-0.0000	0.0000	-0.4501	0.6526		-0.0000	0.0000	-1.8310	0.0671	*
crea	1	-0.0305	0.0234	-1.3032	0.1925		-0.0243	0.0103	-2.3478	0.0189	**
	2	-0.0172	0.0222	-0.7734	0.4393		-0.0032	0.0016	-2.0295	0.0424	**
	3	0.0190	0.0202	0.9370	0.3487		0.0008	0.0005	1.5360	0.1245	
	4	0.0117	0.0146	0.8000	0.4237		0.0003	0.0002	1.6622	0.0965	*
	5	0.0170	0.0273	0.6242	0.5325		0.0264	0.0113	2.3343	0.0196	**
dzgroup	1	-0.0167	0.0100	-1.6654	0.0958	*	-0.0453	0.0073	-6.2305	0.0000	***
	2	-0.0550	0.0174	-3.1661	0.0015	***	-0.0059	0.0017	-3.4993	0.0005	***
	3	-0.0115	0.0144	-0.8012	0.4230		0.0016	0.0008	1.8889	0.0589	*
	4	-0.0042	0.0086	-0.4937	0.6215		0.0005	0.0002	2.1800	0.0293	**
	5	0.0875	0.0293	2.9819	0.0029	***	0.0491	0.0080	6.1109	0.0000	***
hrt	1	-0.0002	0.0007	-0.2297	0.8183		-0.0011	0.0006	-1.9711	0.0487	**
	2	0.0002	0.0006	0.3185	0.7501		-0.0001	0.0001	-1.8058	0.0710	*
	3	-0.0005	0.0010	-0.4462	0.6555		0.0000	0.0000	1.3738	0.1695	
	4	0.0006	0.0006	1.0007	0.3170		0.0000	0.0000	1.4953	0.1348	
	5	-0.0002	0.0016	-0.0971	0.9227		0.0012	0.0006	1.9726	0.0485	**
meanbp	1	0.0003	0.0003	0.8334	0.4046		0.0013	0.0006	2.1434	0.0321	**
	2	-0.0005	0.0005	-0.9325	0.3511		0.0002	0.0001	1.9143	0.0556	*
	3	0.0001	0.0005	0.2744	0.7838		-0.0000	0.0000	-1.4432	0.1490	
	4	0.0002	0.0003	0.5978	0.5500		-0.0000	0.0000	-1.5715	0.1161	
	5	-0.0001	0.0004	-0.2825	0.7776		-0.0014	0.0007	-2.1402	0.0323	**
num.co	1	-0.0025	0.0052	-0.4857	0.6272		-0.0024	0.0128	-0.1846	0.8536	
	2	0.0033	0.0040	0.8202	0.4121		-0.0003	0.0017	-0.1844	0.8537	
	3	-0.0047	0.0079	-0.5898	0.5553		0.0001	0.0004	0.1838	0.8542	
	4	0.0031	0.0053	0.5737	0.5661		0.0000	0.0001	0.1840	0.8540	
	5	0.0008	0.0106	0.0784	0.9375		0.0026	0.0139	0.1846	0.8536	
race	1	-0.0095	0.0071	-1.3378	0.1810		-0.0062	0.0183	-0.3400	0.7339	
	2	0.0068	0.0050	1.3535	0.1759		-0.0008	0.0024	-0.3396	0.7341	
	3	0.0081	0.0055	1.4697	0.1416		0.0002	0.0006	0.3341	0.7383	
	4	-0.0045	0.0039	-1.1592	0.2464		0.0001	0.0002	0.3361	0.7368	
	5	-0.0008	0.0078	-0.1078	0.9141		0.0068	0.0199	0.3401	0.7338	
resp	1	0.0010	0.0013	0.7541	0.4508		-0.0060	0.0018	-3.3087	0.0009	***
	2	-0.0002	0.0025	-0.0955	0.9239		-0.0008	0.0003	-2.5951	0.0095	***
	3	-0.0024	0.0027	-0.8679	0.3855		0.0002	0.0001	1.7077	0.0877	*
	4	-0.0010	0.0017	-0.6190	0.5359		0.0001	0.0000	1.9037	0.0569	*

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Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
	5	0.0027	0.0017	1.5985	0.1099		0.0065	0.0020	3.2860	0.0010	***
scoma	1	0.0000	0.0000	0.0000	1.0000		-0.0044	0.0008	-5.3793	0.0000	***
	2	-0.0001	0.0001	-2.0341	0.0419	**	-0.0006	0.0002	-3.2495	0.0012	***
	3	0.0002	0.0001	1.3658	0.1720		0.0002	0.0001	1.8912	0.0586	*
	4	-0.0005	0.0001	-3.2260	0.0013	***	0.0000	0.0000	2.1553	0.0311	**
	5	0.0004	0.0001	5.1852	0.0000	***	0.0048	0.0009	5.2395	0.0000	***
sex	1	0.0010	0.0077	0.1231	0.9020		-0.0498	0.0326	-1.5286	0.1264	
	2	0.0048	0.0076	0.6314	0.5278		-0.0065	0.0045	-1.4394	0.1500	
	3	-0.0077	0.0066	-1.1632	0.2448		0.0017	0.0014	1.2097	0.2264	
	4	0.0024	0.0056	0.4372	0.6620		0.0005	0.0004	1.2770	0.2016	
	5	-0.0005	0.0124	-0.0367	0.9708		0.0541	0.0354	1.5271	0.1267	
sod	1	0.0026	0.0024	1.0949	0.2735		0.0019	0.0027	0.6870	0.4921	
	2	-0.0012	0.0019	-0.6318	0.5275		0.0002	0.0004	0.6772	0.4983	
	3	-0.0006	0.0019	-0.3051	0.7603		-0.0001	0.0001	-0.6504	0.5155	
	4	0.0005	0.0018	0.2849	0.7757		-0.0000	0.0000	-0.6591	0.5098	
	5	-0.0014	0.0022	-0.6291	0.5293		-0.0020	0.0030	-0.6867	0.4923	
temp	1	0.0044	0.0103	0.4229	0.6724		0.0345	0.0155	2.2297	0.0258	**
	2	-0.0016	0.0080	-0.2039	0.8385		0.0045	0.0023	1.9729	0.0485	**
	3	-0.0001	0.0063	-0.0225	0.9821		-0.0012	0.0008	-1.4786	0.1392	
	4	0.0013	0.0056	0.2235	0.8231		-0.0004	0.0002	-1.6114	0.1071	
	5	-0.0039	0.0051	-0.7534	0.4512		-0.0374	0.0168	-2.2245	0.0261	**
wblc	1	-0.0029	0.0034	-0.8484	0.3962		-0.0007	0.0019	-0.3913	0.6956	
	2	-0.0019	0.0024	-0.7787	0.4361		-0.0001	0.0003	-0.3892	0.6971	
	3	0.0010	0.0023	0.4413	0.6590		0.0000	0.0001	0.3840	0.7010	
	4	0.0023	0.0021	1.0790	0.2806		0.0000	0.0000	0.3858	0.6997	
	5	0.0015	0.0042	0.3494	0.7268		0.0008	0.0021	0.3912	0.6956	

Significance levels correspond to: $***. < 0.01$, $**. < 0.05$, $*. < 0.1$.

Notes: Table shows the comparison of the marginal effects at the covariates means between the *Ordered Forest* and the ordered logit. The effects are estimated for all classes, together with the corresponding standard errors, t-values and p-values. The standard errors for the *Ordered Forest* are estimated using the weight-based inference and the standard errors for the ordered logit are obtained via the delta method.

Table 36: Mean Marginal Effects: Support Study Dataset

Dataset		Ordered Forest					Ordered Logit				
Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
age	1	-0.0003	0.0006	-0.4173	0.6765		-0.0021	0.0009	-2.3352	0.0195	**
	2	0.0004	0.0003	1.2637	0.2063		-0.0000	0.0000	-0.1266	0.8993	
	3	-0.0005	0.0002	-2.0010	0.0454	**	0.0001	0.0000	2.2150	0.0268	**
	4	-0.0000	0.0001	-0.4444	0.6567		0.0000	0.0000	1.7601	0.0784	*
	5	0.0004	0.0006	0.6645	0.5064		0.0020	0.0008	2.3250	0.0201	**
avtisst	1	-0.0038	0.0007	-5.0688	0.0000	***	-0.0145	0.0013	-11.2587	0.0000	***
	2	-0.0001	0.0005	-0.1400	0.8886		-0.0000	0.0002	-0.1273	0.8987	
	3	-0.0011	0.0004	-2.4580	0.0140	**	0.0007	0.0002	4.6390	0.0000	***
	4	-0.0001	0.0002	-0.6888	0.4909		0.0001	0.0001	2.5058	0.0122	**
	5	0.0050	0.0010	4.9959	0.0000	***	0.0137	0.0012	11.6725	0.0000	***
charges	1	-0.0000	0.0000	-1.6278	0.1036		0.0000	0.0000	1.8443	0.0651	*
	2	0.0000	0.0000	2.0991	0.0358	**	0.0000	0.0000	0.1271	0.8989	
	3	0.0000	0.0000	1.1021	0.2704		-0.0000	0.0000	-1.6961	0.0899	*
	4	0.0000	0.0000	0.8566	0.3917		-0.0000	0.0000	-1.4711	0.1413	
	5	0.0000	0.0000	0.8934	0.3717		-0.0000	0.0000	-1.8496	0.0644	*
crea	1	-0.0026	0.0091	-0.2843	0.7762		-0.0205	0.0087	-2.3560	0.0185	**
	2	-0.0149	0.0093	-1.6056	0.1084		-0.0000	0.0003	-0.1271	0.8988	
	3	0.0067	0.0050	1.3302	0.1835		0.0010	0.0005	2.1163	0.0343	**
	4	-0.0017	0.0029	-0.5862	0.5578		0.0002	0.0001	1.7253	0.0845	*
	5	0.0125	0.0144	0.8664	0.3862		0.0193	0.0082	2.3616	0.0182	**
dzgroup	1	-0.0171	0.0062	-2.7672	0.0057	***	-0.0382	0.0058	-6.5550	0.0000	***
	2	-0.0364	0.0110	-3.3151	0.0009	***	-0.0001	0.0006	-0.1270	0.8989	
	3	-0.0023	0.0059	-0.3945	0.6932		0.0019	0.0005	4.1333	0.0000	***
	4	-0.0048	0.0020	-2.3421	0.0192	**	0.0003	0.0001	2.4040	0.0162	**
	5	0.0606	0.0171	3.5517	0.0004	***	0.0360	0.0055	6.5419	0.0000	***
hrt	1	-0.0005	0.0002	-1.9700	0.0488	**	-0.0009	0.0005	-1.9855	0.0471	**
	2	-0.0002	0.0002	-0.9503	0.3420		-0.0000	0.0000	-0.1267	0.8992	
	3	0.0003	0.0001	1.8082	0.0706	*	0.0000	0.0000	1.8762	0.0606	*
	4	0.0001	0.0001	0.9720	0.3311		0.0000	0.0000	1.5835	0.1133	
	5	0.0003	0.0003	1.0541	0.2918		0.0009	0.0004	1.9828	0.0474	**
meanbp	1	0.0006	0.0003	2.1290	0.0333	**	0.0011	0.0005	2.1579	0.0309	**
	2	0.0007	0.0003	2.3052	0.0212	**	0.0000	0.0000	0.1268	0.8991	
	3	-0.0001	0.0002	-0.4364	0.6625		-0.0001	0.0000	-2.0057	0.0449	**
	4	-0.0000	0.0001	-0.0627	0.9500		-0.0000	0.0000	-1.6629	0.0963	*
	5	-0.0012	0.0004	-3.3738	0.0007	***	-0.0011	0.0005	-2.1565	0.0310	**
num.co	1	-0.0018	0.0024	-0.7439	0.4569		-0.0020	0.0108	-0.1845	0.8536	
	2	0.0017	0.0013	1.2945	0.1955		-0.0000	0.0000	-0.1068	0.9149	
	3	-0.0006	0.0021	-0.3091	0.7572		0.0001	0.0005	0.1842	0.8539	
	4	-0.0009	0.0008	-1.1345	0.2566		0.0000	0.0001	0.1839	0.8541	
	5	0.0017	0.0044	0.3824	0.7022		0.0019	0.0102	0.1846	0.8536	
race	1	-0.0052	0.0044	-1.1745	0.2402		-0.0053	0.0154	-0.3402	0.7337	
	2	0.0031	0.0029	1.0629	0.2878		-0.0000	0.0001	-0.1184	0.9057	
	3	0.0019	0.0022	0.8515	0.3945		0.0003	0.0008	0.3402	0.7337	
	4	-0.0005	0.0008	-0.6510	0.5151		0.0000	0.0001	0.3377	0.7356	
	5	0.0007	0.0040	0.1854	0.8529		0.0050	0.0146	0.3401	0.7338	
resp	1	-0.0007	0.0004	-1.9854	0.0471	**	-0.0050	0.0015	-3.3528	0.0008	***
	2	-0.0003	0.0003	-0.9091	0.3633		-0.0000	0.0001	-0.1269	0.8990	
	3	-0.0007	0.0004	-1.6536	0.0982	*	0.0003	0.0001	2.8462	0.0044	***
	4	0.0001	0.0002	0.7965	0.4258		0.0000	0.0000	2.0438	0.0410	**
	5	0.0016	0.0008	2.1418	0.0322	**	0.0048	0.0014	3.3464	0.0008	***
scoma	1	-0.0003	0.0001	-2.5004	0.0124	**	-0.0037	0.0007	-5.4618	0.0000	***
	2	-0.0001	0.0001	-1.3778	0.1683		-0.0000	0.0001	-0.1275	0.8985	

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Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
	3	-0.0004	0.0002	-2.2932	0.0218	**	0.0002	0.0001	3.5064	0.0005	***
	4	-0.0001	0.0001	-0.8367	0.4028		0.0000	0.0000	2.2813	0.0225	**
	5	0.0009	0.0004	2.3601	0.0183	**	0.0035	0.0006	5.5980	0.0000	***
sex	1	0.0000	0.0041	0.0016	0.9987		-0.0420	0.0274	-1.5343	0.1250	
	2	-0.0024	0.0024	-0.9900	0.3222		-0.0001	0.0006	-0.1261	0.8997	
	3	-0.0015	0.0018	-0.8520	0.3942		0.0021	0.0014	1.4833	0.1380	
	4	0.0006	0.0009	0.6029	0.5466		0.0004	0.0003	1.3199	0.1869	
	5	0.0033	0.0047	0.7123	0.4763		0.0396	0.0258	1.5316	0.1256	
sod	1	0.0020	0.0016	1.2382	0.2156		0.0016	0.0023	0.6875	0.4917	
	2	0.0000	0.0008	0.0448	0.9643		0.0000	0.0000	0.1240	0.9013	
	3	-0.0006	0.0008	-0.7257	0.4680		-0.0001	0.0001	-0.6844	0.4938	
	4	-0.0004	0.0003	-1.3647	0.1724		-0.0000	0.0000	-0.6655	0.5057	
	5	-0.0010	0.0013	-0.7664	0.4434		-0.0015	0.0022	-0.6870	0.4921	
temp	1	0.0106	0.0043	2.4824	0.0131	**	0.0291	0.0129	2.2435	0.0249	**
	2	-0.0024	0.0024	-1.0249	0.3054		0.0001	0.0004	0.1269	0.8990	
	3	-0.0039	0.0031	-1.2679	0.2048		-0.0015	0.0007	-2.0536	0.0400	**
	4	-0.0002	0.0008	-0.2289	0.8190		-0.0003	0.0002	-1.6891	0.0912	*
	5	-0.0041	0.0053	-0.7662	0.4436		-0.0274	0.0122	-2.2450	0.0248	**
wblc	1	-0.0013	0.0016	-0.7905	0.4292		-0.0006	0.0016	-0.3914	0.6955	
	2	0.0003	0.0008	0.4374	0.6618		-0.0000	0.0000	-0.1211	0.9036	
	3	0.0000	0.0006	0.0102	0.9919		0.0000	0.0001	0.3900	0.6966	
	4	-0.0000	0.0002	-0.1225	0.9025		0.0000	0.0000	0.3868	0.6989	
	5	0.0010	0.0014	0.6934	0.4880		0.0006	0.0015	0.3915	0.6954	

Significance levels correspond to: $***. < 0.01$, $**. < 0.05$, $*. < 0.1$.

Notes: Table shows the comparison of the mean marginal effects between the *Ordered Forest* and the ordered logit. The effects are estimated for all classes, together with the corresponding standard errors, t-values and p-values. The standard errors for the *Ordered Forest* are estimated using the weight-based inference and the standard errors for the ordered logit are obtained via the delta method.

C.3.4 Data: vlbw

Table 37: Marginal Effects at Mean: Vlbw Dataset

Dataset		Ordered Forest					Ordered Logit				
Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
bwt	1	-0.0000	0.0000	-0.2880	0.7734		-0.0002	0.0001	-3.5903	0.0003	***
	2	0.0000	0.0001	0.1112	0.9115		-0.0001	0.0000	-2.9197	0.0035	***
	3	-0.0002	0.0001	-1.0348	0.3008		-0.0001	0.0000	-2.9034	0.0037	***
	4	0.0000	0.0003	0.0964	0.9232		-0.0001	0.0000	-2.5144	0.0119	**
	5	-0.0004	0.0004	-1.0199	0.3078		-0.0000	0.0000	-1.8416	0.0655	*
	6	0.0002	0.0002	0.6382	0.5234		0.0000	0.0000	1.8320	0.0670	*
	7	0.0005	0.0004	1.2699	0.2041		0.0002	0.0001	3.1962	0.0014	***
	8	0.0000	0.0005	0.0470	0.9625		0.0002	0.0001	3.4810	0.0005	***
	9	-0.0001	0.0005	-0.2597	0.7951		0.0001	0.0000	2.6825	0.0073	***
delivery	1	-0.0226	0.0222	-1.0214	0.3071		-0.0005	0.0261	-0.0210	0.9833	
	2	0.0343	0.0292	1.1764	0.2394		-0.0003	0.0128	-0.0210	0.9833	
	3	0.0043	0.0198	0.2175	0.8278		-0.0003	0.0135	-0.0210	0.9833	
	4	-0.0127	0.0299	-0.4241	0.6715		-0.0002	0.0081	-0.0210	0.9833	
	5	-0.0221	0.0269	-0.8238	0.4100		-0.0001	0.0059	-0.0210	0.9833	
	6	0.0155	0.0212	0.7311	0.4647		0.0001	0.0062	0.0210	0.9833	
	7	0.0077	0.0261	0.2957	0.7675		0.0005	0.0239	0.0210	0.9833	
	8	0.0030	0.0327	0.0913	0.9273		0.0006	0.0282	0.0210	0.9833	
	9	-0.0074	0.0545	-0.1348	0.8928		0.0002	0.0083	0.0210	0.9833	
inout	1	-0.0217	0.0075	-2.8865	0.0039	***	-0.0204	0.0730	-0.2802	0.7794	
	2	0.0842	0.1012	0.8326	0.4051		-0.0100	0.0358	-0.2798	0.7796	
	3	-0.0137	0.0139	-0.9882	0.3230		-0.0106	0.0379	-0.2800	0.7795	
	4	-0.0197	0.0107	-1.8498	0.0643	*	-0.0064	0.0228	-0.2795	0.7798	
	5	-0.0129	0.0309	-0.4190	0.6752		-0.0047	0.0168	-0.2780	0.7810	
	6	0.0036	0.0500	0.0727	0.9420		0.0049	0.0175	0.2781	0.7810	
	7	-0.0140	0.0101	-1.3906	0.1643		0.0187	0.0667	0.2801	0.7794	
	8	-0.0053	0.0905	-0.0589	0.9530		0.0221	0.0788	0.2802	0.7794	
	9	-0.0005	0.0036	-0.1332	0.8940		0.0065	0.0231	0.2797	0.7797	
lol	1	0.0008	0.0037	0.2133	0.8311		0.0013	0.0007	1.8412	0.0656	*
	2	-0.0016	0.0019	-0.8530	0.3937		0.0006	0.0004	1.7499	0.0801	*
	3	-0.0010	0.0019	-0.5272	0.5980		0.0007	0.0004	1.7659	0.0774	*
	4	0.0052	0.0035	1.4858	0.1373		0.0004	0.0002	1.6673	0.0955	*
	5	-0.0034	0.0072	-0.4806	0.6308		0.0003	0.0002	1.4171	0.1564	
	6	-0.0003	0.0048	-0.0611	0.9513		-0.0003	0.0002	-1.4016	0.1610	
	7	-0.0016	0.0059	-0.2727	0.7851		-0.0012	0.0007	-1.7915	0.0732	*
	8	0.0019	0.0105	0.1843	0.8538		-0.0014	0.0007	-1.8566	0.0634	*
	9	0.0000	0.0000	0.0000	1.0000		-0.0004	0.0002	-1.7110	0.0871	*
magsulf	1	0.0127	0.0352	0.3605	0.7185		-0.0333	0.0282	-1.1819	0.2372	
	2	0.0516	0.0628	0.8208	0.4117		-0.0170	0.0153	-1.1156	0.2646	
	3	-0.0185	0.0204	-0.9048	0.3656		-0.0188	0.0175	-1.0759	0.2820	
	4	-0.0283	0.0243	-1.1648	0.2441		-0.0121	0.0123	-0.9917	0.3214	
	5	-0.0248	0.0452	-0.5482	0.5835		-0.0109	0.0130	-0.8400	0.4009	
	6	-0.0074	0.0507	-0.1452	0.8845		0.0053	0.0045	1.1816	0.2374	
	7	0.0135	0.0413	0.3268	0.7438		0.0318	0.0281	1.1345	0.2566	
	8	-0.0012	0.0414	-0.0295	0.9764		0.0422	0.0415	1.0168	0.3092	
	9	0.0025	0.0264	0.0937	0.9253		0.0129	0.0138	0.9353	0.3496	
meth	1	-0.0422	0.0456	-0.9252	0.3548		-0.1064	0.0292	-3.6460	0.0003	***
	2	-0.0480	0.0365	-1.3137	0.1889		-0.0514	0.0169	-3.0480	0.0023	***
	3	-0.0102	0.0264	-0.3876	0.6983		-0.0543	0.0175	-3.1104	0.0019	***
	4	-0.0261	0.0557	-0.4683	0.6396		-0.0334	0.0123	-2.7191	0.0065	***
	5	-0.0937	0.0671	-1.3956	0.1628		-0.0274	0.0128	-2.1359	0.0327	**

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Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
	6	0.0270	0.0502	0.5378	0.5907		0.0183	0.0119	1.5399	0.1236	
	7	0.1090	0.0595	1.8321	0.0669	*	0.0922	0.0266	3.4649	0.0005	***
	8	0.0743	0.0693	1.0716	0.2839		0.1236	0.0344	3.5942	0.0003	***
	9	0.0099	0.0320	0.3106	0.7561		0.0388	0.0148	2.6263	0.0086	***
race	1	0.0283	0.0178	1.5959	0.1105		0.0532	0.0259	2.0587	0.0395	**
	2	0.0380	0.0199	1.9143	0.0556	*	0.0261	0.0137	1.9025	0.0571	*
	3	0.0197	0.0199	0.9883	0.3230		0.0276	0.0147	1.8824	0.0598	*
	4	0.0076	0.0406	0.1875	0.8512		0.0166	0.0094	1.7753	0.0759	*
	5	-0.0580	0.0570	-1.0182	0.3086		0.0121	0.0080	1.5132	0.1302	
	6	-0.0047	0.0448	-0.1045	0.9167		-0.0126	0.0087	-1.4496	0.1472	
	7	-0.0276	0.0471	-0.5853	0.5584		-0.0486	0.0246	-1.9754	0.0482	**
	8	-0.0120	0.0489	-0.2460	0.8057		-0.0575	0.0281	-2.0494	0.0404	**
	9	0.0087	0.0442	0.1960	0.8446		-0.0168	0.0091	-1.8553	0.0635	*
sex	1	-0.0600	0.0360	-1.6665	0.0956	*	0.0124	0.0238	0.5211	0.6023	
	2	-0.0243	0.0379	-0.6421	0.5208		0.0061	0.0117	0.5182	0.6043	
	3	-0.0495	0.0442	-1.1217	0.2620		0.0064	0.0124	0.5169	0.6053	
	4	0.0891	0.0419	2.1275	0.0334	**	0.0039	0.0075	0.5122	0.6085	
	5	0.0051	0.0395	0.1290	0.8974		0.0028	0.0056	0.5008	0.6165	
	6	-0.0065	0.0401	-0.1615	0.8717		-0.0029	0.0058	-0.5050	0.6136	
	7	0.0376	0.0419	0.8975	0.3694		-0.0113	0.0219	-0.5177	0.6047	
	8	0.0023	0.0583	0.0389	0.9690		-0.0134	0.0257	-0.5200	0.6031	
	9	0.0063	0.0273	0.2317	0.8168		-0.0039	0.0076	-0.5157	0.6060	
toc	1	-0.0423	0.0147	-2.8833	0.0039	***	-0.0665	0.0242	-2.7491	0.0060	***
	2	-0.0179	0.0119	-1.5077	0.1316		-0.0345	0.0143	-2.4110	0.0159	**
	3	0.0043	0.0162	0.2630	0.7925		-0.0390	0.0162	-2.4091	0.0160	**
	4	0.0138	0.0628	0.2193	0.8264		-0.0261	0.0122	-2.1452	0.0319	**
	5	0.0111	0.0416	0.2669	0.7896		-0.0261	0.0147	-1.7824	0.0747	*
	6	0.0011	0.0385	0.0294	0.9765		0.0057	0.0088	0.6453	0.5188	
	7	-0.0459	0.0323	-1.4221	0.1550		0.0629	0.0235	2.6741	0.0075	***
	8	0.0662	0.0681	0.9725	0.3308		0.0934	0.0407	2.2964	0.0217	**
	9	0.0097	0.0160	0.6044	0.5456		0.0303	0.0158	1.9213	0.0547	*
twn	1	0.0002	0.0321	0.0056	0.9956		-0.0011	0.0298	-0.0374	0.9702	
	2	-0.0158	0.0147	-1.0786	0.2807		-0.0005	0.0146	-0.0373	0.9702	
	3	-0.0220	0.0104	-2.1152	0.0344	**	-0.0006	0.0155	-0.0373	0.9703	
	4	-0.0160	0.0316	-0.5064	0.6126		-0.0003	0.0094	-0.0372	0.9703	
	5	-0.0090	0.0317	-0.2833	0.7770		-0.0003	0.0069	-0.0370	0.9705	
	6	0.0196	0.0313	0.6262	0.5312		0.0003	0.0070	0.0376	0.9700	
	7	0.0436	0.0303	1.4386	0.1503		0.0010	0.0273	0.0373	0.9703	
	8	0.0046	0.0472	0.0979	0.9220		0.0012	0.0324	0.0372	0.9703	
	9	-0.0052	0.0231	-0.2239	0.8229		0.0004	0.0095	0.0372	0.9703	

Significance levels correspond to: $***. < 0.01$, $**. < 0.05$, $*. < 0.1$.

Notes: Table shows the comparison of the marginal effects at the covariates means between the *Ordered Forest* and the ordered logit. The effects are estimated for all classes, together with the corresponding standard errors, t-values and p-values. The standard errors for the *Ordered Forest* are estimated using the weight-based inference and the standard errors for the ordered logit are obtained via the delta method.

Table 38: Mean Marginal Effects: Vlbw Dataset

Dataset		**Ordered Forest**					**Ordered Logit**				
Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
bwt	1	-0.0001	0.0001	-1.1937	0.2326		-0.0002	0.0001	-3.8018	0.0001	***
	2	0.0001	0.0000	1.7664	0.0773	*	-0.0001	0.0000	-3.1300	0.0017	***
	3	-0.0001	0.0001	-1.3585	0.1743		-0.0001	0.0000	-3.0308	0.0024	***
	4	0.0001	0.0001	0.9957	0.3194		-0.0000	0.0000	-2.3930	0.0167	**
	5	-0.0001	0.0001	-1.3179	0.1876		-0.0000	0.0000	-1.2067	0.2275	
	6	-0.0000	0.0001	-0.3389	0.7347		0.0000	0.0000	2.3521	0.0187	**
	7	0.0000	0.0001	0.3527	0.7243		0.0001	0.0000	3.4928	0.0005	***
	8	0.0002	0.0002	0.9957	0.3194		0.0002	0.0001	3.5488	0.0004	***
	9	-0.0000	0.0002	-0.0719	0.9427		0.0001	0.0000	2.8356	0.0046	***
delivery	1	-0.0074	0.0133	-0.5523	0.5807		-0.0006	0.0300	-0.0210	0.9833	
	2	0.0223	0.0127	1.7486	0.0804	*	-0.0002	0.0094	-0.0210	0.9833	
	3	-0.0049	0.0114	-0.4348	0.6637		-0.0002	0.0078	-0.0210	0.9833	
	4	-0.0118	0.0124	-0.9522	0.3410		-0.0001	0.0038	-0.0210	0.9833	
	5	-0.0044	0.0160	-0.2771	0.7817		-0.0000	0.0019	-0.0210	0.9832	
	6	0.0136	0.0117	1.1587	0.2466		0.0001	0.0036	0.0209	0.9833	
	7	0.0101	0.0218	0.4601	0.6455		0.0003	0.0129	0.0210	0.9833	
	8	-0.0107	0.0279	-0.3829	0.7018		0.0005	0.0240	0.0210	0.9833	
	9	-0.0067	0.0524	-0.1272	0.8988		0.0003	0.0125	0.0210	0.9833	
inout	1	-0.0132	0.0038	-3.4819	0.0005	***	-0.0235	0.0838	-0.2804	0.7792	
	2	0.0432	0.0485	0.8906	0.3731		-0.0074	0.0264	-0.2800	0.7795	
	3	-0.0053	0.0059	-0.8877	0.3747		-0.0061	0.0219	-0.2799	0.7795	
	4	-0.0086	0.0035	-2.4568	0.0140	**	-0.0030	0.0106	-0.2789	0.7803	
	5	-0.0039	0.0090	-0.4380	0.6614		-0.0015	0.0055	-0.2732	0.7847	
	6	0.0013	0.0234	0.0542	0.9568		0.0028	0.0101	0.2791	0.7802	
	7	-0.0067	0.0110	-0.6049	0.5453		0.0101	0.0360	0.2801	0.7794	
	8	-0.0067	0.0475	-0.1409	0.8879		0.0188	0.0670	0.2802	0.7793	
	9	-0.0001	0.0036	-0.0165	0.9868		0.0098	0.0350	0.2798	0.7796	
lol	1	-0.0009	0.0022	-0.4085	0.6829		0.0015	0.0008	1.8907	0.0587	*
	2	0.0011	0.0013	0.7964	0.4258		0.0005	0.0003	1.7359	0.0826	*
	3	-0.0008	0.0019	-0.4226	0.6726		0.0004	0.0002	1.7377	0.0823	*
	4	0.0038	0.0027	1.4203	0.1555		0.0002	0.0001	1.6137	0.1066	
	5	-0.0015	0.0025	-0.5791	0.5625		0.0001	0.0001	1.0961	0.2730	
	6	0.0033	0.0021	1.5589	0.1190		-0.0002	0.0001	-1.4960	0.1346	
	7	0.0006	0.0045	0.1418	0.8872		-0.0006	0.0003	-1.8145	0.0696	*
	8	-0.0052	0.0037	-1.3895	0.1647		-0.0012	0.0006	-1.8742	0.0609	*
	9	-0.0004	0.0058	-0.0723	0.9424		-0.0006	0.0004	-1.7431	0.0813	*
magsulf	1	-0.0002	0.0195	-0.0126	0.9899		-0.0394	0.0340	-1.1589	0.2465	
	2	0.0239	0.0282	0.8481	0.3964		-0.0131	0.0123	-1.0700	0.2846	
	3	-0.0155	0.0165	-0.9342	0.3502		-0.0114	0.0112	-1.0238	0.3059	
	4	-0.0123	0.0147	-0.8370	0.4026		-0.0059	0.0063	-0.9371	0.3487	
	5	-0.0052	0.0315	-0.1645	0.8693		-0.0039	0.0052	-0.7447	0.4564	
	6	0.0076	0.0295	0.2572	0.7971		0.0038	0.0030	1.2364	0.2163	
	7	0.0144	0.0283	0.5075	0.6118		0.0169	0.0146	1.1512	0.2496	
	8	-0.0111	0.0181	-0.6092	0.5424		0.0337	0.0313	1.0766	0.2816	
	9	-0.0016	0.0318	-0.0502	0.9600		0.0194	0.0202	0.9604	0.3369	
meth	1	-0.0597	0.0366	-1.6328	0.1025		-0.1146	0.0292	-3.9267	0.0001	***
	2	-0.0108	0.0289	-0.3743	0.7082		-0.0438	0.0151	-2.9102	0.0036	***
	3	-0.0417	0.0270	-1.5428	0.1229		-0.0417	0.0147	-2.8381	0.0045	***
	4	0.0070	0.0331	0.2133	0.8311		-0.0238	0.0096	-2.4647	0.0137	**
	5	-0.0652	0.0503	-1.2956	0.1951		-0.0189	0.0096	-1.9593	0.0501	*
	6	0.0071	0.0357	0.1989	0.8424		0.0116	0.0083	1.4025	0.1608	
	7	0.1066	0.0459	2.3208	0.0203	**	0.0670	0.0219	3.0645	0.0022	***

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Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
	8	0.0501	0.0481	1.0418	0.2975		0.1162	0.0329	3.5279	0.0004	***
	9	0.0066	0.0340	0.1938	0.8463		0.0478	0.0164	2.9094	0.0036	***
race	1	0.0210	0.0140	1.5040	0.1326		0.0612	0.0294	2.0774	0.0378	**
	2	0.0348	0.0109	3.1908	0.0014	***	0.0193	0.0101	1.9132	0.0557	*
	3	0.0283	0.0191	1.4872	0.1370		0.0160	0.0084	1.8971	0.0578	*
	4	-0.0295	0.0201	-1.4717	0.1411		0.0077	0.0043	1.7708	0.0766	*
	5	-0.0141	0.0249	-0.5666	0.5710		0.0039	0.0033	1.1848	0.2361	
	6	-0.0051	0.0184	-0.2789	0.7803		-0.0073	0.0048	-1.5341	0.1250	
	7	-0.0038	0.0331	-0.1150	0.9085		-0.0263	0.0130	-2.0190	0.0435	**
	8	-0.0323	0.0271	-1.1958	0.2318		-0.0489	0.0231	-2.1119	0.0347	**
	9	0.0007	0.0371	0.0191	0.9847		-0.0255	0.0135	-1.8874	0.0591	*
sex	1	-0.0186	0.0178	-1.0479	0.2947		0.0142	0.0273	0.5206	0.6026	
	2	-0.0258	0.0190	-1.3524	0.1762		0.0045	0.0086	0.5199	0.6031	
	3	-0.0082	0.0224	-0.3668	0.7137		0.0037	0.0072	0.5197	0.6033	
	4	0.0413	0.0194	2.1275	0.0334	**	0.0018	0.0035	0.5143	0.6070	
	5	-0.0008	0.0251	-0.0302	0.9759		0.0009	0.0019	0.4790	0.6320	
	6	-0.0032	0.0237	-0.1365	0.8914		-0.0017	0.0033	-0.5192	0.6037	
	7	0.0106	0.0329	0.3222	0.7473		-0.0061	0.0117	-0.5211	0.6023	
	8	-0.0041	0.0211	-0.1926	0.8473		-0.0114	0.0219	-0.5193	0.6035	
	9	0.0087	0.0250	0.3492	0.7269		-0.0059	0.0115	-0.5167	0.6054	
toc	1	-0.0385	0.0106	-3.6358	0.0003	***	-0.0763	0.0265	-2.8783	0.0040	***
	2	-0.0077	0.0067	-1.1404	0.2541		-0.0279	0.0123	-2.2596	0.0238	**
	3	0.0009	0.0132	0.0666	0.9469		-0.0264	0.0123	-2.1538	0.0313	**
	4	-0.0171	0.0314	-0.5443	0.5863		-0.0155	0.0082	-1.8900	0.0588	*
	5	-0.0123	0.0265	-0.4633	0.6432		-0.0144	0.0091	-1.5752	0.1152	
	6	0.0227	0.0224	1.0118	0.3116		0.0020	0.0049	0.4103	0.6816	
	7	-0.0204	0.0303	-0.6755	0.4994		0.0359	0.0147	2.4322	0.0150	**
	8	0.0620	0.0421	1.4737	0.1406		0.0820	0.0352	2.3331	0.0196	**
	9	0.0104	0.0176	0.5904	0.5549		0.0406	0.0194	2.0953	0.0361	**
twn	1	-0.0131	0.0152	-0.8619	0.3887		-0.0013	0.0342	-0.0373	0.9702	
	2	-0.0041	0.0082	-0.5071	0.6121		-0.0004	0.0108	-0.0372	0.9703	
	3	-0.0178	0.0071	-2.5041	0.0123	**	-0.0003	0.0090	-0.0371	0.9704	
	4	-0.0072	0.0166	-0.4330	0.6650		-0.0002	0.0044	-0.0370	0.9704	
	5	-0.0070	0.0203	-0.3451	0.7300		-0.0001	0.0023	-0.0367	0.9708	
	6	-0.0036	0.0175	-0.2060	0.8368		0.0002	0.0041	0.0374	0.9701	
	7	0.0491	0.0249	1.9748	0.0483	**	0.0005	0.0148	0.0372	0.9703	
	8	0.0072	0.0255	0.2817	0.7782		0.0010	0.0275	0.0372	0.9703	
	9	-0.0035	0.0239	-0.1443	0.8853		0.0005	0.0144	0.0372	0.9703	

Significance levels correspond to: $***. < 0.01$, $**. < 0.05$, $*. < 0.1$.

Notes: Table shows the comparison of the mean marginal effects between the *Ordered Forest* and the ordered logit. The effects are estimated for all classes, together with the corresponding standard errors, t-values and p-values. The standard errors for the *Ordered Forest* are estimated using the weight-based inference and the standard errors for the ordered logit are obtained via the delta method.

C.3.5 Data: winequality

Table 39: Marginal Effects at Mean: Wine Quality Dataset

Dataset		**Ordered Forest**					**Ordered Logit**				
Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
alcohol	1	0.0001	0.0001	1.0274	0.3042		-0.0008	0.0002	-3.5024	0.0005	***
	2	-0.0026	0.0018	-1.4387	0.1502		-0.0074	0.0014	-5.3158	0.0000	***
	3	-0.1941	0.0470	-4.1307	0.0000	***	-0.0816	0.0141	-5.7935	0.0000	***
	4	0.1323	0.0465	2.8428	0.0045	***	0.0323	0.0060	5.3901	0.0000	***
	5	0.0643	0.0405	1.5872	0.1125		0.0496	0.0086	5.7385	0.0000	***
	6	0.0000	0.0017	0.0145	0.9884		0.0080	0.0015	5.3547	0.0000	***
chlorides	1	0.0004	0.0012	0.3431	0.7315		0.0011	0.0026	0.4150	0.6781	
	2	0.0451	0.0404	1.1161	0.2644		0.0098	0.0235	0.4167	0.6769	
	3	3.8315	1.7362	2.2069	0.0273	**	0.1082	0.2595	0.4167	0.6769	
	4	-2.3048	1.8011	-1.2796	0.2007		-0.0428	0.1027	-0.4166	0.6770	
	5	-1.5432	1.5361	-1.0046	0.3151		-0.0656	0.1575	-0.4167	0.6769	
	6	-0.0291	0.1131	-0.2573	0.7969		-0.0106	0.0254	-0.4165	0.6771	
citric acid	1	0.0000	0.0000	0.0000	1.0000		-0.0002	0.0004	-0.4409	0.6593	
	2	0.0029	0.0058	0.5062	0.6127		-0.0018	0.0041	-0.4428	0.6579	
	3	0.1324	0.1416	0.9354	0.3496		-0.0201	0.0454	-0.4430	0.6578	
	4	-0.2522	0.1749	-1.4414	0.1495		0.0080	0.0180	0.4427	0.6580	
	5	0.1150	0.1574	0.7306	0.4651		0.0122	0.0276	0.4430	0.6577	
	6	0.0018	0.0511	0.0361	0.9712		0.0020	0.0044	0.4429	0.6578	
density	1	0.0138	0.0137	1.0063	0.3143		0.8312	0.2166	3.8379	0.0001	***
	2	0.2608	0.1874	1.3918	0.1640		7.6758	1.1808	6.5004	0.0000	***
	3	-3.3364	7.8199	-0.4267	0.6696		84.8474	11.4044	7.4399	0.0000	***
	4	17.9249	9.5652	1.8740	0.0609	*	-33.5574	5.1347	-6.5354	0.0000	***
	5	-14.3207	9.1044	-1.5729	0.1157		-51.4987	6.9465	-7.4136	0.0000	***
	6	-0.5424	1.1946	-0.4540	0.6498		-8.2983	1.2534	-6.6205	0.0000	***
fixed acidity	1	0.0000	0.0000	0.0000	1.0000		-0.0004	0.0001	-2.8330	0.0046	***
	2	0.0025	0.0011	2.3394	0.0193	**	-0.0037	0.0010	-3.5460	0.0004	***
	3	-0.0015	0.0089	-0.1698	0.8651		-0.0409	0.0111	-3.6715	0.0002	***
	4	0.0002	0.0224	0.0081	0.9935		0.0162	0.0046	3.5499	0.0004	***
	5	-0.0011	0.0200	-0.0530	0.9578		0.0248	0.0068	3.6681	0.0002	***
	6	-0.0001	0.0045	-0.0254	0.9797		0.0040	0.0011	3.5485	0.0004	***
free sulfur dioxide	1	0.0000	0.0000	0.0000	1.0000		-0.0000	0.0000	-3.3791	0.0007	***
	2	0.0000	0.0000	0.0000	1.0000		-0.0002	0.0000	-4.8863	0.0000	***
	3	-0.0006	0.0006	-0.9808	0.3267		-0.0022	0.0004	-5.2449	0.0000	***
	4	0.0006	0.0008	0.6812	0.4957		0.0009	0.0002	4.9080	0.0000	***
	5	-0.0000	0.0007	-0.0115	0.9908		0.0013	0.0003	5.2237	0.0000	***
	6	0.0000	0.0001	0.2377	0.8121		0.0002	0.0000	4.9452	0.0000	***
pH	1	0.0000	0.0000	0.0000	1.0000		-0.0037	0.0010	-3.7348	0.0002	***
	2	0.0001	0.0020	0.0390	0.9689		-0.0339	0.0056	-6.1086	0.0000	***
	3	-0.1707	0.1008	-1.6932	0.0904	*	-0.3749	0.0547	-6.8595	0.0000	***
	4	0.1423	0.1060	1.3416	0.1797		0.1483	0.0241	6.1573	0.0000	***
	5	0.0265	0.0587	0.4516	0.6515		0.2276	0.0334	6.8190	0.0000	***
	6	0.0018	0.0090	0.2028	0.8393		0.0367	0.0059	6.1853	0.0000	***
residual sugar	1	0.0000	0.0000	0.0000	1.0000		-0.0004	0.0001	-4.0845	0.0000	***
	2	-0.0002	0.0001	-1.8350	0.0665	*	-0.0038	0.0005	-8.0099	0.0000	***
	3	-0.0024	0.0028	-0.8752	0.3815		-0.0425	0.0043	-9.9279	0.0000	***
	4	0.0033	0.0036	0.9194	0.3579		0.0168	0.0021	8.0748	0.0000	***
	5	-0.0012	0.0033	-0.3574	0.7208		0.0258	0.0026	9.8390	0.0000	***
	6	0.0005	0.0010	0.5263	0.5986		0.0042	0.0005	8.1575	0.0000	***
sulphates	1	-0.0014	0.0008	-1.6619	0.0965	*	-0.0033	0.0009	-3.7477	0.0002	***
	2	0.0020	0.0026	0.7666	0.4433		-0.0308	0.0050	-6.1686	0.0000	***

Continued on next page

Variable	Class	Effect	Std.Error	t-Value	p-Value		Effect	Std.Error	t-Value	p-Value	
	3	-0.2295	0.1674	-1.3716	0.1702		-0.3400	0.0489	-6.9592	0.0000	***
	4	0.1695	0.1850	0.9162	0.3596		0.1345	0.0216	6.2134	0.0000	***
	5	0.0585	0.1656	0.3535	0.7237		0.2064	0.0298	6.9195	0.0000	***
	6	0.0008	0.0025	0.3379	0.7354		0.0333	0.0053	6.3001	0.0000	***
total	1	0.0000	0.0000	0.0000	1.0000		0.0000	0.0000	0.8882	0.3744	
sulfur	2	-0.0000	0.0000	-1.5623	0.1182		0.0000	0.0000	0.9068	0.3645	
dioxide	3	0.0013	0.0008	1.7377	0.0823	*	0.0002	0.0002	0.9097	0.3630	
	4	-0.0010	0.0007	-1.4130	0.1577		-0.0001	0.0001	-0.9080	0.3639	
	5	-0.0003	0.0005	-0.5008	0.6165		-0.0001	0.0001	-0.9094	0.3632	
	6	0.0000	0.0000	0.0043	0.9965		-0.0000	0.0000	-0.9078	0.3640	
volatile	1	0.0000	0.0000	0.0000	1.0000		0.0092	0.0021	4.3528	0.0000	***
acidity	2	0.0125	0.0067	1.8604	0.0628	*	0.0845	0.0079	10.6684	0.0000	***
	3	1.6912	0.6553	2.5809	0.0099	***	0.9342	0.0604	15.4576	0.0000	***
	4	-1.2661	0.5841	-2.1675	0.0302	**	-0.3695	0.0354	-10.4275	0.0000	***
	5	-0.4375	0.3665	-1.1939	0.2325		-0.5670	0.0371	-15.2936	0.0000	***
	6	0.0000	0.0000	0.0000	1.0000		-0.0914	0.0089	-10.2723	0.0000	***

Significance levels correspond to: $***. < 0.01$, $**. < 0.05$, $*. < 0.1$.

Notes: Table shows the comparison of the marginal effects at the covariates means between the *Ordered Forest* and the ordered logit. The effects are estimated for all classes, together with the corresponding standard errors, t-values and p-values. The standard errors for the *Ordered Forest* are estimated using the weight-based inference and the standard errors for the ordered logit are obtained via the delta method.