# Proxy-identification of a structural MGARCH model for asset returns\*

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Version: October 14, 2024

#### Abstract

We extend the multivariate GARCH (MGARCH) specification for volatility modeling by developing a structural MGARCH model that targets the identification of shocks and volatility spillovers in a speculative return system. Similarly to the proxy-SVAR framework, we leverage auxiliary proxy variables to identify the underlying shock system. The estimation of structural parameters, including an orthogonal matrix, is achieved through techniques derived from Riemannian optimization. Our analysis of daily S&P 500 returns, 10-year Treasury yields, and the U.S. Dollar Index, employing news-driven instrument variables, identifies an equity and a bond market shock.

**Keywords:** identification, Riemannian optimization, structural MGARCH, structural modeling, variance decomposition, volatility spillovers

#### JEL: C32, C51, C58, G10

\*We are grateful for financial support by the Swiss National Science Foundation (SNF Grant No: 176684). An earlier version was entitled 'Identifying Structural Shocks to Volatility Through a Proxy-MGARCH Model.' The authors thank the participants of the NBER-NSF Time Series Conference, SoFiE, the 14th Annual Risk Management Conference of NUS RMI, and the Conference of the Swiss Society of Economics and Statistics 2021 for their constructive comments, as well as the research seminar participants at TU Dresden, University of Sussex, UCL, and University of Bologna. We are indebted to Luca Fanelli for his valuable comments, which considerably improved the previous version of this manuscript. Special thanks to Kinane Habra for his research assistance.

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*Knowing the main source of the decline in equity prices (and financial assets in general) may help policymakers understand its persistence and evaluate the policy response.* ECB Economic Bulletin, Issue 4/2020.

# 1 Introduction

A central objective of multivariate volatility models is to accurately describe the stylized facts of asset returns and their second-order moment dynamics, including fat tails, leverage effects, and time-varying cross-asset dependencies (Andersen et al., 2009; Bauwens et al., 2012). Despite their widespread adoption, current multivariate volatility models fall short in providing a clear interpretation of the underlying shock systems due to their reduced-form nature. This limitation curtails their applicability in areas such as studying the transmission of specific economic shocks, investigating counterfactual policies, and analyzing variance error decompositions—domains where structural macroeconomic models excel. Naturally, such applications can also be valuable in applied finance, like in risk management and strategic asset allocation.

In this work, we take a step towards extending reduced-form multivariate volatility models of the GARCH form (MGARCH) to structural models similar to those used in macroeconometrics (Amisano and Giannini, 1997; Kilian and Lütkepohl, 2017). We build upon an MGARCH model recently introduced by Hafner et al. (2022). To estimate its structural representation, we adopt the concepts of proxy-identification, originally developed by Romer and Romer (2010), Mertens and Ravn (2013), and Stock and Watson (2012, 2016, 2018), i.e., we leverage information embedded in external fundamental data—referred to as proxy variables or instruments—to identify the structural parameters and, consequently, the structural shocks. In utilizing meaningful instrumental variables derived from news analytics data, we are also able to label the identified shocks.

For estimation, we formulate an augmented system that incorporates both the reducedform asset return shocks and instrument data and estimate the structural parameters within an expanded parameter vector. As Angelini and Fanelli (2019, 2023) describe, this approach enables partial, and under certain conditions, full identification of the structural parameters, even when not all shocks can be instrumented. The numerical aspect of our estimation poses a significant challenge due to the inclusion of an orthogonal matrix and a symmetric positive-definite matrix in the parameter vector. To address this challenge, we transform the permissible parameter space into a Riemannian matrix manifold—a smooth, locally Euclidean topological space. This transformation facilitates the application of optimization techniques from Riemannian geometry on the constrained sets. The remarkable efficiency of this method even supports the use of bootstrap techniques for inference.

We apply our methodology to study a portfolio of assets, consisting of the S&P 500 index, the constant maturity yield of the U.S. 10-year Treasury notes, and the U.S. Dollar Index. Our goal is to identify two types of shocks: an equity market shock, which refers to events directly impacting the valuation of stocks like shifts in investor risk preferences, and a bond market shock, intended to capture, e.g., changes in inflation expectations or monetary policy adjustments. To identify these shocks, we leverage news analytics data as instruments. The specific data we utilize, the Refinitiv MarketPsych news sentiment indicators (TRMI) for the U.S. stock and U.S. bond markets, are derived from extensive ticker news archives using advanced machine learning algorithms to classify news items by their novelty and relevance. Along with additional over-identifying restrictions, we estimate the structural parameters and identify three structural shocks: the equity market shock, the bond market shock, and a third shock that lacks a priori interpretation.

As an initial insight from the identified model, the orthogonal matrix we estimate significantly deviates from the unit matrix. This finding challenges the common adoption of the principal matrix square root, which assumes symmetric volatility spillovers and is a prevalent assumption in financial modeling. In other words, symmetric volatility spillovers may not effectively capture the complex nature of shock transmission in asset return systems. Next, we leverage the conditional variance dynamics of the structural MGARCH model in two distinct ways. Firstly, we derive the dynamic impact matrix, allowing us to examine two transmission channels crucial for portfolio allocation as well as monetary policy: the transmission of the structural bond market shock into the S&P 500 equation and the structural equity shock into the 10-year Treasury yield return equation. Both generally conform to the signs anticipated by economic theory, with significant time variation observed across our sample period. Interestingly, the bond market shock in the S&P 500 equation occasionally exhibits periods with a reversed sign. While it is of course known that the correlation between reduced-form yield returns and equity returns may switch sign, we reveal this phenomenon at the level of the identified impact matrix and the structural shocks.

Secondly, we analyze volatility spillovers among these markets using conditional zeroorder variance decompositions. Such decompositions can help investors understand how different risk sources contribute to overall portfolio volatility, thereby enabling them to adopt more effective diversification strategies. Once again, our analysis reveals significant variability in both the reception and transmission of volatility. For example, in terms of volatility transmission, our findings indicate a secular decrease in the equity shock's relative volatility contribution to S&P 500 returns from 1998 to 2019, contrasted with an increase in its relative impact on Treasury yield volatility. This trend may reflect a growing interconnectedness in volatility transmission between these key asset classes, beginning well before the 2008 financial crisis, and is also observed in similar contexts (Ehrmann et al., 2011; Diebold and Yilmaz, 2012).

#### Contributions and related literature

We contribute to the literature in several ways. From a methodological standpoint, we introduce a proxy-identification scheme for the structural MGARCH model proposed by Hafner et al. (2022). Unlike their approach, which employs ideas from independent com-

ponent analysis and cannot guarantee economically interpretable structural shocks, we build on proxy-identification and integrate the model within the framework established by Angelini and Fanelli (2019). This approach paves the way for extracting structural shocks that are not only identifiable but also potentially economically meaningful and labeled.

We also add to other recent advancements in proxy-identification, including Fisher and Huh (2019), Angelini et al. (2023), Giacomini et al. (2022a), and Plagborg-Møller and Wolf (2022) in the frequentist domain, alongside Arias et al. (2021) and Giacomini et al. (2022b) in Bayesian econometrics. Similarly to these studies, we identify several orthogonal structural shocks using multiple instruments. This is accomplished by directly estimating a structural orthogonal matrix in a frequentist's sense, exploiting the information provided by the proxy variables. Our modeling approach also applies concepts similar to those in Carriero and Volpicella (2024)'s max share approach. Importantly, like their method, we do not depend on a specific ordering of our variables.

Our modeling approach, focusing on speculative returns, features heteroskedasticity yet fundamentally differs from studies that identify structural shocks through heteroskedasticity. For instance, Primiceri (2005)'s model incorporates a nonstationary covariance structure. Research initiated by Rigobon (2003) and further developed by Lanne et al. (2010), Weber (2010), Lütkepohl and Schlaak (2021), and Lewis (2022), focuses on identifying a dynamic mean equation with a constant impact matrix. Both approaches do not align well with the perspective of return modeling in high-frequency contexts, where one typically assumes a zero mean equation but requires rich, stationary covariance dynamics.

The use of Riemannian optimization to estimate the model parameters represents a novel approach in econometrics. While the estimation of orthogonal matrices and rotation matrices has been explored in similar contexts, it is typically approached through nonlinear parametrizations, such as the Givens rotations, or through simulation, both of which can become cumbersome in high dimensions (Fisher and Huh, 2019; Hafner et al., 2022; Giacomini et al., 2022a; Carriero and Volpicella, 2024). Riemannian optimization, on the other hand, operates directly on the manifold, taking its local geometry into account, and thus scales more effectively to high-dimensional spaces. Recent years have witnessed significant advancements in adapting optimization routines from Euclidean spaces to differentiable manifolds, including techniques like steepest descent, conjugate gradient, and trust region algorithms (Adler et al., 2002; Absil et al., 2007; Huang et al., 2015). The specific techniques we utilize are becoming increasingly popular in certain fields of data science, such as image processing (Pennec et al., 2006), and likely have broad applicability across various areas of econometrics.

From an empirical perspective, we propose an avenue to the daunting task of identifying an asset return system. Indeed, the endogenous determination and rapid response of asset prices render the identification of an asset return system a challenging task because the traditional set of identifying restrictions commonly employed in the SVAR literature, including short- and long-run restrictions, sign restrictions, and exclusion or ordering restrictions, hardly have a justification. While significant advances have been made in understanding the effects of monetary policy decisions on equity prices, thanks to methods like high-frequency identification and identification by heteroskedasticity (Thorbecke, 1997; Rigobon and Sack, 2004; Bernanke and Kuttner, 2005, among others), the influence of equity prices on fixed income returns remains less investigated (Rigobon and Sack, 2003; D'Amico and Farka, 2011). Leveraging high-frequency news items for identification therefore holds untapped potential in asset return systems. Not only does it naturally align with the core principle in financial economics that unexpected fundamental information drives price revisions and volatility, but it also facilitates the labeling of structural shocks, thereby significantly enhancing their interpretability. The rest of the paper is organized as follows. Section 2 outlines our model and identification strategy. We discuss estimation in Section 3. Section 4 presents the empirical results, and Section 5 concludes. Additional information on optimization on Riemannian manifolds and tables supporting our analysis can be found in an appendix.

# 2 Structural GARCH modeling

In this section, we discuss the identification challenges associated with MGARCH models and proceed to introduce the structural GARCH model. Following this, we outline the proxy-identification framework.

#### 2.1 Rotational invariance and identification problem

We consider, for  $t \in \mathbb{Z}$ , the system of *n* speculative (log) returns given by

$$r_t = \mu_t + \varepsilon_t \tag{1}$$

where  $\mu_t = \mathbb{E}[r_t | \mathcal{F}_{t-1}]$  is the conditional mean of returns, with  $\mathcal{F}_t$  denoting the  $\sigma$ -algebra generated by the returns up to and including time t. The n-dimensional innovation vector  $\varepsilon_t$  satisfies  $\mathbb{E}[\varepsilon_t | \mathcal{F}_{t-1}] = 0$  and is conditionally heteroskedastic, i.e.,  $\mathbb{E}[\varepsilon_t \varepsilon_t^\top | \mathcal{F}_{t-1}] = H_t \in \mathbb{R}^{n \times n}$ . The conditional covariance matrix  $H_t$  is assumed to be positive-definite and symmetric with probability one. It displays any type of parametrized dynamics belonging to the class of stationary multivariate generalized autoregressive conditional heterskedasticity models (MGARCH). In our application, we opt for the BEKK(1, 1) model as proposed by Engle and Kroner (1995). In this case, the dynamics of  $H_t$  are represented by the form

$$H_{t} = CC^{\top} + A_{1}^{\top}\varepsilon_{t-1}\varepsilon_{t-1}^{\top}A_{1} + B_{1}^{\top}H_{t-1}B_{1} , \qquad (2)$$

for all  $t \in \mathbb{Z}$ , where *C* is a lower triangular matrix and  $A_1$  and  $B_1$  are coefficient matrices in  $\mathbb{R}^{n \times n}$ . The intercept matrix  $CC^{\top}$  is by construction symmetric and positive-definite if *C* has full rank, ensuring positive-definiteness of  $(H_t)_{t \in \mathbb{Z}}$ . Higher-order extensions to (2) exist; however, this model is typically chosen, as it offers the flexibility and richness in covariance dynamics needed for effective return modeling.

To complete the specification of (1), we let  $\varepsilon_t$  be generated, conditional on  $\mathcal{F}_{t-1}$ , as

$$\varepsilon_t = Q_t \xi_t , \qquad (3)$$

where  $Q_t \in \mathbb{R}^{n \times n}$  satisfies  $Q_t Q_t^{\top} = H_t$  and  $(\xi_t)_{t \in \mathbb{Z}}$  is an *n*-dimensional strict white noise process, i.e., a sequence of iid. shocks with  $\mathbb{E}[\xi_t] = 0$  and  $\mathbb{E}[\xi_t \xi_t^{\top}] = I_n$ , the *n*-dimensional identity matrix.

From the perspective of structural modeling, the matrix  $Q_t$  embodies the transmission mechanism that translates the structural shock  $\xi_t$  into the observable reduced-form innovation  $\varepsilon_t$ . The rows of  $Q_t$  specify the contribution of each element in  $\xi_t$  to the variance of  $\varepsilon_{it}$ , i = 1, ..., n. While  $\varepsilon_t$ , as a composite of various shocks, typically lacks an economic interpretation,  $\xi_t$  ideally can be given one. This characteristic allows for analyzing the impacts of an unforeseen independent event on the asset return system.

As a matter of fact, however, the decomposition of  $H_t = Q_t Q_t^{\top}$  and thus the structural shocks are not identified. To see this, denote by  $\mathcal{M}_o(n) = \{R \in \mathbb{R}^{n \times n} : RR^{\top} = I_n\}$  the set of real orthogonal matrices. For any  $R \in \mathcal{M}_o(n)$ , we obtain an observationally equivalent decomposition by substituting  $Q_t$  with  $Q_t R$  because  $H_t = (Q_t R)(Q_t R)^{\top} = Q_t Q_t^{\top}$ .

For this reason, the matrix decomposition is often determined ad-hoc. Popular choices include  $Q_t = H_t^{1/2}$ , which denotes the principal matrix square root obtained from an eigenvalue decomposition of  $H_t$ , and the Cholesky factorization of  $H_t$ . Both choices can be challenging to justify, yet they carry distinct modeling implications. The principal matrix square root implies symmetric volatility spillovers, while the Cholesky factorization

imposes a specific ordering of the asset returns in (1).

Motivated by these facts, Hafner et al. (2022) suggested the structural MGARCH model

$$\varepsilon_t = H_t^{1/2} R \xi_t , \qquad (4)$$

where  $R \in \mathcal{M}_{rot}(n)$ , the set of proper rotations matrices defined by  $\mathcal{M}_{rot}(n) = \{R \in \mathbb{R}^{n \times n} : RR^{\top} = I_n, \det(R) = +1\}$ , which is a subset of  $\mathcal{M}_o(n)$ . Of course,  $H_t^{1/2}$  is fully identified within the MGARCH model framework, when the principal matrix square root of  $H_t$  is adopted. The identification of R, on the other hand, necessitates additional identifying information. To this end, Hafner et al. (2022) introduce additional structure on (4) by assuming non-Gaussianity for the structural shock process  $(\xi_t)_{t \in \mathbb{Z}}$  and utilizing third and fourth-order moment conditions derived from the observed data. In the following, we develop a scheme for proxy-identification of (4).

#### 2.2 Identification by proxy

Besides adding statistical information or restrictions, a researcher—as highlighted by Stock and Watson (2012), Mertens and Ravn (2013), Stock and Watson (2018), among others—may achieve identification by utilizing further external data, also called instrument or proxy data. In contrast to purely statistical information, instrument data facilitates the labeling of identified shocks.

Under this paradigm, Angelini and Fanelli (2019, 2023) developed a framework for proxy-SVARs. They outline the necessary and sufficient conditions for local point-identification (up to sign normalization), encompassing both partially and fully identified systems, when *g* shocks are of interest and  $r \ge g$  external instruments are available for identification. The main idea is the creation of an augmented system that concurrently consists of both the primary variables of interest and the instruments. We modify this framework for the purpose of identifying the orthogonal matrix *R* and develop the corresponding estimation strategy, which leads to full local point-identification of the structural model in the application presented in Section 4. We comment on partial point-identification in Section 3.4.

We adopt the following assumptions:

#### Assumption 2.1.

(a) The reduced-form innovations  $(\varepsilon_t)_{t\in\mathbb{Z}}$  in (1) follow the *n*-dimensional stationary structural MGARCH process

$$\varepsilon_t = H_t^{1/2} R \xi_t , \qquad (5)$$

with an associated sequence of covariance matrices  $(H_t)_{t\in\mathbb{Z}}$ , where  $H_t \in \mathbb{R}^{n\times n}$  and positive-definite almost surely;  $H_t^{1/2}$  is the principal matrix square root of  $H_t$ ,  $R \in \mathcal{M}_o(n)$ , the set of orthogonal matrices, and  $(\xi_t)_{t\in\mathbb{Z}}$  is a vector-valued strict white noise process of structural shocks.

(b) The parameters of the variance equation of the MGARCH model, denoted by ∂,<sup>1</sup> are identifiable and consistently estimable as the sample size expands, and so is the sequence (*H*<sub>t</sub>)<sub>t∈ℤ</sub> = (*H*<sub>t</sub>(∂))<sub>t∈ℤ</sub>.

For the precise conditions underlying Assumption 2.1 (b) for quasi maximum likelihood estimation of MGARCH models, including the BEKK model as specified in (2), we refer to Comte and Lieberman (2003) and Hafner and Preminger (2009). Notably, our modeling does not rely on the BEKK model; other multivariate GARCH models, such as the diagonal model, the vector GARCH model, or constant and dynamic conditional correlation models, are also applicable—see Bauwens et al. (2012). In line with the approach taken by Hafner et al. (2022), we employ the principal matrix square root. This choice implies symmetric volatility spillovers when  $R = I_n$ , making it straightforward to test for. However,

<sup>&</sup>lt;sup>1</sup>In case of the BEKK(1,1) model in (2), this includes the matrices C,  $A_1$ , and  $B_1$ .

unlike their model, we consider  $R \in M_o(n)$ , as there appears to be no strong economic justification to exclude reflections, which correspond to det(R) = -1.

The fundamental assumption for proxy-identification for *g* structural shocks is

#### Assumption 2.2.

(a) There exists an observable, weakly stationary instrument process  $(Z_t)_{t \in \mathbb{Z}}$  with elements in  $\mathbb{R}^r$ , where  $r \ge g \ge 1$  and g < n, which is generated by

$$Z_t = \Phi \xi_t + v_t \tag{6}$$

where  $\Phi = (\Psi, 0_{r \times (n-g)})$  is an  $r \times n$  and  $\Psi$  an  $r \times g$  full column rank matrix,  $(v_t)_{t \in \mathbb{Z}}$  is a strict white noise process such that  $v_t \sim (0, \Sigma_v)$ , where  $\Sigma_v < \infty$  is its positive-definite covariance matrix.

(b) It holds that  $\xi_t \perp v_t$ , conditional on  $\mathcal{G}_{t-1}$ , where  $\mathcal{G}_t = \sigma(\{\varepsilon_s, Z_s (s \leq t)\})$  is an enlarged filtration.

Partitioning  $\xi_t = (\xi_{1:g,t}^{\top}, \xi_{g+1:n,t}^{\top})^{\top}$  into the *g* instrumented and (n - g) non-instrumented shocks, we see that in  $\Phi$ ,  $E_{t-1}[Z_t\xi_{1:g,t}^{\top}] = \Psi$  embodies the relevance conditions of the instruments, which reflect that the instruments are informative about the structural shocks of interest. In particular, the condition rank $(\Psi) = g$  ensures that the instruments provide non-redundant information about the shocks. The assumption  $E_{t-1}[Z_t\xi_{g+1:n,t}^{\top}] = 0_{r \times (n-g)}$  imposes r(n - g) exogeneity conditions on the structural shocks (Mertens and Ravn, 2013; Stock and Watson, 2018; Angelini and Fanelli, 2019; Giacomini et al., 2022b; Plagborg-Møller and Wolf, 2022). Importantly, this setup does not exclude the significant case where multiple instruments are jointly informative about a structural shock; in other words, we do not require  $\Psi$  to be diagonal.

Without loss of generality and for the sake of clarity, we drop here that  $(Z_t)_{t \in \mathbb{Z}}$  may be a full-fledged VAR process or may depend on other exogenous and deterministic variables

as well as on past structural shocks. It is important to observe that the elements of  $(Z_t)_{t \in \mathbb{Z}}$ do not have to be uncorrelated nor does  $\Sigma_v$  have to be diagonal. Crucially, however, while both  $Z_t$  and  $\varepsilon_t$  are driven by  $\xi_t$ , neither  $Z_t$  nor  $v_t$  must enter the return and variance process  $(\varepsilon_t, H_t)_{t \in \mathbb{Z}}$ , which would violate exogeneity.

Given  $\mathcal{G}_{t-1}$ -measurability of  $H_t$ , define standardized residuals  $u_t$  based on the initial decomposition by

$$u_t = H_t^{-1/2} \varepsilon_t = R\xi_t \,. \tag{7}$$

This allows us to define the enlarged system  $\eta_t = (u_t^{\top}, Z_t^{\top})^{\top}$  taking values in  $\mathbb{R}^m$ , where m = n + r, and

$$\eta_t = \begin{pmatrix} R_{\bullet,1:g} & R_{\bullet,g+1:n} & 0_{n \times r} \\ \Psi & 0_{r \times (n-g)} & \Sigma_v^{1/2} \end{pmatrix} \begin{pmatrix} \xi_t \\ v_t \end{pmatrix} = G \begin{pmatrix} \xi_t \\ v_t \end{pmatrix}.$$
(8)

Here, we partition  $R = (R_{\bullet,1:g}, R_{\bullet,g+1:n})$  into columns which are instrumented and noninstrumented, respectively, in accordance with  $\xi_t$ , and  $\Sigma_v^{1/2}$  is the principal matrix square root of  $\Sigma_v$ . Equation (8) corresponds to a VAR(0), where the first *n* elements of the governing shock vector are the uncorrelated, mean-zero, unit-variance structural shocks. The remaining *r* elements comprise the shocks that drive the instrument process. Importantly, the order of variables within the standardized return system  $(u_t)$  does not affect this setup.

To see the covariance restrictions of the enlarged model, define  $\Sigma_{\eta} = E[\eta_t \eta_t^{\top}] = GG^{\top}$ , where

$$\Sigma_{\eta} = \begin{pmatrix} I_n & R_{\bullet,1:g} \Psi^{\top} \\ \Psi R_{\bullet,1:g}^{\top} & \Psi \Psi^{\top} + \Sigma_v \end{pmatrix} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}.$$
(9)

This shows that the covariance restrictions of the enlarged model are  $\Sigma_{11} = I_n$ , reflecting the orthogonality of *R*, as well as  $\Sigma_{21} = \Sigma_{12}^{\top} = \Psi R_{\bullet,1:g}^{\top}$  and  $\Sigma_{22} = \Psi \Psi^{\top} + \Sigma_v$ , which are provided by the instruments.

It is well-known that the necessary condition for identification requires that m(m-1)/2 constraints be imposed on *G*. This requirement arises because a total of m(m+1)/2 parameters are determined by the orthogonality conditions on *R* and the parameters estimated in  $\Sigma_{\eta}$ . From (8), nr + r(n-g) zero constraints are derived due to the instrument exogeneity conditions. However, this number often falls short of the required m(m-1)/2, thus necessitating the imposition of additional conditions on *R*,  $\Psi$ , or  $\Sigma_v$ . If these additional conditions can be found and provided  $\Psi$  has full column rank, it is possible to identify *R* up to its sign.

**Example 2.1.** Consider the system of n = 3 assets and suppose we have r = 2 instruments that identify g = 2 shocks. In this case, we have

$$G = \begin{pmatrix} R_{11} & R_{12} & R_{13} & 0 & 0 \\ R_{21} & R_{22} & R_{23} & 0 & 0 \\ R_{31} & R_{32} & R_{33} & 0 & 0 \\ \psi_{11} & \psi_{12} & 0 & \sigma_{v,11} & \sigma_{v,12} \\ \psi_{21} & \psi_{22} & 0 & \sigma_{v,21} & \sigma_{v,22} \end{pmatrix},$$
(10)

where the first two columns  $R_{\bullet,1:2}$  refer to those identified directly by the instrumented shocks. Although there are fewer instruments than shocks, we may be able to identify the entire system up to sign normalization. Because m = n + r = 5, we need ten restrictions. Eight arise from the external instruments (nr = 6 from the exogeneity of the instrument process to the asset returns, and another r(n - g) = 2 from the orthogonality of both instruments to the third structural shock). Moreover, imposing symmetry on  $\Sigma_v^{1/2}$ , by requiring  $\sigma_{v,12} = \sigma_{v,21}$ , yields ( $r^2 - r$ )/2 = 1 additional restriction. Consequently, the system would be just-identified by imposing a zero restriction, say, on  $\psi_{12}$ . It becomes overidentified with the addition of another zero restriction, such as on  $\psi_{21}$ .

Both necessary and sufficient conditions for local identification are a consequence of Rothenberg (1971). Let  $\theta$  represent the free parameters to be estimated in  $G(\cdot)$ , and  $\theta_0$ 

their population values. Then, one needs to verify that

$$\operatorname{rank}\{D_m^+(G_0 \otimes I_m)S_G\} = a_G , \qquad (11)$$

where  $G_0 = G(\theta_0)$  must be 'regular' in the sense that its rank does not change in a neighborhood of  $\theta_0$ ; additionally,  $D_m^+$  is the Moore-Penrose generalized inverse of the duplication matrix,<sup>2</sup>  $S_G$  is a full column-rank selection matrix, which maps the  $a_G \leq \frac{1}{2}m(m+1)$  free parameters on vec(G)—see Angelini and Fanelli (2019).

### **3** Estimation strategy

We now present the estimation approach, which requires methods from Riemannian optimization. We briefly outline these concepts, drawing on Absil et al. (2008) and Boumal (2023), but defer further details to Appendix A. Finally, we comment on the estimation of partially identified systems.

#### 3.1 Estimation of the reduced-form and structural model

We first estimate the reduced-form MGARCH model, specifically the BEKK(1, 1) given in (2). To this end, we rely on the standard quasi-maximum likelihood method assuming Gaussianity—see Comte and Lieberman (2003) and Hafner and Preminger (2009) for the underlying theory. Our numerical optimization of the log-likelihood takes advantage of the analytical derivatives provided in Hafner and Herwartz (2008). This yields the estimates for the BEKK parameters  $\hat{\vartheta} = \left( \operatorname{vec} (\hat{A}_1)^\top, \operatorname{vec} (\hat{B}_1)^\top, \operatorname{vech} (\hat{C})^\top \right)^\top$ , as well as  $\hat{H}_t(\hat{\vartheta})$  and the standardized reduced-form residuals  $\hat{u}_t = \hat{u}_t(\hat{\vartheta}) = \hat{H}_t(\hat{\vartheta})^{-1/2}\hat{\varepsilon}_t$ . From these, we construct the augmented system  $\eta_t = (\hat{u}_t^\top, Z_t^\top)^\top$ .

<sup>&</sup>lt;sup>2</sup>The duplication matrix  $D_m$  of size  $m^2 \times \frac{1}{2}m(m+1)$  is defined by the property that  $vec(A) = D_m vech(A)$ , for any symmetric  $m \times m$  matrix A.

In the subsequent step, aiming to fully identify our model, we employ maximum likelihood estimation for (8) as suggested by Angelini and Fanelli (2019). Specifically, we assume joint normality of  $\eta_t$ . Then, the reduced-form estimator for  $\Sigma_{\eta}$  is the sample covariance  $\hat{\Sigma}_{\eta} = \frac{1}{T} \sum_{t=1}^{T} \eta \eta^{\top}$ .

The log-likelihood of the structural model is given by

$$\mathscr{L}^{s}(\theta) = -\frac{mT}{2}\log(2\pi) - \frac{T}{2}\log\det(GG^{\top}) - \frac{T}{2}\operatorname{tr}\{G^{-1}\widehat{\Sigma}_{\eta}(G^{-1})^{\top}\}, \quad (12)$$

where we recall that  $G = G(\theta)$ , i.e., G depends on the parameter vector  $\theta \in \mathbb{R}^{a_G}$ ,  $a_G \leq m(m+1)/2$ , and  $\theta = \left( \operatorname{vec}(R)^{\top}, \psi^{\top}, \operatorname{vec}(\Sigma_v^{1/2})^{\top} \right)^{\top}$ . Specifically, the various elements of  $\theta$  satisfy  $R \in \mathcal{M}_o(n)$ ,  $\psi \in \mathbb{R}^{a_{\psi}}$ ,  $\Sigma_v^{1/2} \in \mathcal{M}_{spd}(r)$ , where  $\mathcal{M}_o(n)$  is the set of  $n \times n$  orthogonal matrices,  $\psi$  is a vector of length  $a_{\psi} \leq rg$ , collecting the free parameters of  $\Psi$ , not constrained to zero, and  $\mathcal{M}_{spd}(r)$  denotes the set of  $r \times r$  symmetric positive-definite matrices. Thus, the structural parameter matrix R will be estimated, up to sign, as part of the arguments that maximize the log-likelihood. This enures that the covariance restrictions outlined in (9) are observed.

As is well-known, optimizing this log-likelihood is challenging due to the non-convex nature of the orthogonality constraints and the high cost associated with maintaining them during the iterations. The key to effective estimation lies in recognizing that  $\mathcal{M}_o(n)$  and  $\mathcal{M}_{spd}(r)$  are not merely sets of matrices but can be transformed into Riemannian (matrix) manifolds. While this insight may not seem profound initially, it opens the door to the application of Riemannian optimization for maximizing the log-likelihood. By leveraging the intrinsic geometric structure of the manifolds during iterations, Riemannian optimization ensures that the estimates remain within the underlying manifold and avoids having to impose cumbersome nonlinearity conditions.

#### 3.2 Optimization on manifolds

A manifold is a topological space that, in an open neighborhood around any given point, can be approximated by a Euclidean space. The specific notion of a manifold we rely on is that of a  $d_1$ -dimensional smooth submanifold embedded in an ambient Euclidean space  $\mathcal{E}$  of dimension  $d_2$ ; for our purposes, we can identify  $\mathcal{E}$  with  $\mathbb{R}^{d_2}$ .

**Definition 3.1.** (Boumal, 2023). A non-empty set  $\mathcal{M}$ , embedded in  $\mathcal{E}$ , is called a smooth submanifold of dimension  $d_1 \leq d_2$  in either of two cases:

- (*i*) if  $d_1 = d_2$  and  $\mathcal{M}$  is open in  $\mathcal{E}$ ; it is then called an open submanifold in  $\mathcal{E}$ .
- (*ii*) If  $d_1 < d_2$  and the set is of form  $\mathcal{M} = \{X \in \mathcal{E} : h(X) = 0_{d_2-d_1}\}$ , where the defining function  $h : \mathcal{E} \to \mathbb{R}^{d_2-d_1}$  is infinitely often differentiable and its Jacobian has rank  $d_2 d_1$ , for all  $X \in \mathcal{M}$ .<sup>3</sup>

As we make specific in Appendix A.2, the set of symmetric positive-definite matrices is an example of case (*i*), while the set of orthogonal matrices is covered by case (*ii*). The two-dimensional sphere embedded in  $\mathbb{R}^3$  is another example of a manifold of case (*ii*), which we use for illustration in Figure 1.

Due to smoothness, we can locally approximate  $\mathcal{M}$  at each point  $X \in \mathcal{M}$  using a linear space known as the tangent space, which contains all vectors that tangentially pass through X. For the sphere in Figure 1,  $\mathcal{T}_{X_k}\mathcal{M}$  is represented by the set of vectors orthogonal to  $X_k$  and depicted as a plane touching the sphere at  $X_k$ . A manifold becomes a Riemannian manifold after endowing all tangent spaces with an inner product that varies smoothly in X. This inner product is called the Riemannian metric.

<sup>&</sup>lt;sup>3</sup>This notion of a manifold is greatly simplified. It is, however, adequate for our work and is still fully compatible with the more general concept of a manifold, which does not require an embedding space and includes charts and atlases.



Figure 1: Illustration of an embedded Riemannian manifold (two-dimensional sphere) with a tangent space  $\mathcal{T}_{X_k}\mathcal{M}$ , Riemannian gradient grad  $f(X_k)$ , and retraction step. Figure 1a demonstrates how to obtain grad  $f(X_k)$  by projecting the Euclidean gradient  $\nabla f(X_k)$  onto the tangent space. We emphasize that this particular projection step is only applicable in certain cases, which apply here for the manifold of orthogonal matrices only—see main text and appendix for more details. Figure 1b depicts the retraction process, returning a multiple of grad  $f(X_k)$  to the manifold.

Given a differentiable function defined on the manifold,  $f : \mathcal{M} \to \mathbb{R}$ , we can define the Riemannian gradient with the help of the Riemannian metric, similarly to the Euclidean space. This gradient represents the direction of steepest ascent of f in  $\mathcal{T}_X \mathcal{M}$  at X, while taking the local geometry of  $\mathcal{M}$  at X into account. If the Euclidean inner product of the ambient space is selected as the Riemannian metric, the Riemannian gradient can be derived by orthogonally projecting the Euclidean gradient of f onto  $\mathcal{T}_X \mathcal{M}$ , as sketched in Figure 1a. In our application, this principle is relevant to  $\mathcal{M}_o(n)$  but not to  $\mathcal{M}_{spd}(r)$  due to the availability of computationally more effective choices for the Riemannian metric—see Appendix A.2 for an explanation.

In gradient-based minimization methods for Euclidean spaces, during the *k*th iteration, one updates the new point  $X_{k+1}$  by adding to  $X_k$  a search step of size  $\alpha_k > 0$  in the direction of the negative gradient. Because this approach would lead to 'leaving'  $\mathcal{M}$  due to its non-Euclidean geometry, this updating scheme cannot be applied. Instead, a

retraction  $\Gamma$  is required, which is a map that takes its argument (the tangent vector) back to the manifold, yet oriented in the direction as specified by the argument—see Figure 1b.

Given a retraction, the blueprint of a gradient descent on a Riemannian manifold, updating  $X_k$  to  $X_{k+1}$ , takes the form:

$$X_{k+1} = \Gamma_{X_k} \left( -\alpha_k \operatorname{grad} f(X_k) \right)$$
,

with the step size  $\alpha_k$  appropriately chosen. While different retractions can be constructed, the key to efficiency lies in having one that is both fast and reliable to compute. In Appendix A.2, we detail the specific Riemannian metrics and retractions we use for the present optimization problem.

Second-order accurate schemes, such as Newton's method and trust region algorithms, have been developed by extending these concepts to Riemannian geometry. This enables effective optimization on manifolds (Adler et al., 2002; Absil et al., 2007; Huang et al., 2015).

#### 3.3 Discussion of the estimator

Building upon the framework detailed in Section 3.2 and further expanded in Appendix A, particularly through the careful selection of suitable ambient spaces and Riemannian metrics, we transform the problem of optimizing the log-likelihood into a task of function maximization over a manifold. Thus, the estimate is given by

$$\widehat{\theta} = \underset{\theta \in \Theta \cap \mathcal{M}}{\operatorname{arg\,max}} \quad \mathscr{L}^{s}(\theta) \tag{13}$$

where  $\mathscr{L}^{s}(\theta)$  is defined in (12) and  $\theta = \left(\operatorname{vec}(R)^{\top}, \psi^{\top}, \operatorname{vec}(\Sigma_{v}^{1/2})\right)^{\top}$ . The manifold  $\mathcal{M}$  is the Cartesian product manifold  $\mathcal{M} = \mathcal{M}_{o}(n) \times \mathcal{M}_{e}(a_{\psi}) \times \mathcal{M}_{spd}(r)$ , constructed from the manifolds of orthogonal matrices, the Euclidean space of dimension  $a_{\psi}$ ,<sup>4</sup> and the mani-

<sup>&</sup>lt;sup>4</sup>The Euclidean space is an example of the (trivial) linear manifold.

fold of symmetric positive-definite matrices. This yields a well-defined submanifold embedded in the product space of the ambient vector spaces: specifically, the vector space of  $n \times n$  matrices, the Euclidean space of dimension  $a_{\psi}$ , and the vector space of  $r \times r$ symmetric matrices. We assume that  $\Theta$  is a compact set within this ambient space and is positive-definite in the coordinates associated with  $\Sigma_v^{1/2}$ .

It is important to realize that (13) is fully equivalent to maximizing the log-likelihood in a Euclidean space, subject to additional linear and nonlinear constraints. Therefore, under Assumptions 2.1 and 2.2, the estimate is consistent and asymptotically normal, assuming the 'usual' regularity conditions and provided  $\theta_0$  resides in  $\Theta \cap \mathcal{M}$  (Newey and McFadden, 1994; Wooldridge, 1994). Note first that  $\Theta \cap \mathcal{M}$  is compact. This follows from the fact that the orthogonality and symmetry constraints can be expressed as  $g(\theta) = 0$ , which forms a closed subset in  $\Theta$ . This closed subset remains compact if  $\Theta$  is compact. Thus, consistency follows, provided the estimator over  $\Theta$  is consistent. Moreover, the function g is twice continuously differentiable and has a full-rank Jacobian—recall the rank condition of Definition 3.1 in case (*ii*), implying non-redundency of any of the constraints. Hence, asymptotic normality is a direct consequence of Rothenberg (1973).

Asymptotic standard errors exploiting the expected efficiency gains can therefore be obtained by evaluating the formulae for the asymptotic variance given in Rothenberg (1973) at the optimizer. However, because our implementation is sufficiently fast and  $\eta_t$  conditionally homoskedastic, we can, as Angelini and Fanelli (2019), take advantage of a residual-based bootstrap, allowing us to efficiently compute the standard errors for the estimates—see Jentsch and Lunsford (2022) for a discussion in the context of proxy identification.<sup>5</sup> In our implementation, we derived the analytical Euclidean gradients of the

<sup>&</sup>lt;sup>5</sup>This approach does not account for the additional error that may arise from estimating the parameters of the MGARCH model. While the bootstrap could, in principle, be extended in this way, it will be computationally cumbersome for three-dimensional MGARCH systems. Therefore, we decided against this extension.

likelihood function and used a trust-region algorithm, taking advantage of the Matlab toolbox Manopt—see Boumal et al. (2014).

#### 3.4 Partial identification

In cases of higher dimensional systems or a shortage of appropriate instruments, partial identification may become necessary. Suppose that g < n shocks are targeted for identification, which are, without any loss of generality, ordered first in the system. Then, one is no longer interested in estimating the entire orthogonal matrix R, but in a subset of its parameters given by its first g columns, denoted  $R_1$ . Within the augmented system, the set of parameters to be estimated consists of the first g columns of the matrix G in (8), i.e.,

$$G_1 = \begin{pmatrix} R_1 \\ \Psi \end{pmatrix} . \tag{14}$$

Following Angelini and Fanelli (2019), the estimation of  $G_1$  can be framed as a classical minimum distance estimator. In the present case, however, the additional complexity is that the estimation must be carried out under the constraint that the tall matrix  $R_1 \in \{X \in \mathbb{R}^{n \times g} : X^\top X = I_g\}$ , where g < n. This set of matrices, known as the Stiefel manifold, can be transformed into a Riemannian manifold by choosing a suitable Riemannian metric—see Absil et al. (2008). Thus, after selecting a retraction, we can employ Riemannian optimization as outlined above to obtain the parameter estimates.

The identification conditions remain standard. If r = g = 1, the single shock is identified. If r = g > 1, identification requires imposing additional  $\frac{1}{2}g(g - 1)$  restrictions on  $G_1$  to identify the *g* shocks, which is only a necessary condition. If r > g, all *g* shocks are identified, or even overidentified, without further restrictions—see Angelini and Fanelli (2019, 2023) for the in-depth discussion, particularly regarding necessary and sufficient conditions. It is therefore straightforward to accommodate a scenario of partial identification within our current framework. In fact, as a proof-of-concept, we have already implemented this estimator in unreported robustness checks.

### 4 An identified asset return system

We implement the structural proxy-MGARCH model on a three-dimensional portfolio comprising daily returns from equity, bond yields, and foreign exchange markets. Our objective is to identify equity and bond market shocks by means of news analytics data.

#### 4.1 Data

The asset triple, ranging from 01/01/1998 to 12/31/2019, consists of the S&P 500 index, the constant maturity yield of U.S. 10-year Treasury notes, and the U.S. Dollar Index, which is a measure of the value of the U.S. Dollar relative to a currency basket of major U.S. trade partners. The top row of Figure 2 displays the demeaned log returns of each series. They exhibit the salient features of daily return data, including heteroskedasticity and volatility clustering—see also the summary statistics in the left panel of Table 1.

We select the instrument data to proxy for the structural shocks of asset returns based on a core concept in financial economics and econometrics: asset prices move in response to new, unexpected fundamental information.<sup>6</sup> The study of this concept has received renewed momentum through the utilization of high-frequency intraday data and the advent of news analytics data, which extract information from a broad spectrum

<sup>&</sup>lt;sup>6</sup>See Grossman and Stiglitz (1980), Milgrom and Stokey (1982), and Tauchen and Pitts (1983) for early theoretical formalizations of this idea, and Clark (1973), Engle (1982), and Engle and Ng (1993) for pioneer-ing econometric advancements.



Figure 2: Upper panel: Demeaned daily log returns of the S&P 500 index, the constant maturity yield of U.S. 10 year Treasury notes, and the U.S. Dollar Index. Lower panel: standardized TRMI U.S. stock index and TRMI U.S. bond sentiment on trading days, LOESS and VAR filtered. Data range: 01/01/1998 to 12/31/2019.

of news sources using advanced machine learning (Groß-Klußmann and Hautsch, 2011; Michaelides et al., 2015; Bollerslev et al., 2018; Boudoukh et al., 2018).

We therefore derive the proxy variables from the Thomson Reuters MarketPsych Indices (TRMI). The TRMIs are constructed based on a proprietary supervised natural language processing scheme applied to a broad range of media outlets, which includes the live content delivered via the Thomson Reuters News Feed Direct and LexisNexis as well as common financial news sites, such as The New York Times, The Wall Street Journal, Financial Times, Seeking Alpha, among others.<sup>7</sup> The algorithm organizes the gathered news items into categories such as company, economic sector, geographical area, country, commodity or energy subjects, and currency. It also assesses and scores all references

<sup>&</sup>lt;sup>7</sup>For more information, see https://www.marketpsych.com/ and Peterson (2016, Appendix A).

		Descripti	ve statistic	cs		
	A	sset return	S		Instrume	ent data
Statistic	S&P 500	Yield	USDX		Equities	Bonds
Min.	-0.095	-0.185	-0.027	-	-3.098	-4.643
Max.	0.109	0.097	0.024		3.204	5.847
Mean	0.000	0.000	0.000		0.000	0.000
Median	0.000	0.000	0.000		-0.013	-0.007
Std. Dev.	0.012	0.018	0.005		1.000	1.000
Skewn.	-0.239	-0.082	-0.042		0.068	0.171
Kurt.	11.100	7.037	4.499		2.486	4.088

Table 1: Descriptive statistics of the asset return system and instrument data. The first three columns show the demeaned daily log returns of the S&P 500 index, the constant maturity yield of the U.S. 10-year Treasury notes, and the U.S. Dollar Index (USDX). The last two columns exhibit the proxy data derived from the U.S. stock index sentiment and U.S. bond sentiment (TRMI MarketPsych indices). The sample period extends from 01/01/1998 to 12/31/2019, encompassing N = 5544 observations. Data sources: Thomson Reuters.

in real time for their relevance, freshness of information, and sentiment—whether it is positive, negative, or neutral.

We use data at daily frequency, which only captures news items published until 3:30 p.m. Eastern Time. While this timing is not perfectly aligned with the end of the core trading session of the NYSE at 4:00 p.m. Eastern Time, it precludes a forward-looking bias. While it is impossible to test whether the selected TRMIs form valid instruments, by the way the TRMIs are constructed from live-feed news ticker data, it appears plausible that the unobservable structural shocks we aim to identify drive returns *and* enter a feed of unexpected sentimentally-charged news items, as required by Assumptions 2.1 and 2.2. TRMIs are employed by a burgeoning empirical literature (Michaelides et al., 2015; Sun et al., 2016; Audrino and Tetereva, 2019; Michaelides et al., 2019).

To identify a structural equity market shock, which may signal shifts in investor risk preferences or significant macroeconomic events influencing stock valuations, we select TRMI's U.S. stock index sentiment as an instrumental variable. To identify a structural bond market shock, aimed at capturing shocks to bond prices such as changes in the real interest rate, inflation expectations, or monetary policy shocks, we select TRMI's U.S. bond sentiment as an instrumental variable. Both sentiment indices aggregate all relevant scored positive references, net of all negative references, made within news items in their respective subject areas, and are normalized to range from [-1, 1]. Thus, positive values indicate upbeat sentiment and negative values negative sentiment.

As outlined in Example 2.1 and discussed in more detail in Section 4.2, two instruments are sufficient for local point-identification. To account for the possibility of a slowly time-varying mean component in the instrument data, we filtered the time series using a local polynomial (LOESS) regression. We also eliminated any (cross-)dynamics in the mean by fitting high-order VAR models.<sup>8</sup> The unexpected innovation to the U.S. stock index sentiment and the U.S. bond sentiment, standardized to unit variance, serves as our instrumental variable—see the right panel of Table 1 and the lower panel of Figure 2.

#### 4.2 Structural model estimates and identified shocks

Given the asset return system, where the S&P 500 equation is ordered first, the 10-year Treasury note second, followed by the U.S. Dollar Index, we first estimate the BEKK(1,1) model. As reported in Table 2, the diagonal parameters and selected off-diagonal parameters of the coefficient matrices are statistically significant at conventional levels. The Akaike criterion speaks in favor of the full BEKK specification, leading us to base the further analysis on this model.

With n = 3 assets and g = 2 shocks instrumented with r = 2 proxy variables, the ap-

<sup>&</sup>lt;sup>8</sup>We also explored alternative approaches, including moving average filtering and ARMA modeling, both with and without prior nonparametric demeaning, which lead to qualitatively very similar results.

plication corresponds to Example 2.1. In our overidentified model, we impose ten zero constraints and enforce symmetry on  $\Sigma_v$ , i.e.,  $\varpi = \sigma_{v,12} = \sigma_{v,21}$ . We thus estimate

$$G = \begin{pmatrix} R_{11} & R_{12} & R_{13} & 0 & 0 \\ R_{21} & R_{22} & R_{23} & 0 & 0 \\ R_{31} & R_{32} & R_{33} & 0 & 0 \\ \psi_{11} & 0 & 0 & \sigma_{v,11} & \varpi \\ 0 & \psi_{22} & 0 & \varpi & \sigma_{v,22} \end{pmatrix}.$$
 (15)

Table 3 displays the estimates.<sup>9</sup> Because the first *n* columns of *G* can only be identified up to sign, we obtain the column signs by specifically considering the economic meaning behind the diagonal elements of R.<sup>10</sup> Given our association of  $\xi_1$  with an equity shock, identified through a market sentiment variable, a positive shock is anticipated to have a positive effect on the S&P 500 equation; hence, we choose the sign of  $\hat{R}_{\bullet 1}$  such that  $\hat{R}_{11} > 0$ . Regarding  $\xi_2$ , a positive realization is expected to inversely affect the yield. Consequently, we set the sign such that  $\hat{R}_{22} < 0$ . Lastly, we make sure  $\hat{R}_{33} > 0$ , consistent with the interpretation that a positive shock increases the U.S. Dollar Index, which, by construction of the index, implies that the U.S. Dollar gains value compared to the currencies of major U.S. trading partners. In Appendix **B**, we analyze financial news from days coinciding with the tail events of the shocks. This analysis bolsters these interpretations and also suggests that  $\xi_3$  may exhibit characteristics of a currency shock.

As Table 3 shows, the orthogonal matrix has the largest entries along the diagonal, indicating that the nth equation is most strongly impacted by the nth structural shock. This appears plausible, considering the interpretation of the shocks. Judged by the boot-

<sup>&</sup>lt;sup>9</sup>To verify the necessary and sufficient condition for identification as described by Rothenberg (1971), we evaluate (11) using the estimate. Examination of its singular values and condition number suggests that rank{ $D_m^+(\hat{G} \otimes I_m)S_G$ } = 15, which supports identification.

<sup>&</sup>lt;sup>10</sup>The signs of column n + 1, ..., n + r are fixed because  $\hat{\Sigma}_{v}^{1/2}$  is estimated as a symmetric positive-definite matrix.

strapped standard errors, the diagonal entries of the matrix are significantly different from either 1 or -1, whereas the off-diagonal elements are clearly non-zero. This suggests that spillovers may not be symmetric, as would be implied by an estimate of *R* close to the unit matrix (up to sign). We test this hypothesis below.

The estimates  $\hat{\psi}_{11}$  and  $\hat{\psi}_{22}$  are significant, implying that the null hypothesis of 'no relevance' of the instruments is soundly rejected. It is worth noting that  $\hat{R}_{22}$  is estimated to have a sign opposite to  $\hat{\psi}_{22}$ , a difference that stems naturally from the instrument being a bond price sentiment, while we model the yield. This is in contrast to  $\hat{R}_{11}$  and  $\hat{\psi}_{11}$ , which have the same sign because the instrument is an equity market sentiment.

In Table 4, we present the descriptive statistics related to the inferred structural shocks, including the correlations among the shocks as well as their correlations with the proxy variables. The structural shocks are mutually uncorrelated, and additional tests, not shown, suggest white noise characteristics. The correlation patterns with the proxy variables conform to the diagonal structure imposed on  $\Psi$ . Notably, the correlations between  $Z_1$  and  $\hat{\xi}_1$ , and between  $Z_2$  and  $\hat{\xi}_2$  are strongly significant. Conversely, the remaining correlations are negligibly small. While this follows by assumption in the overidentified system, we can also confirm it in the just-identified models.

As discussed, the model is specified with one more constraint than necessary for exact identification. To assess overidentification, we employ the likelihood ratio statistic  $LR_T = -2(\mathscr{L}^s(\hat{\theta}) - \mathscr{L}(\tilde{\theta}))$ , which is approximately distributed as a  $\chi^2(1)$  variable, where  $\mathscr{L}(\tilde{\theta})$  denotes the reduced-form log-likelihood, with  $\tilde{\theta} = \operatorname{vech}(\hat{\Sigma}_{\eta})$ . The test statistic, shown in the lower part of Table 3, indicates that the model is well-specified.

	Ĉ			$\hat{A}_1$			$\hat{B}_1$	
0.0012	0.0000	0.0000	0.3013	0.0301	-0.0105	0.9470	-0.0047	0.0036
0.0000	0.0010	0.0000	-0.0072	0.2104	0.0025	0.0032	0.9765	-0.0009
0.0000	0.0001	0.0002	0.0514	-0.0611	0.1459	-0.0126	0.0101	0.9882
(7.2445)	ı	ı	(4.7720)	(0.1861)	(-0.8522)	(50.322)	(-0.0840)	(1.1631)
(-0.1385)	(5.4585)	ı	(0.0209)	(7.6499)	(1.3591)	(0.2268)	(195.32)	(-1.6027)
(0.3106)	(0.8824)	(3.3359)	(1.3258)	(-1.0422)	(13.169)	(-0.6507)	(1.1362)	(677.01)
Spectral rad	dius: 0.9986							
Akaike crit	erion							
full BEKK:	-110,822		diagonal B	3) (EKK: -110, 8)	312			

Table 2: Quasi maximum likelihood (QML) parameter estimates of the unrestricted BEKK(1,1) model of the demeaned daily log returns of the S&P 500, the constant maturity yield of U.S. 10-year Treasury notes, and the U.S. Dollar Index in order of appearance over the time period from 01/01/1998 to 12/31/2019. Entries in parentheses are the robust QML *t*-ratios. Lastly, we evaluate the symmetric spillover hypothesis, which posits that  $R = I_3$ . Naturally, due to sign normalization, this hypothesis is equivalent to flipping any of the signs in  $I_3$ . This test, which can be considered another overidentification test, can be evaluated using the likelihood ratio statistic  $LR_T = -2(\mathscr{L}^s(\check{\theta}) - \mathscr{L}^s(\hat{\theta}))$ , where  $\check{\theta} = (\check{\Psi}, \operatorname{vech}(\check{\Sigma}_v^{1/2}))^{\top}$  is an estimate of the shortened parameter vector obtained under the restriction  $R = I_3$ . Because R, as an orthogonal matrix, has n(n-1)/2 = 3 degrees of freedom, this LR test has an asymptotic  $\chi^2(3)$  distribution. As evidenced in Table 3, the data support the estimated model more than they do a symmetric one. This is in line with what we expected from our earlier analysis of the individual elements in  $\hat{R}$ .

Following our identification strategy, we categorize the targeted shocks  $\xi_1$  and  $\xi_2$  as a structural equity market shock and a structural bond market shock, respectively. To gain deeper insights into these shocks, we also examined the financial news on days linked to their tail events. Our analysis, given in Appendix **B**, reveals that the tail events of these structural shocks are indeed associated with distinct categories of news, specifically affecting either equity or bond valuations. Additionally, in the case of the non-targeted shock  $\xi_3$ , which lacks an a priori interpretation, we observe that it mostly associates with news items about currency markets, but also involves news about the oil and gold markets. Despite that ambiguity, as we discuss in Appendix **B**, the observations lend support to the interpretation that this shock exhibits characteristics of a currency shock, a label we henceforth adopt for brevity.

#### 4.3 Dynamic impact matrix

In Figure 3, we display the elements of the dynamic impact matrix  $Q_t = H_t^{1/2}R$ . To understand the patterns, it useful to observe that  $H_t^{1/2}$  is symmetric, with its diagonal entries being strictly positive and the largest within their respective rows and columns. This fol-

	proxy MGAR	CH model			
	0.9149	0.3622	0.1781	0	0
$\left(\hat{R}_{12},\hat{R}_{2}\right)$	0.3846	-0.9162	-0.1128	0	0
$\hat{G} = \begin{pmatrix} R_{\bullet,1:2} & R_{\bullet,3} & 0.3 \times 2 \\$	-0.1223	-0.1717	0.9775	0	0
$\left( \Psi  0_{2\times 1} \downarrow \Sigma_v^{1/2} \right)$	0.3532	0	0	0.9337	0.0612
	0	0.1698	0	0.0612	0.9838
				1	
	0.0141	0.0346	0.0482	I	
	0.0314	0.0163	0.0720	1	
Bootrapped std. errors	0.0358	0.0783	0.0162		
	0.0116			0.0080	0.0065
		0.0137		0.0065	0.0118
				1	
	$LR_T$	<i>p</i> -value			
Overidentification test, $\chi^2(1)$	1.13	[0.29]			
Symmetry test, $\chi^2(3)$	149.41	[0.00]			

Table 3: Estimation results of the structural MGARCH model of the demeaned daily log returns of the S&P 500, the constant maturity yield of U.S. 10-year Treasury notes, and the U.S. Dollar Index from 01/01/1998 to 12/31/2019, when using the stock market sentiment ( $Z_1$ ) and bond market sentiment ( $Z_2$ ) TRMIs as proxy variables. The upper panel shows the estimated elements in *G*, including the orthogonal matrix, the estimated relevance parameters, and the symmetric matrix decomposition of the instruments error variance. The lower panel exhibits the bootstrapped standard errors obtained with 999 replications.

lows from the positive-definiteness of  $H_t$  and our selection of the principal matrix square root for  $H_t^{1/2}$ . Consequently, and due to the patterns in  $\hat{R}$ , the trajectories of the diagonal elements, displayed in Figures 3a, 3e, and 3i, are the predominant elements in each row and column. For example, as depicted in Figure 3a,  $q_{t,11}$  indicates that a unit equity shock leads to approximately a 1% change in the S&P 500, with the response remaining relatively stable, except in periods of crisis. Figure 3e reveals that a unit bond market shock reduced 10-year yields by 1% until 2007. Since 2008, the yields became significantly more responsive to bond market shocks, particularly so in 2009 and 2012, during which the U.S. Federal Reserve implemented a series of quantitative easing measures. Lastly, a unit shift in the third shock contributes approximately 0.05% to the appreciation of the USD, as reflected by the U.S. Dollar Index.

Comparing the off-diagonal elements of our study with those found in the existing literature offers valuable insights. Although the evidence seems to be scarce, the prevailing view suggests that a positive equity market shock leads to higher yields. This is because monetary authorities, while not directly targeting the stock market, closely watch its impact on economic output and inflation, which may prompt a tightening of monetary policy. Consistent with this perspective, Figure 3d demonstrates that a unit equity shock exerts a positive influence on long-term yield changes. Remarkably, this finding is in concordance with Rigobon and Sack (2003) and D'Amico and Farka (2011), albeit these studies utilize different data types, short-dated yields, and distinct identification strategies. However, unlike their constant estimates, our analysis uncovers considerable time variation in the strength of this impact. This is consistent with Boyd et al. (2005), who find evidence of strong responses in the U.S. equity market to macroeconomic news, varying with economic conditions.

Conversely, Figure 3a illustrates the influence of the bond market shock on the S&P 500, which, when compared to the more pronounced effects of the equity shock, appears subdued. Generally, the impact is positive, suggesting that a positive bond market shock, by lowering yields, favorably affects the stock market. This finding is consistent with a broad literature—see, inter alia, Thorbecke (1997), Rigobon and Sack (2004), and Bernanke and Kuttner (2005). Notably, our dynamic model identifies specific intervals where the impact switches sign, such as after the dot-com crisis and between 2010 and 2013, a nuance not captured by the aforementioned studies.<sup>11</sup>

<sup>&</sup>lt;sup>11</sup>We emphasize that our discussion does not assert a specific sign of the conditional correlation between reduced-form yield changes and S&P 500 returns. This correlation is governed by  $Cov_t[\varepsilon_{t+1,1}, \varepsilon_{t+1,2}] =$ 

Lastly, examining the third structural shock, we note that a positive shock, resulting in an appreciation of the USD, positively affects the S&P 500 return and, generally, has a negative effect on the 10-year yields. This pattern is consistent with a demand-based interpretation, wherein an increased demand for the USD, leading to its appreciation, positively impacts U.S. equity and bond prices. It supports the characterization of  $\xi_3$  as a currency shock. However, as detailed in Section 4.4, the variance contribution of this shock is minor.

#### 4.4 Volatility spillovers

An important application of MGARCH models is the analysis of volatility spillovers between markets (Bauwens et al., 2006). In our identified model, volatility spillovers can be effectively demonstrated through variance decompositions. These decompositions provide valuable insights for investors by breaking down the sources of variability in asset returns. This enables them to identify the factors driving volatility and assess their impact on portfolio performance. Additionally, understanding the variance components can help investors construct hedging strategies to mitigate specific risks.

To quantify volatility spillovers, we define the following two measures for volatility reception and transmission. Let  $i, j \in \{1, ..., n\}$ ; then volatility reception and transmission between *i* and *j* is measured by

$$VR_{t,i\leftarrow j} = \frac{q_{t,ij}^2}{\sum_{l=1}^n q_{t,il}^2} \quad \text{(volatility reception)} \tag{16}$$

$$VT_{t,i\to j} = \frac{q_{t,ij}^2}{\sum_{l=1}^n q_{t,lj}^2} \quad \text{(volatility transmission)} \tag{17}$$

where  $q_{t,ij}$  denote the matrix entries of the identified  $Q_t = H_t^{1/2} R$ . Volatility reception  $\overline{q_{t,11}q_{t,21} + q_{t,12}q_{t,22} + q_{t,13}q_{t,23}}$ , and thus, it depends on the signs and magnitudes of multiple components in  $Q_t$ .

measures the variance share of the impact of the *j*th structural shock on the *i*th return in relation to the variance of *all shocks* on the *i*th return equation. Volatility transmission measures the variance share of the *i*th structural shock on the *j*th return in relation to its variance impact on *all returns* in the system. The volatility reception in (16) corresponds to a *conditional* 0-order forecast error variance decomposition. The definitions follow similar concepts as introduced by Diebold and Yilmaz (2012) and Fengler and Herwartz (2018).

Figures 4 and 5 show the volatility reception and transmission implied by the structural model. We focus first on volatility reception in Figure 4, proceeding row by row. Naturally, the equity shock contributes the most to S&P 500 return fluctuations, about 80% or more—see Figure 4a. This dominance is most pronounced from the financial crisis of 2008 through 2013. In addition to the equity shock, the bond market shock contributes notably, especially during calm market periods (see Figure 4b). Regarding the relative influence of  $\xi_3$ , the currency shock, on the S&P 500 returns, there appears to be a relatively stable base level of volatility reception at around 7% (see Figure 4c). These low contributions from currency shocks are consistent with the U.S. being a large and relatively closed economy; they may also reflect that the USD is the world's most traded currency.

In examining the second row, we note in Figure 4d that the Treasury yield return volatility is significantly influenced by the equity shock, with an average contribution of around 20%. This influence notably intensifies during the DotCom crisis from 2000 to 2003, the financial crisis (2007-2008) and throughout the Eurozone crisis (2009-2014). As depicted in Figure 4e, the bond market shock plays the dominant role in affecting yield movements. In contrast, the currency shock appears to have a negligible impact on yield fluctuations, as shown in Figure 4f. The observation regarding the minimal impact of the currency shock aligns with the findings reported by Cenedese and Mallucci (2016).

The third row, beginning with Figure 4g, indicates a minimal impact of the equity shock on U.S. Dollar Index volatility until 2008. From September 2008 to May 2013—a period

marked by major fluctuations of the USD exchange rate—the impact rises to 20%. Figure 4h shows a sizeable bond market shock effect on U.S. Dollar Index volatility, echoing findings like those in Andersen et al. (2003) about U.S. macroeconomic policies influencing the USD-EUR rate. Figure 4i reveals that the third shock is the primary driver of U.S. Dollar Index return fluctuations, underscoring our interpretation as a currency shock.

Figure 5 documents the volatility transmission. As a salient observation, we note the long-term trends in the volatility transmission of the equity shock to equity and fixed income markets—compare Figures 5a and Figures 5d. Specifically, over the sample period, the relative importance of the equity shock on the S&P 500 return decreases in relation to its impact on all return components in the system and declines from approximately 80% to 50%. Conversely, the relative importance of the equity shock on the Treasury yield increases from 20% to a level of more than 50%. This finding aligns with a body of literature suggesting an increasingly tighter link between fixed income and equity markets over the past decades—see, e.g., Ehrmann and Fratzscher (2009) or Ehrmann et al. (2011).

As regards the bond market shock, its transmission to the S&P 500 return weakens over time, decreasing from levels of around 20% at the turn of the millennium to close to zero by 2009—see Figure 5b. Figure 5e shows that this shift, over time, is accompanied by a stronger transmission of the bond market shock to the Treasury yield return. Plausibly, the contributions of both the equity and bond market shocks to U.S. Dollar Index return fluctuations are negligible, as illustrated in Figures 5g and 5h.

Lastly, the third shock exhibits a distinct volatility transmission pattern on each return component, different from the other two shocks. The strongest total shock contribution is to the U.S. Dollar Index—see Figure 5i, fluctuating around a level of about 70%. In contrast, transmission to S&P 500 and Treasury yield returns is more variable. To the S&P 500 return, it surges during 1998 to 2004, from 2006 to 2009, and towards the end of the sample. While the transmission level to the Treasury yield returns is generally low, it

increases markedly in the aftermath of the financial crisis, reaching a maximum of 60% during the Euro zone crisis in 2012. These observations support the interpretation of the third shock primarily as a currency shock, given the Euro's predominance in the U.S. Dollar Index.

# 5 Conclusion

We have introduced a proxy-based structural MGARCH model that extends the reducedform MGARCH model to a structural model in the macroeconometric sense. It ensures flexible modeling of the multivariate volatility dynamics of daily returns, while simultaneously identifying the underlying shock system. Because it is based on proxy variables serving as instrument data, it offers the potential to deliver labeled structural shocks.

Our model formulation results in an estimation problem characterized by constraints on parts of the parameter vector that correspond to orthogonal and symmetric positivedefinite matrices. To tackle this, we employ sophisticated techniques of Riemannian optimization. We expect these methods, currently underutilized in econometrics, to hold considerable potential for broader application within the field, especially to orderinginvariant identification in structural VAR models.

In the empirical application to a speculative return system consisting of S&P 500, 10 year Treasury yield, and U.S. Dollar Index returns, we obtain structural shocks interpretable as an equity market and a bond market shock. We study the dynamic impact matrix and estimate the structural volatility reception and volatility transmission between the three markets. Notably, the structural orthogonal matrix we estimate deviates significantly from symmetry, casting doubt on the appropriateness of employing the spectral decomposition—the commonly used approach in MGARCH modeling.

In his presidential address, Shiller (2017) highlighted the value of narrative measures to identify exogenous shocks to the economy. Our identification strategy, based on news-related proxy variables, affirms their potential and opens multiple avenues for further research. For example, the shocks we identify are very general, whereas the recent literature on monetary policy emphasizes a variety of different shocks, making a more nuanced analysis essential (Ferreira, 2022). In this regard, news analytics data and identification strategies similar to ours could provide a promising path for future research.

	S	tructural sh	ocks	
		$\hat{\xi}_1$	ξ <sub>2</sub>	Ê3
Mean		-0.0015	0.0106	0.0012
Median		0.0433	0.0361	0.0214
Minimum		-7.6653	-4.6669	-6.9155
Maximum		3.7471	6.9360	4.4969
Std. Dev.		1.0012	0.9941	1.0031
Skewness		-0.5728	-0.0789	-0.1810
Kurtosis		5.1164	4.6004	4.2905
	$\hat{\xi}_1$	1.0000		
Correlations	$\hat{\xi}_2$	0.0060	1.0000	
	$\hat{\xi}_3$	0.0014	0.0009	1.0000
	Ê1			
<i>t</i> -statistics	$\hat{\xi}_2$	0.446		
	Ê3	0.104	0.067	
	$Z_1$	0.3527	-0.0013	0.0006
Correlations	$Z_2$	-0.0068	0.1683	0.0000
	Zı	28.064	-0.097	0.043
<i>t</i> -statistics	$Z_2$	-0.509	12.711	0.003

Table 4: Descriptive statistics of the identified structural shocks of the structural MGARCH model of the demeaned daily log returns of the S&P 500 index, the constant maturity yield of U.S. 10-year Treasury notes, and the U.S. Dollar Index. Regarding the proxy variables,  $Z_1$  denotes the TRMI-based stock market sentiment and  $Z_2$  the TRMI-based bond market sentiment. Sample: from 01/01/1998 to 12/31/2019.





Figure 3: Elements of  $Q_t = H_t^{1/2} R$  as estimated by the structural MGARCH model of the daily log returns of the S&P 500, the constant maturity yield of U.S. 10-year Treasury notes, and the U.S. Dollar Index. Sample from 01/01/1998 to 12/31/2019.



Figure 4: Volatility reception (VR) of the structural MGARCH model of the daily log returns of the S&P 500, the constant maturity yield of U.S. 10-year Treasury notes, and the U.S. Dollar Index. Volatility reception measures the share of the impact of the *j*th structural shock on the *i*th return in relation to the impact of all other shocks on this component. Sample from 01/01/1998 to 12/31/2019.



Figure 5: Volatility transmission (VT) of the structural MGARCH model of the daily log returns of the S&P 500, the constant maturity yield of U.S. 10-year Treasury notes, and the U.S. Dollar Index. Volatility transmission measures the share of the impact of the *i*th structural shock on the *j*th return component in relation to its impact on all return components in the system. Sample from 01/01/1998 to 12/31/2019.

# References

- Absil, P.-A., Baker, C. G. and Gallivan, K. A. (2007). Trust-region methods on Riemannian manifolds, *Foundations of Computational Mathematics* **7**: 303–330.
- Absil, P.-A., Mahony, R. and Sepulchre, R. (2008). *Optimization algorithms on matrix manifolds*, Princeton University Press.
- Adler, R. L., Dedieu, J.-P., Margulies, J. Y., Martens, M. and Shub, M. (2002). Newton's method on Riemannian manifolds and a geometric model for the human spine, *IMA Journal of Numerical Analysis* **22**(3): 359–390.
- Amisano, G. and Giannini, C. (1997). Topics in Structural VAR Econometrics, Springer, Berlin, Heidelberg.
- Andersen, T., Bollerslev, T. and Diebold, F. (2003). Micro effects of macro announcements: Real-time price discovery in foreign exchange, *American Economic Review* **93**: 38–62.
- Andersen, T. G., Davis, R. A., Kreiß, J.-P. and Mikosch, T. V. (2009). *Handbook of financial time series*, Springer Science & Business Media.
- Angelini, G., Caggiano, G., Castelnuovo, E. and Fanelli, L. (2023). Are fiscal multipliers estimated with proxy-SVARs robust?, *Oxford Bulletin of Economics and Statistics* **85**(1): 95–122.
- Angelini, G. and Fanelli, L. (2019). Exogenous uncertainty and the identification of structural vector autoregressions with external instruments, *Journal of Applied Econometrics* **34**(6): 951–971.
- Angelini, G. and Fanelli, L. (2023). Correction to "Exogenous uncertainty and the identification of structural vector autoregressions with external instruments", *Journal of Applied Econometrics* **38**(5): 795–797.
- Arias, J. E., Rubio-Ramírez, J. F. and Waggoner, D. F. (2021). Inference in Bayesian proxy-SVARs, *Journal of Econometrics*.
- Audrino, F. and Tetereva, A. (2019). Sentiment spillover effects for US and European companies, *Journal of Banking & Finance* **106**(C): 542–567.
- Bauwens, L., Hafner, C. and Laurent, S. (2012). *Handbook of Volatility Models and Their Applications*, John Wiley & Sons, Ltd.
- Bauwens, L., Laurent, S. and Rombouts, J. V. K. (2006). Multivariate GARCH models: a survey, *Journal of Applied Econometrics* **21**(1): 79–109.
- Bernanke, B. S. and Kuttner, K. N. (2005). What explains the stock market's reaction to Federal Reserve policy?, *The Journal of Finance* **60**(3): 1221–1257.
- Bhatia, R. (2009). Positive Definite Matrices, Princeton University Press.
- Bollerslev, T., Li, J. and Xue, Y. (2018). Volume, volatility, and public news announcements, *The Review of Economic Studies* **85**(4): 2005–2041.
- Boudoukh, J., Feldman, R., Kogan, S. and Richardson, M. (2018). Information, trading and volatility: Evidence from firm-specific news, *The Review of Financial Studies* **32**(3): 992–1033.
- Boumal, N. (2023). An introduction to optimization on smooth manifolds, Cambridge University Press.

- Boumal, N., Mishra, B., Absil, P.-A. and Sepulchre, R. (2014). Manopt, a Matlab toolbox for optimization on manifolds, *Journal of Machine Learning Research* **15**(42): 1455–1459.
- Boyd, J. H., Hu, J. and Jagannathan, R. (2005). The stock market's reaction to unemployment news: Why bad news is usually good for stocks, *Journal of Finance* **60**(2): 649–672.
- Carriero, A. and Volpicella, A. (2024). *Max Share* identification of multiple shocks: An application to uncertainty and financial conditions, *Journal of Business & Economic Statistics* (just-accepted): 1–13.
- Cenedese, G. and Mallucci, E. (2016). What moves international stock and bond markets?, *Journal of International Money and Finance* **60**: 94–113.
- Clark, P. K. (1973). A subordinated stochastic process model with finite variance for speculative prices, *Econometrica* **41**(1): 135–155.
- Comte, F. and Lieberman, O. (2003). Asymptotic theory for multivariate GARCH processes, *Journal of Multivariate Analysis* pp. 61–84.
- Diebold, F. X. and Yilmaz, K. (2012). Better to give than to receive: Predictive directional measurement of volatility spillovers, *International Journal of Forecasting* **28**(1): 57–66.
- D'Amico, S. and Farka, M. (2011). The fed and the stock market: An identification based on intraday futures data, *Journal of Business & Economic Statistics* **29**(1): 126–137.
- Ehrmann, M. and Fratzscher, M. (2009). Global financial transmission of monetary policy shocks, *Oxford Bulletin of Economics and Statistics* **71**(6): 739–759.
- Ehrmann, M., Fratzscher, M. and Rigobon, R. (2011). Stocks, bonds, money markets and exchange rates: measuring international financial transmission, *Journal of Applied Econometrics* **26**(6): 948–974.
- Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation, *Econometrica* **50**(4): 987–1007.
- Engle, R. F. and Kroner, K. F. (1995). Multivariate simultaneous generalized ARCH, *Econometric Theory* **11**(1): 122–150.
- Engle, R. F. and Ng, V. K. (1993). Measuring and testing the impact of news on volatility, *The Journal of Finance* **48**(5): 1749–1778.
- Fengler, M. and Herwartz, H. (2018). Measuring spotvariance spillovers when (co)variances are timevarying – the case of multivariate GARCH models, *Oxford Bulletin of Economics and Statistics* 80(1): 135– 159.
- Ferreira, L. N. (2022). Forward guidance matters: Disentangling monetary policy shocks, *Journal of Macroe-conomics* **73**: 103423.
- Fisher, L. and Huh, H. S. (2019). Combining sign and parametric restrictions in SVARs by utilising Givens rotations, *Studies in Nonlinear Dynamics and Econometrics* **24**(3): 1–19.
- Giacomini, R., Kitagawa, T. and Read, M. (2022a). Narrative restrictions and proxies, *Journal of Business & Economic Statistics* **40**(4): 1415–1425.
- Giacomini, R., Kitagawa, T. and Read, M. (2022b). Robust Bayesian inference in proxy SVARs, *Journal of Econometrics* 228(1): 107–126.

- Grossman, S. J. and Stiglitz, J. E. (1980). On the impossibility of informationally efficient markets, *The American Economic Review* **70**(3): 393–408.
- Groß-Klußmann, A. and Hautsch, N. (2011). When machines read the news: Using automated text analytics to quantify high frequency news-implied market reactions, *Journal of Empirical Finance* **18**(2): 321 340.
- Hafner, C. M. and Herwartz, H. (2008). Analytical quasi maximum likelihood inference in multivariate volatility models, *Metrika: International Journal for Theoretical and Applied Statistics* **67**(2): 219–239.
- Hafner, C. M., Herwartz, H. and Maxand, S. (2022). Identification of structural multivariate GARCH models, *Journal of Econometrics* 227(1): 212–227.
- Hafner, C. M. and Preminger, A. (2009). On asymptotic theory for multivariate GARCH models, *Journal of Multivariate Analysis* **100**(9): 2044–2054.
- Huang, W., Gallivan, K. A. and Absil, P.-A. (2015). A Broyden class of quasi-newton methods for Riemannian optimization, *SIAM Journal on Optimization* **25**(3): 1660–1685.
- Jentsch, C. and Lunsford, K. G. (2022). Asymptotically valid bootstrap inference for proxy svars, *Journal of Business & Economic Statistics* **40**(4): 1876–1891.
- Jeuris, B., Vandebril, R. and Vandereycken, B. (2012). A survey and comparison of contemporary algorithms for computing the matrix geometric mean, *Electronic Transactions on Numerical Analysis* **39**: 379–402.
- Kilian, L. and Lütkepohl, H. (2017). Structural Vector Autoregressive Analysis, Cambridge University Press.
- Lanne, M., Lütkepohl, H. and Maciejowska, K. (2010). Structural vector autoregressions with Markov switching, *Journal of Economic Dynamics and Control* **34**(2): 121–131.
- Lewis, D. J. (2022). Robust inference in models identified via heteroskedasticity, *Review of Economics and Statistics* **104**(3): 510–524.
- Lütkepohl, H. and Schlaak, T. (2021). Heteroscedastic proxy vector autoregressions, *Journal of Business & Economic Statistics* pp. 1–14.
- Mertens, K. and Ravn, M. O. (2013). The dynamic effects of personal and corporate income tax changes in the United States, *American Economic Review* **103**(4): 1212–1247.
- Michaelides, A., Milidonis, A. and Nishiotis, G. P. (2019). Private information in currency markets, *Journal* of *Financial Economics* **131**(3): 643–665.
- Michaelides, A., Milidonis, A., Nishiotis, G. P. and Papakyriakou, P. (2015). The adverse effects of systematic leakage ahead of official sovereign debt rating announcements, *Journal of Financial Economics* **116**(3): 526–547.
- Milgrom, P. and Stokey, N. (1982). Information, trade and common knowledge, *Journal of Economic Theory* **26**(1): 17–27.
- Moakher, M. (2005). A differential geometric approach to the geometric mean of symmetric positive-definite matrices, *SIAM Journal on Matrix Analysis and Applications* **26**(3): 735–747.
- Newey, W. K. and McFadden, D. (1994). Large sample estimation and hypothesis testing, *Handbook of econometrics* **4**: 2111–2245.

- Pennec, X., Fillard, P. and Ayache, N. (2006). A Riemannian framework for tensor computing, *International Journal of Computer Vision* 66: 41–66.
- Peterson, R. L. (2016). Trading on sentiment: The power of minds over markets, John Wiley & Sons.
- Plagborg-Møller, M. and Wolf, C. K. (2022). Instrumental variable identification of dynamic variance decompositions, *Journal of Political Economy* 130(8): 2164–2202.
- Primiceri, G. (2005). Time varying structural vector autoregressions and monetary policy, *The Review of Economic Studies* **72**(3): 821–852.
- Rigobon, R. (2003). Identification through heteroskedasticity, *Review of Economics and Statistics* **85**(4): 777–792.
- Rigobon, R. and Sack, B. (2003). Measuring the reaction of monetary policy to the stock market, *The Quarterly Journal of Economics* **118**(2): 639–669.
- Rigobon, R. and Sack, B. (2004). The impact of monetary policy on asset prices, *Journal of Monetary Economics* **51**(8): 1553–1575.
- Romer, C. D. and Romer, D. H. (2010). The macroeconomic effects of tax changes: Estimates based on a new measure of fiscal shocks, *American Economic Review* **100**(3): 763–801.
- Rothenberg, T. J. (1971). Identification in parametric models, *Econometrica* 39: 577–591.
- Rothenberg, T. J. (1973). Efficient estimation with a priori information, Yale University Press.
- Shiller, R. J. (2017). Narrative economics, American Economic Review 107(4): 967–1004.
- Sra, S. and Hosseini, R. (2015). Conic geometric optimization on the manifold of positive definite matrices, SIAM Journal on Optimization 25(1): 713–739.
- Stock, J. H. and Watson, M. (2012). Disentangling the channels of the 2007-09 recession, *Brookings Papers on Economic Activity* **43**(1): 81–156.
- Stock, J. H. and Watson, M. (2016). Dynamic factor models, factor-augmented vector autoregressions, and structural vector autoregressions in macroeconomics, *Handbook of Macroeconomics*, Vol. 2, Elsevier, chapter Chapter 8, pp. 415–525.
- Stock, J. H. and Watson, M. W. (2018). Identification and estimation of dynamic causal effects in macroeconomics using external instruments, *The Economic Journal* **128**(610): 917–948.
- Sun, L., Najand, M. and Shen, J. (2016). Stock return predictability and investor sentiment: A high-frequency perspective, *Journal of Banking & Finance* 73: 147–164.
- Tauchen, G. E. and Pitts, M. (1983). The price variability-volume relationship on speculative markets, *Econometrica* pp. 485–505.
- Thorbecke, W. (1997). On stock market returns and monetary policy, The Journal of Finance 52(2): 635–654.
- Weber, E. (2010). Structural conditional correlation, Journal of Financial Econometrics 8(3): 392–407.
- Wooldridge, J. M. (1994). Estimation and inference for dependent processes, *Handbook of Econometrics* 4: 2639–2738.

# A Riemannian manifolds

We provide a concise overview of the essential facts about Riemannian manifolds necessary for understanding optimization on these structures, drawing upon Absil et al. (2008) and Boumal (2023). In Section A.2, we discuss in greater detail the manifolds pertinent to our structural model. We also outline the specific choices of Riemannian metrics and retractions implemented in our optimization procedure.

#### A.1 General framework

The notion of a manifold we adopt is that of a  $d_1$ -dimensional smooth submanifold embedded in an ambient Euclidean space  $\mathcal{E}$  of dimension  $d_2$ . Here,  $\mathcal{E} \cong \mathbb{R}^{d_2}$ , i.e., we can identify  $\mathcal{E}$  with  $\mathbb{R}^{d_2}$ . For precision, we reiterate Definition 3.1 from the main text, refining it with the concept of a differential. For a function  $h : \mathbb{R}^m \to \mathbb{R}^n$ , the differential Dh(X)at a point  $X \in \mathbb{R}^m$  is the linear operator  $Dh(X) : \mathbb{R}^m \to \mathbb{R}^n$ , mapping V to Dh(X)[V], defined as  $Dh(X)[V] = \lim_{s\to 0} \frac{h(X+sV)-h(X)}{s}$ ,  $s \in \mathbb{R}$ . The expression  $Dh(X)[V] \in \mathbb{R}^n$  is called the directional derivative of h along V.

**Definition A.1.** A non-empty set  $\mathcal{M}$ , embedded in  $\mathcal{E}$ , is called a smooth submanifold of dimension  $d_1 \leq d_2$  in either of two cases:

- (*i*) if  $d_1 = d_2$  and  $\mathcal{M}$  is open in  $\mathcal{E}$ ; it is then called an open submanifold in  $\mathcal{E}$ .
- (*ii*) If  $d_1 < d_2$  and the set is of form  $\mathcal{M} = \{X \in \mathcal{E} : h(X) = 0_{d_2-d_1}\}$ , where the defining function  $h : \mathcal{E} \to \mathbb{R}^{d_2-d_1}$  is infinitely often differentiable and its differential Dh(X) has rank  $d_2 d_1$ , for all  $X \in \mathcal{M}$ .

The tangent space  $\mathcal{T}_X \mathcal{M}$  offers a local vector space approximation of  $\mathcal{M}$  at  $X \in \mathcal{M}$ . Formally, given a point  $X \in \mathcal{M}$ , let  $\gamma : \mathbb{R} \to \mathcal{M}$  be a smooth curve such that  $\gamma(0) = X$ and denote its derivative at 0 by  $\dot{\gamma}(0)$ . The space spanned by all vectors  $\dot{\gamma}(0)$  defines the tangent space  $\mathcal{T}_X \mathcal{M}$  at X and it has the same dimension as  $\mathcal{M}$ . For open submanifolds, one identifies the ambient space as the tangent space:  $\mathcal{T}_x \mathcal{M} \cong \mathcal{E}^{12}$  For an embedded submanifold of dimension  $d_1 < d_2$ , the tangent spaces are linear subspaces of  $\mathcal{E}$  and can equivalently be characterized by the kernel of Dh(X)—see Absil et al. (2008).

Because the  $\mathcal{T}_X \mathcal{M}$  is a linear space, we can endow it with an inner product  $\langle \cdot, \cdot \rangle_X$ , where the notation emphasizes that the inner product depends on *X*. If  $\langle \cdot, \cdot \rangle_X$  varies smoothly in  $X \in \mathcal{M}$ , it is called a Riemannian metric and the pair  $(\mathcal{M}, \langle \cdot, \cdot \rangle_X)$  a Riemannian manifold. The Riemannian metric can be—but does not have to be—inherited from the ambient vector space.

Optimization on a manifold requires the definition of a gradient of a differentiable function  $f : \mathcal{M} \to \mathbb{R}$ . To this end, on a Riemannian manifold, the Riemannian metric is used. The Riemannian gradient is the uniquely defined vector grad f(X) such that  $Df(X)[T] = \langle \operatorname{grad} f(X), T \rangle_X$  for all  $T \in \mathcal{T}_X \mathcal{M}$  and all  $X \in \mathcal{M}$ .<sup>13</sup> It represents the direction of steepest ascent of f in  $\mathcal{T}_X \mathcal{M}$  at X, taking into account the local geometry of  $\mathcal{M}$ at X. If the Euclidean inner product of the ambient space is selected as the Riemannian metric, the Riemannian gradient can be derived by orthogonally projecting the Euclidean gradient of f onto  $\mathcal{T}_X \mathcal{M}$ , as sketched in Figure 1a. This approach is viable if f is welldefined in an open neighborhood in  $\mathcal{E}$  around  $X \in \mathcal{M}$ , which is often the case in much applied work. From a computational perspective, however, this is not always the optimal choice—see Section A.2 for further discussion.

Because of the non-Euclidean geometry of  $\mathcal{M}$ , the updating scheme of a Riemannian

 $<sup>^{12}\</sup>mathcal{T}_X\mathcal{M}$  is a subset of  $\mathcal{E}$  by definition. It is easy to show that  $\mathcal{E}$  is also a subset of  $\mathcal{T}_X\mathcal{M}$ , from which the claim follows—see Boumal (2023), Prop. 3.15.

<sup>&</sup>lt;sup>13</sup>Let *V* be a tangent vector at *X*, i.e.,  $V \in \mathcal{T}_X \mathcal{M}$ . Then exists a smooth curve  $\gamma$  on  $\mathcal{M}$  satisfying  $\gamma(0) = X$ and  $\dot{\gamma}(0) = V$ . Moreover, given a smooth function  $f : \mathcal{M} \to \mathbb{R}$ ,  $s \mapsto f(\gamma(s))$  defines a smooth curve on  $\mathbb{R}$  by composition. This curve passes through  $f(X) = f(\gamma(0))$  with a certain velocity, which is a tangent vector of  $\mathbb{R}$  at f(X). This tangent vector is called the differential of f at X along V.

Formally, for a smooth function  $f : \mathcal{M} \to \mathbb{R}$ , the differential D f(X) is a linear operator from  $\mathcal{T}_X \mathcal{M}$  to  $\mathcal{T}_{f(X)}\mathbb{R} \cong \mathbb{R}$  defined by  $D f(X)[V] = \frac{d}{dt}f(\gamma(t))\Big|_{t=0}$ . It can be shown that this definition is independent from  $\gamma$  and that D f(X) is indeed linear (Boumal, 2023).

optimization step requires a retraction. A retraction is a map  $\Gamma_X : \mathcal{T}_X \mathcal{M} \to \mathcal{M}$  that, for each curve  $\gamma(s) = \Gamma_X(sT)$ ,  $s \in \mathbb{R}$  and  $T \in \mathcal{T}_X \mathcal{M}$ , satisfies  $\gamma(0) = X$  and  $\dot{\gamma}(0) = T$ , for all  $X \in \mathcal{M}$ . Given a retraction, the updating step of a gradient descent on a Riemannian manifold, updating  $X_k$  to  $X_{k+1}$ , takes the form  $X_{k+1} = \Gamma_{X_k} (-\alpha_k \operatorname{grad} f(X_k))$ , where the step size  $\alpha_k$  must be appropriately chosen.

Advanced methods of Riemannian optimization also leverage second-order information about f, akin to the classical Newton's method. However, they necessitate a concept of a Riemannian Hessian and, consequently, a more sophisticated toolset. Subject to smoothness of f and further technical conditions, these methods are known to converge to a local minimum of f on  $\mathcal{M}$ . Hence, the optimization on a manifold is not inherently distinct from optimizing in a Euclidean space—see Absil et al. (2008) for an in-depth discussion.

#### A.2 Discussion of the Riemannian manifolds utilized

#### Linear (sub)space of $\mathbb{R}^n$

The Riemannian geometry encompasses the Euclidean geometry as special case. Therefore, as the simplest example, any linear (sub)space of  $\mathbb{R}^n$  admits a linear manifold structure. For any  $X \in \mathbb{R}^n$ , we have  $\mathcal{T}_X \mathbb{R}^n \cong \mathbb{R}^n$  and an obvious retraction is  $\Gamma_X = X + sT$ ,  $T \in \mathcal{T}_X \mathbb{R}^n$  and  $s \in \mathbb{R}$ .

#### The set of symmetric positive-definite matrices $\mathcal{M}_{spd}$

 $\mathcal{M}_{spd}(d_2) = \{X \in \mathcal{S}(d_2) : a^\top Xa > 0, a \in \mathbb{R}^{d_2}\}\$  is an example of an open submanifold. It is an open set in  $\mathcal{S}(d_2) = \{X \in \mathbb{R}^{d_2 \times d_2} : X = X^\top\}\$ , the vector space of symmetric matrices of size  $d_2 \times d_2$ . Therefore,  $\mathcal{T}_X \mathcal{M}_{spd}(d_2) \cong \mathcal{S}(d_2)$ , for any  $X \in \mathcal{M}_{spd}(d_2)$ . Because the natural inner product on  $\mathcal{S}(d_2)$  is the Frobenius inner product  $\langle A, B \rangle = \operatorname{tr}(A^\top B)$ , where  $A, B \in \mathcal{S}(d_2)$ , one could also use the Frobenius inner product on the tangent spaces of the manifold of positive-definite matrices. In our Riemannian optimization, however, we employ a different metric—specifically, the affine invariant metric, given by  $\langle A, B \rangle_X^{\text{aff}} = \text{tr}(X^{-1}AX^{-1}B)$ , for  $X \in \mathcal{M}_{spd}$  and  $A, B \in \mathcal{T}_X \mathcal{M}_{spd}$ —see Moakher (2005) and Pennec et al. (2006). The rationale is that the distance induced by this metric makes  $\mathcal{M}_{spd}$  a complete metric space, by rejecting any element of  $\mathcal{S}(d_2)$  with zero or infinite eigenvalues to an infinite distance from any element in  $\mathcal{M}_{spd}$ .<sup>14</sup> This property effectively prevents the optimizer from 'leaving' the open convex cone  $\mathcal{M}_{spd}$ , which is of considerable convenience during optimization. Because the affine invariant metric is employed instead of the Frobenius inner product, the Riemannian gradient is not accessible through projection. Nevertheless, it can be proved that under the affine invariant metric, grad  $f(X) = X \nabla f(X) X$ , where  $\nabla f(X)$  denotes the Euclidean gradient of f at X—see Boumal (2023, p. 328-329). A good retraction is given by  $\Gamma_X(A) = X + A + \frac{1}{2}AX^{-1}A$ , where  $X \in \mathcal{M}_{spd}$  and  $A \in \mathcal{T}_X \mathcal{M}_{spd}$ —see Jeuris et al. (2012) and Sra and Hosseini (2015).

#### The set of orthogonal matrices $\mathcal{M}_o$

The defining function of  $\mathcal{M}_o(d_2)$  is given by  $h(X) = X^\top X - I_n$ , which maps any  $X \in \mathbb{R}^{d_2 \times d_2}$  into  $\mathcal{S}(d_2)$ . The directional derivative of h at X along Z, where  $Z \in \mathbb{R}^{d_2 \times d_2}$ , is  $Dh(X)[Z] = X^\top Z + Z^\top X$ . Hence, the tangent spaces are given by  $\mathcal{T}_X \mathcal{M}_o(d_2) = \{Z \in \mathbb{R}^{d_2 \times d_2} : X^\top Z + Z^\top X = 0\}$ . Moreover, the rank of Dh(X), seen as the linear map  $\mathbb{R}^{d_2 \times d_2} \to \mathcal{S}(d_2)$ , is  $\frac{1}{2}d_2(d_2 + 1)$ , because Dh(X)[Z] is surjective for any  $Z \in \mathbb{R}^{d_2 \times d_2}$ ; <sup>15</sup> hence, dim $(\mathcal{M}_o(d_2)) = d_1 = d_2^2 - \frac{1}{2}d_2(d_2 + 1) = \frac{1}{2}d_2(d_2 - 1)$ . In this case, one typically inherits the Frobenius inner product from  $\mathbb{R}^{d_2 \times d_2}$  to the Riemannian manifold  $(\mathcal{M}_o(d_2), \langle \cdot, \cdot \rangle)$ , implying that we can obtain the Riemannian gradient by projecting the Euclidean gradient on the tangent space, as we illustrate in Figure 1a for the case of the two-dimensional sphere embedded in  $\mathbb{R}^3$ . A standard retraction for  $\mathcal{M}_o$  is the Q-factor retraction, which is the function that sends a square matrix A onto the orthogonal matrix

<sup>&</sup>lt;sup>14</sup>See Bhatia (2009, Chap. 6). The geodesic distance, i.e., the minimum distance between *A* and *B* while traversing on the manifold, induced by  $\langle \cdot, \cdot \rangle_X^{\text{aff}}$  is given by  $d(A, B) = \left(\sum_{i=1}^d \log^2 \lambda_i (A^{-1}B)\right)^{1/2}$ , where  $\lambda_i(A^{-1}B)$  denote the eigenvalues of  $A^{-1}B$ , which explains this aforementioned property.

<sup>&</sup>lt;sup>15</sup>To see this, take  $Z = \frac{1}{2}XS$ , for any  $S \in \mathcal{S}(d_2)$ .

*Q* obtained by means of its QR decomposition, i.e., by decomposing into A = QR, where *R* is upper triangular with nonnegative diagonal entries. The Q-factor retraction is also applicable when retracting to  $\mathcal{M}_{rot}$  (Absil et al., 2008).

 $M_o$  poses a particular challenge for gradient-based optimization methods because it includes two disconnected parts: the orthogonal matrices with determinant +1 and the orthogonal matrices with determinant -1. Because the gradient descent and its variants make small, continuous adjustments to improve the objective function, it cannot 'leap' from one part to the other. Therefore, the starting point of the optimizer must be carefully selected. For our application, this issue is immaterial because we can identify the columns of the orthogonal matrix only up to sign. We later select these signs based on economic considerations—see the main text.

#### **Product manifolds**

A Riemannian manifold can also be constructed as Cartesian product of Riemannian manifolds. Thus, if  $\mathcal{M}_1$  and  $\mathcal{M}_2$  are two submanifolds embedded in  $\mathcal{E}_1$  and  $\mathcal{E}_2$ , respectively,  $\mathcal{M}_{\Pi} = \mathcal{M}_1 \times \mathcal{M}_2$  is a submanifold of dim $(\mathcal{M}_{\Pi}) = \dim(\mathcal{M}_1) + \dim(\mathcal{M}_2)$ , embedded in  $\mathcal{E}_1 \times \mathcal{E}_2$ , and has tangent spaces  $\mathcal{T}_{(X_1,X_2)}\mathcal{M}_{\Pi} = \mathcal{T}_{X_1}\mathcal{M}_1 \times \mathcal{T}_{X_2}\mathcal{M}_2$ , for any  $(X_1, X_2) \in \mathcal{M}_{\Pi}$ . A Riemannian metric on  $\mathcal{M}_{\Pi}$  is, e.g., the product metric given by  $\langle (T_1, S_1), (T_2, S_2) \rangle_{(X_1,X_2)}^{\mathcal{M}_{\Pi}} = \langle T_1, T_2 \rangle_{X_1}^{\mathcal{M}_1} + \langle S_1, S_2 \rangle_{X_2}^{\mathcal{M}_2}$ , for  $(X_1, X_2) \in \mathcal{M}_{\Pi}$  and  $(T_i, S_i) \in \mathcal{T}_{(X_1,X_2)}\mathcal{M}_{\Pi}$ , where  $\langle \cdot, \cdot \rangle_X^{\mathcal{M}_i}$  are the Riemannian metrics on  $\mathcal{M}_i$ , for i = 1, 2.

For a smooth function  $f : \mathcal{M}_{\Pi} \to \mathbb{R}$ , the gradient is given by the pair grad  $f(X_1, X_2) = (\operatorname{grad}(X_1 \mapsto f(X_1, X_2)), \operatorname{grad}(X_2 \mapsto f(X_1, X_2)))$ , where the barred arrow notation  $X_1 \mapsto f(X_1, X_2)$  denotes the function obtained by fixing the second argument, and vice versa for  $X_2 \mapsto f(X_1, X_2)$ . On the product manifold, a retraction  $\Gamma_{(X_1, X_2)} : \mathcal{T}_{(X_1, X_2)}\mathcal{M}_{\Pi} \to \mathcal{M}_{\Pi}$  can be constructed by  $\Gamma_{(X_1, X_2)}((T_1, T_2)) = (\Gamma_{X_1}(T_1), \Gamma_{X_2}(T_2))$ , for  $(T_1, T_2) \in \mathcal{T}_{(x_1, x_2)}\mathcal{M}_{\Pi}$ .

We employ this framework to construct a product manifold consisting of the Euclidean

space manifold, the orthogonal matrices manifold, and the positive-definite matrices manifold. It is on this combined geometric structure that we derive the estimates presented in the main text.

# **B** Narrative corroboration

We interpret  $\xi_1$  as a structural equity market shock and  $\xi_2$  as a structural bond market shock, while  $\xi_3$  does not readily lend itself to an a priori interpretation. To gain a deeper understanding of these shocks, we analyzed the news archives on days corresponding to the tail events of the shocks. We emphasize that this analysis is not intended to recover the news items that were informative about the structural shock and triggered the subsequent drawdown or price rally. Instead, the objective is to confirm that on these days, the market indeed experienced an event, supporting the suggestion that a structural shock of the indicated type likely impacted the market.

For the purpose of the analysis, we use the lower 0.85%-quantile for  $\xi_1$ , the upper 0.85%quantile for  $\xi_2$ , and—because its nature is unknown—both the lower and upper 0.5%quantiles for  $\xi_3$ . The findings, presented in Tables 5, 6, 7, and 8, show that we can associate the structural shocks' tail events with distinct categories of news.

Table 5 documents that  $\xi_1$  encompasses news items concerning the current and prospective financial solidity of major U.S. banks and companies and as well as entire sectors, including the housing market. News items that shape investor risk preferences, such as global political uncertainties and domestic issues like the U.S. fiscal cliff and trade anxieties, also play a significant role. Furthermore, news items significantly impacting equity valuations, specifically those related to the economic outlook, appear. This corroborates the equity shock interpretation. In line with expectations, the upper tail events of  $\xi_2$  are associated with bond price rallies as well as lower yields and monetary policy rates—see Table 6. A significant portion of the news items pertains to Federal Reserve announcements and key communications from the Fed's top officials. Furthermore, inflation data, along with payroll and employment reports, play a significant role. A smaller portion of news items references events that threaten global economic stability and hence prompted 'flight to safety' responses from investors. This underscores the characterization of this shock as a bond market shock.

Lastly, we consider  $\xi_3$  which is not directly targeted in our identification scheme. As documented in Tables 7 and 8, the tail events of  $\xi_3$  frequently correspond to news reflecting fluctuations in the currency markets. Specifically, extreme negative shocks are linked to declines in the U.S. currency, whereas positive shocks align with appreciations in its value. In addition, we also observe headlines referring to substantial corrections in gold and oil prices. This observation appears plausible, considering gold's near-currency status and the predominant use of the USD in energy trading. Moreover, an oil price increase corresponds to a devaluation of the USD relative to oil. Consequently, Table 7 often records upward movements in oil prices, while Table 8 details declines in oil prices. Similar considerations apply to gold. The findings are, however, not in all instances fully consistent in this respect; e.g., on July 1, 2001, we record a major negative shock and a major gold price loss—see Table 7.

Despite these limitations, it seems plausible that  $\xi_3$  exhibits characteristics of a currency shock.

Date	Shock	Narrative corroboration of the structural equity market shock $\xi_1$ : lower tail events Event
04-Aug-1998	-2.941	U.S. stock markets decline amid growing concerns over Asian economic pressures and political uncertainty around the impeachment of Clinton.
27-Aug-1998	-3.924	U.S. stock markets decline amid concerns over a global economic downturn due to Russian market turmoil and ruble slide.
31-Aug-1998	-4.198	U.S. stock markets decline amid uncertainty over the political and economic future of Russia and signs of an economic slow down in the U.S
04-Jan-2000	-5.807	U.S. stock markets decline amid Fed rate hike fears.
24-Jan-2000	-3.066	U.S. stock markets decline amid Fed rate hike fears.
14-Apr-2000	-4.525	Nasdaq crash and Dot-Com crisis until March 2003; Activation of circuit breakers at the NYSE due to fear sales because of rising inflation and overvalued tech companies.
12-Mar-2001	-3.733	Stocks Fell Across Americas Following Steep Slide in U.S Nasdaq stock market continued its year-long slide on Monday.
08-Aug-2001	-2.901	Black Monday; U.S. and global stock market crash following the credit rating downgrade of the U.S. souvereign debt by S&P300.
17-Sep-2001	-3.616	Trading resumes following 9/11 terrorist attacks; DIJA suffers its worst point-loss in history. Losses in insurance companies facing damage claims send S&P500 500 index to its worst finish since October 1998.
29-Jan-2002	-3.506	U.S. stock markets decline after Tyco International sparkes worries that more companies will downwardly restate financial results.
27-Feb-2007	-7.665	Federal Home Loan Mortgage Corporation (Freddie Mac) announces that it will no longer buy the most risky subprime mortgages and mortgage-related securities.
19-Oct-2007	-3.594	U.S. stock markets decline amid fears about credit and housing sector, earnings, record-high oil prices and the slide in the USD; Wachovias earnings fall beyond forecasts due to credit market turmoil.
01-Nov-2007	-2.963	Citigroup downgrade by CIBC analyst reignites credit concerns fueling fears that other major financial players were harder hit by this summer's subprime crisis than originally anticipated.
06-Jun-2008	-3.030	Crude prices largest one-day advance ever (oil price shock) and the biggest one-month surge in over 20 years in the unemployment rate.
04-Sep-2008	-2.999	U.S. stock markets decline amid mixed retail sales reports, weak job market news and an oil price side magnifying fears about a global slowdown.
15-Sep-2008	-3.745	Lehman Brothers Holdings Inc. bankrupty: Bank of America announces its intent to purchase Merrill Lynch .
29-Sep-2008	-4.176	U.S. House of kepresentatives rejects the imergency contomic Stabilization Act of 2006 (ballout plan); block market crash.
01-Oct-2009	-2.910	Us, stock matrixes decline and Supply Management's September 15M index unexpected tail and weekly policies claims jumping more than expected.
77 App 2010	-3.110	Ush davies decline and China curbing and sovereign dentises in Greece, fortugati and Sparin. U.S. stock markets decline and China curbing and sovereign dentises in Creece, fortugati and Sparin.
26 I 2011	010°C-	Jobal stock markets decime arter Greece sovereign credit rating downgrade powrtown to Junk to market archive. European sovereign debt crisis severes, rice arter markets decime arter Greece sovereign debt crisis severes.
28-Jan-2011	-2.994	U.S. and Japan receive sharp warmings from IMF and rating agencies to tackle their budget deficities. Economic indicators below expectations.
22-Feb-2011	-3.313	Groat stock markets decime amu tensions in the Mindle East and North Africa (Lybra) section gold prices soaring
01-100-110	-3.336	Us, stock markets decline after Moody's citutizenese should rating by three notches to Caal in even lower junk level.
04-Aug-2011	-4.326	Clobal stock markets decline due to continuing high foreclosure rates in the U.S and the European debt crisis (Italy, Spain); Vix spike (uncertainty shock)
08-Aug-2011	-4.160	Global stock markets decime due to the widering eurozone government defit crisis and Finday's U.S. rating downgrade.
2102-2011-/0	101.6-	Construct markets decime and U.S. instal cut and weak economic outlook for economic growth in turbing, market or an economic slow down in Germany in the course of European deof crists.
15-Apr-2013	-3.369	Clobal stock markets decime amu report of boston explosions. China reports seconding given is own in U(1, cold price gluinges.
21-1-1-2014 31-1-1-2014	-3.758	otowa suck matrices sections and uteringing matrix toolud and currency turniou surrounding the frazil, intua, intoriesia, 2A and 1urxey). Obshot stock matrix doctions and a tworkinging that exists.
25-Sem-2014	750 6-	ours accountees entropy and and a second second second in Fundam in Fundam on a more stimulus measures in China) and reconditional hearing of the villa sectors. Use stock measures desting and a second second second is fundam in Fundam on a more stimulus measures in China) and reconditional hearing of the villa sectors.
29-Jun-2015	-3.292	Use stock interface activity of a proving the provingtion of the proving interface standard interface standard interface activity of the spices. Use the Arciel Market for burn that is a proving the Archaeter interface data of the activity of the activity of
21-Aug-2015	-3.054	rug the second material school of the physical second stream in Second material material second stream second stream and the second stream second s
11-Dec-2015	-3.209	contraines on a unit accuration of the providence reaction and the providence of the
24-Jun-2016	-5.921	U.S. stocks recorded their steepest declines since
09-Sep-2016	-3.994	Dow industrials tumble nearly 400 points, as doubts over central banks' willingness or ability to stimulate economic growth sends stocks and bonds tumbling.
17-May-2017	-3.974	Stocks, Dollar Sink on Washington Turmoil.
10-Aug-2017	-3.515	Dow Falls Nearly 205 Points as North Korea Tensions Persist.
05-Feb-2018	-5.247	Rout in Stocks Jolts Traders Used to Heady Days of 2017.
22-Mar-2018	-2.950	U.S. Stocks Sell Off on Trade Concerns.
29-May-2018	-3.639	U.S. Stocks Tumble as Italian Woes Jolt World-Wride Markets.
10-Oct-2018	-5.503	Quickening Retreat From Tech Sinks Market. U.S. Stocks Fall Sharply; Tech Sector Leads Declines.
24-Oct-2018	-3.2/4	Dow, Ser Foul hum Negature for the Year, Nasdaq Drops 4.4%.
04-Dec-2018	-3.093	Bank Stocks Lada Markers Live.
22-Mar-2019	-3.260	Stocks, bond relates rail on chore a concerns. Failing Stocks Lead to Payott on Wagers in the Feat Gauge .
0/-IMay-2019	-3.040	Success Data for involuting that extensions. Success Data for the view of the Andrew Trady Andrew
15-1414y-2019 05-4116-2019		Dow Feler Works Development works to the and throw the form of the
6107-Bny-co	-4.2/0	Dow raits Sharply as tuan keels and irturp Jabs at Crinta. Iyson roods, cold Minels and cun Makels Dely Market Downturn.
Table 5:	Narrativ	e corroboration of the lower 0.85%-quantile of the identified equity market shock $\xi_1$ obtained from a return system comprising
1 - 11 11 11		
daily lo	g returns (	of the S&P 500 index, the constant maturity yield of the U.S. 10-year Ireasury notes, and the U.S. Dollar Index from 01/01/1998 to
12/31/.	2019, whe	n using the TRMI stock market index and TRMI bond market index as proxy variables. News items are sourced from a range of
archiva	l resources	s, primarily from wsi.com and money.cnn.com, among others.

30-Apr-1998 2.531 10-Sep-1998 3.930 05-Oct-1998 2.492 22-Feb-2000 2.511 20-Jul-2000 2.955 05-Dec-2000 3.965 02-Jan-2001 3.056	The bond market leaped after the friendly inflation data. Inflation and unemployment data quell fears of a Fed interest rate hike.
10->ep-1998 3:930 05-Oct-1998 2:492 22-Feb-2000 2:511 20-Jul-2000 2:951 05-Dec-2000 3:965 02-Jan-2001 3:056	
22-Feb-2000 2.511 20-Jul-2000 2.955 05-Dec-2000 3.965 02-Jan-2001 3.050	Economic and political uncertainty over emerging markets, kussia's economic restructuring and U.S. rresident bil Clobal economic melidown over emervine market turmoil (Asia, Russia, Brazil).
20-Jul-2000 2.955 05-Dec-2000 3.962 02-Jan-2001 3.050	Inflation data causes Treasury bond rally.
05-Dec-2000 3.962 02-Jan-2001 3.050	Fed Monetary Policy report and economic testimony to Congress by Alan Greenspan; U.S. stock market declines amid profit and rate concerns.
100-1an-1an-100	recenspan Speech to America's Community Binkers profilering the possibility of interest cuts next year due to economic cool-down; Resolution of presidential election.
18-Anr-2001 2 524	The pleid on the government is 11-year note return to its lowest teeled. The pleid on the government is 11-year-how indexnet pleids and the nerventation is not the accession.
08-Aug-2001 3.124	real para monomentandia na cua monomenta naceo y na nageresare nar percentago pontevaren a recession. Fed Beire Book Report and petter-than-expected action of new 10-year notes.
13-Sep-2001 3.800	First day of official trading after 9/11 pucking thesury yields to historic lows.
31-Jul-2002 2.701	Fed Beige Book Report shows worse-than-expected economic reports. U.S. stock markets decline amid profit warnings in the chip and retail sectors.
07-Nov-2002 2.475	U.S. stock markets decline in a sell-off after a months rally. Previous day's Fed decision to cut interest rates surprisingly sharply signals concerns with weakening U.S. economy.
05-Mar-2004 3.69i 15-Lim-2004 3.331	U.S. payroll and unemployment data has howing slowest pace in Wage growth in 18 Years and presistent unem ployment. Creaersonan testimonus entroceeting that in fulficion uses at factor to constained anough to allow for masering
06-Aug-2004 2.500	U.S. parvel and employment data below estimates and concern about oil supply around Yikos woes causing surger in oil prices.
03-Dec-2004 2.542	U.S. payroll and employment data below estimates.
04-Feb-2005 2.71(	U.S. payroll and employment data below estimates.
31-Aug-2005 3.604	Hurricane Katrina raises concerns about the mounting pressures on the U.S. economy from rising energy prices. Prospect of a flat or inverted Treasury yield curve.
02-Jun-2006 2.466	U.S. payroul data below testingtes. List and nome proces and manutacturing action of cool-down. Doorne of Vieinman Bue mestingtes in direct in Commence and hand according to cool-down.
26-Nov-2007 3.004	U.S. stock maturation but not a more incomparation transmit may vote the conton with a more store and a more and a more store and a more and a more store and a more an
15-Sep-2008 3.255	Lehman Brothers Holdings, Inc. files for Chapter 11 bankruptcy protection. Bank of America announces its intent to purchase Merrill Lynch & Co. causing investors to flock to the safety of government securities.
25-Nov-2008 3.336	Fed announcement of QE1.
16-Dec-2008 3.94 <sup>t</sup>	Fed decision for aggressive rate cut. Longer-term Treasury notes rise amid uncertainty about the ballout of the auto industry. Prospects of Fed using unconventional tools, e.g. buying longer-dated Treasury debt to
10 Mar 2000	keep cash flowing to the corrown are official leading rates fail toward zero. Keeponal manufacturing data below estimates and inflation data leading to deflation concerns.
75 Line 2009 2 471	ted is 2.02. 2000 percegica purculase of any effect inter nexis as information. Educida processos heid in encienda intervientada intervientad
10-Sep-2009 2.461	rectat reservented is bound on intersorates and its researupputcies prate. Fed Beige Book Report raises new on meens hourd consumer steeding. Fed statement that ranid rate hike improbable given weakness of global economy.
16-Nov-2009 2.558	Fed chairman Ben Bernanke speech stating that economic headwinds will restrain the pace of the U.S. recovery and that reduced bank lending and a weak labour market continue to justify low interest rates.
16-Aug-2010 2.61£	Growing concerns about the global economic outlook (Japan) cause investors to flock to the relative safety of government securities.
04-Nov-2010 3.916	Fed announcement of QE2 on 03-Nov-2010 causes yields on U.S. governments bonds to fall.
08-Jul-2011 2.475	U.S. job data for lune below estimates.
29-Jul-2011 3.185	American Delor Crisis and Q2 GJP report below settinates.
09-Mug-2011 0.233	re to statement to keep interest areas coverptionary tow unturat reasts 2013. Fed statements to keep interest states coverption by late 2014.
18-Sep-2013 3.885	requirements on were interesting to solve down the part of the bond burchasses ver-
14-Oct-2014 3.058	Fighting in the Middle East, gepolitical tensions between West and Russia and the spread of the Ebola virus cause investors to flock to the relative safety of government securities.
18-Mar-2015 3.002	Investors Celebrate Gentler Tone From Officials on Rates. Fed Puts Interest-Rate Hikes in Play.
29-Jan-2016 2.591	U.S. Government Bonds Rise After Japan Shifts to Negative Rates.
08-Feb-2016 2.611	N.Y. Fed Survey Shows Inflation Expectations Weakened Again in January. U.S. Government Bond Yields Drop.
29-Mar-2016 2.924	Market views but downward pressure no nlonger-term interest rates, supporting spending, Fed chief says. Dallas Fed's Robert Kaplan Reiterates Gradual Stance on Rate Increases.
15-Mar-2017 2.102	weak ming relates back from CO.S. Overmient both index sur- weak ming relates branch from Ara-th (A.S. Tahinan) and the Showe Contral Bank Has More Room to Manenteer - Rond mines and stroke surres for Reises (hierest from strack th (A.S. Tahinan) and strack and fully Showe Contral Rank Has More Room for the surres
05-Sep-2017 2.469	Fed's Kaplan Urges Preiser, with turther Rate Increases. Array of Threats Strict Up Markets. Stocks Turnble as Beechmark Treasury Mont Processor and a second processor and a
29-May-2018 3.307	U.S. Government Bonds Rally on Italy Concerns.
31-Jan-2019 2.579	U.S. Government Bonds Extend Post-Fed Rally.
03-Jun-2019 2.687	U.S. Government Bonds Extend Gains: Fed's Bullard: Rate Cut 'May Be Warranted Soon'.
01-Aug-2019 2.962	Stocks, bond freids. Uit Fall on New China Tariffs.

Date	Shock	Narrative corroboration of the structural shock $\xi_3$ : lower tail events Event
28-Aug-1998 07-Oct-1998 08-Jun-1999 08-Jun-1999 03-Jan-2000 17-Sep-2001 05-Jun-2003 06-Oct-2005 06-Oct-2005 01-Sep-2005 05-Sep-2005 01-Sep-2005 22-Sep-2005 22-Sep-2005 17-Sep-2005 17-Sep-2016 07-Jun-2015 01-Jan-2015 03-Jun-2015 03-Jun-2016 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun-2010 03-Jun	-4.457 -5.273 -5.273 -3.149 -3.367 -3.140 -3.140 -3.140 -3.164 -3.064 -3.064 -3.064 -3.064 -3.165 -3.064 -3.165 -3.165 -3.165 -3.165 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -3.145 -	Rissian tremors on Wall Street. 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Table 7: log retu 12/31/2 archiva]	Narrativ urns of the 2019, whe resources	e corroboration of the lower 0.5%-quantile of the identified third structural shock $\xi_3$ obtained from a system comprising the daily $\xi_2 \$ S&P 500 index, the constant maturity yield of the U.S. 10-year Treasury notes, and the U.S. Dollar Index from 01/01/1998 to n using the TRMI stock market index and TRMI bond market index as proxy variables. News items are sourced from a range of $\xi_3$ , primarily from wsj.com and money.cnn.com, among others.

Date	Shock	Narrative corroboration of the structural shock $\xi_3$ : upper tail events Event
06-Jan-1999 10-Sen-1999	2.799	The dollar regained ground against the yen, but market experts predicted more pain ahead for the U.S. currency. The greenback also rose against the euro. Dollar Gains on Yan Funo Afrae Taran Bank Action.
04-Sep-2001	3.910	outing out the provide the provided and
23-Jul-2002	3.324	Stocks, Strong Dollar Tarnish Gold. Dollar Rises Against Yen, Euro.
31-May-2005	3.757	Dollar's Recent Advances Could Stall.
25-Jul-2007	2.941	The dollar gained against the euro and the yen. Oil prices surged, U.S. light crude soared \$2.34 to \$75.90 a barrel on the New York Mercantile Exchange.
08-Aug-2008	3.540	Stocks, Dollar Surge as Oil Swoons. Dollar Rallies Against Euro After ECB Admits Growth Risks.
02-Sep-2008	2.884	Crudé's Tumble Lifts the Dollar. Oil's Swoon Lifts European Stocks.
30-Sep-2008	3.695	Oil's Plunge Is Biggest In Dollars Since '91. Dollar Gains on Euro, Falls Against Yen. Dollar rises on euro, pound ahead of vote on U.S. rescue.
10-Oct-2008	2.803	Dollar, euro gain against yen as rate cut alters risk outlook.
04-Dec-2009	3.697	Oil Prices Skid on Jobs Data. Stronger Dollar Sinks Gold Prices.
11-Aug-2010	2.679	Economic Anxiety Sends Investors Fleeing to Dollar.
19-Oct-2010	2.705	Dollar Gains Sharply on China Rate Hike.
05-May-2011	3.380	Commodities Rout Drives Oil Below USD 100. Dollar Rallies as Euro Sinks.
15-Jun-2011	3.268	Crude Tumbles As Global Economic Fears Spread. Dollar Strength Stifles Gold's Rally.
06-Sep-2011	3.124	U.S. Oil Prices Drop. Oil Prices Tumble.
05-Jul-2012	2.926	Oil Falls as Investors Focus on Stronger Dollar.
02-May-2013	2.699	Canadian Dollar Weakens After New BOC Governor Named. Weak U.S. Gas Market Weighs on Statoil.
04-Sep-2014	3.887	Euro Drops to 14-Month Low Against Dollar. Oil Prices Fall on Dollar Strength. Gold Slips on Dollar Strength.
03-Oct-2014	4.370	Dollar Gains to New Highs. Oil Prices Extend Losses.
21-Nov-2014	2.718	The dollar fell against the yen but rose against the pound and gained almost 1% against the euro.
28-Nov-2014	2.938	Oil Prices Tumble to Five-Year Lows. Oil-Linked Currencies Extend Losses.
22-Jan-2015	3.005	Dollar Rises Against Euro, European Currencies. Oil Prices Slide as Supplies Grow.
22-Oct-2015	3.308	Gold Prices Slip. Oil prices fell on Wednesday following another increase in oil stockpiles.
24-Jun-2016	4.497	Pound Plummets While Dollar, Yen Soar. Oil Prices Down, but Pare Losses.
29-May-2018	2.769	Dollar Rises on European Political Turmoil. U.S. Oil Prices Fall as Supply Worries Linger. Stock Declines Put Added Pressure on Oil Prices.
14-Jun-2018	3.265	Dollar Rises Against Euro, Broad Range of Other Currencies. Oil Remains Down Slightly in Asia After Wednesday Bounce.
21-Mar-2019	2.697	Dollar Rebounds on Europe Worries.
Table 8:	Narrativ	s corroboration of the upper 0.5%-quantile of the bond shock of the unrestricted BEKK model of the system of daily log returns of
tho C&-E	SOO indo	the constant maturity of the ITS 10 years. The constant and the ITS Dollar Index from 01 /01 /1008 to 17 /2014 when
יאר אווו		c, the constant maturity yield of the U.S. 10-year measury mores, and the U.S. Donar more more 1011 01/01/1770 to 12/01/2 when

using the stock market index  $(Z_1)$  and bond market sentiment  $(Z_2)$  TRMIs as proxy variables. News items are sourced from a range of archival resources, primarily from wsj.com and money.cnn.com, among others.