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Abstract

We develop a model in which large risk-neutral firms and individual risk-averse consumers compete to employ heterogeneous workers by posting compensation menus. Production takes time, and we analyze how screening motives interact with the desire to smooth consumption. There is a unique symmetric separating equilibrium that is also efficient. In equilibrium, the extent to which the compensation scheme delays payment until the production quality becomes known depends on whether, and to which extent, the consumers are financially constrained. We discuss how our model relates to the design of compensation schemes in current online peer-to-peer markets.

Keywords

Adverse selection, Self selection, Peer-to-peer markets, Labor markets, Capital market imperfections.

JEL Classification

D15, D82, D86, E24, J33, M52.

1 Introduction

Traditionally, firms act as intermediaries between consumers and workers. On the one hand, firms select and hire workers for production, and on the other hand, they sell their goods and services to consumers. In modern labor markets, however, it has become increasingly common for goods and services to be exchanged directly between consumers and workers in a peer-to-peer manner (for example through online labor markets such as Fiverr, Amazon MTurk, Guru, or UpWork).

In these direct exchanges, the consumers act as individual principals whose payoffs are directly dependent on the value of production and the payments they make to their workers. Often, production takes some time to complete. Then, the motives of selecting workers by offering specific payment schemes and the desire to smooth consumption over time may interact. Moreover, the direct exchanges occur in the context of the traditional, still much larger labor market, which constrains the contracts individual consumers can offer.

In this paper, we develop a competitive market model to analyze the interdependence between traditional labor markets and their more modern peer-to-peer counterpart. We are especially interested in how this affects the balance between contractual screening and consumption smoothing for individual consumers. We model a situation where large firms and individual consumers coexist and draw from the same pool of workers. Firms and consumers compete for workers by offering screening contracts. Firms are risk-neutral and can hire any positive number of workers, while individual consumers are risk-averse and employ one worker for a specific task. Firms and consumers simultaneously post contracts, and workers apply to the jobs that offer them the highest utility, choosing randomly if they are indifferent between options.

Production occurs over two periods and can either succeed or fail. Successful production yields consumption value in both periods. Failed production, in addition, incurs a loss in the second period. Workers have private information about their probability of producing a failure. A relevant constraint is that workers are protected by limited liability in the second period, which is motivated by the asymmetric enforceability of payments between workers on the one hand and firms and consumers on the other hand. Indeed, empirically, consumers in peer-to-peer markets tend to live in countries with better contract enforcement than workers (ILO, 2021).

An equilibrium in this setup consists of a profile of compensation menus offered by the firms and consumers. Throughout, we restrict attention to symmetric profiles, which means that all firms and consumers offer the same compensation menu. In a preliminary result, we show that, due to competition, a pooling equilibrium in which both worker types obtain the same contract cannot exist. Rather, all equilibria are of the separating kind.

In the first major step of our analysis, we then characterize the separating equilibrium in the traditional labor market. We show that a separating equilibrium always exists in which bad types accept a flat wage and good types accept a scheme involving, in addition to the wage, a down payment before employment (a *stake*) combined with deferred compensation in case of success (a *bonus*). Posting their wages entails elements of a coordination problem for the firms, so there might be inefficient equilibria without full employment. Yet, the firm-optimal equilibrium — in which no workers remain unemployed — coincides with the efficient equilibrium.

Because firms and workers are risk-neutral, multiple payment schemes implement the firm-optimal equilibrium. A unique firm-optimal equilibrium arises if we consider the full model in which individual risk-averse consumers compete with the firms for workers. Concavity of the utility function means that the consumers have an imperfect elasticity of substitution between consumption across states and time. This implies a unique optimal scheme, which pins down the unique separating equilibrium. In this equilibrium, the firms' hiring decisions are the same as in the absence of the individual consumers, and the consumers offer a compensation scheme that is only accepted by the good types. With the equilibrium stake and the bonus payments, the consumers optimally smooth their consumption over employment and screen for the good types.

Finally, we investigate how the equilibrium compensation scheme depends on the consumers' access to financial markets and other exogenous parameters. Many consumers, even in developed countries and over relatively short times, face binding borrowing constraints (see, e.g., Jappelli and Pistaferri, 2017).[1] Intuitively, the stake that consumers ask workers to put down at the beginning of employment increases in the severeness of the borrowing constraint, as does the bonus payment in case of success. If the borrowing constraint does not bind, both the stake and the bonus increase in the loss caused by failure, and they decrease in the probability that the bad types fail.

Our analysis is structured as follows. After discussing the related literature in Section 2, we present the model in Section 3. We start the analysis in Section 4 with two special cases, which (i) highlight the risk-sharing between consumers and the good worker type, and (ii) analyze the properties of the competitive equilibrium if only firms employ workers. Section 5 then characterizes an implementation of the optimal compensation scheme in the full model with heterogeneous employers and workers. We discuss how our model and findings relate to existing online markets in Section 6. In Appendix A we provide the proofs for results not derived in the main text, Appendix B analyzes additional equilibria, which may arise in specific circumstances, and Appendix C discusses model extensions.

[1] See the remarks on financial constraints and payday loans in Section 2.

2 Related Literature

Our paper contributes to the literature on principal-agent problems in the presence of adverse selection or moral hazard in the labor market (e.g., Burdett and Mortensen, 1981; Davoodalhosseini, 2019; Starmans, 2024). Oyer and Schaefer (2011) provide a comprehensive survey.

Prescott and Townsend (1984) and Guerrieri et al. (2010) analyze adverse selection in competitive equilibrium and competitive search equilibrium, respectively. They focus on applications such as the classic insurance-incentive trade-off when a risk-neutral principal tries to insure a risk-averse agent in the presence of asymmetric information. Our setting differs in at least two important aspects. Firstly, we focus on how the smoothing motive of a risk-averse principal affects optimal incentive provision. Secondly, we analyze the competitive equilibrium with heterogeneous employers: risk-neutral firms and risk-averse consumers.

We build on Salop and Salop (1976) who analyze a principal-agent problem between firms and employees and introduce a compensation scheme that incentivizes self selection to reduce labor market turnover.[2] Our work differs in various aspects. We consider the selection problem in a setting where firms do not always intermediate the exchange of goods and services between workers and consumers. The presence of risk-averse consumers, who directly employ workers, implies that incentive provision and consumption smoothing motives are intertwined. We show that a compensation scheme, consisting of a wage, bonus and stake, can implement the unique firm-optimal separating equilibrium.

If the stake in the optimal compensation scheme in our model exceeds the fixed wage, the compensation scheme resembles a performance bond. There is a crucial difference, however, relative to the literature on performance bonds in moral-hazard settings (see, e.g., Becker and Stigler, 1974; Lazear, 1979, 1981). Because of objective verification and commitment technologies, which are available in peer-to-peer labor markets discussed further in Section 3, implementation in our setting may not be hampered by bankruptcy risk or double moral hazard. The latter arises in the classic analyses of performance bonds. Because of double moral hazard, principals may pretend malfeasance of agents to confiscate their collateral (see, for example, Eaton and White, 1982; Shapiro and Stiglitz, 1984; Ritter and Taylor, 1994).

Our work further relates to recent work by Hoffmann et al. (2021) who examine the timing of pay in a principal-agent setting with moral hazard and an impatient, possibly risk-averse agent. In this context, Hoffmann et al. (2021) show that a trade-off arises between the backloading of payments and the agent's resources for consumption. Our

[2] Lazear (2000) and Dohmen and Falk (2011) provide empirical evidence that performance pay matters for self selection.

problem is different from theirs in that we analyze the interactions between incentive provision and consumption smoothing that arise from risk aversion of the principal rather than the agent in a setting with adverse selection.

Our analysis is also related to the literature on incentives in the context of trade credit. Smith (1987) emphasizes the role of interest rates in trade-credit arrangements to screen buyers for default risk. Kouvelis and Zhao (2012) analyze how trade credit may emerge through strategic interactions between suppliers and retailers in a setting with imperfect competition and liquidity constraints. Rui and Lai (2015) analyze the role of deferred payments to ensure product quality in a moral-hazard setting. Complementary to this line of research, we analyze which compensation scheme can ensure selection of good-quality workers in competitive markets and allow potentially liquidity constrained consumers to smooth consumption. Delayed payments through stakes also feature in our optimal compensation scheme but are generally accompanied by a state-contingent bonus.

Our analysis of the interaction between worker incentives and the timing of payments to and from consumers contributes to the growing literature on online labor markets. Li et al. (2022) analyze how insurance against bad quality services affects the demand for these services. Liang et al. (2022) show that monitoring is accepted by workers in online labor markets only at substantial cost (30% of the average hourly wage). Burtch et al. (2018) as well as Huang et al. (2020) consider how the growth of gig-economy platforms impacts local entrepreneurial activity and unemployment. Buchak (2024) analyzes how financial constraints of workers affect growth of the gig economy.

Financial constraints of consumers imply in our analysis that the compensation of workers is delayed relatively more, within the constraints imposed by incentive provision. Empirically, a sizable part of the population faces severe liquidity constraints. A report on the economic well-being of U.S. households (Federal Reserve System, 2018) finds that a third of all adults needs to sell assets, borrow from friends or family, use bank overdrafts or take out payday loans when confronted with an unforeseen expense of $400. 12% would not be able to cover such an expense at all. This is reflected by the size and importance of payday lending markets in developed countries, such as the U.S. and the U.K. (Stegman, 2007; Gathergood et al., 2018). Similarly, Kaplan et al. (2014) report that 20 – 30% of households in developed countries are hand-to-mouth consumers. For members of these households, consumption smoothing is very costly, even over short horizons. A related literature on consumer finance, surveyed in Tufano (2009), analyzes financial constraints of consumers and the provision of consumer credit (e.g., Bertola et al., 2005).

3 The Model

There are $N \in \mathbb{N}_+$ homogeneous firms, $K \in \mathbb{N}_+$ individual consumers and a mass $W > 0$ of workers. We index firms with $i \in \{1, ..., N\}$ and consumers with $i \in \{N+1, ..., N+K\}$. Firms and consumers hire workers for a productive task. Firms can hire mass $\ell \leq W$ of available workers. Individual consumers are atomistic, and each consumer can hire one worker.

Production Workers are risk neutral.[3] Each worker produces one unit of output, whose value might depend on whether the worker is hired by a firm or a consumer, as discussed further below. Output is produced over two periods. A share $\alpha \in [0,1]$ of the total output is produced in the first period; a share $1-\alpha$ in the second period.

Output can be of high quality or low quality. A low-quality output is associated with a loss of size $L > 0$ to the employer in the second period. Workers can be of good or bad type. The probability of failing at the task is higher for a bad than a good worker: $q_b > q_g$. The probability that a given worker is of good type is $\phi \in [0,1]$, and for a bad type, it is $1-\phi$. We write $\bar{q}$ for the average failing probability of a randomly drawn worker, $\bar{q} = \phi q_g + (1-\phi) q_b$. For simplicity, we normalize the workers' cost of working and their outside options to zero.

Firms Hiring a mass ℓ of workers yields total output $F(\ell)$ to a firm. Throughout, we take $F(\ell)$ to be bounded and assume $F(0) = 0$. In the following, we call $f(\ell) \equiv F'(\ell)$ the marginal productivity of labor and assume that it strictly decreases in the mass of hired workers, ℓ. The real interest rate is $r \geq 0$.

So, if a firm hires a mass ℓ of workers and mass $\kappa \leq \ell$ of them provide a bad contribution to production, the firm's profit net of possible transfers to the agent is

$$\alpha F(\ell) + \frac{(1-\alpha)F(\ell) - \kappa L}{1+r}.$$

The firm's losses are proportional to the mass of workers it hires whereas the output produced is not. Each worker is hired to perform a specific task, and there are diseconomies of scale. The damage caused by a failed task execution scales linearly.

Throughout, we assume

$$f(W/N)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_b \frac{L}{1+r} \geq 0. \tag{A1}$$

Assumption (A1) says that when all firms employ an equal share of workers, and there is full employment, the marginal profit of hiring an additional bad worker is positive. As

[3] In Appendix C.1, we discuss how the analysis would modify if workers were risk averse.

we will see, the assumption is necessary for a separating full-employment equilibrium to exist.

Consumers Individual consumers are risk averse. They are characterized by a strictly increasing and strictly concave utility function $u(x)$ satisfying $\lim_{x\to 0} u'(x) = \infty$ and $\lim_{x\to\infty} u'(x) = 0$. The value of hiring a worker is represented by a positive match value V that accrues over two periods, similar to the firms. Each consumer enjoys the output αV from production in the first period, and $(1-\alpha)V$ in the second period, where they may also incur a loss L. We assume that the discount rate of consumers equals the real interest rate of the firm.

Compensation Schemes At the outset, firms $i \in \{1,...,N\}$ and consumers $i \in \{N+1,...,N+K\}$ publicly post a compensation scheme

$$\{(t_{i1\theta}, t_{i2f\theta}, t_{i2s\theta})\}_{\theta=b,g},$$

determining the payment $t_{i1\theta}$ from the firm or consumer i to the worker of type θ in the first period, the payment $t_{i2f\theta}$ in case of production failure in the second period, the payment $t_{i2s\theta}$ in case of production success in the second period.

Although the worker can be asked for a down payment in the first period (i.e., $t_{i1\theta}$ can be either positive or negative), there is a limited liability constraint on the second-period payments,

$$t_{i2s\theta}, t_{i2f\theta} \geq 0. \tag{A2}$$

As mentioned in the introduction, this is motivated by asymmetric legal frictions. Workers, firms, and consumers live in different jurisdictions so that securing a claim from the firm or the consumer is easier for the workers but not necessarily the other way around. The assumption (A2) captures this asymmetry in a stylized way. We discuss in Appendix C.2 how firms and consumers may commit to honor the promises implied by the compensation scheme.

The Labor Market We assume that firms and consumers compete for workers by offering compensation schemes. The timing in the labor market is as follows.

1. Firms and consumers simultaneously publish a menu of compensation schemes.

2. Workers apply to at most one firm or consumer, indicating which menu option they want. Each worker applies to a firm or consumer that yields the highest utility conditional on acceptance.

3. Firms and consumers randomly choose from their applicants and hire them according to their chosen menu options.

If workers are indifferent between firms or between applying and not, we will assume that they randomize symmetrically.[4] If, in addition, workers are indifferent between firms and consumers, then workers coordinate so that each consumer obtains exactly one application from a different worker. In case of indifference between the menu options, workers choose the option that is intended for their type.

Equilibrium Throughout the main text, we look at separating equilibria in which both worker types apply for a job with probability one.[5] An equilibrium is thus fully described by a profile of compensation scheme menus

$$\left\{(t^*_{i1b}, t^*_{i2fb}, t^*_{i2sb}), (t^*_{i1g}, t^*_{i2fg}, t^*_{i2sg})\right\}_{i\in\{1,\dots,N+K\}}$$

by the firms $i \in \{1,...,N\}$ and consumers $i \in \{N+1,...N+K\}$, so that no firm or consumer has the incentive to offer a different menu of compensation schemes. We focus on symmetric equilibria, in which all firms and consumers choose to offer the same menus.

4 Preliminary Analysis of two Illustrative Special Cases

In this section, we discuss two special cases that lay the ground for our analysis of the full model in Section 5, in which optimal compensation is jointly determined by consumption smoothing and incentive provision. First, we analyze the situation in which one consumer hires one good worker. Doing so allows us to delineate how the consumption-smoothing motive of the consumers affects compensation. Second, we analyze the equilibrium between firms when no individual consumers also employ workers. This allows us to isolate how incentive provision by firms affects compensation and to work out the details of the wage-stake-bonus compensation scheme that we will work with in Section 5. In either example, we assume that consumers do not have access to capital markets – which we then analyze in Section 5.2.

[4]That is, if they are indifferent between firms, workers will randomize between the firms with uniform probability. If, in addition, they are indifferent between applying and not, all workers apply with the same probability and, conditional on having applied, mix uniformly between firms.

[5]Under (A1), such equilibria always exist. When (A1) fails, there might be additional equilibria, in which firms offer contracts such that all the good-type workers apply, yet only a subset or none of the bad-type workers do. We provide a discussion of these equilibria in Appendix B.

4.1 Efficient Risk-Sharing between a Consumer and a Good Worker

Suppose one consumer wants to hire one worker, and there are no firms. Furthermore, the worker has an outside option of zero and is of good type with probability one, $\phi = 1$. Let (t_1, t_{2f}, t_{2s}) be the compensation scheme that the consumer offers to the worker. The problem of the consumer is

$$\max_{t_1, t_{2f}, t_{2s}} \quad u(\alpha V - t_1) + \frac{1}{1+r}\left[q_g u((1-\alpha)V - L - t_{2f}) + (1-q_g)u((1-\alpha)V - t_{2s})\right]$$

subject to

$$t_1 + \frac{1}{1+r}\left[q_g t_{2f} + (1-q_g)t_{2s}\right] \geq 0 \tag{1}$$

$$t_{2f}, t_{2s} \geq 0, \tag{2}$$

where (1) is the participation constraint and (2) restricts the compensation scheme because of limited liability (A2).

Because the consumer can always increase her payoff by reducing t_1, t_{2f}, or t_{2s} (whichever is strictly positive) when the participation constraint (1) is slack, the participation constraint must bind. Moreover, from the concavity of the utility function $u(.)$ we obtain that $t_{2f} = 0$: if $t_{2f} > 0$, it would be optimal for the consumer to reduce t_{2f} to increase utility while leaving the utility of the worker unchanged. Then, either $t_1 = t_{2s} = 0$ or $t_1 < 0 < t_{2s}$. In the latter case, using the binding participation constraint (1) to substitute t_{2s} in the objective function, we obtain that the optimal t_1 satisfies

$$u'(\alpha V - t_1) = u'\left((1-\alpha)V + \frac{1+r}{1-q_g}t_1\right),$$

giving us the following result:

Proposition 1 *Suppose there is one consumer who wants to hire one worker. The worker is of good type with probability one. In the optimum,*

$$t_1^* = \min\left\{0, (2\alpha - 1)V\frac{1-q_g}{2+r-q_g}\right\}, \quad t_{2f}^* = 0, \quad t_{2s}^* = \max\left\{0, (1-2\alpha)V\frac{1+r}{2+r-q_g}\right\}.$$

Because of risk aversion, the consumer prefers to not pay anything in case of a loss, and the limited liability constraint binds: $t_{2f}^* = 0$. In the optimum, the transfer in the first period, t_1^*, is either negative or zero, which depends on the production path. In particular, if production is back-loaded, $\alpha < 1/2$, then the consumer asks the worker to put down a positive stake. In return, the worker is promised a bonus in case of success.

Such a compensation scheme allows the consumer to smooth consumption over time. In the case of front-loaded production, $\alpha > 1/2$, the optimal compensation scheme entails no payments, and the worker is kept at her outside option. In this case, the limited liability constraint on t_{2s} in conjunction with the participation constraint prevents the consumer from smoothing consumption with the contract.

4.2 Competitive Equilibrium

Next, we assume $N > K = 0$, i.e., there are multiple firms but no consumers, allowing us to focus on the equilibrium with only firms.

We say that a symmetric equilibrium is pooling if firms offer identical compensation schemes to the respective worker types — $t_{i1b} = t_{i1g}$, $t_{i2fb} = t_{i2fg}$, and $t_{i2sb} = t_{i2sg}$ for all firms i — which both worker types find acceptable. The proof of the following first result formalizes the arguments in Salop and Salop (1976), showing that for the class of contracts we consider, pooling equilibria under which $t^*_{i1\theta} > t^*_{i2s\theta} = t^*_{i2f\theta} = 0$ for all i and θ never exist. We call such equilibria pooling equilibria in pure wage contracts.

Lemma 1 *No symmetric pooling equilibrium in pure wage contracts ($t^*_{i1\theta} > t^*_{i2s\theta} = t^*_{i2f\theta} = 0$ for all i and θ) exists.*

4.2.1 Separating Equilibria

Lemma 1 prompts us to look at separating equilibria. Our next result establishes that, for the class of symmetric separating equilibria in which both types apply with certainty, it is without loss of generality to focus on equilibria in which the firms offer contracts that give the bad types a fixed wage, $t_{i1b} > t_{i2fb} = t_{i2sb} = 0$, while the good types select themselves into a scheme that also offers a stake/bonus component. Formally,

Lemma 2 *In the class of symmetric separating equilibria in which both types apply with certainty, it is without loss of generality to consider equilibria in which $t_{i2fg} = t_{i2fb} = t_{i2sb} = 0$.*

The proof of Lemma 2 exploits that all relevant constraints of the firms are linear in the compensation components. The result allows us to focus on the class of symmetric separating equilibria in which the bad types receive a flat payment and the good types receive a flat payment together with a bonus in case of successful production.

Proposition 2 *Suppose $N > K = 0$. There are multiple symmetric separating equilibria. The equilibria differ in the mass $\ell^* \in [0, W/N]$ of workers that each firm hires and in the compensation schemes that they offer. Specifically, each $\ell^* \in [0, W/N]$ corresponds to a continuum* *of equilibria: For any given $\ell^* \in [0, W/N]$, each firm offers a menu $\{(t^*_{1\theta}, t^*_{2s\theta}, t^*_{2f\theta})\}_{\theta=b,g}$ with $t^*_{2sb} = t^*_{2fb} = t^*_{2fg} = 0$, t^*_{1b} given by*

$$t^*_{1b} = f(\ell^*)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_b \frac{L}{1+r}, \tag{3}$$

and (t^*_{1g}, t^*_{2sg}) satisfying

$$t^*_{2sg} \geq L \quad \textit{and} \quad t^*_{1b} = t^*_{1g} + \frac{1-q_g}{1+r} t^*_{2sg} - \frac{q_b - q_g}{1+r} L. \tag{4}$$

Equation (3) stems from the firms' problem of optimally hiring given the compensation schemes offered, stating that the marginal costs from hiring a bad-type worker is equal to the marginal benefits of doing so. The equality in (4) is an indifference condition for the firms, saying that hiring a good worker is equally costly as hiring a bad worker. The inequality in (4) ensures that the incentive compatibility constraint of the bad-type worker is satisfied. Because the incentive compatibiliy constraint of the good-type worker is always satisfied, this inequality implies that we obtain separation.

There are two forms of equilibrium multiplicity described in Proposition 2. First, for a given mass ℓ^* of workers the firms hire, there is a continuum of equilibrium compensation schemes for the good types. Specifically, because both the workers and the firms are risk-neutral there are multiple transfers (t^*_{1g}, t^*_{2sg}) with which the firms achieve separation. This is reflected in the conditions (4). As we show in the analysis of the full model in Section 5, this source of multiplicity vanishes when we introduce risk-averse consumers.

The second source of multiplicity stems from a coordination problem that the firms face when offering contracts. Recall that the workers exclusively apply to the firms offering the highest utility. Hence, for any equilibrium mass ℓ^* of workers, firms do not obtain any applications when they offer a contract that provides less utility than at other firms. On the other hand, no firm has an incentive to offer a contract that provides more utility either, as this would only lower profits (even if adjusting downward the mass of hired workers.) In the following, we focus on the firm-optimal equilibrium.

4.2.2 The Firm-Optimal Equilibrium

Using the usual terminology, we say that an equilibrium is *firm-optimal* if the firms' profits are maximal among all the (symmetric) equilibria described in Proposition 2.[6] When each

[6] In other words, an equilibrium is firm-optimal if not only the firms choose mutually profit-maximizing contracts but also the equilibrium profits are maximal across all equilibria.

firm hires ℓ^* workers, the profit of any firm i is

$$\Pi_i(\ell^*) = F(\ell^*)\left[\alpha + \frac{1-\alpha}{1+r}\right] - \left[t_{1b}^* + q_b\frac{L}{1+r}\right]\ell^*.$$

Using the indifference condition (3), firm profit can be expressed as

$$\Pi_i(\ell^*) = [F(\ell^*) - f(\ell^*)\ell^*]\left[\alpha + \frac{1-\alpha}{1+r}\right],$$

which is strictly increasing in ℓ^* because F is concave. Consequently, the equilibria (and only those) involving full employment, $\ell^* = W/N$, are firm optimal.

Proposition 3 *An equilibrium described in Proposition 2 is firm-optimal if and only if* $\ell^* = W/N$.

The firm-optimal equilibrium is not only focal because it provides maximal rent to the firms but also because it is efficient. To see this, we consider the problem of a social planner with complete information about workers' types deciding the mass of workers each firm will hire.

Let $a_g \geq 0$ be the mass of good types hired and let $a_b \geq 0$ be the mass of bad types being hired. Because F is concave, in the social optimum, each firm will hire the same mass of workers, and we may write the planner's problem as

$$\max_{a_g, a_b \geq 0} \; F(a_g + a_b)\left[\alpha + \frac{1-\alpha}{1+r}\right] - \left[a_g q_g + a_b q_b\right]\frac{L}{1+r} \tag{5}$$

subject to

$$a_g \leq \phi W/N$$
$$a_b \leq (1-\phi)W/N.$$

Under (A1) the solution to the planner's problem is $a_g = \phi W/N$ and $a_b = (1-\phi)W/N$. In other words, firms hire all available workers in the social optimum. We may thus state, without further proof,

Proposition 4 *The firm-optimal equilibrium in the class of equilibria described with Proposition 2 is efficient.*

We conclude this section by discussing what happens if Assumption (A1) fails. If the inequality (A1) does not hold (but holds instead if we replace W/N in that inequality with zero), then the maximum mass of workers that will be hired in any symmetric separating equilibrium is equal to

$$\bar{\ell} \equiv \max\left\{\ell : f(\ell)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_b \frac{L}{1+r} \geq 0\right\} \in [0, W/N). \tag{6}$$

Any separating equilibrium with $\ell^* \leq \bar{\ell}$ is inefficient because the total output can always be increased by replacing some of the employed bad workers with unemployed good workers.[7] We show in Appendix B that the efficient workforce composition can nevertheless be supported in an equilibrium where bad-type workers do not apply with probability one.

4.2.3 Implementation

To finish this section, we discuss how the equilibrium compensation schemes in the competitive model may be implemented in practice. Following the discussion after Proposition 1, a salient implementation is that of a fixed wage scheme together with a bonus/stake option for the good workers. More precisely, the equilibrium compensation schemes characterized in Proposition 2 can be implemented by the firms as follows:

(1) Offer a flat wage $w^* = t^*_{1b}$ to all workers.

(2) In addition, ask the good types to put down a stake $S^* = t^*_{1b} - t^*_{1g}$ upfront, which is paid back together with a bonus $B^* = t^*_{2sg} - S^*$ in case of successful production.

The equilibrium wage-stake-bonus scheme (w^*, S^*, B^*) is then characterized as follows, which is obtained by combining the above definitions of (w^*, S^*, B^*) with the characterizations (3) and (4) in Proposition 2. Specifically, we say that a wage-stake-bonus scheme (w^*, S^*, B^*) implements an equilibrium menu of compensation schemes if there is an equilibrium menu $\{(t^*_{1\theta}, t^*_{2s\theta}, t^*_{2f\theta})\}_{\theta=b,g}$ that satisfies points (1) and (2) above.

Corollary 1 (Implementation) *Any wage-stake-bonus scheme* (w^*, S^*, B^*) *that satisfies*

$$w^* = f(\ell^*)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_b \frac{L}{1+r}, \tag{7}$$

$$B^* = \frac{q_b - q_g}{1-q_g} L + \frac{q_g + r}{1-q_g} S^*, \tag{8}$$

$$S^* \geq (1-q_b)\frac{L}{1+r} \tag{9}$$

implements an equilibrium menu of compensation schemes.

[7] The source of inefficiency is quite different from the inefficiency that Guerrieri et al. (2010) find under directed search. Their firms have unit demand so that the kind of equilibria in which each firm hires inefficiently few workers do not arise.

The wage w^* corresponds to the marginal gain from hiring an additional worker that is of a bad type. The stake and bonus pair offered serves to separate good workers from bad ones. Under any of the equilibrium bonus schemes, the good types obtain a strictly positive information rent

$$\Delta(L) \equiv \frac{q_b - q_g}{1 - q_g} L. \tag{10}$$

The condition (9) ensures that the bad types receive a strictly negative utility from the bonus-stake pair intended for the good types so that they accept the option that comprises the flat wage w^* alone.[8]

The bonus in the indifference condition (8) consists of the information rent as well as the expected loss of the stake $q_g S$ plus interest on the stake rS, scaled by the probability of success $1 - q_g$. Workers receive a higher bonus for a given stake if the interest rate r is higher. This term premium serves as compensation for the stake being locked in during the production process. The information rent in (10) increases in the size of the loss and the unobservable heterogeneity in the workforce, as reflected by the difference between q_b and q_g. The interaction between these two magnitudes implies that smaller potential losses make the trait of having a lower loss probability less valuable.

5 Analysis of the Full Model

We now turn to the equilibrium analysis of the full model, in which we have $N > 0$ firms and $K > 0$ individual consumers. Adding a finite number of individual consumers to the model analyzed in the last section does not change the behavior of the firms, but it will pin down a unique equilibrium. Throughout this section, we consider the wage-stake-bonus scheme representation (w, S, B) of the equilibrium compensation schemes, as outlined in Corollary 1 above.

We will analyze two cases, one in which the consumers have no access to financial markets and one in which they have partial access. In either of the two cases, it is optimal for the individual consumers to hire a good-type rather than a bad-type worker. Thus, the consumer will offer a single compensation scheme rather than a menu, which is only acceptable to a good-type worker.[9] This is a consequence of the fact that the (expected) marginal cost of employing either worker is the same for the risk-neutral firm. Consequently, the risk-averse consumer will always strictly prefer a good-type worker.

[8] It can be easily verified that for the smallest equilibrium stake implied by (9), $B + S = L$. Hence, $B + S = t_{2sg} \geq L$ as in (4) always holds.

[9] Technically, the consumers do offer a menu, where the the compensation scheme for the bad-type workers is such that none will apply (e.g., involving zero expected payments).

5.1 No access to capital markets

For (w^*, S^*, B^*) to correspond to a symmetric equilibrium, the individual consumers must optimally choose a wage-stake-bonus scheme among those satisfying (7), (8), and (9). Without access to capital markets, the maximization problem of the consumer is thus

$$\max_S \quad u(\alpha V - (w^* - S)) + \tag{11}$$

$$+\frac{1}{1+r}\left[q_g u((1-\alpha)V - L) + (1-q_g)u\left((1-\alpha)V - \Delta(L) - \frac{1+r}{1-q_g}S\right)\right]$$

$$\text{s.t. } S \geq (1-q_b)\frac{L}{1+r} \equiv S^{\min}. \tag{12}$$

The consumer selects the stake above $S^{\min}$ that maximizes (expected) utility.[10] The first-order condition for S implies that, at an interior optimum,

$$u'(\alpha V - (w^* - S)) = u'\left((1-\alpha)V - \Delta(L) - \frac{1+r}{1-q_g}S\right). \tag{13}$$

Given the constraint (12), we thus have the following result.

Proposition 5 *If consumers do not have access to capital markets, then the stake that consumers require in the unique firm-optimal equilibrium is* $S^\dagger = \max\left\{S^{\min}, S^*\right\}$*, where*

$$S^* = \frac{1-q_g}{2+r-q_g}\left[(1-2\alpha)V + w^* - \Delta(L)\right]. \tag{14}$$

The bonus $B^\dagger$ associated with this optimal stake $S^\dagger$ follows from (8).

Intuitively, the risk-averse consumer uses S and B to smooth consumption across time and states, given the lack of access to capital markets. The stake delays payment of the worker to the second period and thus helps to smooth consumption across time. Because the stake affects second-period consumption only in the state without failure, the bonus is an additional instrument to smooth second-period consumption across the states with and without failure.[11] Figure 1 provides a graphical illustration for how the equilibrium bonus-stake combination is determined.

[10] If the stake S exceeds the non-contingent wage w, possible commitment problems may be addressed by pledging illiquid collateral, as discussed in Appendix C.2.

[11] At an interior optimum, the optimal stake in (14) does not explicitly depend on risk aversion. As discussed in Section 4.1, risk aversion matters for the optimal stake; however, risk-averse consumers prefer not to pay anything to the worker in case of failure (a corner solution for the payment in the failure state). The optimality condition (13) thus depends on the risk aversion of consumers only implicitly.

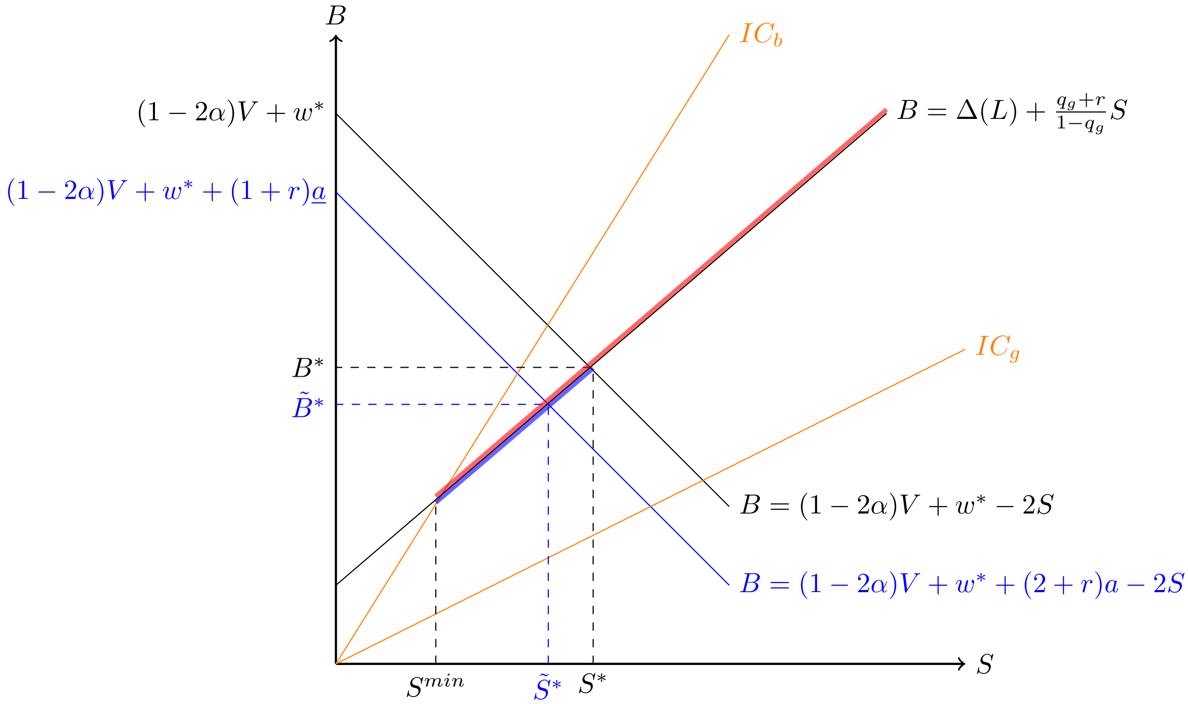

Figure 1: *Equilibrium bonus-stake combination in the full model with and without consumers having access to capital markets.* Explanation: All points to the right of the line IC_b — which is constructed from (A.11) given the implementation discussed in Section 4.2.3 — correspond to (S, B) combinations that satisfy the bad types' incentive compatibility constraint. All points north of the line IC_g — which is constructed from (A.12) — correspond to (S, B) combinations that satisfy the good types' incentive compatibility constraint. The black upward-sloping line is (8), describing the (S, B) combinations for which the firms are indifferent between hiring a good and a bad worker. Among these, all (S, B) combinations to the right of $S^{\min}$ achieve separation. This gives us the set of of (S, B) combinations that can be supported in a firm-optimal equilibrium in the absence of consumers, as highlighted in red. The black downward-sloping line is constructed from (13), writing the argument in the marginal utility on the right as $(1-\alpha)V - (B+S)$. In the unique firm-optimal equilibrium, the consumers equalize marginal utility in the first period and the second period in a state without loss, which gives us $(S^\dagger, B^\dagger) = (S^*, B^*)$, lying at the intersection with (8). If $S^{\min} \geq S^*$, then the equilibrium stake $B^\dagger$ is at the intersection of the vertical line at $S^{\min}$ and (8). The blue downward-sloping line describes the (S, B) combinations that equalize consumers' marginal utilities when they have access to a financial market and their borrowing constraint binds. The ensuing equilibrium stake-bonus combination is $(\tilde{S}^\dagger, \tilde{B}^\dagger)$, with $\tilde{S}^\dagger$ moving to the left as we relax the borrowing constraint (i.e., lower $\underline{a}$). Consequently, the set of $(\tilde{S}^\dagger, \tilde{B}^\dagger)$ that may occur when firms have access to financial markets corresponds to the blue-shaded segment of (8).

The expression in (14) shows that the optimal stake decreases in α. The parameter α measures how front-loaded the production by the worker is. Intuitively, the higher the resources in the first period, the lower the motive to smooth consumption by asking the worker to provide a stake, i.e., the lower the stake. We postpone the discussion of additional comparative statics to Section 5.3.

Compared to the efficient risk-sharing between the consumer and the good worker without asymmetric information analyzed in Section 4.1, the optimal stake accounts for the equilibrium wage paid by the firms in the competitive equilibrium and the information rent paid to the good type. Otherwise, the optimal stake S^* in Proposition 5 is the same as implied by t_1^* at an interior optimum in Proposition 1.

5.2 Access to capital markets

Assume that consumption can be smoothed across time using an asset a with non-contingent return r. However, the consumer faces a borrowing constraint $a \geq \underline{a}$ (where $\underline{a} \leq 0$). Then, the problem of the consumer is

$$\max_{S,a} \quad u(\alpha V - a - (w^* - S)) + \frac{1}{1+r}\Bigg[q_g u((1-\alpha)V + (1+r)a - L) \qquad (15)$$

$$+(1-q_g)u\left((1-\alpha)V + (1+r)a - \Delta(L) - \frac{1+r}{1-q_g}S\right)\Bigg]$$

$$\text{s.t. } S \geq (1-q_b)\frac{L}{1+r} \equiv S^{\min}. \qquad (16)$$

$$a \geq \underline{a}\ . \qquad (17)$$

We may derive the solution to this program by first considering the case in which the borrowing constraint (17) is slack. In this case, the agent can fully insure against losses in the second period, in the sense that consumption in the second period is independent of the realization of the loss. The optimality conditions for a imply

$$L = \Delta(L) + \frac{1+r}{1-q_g}S \implies (1-q_b)L = (1+r)S \implies S = S^{\min}. \qquad (18)$$

Having pinned down the stake S, the optimality condition for a further yields

$$u'\left(\alpha V - a - w^* + \frac{1-q_b}{1+r}L\right) = (1+r)q_g u'((1-\alpha)V + (1+r)a - L). \qquad (19)$$

The right side of the above equality strictly increases in a, while the left side strictly decreases. Moreover, by assumption, the left side diverges to infinity when a becomes large, while the right side approaches zero. Thus we have an interior solution for a, $a > \underline{a}$, if and only if

$$u'\left(\alpha V - \underline{a} - w^* + \frac{1-q_b}{1+r}L\right) < (1+r)q_g u'((1-\alpha)V + (1+r)\underline{a} - L). \tag{20}$$

This allows us to state the following result.

Proposition 6 *Suppose consumers have access to capital markets. There is a unique firm-optimal equilibrium.*

(a) *Suppose (20) holds. Then, the consumers fully insure themselves against the potential loss in the second period. The stake that consumers require is $S^\dagger = S^{min}$ and the asset $a^\dagger$ is equal to the a that satisfies (19).*

(b) *Suppose (20) fails. Then, the consumers can only partially insure themselves, the stake that consumers require is $S^\dagger = \max\left\{S^{\min}, \tilde{S}^*\right\}$, where*

$$\tilde{S}^* = \frac{1-q_g}{2+r-q_g}\left[(1-2\alpha)V + (2+r)\underline{a} + w^* - \Delta(L)\right], \tag{21}$$

and the borrowing constraint binds, $a^\dagger = \underline{a}$.

(c) *In either case, the bonus $B^\dagger$ associated with this optimal stake $S^\dagger$ follows from (8).*

In comparison with the optimal stake derived in Proposition 5 for the case without access to capital markets, Proposition 6 shows that access to capital markets reduces the size of the optimal stake because the stake serves less as a consumption smoothing device. Thus, payments to the worker are delayed less to the second period when production quality becomes known. If the consumer is unconstrained, then the stake is at its lower bound $S^{\min}$. If the consumer is constrained, the optimal stake can be larger than that lower bound. In this case, the optimal stake depends negatively on the available borrowing opportunities $\underline{a} \leq 0$, in a proportional fashion. Figure 1 illustrates these observations.

5.3 Comparative statics

We now analyze how the optimal stake varies with the characteristics of the workers and the production technology, conditional on being in the separating equilibrium we have focused on in our analysis.[12] We begin by analyzing the impact of changes in the production technology, α and L.

[12]The parameter ϕ, denoting the incidence of good workers, matters for the type of equilibrium which can be sustained but is irrelevant for the optimal compensation conditional on being in the separating equilibrium.

Proposition 7 (Comparative Statics: Production Characteristics) *In the optimal compensation scheme with stake $S^{\dagger}$ and bonus $B^{\dagger}$,*

(a) *the optimal stake and bonus both (weakly) decrease in α, irrespective of whether consumers are constrained in their access to capital markets or not;*

(b) *the optimal stake decreases in L for consumers constrained in their access to capital markets and increases in L for unconstrained consumers. The bonus always increases in L.*

Intuitively, if production is more front-loaded (i.e., if α is high), then the consumer asks for a lower stake because there is less need to transfer consumption from the second period to the first. If the consumer is unconstrained, in which case the stake equals $S^{\min}$, the stake does not depend on α because the agent is not constrained in smoothing resources using the non-contingent asset. This explains part (a) of the proposition.

Similarly in part (b) of the proposition, the need to transfer utility from the second period to the first period by using the stake decreases for the constrained consumer if the loss from faulty production, expected to occur in the second period, increases. With constrained access to capital markets, a larger loss size leads to a lower optimal stake. The higher loss increases the information rent that can be extracted by good-type workers, implying a larger bonus for any given stake. This causes the consumer to rebalance resources between the two states in the second period. A constrained consumer is prepared to sacrifice consumption in the first period in favor of consumption in the no-loss state in the second period. The consumer does so by choosing a lower stake.

If the consumer is unconstrained in the access to capital markets, a greater loss size L increases the equilibrium stake instead. Intuitively, the consumer then relies on the non-contingent asset to smooth consumption across time and can therefore use the stake exclusively to balance consumption across states so that (18) holds. For a higher loss, this means reducing consumption in the no loss state, which can be achieved by paying back a higher stake (plus bonus).

As is intuitive, the bonus increases in L whether consumers are constrained or not. Next, we turn to the comparative statics of parameters related to workers.

Proposition 8 (Comparative Statics: Worker Characteristics) *In the optimal compensation scheme with stake $S^{\dagger}$ and bonus $B^{\dagger}$,*

(a) *the optimal stake decreases in q_b and the bonus increases in q_b, irrespective of whether consumers are constrained in their access to capital markets or not;*

(b) *the optimal stake is independent of q_g and the optimal bonus decreases in q_g if the consumer is not constrained in the access to capital markets; the optimal stake and bonus both may decrease or increase in q_g if the consumer is constrained.*

The intuition for these results is as follows.The equilibrium stake demanded by a constrained consumer decreases with a higher loss probability for the bad workers q_b because a higher q_b leads to a higher bonus for a given stake as good workers can extract a larger information rent. Intuitively, the larger bonus (for a given stake) reduces consumption in the no-loss state in the second period. Hence, the optimal stake falls to counterbalance this effect by shifting resources from the first period to the no-loss state in the second period. For an unconstrained consumer, the stake equals $S^{\min}$, ensuring bad workers do not select into the wage-stake-bonus scheme. $S^{\min}$ decreases in q_b, as is intuitive because the wage-stake-bonus scheme becomes less attractive for bad workers if their loss probability is higher. A higher q_b increases the bonus for both the constrained and unconstrained consumer because, as shown in the proof, the larger implied information rent (the first term in (8)) dominates the decrease of the stake (which enters the second term in (8)).

A higher loss probability of the good workers q_g reduces the information rent. This increases the stake for constrained workers ceteris paribus, as made explicit in (14) and (21). At the same time, a higher q_g also implies for a constrained consumer that the optimal stake that smooths consumption in (13) is smaller because the consumer has to pay back the stake to the worker with smaller probability in the second period. Either of the two effects may dominate. For unconstrained workers, the stake $S^{\min}$ is independent of q_g. The optimal bonus then decreases in q_g because the information rent becomes smaller (the first term in (8) depends negatively on q_g).

6 Discussion

6.1 Applications

As mentioned in the introduction, our model applies to peer-to-peer markets that co-exist with traditional, more centralized markets intermediated by large firms. We discuss two pertinent examples in turn.

Blockchain-based Organizations Blockchain-based decentralized autonomous organizations (DAOs) are an emerging institutional arrangement that allows for disintermediated economic exchanges with full anonymity and permissionless entry and exit of agents (see, e.g., Buterin, 2013; Braun and Haeusle, 2023). Such organizations operate without hierarchy or human management intermediating economic exchanges, and decision-making happens through the voting of participants. Business rules and governance mechanisms are stored in so-called smart contracts, pieces of software code that can be executed on the blockchain.

There are many DAOs that are peer-oriented and focus on services or freelancing. For instance, members of LexDAO provide legal services through a decentralized network of legal experts who participate in creating and enforcing digitally and legally compliant contracts on the blockchain. Other DAOs, like Dework, focus on a more general peer-to-peer decentralized work market, including Web3 developers, marketers, and designers. In these DAOs, freelancers can offer their skills and services in exchange for cryptocurrency payments. Clients can browse the services offered and engage freelancers, ensuring payment upon job completion through smart contracts. The freelancers are usually anonymous and distributed across the globe. This exacerbates adverse selection and limits traditional enforcement mechanisms. Furthermore, the economic exchanges are often on a one-off basis, which complicates relational contracting.

The insights from our analysis could allow to devise optimal smart contracts that mitigate adverse selection in such an environment and allow for consumption smoothing over time and across states. Indeed, the idea of putting down a stake is already very prevalent in blockchain-based platforms, albeit more related to the infrastructure by securing the network through a proof-of-stake consensus mechanism (e.g., Halaburda et al., 2022).

Online Labor Markets Online labor markets such as Fiverr, Amazon MTurk, Guru, or UpWork allow consumers, self employed or small businesses with financing constraints to source freelancers for a multitude of services, such as website design, data analysis or the writing of documents.[13] Whereas some gig-economy business models, such as ride sharing, require buyer and service provider to be co-located, many other tasks can be delivered remotely. Because production or service provision in online labor markets is disintermediated, tasks which require some time imply that consumption smoothing motives directly interact with worker incentives.[14]

The adverse selection issue examined in this paper is especially important for the typical gig-economy tasks mentioned above since worker quality is (at least partially) unobservable and ex ante information asymmetries are severe in an online setting, leading to high quality uncertainty (Autor, 2001; Dimoka et al., 2012; Stanton and Thomas, 2016, 2020). Reasons include the anonymity or pseudonymity of market participants, the shortcomings of existing signals such as reputation-based rating systems, and the dominance

[13] Despite its relatively small size, the potential of the gig economy is substantial. According to estimates by Blinder and Krueger (2013) for the U.S., 20-25% of occupations are suitable for online production. Moreover, the recent COVID-19 pandemic has accelerated the trend toward remote work, potentially ushering in a new era of globalization in services. See the interview with Nick Bloom in the "Future of Work" series of the Financial Times on December 22, 2021.

[14] Note that only production is disintermediated. Contrary to blockchain-based labor markets, traditional online markets themselves are intermediated through platform providers that match the transaction partners. The platform providers, however, typically neither produce the goods or services nor bear any liability for their quality.

of one-off assignments. For instance, Stanton and Thomas (2020) analyze data from five major English-speaking websites, representing almost three quarters of the online labor market, and find that more than half of all job postings between 2016 and 2020 did not strive for repeat transactions.

Furthermore, the recent boom of buy now pay later (BNPL) schemes on online labor market platforms, offered by firms such as Zip on Fiverr, underlines that liquidity-constrained individuals are present in online labor markets. Their current reliance on the services of BNPL intermediaries is very costly, because the latter collect sizeable fees for their services.[15]

Our analysis suggests that at least some of these worker-consumer pairs may find it beneficial to self-select into compensation schemes instead of using the BNPL services. This would allow these worker-consumer pairs to choose a compensation scheme without having to rely on costly financial intermediation and to benefit from different willingness between consumers and workers to shift payments across time. Put differently, bonus-stake schemes could be used to redistribute the margins currently earned by BNPL service providers either to the consumer-worker pair or the platform itself. We leave further analysis of these issues to future research.

6.2 Concluding Remarks

We analyze a novel joint principal-agent and consumption-smoothing problem, in which workers of unobservable quality (the agents) may transact with risk-averse and possibly borrowing constrained consumers (the principals). We show that the timing of compensation payments does not only affect the incentives of good-type workers to self select into jobs and for the bad-type workers to stay away, but also the extent of consumption smoothing achievable by consumers.

We characterize a separating equilibrium, in which high-quality workers self select into a compensation scheme that, at the same time, allows constrained consumers to shift resources for consumption across time and states. We characterize an implementation with a three-part compensation scheme. Our analysis shows that the optimal size of the variable payments in this compensation scheme depends on the characteristics of the consumer, the worker and the task. We argue that such compensation schemes are relevant in various contexts.

Future research could extend our analysis in several directions. From a theoretical perspective, one could investigate whether optimal compensation schemes can provide workers with insurance against the risk of task completion. Moral hazard, which prevents

[15] For the services on Fiverr, Zip, e.g., charges 4% of the transacted amount.

full insurance, in this case may not only interact with adverse selection but also with the consumption smoothing motive of the purchaser. From an empirical perspective, it would be interesting to provide evidence on the response of the demand for goods and services to the type of available payment schemes.

A Proofs

Proof of Lemma 1. By contradiction. Suppose there is a pooling equilibrium in which all workers are offered a compensation scheme (t_1, t_{2s}, t_{2f}) with $t_1 > t_{2s} = t_{2f} = 0$. In such an equilibrium, all workers apply to all firms, and each firm hires $\ell \leq L/N$ workers. Then, firm profits are

$$F(\ell)\left[\alpha + \frac{1-\alpha}{1+r}\right] - \ell\left[t_1 + \bar{q}\frac{L}{1+r}\right]. \tag{A.1}$$

If firm i can offer an additional compensation scheme (t_1', t_{2s}', t_{2f}') that leaves the good types slightly better off and that is accepted only by the good types, then the all the good types will apply with firm i while the bad types still distribute their applications uniformly across firms. In such a case, firm i can still hire at least ℓ workers. When doing so, it will optimally hire as many good workers as possible, and its profit will be

$$\begin{aligned} F(\ell)\left[\alpha + \frac{1-\alpha}{1+r}\right] \quad & - \max\{0, \ell - \phi\}\left[t_1 + q_b\frac{L}{1+r}\right] \\ & - \min\{\ell, \phi\}\left[t_1' + \frac{1}{1+r}\left[q_g t_{2f}' + (1-q_g)t_{2s}'\right] + q_g\frac{L}{1+r}\right]. \end{aligned}$$

So, we need to argue that an additional compensation scheme (t_1', t_{2s}', t_{2f}') exists such that this payoff is higher than the conjectured equilibrium payoff. Indeed, consider $\epsilon > 0$ and consider the scheme (t_1', t_{2s}', t_{2f}') with $t_{2f}' = 0$, satisfying

$$t_1' + \frac{1-q_g}{1+r}t_{2s}' = t_1 + \epsilon, \tag{A.2}$$

$$t_1' + \frac{1-q_b}{1+r}t_{2s}' < t_1. \tag{A.3}$$

Clearly, such a scheme exists whenever $\epsilon > 0$ is small enough because $q_b > q_g$ by assumption. The equality ensures that the good types are strictly better off under the alternative scheme, while the inequality ensures that bad types will not choose it. Under such a scheme, the profit of the firm becomes

$$F(\ell)\left[\alpha + \frac{1-\alpha}{1+r}\right] - \max\{0, \ell - \phi\}\left[t_1 + q_b\frac{L}{1+r}\right] - \min\{\ell, \phi\}\left[t_1 + \epsilon + q_g\frac{L}{1+r}\right],$$

which is strictly greater than (A.1) whenever ϵ is small enough, again because $q_b > q_g$ by assumption. ■

Proof of Lemma 2. There are three relevant equilibrium conditions in a separating equilibrium. First, incentive compatibility requires

$$t_{1b} + \frac{1}{1+r}\left[q_b t_{2fb} + (1-q_b)t_{2sb}\right] \geq t_{1g} + \frac{1}{1+r}\left[q_b t_{2fg} + (1-q_b)t_{2sg}\right] \tag{A.4}$$

$$t_{1g} + \frac{1}{1+r}\left[q_g t_{2fg} + (1-q_g)t_{2sg}\right] \geq t_{1b} + \frac{1}{1+r}\left[q_g t_{2fb} + (1-q_g)t_{2sb}\right]. \tag{A.5}$$

The first incentive compatibility constraint, (A.4), ensures that bad types report truthfully, while the second one, (A.5), ensures that good types report truthfully.

The third relevant equilibrium condition is the indifference constraint of the firm, requiring that the firms be indifferent between hiring either of the two types of workers. Indifference amounts to equal marginal costs of hiring a given type,

$$t_{1b} + \frac{1}{1+r}\left[q_b t_{2fb} + (1-q_b)t_{2sb} + q_b L\right] = t_{1g} + \frac{1}{1+r}\left[q_g t_{2fg} + (1-q_g)t_{2sg} + q_g L\right]. \tag{A.6}$$

To continue, we proceed in two steps. First, we show that to any menu of compensation schemes $\{(t_{1\theta}, t_{2s\theta}, t_{2f\theta})\}_{\theta=b,g}$ satisfying (A.4), (A.5), and (A.6), there is an alternative menu $\{(\hat{t}_{1\theta}, \hat{t}_{2s\theta}, \hat{t}_{2f\theta})\}_{\theta=b,g}$ that leaves the utilities of the firms and the workers unchanged, still satisfies the constraints, and satisfies $\hat{t}_{2sb} = \hat{t}_{2fg} = 0$. To this end, consider an alternative scheme $\{(\hat{t}_{1\theta}, \hat{t}_{2s\theta}, \hat{t}_{2f\theta})\}_{\theta=b,g}$ satisfying $\hat{t}_{2sb} = \hat{t}_{2fg} = 0$, $\hat{t}_{2fb} = t_{2fb}$, $\hat{t}_{2sg} = t_{2sg}$ and

$$\hat{t}_{1b} = t_{1b} + \frac{1-q_b}{1+r}t_{2sb}$$

$$\hat{t}_{1g} = t_{1g} + \frac{q_g}{1+r}t_{2fg}.$$

By construction, the expected payments to the workers remain unchanged, so the indifference condition (A.6) continues to hold. Moreover, observe that

$$\begin{aligned}\hat{t}_{1g} + \frac{1-q_b}{1+r}t_{2sg} &= t_{1g} + \frac{1}{1+r}\left[q_b t_{2fg} + (1-q_b)t_{2sg}\right] - \frac{q_b - q_g}{1+r}t_{2fg} \\ &< t_{1g} + \frac{1}{1+r}\left[q_b t_{2fg} + (1-q_b)t_{2sg}\right],\end{aligned}$$

where the inequality follows from $q_b > q_g$. Because the utility of the bad types is the same under the new menu as under the old menu, it follows from (A.4) that bad types do not have an incentive to mimic good types under the new menu, either. Analogously, we have

$$\hat{t}_{1b} + \frac{q_g}{1+r} t_{2fb} = t_{1b} + \frac{1}{1+r}\Big[q_g t_{2fb} + (1-q_g) t_{2sb}\Big] - \frac{q_b - q_g}{1+r} t_{2sb}$$
$$< t_{1b} + \frac{1}{1+r}\Big[q_g t_{2fb} + (1-q_g) t_{2sb}\Big],$$

where the inequality follows from $q_b > q_g$. Because the utility of the good types is the same under the new menu as under the old menu, it follows from (A.5) that bad types do not have an incentive to mimic good types under the new menu, either.

In a second step, we argue that from the menu $\{(\hat{t}_{1\theta}, \hat{t}_{2s\theta}, \hat{t}_{2f\theta})\}_{\theta=b,g}$ we can construct yet another menu $\{(\tilde{t}_{1\theta}, \tilde{t}_{2s\theta}, \tilde{t}_{2f\theta})\}_{\theta=b,g}$ that satisfies all relevant constraints and involves $\tilde{t}_{2fg} = \tilde{t}_{2sb} = \tilde{t}_{2fb} = 0$.

We know that under the menu $\{(\hat{t}_{1\theta}, \hat{t}_{2s\theta}, \hat{t}_{2f\theta})\}_{\theta=b,g}$ we have the following incentive compatibility constraints,

$$\hat{t}_{1b} + \frac{q_b}{1+r} \hat{t}_{2fb} \geq \hat{t}_{1g} + \frac{1-q_b}{1+r} \hat{t}_{2sg} \tag{A.7}$$

$$\hat{t}_{1g} + \frac{1-q_g}{1+r} \hat{t}_{2sg} \geq \hat{t}_{1b} + \frac{q_g}{1+r} \hat{t}_{2fb}. \tag{A.8}$$

and the indifference condition

$$\hat{t}_{1b} + \frac{1}{1+r}\Big[q_b \hat{t}_{2fb} + q_b L\Big] = \hat{t}_{1g} + \frac{1}{1+r}\Big[(1-q_g)\hat{t}_{2sg} + q_g L\Big]. \tag{A.9}$$

Now, suppose the menu $\{(\tilde{t}_{1\theta}, \tilde{t}_{2s\theta}, \tilde{t}_{2f\theta})\}_{\theta=b,g}$ equals $\{(\hat{t}_{1\theta}, \hat{t}_{2s\theta}, \hat{t}_{2f\theta})\}_{\theta=b,g}$ except for $\tilde{t}_{1b}$ and $\tilde{t}_{2fb}$. Specifically, consider $\tilde{t}_{2fb} = 0$ and

$$\tilde{t}_{1b} = \hat{t}_{1b} + \frac{q_b}{1+r} \hat{t}_{2fb}.$$

Clearly, the indifference condition continues to hold. In particular, we may re-write it as

$$\tilde{t}_{1b} = \tilde{t}_{1g} + \frac{1}{1+r}\Big[(1-q_g)\tilde{t}_{2sg} - (q_b - q_g)L\Big].$$

As the utility of the bad types remains the same and the transfers for the good types are unchanged, the bad types still have no incentive to misrepresent their type. To see that good types do not have an incentive to misrepresent their type, either, observe that we may use the above indifference condition to write

$$\tilde{t}_{1g} + \frac{1-q_g}{1+r}\tilde{t}_{2sg} = \tilde{t}_{1b} - \frac{1}{1+r}\Big[(1-q_g)\tilde{t}_{2sg} - (q_b - q_g)L\Big] + \frac{1-q_g}{1+r}\tilde{t}_{2sg}$$
$$= \tilde{t}_{1b} + \frac{q_b - q_g}{1+r}L > \tilde{t}_{1b}.$$

But this implies that good types fare strictly worse when pretending to be of bad type.

To sum up, we have shown that, to any menu of compensation schemes that satisfy incentive compatibility and firm indifference, there is an alternative menu of compensation schemes that leave the utilities of the firms and workers unchanged, still satisfy incentive compatibility and firm indifference, and involve zero contingent payments for bad types and no payment in case of failure for good types. ■

Proof of Proposition 2. In any separating equilibrium with both worker types being hired, all firms must be indifferent at the margin between hiring a good worker and a bad worker,

$$t_{1b} + \frac{1}{1+r}q_b L = t_{1g} + \frac{1}{1+r}\Big[(1-q_g)t_{2sg} + q_g L\Big]. \tag{A.10}$$

Rearranging yields the second equality in (4). Furthermore, incentive compatibility requires

$$t_{1b} \geq t_{1g} + \frac{1-q_b}{1+r}t_{2sg} \tag{A.11}$$
$$t_{1g} + \frac{1-q_g}{1+r}t_{2sg} \geq t_{1b}. \tag{A.12}$$

Combining (A.6) with (A.12), we obtain that (A.12) is satisfied with strict inequality whenever (A.10) holds. Combining (A.6) with (A.11) yields the first equality in (4).

To obtain (3), suppose first that

$$f(\ell^*)\left[\alpha + \frac{1-\alpha}{1+r}\right] < t^*_{1b} + q_b\frac{L}{1+r}.$$

The left side of the above inequality is the marginal gain from hiring another worker, whereas the right side is the marginal cost of hiring another worker (given the firm is indifferent between hiring good workers and hiring bad workers). By the concavity of F, each firm would then do strictly better by reducing the mass of workers it hires.

On the other hand, suppose

$$f(\ell^*)\left[\alpha + \frac{1-\alpha}{1+r}\right] > t^*_{1b} + q_b\frac{L}{1+r}.$$

Then, the firms could strictly gain by hiring more workers (that are of good type with probability ϕ). To attract both good-type and bad-type workers, the deviating firm needs to improve the contractual terms for both types while keeping separation. Consider $\epsilon_g > 0$ and $\epsilon_b > 0$ satisfying

$$(1+\epsilon_b)t_{1b}^* = (1+\epsilon_g)\left[t_{1g}^* + \frac{1-q_g}{1+r}t_{2sg}^*\right] - L\frac{q_b - q_g}{1+r}. \tag{A.13}$$

So, under any alternative compensation menu where the first-period transfer to the bad types is $t_{1b}^*(1+\epsilon_b)$, the first-period transfer to the good types is $t_{1g}^*(1+\epsilon_g)$, the second-period transfer in case of successful production is $t_{2sg}^*(1+\epsilon_g)$, and the second-period transfer in case of a failure is zero, the firm remains indifferent between the two worker types. The incentive constraints (A.11) – (A.12) continue to hold under such an alternative compensation scheme. Because for all sufficiently small $\epsilon_b > 0$,

$$f(\ell^*)\left[\alpha + \frac{1-\alpha}{1+r}\right] > t_{1b}^*(1+\epsilon_b) + q_b\frac{L}{1+r},$$

the deviating firm can strictly gain by increasing the mass of workers it hires despite marginally increasing payment. But then (3) follows.

Finally, we show that $\ell^* \leq W/N$ holds in any symmetric equilibrium. Clearly, $\ell^* > W/N$ is not feasible. So, suppose firms hire $\ell^* \leq W/N$. Lowering the payment to the workers is not possible as no one will apply. Increasing payment to the workers cannot strictly increase profits, because firms already choose optimally by (3).

To finish the proof, it remains to show that no firm ever wants to change its compensation menu in order to attract a particular type. If the firm makes the menu for a particular type slightly more attractive (i.e., if it pays more to a particular type) it will obtain all applications from these types. But then (A.10) ensures that hiring them is less attractive than hiring the other type. If the firm makes the menu less attractive for a particular type, it will not receive any applications from that type, which cannot increase profits. ■

Proposition 7. Concerning part (a) of the proposition, (14) and (21) imply that the optimal stake is decreasing in α if the consumer is constrained. (16) implies that the stake is independent of α if the consumer is unconstrained. (8) shows that these properties also apply to the bonus. Concerning part (b) of the proposition, differentiating (14) or (21) using (10) implies

$$\frac{\partial S^*}{\partial L} = \frac{\partial \tilde{S}^*}{\partial L} = -\frac{q_b - q_g}{2+r-q_g} < 0. \tag{A.14}$$

Substituting (14) or (21) into (8), we observe that the bonus depends positively on the loss for these optimal stakes given that $0 \leq r \leq 1$ and $0 \leq q_g \leq 1$.

Differentiating (16), we obtain

$$\frac{\partial S^{\min}}{\partial L} = \frac{1-q_b}{1+r} > 0. \tag{A.15}$$

Then (8) implies that also the bonus increases. ∎

Proposition 8. Note that the information rent (10) increases in q_b. Inspecting the optimal stakes characterized by (14), (16) or (21) then reveals that the optimal stake $S^\dagger$ decreases in q_b. Specifically, we obtain

$$\frac{\partial S^*}{\partial q_B} = \frac{\partial \tilde{S}^*}{\partial q_B} = -\frac{1}{2+r-q_g}L < 0 \text{ and } \frac{\partial S^{\min}}{\partial q_b} = -\frac{1}{1+r}L < 0.$$

Then, we obtain

$$\frac{\partial B^\dagger}{\partial q_B} = \frac{1}{1-q_g}L - \frac{q_g+r}{1-q_g}\frac{1}{2+r-q_g}L = \frac{1}{1-q_g}L\left[1-\frac{q_g+r}{2+r-q_g}\right] = \frac{2L}{2+r-q_g} > 0$$

for an interior optimum of the stake and

$$\frac{\partial B^\dagger}{\partial q_B} = \frac{L}{1+r} > 0$$

if $S = S^{\min}$.

Further, (16) implies that a change in the loss probability of good types q_g has no effect on the optimal stake for unconstrained consumers. The bonus in (8) then decreases in q_g as the information rent decreases. The effect for constrained consumers is more intricate. Differentiating (14) with respect to q_g, accounting for how the information rent in (10) depends on q_g, we obtain

$$\begin{aligned}\frac{\partial S^*}{\partial q_g} &= -\frac{1+r}{(2+r-q_g)^2}\left[(1-2\alpha)V + w^* - \Delta(L)\right] \\ &\quad + \frac{L}{2+r-q_g}.\end{aligned} \tag{A.16}$$

The sign of this derivative depends on the parameter values. Similarly, also the derivative of (21) can be positive or negative, depending on the parameter values. It follows that also the bonus in (8) may then decrease or increase in q_g. ∎

B Additional Equilibria

In case the condition (A1) holds, we know that the separating equilibrium, in which both types apply to jobs and each firm hires $\ell^* = W/N$ workers, is efficient. Here, we show that if condition (A1) fails,

$$f(W/N)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_b \frac{L}{1+r} < 0,$$

but the following inequality holds,

$$f(0)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_b \frac{L}{1+r} > 0,$$

then we can always construct an efficient equilibrium in which (i) the firms pay nothing to bad workers, (ii) they offer a bonus/stake scheme to good workers, and (iii) workers coordinate on the efficient composition of the workforce.[16] Moreover, such equilibria do not exist when (A1) holds with strict inequality. We distinguish three cases.

Case I: First, we consider the case

$$f(\phi W/N)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_g \frac{L}{1+r} \leq 0.$$

In this case, (5) implies that, in the social optimum, no bad-type workers are hired, $a_b = 0$, and good-type workers are hired such that $f(a_g)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_g \frac{L}{1+r} = 0$, where $a_g < \phi W/N$.

This workforce composition can be implemented in an equilibrium as follows: Firms pay zero $t_{i1\theta} = t_{i2f\theta} = t_{i2s\theta} = 0$, the bad types abstain from applying to jobs, and the good types coordinate such that each firm obtains exactly a_g applications. All workers are indifferent between applying and not applying, no firm wants to hire bad workers, and no firm wants to change the mass of good workers it hires.

Case II: Consider the case

$$f(\phi W/N)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_g \frac{L}{1+r} > 0$$

and

$$f(\phi W/N)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_b \frac{L}{1+r} < 0.$$

In this case, (5) implies that, in the social optimum, each firm hires $a_g = \phi W/N$ good workers, and does not hire any bad-type workers, $a_b = 0$.

[16] Such coordination among workers could be achieved by a union.

This workforce composition can be implemented in a competitive equilibrium as follows:

1. Firms pay zero $t_{i1b} = t_{i2fb} = t_{i2sb} = 0$ to bad workers;
2. Firms offer $t_{i2fg} = 0$ and any t_{i2sg} together with t_{i1g} satisfying
$$t_{i1g} + \frac{1}{1+r} t_{i2sg} = f(\phi W/N)\left[\alpha + \frac{1-\alpha}{1+r}\right];$$
3. All good-type workers apply for a job, and the bad types do not.

Under the offered contract, the good-type workers obtain a positive rent,

$$t_{i1g} + \frac{1}{1+r}(1-q_g) t_{i2sg} = f(\phi W/N)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_g \frac{L}{1+r} > 0,$$

and, because bad types are not offered any transfers, the good-type workers incentive compatibility constraint is satisfied. Further, the incentive compatibility constraint of the bad-type workers also holds, which follows from

$$\begin{aligned} t_{i1g} + \frac{1}{1+r}(1-q_b) t_{i2sg} &= t_{i1g} + \frac{1}{1+r}(1-q_g) t_{i2sg} - \frac{1}{1+r}(q_b - q_g) t_{i2sg} \\ &= f(\phi W/N)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_b \frac{L}{1+r} < 0. \end{aligned}$$

Last, no firm wants to hire any bad workers because

$$f(\phi W/N)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_b \frac{L}{1+r} < 0.$$

Also, no firm wants to change the mass of good workers it hires because

$$f(\phi W/N)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_g \frac{L}{1+r} = t_{i1g} + \frac{1}{1+r}(1-q_b) t_{i2sg}.$$

Case III: Consider the case

$$f(\phi W/N)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_g \frac{L}{1+r} > 0$$

and

$$f(\phi W/N)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_b \frac{L}{1+r} \geq 0.$$

In this case, (5) implies that, in the social optimum, each firm hires $a_g = \phi W/N$ good workers, and hires bad-type workers $a_b \geq 0$ such that $f(\phi W/N + a_b)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_b \frac{L}{1+r} = 0$.

This workforce composition can be implemented in an equilibrium as follows:

1. Firms pay zero $t_{i1b} = t_{i2fb} = t_{i2sb} = 0$ to bad workers;
2. Firms offer $t_{i2fg} = 0$ and any $t_{i2sg} > L$ together with $t_{1g} = (q_b - q_g)\frac{L}{1+r} - (1-q_g)\frac{t_{i2sg}}{1+r}$;
3. All good-type workers apply for a job, and the bad types coordinate such that each firm obtains exactly a_b applications.

Under the offered contract, the good-type workers obtain a positive rent,

$$t_{i1g} + \frac{1}{1+r}(1-q_g)t_{i2sg} = (q_b - q_g)\frac{L}{1+r} > 0,$$

and, because bad types are not offered any transfers, the good-type workers' incentive compatibility constraint is satisfied. Further, the incentive compatibility constraint of the bad-type workers also holds, which follows from

$$\begin{aligned} t_{i1g} + \frac{1}{1+r}(1-q_b)t_{i2sg} &= t_{i1g} + \frac{1}{1+r}(1-q_g)t_{i2sg} - \frac{1}{1+r}(q_b - q_g)t_{i2sg} \\ &= (q_b - q_g)\frac{1}{1+r}\left[L - t_{i2sg}\right] \leq 0. \end{aligned}$$

Last, no firm wants to change the mass of bad workers it hires because

$$f(\phi W/N + a_b)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_b\frac{L}{1+r} = 0.$$

Also, no firm wants to change the mass of good workers it hires because

$$f(\phi W/N + a_b)\left[\alpha + \frac{1-\alpha}{1+r}\right] - q_g\frac{L}{1+r} = (q_b - q_g)\frac{L}{1+r} = t_{i1g} + \frac{1}{1+r}(1-q_b)t_{i2sg}.$$

Now, obviously if (A1) holds and $\phi \in (0,1)$, then it always pays to hire some bad-type workers when all good-type workers are hired, and not offering any transfers to the bad types can never be an equilibrium, implying that the kind of equilibria discussed in the cases I–III above can never exist if (A1) holds with strict inequality.[17]

C Model Extensions

C.1 Risk-averse workers

In this appendix, we sketch how risk aversion of workers reduces the scope for consumption smoothing for the consumer. For this purpose, we show how the incentive compatibility constraints of risk-averse workers change, given the same implemented type of the

[17] If (A1) holds with equality, then the equilibrium described under Case III above coincides with that in the main text for $\ell^* = W/N$.

compensation scheme consisting of a wage, bonus and stake characterized in the main text.

To this end, we derive the slope of the incentive compatibility constraint of the workers in the (B,S)-space. Recall that the worker has to pay S in period 1 and that the worker receives the bonus B and the stake S in period 2 in the state without failure. Assuming, as before, that the worker discounts the future at rate r, risk aversion of workers modifies the incentive compatibility constraints (A.11) and (A.12), after substituting in the transfers specified in the implementation in Section 4.2.3, as follows:

$$u(x+w) \geq u(\omega x + w - S) + \frac{1-q_b}{1+r} u((1-\omega)x + B + S), \quad \text{(C.1)}$$

$$u(\omega x + w - S) + \frac{1-q_g}{1+r} u((1-\omega)x + B + S) \geq u(x+w), \quad \text{(C.2)}$$

where x denotes other resources that the worker may have access to and ω specifies the fraction of these resources that are available when the different payments of the compensation package are made. (C.1) is the incentive compatibility constraint for the risk-averse bad-quality workers and (C.2) is the incentive compatibility constraint for the risk-averse good-quality workers. Totally differentiating with respect to B and S, we thus obtain the combinations of the bonus and the stake that maintain incentive compatibility of the respective risk-averse worker type:

$$-u'(\omega x + w - S)dS + \frac{1-q_\theta}{1+r} u'((1-\omega)x + B + S)(dB + dS) = 0 \text{ , for } \theta = b, g. \quad \text{(C.3)}$$

The slope of the incentive compatibility constraint for type $\theta = b,g$ is thus

$$\frac{dB}{dS} = \frac{1+r}{1-q_\theta} \frac{u'(\omega x + w - S)}{u'((1-\omega)x + B + S)} - 1 \; . \quad \text{(C.4)}$$

For risk-neutral workers with constant marginal utility, the slope simplifies to $(1 + r)(1 - q_\theta) - 1$, as implied by the incentive compatibility constraints (A.11) and (A.12) of risk-neutral workers after substituting in the transfers specified in the implementation in Section 4.2.3. For risk-averse workers of type θ, the strictly concave utility function implies that a higher stake increases the marginal utility of consumption in period 1 (the marginal utility in the numerator of (C.4)) and it reduces the marginal utility in the state without failure in period 2 (the marginal utility in the denominator of (C.4)). Thus, a higher stake increases the slope of the incentive-compatibility constraint, implying a convex shape in the (B,S)-space. We now illustrate the implications of this convex shape for

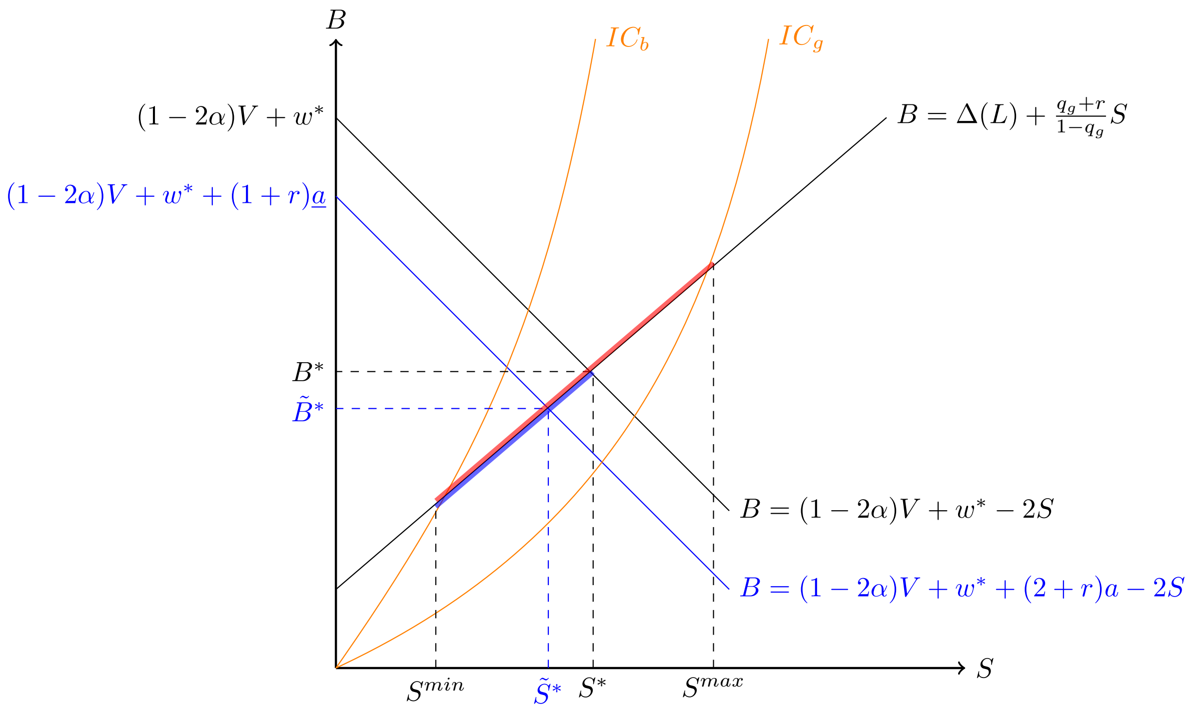

Figure 2: *Equilibrium bonus-stake combination with risk averse workers.* Explanation: The figure shows the feasible (S,B)-combinations that can be supported in equilibrium in red. Compared to the case with risk neutrality, the IC constraints are convex and upward sloping. This implies an upper bound $S^{\max}$ on the stakes that can be supported in equilibrium. See also the explanation to Figure 1.

our analysis, relative to the linear shape resulting for risk-neutral workers analyzed in the main text.

For illustration purposes, we assume in Figure 2 that the ratio of marginal utilities in (C.4) approaches one for a small stake and bonus, i.e., $\omega = 1/2$ and the wage is small relative to the other available resources x. Figure 2 shows that, because of the convexity of the incentive compatibility constraint for good workers, there are now additional stake-bonus combinations on the firms' indifference condition (8) that violate incentive compatibility, namely the good types' incentive compatibility constraint. Specifically, there is now a strictly positive $S^{\max}$ such that stakes $S > S^{\max}$ cannot be part of an equilibrium with risk-averse consumers. In other words, the presence of risk-averse workers shrinks the set of feasible (S,B)-schemes that can be supported in a symmetric equilibrium.

C.2 Commitment

We discuss how firms and consumers may commit based on the implementation of the optimal compensation scheme analyzed in Section 5. For a stake $S \leq w$, the stake resembles a trade credit or consumer credit. For $S > w$, there may be commitment issues similar to the literature on double moral hazard in the context of bonds issued by workers before executing work at a firm.

Posting of illiquid collateral of value $S - w$ by the consumer, for example by putting the collateral into an escrow account, may prevent reneging on the promised payments at the same time as the credit provides the borrowing constrained consumer with additional liquidity. In certain contexts, the execution of the credit contract can be fully committed to, for example by using smart contracts on a blockchain.

If the consumer has no collateral to post, the stake is bounded above by w. The maximization problems (11) without access to capital markets and (15) with access to capital markets then feature the additional constraint $S \leq w$. The optimal stake of the consumer without access to capital markets is thus given by

$$\tilde{S}^{\dagger} = \max\left\{S^{\min}, \min\left\{S^{*}, w\right\}\right\} \tag{C.5}$$

and the optimal stake of the consumer with access to capital markets is given by

$$\tilde{S}^{\ddagger} = \max\left\{S^{\min}, \min\left\{\tilde{S}^{*}, w\right\}\right\}. \tag{C.6}$$

Existence of a separating equilibrium, which ensures self selection of good workers, thus requires $w > S^{\min}$ in this case.

References

Autor, D. (2001). Wiring the labor market. *Journal of Economic Perspectives*, 15(1):25–40.

Becker, G. S. and Stigler, G. J. (1974). Law enforcement, malfeasance, and the compensation of enforcers. *Journal of Legal Studies*, 3:1–18.

Bertola, G., Hochguertel, S., and Koeniger, W. (2005). Dealer pricing of consumer credit. *International Economic Review*, 46(4):1103–1142.

Blinder, A. and Krueger, A. B. (2013). Alternative measures of offshorability: a survey approach. *Journal of Labor Economics*, 31(2):S97–S128.

Braun, A. and Haeusle, N. (2023). Collusion-proof decentralized autonomous organizations. *SSRN*.

Buchak, G. (2024). Financing the gig economy. *The Journal of Finance*, 79(1):219–256.

Burdett, K. and Mortensen, D. T. (1981). Testing for ability in a competitive labor market. *Journal of Economic Theory*, 25(1):42–66.

Burtch, G., Carnahan, S., and Greenwood, B. N. (2018). Can you gig it? An empirical examination of the gig economy and entrepreneurial activity. *Management Science*, 64(12):5497–5520.

Buterin, V. (2013). A Next-Generation Smart Contract and Decentralized Application Platform. *Ethereum White Paper*, ethereum.org.

Davoodalhosseini, S. M. (2019). Constrained efficiency with adverse selection and directed search. *Journal of Economic Theory*, 183:568–593.

Dimoka, A., Hong, Y., and Pavlou, P. A. (2012). On product uncertainty in online markets: Theory and evidence. *MIS Quarterly*, 36(2):395–426.

Dohmen, T. and Falk, A. (2011). Performance pay and multidimensional sorting: Productivity, preferences, and gender. *American Economic Review*, 101(2):556–590.

Eaton, C. and White, W. D. (1982). Agent compensation and the limits of bonding. *Economic Inquiry*, 20:330–343.

Federal Reserve System (2018). Report on the economic well-being of U.S. households in 2018. Technical report, Board of Governors of the Federal Reserve System.

Gathergood, J., Guttman-Kenney, B., and Hunt, S. (2018). How do payday loans affect borrowers? Evidence from the U.K. market. *The Review of Financial Studies*, 32(2):496–523.

Guerrieri, V., Shimer, R., and Wright, R. (2010). Adverse selection in competitive search equilibrium. *Econometrica*, 78(6):1823–1862.

Halaburda, H., Sarvary, M., and Haeringer, G. (2022). *Beyond Bitcoin*. Springer.

Hoffmann, F., Inderst, R., and Opp, M. (2021). Only time will tell: A theory of deferred compensation. *The Review of Economic Studies*, 88:1253–1278.

Huang, N., Burtch, G., Hong, Y., and Pavlou, P. A. (2020). Unemployment and worker participation in the gig economy: Evidence from an online labor market. *Information Systems Research*, 31(2):431–448.

ILO (2021). *World Employment and Social Outlook: The role of digital labour platforms in transforming the world of work*. International Labour Organization, Geneva.

Jappelli, T. and Pistaferri, L. (2017). *The Economics of Consumption*. Oxford University Press.

Kaplan, G., Violante, G. L., and Weidner, J. (2014). The wealthy hand-to-mouth. *Brookings Papers on Economic Activity*, 45(1):77–153.

Kouvelis, P. and Zhao, W. (2012). Financing the newsvendor: Supplier vs. bank, and the structure of optimal trade credit contracts. *Operations Research*, 60(3):566–580.

Lazear, E. P. (1979). Why is there mandatory retirement? *Journal of Political Economy*, 87(6):1261–1284.

Lazear, E. P. (1981). Agency, earnings profiles, productivity, and hours restrictions. *American Economic Review*, 71:606–620.

Lazear, E. P. (2000). Performance pay and productivity. *American Economic Review*, 90(5):1346–1361.

Li, H., Peng, J., Li, X., and Stallaert, J. (2022). When more can be less: The effect of add-on insurance on the consumption of professional services. *Information Systems Research*.

Liang, C., Peng, J., Hong, Y., and Gu, B. (2022). The hidden costs and benefits of monitoring in the gig economy. *Information Systems Research*.

Oyer, P. and Schaefer, S. (2011). Personnel economics: Hiring and incentives. In Ashenfelter, O. and Card, D., editors, *Handbook of Labor Economics*, volume 4B, chapter 20, pages 1769–1823. North Holland, Elsevier.

Prescott, E. C. and Townsend, R. M. (1984). Pareto optima and competitive equilibria with adverse selection and moral hazard. *Econometrica*, 52(1):21–46.

Ritter, J. A. and Taylor, L. J. (1994). Workers as creditors: Performance bonds and efficiency wages. *American Economic Review*, 84(3):694–704.

Rui, H. and Lai, G. (2015). Sourcing with deferred payment and inspection under supplier product adulteration risk. *Production and Operations Management*, 24(6):934—-946.

Salop, J. and Salop, S. (1976). Self-selection and turnover in the labor market. *The Quarterly Journal of Economics*, 90(4):619–627.

Shapiro, C. and Stiglitz, J. E. (1984). Equilibrium unemployment as a worker discipline device. *American Economic Review*, 74(3):433–444.

Smith, J. K. (1987). Trade credit and informational asymmetry. *Journal of Finance*, 42(4):25–40.

Stanton, C. T. and Thomas, C. (2016). Landing the first job: The value of intermediaries in online hiring. *The Review of Economic Studies*, 83(2):810–854.

Stanton, C. T. and Thomas, C. (2020). The gig economy beyond local services and transportation. *CESifo Forum*, 21(6):21–26.

Starmans, J. (2024). Contracting and search with heterogeneous principals and agents. *Journal of Economic Theory*, 217:105761.

Stegman, M. A. (2007). Payday lending. *Journal of Economic Perspectives*, 21(1):169–190.

Tufano, P. (2009). Consumer finance. *Annual Review of Financial Economics*, 1:227–247.