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INTERNATIONAL FINANCIAL TRANSMISSION OF THE US MONETARY POLICY: AN EMPIRICAL ASSESSMENT

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International Financial Transmission of the US Monetary Policy: An Empirical Assessment[*]

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Abstract

This paper proposes a way to study the transmission mechanism of the US monetary policy to foreign yield curves. It elaborates the high-frequency identification of monetary policy shocks from Piazzesi (2005) in an international setting and uses a sample of 125 policy rate decisions of the Fed to extract *realised* policy shocks. The Fed decisions span from February 1994 to December 2008 and are divided according to the direction of the policy rate move and weather they were anticipated by the Fed funds futures market. A consistent, two-country term structure model is estimated on daily data and used to assess both instantaneous and lagged reaction of foreign interest rates and forward term premia to the Fed policy rate decisions. Empirical analysis of the US - UK model shows that the most of the movement in the UK yields around policy action days results from estimated term premia. A surprise policy action seems to produce a spike in the UK premia around the short- and mid-range maturities, independently from the direction of the policy rate move. The estimated lagged reaction of the UK yields to a policy decision of the Fed is also negative, after both hikes and cuts of the policy rate. The results hold for different market price of risk specifications, after two robustness checks and for both two-country and single-country model output.

Keywords: term premia, two countries, Fed, policy actions, principal components

JEL Classifications: E43, E52, C11, G12, F31

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1 Introduction

Increasingly integrated financial markets are one of the key transmission channels of international macroeconomic and monetary shocks.[1] The transmission mechanism of the US monetary policy is particularly researched, where usually a vector autoregression (VAR) - type analysis is used to enhance our understanding of how monetary policy affects equity markets[2], interest rates[3] or both[4].

Yet, as pointed out in Cochrane and Piazzesi (2002), the VARs may not be sufficiently flexible to accommodate the time-varying preferences of the Fed, nor able to provide a solid identification of the Fed's reaction to the interest rates from the interest-rate reactions to the Fed. Consequently, a high-frequency identification strategy from Piazzesi (2005), together with monetary policy shocks extracted from the state variables' residuals around policy action days, can be used to analyse the impact of the US monetary policy decisions on foreign interest rates and term premia. The Fed decisions in the sample are split into two different groups, depending on the direction of the policy rate move and wheater the move was anticipated.[5] Allowing for different types of policy actions to represent different policy shocks, explicitly accounts for possible asymmetries in yields response mentioned in Bernanke and Kuttner (2005). The VAR framework might not be able to capture such asymmetries.

The model used in the assessment is a two-country Gaussian term structure model with observable risk factors from Joslin, Singleton and Zhu (2011) (JSZ).[6] Given the reduced-form nature of the model, the two economies are "connected" through the exchange rate between them. Following Backus, Foresi and Telmer (2001) and Dong (2006), both pricing kernels are defined and the implied depreciation rate is used to confirm that the model satisfies the widely acknowledged empirical finding[7] according to which the high interest rate currencies tend to appreciate. Fama (1984) attributes such behavior of the exchange rates to the time-varying risk premium and imposes two necessary conditions, which the proposed model satisfies.

The idea of identifying US monetary policy shocks to a foreign yield curve is illustrated on the UK term structure of interest rates. The UK is one of the main

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[1] See Canova (2005), Cooley and Quadrini (2001) and Ehrmann and Fratzscher (2006).

[2] See Bernanke and Kuttner (2005) and Ehrmann and Fratzscher (2004).

[3] See Taylor (1995) and Canova (2005).

[4] See Rigobon and Sack (2004).

[5] Following the ideas in Kuttner (2001).

[6] The previous studies that modelled the JSZ canonical form in a multi-country setting are Graveline and Joslin (2011), Jotikasthira, Le and Lundblad (2010) and Bauer and de los Rios (2011).

[7] See Hansen and Hodrick (1983), Fama (1984), Cumby and Obstfeld (1985), Hodrick (1987), Engel (1996), Bansal (1997), Dong (2006) and Graveline (2006) among others.

trading partners of the US[8] and the financial markets between the two countries are arguably highly integrated.[9] Thus the US and the UK yield curves are jointly fitted and every one-day change of the yields, following a policy rate decision of the Fed, is decomposed to expected future short-rate change and the term premia change.

There are arguably three important conclusions of this study. First, the estimated UK term premia are especially responsive to the Fed decisions. The forward term premia seem to be the main driver of economically interesting variation in yields around policy days and in particular, around anticipated increases in the Federal funds rate. Widely expected increase in the Fed funds rate is estimated to cause a decline in the medium and long-term yields, as the implicit term premia fall.[10]

Secondly, the average estimated reaction of the short- and medium term UK premia to a surprise decision of the Fed seem to be positive and independent of the direction of the policy rate move. The result is independent of the market price of risk specification, i.e. of weather some or all the "risks" (namely level, slope and curvature) in the economy are priced in the yield curve. They also hold after using the single-country model for the UK, instead of the two-country model in the assessment, and after performing two additional robustness checks.

Finally, dynamic response of the UK yield curve to the Fed funds rate decisions is estimated to be negative, again independently of weather the Fed delivered an interest rate hike or cut. The response of the UK long-term yield to a surprise cut indicates that the estimated instantaneous rise in the term premia is short-lived and quickly followed by a decline in the yield. Interest rate hikes seem to provoke a parallel movement of the UK curve, while the interest rate cuts of the US policy rate are estimated to cause the steepening of the UK curve.

The rest of the paper is organised as follows. The next Section illustrates the dataset and explains how the Fed decisions are split. Section 3 introduces the model and the Fama (1984) conditions, while the estimation details could be found in Section 4. Finally, Section 5 discusses the results and Section 6 concludes.

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[8]Source: U.S. Census Bureau, Foreign Trade Statistics.

[9]See for example Fraser and Oyefeso (2005).

[10]This result is very much in line with Favero and Giavazzi (2008), who estimate a negative response of the interest rates in the Euro area to monetary policy tightening in the US. In contrast to this, Canova (2005) estimates that a contractionary US monetary shock induces an instantaneous increase in Latin American interest rates.

2 Dataset

2.1 Yields

The dataset spans from the beginning of January 1994 to the end of December 2008 and contains 3912 daily observations of the 6-month U.S. Dollar and G.B. Pound Libor rates, and plain vanilla fixed-for-floating interest rate swap rates from the two countries with maturities of 2, 3, 5, 7 and 10 years.[11] All the yields are converted to continuously compounded assuming semi-annual compounding.[12] The two curves are illustrated in Figure 1.

On the short end, the 6-month Libor rates are corrected for the consequences of the credit disruption initiated in August 2007 and lasted until the end of the sample. For this time period, I simply use the 6-month Overnight Indexed Swap (OIS) rates in two currencies plus the average OIS - Libor spread for the entire sample.[13] In such a way, the short rate in the sample reflects the average credit conditions throughout the sample and excludes the spike in the Libor rates after the Lehman Brothers bankruptcy. During the considered time period, there were indeed several other episodes with particularly tight credit conditions in both the U.S. and the U.K., most notably the "Asian crisis" in July 1997, the "Russian crisis" in August 1998 and the "Dot-com bubble" burst in early 2000. Yet, on all these occasions there was no significant divergence of the Libor rates from the respective OIS rates in the two countries, nor from the respective Treasuries securities' yields.

Regarding the mid- and longer-term maturities, the swap rates are used mainly for two reasons. First, they are often regarded as "true" constant maturity yield data[14] and thus not a subject to approximation error of bootstrapping and interpolation techniques. In addition, the swap rates imply a limited credit risk premium, as in most cases only the intermediate cash-flows are exchanged. The preliminary data inspection shows that the spread, as much as the change in the spread, between the swap rates and off-the-run treasuries (in the U.S.) and the gilts (in the U.K.) of the corresponding maturity is minor, also around the Lehman Brothers bankruptcy and the subsequent credit disruption in October 2008.

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[11]The Libor rates are obtained from daily fixings by the British Bankers Association while the swap rates are indicative mid-quotes averaged across many data providers. Both series are available on Bloomberg and the fixing time for the swap rates is set to 17:00 hours New York time.

[12]See Hull (2008).

[13]The OIS rates are also available on Bloomberg from beginning of 2001. The average OIS - Libor spread in the U.S. case was 11 basis points, and in the U.K. case 29 basis points.

[14]See Dai and Singleton (2000).

2.2 Fed policy actions

The dataset includes 125 policy meetings of the Federal Open Market Committee (FOMC) that resulted in an interest rate decision.[15] The starting policy action was an interest rate hike delivered on the 4th of February 1994. With this particular decision, the Fed started communicating the policy rate at the end of each meeting and the procedure has not been changed ever since.[16] The last decision in the sample was made on the 16th December 2008 in the midst of the recent financial crisis, when the Fed decided to cut the reference rate by 75 basis points to the target range 0 - 1/4 percent.

Out of 125 FOMC meetings, 15 decisions are identified as "surprise changes" of the Federal Funds target rate. Following Kuttner (2001)[17], I first construct a measure of the "surprise element" in Federal Funds target changes using the Federal Funds futures data from Chicago Mercantile Exchange. Secondly, different policy actions are characterised as expected or unexpected.

In the construction of the policy surprise indicator, the change in the Fed target rate implied by the current-month futures contract on (monthly) average Federal Funds rate is considered. For a Fed decision that took place at day d of the month m, the unexpected change in the policy rate, scaled up by the factor that takes into account the number of days in the month affected by the change is calculated as:

$$\Delta i^{unexpected} = \frac{D}{D-d}(F_{m,d} - F_{m,d-1}) \tag{1}$$

where D is the number of days in the current month and $F_{m,d}$ is the Fed Funds rate implied by the current-month futures contract value. If a policy decision was widely expected, the above change should be close to zero. In order to minimise the effect of month-end noise, I calculate an unscaled change for any decisions that came in place in the last 10 calendar days of any month.[18] Results are shown in Table 1 in the Appendix.

Once constructed the surprise index, a "surprise change" is considered to be any difference calculated in (1) that exceeds a two thirds of the usual 25 basis points move in any direction, namely under -16 and above +16 basis points.

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[15]During the period, the FOMC delivered 126 policy rate decisions, out of which the interest rate cut delivered on the 8th of October 2008 was coordinated with, among others, the Bank of England (BoE). Consequently, this particular decision is excluded from the set.

[16]See Piazzesi (2005) and Gurkaynak, Sack and Swanson (2005). The starting date in the sample has been chosen accordingly.

[17]See also Bernanke and Kuttner (2005) and Gurkaynak et al. (2005).

[18]Kuttner (2001) proposes 3 days for the same purpose. 10 days are chosen to bring the measure closer to what previous studies using the tick-by-tick data produced, most notably Fleming and Piazzesi (2005).

The two-thirds threshold was chosen as an arguably reasonable portion of the usual policy move, above which the move might be considered as a surprise one.[19] Out of 125 policy actions, 15 decisions are classified as "surprise moves" of the Fed.

Specifically, out of 31 decisions opting for an interest rate hike, four seem to have surprised the markets. The three of them were brought in 1994 and one was delivered on 22nd of March 2005, in a series of rate hikes lasting from June 2004 to June 2006. Two decisions are considered as unexpected holds, namely the one delivered on the 19th of March 2002 and the one on the 18th of September 2008. The remaining 9 policy actions are considered as surprise target rate cuts and are equally spaced between the dot-com crisis at the beginning of 2000's and the sub-prime crisis. Roughly half of these are delivered after an unscheduled meeting of the Fed.[20] There were overall 28 Fed decisions to cut the target rate.

To illustrate the splits, Figure 2 reports the histograms of the size of policy rate changes for single "types" of policy decisions. It shows that the moves larger than 25 basis points tend to be classified as surprise moves, especially for the interest rate cuts. Also to notice is that expantionary policy decisions seem more likely to come at a surprise and that those decisions in the sample were on average higher in magnitude than the hiking decisions. Figure 3 illustrates the distributions of the surprise indicator, again conditional on different policy actions. The magnitude of the indicator seem to be again much higher around the policy rate cuts. This might not be surprising, as the decisions to cut the policy rate are usually delivered in times of elevated uncertainty and sometimes after an unscheduled meeting.[21]

Finally, a brief comment regarding the FOMC decision on the 16th of December 2008, when the policy rate reached the target range 0 - 25 basis points, is warranted. It seems that the futures market was actually "surprised" only by the magnitude of this final rate cut, where the scaled one day changed of the Fed futures was 35 basis points. Considering the size of the reserve balances of depository institutions at Federal Reserve banks at that time, the amount of monetary easing seem to have front-run the effective Federal funds rate.[22] For this reason, I do not consider this last Fed decision in the sample as a "surprise move" and re-classifying it to "anticipated" does not change the results.

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[19]Altering the threshold to 13 bp (assuming a "more-then-a-half" rule) makes the Fed funds cuts from 2nd October 2001 and 6th November 2001 become surprise cuts. Changing the cut-off around 16bp also does not alter the split significantly. The key results remain in both cases. Finally, increasing the cut-off to one entire move (25bp) would classify only few decisions as surprise.

[20]Namely, on the 15th of October 1998, 3rd of January 2001, the 18th of April 2001, the 17th of September 2001, and the 22nd of January 2008.

[21]The highest reading of the indicator of minus 68 basis points followed from the unscheduled meeting of the Federal Open Market Committee (FOMC) on the 22nd of January 2008, the details are reported in Table 1.

[22]See Taylor (2010).

3 Model

The following Section presents the two-country model where the home country (e.g. the United States) market prices of risk are priced into foreign bond markets (e.g. the United Kingdom). The key assumption is that the financial markets are perfectly integrated[23] and complete[24].

3.1 General Pricing Equation

Let $B_{n,t}^F$ be the price of an n-days-to-maturity bond denominated in foreign currency (e.g. British Pounds) at time t. The general pricing formula reads:

$$B_{n,t}^F = E_t \left[M_{t+1}^F B_{n-1,t+1}^F \right] \tag{2}$$

where M_{t+1}^F is a minimum variance and strictly positive stochastic discount factor (SDF) in the foreign country. In a risk-neutral world where investors request no risk compensation, the price of the bond $B_{n,t}^F$ equals:

$$B_{n,t}^F = E_t^Q \left[exp(-y_{1,t}^F) B_{n-1,t+1}^F \right] \tag{3}$$

and $y_{1,t}^F$ is the one-period interest rate. If the bond market in the foreign country is opened to home investors, the same bond denominated in domestic currency (e.g. US Dollars) follows:

$$B_{n,t}^F S_t = E_t \left[M_{t+1}^H B_{n-1,t+1}^F S_{t+1} \right]$$

where S_t is the exchange rate (e.g. the amount of US dollars for one British pound) and M_{t+1} is the home country minimum variance SDF. We can rearrange the above equation as:

$$B_{n,t}^F = E_t \left[M_{t+1}^H \frac{S_{t+1}}{S_t} B_{n-1,t+1}^F \right] \tag{4}$$

Intuitively, the home country risk factors, together with the adequate depreciation rate S_{t+1}/S_t, are priced in foreign bonds, as long as all the bonds and cur-

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[23]See Brennan and Xia (2006) and Dong (2006).
[24]See Brandt and Santa-Clara (2002) for implications of the incomplete-markets assumption.

rencies can be traded. To preclude arbitrage opportunities in the international markets, the bond prices in (2) and (4) need to be equal, i.e. it must be that:[25]

$$\frac{M_{t+1}^F}{M_{t+1}^H} = \frac{S_t}{S_{t+1}}$$

or expressed in logs:

$$\Delta s_{t+1} = m_{t+1}^H - m_{t+1}^F \tag{5}$$

The relation in (5) basically states that, if the minimum variance SDFs in the two countries exist and if the no-arbitrage assumption holds, the implied expected depreciation rate can be derived from the two SDFs. Exchange rate dynamics are completely driven by the factors which determine the SDFs' dynamics. Put differently, one out of three random variables, m_{t+1}^H, m_{t+1}^F and Δs_{t+1} is redundant and can be constructed from the other two.

Following Backus et al. (2001) and Dong (2006), I define the two pricing kernels and use the implied depreciation rate to perform a sort of "model consistency" check, namely weather the time-varying forward risk premium (implicit in the model-generated depreciation rate) satisfies the Fama (1984) conditions. Details are reported in Section A of the Appendix.

3.2 Mechanics

3.2.1 Setting

Following Duffie and Kan (1996) and Graveline and Joslin (2011), the short interest rates in two countries are affine functions of Z-dimensional risk factors X_t^H and X_t^F: [26]

$$\begin{bmatrix} y_{1,t}^H \\ y_{1,t}^F \end{bmatrix} = \begin{bmatrix} \rho_{0X}^H \\ \rho_{0X}^F \end{bmatrix} + \begin{bmatrix} \rho_{1X}^H & 0 \\ 0 & \rho_{1X}^F \end{bmatrix} \begin{bmatrix} X_t^H \\ X_t^F \end{bmatrix} \tag{6}$$

where ρ_{0X}^C, $C = \{H, F\}$ is a scalar proportional to the average long-run one-period yield, ρ_{1X}^C is a $1 \times Z$ vector of loadings of state variables on $y_{1,t}^C$, and 0 is a

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[25]Backus et al. (2001) derive this relation under the complete market assumption. See Brandt and Santa-Clara (2002) for the case when markets are incomplete.

[26]Naturaly, the ideas also apply to a model with different number of state variables in single countries. Most of the notation is taken from Joslin et al. (2011).

$1 \times Z$ vector of zeros. The state variables follow an AR(1) process under the risk neutral measure $\mathbb{Q}$:

$$
\begin{bmatrix} X_{t+1}^H \\ X_{t+1}^F \end{bmatrix} = \begin{bmatrix} K_{0X}^{H,\mathbb{Q}} \\ K_{0X}^{F,\mathbb{Q}} \end{bmatrix} + \begin{bmatrix} K_{1X}^{H,\mathbb{Q}} & 0 \\ 0 & K_{1X}^{F,\mathbb{Q}} \end{bmatrix} \begin{bmatrix} X_t^H \\ X_t^F \end{bmatrix} + \begin{bmatrix} \Sigma_X^H & 0 \\ 0 & \Sigma_X^F \end{bmatrix} \begin{bmatrix} \varepsilon_t^{H,\mathbb{Q}} \\ \varepsilon_t^{F,\mathbb{Q}} \end{bmatrix} \tag{7}
$$

where $K_{1X}^{C,\mathbb{Q}}$ is the feedback matrix, Σ_X^C is the variance-covariance matrix of the normally distributed error term $\varepsilon_t^{C,\mathbb{Q}} \sim N(0,1)$. The zero restrictions in equations (6) and (7) have two important implications. First, the state variables in single countries under $\mathbb{Q}$ drive one-period yields in those countries only. As the short-rate is closely related to the monetary policy rate instrument, the zero restrictions on the one-period loadings intuitively imply, that the respective monetary policy makers mostly regard domestic variables of interest, when delivering a policy rate decision.

Secondly, the co-movement between the risk factors in two countries is not allowed under the $\mathbb{Q}$ measure. Both implications result in single countries cross-sections of yields being driven by the domestic state-variables only. An obvious disadvantage of it is that the model can not accomodate common risk factors for the two countries yield curves.[27] Yet the zero restrictions in (6) and (7) might be necessary for econometric identification of the model.[28] Furthermore, they prove to be useful in the analysis presented here, as the model assigns a minor role of the US factors in explaining the UK yields and term premia. Since one would expect that the US yields (and not the UK yields) are particularly responsive to the Fed decisions, minimising their role in the model might offer more "conservative" results in assessing the reaction of the UK yield curve to policy rate decisions in the US.

Combining the two equations and assuming joint log-normality of the stochastic discount factor and the bond prices in the general pricing equation (2), it can be shown that the n-days to maturity zero-coupon yields in the two countries are functions of the respective state variables:

$$
\begin{bmatrix} y_{n,t}^H \\ y_{n,t}^F \end{bmatrix} = \begin{bmatrix} A_{n,X}^{H,\mathbb{Q}} \\ A_{n,X}^{F,\mathbb{Q}} \end{bmatrix} + \begin{bmatrix} B_{n,X}^{H,\mathbb{Q}} & 0 \\ 0 & B_{n,X}^{F,\mathbb{Q}} \end{bmatrix} \begin{bmatrix} X_t^H \\ X_t^F \end{bmatrix} \tag{8}
$$

where:

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[27]Diebold and Li (2006) for example find a strong empirical support of a common level factor in international bond markets. See also Leippold and Wu (2002) and Dong (2006).

[28]A model with common factors across countries might not be econometrically identified in the JSZ setting. An informal illustration is provided in the B Section of the Appendix.

$$B_{n,X}^{C,Q} = K_{1X}^{C,Q} B_{n-1,X}^{C,Q} - \rho_{1X}^C$$

$$A_{n,X}^{C,Q} = K_{0X}^{C,Q} B_{n-1,X}^{C,Q} + \frac{1}{2}(B_{n-1,X}^{C,Q})' \Sigma_X^C B_{n-1,X}^{C,Q} + A_{n-1,X}^{C,Q} - \rho_{0X}^C$$

Differently from $\mathbb{Q}$ dynamics, the pricing factors' under physical measure $\mathbb{P}$ are allowed to co-move. Once more, the state variables follow the AR(1) process:

$$\begin{bmatrix} X_{t+1}^H \\ X_{t+1}^F \end{bmatrix} = K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} \begin{bmatrix} X_t^H \\ X_t^F \end{bmatrix} + \Sigma_X^{\mathbb{P}} \begin{bmatrix} \varepsilon_t^{H,\mathbb{P}} \\ \varepsilon_t^{F,\mathbb{P}} \end{bmatrix} \tag{9}$$

where the upper-left and the lower-right blocks of the matrix $\Sigma_X^{\mathbb{P}}$ are equal to the matrices Σ_X^H and Σ_X^F, respectively. Yet the off-diagonal blocks of $K_{1X}^{\mathbb{P}}$ and $\Sigma_X^{\mathbb{P}}$ matrices are no more zero matrices.[29] Allowing for co-movement between the pricing factors from two countries under $\mathbb{P}$ implies that the risk factors in one country affect the shape of market prices of risk[30] in the other:

$$\Lambda_t^X = \left(K_{0X}^{\mathbb{P}} - \begin{bmatrix} K_{0X}^{H,Q} \\ K_{0X}^{F,Q} \end{bmatrix} \right) + \left(K_{1X}^{\mathbb{P}} - \begin{bmatrix} K_{1X}^{H,Q} & 0 \\ 0 & K_{1X}^{F,Q} \end{bmatrix} \right) \begin{bmatrix} X_t^H \\ X_t^F \end{bmatrix} \tag{10}$$

As we will see in Section 3.2.3, the forward term premia in the foreign country will be consequently driven by the domestic risk factors also. Explicitly accounting for "shared risks" in international term premia is arguably in line with what previous studies have estimated.[31]

Finally, the market prices of risk process in (10) can be constrained to allow for a small number of "priced" risk factors. Specifically, Cochrane and Piazzesi (2008) show that the level shock only is priced in yields, while Joslin et al. (2010) argue that both level and slope factors are responsible for driving excess returns in bond yields.

3.2.2 JSZ Rotation

This section describes how the Joslin et al. (2011) (JSZ) rotation is implemented in a two-country setting to obtain a canonical term structure model with *observed* yield factors. The rotation proves to be especially useful in estimation, where

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[29] As in Graveline and Joslin (2011).
[30] See Joslin, Priebsch and Singleton (2010).
[31] See for instance Sutton (2000) and Kose, Otrok and Whiteman (2003).

even standard maximum likelihood algorithms converge to the global optimum almost instantaneously.[32] Let $\mathcal{R}_t^H$ and $\mathcal{R}_t^F$ be the vectors of rotated cross sections of domestic and foreign yields Y_t^H and Y_t^F in time t as:

$$\begin{bmatrix} \mathcal{R}_t^H \\ \mathcal{R}_t^F \end{bmatrix} = \begin{bmatrix} W_{N\times N}^H & 0_{N\times N} \\ 0_{N\times N} & W_{N\times N}^F \end{bmatrix} \begin{bmatrix} Y_t^H \\ Y_t^F \end{bmatrix}$$

where N is the number of maturities in the term structures and $W_{N\times N}^C$, $C = \{H, F\}$ is a full-rank matrix of loadings obtained from an eigenvalue decomposition of the variance-covariance matrix of yields.[33] Assume that the first Z principal components explain the most of the variation in the cross-section of the yields in the domestic and the foreign country, respectively:

$$\begin{bmatrix} \mathcal{P}_t^H \\ \mathcal{P}_t^F \end{bmatrix} = \mathbf{W} \begin{bmatrix} Y_t^H \\ Y_t^F \end{bmatrix}$$

with:

$$\mathbf{W} = \begin{bmatrix} W_{Z\times N}^H & 0_{Z\times N} \\ 0_{Z\times N} & W_{Z\times N}^F \end{bmatrix}$$

Pre-multiply the equation (8) for the entire cross-section of yields with the rotation matrix $\mathbf{W}$:

$$\begin{bmatrix} \mathcal{P}_t^H \\ \mathcal{P}_t^F \end{bmatrix} = \mathbf{W} \begin{bmatrix} A_X^{H,\mathbb{Q}} \\ A_X^{F,\mathbb{Q}} \end{bmatrix} + \mathbf{W} \begin{bmatrix} B_X^{H,\mathbb{Q}} & 0 \\ 0 & B_X^{F,\mathbb{Q}} \end{bmatrix} \begin{bmatrix} X_t^H \\ X_t^F \end{bmatrix} \tag{11}$$

and express the latent factors in terms of the observable factors and the parameters. Plugging it back into equation (8) yields the rotated measurement equation:

$$\begin{bmatrix} Y_t^H \\ Y_t^F \end{bmatrix} = \mathcal{A}_\mathcal{P} + \mathcal{B}_\mathcal{P} \begin{bmatrix} \mathcal{P}_t^H \\ \mathcal{P}_t^F \end{bmatrix} \tag{12}$$

where:

$$\mathcal{A}_\mathcal{P} = \left(\mathbf{I} - \begin{bmatrix} B_X^{H,\mathbb{Q}} & 0 \\ 0 & B_X^{F,\mathbb{Q}} \end{bmatrix} \left(\mathbf{W} \begin{bmatrix} B_X^{H,\mathbb{Q}} & 0 \\ 0 & B_X^{F,\mathbb{Q}} \end{bmatrix} \right)^{-1} \mathbf{W} \right) \begin{bmatrix} A_X^{H,\mathbb{Q}} \\ A_X^{F,\mathbb{Q}} \end{bmatrix}$$

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[32]See Joslin et al. (2011) and Joslin et al. (2010).

[33]Alternatively, one could think of extracting the pricing factors from international cross-section of yields, as illustrated in Leippold and Wu (2002). Yet, as illustrated in the Appendix B, a model with common factors in the JSZ setting might not be well-identified.

and

$$\mathcal{B}_{\mathcal{P}} = \begin{bmatrix} B_X^{H,Q} & 0 \\ 0 & B_X^{F,Q} \end{bmatrix} \left(\mathbf{W} \begin{bmatrix} B_X^{H,Q} & 0 \\ 0 & B_X^{F,Q} \end{bmatrix} \right)^{-1}$$

Applying the same idea to the short rates in (6), to the state variables dynamics under the risk neutral measure (7) and under the physical measure (9) yields the JSZ canonical Gaussian dynamic term structure model:

$$\begin{bmatrix} y_{1,t}^{H} \\ y_{1,t}^{F} \end{bmatrix} = \begin{bmatrix} \rho_{0\mathcal{P}}^{H} \\ \rho_{0\mathcal{P}}^{F} \end{bmatrix} + \begin{bmatrix} \rho_{1\mathcal{P}}^{H} & 0 \\ 0 & \rho_{1\mathcal{P}}^{F} \end{bmatrix} \begin{bmatrix} \mathcal{P}_{t}^{H} \\ \mathcal{P}_{t}^{F} \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} \mathcal{P}_{t+1}^{H} \\ \mathcal{P}_{t+1}^{F} \end{bmatrix} = \begin{bmatrix} K_{0\mathcal{P}}^{H,Q} \\ K_{0\mathcal{P}}^{F,Q} \end{bmatrix} + \begin{bmatrix} K_{1\mathcal{P}}^{H,Q} & 0 \\ 0 & K_{1\mathcal{P}}^{F,Q} \end{bmatrix} \begin{bmatrix} \mathcal{P}_{t}^{H} \\ \mathcal{P}_{t}^{F} \end{bmatrix} + \begin{bmatrix} \Sigma_{\mathcal{P}}^{H} & 0 \\ 0 & \Sigma_{\mathcal{P}}^{F} \end{bmatrix} \begin{bmatrix} \varepsilon_{t}^{H,Q} \\ \varepsilon_{t}^{F,Q} \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} \mathcal{P}_{t+1}^{H} \\ \mathcal{P}_{t+1}^{F} \end{bmatrix} = K_{0\mathcal{P}}^{\mathbb{P}} + K_{1\mathcal{P}}^{\mathbb{P}} \begin{bmatrix} \mathcal{P}_{t}^{H} \\ \mathcal{P}_{t}^{F} \end{bmatrix} + \Sigma_{\mathcal{P}}^{\mathbb{P}} \begin{bmatrix} \varepsilon_{t}^{H,Q} \\ \varepsilon_{t}^{F,\mathbb{P}} \end{bmatrix} \tag{15}$$

Given the rotation matrix $\mathbf{W}$, the invariant transformations of the single parameters of the model are equivalent to those in a single-country setting and thus can be found in Joslin et al. (2011).[34] In addition, note that the dynamics under $\mathbb{P}$ is entirely driven by the parameters from the $\mathbb{P}$ distribution, i.e. $K_{0\mathcal{P}}^{\mathbb{P}}$ and $K_{1\mathcal{P}}^{\mathbb{P}}$. This is the so called "separation property" of the JSZ normalisation and it proves to be very helpful in estimation, because if the pricing factors $\mathcal{P}_{t}^{C}$ are observed, the $K_{1\mathcal{P}}^{\mathbb{P}}$ matrix can be estimated with the ordinary least squares.

Finally, the matrix $K_{1\mathcal{P}}^{\mathbb{P}}$ as already mentioned shapes the market prices of risk process, which in the rotated form reads:

$$\Lambda_{t}^{\mathcal{P}} = \Lambda_{0t} + \Lambda_{1t}^{\mathcal{P}} \begin{bmatrix} \mathcal{P}_{t}^{H} \\ \mathcal{P}_{t}^{F} \end{bmatrix} \tag{16}$$

where:

$$\Lambda_{0t}^{\mathcal{P}} = \left(K_{0\mathcal{P}}^{\mathbb{P}} - \begin{bmatrix} K_{0\mathcal{P}}^{H,Q} \\ K_{0\mathcal{P}}^{F,Q} \end{bmatrix} \right) \quad \text{and} \quad \Lambda_{1}^{\mathcal{P}} = \left(K_{1\mathcal{P}}^{\mathbb{P}} - \begin{bmatrix} K_{1\mathcal{P}}^{H,Q} & 0 \\ 0 & K_{1\mathcal{P}}^{F,Q} \end{bmatrix} \right)$$

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[34]See Appendix B of the article.

Constraining the number of priced risks amounts to constraining the rank of the matrix $\begin{bmatrix} \Lambda_0^{\mathcal{P}} & \Lambda_1^{\mathcal{P}} \end{bmatrix}$.[35] The next section defines the likelihood function and explains how the market prices of risk can be constrained.

3.2.3 Forward Term Premia

In this section, the model-implied forward term premia are derived. The reason for focusing on this particular definition of the risk premium[36] is that most of the studies of the U.S. term premia report the forward term premia.[37] According to the expectation theory of the term structure[38], an "$n - m$ period" forward rate n periods ahead is equal to the expected future short rate plus the term premium:

$$
\begin{bmatrix} fwd_{mn,t}^H \\ fwd_{mn,t}^F \end{bmatrix} = E_t^{\mathbb{P}} \begin{bmatrix} y_{1,t+n-1}^H \\ y_{1,t+n-1}^F \end{bmatrix} + \begin{bmatrix} FTP_{1,n}^H \\ FTP_{1,n}^F \end{bmatrix} \tag{17}
$$

where the continuously compounded $fwd_{mn,t}^C$, $C = \{H, F\}$ equals:

$$
fwd_{mn,t}^C = ny_{n,t}^C - my_{m,t}^C \tag{18}
$$

The corresponding expected one-period rate n periods in advance is:

$$
\begin{aligned}
& E_t^{\mathbb{P}} \begin{bmatrix} y_{1,t+n-1}^H \\ y_{1,t+n-1}^F \end{bmatrix} \\
= & \begin{bmatrix} \rho_{0\mathcal{P}}^H \\ \rho_{0\mathcal{P}}^F \end{bmatrix} + \begin{bmatrix} \rho_{1\mathcal{P}}^H & 0 \\ 0 & \rho_{1\mathcal{P}}^F \end{bmatrix} E_t^{\mathbb{P}} \begin{bmatrix} \mathcal{P}_{t+n-1}^H \\ \mathcal{P}_{t+n-1}^F \end{bmatrix} \\
= & \begin{bmatrix} \rho_{0\mathcal{P}}^H \\ \rho_{0\mathcal{P}}^F \end{bmatrix} + \begin{bmatrix} \rho_{1\mathcal{P}}^H & 0 \\ 0 & \rho_{1\mathcal{P}}^F \end{bmatrix} \left(\left(I + K_{1\mathcal{P}}^{\mathbb{P}} + ... + (K_{1\mathcal{P}}^{\mathbb{P}})^{n-1} \right) K_{0\mathcal{P}}^{\mathbb{P}} + (K_{1\mathcal{P}}^{\mathbb{P}})^{n-1} \begin{bmatrix} \mathcal{P}_t^H \\ \mathcal{P}_t^F \end{bmatrix} \right)
\end{aligned}
$$

I subtract the obtained expectation part from the forward rates to get the forward term premia. As it can be noticed, the home country risk factors do not affect the yields, but do impact the decomposition of the yields and most importantly the term premia in the foreign country.[39]

---

[35] A less restrictive constraint would be to impose the number of ranks on the $\Lambda_1^{\mathcal{P}}$ matrix alone, yet this would constrain the number of *time varying* market prices of risk. See Joslin et al. (2011).

[36] The term premium or the *risk* premium can be equivalently defined as a yield risk premium, a forward risk premium and a return risk premium. For a detailed discussion see Cochrane and Piazzesi (2008)

[37] See Kim and Wright (2005), Cochrane and Piazzesi (2008), and Joslin et al. (2010) among others.

[38] See Campbell and Shiller (1991) for an insightful discussion of the expectation theory of the term structure.

[39] Yet when the market prices of risk are constrained, the parameters in $K_{1\mathcal{P}}^{\mathbb{P}}$ exercise some

4 Econometric Identification and Estimation

4.1 Parameter Identification

Solid identification of parameters is an essential part of dynamic term structure models estimation. Before defining the likelihood function and providing estimation details, this section explains the identification strategy used, which is mostly based on ideas from JSZ work.

Following Hamilton and Wu (2010), Calvet, Fisher and Wu (2010) and Bauer and de los Rios (2011), the $K_{1X}^{C,\mathbb{Q}}$ matrix, $C = \{H, F\}$ is set to be a power law structure, with zero non-diagonal elements and the following power relation on the matrix' diagonal:

$$\lambda_{zz}^{C,\mathbb{Q}} = \lambda_{11}^{C,\mathbb{Q}} \alpha^{C,z-1}$$

where $\lambda_{11}^{C,\mathbb{Q}}$ is the largest eigenvalue of the matrix $K_{1X}^{C,\mathbb{Q}}$, α^C is a scaling parameter controlling the distance between the eigenvalues, and finally $z = 2, ...Z$. Given the pricing factors' dynamics under the risk neutral measure $\mathbb{Q}$ in (14), the pricing factors might not be necessarily stationary under $\mathbb{Q}$, i.e. the eigenvalues of $K_{1X}^{C,\mathbb{Q}}$ might be equal or larger than one. As noted in Joslin et al. (2011), the long-run means[40] of the one-period rates in such case are not well-defined or negative, respectivelly. Consequently, the authors propose the following identification tactic. The $\rho_{0\mathcal{P}}^C$ is set to zero and the drift of the most persistent factor $\mathcal{P}_{1,t}^C$ is set to be a constant:

$$K_{0X}^{C,\mathbb{Q}} = \begin{pmatrix} k_\infty^{C,\mathbb{Q}} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

where $k_\infty^{C,\mathbb{Q}}$ is a derived parameter[41]. Finally, the scale of the pricing factors $\rho_{1\mathcal{P}}^C$ is set to be a unit vector and the $\Sigma_{\mathcal{P}}^{\mathbb{P}}$ is Cholesky-decomposed to a lower triangular matrix $L_{\Sigma_{\mathcal{P}}^{\mathbb{P}}}$ with $2Z(2Z+1)/2$ parameters to estimate.

That said, the parameters $\lambda_{11}^{C,\mathbb{Q}}$, α^C and the two blocks on the diagonal of $\Sigma_{\mathcal{P}}^{\mathbb{P}}$ entirely caracterise the $\mathbb{Q}$ distribution of yields. The physical dynamics $\mathbb{P}$, on

---

impact on the cross-section of yields as well, see the last paragraph of Section 4.2.

[40]The intercept term in the equation (13).

[41]Calculated in such a way that, a particular value of $k_\infty^{C,\mathbb{Q}}$ corresponds to the zero vector $\rho_{0\mathcal{P}}^C$, given $K_{1\mathcal{P}}^{C,\mathbb{Q}}$, see the appendix of Joslin et al. (2011).

the other side, is determined by the $(K_{0\mathcal{P}}^{\mathbb{P}}, K_{1\mathcal{P}}^{\mathbb{P}}, \Sigma_{\mathcal{P}}^{\mathbb{P}})$ parameter set. The complete parameters' vector is:

$$\Theta = \left\{ \lambda_{11}^{H,\mathbb{Q}}, \lambda_{11}^{F,\mathbb{Q}}, \alpha^H, \alpha^F, K_{0\mathcal{P}}^{\mathbb{P}}, K_{1\mathcal{P}}^{\mathbb{P}}, \Sigma_{\mathcal{P}}^{\mathbb{P}} \right\}$$

4.2 Estimation

Let us now define the likelihood function. Following Chen and Scott (1993) and Joslin et al. (2010), it is assumed that the first Z principal components $\mathcal{P}_t^C$, $C = \{H, F\}$ are observed without error and the remaining $(N - Z)$ components $\mathcal{P}_t^{C,u}$ are measured with error:

$$\begin{bmatrix} \mathcal{P}_t^{H,u} \\ \mathcal{P}_t^{F,u} \end{bmatrix} = \mathbf{W}^u \mathcal{A}_{\mathcal{P}} + \mathbf{W}^u \mathcal{B}_{\mathcal{P}} \begin{bmatrix} \mathcal{P}_t^H \\ \mathcal{P}_t^F \end{bmatrix} + \Sigma_{\xi} \begin{bmatrix} \xi_t^H \\ \xi_t^F \end{bmatrix} \tag{19}$$

where:

$$\mathbf{W}^u = \begin{bmatrix} W_{(N-Z)\times N}^H & 0_{(N-Z)\times N} \\ 0_{(N-Z)\times N} & W_{(N-Z)\times N}^F \end{bmatrix} \quad \text{and} \quad \Sigma_{\xi} = \begin{bmatrix} \Sigma_{\xi}^H & 0_{(N-Z)\times N} \\ 0_{(N-Z)\times N} & \Sigma_{\xi}^F \end{bmatrix}$$

and the variance-covariance matrix of the pricing errors $\Sigma_{\xi}^{\bullet}$ is diagonal, while the error term is a multivariate normal $\xi_t^{\bullet} \sim N(0,1)$. The conditional joint density of the state vector and the $\mathcal{P}_t^{\bullet u}$ unobserved components is:

$$pdf\left(\mathcal{P}_t^H, \mathcal{P}_t^F, \mathcal{P}_t^{H,u}, \mathcal{P}_t^{F,u} \mid \mathcal{P}_{t-1}^H, \mathcal{P}_{t-1}^F, \Theta\right) =$$

$$pdf\left(\mathcal{P}_t^H, \mathcal{P}_t^F \mid \mathcal{P}_{t-1}^H, \mathcal{P}_{t-1}^F, K_{0\mathcal{P}}^{\mathbb{P}}, K_{1\mathcal{P}}^{\mathbb{P}}, \Sigma_{\mathcal{P}}^{\mathbb{P}}\right)$$

$$\times \quad pdf\left(\mathcal{P}_t^{H,u} \mid \mathcal{P}_t^H, \lambda_{11}^{H,\mathbb{Q}}, \alpha^H, \Sigma_{\mathcal{P}}^H\right)$$

$$\times \quad pdf\left(\mathcal{P}_t^{F,u} \mid \mathcal{P}_t^F, \lambda_{11}^{F,\mathbb{Q}}, \alpha^F, \Sigma_{\mathcal{P}}^F\right)$$

$$\tag{20}$$

The $\mathbb{Q}$ parameters, $\left\{ \lambda_{11}^{H,\mathbb{Q}}, \lambda_{11}^{F,\mathbb{Q}}, \alpha^H, \alpha^F, \Sigma_{\mathcal{P}}^{\mathbb{P}} \right\}$, are estimated using the maximum likelihood (ML) estimation. In a constrained optimisation[42], a standard line-search algorithm is used where the descent direction is calculated with Quasi-Newton method. The starting values for the covariance matrix are taken from

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[42] Only the non-negativity constraint on the diagonal elements of the covariance matrix is imposed.

the uncontrained VAR(1) estimation of the pricing factors. Departing from randomly chosen values of parameters $\left\{\lambda_{11}^{H,\mathbb{Q}}, \lambda_{11}^{F,\mathbb{Q}}, \alpha^H, \alpha^F\right\}$, the algorithm converges almost instantaneously to the same solution to the 6th decimal.

The parameters of the physical distribution, $\{K_{0\mathcal{P}}^{\mathbb{P}}, K_{1\mathcal{P}}^{\mathbb{P}}\}$, are estimated using the OLS. Yet, as already mentioned in the previous section, the market prices of risk can be also constrained by reducing the rank of the matrix $\begin{bmatrix} \Lambda_{0t}^{\mathcal{P}} & \Lambda_{1t}^{\mathcal{P}} \end{bmatrix}$ in equation (16). In that case, the parameters of the $\mathbb{P}$ distribution are computed as if they were ML estimates in the following way. The idea is to first perform the following reduced-rank regression:

$$
\begin{bmatrix} \mathcal{P}_{t+1}^H \\ \mathcal{P}_{t+1}^F \end{bmatrix} - \left(\begin{bmatrix} K_{0\mathcal{P}}^{H,\mathbb{Q}} \\ K_{0\mathcal{P}}^{F,\mathbb{Q}} \end{bmatrix} + \begin{bmatrix} K_{1\mathcal{P}}^{H,\mathbb{Q}} & 0 \\ 0 & K_{1\mathcal{P}}^{F,\mathbb{Q}} \end{bmatrix} \begin{bmatrix} \mathcal{P}_t^H \\ \mathcal{P}_t^F \end{bmatrix} \right) = \beta_0 + \beta_1 \begin{bmatrix} \mathcal{P}_t^H \\ \mathcal{P}_t^F \end{bmatrix} + \epsilon_t^{\mathcal{P}}
$$

where β_1 is restricted to have a rank lower than the number of pricing factors.[43]

Given the parameters $\left\{\lambda_{11}^{H,\mathbb{Q}}, \lambda_{11}^{F,\mathbb{Q}}, \alpha^H, \alpha^F, \Sigma_{\mathcal{P}}^{\mathbb{P}}\right\}$, the ML estimates of the $\mathbb{P}$ parameters are then given by:

$$
K_{0\mathcal{P}}^{\mathbb{P}} = \begin{bmatrix} K_{0\mathcal{P}}^{H,\mathbb{Q}} \\ K_{0\mathcal{P}}^{F,\mathbb{Q}} \end{bmatrix} + \widehat{\beta_0} \quad \text{and} \quad K_{1\mathcal{P}}^{\mathbb{P}} = \begin{bmatrix} K_{1\mathcal{P}}^{H,\mathbb{Q}} & 0 \\ 0 & K_{1\mathcal{P}}^{F,\mathbb{Q}} \end{bmatrix} + \widehat{\beta_1}
$$

As it can be noticed, the ML estimates of the $\mathbb{P}$ parameters when the rank of the risk premium is reduced will be no longer given by their OLS counterparts. In other words, the assumptions on the parameters from the $\mathbb{P}$ measure directly affect the estimates of the $\mathbb{P}$ and the "separation property" of the JSZ normalisation does not hold anymore. In addition, the reduced rank idea allows us to understand the nature (and the number) of priced factors in a two-country setting.

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[43] As it is shown in Joslin et al. (2011), the solution for $\widehat{\beta_1}$ is singular value decomposition of β_1, namely, $\widehat{\beta_1} = U D_r^* V'$, where the matrix D_r^* is obtained by setting to zero all the singular values of D with index $n > r$.

5 Results

5.1 Parameters

The estimated parameters are reported in the Table 2 together with asymptotic and bootstrapped[44] standard errors. As it can be noticed, the values of standard errors are comparable for most of the parameters.[45] The 3×3 matrices on the diagonal of $L_{\Sigma_{\mathcal{P}}^{\mathbb{P}}}$ are covariance matrices of single countries' pricing factors, while the remaining parameters are covariances of state variables across the two countries. Few of the residuals seem to significantly co-move. On the other side, the estimates of the $K_{1\mathcal{P}}^{\mathbb{P}}$ matrix point to a statistically significant relation among the international pricing factors.

As expected, the UK level factor has no predictive power on the US level factor, but the US level factor in one period does explain a portion of the UK level factor in the next period. What is more, all the US factors can help explain the UK level factor, where the US curvature factor seem to have the strongest predictive power. One percentage point increase in the US curvature factor in t is on average followed by a 2.6 basis points increase in UK level factor in $t + 1$.

Finally, the lower-left non-zero block of $K_{1\mathcal{P}}^{\mathbb{P}}$ matrix allows for the US risk factors to affect the term premia in the UK. The Figure 4 illustrates the decomposition of the 10-year UK term premia to the parts driven by the UK factors and the US factors. By construction, the most of the variation in premia is produced by the UK factors, whereas the US factors capture the variation in the UK premia not contained in the shape of the UK yield curve.[46]

5.2 Forward premium conditions

As discussed in the Section 3.1 and Appendix A, the two-country model needs to generate the foreign exchange risk premium in line with Fama (1984): it should be negatively correlated with the interest rate differential and its variance should be higher than the variance of the interest rate differential. The upper panel of the Figure 5 plots the model-implied depreciation rate, together with its two components. Most of the variation in the depreciation rate indeed

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[44]Bootstrapped standard errors are calculated as follows. A starting value for pricing factors is randomly chosen from the dataset. The estimated parameters are then used to simulate a time series of pricing factors with 3,000 observations. The parameters and the simulated path of pricing factors produce a simulated path of two yield curves. The model is estimated on such simulated paths for 1,000 times.

[45]Bigger differences between the asymptotic and the standard errors are estimated for the variance parameters of the slope and curvature factors in the two countries.

[46]Similarly to Joslin et al. (2010).

comes from the variation in the foreign exchange risk premium and the standard deviations of the two are 15.13 and 16.17 percent, respectively. Nonetheless, there is a negative correlation of -.83 between the foreign exchange premium and the interest rate differential.

Finally, the lower panel of the Figure 5 plots the model-generated foreign exchange premium against the one from the data. As it can be noticed, the model explains some variation in the observed depreciation rate, where the correlation of the two series is 0.21 and the standard deviations of the modelled and the observed depreciation rates are 15.13 and 10.71 percent, respectively. A rather poor fit, yet correctly estimated moments of the single elements of model-implied depreciation rate, might be enough to confirm the validity of the two-country model.

5.3 Pricing performance

The upper panel of the Table 3 reports the mean absolute pricing errors of the single- and the two-country model. The single-country models for the UK and for the US are estimated under the full-rank $\begin{bmatrix} \Lambda_{0t}^{\mathcal{P}} & \Lambda_{1t}^{\mathcal{P}} \end{bmatrix}$ matrix. The two-country model, with the market price of risk matrix having the rank of 2, corresponds to the notion that only the level risks are priced in the yield curve.[47] If the matrix has the rank of 4, both level and slope risks are priced in yields.[48] As it can be noticed, the performance of the two-country model is comparable to the single-country model, whereas the two-country model marginally improves the fit of the US yield curve.

The Table 3 in the lower panel reports the means and the standard deviations of the 1-day-ahead forecasting errors of the two-country model on different policy action days and for the selected yields. The independent two-sample means t-test shows that some of the forecasting errors' means are statistically different from zero. Specifically, the yields on the short-end of the US yield curve and around the Fed funds rate cuts are sistematically over-priced by 4.4 basis points. This is also the case for the longer-end of the UK yield curve and around anticipated Fed funds hikes, where the yields are on average over-priced by 2.2 basis points.

Finally, one can also notice that the magnitude and the volatility of the US forecasting errors is much higher around interest rate cuts, than around interest rate hikes. The lower forecasting performance might be due to elevated macroeconomic uncertainty during the circumstances in which the decisions to cut the Fed funds rate are usually delivered. The model is thus more likely to be "wrong" around those decisions. The same pattern does not seem to hold for the UK yields.

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[47]As in Cochrane and Piazzesi (2008).
[48]Similarly to Duffee (2010) and Joslin et al. (2010).

5.4 Reactions to the Fed decisions

As already mentioned, different policy rate decisions of the Fed are classified across two dimensions and used to analyse the reaction of the yields to those decisions. This allows for a sort of generalisation of the notion of a policy shock and possibly accounts for asymmetries in the market response to different interest rate moves. This section reports both instantaneous and lagged reaction of the UK yield curve to the Fed policy actions and shows that the asymetries are indeed non-trivial.

5.4.1 Instantaneous reactions

Table 4 reports one-day average change in the UK yields followed by different policy rate decisions of the Fed. The changes are expressed in basis points and compared with the average one-day changes on non-policy days in an independent two-sample t-test of means. The values in brackets are corresponding p-values of the test statistic. As it can be noticed, there is a statistically significant decrease in the UK yields on the long-end of the curve, as an average change after anticipated decisions to hike the Fed funds rate.

This might not sound intuitive, because the "anticipated" decisions should be priced in yields. Yet, if the Fed funds futures market correctly anticipated a policy decision, it does not necessarily mean that the rates market followed suit. The decisions are classified into expected or surprise policy actions by only looking at the Fed funds futures quotes and the corresponding implicit Fed funds rate "expectation". In addition to this, Bernanke and Kuttner (2005) notice that asset prices need not to respond only to surprise moves of the Fed, but also to revisions in expectation about future policy, which may also result from a policy decision.[49]

Why do the long-term UK yields fall after an anticipated hike of the Federal funds rate? As the Table 5 reports, the fall in yields seem to be given mostly by the decrease in the term premia. If the estimated time-varying premia can be regarded as uncertainty around future short-rate expectation i.e. a "deviation" from expectation hypothesis[50], then the anticipated hike decision of the Fed seem to reduce that uncertainty. The negative reaction of the premia is almost equal in magnitude across the maturity spectrum. Since the future short-rate expectations for maturities under 5Y slightly rise[51], only the yields on the longer-end seem to be affected by the shift. The average increase of the Fed

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[49]Every policy rate decision is announced together with a brief communiqué on the general economic assessment, in the form of so called Federal Open Market Committee (FOMC) Statement.

[50]See Kim and Orphanides (2007)

[51]Unfortunately, the changes in future short-rate expectations are not statistically significant and thus not reported.

funds rate by 29bp (on 27 anticipated hike decisions) is estimated to cause on average a 2bp fall in 5 to 10Y maturities yields. Almost entire reaction is estimated to be driven by the fall in longer term premia by approximately the same amount.

If anticipated rate decisions provoke a decrease in the premia, a suprise policy actions should have an opposite effect. Indeed, the Table 5 reports statistically significant and positive 1-day change of the premia after both unexpected policy rate hikes and cuts of the Fed funds rate. There is on average a 4.5 bp increase in the premia around short- and medium term maturities after an unexpected interest rate cut. As only 4 decisions to increase the Fed funds rate are labelled as unexpected, the reaction of the premia around those days is not statistically significant. Still, the average change of the premia after all surprises are statistically different from the average change on a non-policy day. The Table 6 confirms these conclusions for the single-country model for the UK yield curve and the results do not change when the market prices of risk are constrained.

5.4.2 Impulse response functions

Usually in the term structure models' analysis, the general impulse response function of Pesaran and Shin (1998) is used to describe the reaction of the state variables or yields to one standard deviation shock in another state variable.[52] In such a case, the dynamic reaction of yields to a monetary policy shock could be analysed by considering the one-period interest rate in the sample as the monetary policy instrument and then using it as one of the state variables.[53] In this study, the one-period US interest rate is the 6-month USD Libor. Even though the short-term Libor rates closely co-move with the Fed funds rate, the spread between the funds rate and the Libor rates might not be necessarily constant, because the latter include credit risk premium.[54]

Alternatively, the idea here would be to extract the shocks from the models' residuals around policy action days. From every realisation of the state variable vector P_d on a policy action day d, its ex-ante expectation $E\left[P_d | \mathbb{I}_{d-1}\right]$ is subtracted. The residuals obtained in this way are then grouped to the mentioned classess (e.g. surprise hikes) and the means of the residuals for each group are calculated. The Appendix C illustrates the idea and the Figure 6 reports the response functions of the UK yields to the extracted shocks together with 90 percent confidence intervals.

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[52]See for instance Söderlind (2010) or Kaminska (2008).

[53]In the JSZ framework, this is done by setting the element (1,1) of the rotation matrix $W_{Z \times N}$ in (11) to 1 and other elements of the first row of the matrix to 0.

[54]An interesting way of including the Fed policy rate to the model estimation is proposed in Piazzesi (2005), who uses the effective Fed funds rate as the state variable.

The first important result is that there seem to be a negative reaction of the UK yields to both interest rate hikes and interest rate cuts. The reaction of the UK one-period rate seem to be higher in magnitude, but also somewhat more persistant, than the reaction of the 10Y rate. Secondly, anticipated policy actions seem to produce less persistent reactions. The 2 basis points instantaneous decrease in the UK long-term yields, after an anticipated hike of the Fed funds rate, is estimated to die off relativelly quickly respect to other responses.[55] Finally, the UK long-term yields' response to a surprise cut seem to be negative, even though there is an instantaneous positive shift, presumably connected with the findings in the Table 5.

5.4.3 Robustness check: Post-2007 period

On the 9th of August 2007, the interbank markets of the United States and the euro area came under unexpected and severe strains,[56] after months of falling house prices and adverse events in the US sub-prime mortgage market. The US policy-maker, concerned about the tightening of credit conditions, lowered the Federal funds rate by 50 basis points on the 18th of September 2007 and embarked on a stream of interest rate cuts. In the UK, the Bank of England started to decrease the reference rate in December 2007 and continued to do so on several occasions until the end of the sample.[57]

Highly correlated policy paths during this period, together with globally deteriorating growth prospects and dire credit conditions, might be excessively driving the results presented above. For this reason, I re-estimate the two-country model on the sub-sample excluding the period from the beginning of August 2007 until the end of the sample. The estimated term-premia average changes are reported in the Table 7. As it can be noticed, the main result remains. The one-day average change in the term premia, after an anticipated hike of the Federal funds rate, is statistically different from zero. Nonetheless, there seem to be an increase in premia on short- and medium-term maturities, after the surprise policy actions of the Fed.

5.4.4 Robustness check: Weighted average response

Another important check would be to control for the size heterogeneity of policy moves across and within different groups of policy actions. As we have seen in the Section 2.2, larger changes of the Fed funds rate are usually communi-

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[55]Given that the data are daily, the estimated state variables' dynamics are substantially persistent. Consequently, the reported duration of the shocks should by no means be taken as guiding.

[56]See Borio (2008).

[57]The bank rate was reduced on 7th of February, 10th of April, 8th of October, 6th of November and 4th of December 2008. The details about the decisions can be found here.

cated after expantionary decisions. In addition, most of the surprise decisions are changes in the policy rate of more than 25 basis points, see Figure 3. Accordingly, the results might be driven by several extreme cases of substantial change in the policy rate and the subsequent reaction in the yields.

To control for such effects, I calculate a weighted average reaction of the UK premia to different policy actions in the following way. All the one-day changes are multiplied by the inverse of the corresponding policy rate move times 25 (e.g. for a 50 basis points hike, the "weight" would be 25/50). The re-scaled changes are then summed up within a group of policy actions and divided by the number of decisions in the group. The outcome is an estimated one-day average change of the term premia, as a result of *25 basis points change* in the policy rate and it is reported in Table 8. As expected, the magnitude of changes is somewhat lower, especially after the interest rate cuts. Still, the reaction of the premia to surprise policy moves is still statistically different from zero and independent of the direction of the policy move and there is a decrease in the premia at the medium and long-end of the curve as a result of anticipated hikes of the Fed funds rate.

6   Conclusion

The study proposes a way of analysing international financial transmission mechanism of a globally relevant policy-maker by using the high-frequency indentification from Piazzesi (2005) and extracting "realised" policy shocks from the sample of 125 interest rate decisions. All the Fed decisions are divided according to direction of the move and weather the markets anticipated them. The reaction of yields to separate groups of policy actions is analysed, which explicitly accounts for possible asymmetries in the response mentioned in Bernanke and Kuttner (2005). By estimating a two-country yield curve model on the US-UK data, this study shows that such asymmetries are non-trivial and might not be captured in a standard VAR analysis.

There are many ways to go from here. One could use a latent-factor framework to give the home countries risk factors a "fair chance" in explaining the movement in foreign term premia but especially in the foreign cross-section of yields. The estimated factor loadings on one-period rates could provide insight into the nature of information used by the central banks in respective countries. Furthermore, it might be interesting to explore the extent to which a highly correlated or even common level and slope factors can be connected to the convergence in medium-term inflation expectations in the two countries, or to similar business cycles or policy instruments paths. This study shows that the mean forecasting errors for some decision days are statistically non-zero, and a proper regime-switching model could offer a better performance. This all might be the subject of author's future research.

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Appendix

A. Fama conditions

The depreciation rate generated by the two-countries model needs to be in line with the so-called "forward premium anomaly", i.e. the widely acknowledged empirical finding[58] according to which the high interest rate currencies tend to appreciate. Fama (1984) imposes two necessary conditions on the time-varying forward risk premium. First, it must be negatively correlated with its expected rate of depreciation. Secondly, it must have greater variance than the expected depreciation rate. The conditions are tested as follows.

First, define the log-pricing kernel from equation (5) as:

$$m_{t+1}^C = -y_{1,t}^C - \frac{1}{2}(\Lambda_t^C)'\Lambda_t^C - (\Lambda_t^C)'\varepsilon_{t+1} \tag{21}$$

where $C = \{H, F\}$ and Λ_t^C is a time-varying market price of risk defined in the following sub-section. Given (5) and (21), the *expected* depreciation rate consists of the interest rate differential in the two countries and the foreign exchange risk premium:

$$\Delta s_{t+1} = ird_t + frp_t$$

$$ird_t = y_{1,t}^F - y_{1,t}^H$$

and

$$frp_t = \frac{1}{2}\left((\Lambda_t^F)'\Lambda_t^F - (\Lambda_t^H)'\Lambda_t^H\right)$$

Under the risk neutral measure $\mathbb{Q}$, the depreciation rate equals the interest rate differential:

$$\Delta s_{t+1} = y_{1,t}^F - y_{1,t}^H \tag{22}$$

or, in other words, the uncovered interest rate parity (UIRP) should hold. To test for the "forward premium anomaly", regress the ex-post depreciation rate against the rate differential:

[58]See Hansen and Hodrick (1983), Fama (1984), Cumby and Obstfeld (1985), Hodrick (1987), Engel (1996), Bansal (1997), Dong (2006) and Graveline (2006) among others.

$$\Delta s_{t+1} = a + b(y^F_{1,t} - y^H_{1,t}) + \varepsilon_{t+1} \tag{23}$$

where the slope coefficient of the regression is broadly found to be negative, instead of being 1, as the UIRP would suggest. According to Fama (1984), the deviations from the UIRP can be expressed as two conditions on the forward premium anomaly. First, there is a negative correlation between the forward risk premium and the interest rate differential:

$$
\begin{aligned}
b &= \frac{cov\,(\Delta s_{t+1}, ird_t)}{var(ird_t)} \\
&=> \quad cov(frp_t, ird_t) + var(ird_t) < 0 \\
&\qquad\quad - \qquad\qquad\quad +
\end{aligned} \tag{24}
$$

and, secondly, the variance of the foreign exchange risk premium should be higher than the variance of the interest rate differential. Specifically, Fama (1984) performs the two following regressions:

$$F_t - S_{t+1} = a1 + b1(F_t - S_t) + \varepsilon_{1,t+1}$$

and

$$S_{t+1} - S_t = a2 + b2(F_t - S_t) + \varepsilon_{1,t+1}$$

where F_t and S_t are the forward- and the spot exchange rate, respectively. He estimates the distance between the coefficients $b1$ and $b2$:

$$b1 - b2 = \frac{var(frp_t) - var(E[S_{t+1} - S_t])}{var(F_t - S_t)}$$

to be positive for all the considered currency pairs, from where he concludes that the $var(frp_t)$ is larger than $var(E(S_{t+1} - S_t))$. Consequently, it follows that $var(frp_t)$ is larger than $var(ird_t)$ as well.[59] In other words, most of the variation in the depreciation rate should come from variation in the foreign exchange risk premium. As it is shown in the Section Results, the model satisfies both conditions.

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[59] See Fama (1984) for details.

B. Common factors in the JSZ setting

A two-country yield curve model with common observed factors might not be well-identified in the Joslin et al. (2011) setting. This part of the Appendix offers an illustration of this claim.

For the sake of simplicity, let us assume that the short rates in two countries are affine functions of one latent state variable x_t, e.g. a global level factor:

$$
\begin{bmatrix} y_{1,t}^H \\ y_{1,t}^F \end{bmatrix} = \begin{bmatrix} \rho_{0X}^H \\ \rho_{0X}^F \end{bmatrix} + \begin{bmatrix} \rho_{1X}^H \\ \rho_{1X}^F \end{bmatrix} x_t
$$

where ρ_{0X}^C and ρ_{1X}^C, $C = \{H, F\}$ are scalars. The state variable follows a Gaussian AR(1) process as:

$$
x_{t+1} = k_{0X}^Q + k_{1X}^Q x_t + \sigma_X \varepsilon_t^Q \tag{25}
$$

Solving for the bond prices recursively, the entire cross-section of the two yield curves is an affine function of the state variable:

$$
\begin{bmatrix} Y_t^H \\ Y_t^F \end{bmatrix} = \begin{bmatrix} A_X^H \\ A_X^F \end{bmatrix} + \begin{bmatrix} B_X^H \\ B_X^F \end{bmatrix} x_t \tag{26}
$$

where:

$$
b_{n,X}^C = k_{1X}^Q b_{n-1,X}^C - \rho_{1X}^C
$$

$$
a_{n,X}^C = k_{0X}^Q b_{n-1,X}^C + \frac{1}{2}(b_{n-1,X}^C)' \sigma_X b_{n-1,X}^C + a_{n-1,X}^C - \rho_{0X}^C
$$

and $n = 1, 2, \ldots N$. Now, let us rotate the measurement equation (26) by pre-multiplying it with a $1 \times (N + N)$ vector W selected by the modeller, e.g. the loadings of a first principal components on the entire yield matrix:

$$
\mathcal{P}_t = W \begin{bmatrix} A_X^H \\ A_X^F \end{bmatrix} + W \begin{bmatrix} B_X^H \\ B_X^F \end{bmatrix} x_t
$$

where the observed pricing factor $\mathcal{P}_t = W \begin{bmatrix} Y_t^H \\ Y_t^F \end{bmatrix}$. Solve the above equation for x_t and plug the expression back into (26) to obtain:

$$\left[\begin{array}{c} Y_t^H \\ Y_t^F \end{array} \right] = \left[\begin{array}{c} A_X^H \\ A_X^F \end{array} \right] + \left[\begin{array}{c} B_X^H \\ B_X^F \end{array} \right] (WB_X)^{-1} \left(\mathcal{P}_t - W \left[\begin{array}{c} A_X^H \\ A_X^F \end{array} \right] \right)$$

where WB_X is the scalar $W \left[\begin{array}{c} B_X^H \\ B_X^F \end{array} \right]$. Rearranging the above yields:

$$\left[\begin{array}{c} Y_t^H \\ Y_t^F \end{array} \right] = \mathcal{A}_{\mathcal{P}} + \mathcal{B}_{\mathcal{P}} \mathcal{P}_t \qquad (27)$$

where:

$$\mathcal{A}_{\mathcal{P}} = \left(\mathbf{I} - \left[\begin{array}{c} B_X^H \\ B_X^F \end{array} \right] (WB_X)^{-1} W \right) \left[\begin{array}{c} A_X^H \\ A_X^F \end{array} \right]$$

and

$$\mathcal{B}_{\mathcal{P}} = \left[\begin{array}{c} B_X^H \\ B_X^F \end{array} \right] (WB_X)^{-1}$$

where $\mathbf{I}$ is an $(N+N) \times (N+N)$ unity matrix. Finally, insert the same expression into the transition equation (25) under physical measure $\mathbb{P}$ to obtain:

$$\mathcal{P}_{t+1} = k_{0\mathcal{P}}^{\mathbb{P}} + k_{1\mathcal{P}}^{\mathbb{P}} \mathcal{P}_t + \sigma_{\mathcal{P}} \varepsilon_t^{\mathbb{P}} \qquad (28)$$

where it is important to notice that: $\sigma_{\mathcal{P}} = WB_X \sigma_X (WB_X)'$.

Now, suppose we want to estimate (27) using the parameters for the cross-sectional fit $\{k_{1X}^Q, \rho_{1X}^H, \rho_{1X}^F\}$[60] and derive the parameter σ_X by estimating the (27) and the transition equation (28). Given that the parameter k_{1X} does not show up in the transition equation, the roles of k_{1X}^Q, ρ_{1X}^H and ρ_{1X}^F are interchangable. Futhermore, if one fixes the parameter k_{1X}^Q and estimates the parameters ρ_{1X}^H and ρ_{1X}^F, the model might still not be well identified, given that both enter the scalar $(WB_X)^{-1}$ that mutiplies all the single elements of $\mathcal{A}_{\mathcal{P}}$ and $\mathcal{B}_{\mathcal{P}}$.

Consequently, the two-country model with common factors might not be well-identified because of the rotation matrix (scalar). For this reason, a possible zero restrictions on the short-rate yields' loadings of single countries[61] might also not work in the JSZ setting.

---

[60]For simplicity, assume we fix the parameters ρ_{0X} and ρ_{0X}^F to some values.

[61]One could think of a setting in which certain local factors do not load on the opposite contries' yields.

C. Impulse response functions

Let us re-write the transition equation (15) as:

$$\boldsymbol{P}_d = E\left[\boldsymbol{P}_d | \mathbb{I}_{d-1}\right] + \Sigma_{\mathcal{P}}^{\mathbb{P}} \begin{bmatrix} \varepsilon_d^{H,\mathbb{P}} \\ \varepsilon_d^{F,\mathbb{P}} \end{bmatrix} \qquad (29)$$

where d is the day of a Fed decision, $\mathbb{I}_{d-1}$ is the information set in $d-1$ and:

$$\boldsymbol{P}_d = \begin{bmatrix} \mathcal{P}_d^H \\ \mathcal{P}_d^F \end{bmatrix}$$

$$E\left[\boldsymbol{P}_d | \mathbb{I}_{d-1}\right] = K_{0\mathcal{P}}^{\mathbb{P}} + K_{1\mathcal{P}}^{\mathbb{P}} \begin{bmatrix} \mathcal{P}_{d-1}^H \\ \mathcal{P}_{d-1}^F \end{bmatrix}$$

Iterating (29) forward, it can be shown that:

$$\boldsymbol{P}_{d+n} - E\left[\boldsymbol{P}_{d+n} | \mathbb{I}_{d-1}\right] = \left(K_{1\mathcal{P}}^{\mathbb{P}}\right)^{n-1} \Sigma_{\mathcal{P}}^{\mathbb{P}} \begin{bmatrix} \varepsilon_d^{H,\mathbb{P}} \\ \varepsilon_d^{F,\mathbb{P}} \end{bmatrix} \qquad (30)$$

Pre-multiplying the right-hand side of the above equation with the factor loadings matrix $\mathcal{B}_{\mathcal{P}}$ gets the impulse response function of single yields to the shock $\boldsymbol{P}_d - E\left[\boldsymbol{P}_d | \mathbb{I}_{d-1}\right]$. For every class of the Fed decisions (e.g. surprise cuts), an average value of residuals is calculated:

$$\frac{1}{N_d} \sum_{n_d=1}^{N_d} \left(\boldsymbol{P}_d^{(n_d)} - E\left[\boldsymbol{P}_d^{(n_d)} | \mathbb{I}_{d-1}\right]\right)$$

and used to "shock" the system. N_d is the number of certain decisions (e.g. there are in total 8 interest rate cuts, sorted as surprise cuts) in the sample.

D. Tables and Graphs

Table 1: The reported FOMC meetings that resulted in an interest rate decision include both scheduled and unscheduled meetings. The sample covers 31 decisions to hike the policy rate, 29 cut and 66 hold decisions. Column Surprise (bp) reports the unexpected element of every decision extracted from the Fed futures market and following Kuttner (2001).

Day	Month	Year	Decision (bp)	Suprise (bp)	Surprise Hike	Surprise Cut
4	Feb	1994	25	16	*	
22	Mar	1994	25	11		
18	Apr	1994	25	25	*	
17	May	1994	50	12		
6	Jul	1994	0	-2		
16	Aug	1994	50	21	*	
27	Sep	1994	0	0		
15	Nov	1994	75	0		
20	Dec	1994	0	-23		
1	Feb	1995	50	2		
28	Mar	1995	0	2		
23	May	1995	0	-1		
6	Jul	1995	-25	-9		
22	Aug	1995	0	-4		
26	Sep	1995	0	4		
15	Nov	1995	0	2		
19	Dec	1995	-25	-7		
31	Jan	1996	-25	3		
26	Mar	1996	0	1		
21	May	1996	0	1		
3	Jul	1996	0	-6		
20	Aug	1996	0	-1		
24	Sep	1996	0	-12		
13	Nov	1996	0	2		

Day	Month	Year	Decision (bp)	Suprise (bp)	Surprise Hike	Surprise Cut
17	Dec	1996	0	0		
5	Feb	1997	0	-2		
25	Mar	1997	25	2		
20	May	1997	0	-9		
2	Jul	1997	0	-1		
19	Aug	1997	0	3		
30	Sep	1997	0	0		
12	Nov	1997	0	-3		
16	Dec	1997	0	-2		
4	Feb	1998	0	0		
31	Mar	1998	0	0		
19	May	1998	0	-5		
1	Jul	1998	0	-2		
18	Aug	1998	0	3		
29	Sep	1998	-25	4		
15	Oct	1998	-25	-24		*
17	Nov	1998	-25	-12		
22	Dec	1998	0	-4		
3	Feb	1999	0	1		
30	Mar	1999	0	-1		
18	May	1999	0	10		
30	Jun	1999	25	-8		
24	Aug	1999	25	2		
5	Oct	1999	0	6		
16	Nov	1999	25	11		
21	Dec	1999	0	12		
2	Feb	2000	25	2		
21	Mar	2000	25	-1		
16	May	2000	50	8		
28	Jun	2000	0	-2		
22	Aug	2000	0	3		
3	Oct	2000	0	2		
15	Nov	2000	0	0		
19	Dec	2000	0	3		
3	Jan	2001	-50	-24		*
31	Jan	2001	-50	-5		
20	Mar	2001	-50	-7		
18	Apr	2001	-50	-79		*
15	May	2001	-50	-22		*

Day	Month	Year	Decision (bp)	Suprise (bp)	Surprise Hike	Surprise Cut
27	Jun	2001	-25	9		
21	Aug	2001	-25	-3		
17	Sep	2001	-50	-28		*
2	Oct	2001	-50	-14		
6	Nov	2001	-50	-16		
11	Dec	2001	-25	-7		
30	Jan	2002	0	3		
19	Mar	2002	0	53		
7	May	2002	0	-4		
26	Jun	2002	0	-9		
13	Aug	2002	0	-7		
24	Sep	2002	0	-2		
6	Nov	2002	-50	-12		
10	Dec	2002	0	0		
29	Jan	2003	0	2		
18	Mar	2003	0	4		
6	May	2003	0	-7		
25	Jun	2003	-25	11		
12	Aug	2003	0	0		
16	Sep	2003	0	0		
28	Oct	2003	0	-1		
9	Dec	2003	0	1		
28	Jan	2004	0	3		
16	Mar	2004	0	0		
4	May	2004	0	1		
30	Jun	2004	25	-8		
10	Aug	2004	25	7		
21	Sep	2004	25	8		
10	Nov	2004	25	1		
14	Dec	2004	25	0		
2	Feb	2005	25	1		
22	Mar	2005	25	29	*	
3	May	2005	25	2		
30	Jun	2005	25	2		
9	Aug	2005	25	0		
20	Sep	2005	25	7		
1	Nov	2005	25	1		
13	Dec	2005	25	0		
31	Jan	2006	25	1		

Day	Month	Year	Decision (bp)	Suprise (bp)	Surprise Hike	Surprise Cut
28	Mar	2006	25	6		
10	May	2006	25	1		
29	Jun	2006	25	-7		
8	Aug	2006	0	-5		
20	Sep	2006	0	0		
25	Oct	2006	0	-2		
12	Dec	2006	0	-1		
31	Jan	2007	0	0		
9	May	2007	0	1		
28	Jun	2007	0	2		
7	Aug	2007	0	8		
18	Sep	2007	-50	-44		*
31	Oct	2007	-25	9		
11	Dec	2007	-25	0		
22	Jan	2008	-75	-63		*
30	Jan	2008	-50	-10		
18	Mar	2008	-75	49		*
30	Apr	2008	-25	-7		
25	Jun	2008	0	-4		
5	Aug	2008	0	-3		
16	Sep	2008	0	21		
8	Oct	2008	-50	-17		coordinated cut
29	Oct	2008	-50	-10		
16	Dec	2008	-75	-35		*as anticipated

Table 2: The table reports the estimated parameters. The standard errors for $K_{1\mathcal{P}}^{\mathbb{P}}$ are calculated from the output of an uncontrained VAR(1). The standard errors of the Q parameters are assymptotic standard errors in $(\bullet)$ brackets and bootstrapped standard errors in $\{\bullet\}$ brackets.

$$K_{1\mathcal{P}}^{\mathbb{P}}$$

US level	0.998	0.004	0.006	0.001	-0.007	0.004
	(0.002)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
US slope	0.001	0.998	-0.018	-0.001	0.001	-0.003
	(0.003)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)
US curvature	0.001	0.000	0.985	-0.001	0.001	-0.005
	(0.012)	(0.004)	(0.003)	(0.010)	(0.004)	(0.002)
UK level	0.004	-0.006	0.026	0.997	-0.004	0.002
	(0.002)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
UK slope	-0.001	0.001	0.009	0.002	0.997	0.018
	(0.003)	(0.001)	(0.001)	(0.003)	(0.001)	(0.001)
UK curvature	0.001	-0.001	-0.006	-0.001	0.000	0.988
	(0.011)	(0.004)	(0.003)	(0.009)	(0.004)	(0.002)

$$L_{\Sigma_{\mathcal{P}}^{\mathbb{P}}}$$

US (Home Country)

	0.131					
$\lambda_{11}^{F,Q}$	(0.082)					
0.991	{0.089}					
(0.005)	-0.030	0.035				
{0.014}	(0.053)	(0.022)				
α^F	{0.017}	{0.005}				
0.748	-0.025	0.005	0.026			
(0.014)	(0.029)	(0.025)	(0.016)			
{0.018}	{0.035}	{0.012}	{0.002}	UK (Foreign Country)		
λ_{11}^{Q}	0.059	0.002	-0.002	0.090		
0.997	(0.156)	(0.072)	(0.065)	(0.056)		
(0.003)	{0.059}	{0.021}	{0.072}	{0.020}		
{0.003}	-0.014	0.010	0.002	-0.016	0.037	
α	(0.059)	(0.033)	(0.034)	(0.032)	(0.022)	
0.813	{0.010}	{0.068}	{0.018}	{0.091}	{0.005}	
(0.011)	0.007	0.002	-0.004	0.010	0.004	0.025
{0.007}	(0.035)	(0.024)	(0.025)	(0.020)	(0.024)	(0.016)
	{0.028}	{0.016}	{0.041}	{0.016}	{0.061}	{0.007}

Table 3: The upper panel of the table reports in-sample mean absolute pricing errors calculated on the entire sample and expressed in basis points. The lower panel reports the means and the standard deviations of 1-day ahead forecasting errors on different policy days and for the selected 6-month US and the 10Y UK yields. The p-values reported in brackets come from an independent two-sample means t-test and the levels of significance of .10, .05 and .01 are denoted with *, ** and ***, respectivelly.

Single-country Model						
	6m	2Y	3Y	5Y	7Y	10Y
UK	0.5	2.4	1.3	2.3	1.2	2.0
US	0.4	2.4	1.7	1.7	1.0	1.4

Two-country Model						
			UK			
Rank	6m	2Y	3Y	5Y	7Y	10Y
2	0.5	2.5	1.4	2.3	1.2	2.1
4	0.7	3.1	2.0	2.9	1.6	2.7
			US			
Rank	6m	2Y	3Y	5Y	7Y	10Y
2	0.3	1.8	1.1	1.5	0.8	1.2
4	0.3	1.6	1.0	1.4	0.8	1.2

Decisions	6m US		10Y UK	
	mean	std	mean	std
Hikes (31)	0.4	2.2	-1.4	5.2
	(0.15)		(0.31)	
- Anticipated Hikes (27)	0.4	2.1	-2.2***	3.3
	(0.24)		(0.01)	
- Surprise Hikes (4)	1.1	3.1	3.6	11.9
	(0.49)		(0.53)	
Cuts (28)	-4.4**	11.4	0.1	4.6
	(0.05)		(0.44)	
- Anticipated Cuts (20)	-2.3*	4.8	-0.7	4.0
	(0.06)		(0.77)	
- Surprise Cuts (8)	-8.1	19.3	2.2	5.5
	(0.24)		(0.18)	

Table 4: One-day average changes in the UK yields after different policy actions. The p-values reported in brackets come from an independent two-sample means t-test.

	6M	1Y	3Y	5Y	7Y	10Y
All Decisions (125)	-0.4	0.3	0.4	-0.1	0.1	0.0
	(0.16)	(0.35)	(0.22)	(0.92)	(0.58)	(0.82)
All Surprises (15)	-2.1	0.7	1.6	1.7	2.1	2.4
	(0.14)	(0.54)	(0.20)	(0.30)	(0.18)	(0.15)
Hikes (31)						
- Anticipated Hikes (27)	-0.1	-1.1	-1.3**	-1.9***	-1.7**	-1.8***
	(0.98)	(0.11)	(0.05)	(0.01)	(0.02)	(0.01)
- Surprise Hikes (4)	-5.1	0.9	1.2	2.9	3.1	4.6
	(0.16)	(0.73)	(0.65)	(0.63)	(0.60)	(0.49)
Cuts (28)						
- Anticipated Cuts (20)	-0.7	-0.4	-0.1	-0.6	-0.3	-0.2
	(0.22)	(0.68)	(0.99)	(0.55)	(0.83)	(0.90)
- Surprise Cuts (8)	-1.9	0.9	2.1	1.8	2.2	2.6
	(0.32)	(0.65)	(0.28)	(0.32)	(0.18)	(0.14)

Table 5: One-Day average change in the UK forward term premia in basis points. The estimated premia comes from the two-country model. The p-values reported in brackets come from an independent two-sample means t-test.

	6M - 2Y	2Y - 3Y	3Y - 5Y	5Y - 7Y	7Y - 10Y
All Decisions (125)	0.9**	0.8	0.4	0.1	-0.1
	(0.04)	(0.12)	(0.44)	(0.93)	(0.82)
All Surprises (15)	4.2**	5.0**	4.5*	4.0	3.6
	(0.03)	(0.05)	(0.09)	(0.15)	(0.20)
Hikes (31)					
- Anticipated Hikes (27)	-1.4**	-1.9***	-1.9***	-1.9**	-1.9**
	(0.03)	(0.01)	(0.01)	(0.03)	(0.05)
- Surprise Hikes (4)	6.8	9.3	9.7	9.5	9.2
	(0.30)	(0.33)	(0.36)	(0.38)	(0.39)
Cuts (28)					
- Anticipated Cuts (20)	0.4	0.4	0.4	0.3	0.2
	(0.70)	(0.71)	(0.74)	(0.79)	(0.83)
- Surprise Cuts (8)	4.7**	5.3**	4.4**	3.5	3.0
	(0.03)	(0.02)	(0.05)	(0.16)	(0.26)

Table 6: One-Day average change in the UK forward term premia in basis points. The estimated premia comes from the single-country model for the UK. The p-values reported in brackets come from an independent two-sample means t-test.

	6M - 2Y	2Y - 3Y	3Y - 5Y	5Y - 7Y	7Y - 10Y
All Decisions (125)	0.9**	0.8	0.4	0.1	-0.1
	(0.04)	(0.12)	(0.44)	(0.92)	(0.83)
All Surprises (15)	4.2**	5.0**	4.5*	4.0	3.6
	(0.03)	(0.05)	(0.09)	(0.15)	(0.20)
Hikes (31)					
- Anticipated Hikes (27)	-1.4**	-1.9***	-1.9***	-1.9**	-1.8*
	(0.03)	(0.01)	(0.01)	(0.03)	(0.06)
- Surprise Hikes (4)	6.8	9.4	9.8	9.7	9.4
	(0.30)	(0.33)	(0.35)	(0.37)	(0.38)
Cuts (28)					
- Anticipated Cuts (20)	0.4	0.5	0.4	0.3	0.3
	(0.70)	(0.70)	(0.72)	(0.75)	(0.78)
- Surprise Cuts (8)	4.7**	5.3**	4.4*	3.4	2.9
	(0.03)	(0.02)	(0.06)	(0.17)	(0.28)

Table 7: One-Day average change in the UK forward term premia in basis points. The premia reaction is estimated on the sub-sample from the beginning of January 1994 until the end of July 2007 and using the two-countries model. The p-values reported in brackets come from an independent two-sample means t-test.

	6M - 2Y	2Y - 3Y	3Y - 5Y	5Y - 7Y	7Y - 10Y
All Decisions (115)	0.7**	0.7	0.4	0.0	-0.2
	(0.05)	(0.11)	(0.39)	(0.91)	(0.78)
All Surprises (12)	3.8**	4.81*	4.5	3.9	3.5
	(0.05)	(0.08)	(0.15)	(0.27)	(0.35)
Hikes					
- Anticipated Hikes (27)	-1.3*	-1.7***	-1.8***	-1.7*	-1.7
	(0.06)	(0.01)	(0.01)	(0.07)	(0.15)
- Surprise Hikes (4)	6.2	9.3	10.4	10.8	10.9
	(0.24)	(0.26)	(0.29)	(0.32)	(0.34)
Cuts (21)					
- Anticipated Cuts (15)	0.3	0.3	0.3	0.2	0.1
	(0.79)	(0.79)	(0.80)	(0.83)	(0.86)
- Surprise Cuts (6)	2.9	3.4*	2.7	1.9	1.4
	(0.14)	(0.10)	(0.16)	(0.40)	(0.60)

Table 8: One-Day *weighted* average change in the UK forward term premia in basis points. The single reactions are weighted with the inverse of the size of policy rate move multiplied by 25, e.g. the reaction to a 75 basis point move is weighted by 25/75. The p-values reported in brackets come from an independent two-sample means t-test.

	6M - 2Y	2Y - 3Y	3Y - 5Y	5Y - 7Y	7Y - 10Y
All Decisions (125)	0.1	0.2	0.2	0.2	0.2
	(0.67)	(0.58)	(0.54)	(0.54)	(0.54)
All Surprises (17)	2.5**	3.6*	3.8*	3.9	3.9
	(0.05)	(0.07)	(0.10)	(0.14)	(0.16)
Hikes (31)					
- Anticipated Hikes (27)	-1.1*	-1.5***	-1.5**	-1.4*	-1.4
	(0.07)	(0.01)	(0.02)	(0.10)	(0.20)
- Surprise Hikes (4)	5.6	8.99	10.5	11.4	11.8
	(0.29)	(0.30)	(0.31)	(0.31)	(0.32)
Cuts (28)					
- Anticipated Cuts (20)	0.5	0.7	0.6	0.6	0.6
	(0.53)	(0.47)	(0.46)	(0.48)	(0.51)
- Surprise Cuts (8)	2.0**	2.4***	2.1**	1.8	1.6
	(0.02)	(0.01)	(0.04)	(0.13)	(0.23)

Figure 1: The US (Upper panel) and the UK (right panel) yield curves are plotted with the FOMC policy rate decisions to hike (solid green) or cut (dashed red) the Federal funds rate. The gray areas are NBER recessions in the US.
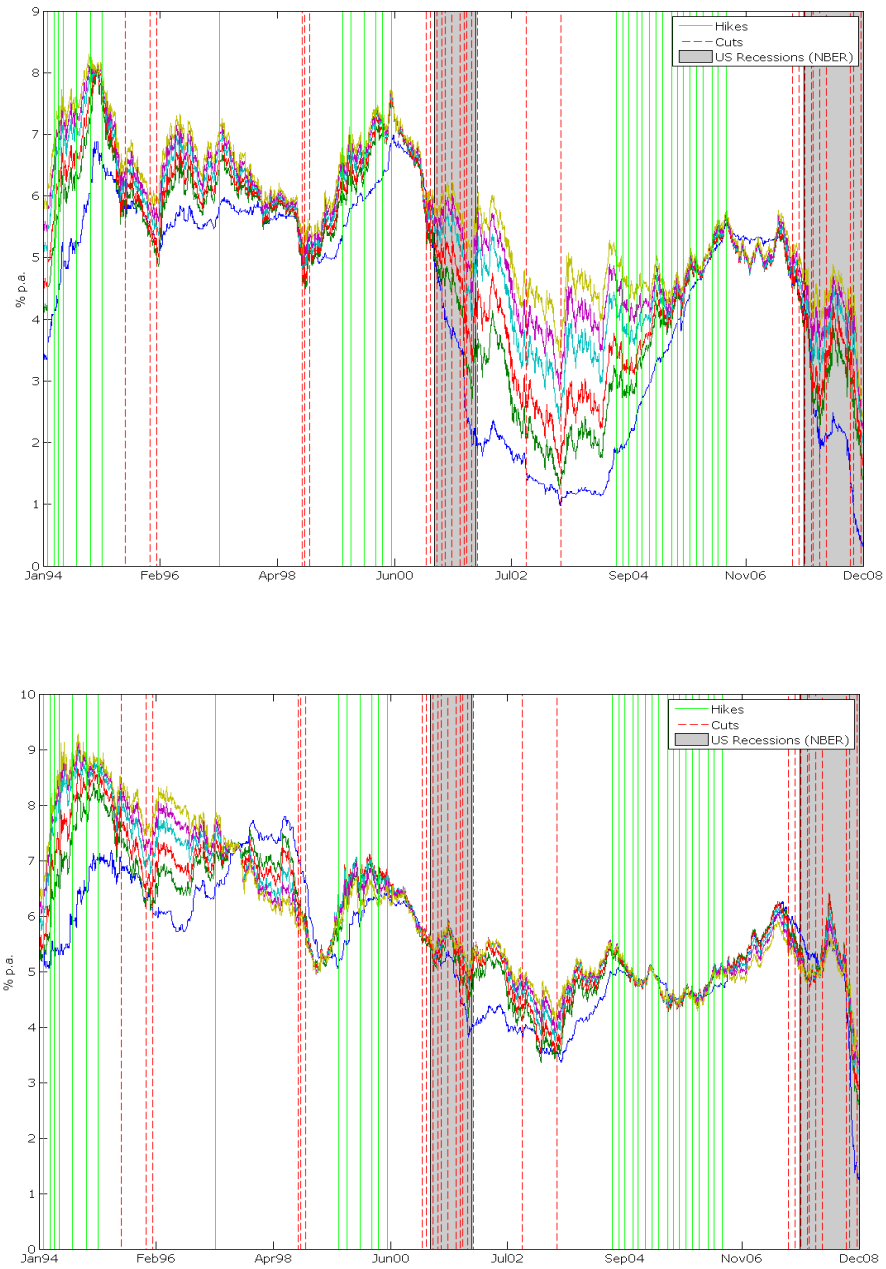
Figure 2: The Figure reports the histograms of sizes of the Fed funds rate increases/decreases for different policy actions. The x-axis is expressed in basis points and the y-axis shows the number of corresponding decisions.
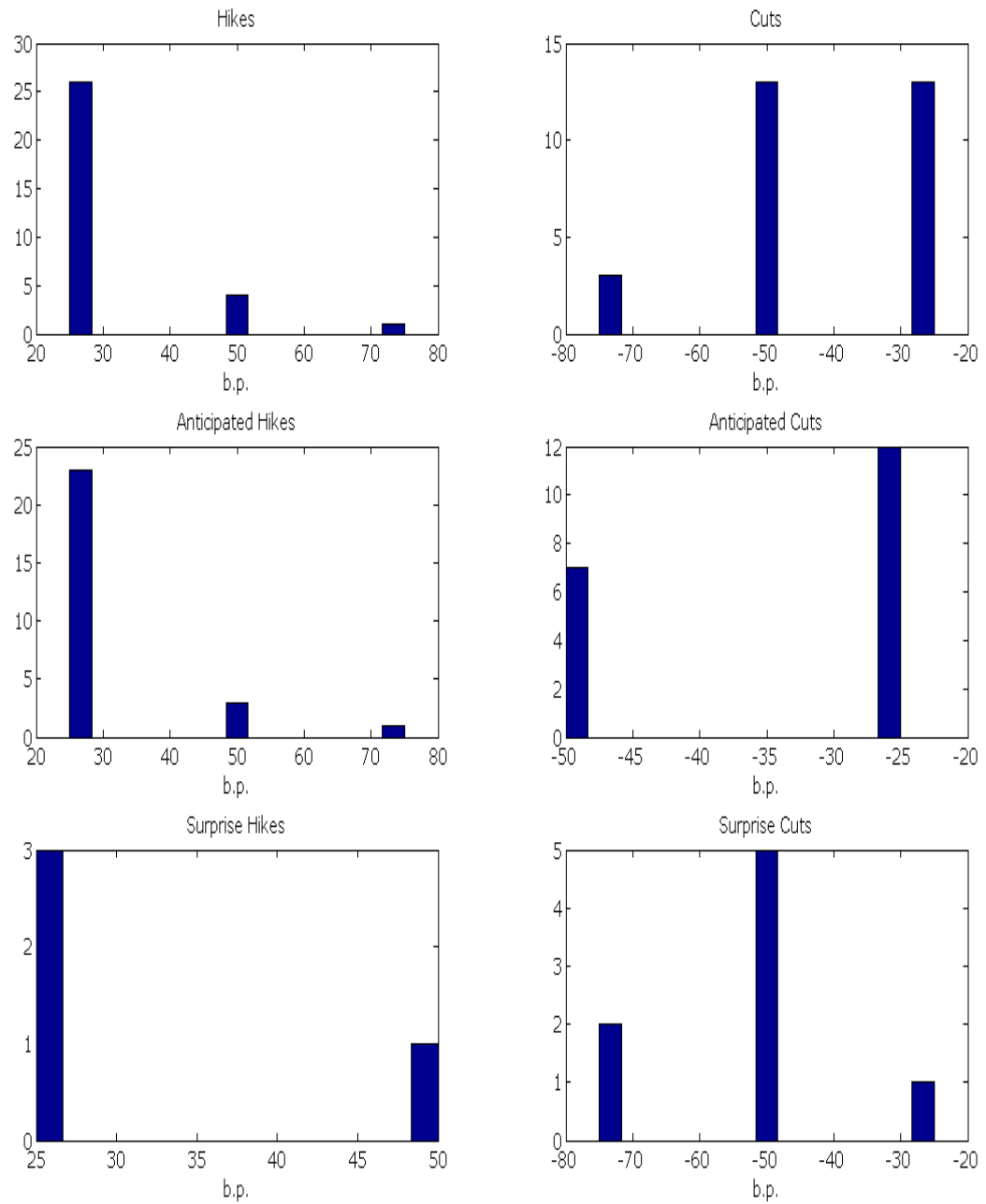
Figure 3: The Figure reports the histograms of surprise indicator calculated in equation (1) for different policy actions. The x-axis is expressed in basis points and the y-axis shows the number of corresponding decisions.
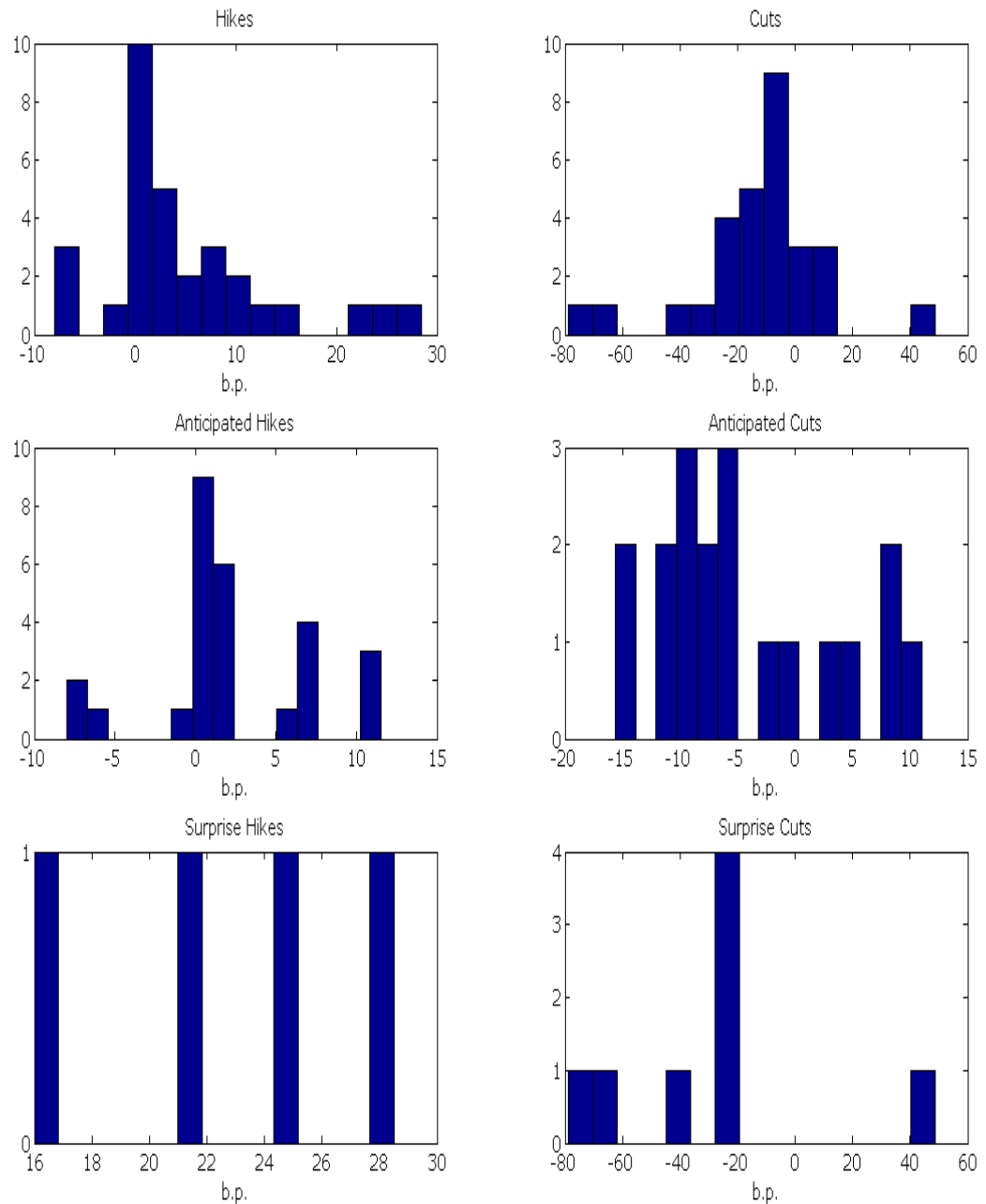
Figure 4: The Figure reports the decomposition of the UK term premia (solid red) to the part driven by the three UK factors (dashed blue) and US factors (solid black line).
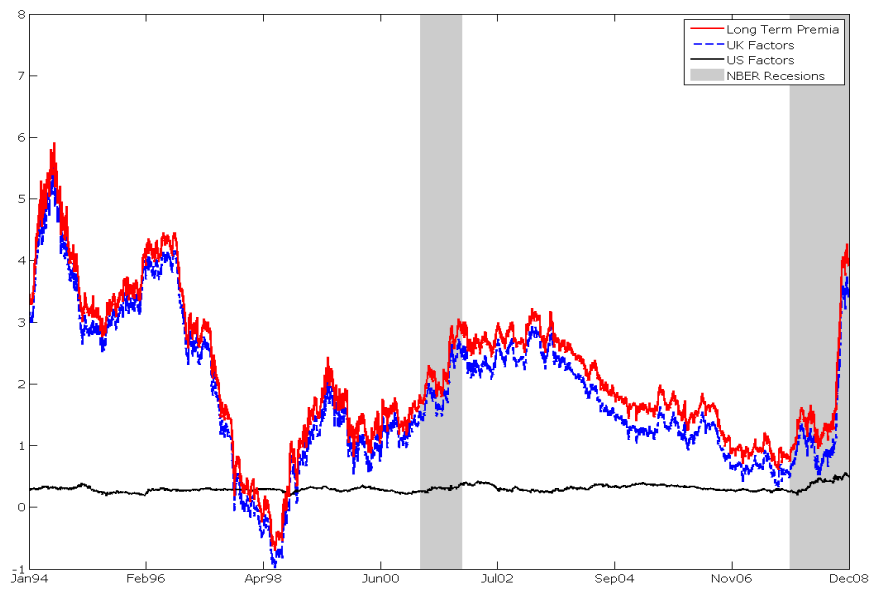
Figure 5: The upper panel illustrates the decomposition of the model-implied depreciation rate (solid black) to interest rate differential (solid blue) and foreign exchange risk premium (dashed red line). The lower panel plots the modeled depreciation rate (solid black) against the data (dashed green). The data depreciation rate is annualised and expressed in percentages.
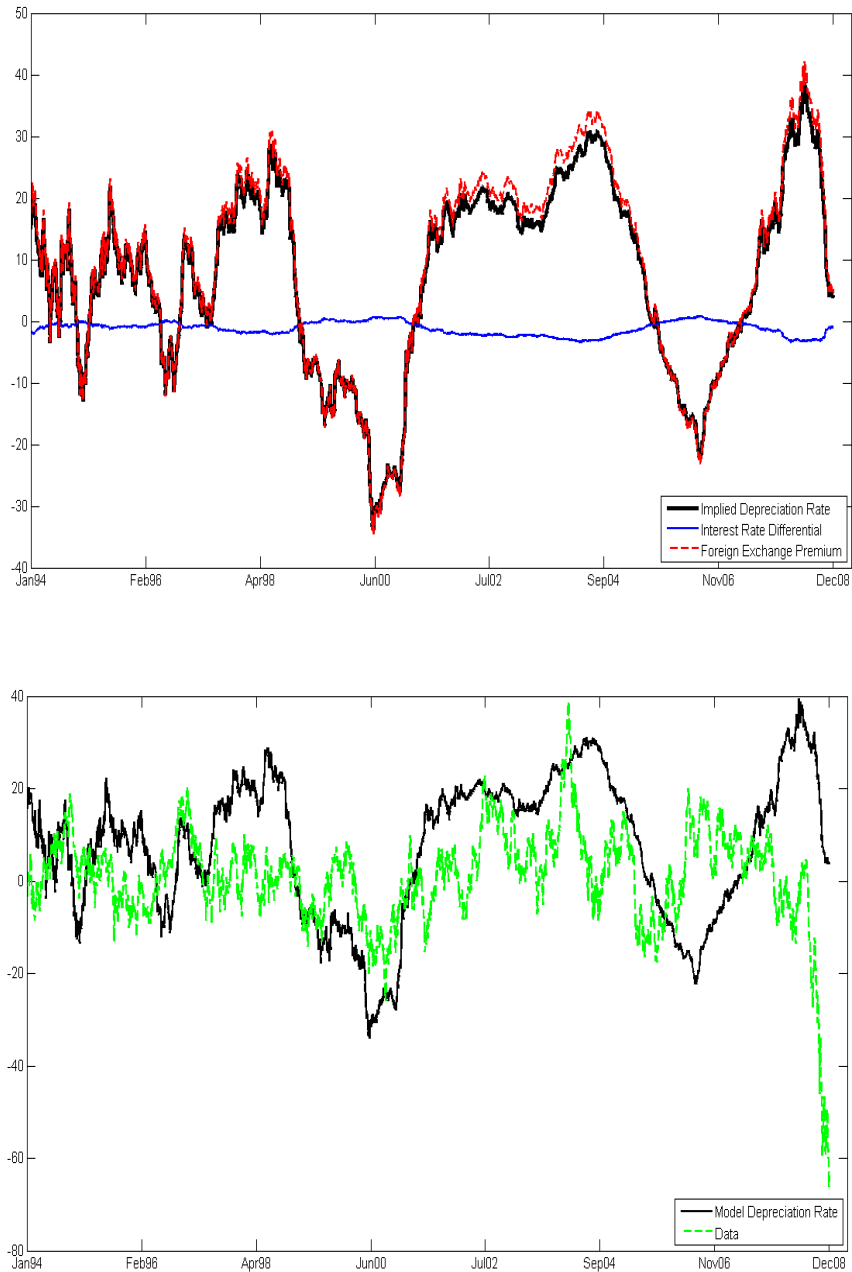
Figure 6: Table reports the impulse response functions of 6-month (solid line) and the 10-year (dashed line) UK yields to different policy shocks, together with 90 percent confidence intervals (grey areas). The confidence intervals are calculated by using the bootstrapping technique explained in the Section 5.