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Abstract

This paper analyzes the reaction of the S&P 500 returns to changes in implied volatility given by the VIX index, using a daily data sample from 1990 to 2012. We found that in normal regimes increases (declines) in the expected market volatility result in lower (higher) subsequent stock market returns. Thus, investors enter into selling positions upon a perception of increased risk for their equity investments, while they enter into long positions when they perceive an improved environment for those investments. However, for extreme regimes investors' reaction to increasing risk is ambiguous. We found that VIX variation significantly influences investment strategies for holding periods up to one month. Additionally we propose an investment rule for short-term oriented investors.

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1 Introduction

The interest of researchers and market practitioners in implied volatility, particularly the implied volatility index VIX,[2] has increased significantly over the past decade, based on the belief that it can be a good indicator or signalling instrument for adopting a certain position (i.e. long or short) in the equity markets.

Giot (2002) shows a clear negative correlation between the VIX and the S&P 100,[3] although he finds that periods of extremely high observations of the implied volatility index are subsequently followed by average positive returns, which are convenient from a mean-variance perspective only in the very short run. In a later work, Giot (2003) defines a more precise way of determining whether the VIX is 'sufficiently high' or not, by splitting the sample in percentiles over a rolling time window of two years, and proposes a trading rule according to the results found.

Banerjee, Doran & Peterson (2007) find that the VIX is significant to explain the expected return of the S&P 500. In particular, they analyze the relation between the VIX and the future return of portfolios sorted by several stock characteristics (i.e. Beta, size, valuation). Their main finding is that high Beta stocks' returns are more responsive to the VIX levels compared to low Beta stocks. Similarly, Ammann, Verhofen & Süss (2010) analyze the relation between implied volatility and future returns for the cross section, evaluating individual equities which are considered according to several characteristics. Consistent with Giot (2002), they show a positive relation between returns and lagged implied volatility. Particularly, they find that the relationship is stronger for small firms, as well as for stocks with lower degree of research analysts' coverage.

In accordance with these findings, many practitioners see a high reading of the VIX as a signal that the market is undervalued, possibly because of an overselling behavior of those investors who are more risk averse and who try to limit loses early. Thus, extremely high levels of the VIX could be probably signalling a market *floor*. Not surprisingly, the VIX is often referred to as the *fear index*.

[2] This index, computed by the Chicago Board Options Exchange (CBOE), intends to measure the implied volatility on the S&P 500. For more details please refer to the Methodology section of this paper.

[3] By that time, the VIX index provided the implied volatility on the S&P 100. In 2003, the CBOE changed its methodology and, among other modifications, switched to the S&P 500 instead.

The methodology of this paper has a starting point based on the aforementioned research studies, particularly Giot (2002) and Giot (2003). However, it incorporates the following main innovations:

i) We analyse the return realization of the S&P 500 subsequent to the *changes* or *variations* of the VIX, as opposed to analyzing the returns that follow high or low *levels* of this index. The motivation of this paper is to determine how investors react to strong fluctuations in expected volatility. Do they quickly enter into selling positions after observing higher future expected volatility in the markets, and vice versa? Consequently, we test if strong short-term variations of the VIX can provide a consistent trading signal, and develop a trading rule accordingly.

ii) We use the VIX index that has been reformulated by the CBOE in 2003. The index is now computed from a much larger options set on the equity index (including those out-of-the-money), rather than a set of just 8 in-the-money options used before, which we believe provides an enhanced informational content and is less susceptible to outliers. Another innovation is that the implied volatility is now computed on the S&P 500 rather than the S&P 100, while we consider this is beneficial since the S&P 500 might capture a broader and diversified investor base.

iii) We enjoy a much larger data set, with daily information from 1990 to 2012, which equals to over 5,600 observations. Our data includes the last financial crisis, which we believe increases the robustness of any results found.

This paper continues in the following way: Section 2 describes the data set. Section 3 presents the methodology employed and results obtained, together with their statistical and economic interpretation. Section 4 defines an investment rule based on those results and reports its computed performance. Section 5 concludes.

2 Data

We use daily data for the period running from 2-Jan-1990 to 31-Jul-2012, which equals to over 5,600 observations, for the series reflecting VIX and S&P 500 closing prices. Even though the CBOE changed its methodology for computing the daily VIX in 2003, it is possible to download the newly computed series from 1990 onwards from its website.[4] The data for the S&P 500 historic prices were also obtained from CBOE's website.[5]

The VIX is a volatility index introduced and computed by the Chicago Board Options Exchange (CBOE). It provides a measure of short term expected volatility in the equity market. It is worth pointing out that the VIX is forward-looking, so that it can be interpreted as the expectation that investors have on the S&P 500's volatility over the subsequent 30-day period.

It is further important to note that the index is *implied*. As opposed to traditional price indexes, the VIX is computed as a weighted average of the qualifying call and put options on the S&P 500. There is a defined criteria for determining the set of options to be considered, which can change every day and even intraday. Finally, the level of the index represents the expected volatility in percentage terms (i.e. a VIX level of 20 indicates an implied volatility of 20%).[6]

Using the VIX for our study is functional for the following main reason: we want to analyze the relation between sudden changes in the investors' perception of future volatility against the future realized returns in the equity market. We also benefit from the VIX being forward-looking and measuring investors' expectations (i.e. as opposed to using historical volatility). Finally, as opposed to the implied volatility that can be calculated from option prices (for instance, through the Black-Scholes model), the VIX is observable and publicly available to all market participants. Therefore, it is likely that the index is effectively considered by investors for making investment decisions.

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[4] http://www.cboe.com/VIX

[5] http://www.cboe.com/micro/IndexSites.aspx

[6] For a complete and detailed description of the VIX computation process and methodology please refer to Chicago Board Options Exchange, *VIX: The CBOE Volatility Index*, White Paper, 2009.

3 Methodology and Results

3.1 Contemporaneous analysis

Based on the methodology applied by Giot (2002), we study the contemporaneous relation between the VIX and the S&P 500 indices' variation. In other words, we analyze the simultaneous evolution of both indices during time periods of the same length, considering periods of 1, 5, 10, 22 and 44 days. The days referred are *trading days*, so that 5 days would represent a calendar week and 22 days a calendar month. We include different time periods in our analysis since we do not know in advance which of them (if any) would show the greatest significance. The longest time period considered (44 days, or 2 calendar months) is based on the findings of Banerjee, et al. (2007), who state that their results are stronger for this time horizon given that the VIX takes about 60 days to complete a mean reversion process.

The main variables are defined in the following way:

VIX_t is the daily closing value of the VIX index on day t

SP_t is the daily closing value of the S&P 500 index on day t

$$VIX_{i,t} = \ln(VIX_t) - \ln(VIX_{t-i}) \quad (1)$$

represents the relative change of the VIX over a period of i days (where i takes values 1, 5, 10, 22 and 44).

Similarly,

$$SP_{i,t} = \ln(SP_t) - \ln(SP_{t-i}) \quad (2)$$

Having defined these variables, the first series of linear regressions is the following:

$$SP_{1,t} = \beta_0 + \beta_1 \times VIX_{1,t} + \varepsilon_t \quad (3)$$

$$SP_{5,t} = \beta_0 + \beta_1 \times VIX_{5,t} + \varepsilon_t \quad (4)$$

$$SP_{10,t} = \beta_0 + \beta_1 \times VIX_{10,t} + \varepsilon_t \quad (5)$$

$$SP_{22,t} = \beta_0 + \beta_1 \times VIX_{22,t} + \varepsilon_t \quad (6)$$

$$SP_{44,t} = \beta_0 + \beta_1 \times VIX_{44,t} + \varepsilon_t \quad (7)$$

The series $VIX_{i,t}$ and $SP_{i,t}$ have been checked for stationarity for all the period lengths considered (denoted by i in the formulas). This ensures that the models tested are stable and allows making valid inferences from the results obtained. Augmented Dickey-Fuller tests results are exhibited in Table A.1 in the Appendix.

Table 1 presents the initial results, in which the existence of a negative relation between the two indices is clear. In order to make the slope coefficients (β_1) comparable across Table 1, we have standardized them. All the slope coefficients are negative and statistically significant, irrespective of the length of the time periods considered. This implies that increases of the VIX are associated to negative stock returns, while declines of the volatility index are associated to positive stock returns. This fact is not surprising, since it might represent the well known *fear factor* of the investors given by the VIX. Assuming that these are risk averse, once they perceive an increase in the risk of their investments (materialized by a volatility increase) they would turn into selling positions and therefore generate a decline in stock prices. As a consequence, we would expect that an increase in expected volatility would be more associated to a bearish stock market than to a bullish one. The results are consistent with those found by Giot (2002) performing a similar analysis. In order to illustrate this negative relation, Figure A.1 in the Appendix shows the historic evolution of the S&P 500 and VIX indices, in levels, for the whole sample period. Figure A.2 shows the same for the sub-sample 2008-2009.

We observe that the negative relation is bigger in magnitude for longer periods of time. The R^2 is also monotonically increasing with the time length considered, being 20.3% for the first case and 42.5% for the latter. Our interpretation is that longer periods of time allow more room for this negative relationship to materialize and thus become more evident and statistically significant. On the other hand, shorter periods of time (for instance, daily periods) might be more affected by noise, showing the negative relation as less clear or weaker. Figures A.3 and A.4 expose this fact showing a scatter plot for changes in the series for time horizons of 1 day and 44 days. The results are similar for the intermediate time horizons.

Table 1 also reports the constant of the regression equations (β_0), which represents the average return of the S&P 500 over the time horizons considered without the impact of the VIX changes. We note this constant is roughly proportionally increasing with the time horizons considered since it reflects the positive average return of the equity index over a longer time period, which will naturally be proportionally larger as well.

Finally, Table 1 reports the Durbin-Watson statistics and the t-statistics for the Augmented Dickey-Fuller tests ran on the residuals of each regression.[7] For all the cases, the Dickey-Fuller tests results obtained suggest the stationarity of the residuals, which imply that the model used is stable. Moreover, the presence of unit roots has been verified on the series of first differences for VIX and S&P 500 data and considering the different time lags used in the study. As shown in Table A.1 in the Appendix, in all cases the null hypothesis of unit root presence is rejected, suggesting the stationarity of the time series. On the other hand, the low Durbin-Watson statistics in Table 1 indicate the presence of positive serial autocorrelation in the residuals, which arise given that we compute the S&P 500 returns on a rolling basis using daily observations. To address this issue, we have considered the Newey-West standard errors of the estimated coefficients in order to better assess their individual significance.

[7] The Null Hypothesis of the tests is that there exists a unit root in the residuals series.

Table 1

Variable (% change)			VIX (independent)				
			1D	5D	10D	22D	44D
S&P 500 (dependent)	1D	β_0	0.0002**				
		β_1	-0.4512***				
		R^2	20.3%				
		DW	2.08				
		ADF*	-63.63				
	5D	β_0		0.0012**			
		β_1		-0.5563***			
		R^2		30.9%			
		DW		0.57			
		ADF		-12.04			
	10D	β_0			0.0025***		
		β_1			-0.5834***		
		R^2			34.1%		
		DW			0.34		
		ADF			-11.15		
	22D	β_0				0.0053***	
		β_1				-0.6402***	
		R^2				41.0%	
		DW				0.19	
		ADF				-9.40	
	44D	β_0					0.0106***
		β_1					-0.6517***
		R^2					42.5%
		DW					0.10
		ADF					-9.42

* Critical test values for 1% level are -3.43. This table shows the results of the regressions between the contemporaneous changes of the VIX and S&P 500 indices. The stars notation ***, ** and * indicate statistical significance of the coefficients at 1%, 5% and 10% levels, respectively.

3.2 Forward-looking analysis

In the second step of our study we perform a similar analysis but lagging the changes in implied volatility given by the VIX. Our objective is to determine the existence of a significant and constant relation between the innovations in the VIX and the return[8] of any investment made in the S&P 500 in the subsequent days, for all different time period lengths considered. The question we try to answer is the following: do the VIX *changes* provide a consistent trading signal for being long or

[8] Total holding period return

short in the equity market? To try to answer this question, we set up similar linear regressions, but lagging the changes in the VIX and also considering different period lengths for both the VIX and the S&P 500 variation. The same as in the prior section, we consider different time periods duration since we do not know in advance which of them will show the greatest significance. The linear regressions have the following form:

$$\mathrm{SP}_{\mathrm{i,t+i}} = \beta_0 + \beta_1 \times \mathrm{VIX}_{\mathrm{j,t}} + \varepsilon_t \qquad (8)$$

with i and j taking values of 1, 5, 10, 22 and 44 (and not necessarily the same).

As an example, with i = 5 and j = 1, we use the 1 day variation of the VIX to try to explain the subsequent 5 days holding period return of the S&P 500, *after* the 1-day VIX variation has already occurred.

Table 2 summarizes the results for this section. For comparison among the different time lags we present again standardized coefficients. For most of the time period combinations tested, the slope coefficients (β_1) are negative (23 out of 25), with many of them statistically significant at 1% or 5% confidence levels. In other words, for most of the time periods combinations considered, positive variations of the VIX are followed by subsequent negative stock returns on average, and vice-versa.

In terms of magnitude, we note that the impact of the VIX variation on the returns (given by β_1) is much smaller than the contemporaneous relation shown in Table 1. A possible explanation for this fact would be that the aforementioned *fear factor* effect is strongest at the same moment of the perceived risk increase. After this moment, the effect could lose its strength during the subsequent days, although it could still remain significant. It is worth noting that most of the slope coefficients lose significance for time periods of 1 and 2 months, both for the holding period and for the VIX variation period (lower-right section of Table 2). This suggests that investors concerned about implied volatility might consider only very short term horizons to react to this variable. Thus, it could be expected that the VIX variation does not carry any more informative content for explaining stock returns for time periods exceeding one month.

Even though the R^2 obtained are very low, we note that the highest among the results (0.7%) is found for 10-day time periods, both for the VIX changes and for the subsequent S&P 500 holding period. We interpret that equity investors who react to implied volatility changes are mostly concerned about bi-weekly periods, both for reading the changes in the VIX (and reacting accordingly) and for holding their investments afterwards. We further observe that for the 1-day returns the highest R^2 is obtained for 5-day VIX change periods, while for 5-day returns the highest is obtained with 1-day VIX changes. This finding suggests that investors might be typically concerned about weekly time periods, either for reacting to VIX changes or for holding their investments for that time length. We note, however, that the relationship is not significant for 5-day variation in the VIX against the subsequent 5-day holding periods returns. Finally, the R2 drops to 0 when considering time periods of over 1 month, consistent with the prior findings and again suggesting the short-term nature of investors' consideration of the VIX fluctuation.

In terms of magnitude, looking at the standardized slope coefficients (β_1) we observe that the time period combination with the highest impact on returns is that one for 10-day periods (both for VIX and S&P 500 changes), which is the combination that exhibited the highest R2, reinforcing our consideration about bi-weekly periods. We further note that the impact seems to extinguish when considering periods of 1 and 2 months (lower-right section of the Table), again in concordance with our prior inferences based on the coefficients' significance.

The same as in the prior section, Table 2 reports the Durbin-Watson statistics and the t-statistics for the Augmented Dickey-Fuller tests ran on the residuals of each regression. For all cases the Dickey-Fuller results obtained show stationary residuals. On the other hand, the low Durbin-Watson statistics indicate the presence of positive serial autocorrelation in the residuals, which we address by considering the Newey-West standard errors of the estimated coefficients in order to make inferences about their statistical significance.

Table 2

Variable (% change)			VIX (independent)				
			1D	5D	10D	22D	44D
S&P 500 (dependent)	1D (1 lead)	β_0	0.0002*	0.0002*	0.0002*	0.0002*	0.0002*
		β_1	0.0190	-0.0692***	-0.0624***	-0.0539***	-0.0375**
		R^2	0.0%	0.5%	0.4%	0.3%	0.1%
		DW	2.09	2.16	2.15	2.14	2.13
		ADF	-56.93	-59.06	-33.16	-58.06	-57.67
	5D (5 leads)	β_0	0.0012**	0.0012**	0.0012**	0.0012**	0.0012**
		β_1	-0.0629***	-0.0176	-0.0553**	-0.0477	-0.0333
		R^2	0.4%	0.0%	0.3%	0.2%	0.1%
		DW	0.53	0.49	0.50	0.50	0.49
		ADF	-12.37	-12.24	-12.25	-12.46	-12.59
	10D (10 leads)	β_0	0.0025**	0.0025**	0.0025**	0.0025**	0.0025**
		β_1	-0.0529***	-0.0516*	-0.0819**	-0.0654	-0.0414
		R^2	0.3%	0.3%	0.7%	0.4%	0.2%
		DW	0.28	0.26	0.27	0.26	0.26
		ADF	-11.01	-10.94	-10.90	-11.15	-11.26
	22D (22 leads)	β_0	0.0054***	0.0054***	0.0054***	0.0054***	0.0054***
		β_1	-0.0424***	-0.0415	-0.0606	-0.0497	-0.0100
		R^2	0.2%	0.2%	0.4%	0.2%	0.0%
		DW	0.13	0.13	0.13	0.12	0.12
		ADF	-10.00	-9.82	-9.84	-9.82	-9.85
	44D (44 leads)	β_0	0.0108***	0.0108***	0.0108***	0.0108***	0.0108***
		β_1	-0.0274**	-0.0272	-0.0354	-0.0112	0.0032
		R^2	0.1%	0.1%	0.1%	0.0%	0.0%
		DW	0.07	0.06	0.06	0.06	0.06
		ADF	-8.84	-9.52	-9.49	-9.34	-9.29

This table shows the results of the regressions between the changes of the VIX and the subsequent changes of the S&P 500 for different time periods. As an example, cell 5D-10D refers to the VIX variation in 10-day periods "t" (columns), against the S&P 500 variation in 5-day periods "t+5" (rows). The stars notation ***, ** and * indicate statistical significance of the coefficients at 1%, 5% and 10% levels, respectively.

3.3 Extreme events

In this section we consider the series of VIX variation for each of the specified time horizons and observe their lowest and highest 10% percentiles observations. Next, we compute the returns of the S&P 500 in the subsequent days after those extreme VIX change observations, also considering different holding periods. In addition, we compute the standard deviation of those returns in order to assess the convenience of those potential investments from the perspective of a mean-variance investor.

We perform this analysis as a stress testing study, since we want to understand the implication of extreme VIX events on the subsequent S&P 500 returns. In other words, what happens in the equity markets if the VIX increases by 50% in just a day or a week? Conversely, what happens to the S&P 500 if the VIX declines rapidly by that magnitude? Stress testing has been increasingly implemented by economic and financial studies, and is also being ever more used and demanded by regulatory agencies in order to better understand and measure the risk exposure of companies (particularly banks), industries and economies. The occurrence of significant extreme events for the financial markets such as the Russian default in 1998, and most importantly the financial crisis which started in 2008, has evidenced that the typical risk analysis techniques used so far were not enough to predict or even expect such extreme scenarios. Of particular relevance for the financial sector has been the consideration of this topic in the amended Basel II and in Basel III framework, which for instance stipulate that banks "must conduct stress tests that include widening credit spreads in recessionary scenarios."[9, 10]

The results of this exercise are presented in Table 3. These clearly confirm the findings from the prior analysis: the extreme negative variations of VIX (the bottom 10% of the variations sample) were always followed by positive returns on average. Further, for nearly all time periods considered the returns and Sharpe ratios obtained are larger than the S&P 500 average over the entire sample. The opposite (i.e. for extreme positive VIX variations) is not necessarily true, since the subsequent S&P 500 returns are mixed between positive and negative. However, in line with our expectations the returns are consistently lower than those of the equity index over the entire sample. Figures A.5 and A.6 expose these findings.

[9] World Bank, *Global Financial Development Report 2013: Rethinking the Role of the State in Finance*, World Bank Publications, Sep 17, 2012 - p. 59

[10] See Aepli, Matthias (2011), *On the Design of Stress Tests,* Masters' Thesis, University of St. Gallen

The results found could have a logical explanation from the perspective of the investors' risk aversion. Upon perceived increases in risk, it may be clear that investors would enter into selling positions generating stock prices to fall. Conversely, a reduction in perceived risk would encourage investors to enter into long positions, as the expected environment for investments would have improved. As a consequence, we would expect stock prices move down after risk perception increases and move up after such risk perception declines. Our results indicate however that extreme negative shock to implied volatility translate into subsequent positive stock returns, while positive shocks result in mixed returns. Intuitively, investors might increase their confidence in the investment scenario once they perceive declining expected volatility more than they lose confidence when they perceive increasing risk.

The results found are quite surprising when put together with the conclusions of prior studies such as Giot (2002, 2003). In these studies, a very high *level* of the VIX is associated to positive subsequent returns on average for the very short-run, since a high level of the index might indicate an oversold equity market. However, the results found here could be considered complementary, as they assess the returns that occur after large *increases* of the volatility index.

We further observe that the standard deviations computed tend to be large compared to the returns achieved. For instance, in all the cases the standard deviation of the returns is larger than the corresponding average return. As a consequence, the Sharpe ratios obtained from the sample are close to 0, with a maximum of 0.24. This is to say that, in the best of the cases, the risk assumed (standard deviation) is 4 times the magnitude of the expected return, or that each unit of risk assumed is compensated by just 0.24 units of expected return. Although considering longer holding periods (i.e. yearly), a standard investment in the S&P 500 reports an average historic return of 7.7% with standard deviation of 18.7%, resulting in a 0.41 Sharpe ratio.[11] However, the results have been directionally consistent, and the criteria might be useful for short-term investors who enjoy low transaction costs, such as banks or hedge funds. The highest Sharpe ratios in absolute terms were found for returns that followed periods of 5 days of

[11] Considering yearly returns for our sample period, 1990-2011.

duration (i.e. 1 trading week) of extreme VIX variation. Again, the results are more consistent for negative VIX changes (and subsequent positive S&P 500 returns) than for positive VIX variation.

Table 3

Period Δ		Bottom 10% of VIX Change			Top 10% of VIX Change		
VIX	SP	Avg Return	StDev	Sharpe	Avg Return	StDev	Sharpe
1d	1d	0.00%	1.35%	0.00	-0.03%	1.48%	-0.02
1d	5d	0.42%	2.97%	0.14	-0.17%	3.04%	-0.06
1d	10d	0.49%	3.97%	0.12	0.00%	3.91%	0.00
1d	22d	0.91%	5.63%	0.16	0.18%	5.36%	0.03
1d	44d	1.32%	7.17%	0.18	0.57%	7.24%	0.08
5d	1d	0.18%	1.21%	0.15	-0.14%	1.73%	-0.08
5d	5d	0.20%	2.48%	0.08	-0.10%	3.31%	-0.03
5d	10d	0.60%	3.42%	0.18	-0.27%	4.15%	-0.06
5d	22d	1.13%	4.67%	0.24	0.20%	6.00%	0.03
5d	44d	1.60%	6.61%	0.24	0.49%	8.26%	0.06
10d	1d	0.13%	1.09%	0.12	-0.11%	1.77%	-0.06
10d	5d	0.20%	2.23%	0.09	-0.09%	3.32%	-0.03
10d	10d	0.58%	3.14%	0.18	-0.19%	4.31%	-0.04
10d	22d	0.86%	4.39%	0.20	-0.20%	6.91%	-0.03
10d	44d	1.14%	7.19%	0.16	0.24%	8.73%	0.03
22d	1d	0.11%	1.02%	0.11	-0.04%	1.67%	-0.03
22d	5d	0.21%	2.06%	0.10	-0.08%	3.27%	-0.02
22d	10d	0.37%	2.74%	0.13	-0.05%	4.36%	-0.01
22d	22d	0.59%	4.25%	0.14	0.18%	6.28%	0.03
22d	44d	0.73%	7.20%	0.10	0.74%	7.93%	0.09
44d	1d	0.05%	0.99%	0.05	-0.07%	1.71%	-0.04
44d	5d	0.13%	2.11%	0.06	-0.08%	3.42%	-0.02
44d	10d	0.25%	2.92%	0.09	-0.01%	4.39%	0.00
44d	22d	0.29%	4.42%	0.07	0.47%	5.85%	0.08
44d	44d	0.92%	6.79%	0.13	1.18%	7.38%	0.16

S&P 500 - Full Sample			
Holding period	Avg Return	StDev	Sharpe
1d	0.02%	1.18%	0.02
5d	0.12%	2.44%	0.05
10d	0.24%	3.27%	0.07
22d	0.54%	4.79%	0.11
44d	1.08%	6.73%	0.16

Table 3 exposes the mean and standard deviation of the returns obtained after the extreme variations of the VIX (top and bottom deciles). The left panel shows the results corresponding to the bottom 10% of VIX variation (i.e. extreme negative changes). Within the columns corresponding to the periods, the left figure refers to the VIX variation period, while the figure on the right refers to the S&P 500 return period. As an example, the 10d - 22d row denotes the mean and standard deviation of the 22-day returns that followed extreme negative 10-day variations of the VIX. For benchmarking purposes, the S&P 500 statistics for the entire sample are reported in the lower section.

3.4 Exclusion of 2008-2012 period

The year 2008 was nearly unprecedented for the financial markets in terms of collapses in asset prices and spiking volatility levels. The turmoil and nervousness originated in that period might not have been extinguished even today. For that reason, we conduct the same analysis done in sections 3.1, 3.2 and 3.3, this time excluding the period 2008-2012 from our sample. With this, we intend to check whether the results found would have been the same provided that the unusual market conditions that started in 2008 would have not occurred. On the contrary, it might be the case that a structural change had taken place after this period.

Table A.2 in the Appendix presents the results of the first analysis. It can be observed that they do not differ materially from those shown in Table 1. The relation between the VIX variation and the S&P 500 returns for contemporaneous changes is negative for all time periods considered and the coefficients are significant at 1% level. However, the magnitude of the slope coefficients is slightly smaller in this exercise compared to those in Table 1, which suggests that the contemporaneous relation between implied volatility movements and market returns might have become stronger with the financial crisis. The same as in Section 3.1, we keep the observation that the correlation between the series is larger for longer periods of time. Naturally, the constants are higher in this case since they represent the average return of the S&P 500 for a sample period which excludes the financial crisis of 2008.

Similar findings result from observing Table A.3, which shows the coefficients for the relation between the VIX changes and the forward-looking S&P 500 returns. Despite the magnitude of the slope coefficients differ slightly from that found in Table 2, they are again mostly negative (21 out of 25) and in many cases significant at 1% or 5% levels. Hence, we keep the interpretation of the results provided in the prior sections and conclude that the results obtained have not been materially affected by the inclusion (or exclusion) of the period 2008-2012 in our sample.

Finally, Table A.4 presents the S&P 500 performance for the periods related to those cases of extreme VIX variation. Compared to Table 3 seen before, we observe the same pattern: extreme negative VIX variation is always followed by positive stock returns on average, while equity returns after positive extreme VIX changes are mixed. For both the left section of the table (negative VIX changes) and the right section (positive VIX changes) the average returns and Sharpe ratios obtained tend to be higher than those computed using the complete sample, given that the financial crisis period is now excluded. Further, we note that the time horizons which present the highest Sharpe ratios are the same as those found considering the whole sample (the periods following extreme VIX variations of 5 trading days).

4 Investment rule development

Based on the results found in the prior section we establish an investment rule to determine the most convenient trades from a mean-variance perspective. We have found that large negative VIX variations are always followed by positive returns on average, while the conclusion is not so clear for large positive VIX changes. Thus, we focus on the first section in order to look for the best expected investment results.

In order to assess what is a sufficiently large VIX change which will trigger a suggested trade, we consider the bottom 1% and 10% percentiles of variation. Similar to the procedure followed by Giot (2003), our sample at every point in time consists of a rolling and backwards-looking 2-year time window. In other words, we compare the VIX variation of each period against the sample of VIX variation over the last 2 years at that point in time, and we do this on a rolling basis. We arbitrarily set a 2-year size in order to capture sufficient enough changes, and at the same time do not consider a larger time frame in order to allow for structural changes in the VIX volatility and behavior.

As we have seen in the prior section, the highest Sharpe ratios are obtained for investments that follow extreme VIX variations of 5 days (i.e. one trading week) and hold the position over a period of 2 weeks, 1 month and 2 months. Therefore, we would expect to find similar results with our proposed investment rule.

Table 4 shows these results for the full data set. Years 1990 and 1991 were used as the formation period, while the results obtained correspond to the period 1992-

2012. As expected, the returns that follow large negative variations of the VIX are positive on average. For the 10% extreme negative observations, the average returns obtained are increasing with the length of the holding period. 10-day holding periods resulted in a 0.6% return on average, while 44-day holding periods (2 months) showed a 1.8% average return. Standard deviation is also increasing with the holding period, although the Sharpe ratio computed is increasing as well. The increment of the average returns more than offset the increase of the standard deviation of those returns.

The middle section of the table shows the results that followed the 1% lower tail of VIX changes. Consistent with our expectations, the average returns obtained after these extreme occurrences are larger than those considering the 10% lower tail. 10-day holding periods resulted in a 0.9% return on average, while 44-day periods represented a 2.4% average gain. The standard deviation of these returns is not materially larger than those from the first sample, which result in increased Sharpe ratios: 0.24 for 10 days and 0.39 for 44 days. The drawback is that the occurrence of these events is by definition much less frequent, since we are considering just the lowest 1% percentile of the VIX variation series.

For benchmarking purposes, we have included the return and standard deviation figures for the S&P 500 over the whole sample (i.e. not just the periods corresponding to VIX tail events). It can be observed that the average returns are lower and the standard deviations higher than for the periods considered by our investing rule, thus clearly showing that the rule dominates the position of being always long on the S&P 500.

Despite the higher Sharpe ratios obtained out of the defined investment rule we still consider that they are low in absolute terms. This is to say that the risk associated to the suggested investments is still large compared to the expected returns. For instance, the returns obtained are always significantly smaller than their standard deviation. Therefore, the convenience of an investment recommendation based solely on this rule is still not clear. In addition, potential transaction costs would mitigate or even cancel off any realized gains from the investments, in particular considering that the suggested investments should occur within holding periods of just 2, 4 or 8 weeks. Thus, we reinforce that this investment criteria could be useful only for short-term oriented investors who enjoy low or negligible transaction costs.

Table 4

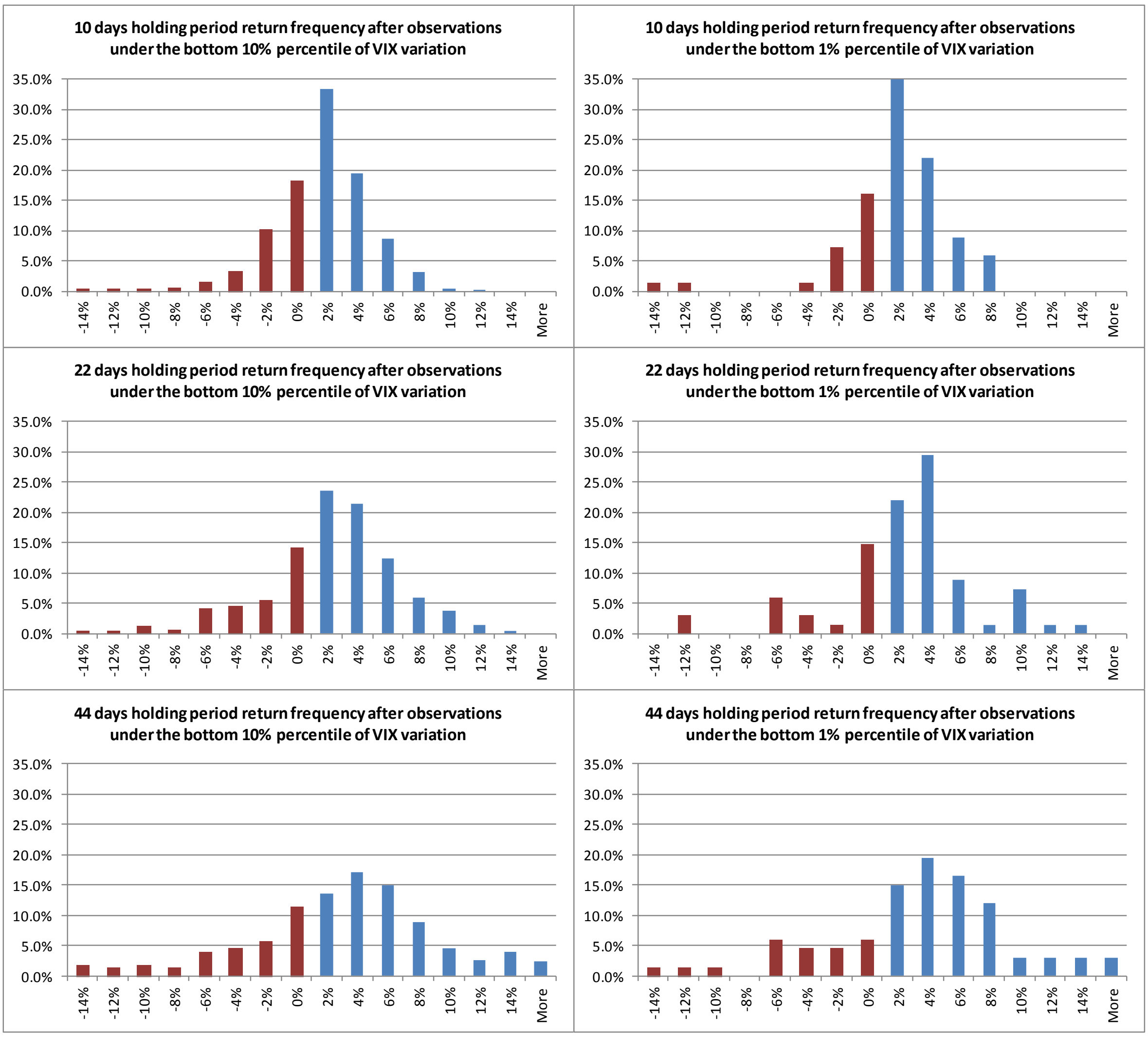
		Forward-looking S&P 500 Holding Period Return			
			10D	22D	44D
VIX Variation Percentile (bottom tail)	10%	Mean	0.6%	1.2%	1.8%
		St. Dev	3.3%	4.5%	6.4%
		Sharpe	0.18	0.26	0.29
		#	538	536	532
	1%	Mean	0.9%	1.6%	2.4%
		St. Dev	3.7%	4.8%	6.3%
		Sharpe	0.24	0.32	0.39
		#	68	68	67
Benchmark	S&P 500 full sample	Mean	0.2%	0.5%	1.1%
		St. Dev	3.3%	4.8%	6.7%
		Sharpe	0.07	0.11	0.16
		#	5680	5668	5646

Table 4 shows the holding period returns that followed the extreme (negative) variations of the VIX over our sample, considering the 10% tail events and the 1% tail events separately. The returns that follow the 1% VIX tail events show consistently higher average returns and Sharpe ratios for every holding period considered. However, naturally the VIX 1% tail events are much less frequent than the 10% cases. As a benchmark, the lower section of the table shows the returns and standard deviation of the S&P for the whole sample, considering the same holding periods.

Figure 1 provides a histogram of the returns observations for the three different holding periods considered and for the two scenarios considered for negative VIX variation (lower 10% tail and lower 1% tail). As previously seen, longer holding periods show higher returns on average (positive returns are more frequent, hence the histogram is shifted to the right). On the other hand, the dispersion is also higher and more extreme returns are present. This is mitigated when considering the whole sample average and Sharpe ratio. However, this would be a negative fact for investors concerned about maximum possible losses.

Another observation, as previously seen, is that the returns that follow from the 1% extreme VIX changes are on average higher than those that follow the 10% VIX variation tail, as the histograms are slightly more geared towards the right. The dispersion is also generally smaller, which results in the aforementioned higher Sharpe ratios, although some extreme negative outliers are still present for all the holding periods considered.

Figure 1



5 Conclusion

This study proposed an alternative approach to the analysis that previous research has conducted on the relation between the implied volatility index VIX and the S&P 500 returns. In contrast to what has been the centre of attention of existing research studies to date, the main contribution and focus of this project has been to assess the reaction of market participants to changes in their expected volatility on the equity market, rather than evaluating the consequences of the current levels of implied volatility.

The analysis was carried on through a variety of approaches. First, we studied the immediate or contemporaneous relation between the evolution of the VIX and the S&P 500 returns. Second, we applied a lag to the VIX changes in order to determine the subsequent reaction of equity prices to realized changes in implied volatilities, considering several time period combinations for the computation of both the VIX changes and the S&P 500 returns. Finally, we carried on an extreme events analysis to observe the reaction of market participants to extreme observations of implied volatility changes (both positive and negative).

All the methodologies applied converged to a single direction in the results. The major finding has been that positive changes in implied volatility lead to subsequent lower stock returns on average, while declines in expected volatility result in increased stock returns. This is consistent with an intuitive explanation which assumes investors' risk aversion. However, when looking at extreme fluctuations in the risk level investing strategies are ambiguous: investors do not necessarily lose confidence when they perceive extreme increases in risk, while they seem to consistently increase confidence upon the perception of sharp declines in risk.

Another major finding has been that the impact of VIX fluctuations vanishes when considering time periods beyond 1 month of duration. We concluded, in contrast, that investors are typically concerned about weekly and bi-weekly periods for observing implied volatility changes and holding period returns. Hence, the VIX fluctuation should be considered as a short term oriented indicator when assessing its impact on subsequent investment returns.

The inclusion of the 2008 financial crisis and subsequent period has not materially affected our results. We note, however, a slight increase in the contemporaneous correlation between VIX changes and S&P 500 returns when considering the complete sample (i.e. including the crisis), which might suggest that market participants are assigning more relevance to the evolution of this volatility indicator since the occurrence of such stress period.

Based on the consistency of the results found, we have proposed an investment rule which exploits the main findings on the relation between implied volatility changes and subsequent stock returns. The magnitude of the expected returns is not large compared to their standard deviation, while the holding period evaluated is for the short run (i.e. 2, 4 and 8 weeks). Thus, the investment rule developed might be only exploitable by short-term oriented investors who enjoy sufficiently low or insignificant transaction costs, such as institutional investors.

A possible extension to the analysis done would be to assess the impact of the VIX changes on the equity market returns when other significant variables are included in the models. Further, a natural extension would imply analyzing the impact of implied volatility *changes* on stocks' returns for the cross section, evaluating individual equities.

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Appendix: Tables and Figures

Table A.1

Augmented Dickey-Fuller tests for ΔSP and ΔVIX

		ΔSP_1	ΔVIX_1
ADF test statistic:		-57.306	-25.954
Critical values:	1% level	-3.431	-3.431
	5% level	-2.862	-2.862
	10% level	-2.567	-2.567
p-values*		0.000	0.000

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller tests for SP_5, VIX_5, SP_{10}, VIX_{10}, SP_{22}, VIX_{22}

		ΔSP_5	ΔVIX_5	ΔSP_{10}	ΔVIX_{10}	ΔSP_{22}	ΔVIX_{22}
ADF test statistic:		-12.749	-13.355	-11.331	-12.176	-10.032	-11.919
Critical values:	1% level	-3.431	-3.431	-3.431	-3.431	-3.431	-3.431
	5% level	-2.862	-2.862	-2.862	-2.862	-2.862	-2.862
	10% level	-2.567	-2.567	-2.567	-2.567	-2.567	-2.567
p-values*		0.000	0.000	0.000	0.000	0.000	0.000

*MacKinnon (1996) one-sided p-values.

Table A.2

Variable (% change)			VIX (independent)				
			1D	5D	10D	22D	44D
S&P 500 (dependent)	1D	β_0	0.0003***				
		β_1	-0.2741***				
		R^2	7.5%				
		DW	2.01				
		ADF	-53.53				
	5D	β_0		0.0017***			
		β_1		-0.4637***			
		R^2		21.5%			
		DW		0.55			
		ADF		-11.50			
	10D	β_0			0.0033***		
		β_1			-0.5112***		
		R^2			26.2%		
		DW			0.35		
		ADF			-10.72		
	22D	β_0				0.0073***	
		β_1				-0.5786***	
		R^2				33.5%	
		DW				0.20	
		ADF				-9.06	
	44D	β_0					0.0145***
		β_1					-0.5846***
		R^2					34.2%
		DW					0.12
		ADF					-8.83

Table A.2 shows the results analogous to those shown in Table 1 (contemporaneous relation between VIX changes and S&P 500 returns), in this case excluding the period 2008-2012 from the data sample. The results do not differ materially, being all the coefficients negative and statistically significant. However, the slope coefficients are lower than those shown in Table 1 for every time period considered. The stars notation ***, ** and * indicate statistical significance of the coefficients at 1%, 5% and 10% levels, respectively.

Table A.3

Variable (% change)			VIX (independent) 1D	5D	10D	22D	44D
S&P 500 (dependent)	1D (1 lead)	β_0	0.0003**	0.0003***	0.0003***	0.0003**	0.0003**
		β_1	-0.0265	-0.1621***	-0.1176***	-0.0999***	-0.0600***
		R^2	0.1%	2.6%	1.4%	1.0%	0.4%
		DW	2.05	2.14	2.08	2.06	2.04
		ADF	-68.82	-83.87	-30.63	-69.11	-68.27
	5D (5)	β_0	0.0016***	0.0016***	0.0016***	0.0016***	0.0016***
		β_1	-0.1433***	-0.0682**	-0.0765**	-0.0510	-0.0135
		R^2	2.1%	0.5%	0.6%	0.3%	0.0%
		DW	0.53	0.46	0.46	0.45	0.45
		ADF	-14.89	-11.45	-11.48	-11.70	-11.64
	10D (10)	β_0	0.0033***	0.0033***	0.0033***	0.0033***	0.0033***
		β_1	-0.0961***	-0.0710**	-0.0778**	-0.0308	-0.0088
		R^2	0.9%	0.5%	0.6%	0.1%	0.0%
		DW	0.28	0.25	0.25	0.24	0.24
		ADF	-10.94	-10.54	-10.58	-10.68	-10.70
	22D (22)	β_0	0.0072***	0.0072***	0.0072***	0.0072***	0.0072***
		β_1	-0.0737***	-0.0430	-0.0289	0.0164	0.0202
		R^2	0.5%	0.2%	0.1%	0.0%	0.0%
		DW	0.14	0.12	0.12	0.11	0.11
		ADF	-9.29	-9.30	-9.30	-9.32	-9.27
	44D (44)	β_0	0.0146***	0.0146***	0.0146***	0.0146***	0.0146***
		β_1	-0.0417***	-0.0110	-0.0080	0.0173	0.0591
		R^2	0.2%	0.0%	0.0%	0.0%	0.3%
		DW	0.07	0.06	0.06	0.06	0.06
		ADF	-7.91	-8.33	-8.31	-8.25	-8.23

This table is analogous to Table 2 (relation between VIX changes and forward-looking S&P 500 returns), in this case excluding the period 2008-2012. Again, the results do not differ materially from those found before, being all the coefficients consistent with respect to their sign and significance. However, the magnitude of the coefficients is slightly different than those in Table 2. The stars notation ***, ** and * indicate statistical significance of the coefficients at 1%, 5% and 10% levels, respectively.

Table A.4

Period Δ		Bottom 10% of VIX Change			Top 10% of VIX Change		
VIX	SP	Avg Return	StDev	Sharpe	Avg Return	StDev	Sharpe
1d	1d	0.01%	1.20%	0.01	-0.11%	1.08%	-0.10
1d	5d	0.75%	2.40%	0.31	-0.42%	2.48%	-0.17
1d	10d	0.86%	3.17%	0.27	-0.12%	3.36%	-0.04
1d	22d	1.47%	4.67%	0.32	0.23%	4.52%	0.05
1d	44d	1.83%	5.94%	0.31	1.00%	5.86%	0.17
5d	1d	0.28%	0.99%	0.28	-0.29%	1.35%	-0.21
5d	5d	0.46%	2.19%	0.21	-0.06%	2.67%	-0.02
5d	10d	0.84%	2.91%	0.29	0.01%	3.63%	0.00
5d	22d	1.49%	4.08%	0.36	0.84%	4.86%	0.17
5d	44d	1.92%	5.89%	0.33	1.79%	6.19%	0.29
10d	1d	0.23%	0.91%	0.25	-0.13%	1.32%	-0.10
10d	5d	0.32%	2.00%	0.16	0.11%	2.68%	0.04
10d	10d	0.70%	2.78%	0.25	0.20%	3.37%	0.06
10d	22d	0.97%	4.07%	0.24	0.74%	5.39%	0.14
10d	44d	1.62%	6.15%	0.26	1.50%	6.55%	0.23
22d	1d	0.16%	0.84%	0.19	-0.07%	1.31%	-0.05
22d	5d	0.31%	1.86%	0.16	0.04%	2.71%	0.01
22d	10d	0.41%	2.45%	0.17	0.25%	3.68%	0.07
22d	22d	0.58%	3.99%	0.15	0.88%	5.09%	0.17
22d	44d	1.39%	5.74%	0.24	1.74%	6.12%	0.28
44d	1d	0.07%	0.85%	0.09	-0.06%	1.33%	-0.04
44d	5d	0.11%	1.83%	0.06	0.04%	2.81%	0.01
44d	10d	0.30%	2.49%	0.12	0.20%	3.58%	0.06
44d	22d	0.44%	3.78%	0.12	0.82%	4.99%	0.17
44d	44d	0.91%	5.98%	0.15	1.93%	5.98%	0.32

S&P 500 - Full Sample			
Holding period	Avg Return	StDev	Sharpe
1d	0.03%	1.00%	0.03
5d	0.16%	2.15%	0.07
10d	0.32%	2.86%	0.11
22d	0.72%	4.18%	0.17
44d	1.45%	5.63%	0.26

The same as Table 3, this table exposes the mean and standard deviation of the returns obtained after the extreme variations of the VIX (top and bottom deciles), **for the sample that excludes the period 2008-2012**. The left panel shows the results corresponding to the bottom 10% of VIX variation (i.e. extreme negative changes). Within the columns corresponding to the periods, the left figure refers to the VIX variation period, while the figure on the right refers to the S&P 500 return period. As an example, the 1d - 22d row denotes the mean and standard deviation of the 22-day returns that followed extreme 1-day variations of the VIX. For benchmarking purposes, the S&P 500 statistics for the entire sample (1990-2007) are reported in the lower section.

Figure A.1

S&P500 vs VIX - 1990/2012 - Daily observations

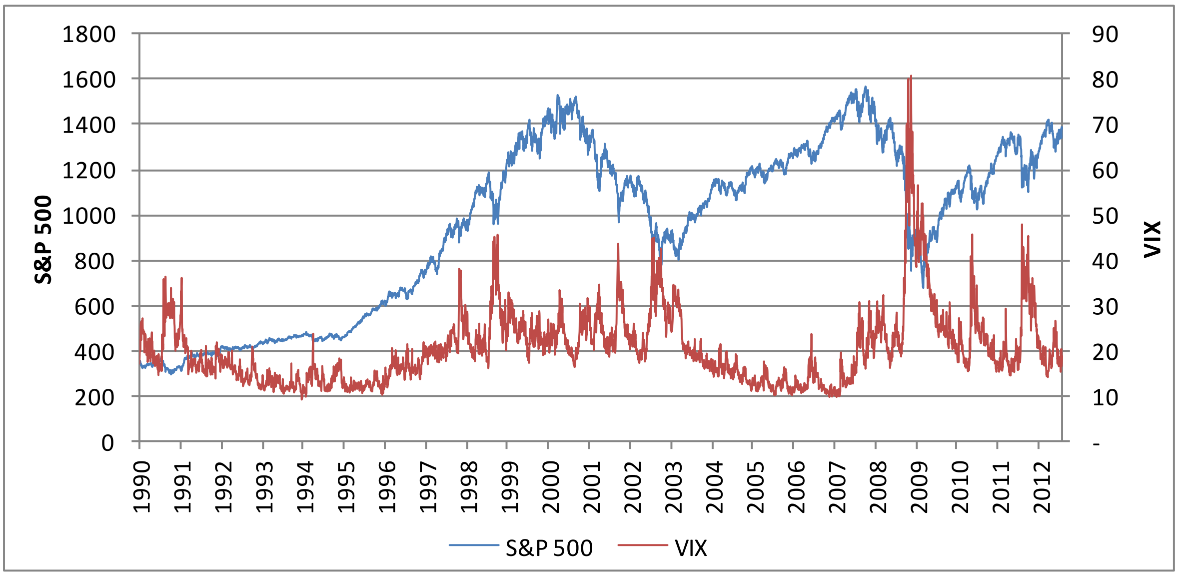

Figure A.2

S&P500 vs VIX - 2008/2009 - Daily observations

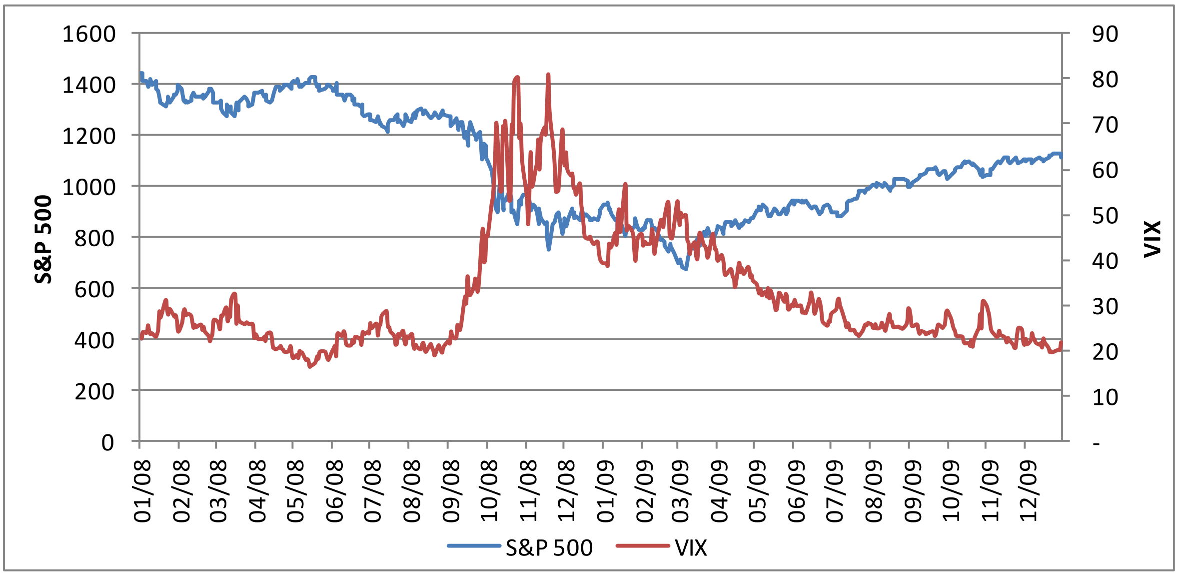

Figure A.3

1 Day Change Relation - S&P 500 vs VIX

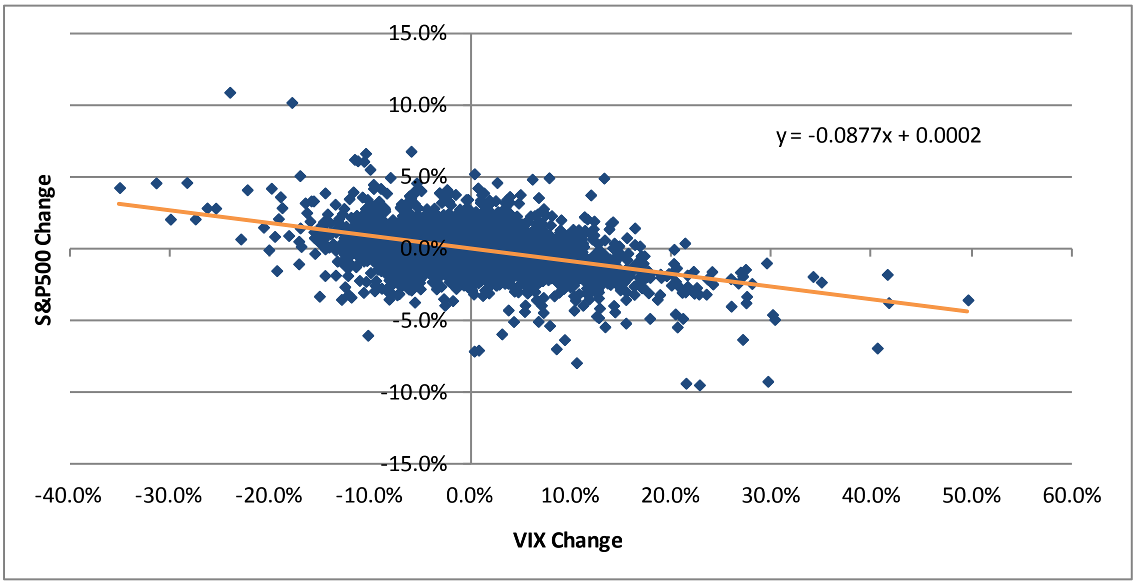

Figure A.4

44 Days Change Relation - S&P 500 vs VIX

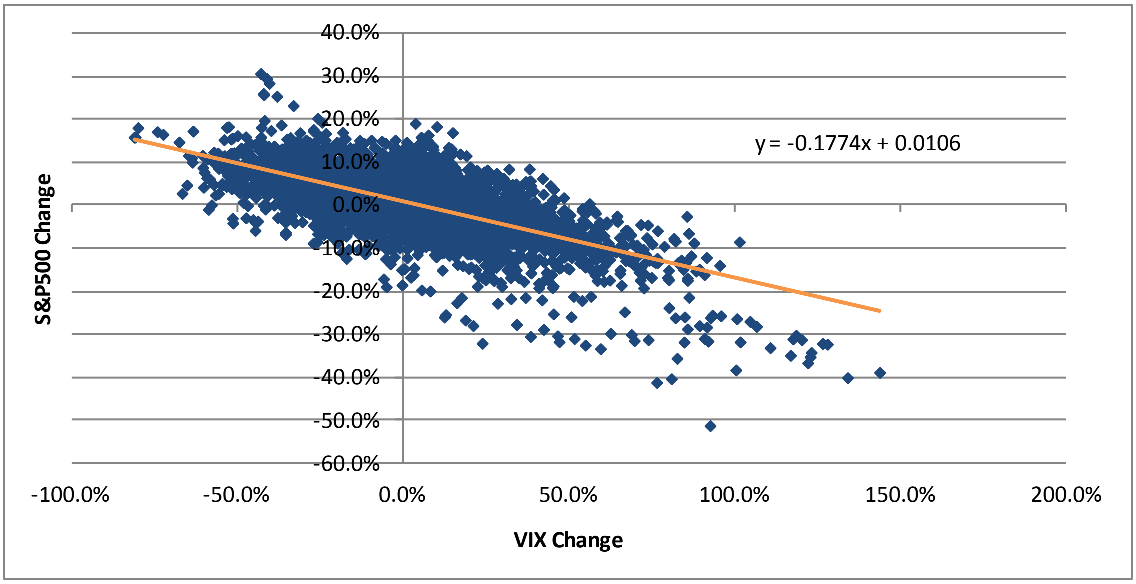

Figure A.5

Average Returns vs. Standard Deviation for the bottom VIX variation decile

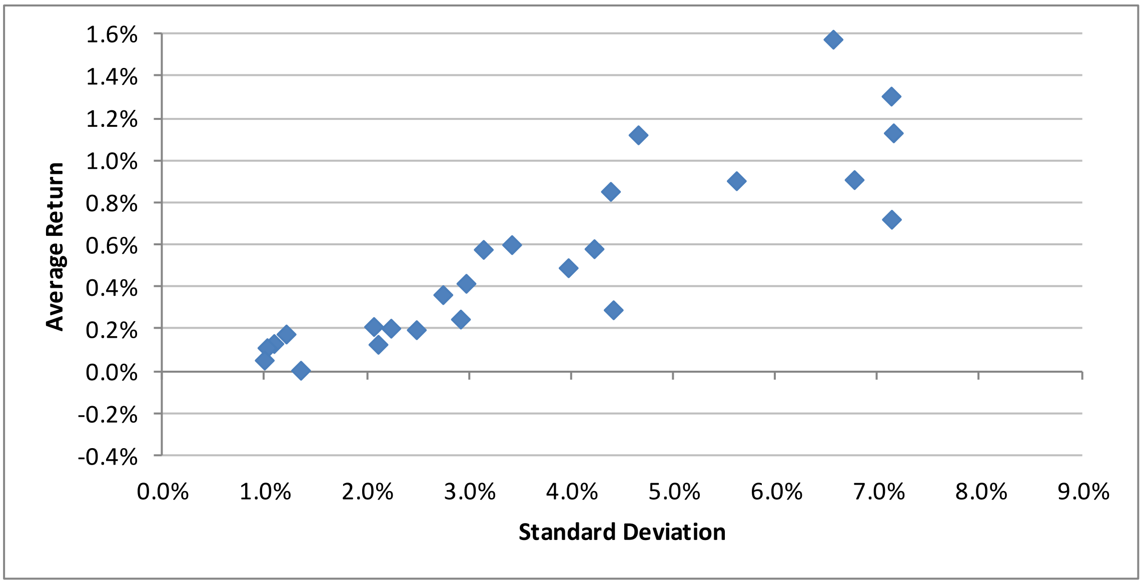

Figure A.6

Average Returns vs. Standard Deviation for the upper VIX variation decile