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Financing Asset Sales and Business Cycles*

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Abstract

This paper analyzes the decision of firms to sell assets to fund investments (financing asset sales). For a sample of U.S. manufacturing firms during the 1971–2010 period, we document new stylized facts about financing asset sales that cannot be explained by traditional motives for selling assets, such as financial distress or financing constraints. Using a structural model of financing, investment, and macroeconomic risk, we show that financing asset sales attenuate the debt overhang problem, because asset sale financed investments imply lower wealth transfers from equity to debt than otherwise identical but equity financed investments. This novel motive to reduce the debt overhang problem can explain how financing asset sales relate to firm characteristics and business cycles. We also confirm with simulated panels of model firms that are structurally similar to their empirical counterpart that they indeed feature the dynamic patterns of financing asset sales we observe in the data for real firms.

JEL Classification Numbers: D92, E32, E44, G12, G32, G33.

Keywords: Asset Sales, Business Cycles, Corporate Investment, Debt Overhang, Financial Leverage, Real Options.

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1. Introduction

A crucial component of corporate investment decisions is the choice of the source of funding. In practice, asset sales play an important role for investment financing. For instance in 2011, the French cement giant Lafarge targeted EUR 750 million (USD 1.1 billion) of asset sales to refinance parts of its debt for the 2007 purchase of Egyptian Orascom Cement. In the same year, Thomson Reuter's announced to raise about USD 1 billion by selling two businesses to fund further investments. In fall 2012, Petrobras announced large asset sales to contribute to the financing needs of nearly USD 15 billion to fund its five-year investment plan. While debt and equity are widely studied sources of investment financing, asset sales are rarely considered. This is surprising, given that the average proceeds from asset sales correspond to roughly 44% of the average net amount of newly issued equity for U.S. manufacturing firms in Compustat between 1971 and 2010.

This paper analyzes the decision of firms to sell assets to fund investments (financing asset sales). We uncover a novel aspect of this decision. We show that this relation can explain stylized facts of empirically observed asset sale patterns. Recognizing that investment may be financed with asset sales also has important consequences for corporate investment policy and firm valuation, in particular, when business cycle shocks influence investment and asset sale decisions of firms. We incorporate business cycles in our analysis for two reasons. First, while the cyclicality of external financing is intensively studied in recent papers, the cyclicality of financing asset sales is not discussed (e.g. Korajczyk and Levy 2003). Second, previous work finds that business cyclicality is crucial to understand financing and investment decisions (e.g., Chen and Manso 2010).

We document empirical facts for a sample of U.S. manufacturing firms that cannot jointly be explained with traditional motivations for asset sales, such as distress or financial constraints (e.g., Bates 2005, Hovakimian and Titman 2006). As the incentive for asset sales is unobservable in our data, we focus on the correlation between asset sales and investment. The idea behind this approach is that financing asset sales should be reflected in the correlation between asset sales and investment. We explore firm and business cycle variables as determinants of financing asset sales. At the same time, the regression set-up allows us to control for other firm and industry characteristics that are potentially correlated with asset sales. We find that the correlation between asset sales and investment is significantly higher (i) for firms with higher leverage, (ii) in bad business cycle states, and (iii) for firms with a low cyclicality of growth options in bad business cycle states.

Motivated by these stylized facts on financing asset sales, we derive the implications of a structural model with intertemporal macroeconomic risk, embedded inside a representative agent consumption-based asset pricing framework in the spirit of Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010). This model environment allows us to derive endogenous investment, equity financing, and financing asset sale decisions over the business cycles by linking these decisions to asset prices and economic fundamentals. To generate financing needs, model firms do not only consist of invested assets but, following Arnold, Wagner, and Westermann (2013), also have a growth option that is costly to exercise. We augment this model environment by incorporating business cycle dependency of the equity issuance cost, the asset liquidity, and the growth option. Using the model, we establish the link between financing asset sales and the debt overhang (or wealth transfer) problem between equityholders and debtholders (Myers 1977) by analyzing equityholders' endogenous choice between issuing new equity and selling assets to finance the exercise of the growth option.

Our analysis starts with a typical firm at time zero that consists of assets in place and a growth option and that is optimally financed with equity and risky debt. When the firm exercises its growth option, the total asset volatility decreases, and total earnings increase. Hence, the exercise creates a wealth transfer from equityholders to debtholders because debt becomes less risky. Due to this agency problem, equityholders invest too late compared to an investment policy that maximizes the value of the expansion option (underinvestment).

Equityholders can also select the optimal funding source for the exercise cost of the growth option. Selling assets when exercising the growth option increases leverage, which makes debt more risky. The increase in the riskiness of the firm's debt associated with the asset sale causes a reverse wealth transfer from debtholders to equityholders that mitigates underinvestment. Thus, asset sales can be relatively more attractive than equity issuances for firms that are more exposed to underinvestment.

The wealth transfer problem is larger for more leveraged firms because debt is riskier and hence more sensitive to earnings and asset volatility changes. As a consequence, equityholders of more leveraged firms have a stronger incentive to use financing asset sales. This insight provides a compelling explanation to our first stylized fact that the correlation between asset sales and investment is significantly higher for firms with larger leverage. Moreover, our model allows us to examine the endogenous relation between business cycles and financing asset sales. In bad business cycle states, leverage increases for a given level of earnings because the decrease in the asset value of a firm is larger than the decrease in the debt value. At the same time, however, equityholders optimally invest at a higher earnings level than in good business cycle states, which induces a lower leverage at investment. Our results show that the first effect dominates i.e. leverage at investment is higher in bad business cycle states. Since the wealth transfer problem at option exercise is larger for higher levels of leverage, our model predicts that equityholders tend to prefer financing asset sales during bad business cycle states. These results provide an explanation for the first two stylized facts. Finally, the model also shows that the more valuable a firm's growth option is in bad states, the lower is the earnings level at which it optimally invests during bad states. A lower earnings threshold for investment entails a higher leverage at investment. So, conditional on investing in bad states, the wealth transfer problem is more pronounced for a higher leverage, which explains the third stylized fact of a higher correlation between asset sales and investment for firms with less cyclical growth options.

To explore the dynamic features of our model, we simulate panels of model-implied firms that are structurally similar to the Compustat sample. Each simulation generates a time series of investment, financing, and default observations over the business cycles. We compare these simulated observations to the empirical patterns to validate the model. The model-implied dynamic patterns of financing asset sales provide an explanation to the stylized facts on asset sales and investment that we document in the empirical analysis. In particular, we find that, on average, 42% of the investment in the simulations are financed with asset sales. The simulated samples reconcile the empirical regularity (i) that the correlation between asset sales and investment rises with leverage. The number of firms that use financing asset sales conditional on investment increases to roughly 64% for firms in the highest leverage tercile compared to 35% in the lowest tercile. Investment and financing asset sales in the simulated samples are procyclical. The fraction of firms that use financing asset sales to invest increases to 54% during bad states, and decreases to 38% during good states, which reflects the empirical pattern (ii) that the correlation between asset sales and investment is higher in bad business cycle states. Finally, we also obtain stylized fact (iii) in the simulated panels. The cyclicality of the growth opportunity is important in that the fraction of firms that use financing asset sales to invest during bad states is particularly large for firms that have a less cyclical growth option. In sum, the simulation results show that our model, in which the wealth transfer problem drives the decision of firms to sell assets, yields dynamic patterns of financing asset sales that explain the stylized facts in the Compustat data.

Our contribution is three-fold. First, we develop a dynamic model of investment and financing that endogenizes the choice between equity and asset sales as funding source. The model yields a set of novel insights and testable predictions that improve our understanding about asset sale motives of firms. More specifically, we provide theoretical and empirical evidence for agency conflicts between debt and equity as an important and heretofore neglected motive for asset sales. Our findings complement previous work that associates asset sales with alternative motives. Asquith, Gertner, and Scharfstein (1994), Brown, James, and Mooradian (1994), and Weiss and Wruck (1998) analyze the role of financial distress for asset sales. Investment funding needs of financially constrained firms as a motive for asset sales are discussed in Hovakimian and Titman (2006), and Campello, Graham, and Harvey (2010). Warusawitharana (2008) argues that asset reallocations are mainly driven by firm-specific productivity shocks. More recently, Edmans and Mann (2013) revisit the pecking order theory by examining the relative information asymmetry associated with issuing equity and selling assets. Lang, Poulsen, and Stulz (1995), and Bates (2005) focus on the trade-off between investment efficiency and agency costs of managerial discretion associated with selling assets. Morellec (2001) also considers agency conflicts between debt and equity in the context of asset sales. He highlights that asset liquidity increases the debt capacity only when bond covenants restrict the disposition of assets close to bankruptcy. In contrast, we model asset sales to finance investment and show that it is optimal for equityholders to negotiate debt covenants that admit asset sales if their proceeds are used to purchase new assets (e.g., Smith and Warner 1979, Bradley and Roberts 2004, Nini, Smith, and Sufi 2009).

Second, we contribute to the literature on role of cyclicality for capital structure and credit risk (see e.g. Hackbarth, Miao, and Morellec 2006). We show that incorporating the impact of business cycle shocks is crucial to jointly explain the corporate investment and financing asset sale decisions. While the effect of cyclicality on asset sales through the productivity channel is already explored (e.g. Maksimovic and Phillips 2001, Yang 2008), the impact of cyclicality through the financing channel has so far been neglected. A closely related paper that considers macroeconomic risk and the debt overhang problem is Chen and Manso (2010). Their results emphasize the cyclical nature of growth opportunities, and the increase of debt overhang in bad states. However, they do not consider the role of asset sales. Our findings on the cyclical nature of financing asset sales also complement the empirical literature that suggests internal resources are more important during worse economic times (e.g., Duchin, Ozbas, and Sensoy 2010, Lemmon and Roberts 2010, Campello, Graham, and Harvey 2010, Campello, Giambona, Graham, and Harvey 2011).

Third, this paper integrates to a growing literature in corporate finance that uses simulated panels based on structural models to explain stylized facts in real firm data (see e.g. Gomes and Livdan 2004, Hennessy and Whited 2007, Strebulaev 2007). As endogeneity problems are hard to resolve with an appropriate empirical identification strategy, we use our structural model to rationalize and support the stylized patterns about the relation between financing asset sales and investment that we observe in the real data.

The paper proceeds as follow. In Section 2, we establish empirical facts on the correlation between asset sales and investment. Section 3 introduces a structural model that to explain these stylized facts. Section 4 presents the model solution, and Section 5 derives the predictions generated by our model for a typical firm at initiation. Finally, we simulate model-implied economies of firms to analyze the aggregate dynamics of financing asset sales in Section 6. Section 7 concludes.

2. Stylized Facts

In this section, we document empirical patterns of financing asset sales for a sample of 3,022 U.S. manufacturing firms over the 1971–2010 period.[1] The data on asset sales (Compustat item SPPE) does not reveal the motive behind the asset sales. We can also not exploit a (quasi-) natural experiment or a discontinuity. Hence, we try to identify firm characteristics and business cycle related factors that increase the correlation between asset sales and investment. The idea behind this approach is that more financing asset sales should result in an increased correlation between contemporaneous investment and asset sale. Moreover, focusing on this correlation allows us to abstract away from fire sales of financially distressed firms. The reason is that it is unlikely that distressed firms tend to invest heavily in those periods, in which they are forced to sell assets to repay their debt. We document that the correlation between asset sales and investment is higher (i) for firms with higher leverage, (ii) in bad business cycle states, and (iii) for firms with less cyclical growth opportunities in bad business cycle states. Thus, we show that business cycle conditions, corporate investment, and time-variation of growth opportunities are key determinants of financing asset sales.

Table 1 reports results for OLS panel regressions that explore the correlation of asset sales with investment, leverage, the cyclicality of a firm's growth opportunities, financial constraints and other controls for various firm characteristics. We include industry fixed effects. The standard errors are autocorrelation robust and clustered at the industry level, and the R^2s are adjusted for the number of variables in the regression.[2]

INSERT TABLE 1 NEAR HERE

Column (I) investigates the relation of asset sales and investment controlling for Tobin's q, financial flexibility (cash flow and financial slack), coverage ratio, leverage, and asset volatility. The estimation shows that asset sale and investment are reliably positively correlated. Cash flow, asset volatility, and q exhibit a negative significant regression coefficient, while financial slack and coverage ratio are not significantly correlated with asset sale. The positive significant association between asset sale and investment suggests that financing asset sales are a potential source of investment funding. However, we cannot interpret this correlation by itself as an indicator of what may be a potential motive for firms to use financing asset sales.

To explore this question, we first investigate factors known to be related to the wealth transfer problem. For instance, the wealth transfer problem increases with leverage (see e.g. Myers 1977). Hence, in column (II), we explore the impact of leverage on the relationship between asset sale and investment by using an

[1] All variable definitions, data cleaning filters and summary statistics for the Compustat sample are provided in Appendix B.

[2] The quality of our results remains unaffected if we use e.g. two-step GMM estimations or two-way clustering at the year and at the industry level or alternatively at the year and at the firm level.

interaction term of investment and leverage. If higher leverage is a motive for financing asset sales, we expect a positive coefficient for this interaction term. The result in column (II) confirms that the correlation between asset sale and investment increases with leverage. Moreover, investment and leverage coefficients become insignificant when we add the interaction term between investment and leverage.[3]

Chen and Manso (2010) show that the wealth transfer problem is more severe in bad states of the business cycle. Hence, we are interested in how the correlation between asset sale and investment is related to macroeconomic conditions. In column (III) of Table 1, we additionally incorporate the interaction between investment and a dummy that is equal to one in a bad business cycle state.[4] The positive significant coefficient on this interaction term shows that the correlation between investment and asset sales is higher in downturns. This finding emphasizes the importance of recognizing business cycle dynamics when explaining the positive correlation of investment and asset sale.

Moreover, we next link financing asset sales to the cyclicality of growth opportunities. In particular, we investigate whether firms, that have relatively valuable growth opportunities in economic downturns, exhibit an increased correlation between asset sales and investments during bad states. To this end, we add in column (IV) an interaction term that is the product of three variables: investment, a dummy that is equal to one if the sample economy is in a bad state and zero otherwise, and the correlation between a firm's growth opportunity and the aggregate business cycle state. To construct the correlation measure, we estimate 5-year rolling window correlations between the firm individual q and the aggregate sales growth in our entire sample.[5] The intuition for this correlation measure is that firms that exhibit a relatively lower value of this measure tend to have relatively more valuable growth opportunities in bad states of the business cycle; on the other hand higher values of the measure indicate that a firm has more cyclical growth opportunities.[6] We find a negative coefficient for the interaction term between investment, business cycle states and the cyclicality of a firm's growth opportunity, implying that in bad states of the business cycle, firms with a relatively lower cyclicality exhibit a stronger relationship between investments and asset sales.[7]

[3] In unreported regressions, we replace the dependent variable by net equity issuance. We find that the coefficient estimate of the interaction term of investment and leverage is negative and not significant.

[4] For a bad business cycle year, the aggregate sales growth and the average annual equity return across sample firms are both in the bottom 25% of all years. We choose this definition of a downturn because sales growth combined with market based downturn measures are a direct measure of the propagation of positive and negative shocks from the aggregate economy onto the corporate level (see also the downturn definitions in e.g. Opler and Titman 1994, Gilson, John, and Lang 1990).

[5] We scale the firm individual q by the SIC3-industry average q to control for industry effects. Using larger windows for the correlation measure within a reasonable range (e.g., seven years) has no qualitative effect on the results.

[6] The 25% quantile of the correlation distribution for all firms is -0.5, the median is 0.02, and the 75% quantile is 0.56.

[7] In unreported results, we additionally incorporate the interaction between the bad state dummy and leverage, and the triple interaction between the bad state dummy, leverage, and investment. The coefficient on this triple interaction is positive and significant, indicating that particularly high leverage firms have a stronger relationship between investments and asset sales during bad states of the business cycle, which provides additional support to the wealth transfer problem as an important driver of the positive relation between asset sales and investments.

The above results indicate along several dimensions that the correlation between asset sales and investment increases with firm characteristics that are directly linked to an increased wealth transfer problem. An alternative explanation could be that the positive relationship of leverage with the correlation between asset sales and investments is driven by external financing constraints (e.g. Lang, Poulsen, and Stulz 1995, Hovakimian and Titman 2006, Bates 2005). To analyze the potential role of financing constraints for asset sales, we add in column (V) the SA-index as a proxy for the financial constraint of firms.[8] Higher values of this index indicate lower financial constraints of a firm. The coefficient for the SA-index is positive and significant, implying that unconstrained firms sell more assets.

This result does not answer the question whether financing constraints affect financing asset sales. Additionally, it is well-known that less financially constrained firms have a higher debt capacity, i.e., they can lever up their firm more easily (e.g. Kiyotaki and Moore 1997, Almeida and Campello 2007, Hahn and Lee 2012, Hart and Moore 1994). Hence, our primary result that high leverage firms conduct more financing asset sales may be, in fact, driven by financing constraints. To address this concern, we incorporate the interaction of the SA-index with investment as an additional independent variable in column (VI). The interaction term is insignificant, and the interaction of leverage with investment is only marginally affected by the new controls compared to column (II). The result suggests that leverage (i.e., the wealth transfer problem instead of financing constraints) is the driving force behind the correlation between asset sales and investment.

An alternative motive for asset sales besides investment financing needs is financial distress (e.g. Shleifer and Vishny 1992, Weiss and Wruck 1998, Lang, Poulsen, and Stulz 1995). We test in the final regression models whether financial distress has an impact on the correlation between asset sales and capital expenditures. We do so by including in column (VII) an interaction term of investment and a dummy that indicates if the firm individual Altman (1968) Z-score is below a value of three. Values below three indicate that a firm is likely to be financially distressed. If financial distress were a driver of the asset sale and investment correlation, we would expect a positive significant coefficient for the interaction term. However, the table reveals an insignificant coefficient estimate. Thus, our finding on the correlation between asset sales and investment is unlikely to be caused by financial distress.

Finally, the results for the interaction between investment and leverage could also be driven by financial distress. To address this concern, we include a new interaction term of investment, leverage, and the Z-score dummy in column (VIII). If financial distress were to matter for the coefficient on the interaction between

[8] According to Hadlock and Pierce (2010), the SA-index is useful to measure the financial constraint. Related work supports the view that the ingredients of this index, i.e. size and age, capture the financial constraint of a firm (see e.g. Hennessy and Whited 2007, Fee, Hadlock, and Pierce 2009). Furthermore, size and age are also often interpreted as information asymmetry measures (see e.g. Leary and Roberts 2010).

investment and leverage, we would expect a positive coefficient on this new interaction term. However, the coefficient is insignificant, and the interaction between investment and leverage by itself is hardly affected (compared to column II) by the inclusion of the new interaction term. This finding establishes that fire sales are not the driver of the positive impact of leverage on the correlation between investment and asset sales.

We conduct several robustness tests. For instance, our results are not driven by small observations of asset sales or investments. The coefficients and t-statistics hardly change when we drop the smallest 10% or 20% of the absolute values of asset sales and capital expenditures from our sample. Moreover, if we focus on larger property, plant, and equipment values, our results become much stronger. For example, for observations with property, plant, and equipment above its median value, the coefficient for the interaction term of leverage and investment is 0.103 (t-statistic of 4.34), the coefficient for the interaction term of bad state and investment is 0.031 (t-statistic of 2.26), and the coefficient for the triple interaction of investment, bad state, and correlation is -0.039 (t-statistic of -3.32). At the same time, the coefficients for the interaction of investment and the SA-index, and for the interaction of investment and the Z-score dummy remain insignificant. These findings reinforce the robustness of the above stylized facts in that firms with plenty of property, plant, and equipment may have better access to the asset sale market.

To summarize, our novel stylized facts for the correlation between investments and asset sales cannot be explained by traditional motives for asset sales, such as financial constraints or financial distress. Leverage, rather than proxies for financial constraints or financial distress, drives the results. Hence, the wealth transfer problem between debt and equity is a potentially important driver of financing asset sales.

3. Model setup

In this section, we study a structural model with time-varying macroeconomic conditions, embedded inside a representative agent consumption-based asset pricing framework in the spirit of Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010). This framework determines how aggregate risk and risk prices change with the business cycle. It links the fluctuations in the first and second moments of aggregate growth rates to the values of corporate securities. The model is well suited to explore the role of financing asset sales over the business cycles, as it allows us to endogenize the effect of cyclicality in a simple and realistic fashion. Moreover, it shows how the values of equity, debt, and growth options that determine firms' external financing decisions are endogenously affected by time-varying business cycle conditions.

Following Arnold, Wagner, and Westermann (2013), each firm has one growth option that is costly to exercise. The key innovation in our paper is that we allow firms to endogenously choose between financing the investment cost with the proceeds from the asset sales or the issuance of new equity. Moreover, we incorporate business cycle dependent equity issuance cost, asset liquidity, and cyclicality of the growth option. The structural model approach allows us to analyze equityholders' endogenous choice between issuing new equity and selling assets to finance the exercise of the growth option in an economy with external financing frictions. In addition, it easily lends itself to analyzing firm behavior in simulated panels to explore the dynamic predictions of our model.

A potential caveat for financing asset sales is the existence of covenants in credit contracts that could restrict both investment and asset sales as a source of internal financing. The covenant literature suggests, however, that covenants observed for real firms provide substantial flexibility for expansion investments, and with respect to the choice between new equity or asset sales as source of investment financing.[9]

3.1. Firm Earnings, Investment Financing and Time-Varying Business Cycle Conditions

The economy consists of N different firms with assets in place and a growth option, a large number of identical infinitely lived households, and a government serving as a tax authority. There are two different aggregate states, namely, good (G) and bad (B) states. Aggregate output, corporate earnings, and external financing frictions depend on the current state. To model time-varying aggregate conditions, we define a time-homogeneous observable Markov chain $I_{t\geq 0}$ with state space $\{G, B\}$ and generator $Q := \begin{bmatrix} -\lambda_G & \lambda_G \\ \lambda_B & -\lambda_B \end{bmatrix}$, in which $\lambda_i \in (0,1)$ is the rate of leaving state i.

The aggregate output C_t follows a regime-switching geometric Brownian motion

$$\frac{\mathrm{d}C_t}{C_t} = \theta_i \mathrm{d}t + \sigma_i^C \mathrm{d}W_t^C, \; i = G, B, \tag{1}$$

in which W_t^C is a Brownian motion independent of the Markov chain. The parameters θ_i and σ_i^C are the growth rates and volatilities of the aggregate output, respectively. To incorporate the impact of time-varying aggregate conditions, they are both regime-dependent. In equilibrium, aggregate consumption equals aggregate output. The representative agent has the continuous-time analog of Epstein-Zin-Weil preferences of stochastic differential utility type (e.g. Duffie and Epstein 1992a, Duffie and Epstein 1992b). The dynamics

[9] Nini, Smith, and Sufi (2009) provide evidence of a widespread use of covenants that restrict investment in private credit agreements. Their results, however, suggest that capital expenditure covenants address asset substitution and fire sales rather than investment opportunities. In particular, the authors find that capital expenditure restrictions are less likely in credit agreements of firms with more favorable investment opportunities. They also show that banks and borrowers tend to leave the investment policy unrestricted when credit quality is high, or as long as covenants are not violated. Chava, Kumar, and Warga (2010) show that bond covenants that restrict stock issuance are relatively rare compared to covenants that restrict the issuance of debt. While covenants on asset sales are frequently used, they often explicitly allow firms to sell assets in the ordinary course of business, or as long as the proceeds from the asset sale are used to purchase new fixed assets (Smith and Warner 1979). Other common provisions in asset covenants of private debt contracts are restrictions on asset sales above a fixed amount or requirements to pay down debt with the proceeds from asset sales (Bradley and Roberts 2004).

of the stochastic discount factor, the risk-free rates, r_i, the market prices of consumption risk, η_i, and the market prices of jump risk, κ_i, are derived in Appendix A.1.

At any time, the earnings process of a firm follows

$$\frac{\mathrm{d}X_t}{X_t} = \mu_i \mathrm{d}t + \sigma_i^{X,C} \mathrm{d}W_t^C + \sigma^{X,id} \mathrm{d}W_t^X, \quad i = G, B, \tag{2}$$

in which W_t^X is a standard Brownian motion describing an idiosyncratic shock, independent of the aggregate output shock W_t^C and the Markov chain. μ_i are the regime-dependent drifts; $\sigma_i^{X,C} > 0$, the firm-specific regime-dependent volatilities associated with the aggregate output process; and $\sigma^{X,id} > 0$, the firm-specific volatility associated with the idiosyncratic Brownian shock.

Denote the risk-neutral measure by $\mathbb{Q}$. The expected growth rates, $\tilde{\mu}_i$, of a firm's earnings under the risk-neutral measure are given by

$$\tilde{\mu}_i := \mu_i - \sigma_i^{X,C} \eta_i, \tag{3}$$

and the risk-neutral transition intensities, $\tilde{\lambda}_i$, by

$$\tilde{\lambda}_i = e^{\kappa_i} \lambda_i. \tag{4}$$

Intuitively, in bad times when marginal utility is higher, bad news about future earnings are worse. Hence, by incorporating jump-risk into the expression in Equation (4), we link the historical probabilities of a change in the regime with the risk-neutral probabilities. The main effect for the security prices is that, under the risk neutral measure, bad states last longer and the economy switches faster from a good to a bad state.

Corporate taxes need to be paid at a constant rate τ, and full offsets of corporate losses are allowed. Following Hackbarth, Miao, and Morellec (2006), Chen (2010), and Bhamra, Kuehn, and Strebulaev (2010), the unleveraged after-tax asset value of a firm can then be written as

$$V_t = (1 - \tau) X_t y_i, \quad i = G, B, \tag{5}$$

with y_i being the price-earnings ratio in state i determined by

$$y_i^{-1} = r_i - \tilde{\mu}_i + \frac{(r_i - \tilde{\mu}_j) - (r_i - \tilde{\mu}_i)}{r_j - \tilde{\mu}_j + \tilde{p}} \tilde{p} \tilde{f}_j. \tag{6}$$

$\tilde{p} := \tilde{\lambda}_i + \tilde{\lambda}_j$ is the risk-neutral rate of news arrival, and $\left(\tilde{f}_G, \tilde{f}_B\right) = \left(\frac{\tilde{\lambda}_B}{\tilde{p}}, \frac{\tilde{\lambda}_G}{\tilde{p}}\right)$ is the long-run risk-neutral distribution. y^{-1} can be interpreted as a discount rate, in which the first two terms constitute the standard expression if the economy stayed in regime i forever, and the last term accounts for future time spent in regime j. As in Bhamra, Kuehn, and Strebulaev (2010), the price-earnings ratio in the main analysis is higher in good states than in bad states i.e. $y_G > y_B$.

Finally, the volatility of the earnings process in regime i can be expressed as

$$\bar{\sigma}_i = \sqrt{\left(\sigma_i^{X,C}\right)^2 + \left(\sigma^{X,id}\right)^2}. \tag{7}$$

A firm's expansion (growth) option is modeled as an American call option on its earnings. In particular, a firm (i) can irreversibly exercise this option at any time $\bar{t}$, (ii) needs to pay the exercise cost $K_{\bar{i}}$, and (iii) achieves additional future earnings of $s_{\bar{i}}X_t$ for all $t \geq \bar{t}$ for some factor $s_{\bar{i}} > 0$, in which $\bar{i}$ is the realized state of the economy at the time of exercise. In contrast to Arnold, Wagner, and Westermann (2013), both the exercise cost $K_{\bar{i}}$ and the factor $s_{\bar{i}}$ are regime-dependent to model firms with varying degrees of the cyclicality of their growth option. If an expansion option is exercised, it is once and for all converted into assets in place, so the firm consists of only invested assets.

The exercise cost $K_{\bar{i}}$ can be financed by either issuing new equity or by selling assets in place.[10] Both sources of financing impose frictions on the firm. First, we explicitly consider external financing frictions i.e. that new equity financing is costly, as suggested by the literature (e.g. Campello and Hackbarth 2012). In particular, each equity-financed \$1 leads to a regime-dependent issue cost of $\Upsilon_{\bar{i}}$. The regime dependency of Υ_i allows us to capture the notion that external equity financing is more restricted during bad states (e.g. Erel, Julio, Kim, and Weisbach 2011). The cost Υ_i can be interpreted as the linear component of the equity issuance cost. Hence, a firm with access to equity financing in a given regime can finance the exercise cost $K_{\bar{i}}$ by issuing new equity of $K_{\bar{i}}(1+\Upsilon_{\bar{i}})$.

Second, Pulvino (1998) and Jovanovic and Rousseau (2002) argue that selling assets is costly. The cost occurs because assets are partially firm-specific and the firm-specific component is irreversibly lost in asset transfers, or because existing assets are not made-to-order and, therefore, may require additional disassembling costs to tailor the assets to the buyer's specific needs. We incorporate this friction by stating that the proceeds from selling assets on the market correspond to $0 \leq \Lambda_i \leq 1$ times the value of the assets to the firm. Consistent with Shleifer and Vishny (1992), the parameter Λ_i can be interpreted as the regime-dependent liquidity of the firm's assets in place, and is calibrated such that $\Lambda_G > \Lambda_B$. After exercising the expansion option, the firm obtains current earnings of $(s_{\bar{i}}+1)X_t$ i.e. $s_{\bar{i}}X_t$ from the expansion option, and

[10] We neglect internal cash or additional debt upon investment as financing sources. Both elements would introduce an additional layer of incentive problems that are beyond the scope of this paper. For example, with new debt issuance upon investment, equityholders overlever and overinvest, because they can transfer wealth from initial debt to themselves, which would drive the results. As it is initially not known whether the growth option will be exercised in a good or in a bad state, the incentive problems cannot simply be solved with a priority structure as suggested in Hackbarth and Mauer (2012) for a one regime model.

X_t from existing assets in place. The value of the existing assets in place at option exercise corresponds to $(1-\tau)X_{\bar{t}}y_{\bar{i}}$. The value of the assets required to be sold to finance the exercise cost of the expansion option is given by $K_{\bar{i}}/\Lambda_{\bar{i}}$ or by $\frac{K_{\bar{i}}/\Lambda_{\bar{i}}}{(1-\tau)X_{\bar{t}}y_{\bar{i}}}$ as a fraction of current earnings. As a result, total earnings of a firm at any point in time after financing the exercise cost by selling assets correspond to

$$\left(s_{\bar{i}}+1-\frac{K_{\bar{i}}/\Lambda_{\bar{i}}}{(1-\tau)X_{\bar{t}}y_{\bar{i}}}\right)X_t. \tag{8}$$

Firms take on (risky) debt because it allows them to shield part of the corporate income from taxation. The debt maturity is assumed to be infinite. Once debt has been issued, a firm pays a coupon c at each moment in time. Shareholders have the option to default on their debt obligations. Default is triggered when shareholders are no longer willing to inject additional equity capital to meet net debt service requirements (e.g. Leland 1998). If default occurs, the firm is immediately liquidated. Debtholders receive the liquidation value of the total unleveraged asset value i.e. of the unleveraged assets in place plus the unleveraged growth option, less bankruptcy costs. The proceeds from liquidating the firm upon default correspond to Λ_i times the total unleveraged asset value. The bankruptcy costs include, for example, lawyers' and accountants' fees, or the value of the managerial time spent in administering the bankruptcy. They correspond to a fraction $1-\alpha_i$ of the proceeds from liquidation, with $\alpha_i \in (0,1]$. Hence, the recovery rates to debtholders correspond to $\Lambda_i\alpha_i$ times the unleveraged asset value upon default. The assumption that debtholders also recover a fraction of the unleveraged expansion option implies that the option is transferrable. Upon default, however, the expansion opportunities are far out-of-the-money and have, consequently, only limited value. Hence, assumptions concerning their transferability or recovery rates have a negligible impact on our results.

Equityholders face the following decisions. First, once debt has been issued, they select the default, expansion, and investment financing policies that maximize the equity value. Second, they determine the initially optimal capital structure by choosing a coupon that maximizes the firm value. We do not incorporate debt restructuring neither when the option is exercised nor at endogenous restructuring points.

4. Model solution

Firms can finance investments by selling assets or by issuing equity in each regime, which leaves us with four different funding strategies: financing by issuing equity in good states and selling assets in bad states, financing by issuing equity in both good states and bad states, financing by selling assets in good states and issuing equity in bad states, and financing by selling assets in both good and bad times. In what follows, we derive the solution for a firm that applies the first funding strategy i.e. financing by issuing equity in good states and selling assets in bad states. The solutions for the second to fourth funding strategies can be derived similarly. We first present the values of corporate securities after investment, and for the growth option. We then solve for the values of corporate securities before investment by backward induction.

4.1. Value of corporate securities after investment

After exercising the expansion option, a firm consists of only invested assets, endowed with the initially determined optimal coupon level. Let $\hat{d}_i(X)$ denote the value of corporate debt, $\hat{t}_i(X)$ the value of the tax shield, and $\hat{b}_i(X)$ the value of bankruptcy costs of a firm with only invested assets. The standard solutions for the values of these securities are derived in Appendix A.2. The firm value after investment, $\hat{v}_i(X)$, can be expressed as the value of assets in place plus the tax shield minus bankruptcy costs:

$$\hat{v}_i(X) = (1-\tau)y_i X + \hat{t}_i(X) - \hat{b}_i(X). \tag{9}$$

The total firm value equals the sum of debt and equity values. Hence, the equity value after investment, $\hat{e}_i(X)$, can be written as

$$\hat{e}_i(X) = \hat{v}_i(X) - \hat{d}_i(X). \tag{10}$$

The default policy is chosen by equityholders to maximize the ex post value of equity. As the equity value at the time of default corresponds to zero, this policy can be calculated by equating the first derivative of the equity value to zero at the default boundary in each regime:

$$\begin{cases} \hat{e}'_G(D^*_G) & = & 0 \\ \hat{e}'_B(D^*_B) & = & 0 \end{cases} \tag{11}$$

We solve this system numerically. The value of corporate securities is solved similarly for a firm with a scaled level of earnings after investment. The default policy is then expressed as a scaled earnings levels.

4.2. The value of the growth option

To study cyclicality of expansion options, we extend the model of Arnold, Wagner, and Westermann (2013) by allowing regime-dependency of the additional earnings factor s_i, and the exercise cost K_i of the option. For each regime i, a growth option is exercised immediately whenever $X \geq X_i$ (option exercise region); otherwise, it is optimal to wait (option continuation region). This structure results in a system of ordinary differential equations (ODEs) with associated boundary conditions given in Appendix A.3. The following proposition presents the value of the growth option, $G_i(X)$, in a leveraged firm (leveraged growth option) that finances the exercise cost by issuing equity in good states, and by selling assets in bad states for $X_G \leq X_B$.

Proposition 1. *For any given pair of exercise boundaries $[X_G, X_B]$, the value of the leveraged growth option in regime i is given by*

$$G_i(X) = \left\{ \begin{array}{lll} \bar{A}_{i3}X^{\gamma_3} + \bar{A}_{i4}X^{\gamma_4} & 0 \leq X < X_G, & i = G,B \\ \bar{C}_1 X^{\beta_1^B} + \bar{C}_2 X^{\beta_2^B} + \bar{C}_3 X + \bar{C}_4 & X_G \leq X < X_B, & i = B \\ (1-\tau)s_B X y_B - K_B/\Lambda_B & X \geq X_B & i = B \\ (1-\tau)s_G X y_G - K_G(1+\Upsilon_G) & X \geq X_G & i = G \end{array} \right\}, \tag{12}$$

where

$$\begin{aligned} \beta_{1,2}^B &= \frac{1}{2} - \frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2}\right)^2 + \frac{2(r_B + \tilde{\lambda}_B)}{\tilde{\sigma}_B^2}}, \\ \bar{C}_3 &= \tilde{\lambda}_B \frac{(1-\tau)s_G y_G}{r_B - \tilde{\mu}_B + \tilde{\lambda}_B}, \\ \bar{C}_4 &= -\tilde{\lambda}_B \frac{K_B/\Lambda_B}{r_B + \tilde{\lambda}_B}. \end{aligned} \tag{13}$$

The parameters γ_3 and γ_4 correspond to the positive roots of the quadratic equation

$$(\tilde{\mu}_B\gamma + \frac{1}{2}\tilde{\sigma}_B^2\gamma(\gamma-1) - \tilde{\lambda}_B - r_B)(\tilde{\mu}_G\gamma + \frac{1}{2}\tilde{\sigma}_G^2\gamma(\gamma-1) - \tilde{\lambda}_G - r_G) = \tilde{\lambda}_B\tilde{\lambda}_G. \tag{14}$$

$\bar{A}_{Gk}$ is a multiple of $\bar{A}_{Bk}$, $k = 3,4$, with the factor $\bar{l}_k := \frac{1}{\tilde{\lambda}_G}(r_G + \tilde{\lambda}_G - \tilde{\mu}_G\gamma_k - \frac{1}{2}\tilde{\sigma}_G^2\gamma_k(\gamma_k - 1))$ i.e. $\bar{A}_{Bk} = \bar{l}_k\bar{A}_{Gk}$, and r_i^p is the perpetual risk-free rate given by

$$r_i^p = r_i + \frac{r_j - r_i}{\tilde{p} + r_j}\tilde{p}\tilde{f}_j, \tag{15}$$

in which $\tilde{p} = \tilde{\lambda}_1 + \tilde{\lambda}_2$ is the risk-neutral rate of news arrival and $(\tilde{f}_G, \tilde{f}_B) = \left(\frac{\tilde{\lambda}_B}{\tilde{p}}, \frac{\tilde{\lambda}_G}{\tilde{p}}\right)$ is the long-run risk-neutral distribution. $\left[\bar{A}_{G3}, \bar{A}_{G4}, \bar{C}_1, \bar{C}_2\right]$ solve a linear system given in Section Appendix A.3.

Proposition 1 determines the value of the growth option for any given pair of exercise boundaries $X_G \leq X_B$. The optimal exercise boundaries of the leveraged growth option will be determined in the next section, as they depend on the capital structure of the firm holding the option. Additionally, note that the value of the growth option also depends on both the asset liquidity and the equity issuance cost.

For the derivation of the values of corporate securities before investment, we also need the value of an unleveraged option G_i^{unlev} that corresponds to the value of an option in an all equity financed firm. This value does not depend on the capital structure of a firm. Hence, the optimal exercise boundaries simply maximize the value of the option. They can, therefore, be directly derived by additionally imposing smooth-pasting conditions at the corresponding option exercise boundaries as shown in Appendix A.3.

As we consider a regime-dependent additional earnings factor s_i and exercise cost K_i of the option, we also encounter the case in which the exercise boundary in good states, X_G, is larger than the exercise boundary in bad states, X_B. It occurs when s_B is considerably larger than s_G, or when K_B is much smaller than K_G. The solution of this case can be obtained immediately by interchanging the regime names in the derivation of the presented solution with $X_G \leq X_B$.

4.3. Value of corporate securities before investment

Once the values of corporate securities after investment and of the growth option are known, we can determine the values of corporate securities before investment of a firm that finances the exercise cost by issuing equity in good times, and by selling assets in bad times. Let $d_i(X)$ denote the debt value of a firm with invested assets and an expansion option in regime $i = G, B$, and G_i^{unlev} the value of an unleveraged option derived in the Appendix A.3. Proposition 2 states the value of debt before investment.

Proposition 2. *For any given set of default and exercise boundaries $[D_G, D_B, X_G, X_B]$, the value of infinite maturity debt in regime i is given by*

$$d_i(X) = \begin{cases} \alpha_i \Lambda_i \left((1-\tau)Xy_i + G_i^{unlev}(X)\right) & X \leq D_i, \quad i = G, B, \\ C_1 X^{\beta_1^G} + C_2 X^{\beta_2^G} + C_5 X^{\gamma_3} + C_6 X^{\gamma_4} \\ \quad + \tilde{\lambda}_G \frac{\alpha_B \Lambda_B y_B (1-\tau)}{r_G - \tilde{\mu}_G + \tilde{\lambda}_G} X + \frac{c}{r_G + \tilde{\lambda}_G} & D_G < X \leq D_B, \quad i = G \\ A_{i1} X^{\gamma_1} + A_{i2} X^{\gamma_2} + A_{i3} X^{\gamma_3} + A_{i4} X^{\gamma_4} + \frac{c}{r_i^P} & D_B < X \leq X_G, \quad i = G, B \\ B_1 X^{\beta_1^B} + B_2 X^{\beta_2^B} + Z(X) + \tilde{\lambda}_B \frac{c}{r_i^P (r_B + \tilde{\lambda}_B)} + \frac{c}{r_B + \tilde{\lambda}_B} & X_G < X \leq X_B, \quad i = B \\ \hat{d}_G((s_G + 1)X) & X > X_G, \quad i = G \\ \hat{d}_B\left((s_B + 1 - \frac{K_B/\Lambda_B}{(1-\tau)X_i y_B})X\right) & X > X_B, \quad i = B, \end{cases} \tag{16}$$

where

$$\beta_{1,2}^i = \frac{1}{2} - \frac{\tilde{\mu}_i}{\tilde{\sigma}_i^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_i}{\tilde{\sigma}_i^2}\right)^2 + \frac{2(r_i + \tilde{\lambda}_i)}{\tilde{\sigma}_i^2}} \tag{17}$$

$$C_5 = \alpha_B \Lambda_B \frac{\bar{l}_3}{l_3} \bar{A}_{G3}^{unlev}, \tag{18}$$

$$C_6 = \alpha_B \Lambda_B \frac{\bar{l}_4}{l_4} \bar{A}_{G4}^{unlev}, \tag{19}$$

and

$$Z(X) = \tilde{\lambda}_B B_5 X^{\gamma_1} + \tilde{\lambda}_B B_6 X^{\gamma_2}. \tag{20}$$

The parameters B_5 and B_6 are given by

$$B_5 = \frac{(s_B+1)^{\gamma_1}\hat{A}_{G1}}{r_B - \tilde{\mu}_B\gamma_1 - \frac{1}{2}\tilde{\sigma}_B^2\gamma_1(\gamma_1-1)+\tilde{\lambda}_B} \tag{21}$$

and

$$B_6 = \frac{(s_B+1)^{\gamma_2}\hat{A}_{G2}}{r_B - \tilde{\mu}_B\gamma_2 - \frac{1}{2}\tilde{\sigma}_B^2\gamma_2(\gamma_2-1)+\tilde{\lambda}_B}, \tag{22}$$

and $\gamma_k, k=1,2,3,4$ are the roots of the quadratic equation

$$\left(\tilde{\mu}_B\gamma+\frac{1}{2}\tilde{\sigma}_B^2\gamma(\gamma-1)-\tilde{\lambda}_B-r_B\right)\left(\tilde{\mu}_G\gamma+\frac{1}{2}\tilde{\sigma}_G^2\gamma(\gamma-1)-\tilde{\lambda}_G-r_G\right)=\tilde{\lambda}_B\tilde{\lambda}_G. \tag{23}$$

A_{Bk}, $k=1,2,3,4$, is a multiple of A_{Gk} with the factor

$$l_k := \frac{1}{\tilde{\lambda}_G}\left(r_G+\tilde{\lambda}_G-\tilde{\mu}_G\gamma_k-\frac{1}{2}\tilde{\sigma}_G^2\gamma_k(\gamma_k-1)\right), \tag{24}$$

and r_i^p denotes the perpetual risk-free rate given by

$$r_i^p = r_i + \frac{r_j - r_i}{\tilde{p}+r_j}\tilde{p}\tilde{f}_j, \tag{25}$$

in which $\tilde{p}=\tilde{\lambda}_1+\tilde{\lambda}_2$ is the risk-neutral rate of news arrival and $\left(\tilde{f}_G,\tilde{f}_B\right)=\left(\frac{\tilde{\lambda}_B}{\tilde{p}},\frac{\tilde{\lambda}_G}{\tilde{p}}\right)$ is the long-run risk-neutral distribution. $\hat{d}_i(\cdot)$ denotes the value of debt of a firm with only invested assets. $[A_{G1},A_{G2},A_{G3},A_{G4},C_1,C_2,B_1,B_2]$ solve a linear system given in Section Appendix A.4.

Proof. See Appendix A.4. □

Proposition 2 shows that the firm faces three different regions depending on the value of X. Below the default threshold i.e. $X \leq D_i$, the firm is in the default region in which it defaults immediately. Debtholders receive a fraction $\alpha_i\Lambda_i$ of the total after tax asset value.

The firm is in the continuation region if X is between the default threshold and the exercise boundary i.e. if $D_i < X \leq X_i$. In this region, debt value is determined by three components. The first component is the value of a risk-free claim to the perpetual stream of coupon. The second and third components reflect the changes in the value of debt that occur either due to the idiosyncratic shock reaching a boundary or due to a regime switch. For the region $D_B < X \leq X_G$, where the firm is in the continuation region in both good states and bad states, the solution consists of five terms. The value of the risk-free claim to the coupon is given by the last term. The coupon needs to be discounted by the perpetual risk-free rate r_i^p that incorporates the expected future time spent in each regime. The first four terms capture the changes in value due to the idiosyncratic shock X hitting a region boundary or due to a change of regime. When $D_G < X \leq D_B$, the firm is in the continuation region only in good states, and the solution consists of six terms. The last term is the value of the risk-free claim to the coupon, in which the discount rate is given by the interest rate in good states, r_G, increased by $\tilde{\lambda}_G$ to reflect the possibility of a regime switch to the bad state. The first five terms capture the changes in debt value that occur when the idiosyncratic shock reaches a boundary or when the regime switches to the bad state triggering immediate default. For the region $X_G < X \leq X_B$ where the firm is in the continuation region only in bad states, the solution consists of five terms. The last term is the value of a risk-free perpetual claim to the coupon. To account for a possible regime switch to the good state, the discount rate is given by the interest rate in the bad state, r_B, increased by $\tilde{\lambda}_B$. The remaining four terms capture the value changes due to reaching a region boundary, either X_G from above or X_B from below, or due to a regime switch to a good state triggering immediate option exercise financed with equity.

Finally, the debt value in the exercise region, reached when $X > X_i$, incorporates the financing source for the option exercise cost. In the good states, the option exercise cost K_G is financed by issuing new equity of $K_G(1+\Upsilon_G)$. Hence, the earnings of the firm are scaled by $s_G + 1$. In the bad states, the exercise cost K_B is financed by selling $\frac{K_B/\Lambda_B}{(1-\tau)X_{\bar{i}}y_B}$ of the assets in place, such that the earnings of the firm are scaled by $(s_B + 1 - \frac{K_B/\Lambda_B}{(1-\tau)X_{\bar{i}}y_B})$.

The value of the tax shield before investment can be calculated by using the solution (16) in Proposition 2, in which c and α are replaced by $c\tau$ and zero, respectively, and $\hat{d}_i$ in the last line line of (16) is replaced by $\hat{t}_i$. The value of bankruptcy costs before investment is derived by using the same steps as for debt value with two simple modifications. First, c and α need to be replaced by zero and $(1-\alpha)$, respectively. Second, while the going concern value of the expansion option is given by its leveraged value, the value of the option at default corresponds to its unleveraged value. Therefore, the expansion option's value switches from $G_i(X)$ to $\alpha_i \Lambda_i G_i^{unlev}(X)$ upon default. As a consequence, the functional form of the solution (16) in the default region $X \leq D_i$ needs to be adapted to $(1-\alpha_i\Lambda_i)y_iX(1-\tau) - \alpha_i\Lambda_i G_i^{unlev}(X) + G_i(X)$. The Appendix A.5 shows the resulting solution for the value of bankruptcy costs $b_i(X)$.

Next, firm value before investment, f_i, in regime $i = G, B$ is given by the value of assets in place $(1-\tau)y_iX$, plus the growth option value $G_i(X)$ and the value of tax benefits from debt $t_i(X)$, minus the value of default costs $b_i(X)$ i.e.

$$f_i(X) = (1-\tau)y_iX + G_i(X) + t_i(X) - b_i(X). \tag{26}$$

As firm value equals the sum of debt and equity values, equity values before investment of a firm that finances investment by issuing equity in good states and selling assets in bad states, $e_i^{ES}(X,c)$, $i = G,B$, is

$$e_i(X,c) = f_i(X) - d_i(X) = (1-\tau)y_i X + G_i(X) + t_i(X) - b_i(X) - d_i(X). \tag{27}$$

Equity-holders select the default and investment policies that maximize the ex post value of equity. Denote these policies by D_i^* and X_i^*, respectively. The default policy that maximizes the equity value is determined by setting the first derivative of the equity values to zero at the default boundary in each regime. Simultaneously, optimality of the option exercise thresholds is achieved by equating the first derivative of the equity values at the exercise thresholds to the first derivative of the equity values of a firm with only invested assets after expansion, evaluated at the corresponding earnings in both regimes. These four optimality conditions represent smooth-pasting conditions for equity of a firm that finances the option exercise cost by issuing equity in good states and selling assets in bad states at the respective boundaries:

$$\begin{cases} e'_G(D_G^*,c) & = & 0 \\ e'_B(D_B^*,c) & = & 0 \\ e'_G(X_G^*,c) & = & \hat{e}'_G\left((s_G+1)X_G^*,c\right) \\ e'_B(X_B^*,c) & = & \hat{e}'_B\left(((s_B+1-\frac{K_B/\Lambda_B}{(1-\tau)X_i^* y_B})X_B^*),c\right). \end{cases} \tag{28}$$

The system is solved numerically. As the equations in system Eqs. (28) are evaluated simultaneously, the four conditions are interdependent. Similar systems can be derived for a firm that finances the option exercise cost by issuing equity in both good states and bad states, by selling assets in good states and issuing equity in bad states, or by selling assets in both good states and bad states.

Denote by $e_i^{m,n*}(X,c)$ the equity value given optimal ex post default and expansion thresholds. The exponents $m \in \{E,S\}$ and $n \in \{E,S\}$ indicate the funding sources in good states and bad states, respectively. E denotes equity financing and S selling assets. For each coupon level c, equityholders select the ex post optimal funding source Ω_i^* that maximizes the value of equity, i.e.,

$$\Omega_i^* := \text{argmax}_{m,n}\left(e_i^{m,n*}(X,c)\right). \tag{29}$$

Debtholders anticipate the ex post optimal default and expansion policies, as well as the optimal funding source chosen by shareholders. As debt-issue proceeds accrue to shareholders, they do not only care about the value of equity, but also about the initial valuation of debt. Hence, the optimal capital structure is determined ex ante by the coupon level c^* that maximizes the value of equity and debt, i.e., the value of the firm. Denote by $f_i^*(X)$ the firm value given optimal default boundaries, expansion thresholds, and the optimal funding source. The ex ante optimal coupon of the firm solves

$$c_i^* := \text{argmax}_c f_i^*(X). \tag{30}$$

To summarize, equityholders face the following decisions: First, they choose the default and expansion thresholds that maximize the ex post value of equity for each coupon and funding source. Second, equityholders select the funding strategy that maximizes the ex post value of equity for each coupon. Finally, they determine the initial capital structure that maximizes the ex ante value of equity.

5. Results

In this section, we study the implications of the model for a typical model firm. We start by describing parameter choices for our baseline calibration before we derive the hypothesis in Section 5.2.

5.1. Parameter choice

We summarize our parameter choices in Table 2. Panel A shows the firm characteristics. The initial value of the idiosyncratic earnings X is set to 10. While the starting value for earnings is arbitrary, our results do not depend on this choice. We set the tax advantage of debt to $\tau = 0.15$ as suggested in the literature (e.g., Hackbarth, Miao, and Morellec 2006). Bhamra, Kuehn, and Strebulaev (2010) estimate growth rates and systematic volatilities of earnings in a two-regime model. Their estimates are similar to those obtained by other authors who jointly estimate consumption and dividends with a state-dependent drift and volatility (e.g., Bonomo and Garcia 1996). Hence, we set earnings growth rates (μ_i) and volatilities ($\sigma_i^{X,C}$) to their empirical counterparts reported in Bhamra, Kuehn, and Strebulaev (2010). The idiosyncratic volatility is set to 0.168. Arnold, Wagner, and Westermann (2013) show that using this volatility calibration, a simulated sample of firms with growth options has an average asset volatility of approximately 25%, which corresponds to the average asset volatility of firms with rated debt outstanding (see Schaefer and Strebulaev 2008).

INSERT TABLE 2 NEAR HERE

The main costs of external equity discussed by Fazzari, Hubbard, Petersen, Blinder, and Poterba (1988) are tax costs, adverse selection premia, and flotation costs. Hansen (2001) and Corwin (2003) estimate equity issuance costs around 7% for IPOs and SEOs, respectively. Altinkilic and Hansen (2000) argue that equity costs derive mainly from the variable component. The linear variable component estimated in Hennessy and Whited (2007) is 9.1%. Concerning cyclicality, Bayless and Caplinsky (1996) find that a typical hot market issuer would forego up to 2.33% in additional equity value if he would issue in a cold market instead. To reflect these empirical quantities, we choose as a benchmark case $\Upsilon_G = 0.08$ and $\Upsilon_B = 0.1$. This setting gives us a cyclicality for the equity issuance cost of a two percentage points difference between good and bad states, and an average total equity issuance cost of 8.71%.[11] In the comparative statics we vary the equity issuance cost to analyze how they affect the decision of firms to sell assets to finance the investment cost.

There are only a few empirical studies that estimate the cost of selling assets. Pulvino (1998) finds costs of selling commercial aircrafts between zero and 14%. Strebulaev (2007) assumes that the cost of selling assets lies between 0.05 to 0.25%. Acharya, Bharath, and Srinivasan (2007) show that creditors of defaulted firms recover 10 to 15 percentage points less in a distressed state of the industry than in a healthy state of the industry, i.e., that asset liquidity is cyclical. Overall, we only have vague empirical evidence on the appropriate choice of the parameters for the cost of selling assets. Hence, to avoid that our results are driven by this choice when analyzing firms' endogenous financing decisions, we set $\Lambda_{\bar{i}}$ such that $K_{\bar{i}}/\Lambda_{\bar{i}} = K_{\bar{i}}(1+\Upsilon_{\bar{i}})$, i.e., the friction adjusted cost of exercising the expansion option by selling assets corresponds to the one of exercising the expansion option by issuing new equity. This calibration yields $\Lambda_G = 0.9259$ and $\Lambda_B = 0.9091$.

One caveat is that equity issuance cost and asset liquidity are hard to estimate. We address this issue in two different ways. First, we base our parameter choice on empirical results of previous works. Second, we perform numerous robustness checks with alternative equity issuance cost and asset liquidity parameters. We find that our qualitative predictions are not affected by varying these parameters within plausible ranges.

Bankruptcy costs are assumed to be 30% of the unleveraged assets' liquidation proceeds. Recovery rates are $\Lambda_i(1-0.3)$, so they are 0.63 in good states and 0.57 in bad states. These values are in accordance with the unconditional standard of 0.6 used in the literature (e.g. Chen 2010), and with the notion in e.g. Acharya, Bharath, and Srinivasan (2007) that recovery rates fall during bad states.

Panel B of Table 2 shows the parameters we use to capture growth options. We select exercise prices of $K_G = 183.13$ and $K_B = 160$, respectively. The decline from K_G to K_B corresponds to the relative decline in the value of invested assets following a shift from good to bad states of 12.61%, which is similar to the one assumed in e.g. Hackbarth, Miao, and Morellec (2006). We validate the robustness of our predictions by presenting the results for alternative choices of the absolute level of K_i.

The scale parameter s_i depends on the cyclicality of the firm's option. We use baseline scale parameters of $s_G = 1.0925$ and $s_B = 1.03$. These parameters imply that, given optimal financing at initiation, the average q is 1.3. The q of a model firm is obtained by dividing the value of the firm by the value of its invested assets.

[11] The weights for this average correspond to the long-run, risk-neutral distribution of the Markov chain. One could also simulate a large sample of firms and determine the weights according to the occurrence of equity issues in the two states.

To calculate the average q, the initial q in good and bad states is weighted by the long-run distribution of the Markov chain. To generate typical firms with different degrees of the cyclicality of the expansion option, we alter s_G and s_B while keeping the size of the average q at initiation fixed at its empirical counterpart.

Finally, Panel C, lists the variables describing the underlying economy. The rates of leaving regime i (λ_i), the consumption growth rates (θ_i), and the consumption growth volatilities σ_i^C are estimated in Bhamra, Kuehn, and Strebulaev (2010). In the model economy, the expected duration of regime B (R) is 3.68 (2.03) years, and the average fraction of time spent in regime B (R) is 64% (36%). The annualized rate of time preference, ρ, is 0.015; the relative risk aversion, γ, is equal to 10; and the elasticity of intertemporal substitution, Ψ, is set to 1.5. This parameter choice is commonly used in the literature (e.g., Bansal and Yaron 2004, Chen 2010). It implies that the nominal interest rates are $r_G = 0.0736$ and $r_B = 0.0546$.

5.2. Derivation of the model predictions

Exercising an expansion option has two implications for a firm that finances the exercise cost of the option by issuing equity. First, it increases total earnings. Second, the total asset volatility decreases because the expansion option is riskier than the assets in place (see e.g. Arnold, Wagner, and Westermann 2013). Both effects induce a wealth transfer from equityholders to debtholders as debt becomes less risky. This wealth transfer problem is more severe for firms with larger leverage. The reason is that if leverage increases, debt becomes riskier and, consequently, more sensitive to earnings and asset volatility changes.

The wealth transfer problem has an impact on equityholders' investment timing. The equity value maximizing earnings thresholds for the option exercise of a firm that finances the investment cost by issuing equity are plotted in Figure 1. The lower solid line depicts the optimal investment threshold for various levels of leverage in the good state. The higher solid line is the corresponding threshold in the bad state. As expected, the firm invests earlier in the good state. We refer to the investment thresholds without debt (at zero leverage) as the option value-maximizing threshold. The larger the leverage, the later the equityholders invest compared to the option value-maximizing threshold due to the wealth transfer problem (underinvestment). The dashed lines in Figure 1 depict the optimal investment thresholds of a firm that sells assets to finance the exercise cost of the option. The lower dashed line is the threshold in the good state, the higher dashed line the one in bad times. The option exercise thresholds for financing asset sales in Figure 1 are

INSERT FIGURE 1 NEAR HERE

closer to the option value-maximizing threshold, particularly for large leverage firms in which the wealth transfer problem is more pronounced.[12] Hence, financing asset sales mitigate the underinvestment problem compared to equity financed investment.

In the simulations, the cost of exercising the expansion option by selling assets can be larger than the ones of exercising the expansion option by issuing new equity, i.e., $K_{\bar{i}}/\Lambda_{\bar{i}}$ may be larger than $K_{\bar{i}}(1+\Upsilon_{\bar{i}})$. At the same time, however, selling assets upon investment increases leverage which renders debt more risky. The corresponding wealth transfer from debtholders to equityholders ameliorates the initial wealth transfer problem from the exercise of the expansion option. Hence, equityholders trade off the incremental friction cost of selling assets over the equity issuance cost against the reduction in the wealth transfer when deciding whether to sell assets or to issue equity to finance the exercise of the expansion option. As the wealth transfer problem is more severe for firms with larger leverage, equityholders of such firms tend to finance the expansion option by selling assets. This insight leads to our first model prediction.

Prediction 1. *Equityholders of firms with a larger leverage have a higher tendency to finance the exercise cost of the expansion option by selling assets.*

Prediction 1 explains why we find that, empirically, the correlation between asset sales and investment is higher for firms with larger leverage.

Figure 2 illustrates the quantitative effect of allowing equityholders into financing asset sales on the value of a firm that starts in the good state. The solid line shows the relationship between the increase in

INSERT FIGURE 2 NEAR HERE

the value of a firm from permitting financing asset sales and the scale parameter s_G for high leverage firms. High leverage firms have an initial leverage ratio of 0.75. The remaining parameters are set according to the baseline firm. In particular, the friction cost of selling assets corresponds to the one of issuing new equity. Hence, we measure the pure impact of permitting financing asset sales on firm value from a mitigation of the underinvestment problem. The dashed and dotted lines plot the relationship for medium leverage firms with an initial leverage of 0.5 and for low leverage firms with an initial leverage of 0.35, respectively. Firm value always increases with financing asset sales because they enable equityholders to follow a better option exercise policy. The larger the scale parameter and the higher the leverage in Figure 2, the stronger the posi-

[12] For certain parameter combinations with a high scale parameter s_i, and a low exercise cost K_i, financing asset sales can also induce overinvestment such that the dashed thresholds in Figure 1 decrease with leverage. The reason is that the expansion option is almost immediately exercised for these parameter combinations. As a consequence, the investment cost constitutes a much larger fraction of the asset value upon exercise than for parameter combinations for which the option is exercised at a larger level of X. Hence, the impact on leverage from financing asset sales is also larger, and the corresponding higher wealth transfer can induce equityholders to exercise the option at an earnings level below the option value-maximizing threshold.

tive impact of permitting financing asset sales on the value of a firm. The reason is that the mitigation of the underinvestment problem from financing asset sales is particularly valuable if the value of the growth option is high, and the wealth transfer problem is large due to high leverage. Our results speak to the covenant literature. First, they provide an intuition for why firms negotiate conventions in asset sale covenants that still allow them to use financing asset sales as described in the literature (e.g., Smith and Warner 1979, Bradley and Roberts 2004, Nini, Smith, and Sufi 2009). Second, they explain why high growth firms are typically less likely to include restrictive asset sale covenants (e.g. Kahan and Yermack 1998, Nash, Netter, and Poulsen 2003, Chava, Kumar, and Warga 2010, Reisel 2014). Finally, the results also induce new testable predictions. In particular, the propensity of firms to implement restrictive asset sale covenants should be specifically low if they have both high leverage and high growth opportunities.

We now investigate how the wealth transfer problem of the baseline firm depends on different states of the business cycle. During bad times, leverage increases because the assets of a firm lose more value relative to the decrease in value of the outstanding debt. At the same time, Figure 1 shows that equityholders optimally invest at a higher level of earnings in the bad state. A higher investment threshold induces a larger asset value upon investment and hence lower leverage. To see which effect dominates, Figure 3 plots leverage upon investment for a baseline firm with an initially optimal capital structure and endogenous choice of the funding source, in which the equity issuance cost parameter in good states, Υ_G, is on the x-axis. The corresponding equity issuance cost parameter in bad states is determined by adding 0.02. In this way, we maintain the same difference between the equity issuance costs in good states and bad states as in the baseline parameter specification. The dashed line depicts the leverage at investment during bad states, the solid line is the one during good states. The bumps around $\Upsilon_G = 0.075$ occur due to the switch in the firm's optimal financing strategy. Figure 3 shows that leverage at investment is larger during bad states than during good states. As the wealth transfer problem is more severe for higher leverage, and because asset sales ameliorate this problem, equityholders' trade-off between the cost of financial frictions and the wealth transfer leads to the second model prediction.

Prediction 2. *Firms are more likely to fund investments by selling assets during bad business cycle states.*

Prediction 2 provides an explanation for why the correlation between asset sales and investment is significantly higher during bad business cycle states in our Compustat sample.

INSERT FIGURE 3 NEAR HERE

The procyclical nature of aggregate investment (see e.g. Barro 1990) suggests that growth options are more valuable during good times than during bad times. We argue that the degree of this cyclicality of the growth option is different across firms. To model a firm with a relatively higher value of the expansion option in the good states, i.e., with a higher cyclicality of the expansion option, relative to the baseline firm, we increase the scale parameter in good times, s_G, from 1.0925 to 1.099, and decrease the scale parameter in bad states, s_B, from 1.03 to 1.005, leaving the average q at initiation unchanged at 1.3.[13] A higher scale parameter in good times, and a lower scale parameter in bad times make it relatively more (less) attractive to exercise the option in the good (bad) state compared to the baseline firm. The optimal investment threshold in the good state decreases from 20.18 to 19.67, and the one in the bad state increases from 20.48 to 22.23. Hence, firms with a relatively higher value of the expansion option in good times have a lower probability to invest during bad times. Additionally, Figure 4 compares leverage levels upon investment of the baseline firm to the ones of the firm with a more valuable growth option in good states. The dotted and dashed-dotted lines depict leverage ratios upon investment in good and bad times of the firm with a more valuable growth option in good states. The expansion option of the baseline firm has a relatively higher value during bad times than the one of the firm with a more cyclical growth option. Hence, the baseline firm optimally invests at a lower earnings threshold in bad times, which induces that the asset value is lower and the leverage at investment is higher. As the wealth transfer problem is more severe for firms with higher leverage, and because equityholders trade off the financing cost differential between equity issuance and asset sales against the reduction in the wealth transfer problem when selecting the funding source, we can phrase our third model prediction.

Prediction 3. *Firms with a more valuable expansion option during bad business cycle states are more likely to finance investments by selling assets during bad business cycle states than firms with a more cyclical expansion option.*

This prediction explains our empirical finding that the correlation between asset sales and investment is higher for firms with a low cyclicality of the expansion option during bad times.

INSERT FIGURE 4 NEAR HERE

Figure 5 summarizes the implications of the wealth transfer problem upon investment on equityholders' endogenous financing choice for the baseline firm. On the x-axis, we again plot the equity issuance cost in good states. The y-axis shows the coupon payments that determine a firm's leverage ratio. We are interested in a wide range of coupon payments to replicate the cross-section of real firms that may deviate from an

[13] The cyclicality of the expansion option can also be altered by changing the investment cost K_i. The qualitative predictions from our model also hold in this case.

optimal leverage ratio (see Section 6). On the left hand side of the solid line, equityholders optimally select equity financing in both regimes. On the right hand side of the dashed line, firms prefer financing asset sales in both regimes. Between the two lines the firms' optimal financing strategy yields the issuance of equity in good times, and selling assets in bad times. Recall that in case of the baseline firm with an equity issuance cost of 0.08 in good states and 0.1 in bad states, the cost of selling assets, Λ_i, is calibrated such that the friction cost of issuing equity corresponds to the friction cost of selling assets. In an unleveraged firm as shown on the x-axis, equityholders simply select the funding source based on this financing friction cost: If the equity issuance cost in good states is smaller than 0.08, they finance the exercise cost of the option by issuing equity; otherwise, they finance this cost by selling assets. The figure shows that for larger coupon payments, the range of equity issuance costs for which equityholders prefer equity financing in both regimes declines, and the range for which they prefer selling assets increases. The reason is that asset sales reduce the wealth transfer problem in particular for high leverage firms, and equityholders trade off this reduction against the incremental friction cost of selling assets over issuing equity when selecting the funding source.

INSERT FIGURE 5 NEAR HERE

The figure also shows the higher propensity of financing assets sales in bad business cycle states. The region in which they select financing asset sales in both regimes (on the right side of the dashed line) is smaller than the region in which they optimally sell assets during bad states (on the right side of the solid line).

6. Aggregate Dynamics of Simulated Samples

The analysis of a typical firm at initiation in Section 5.2 contributes to our understanding of the optimal choice between asset sales and equity issuance as sources of investment financing. In this section, we follow Strebulaev (2007) and study the aggregate dynamics of simulated model-implied economies by investigating the cross sectional properties of corporate policies in a way that makes our results comparable to empirical evidence. The simulation approach is important for two reasons. First, the analysis of a typical firm at initiation in Section 5.2 does not allow us to analyze the dynamic features predicted by our model. We need to simulate the model to generate time series of investment, financing, and default observations over the business cycles. Comparing the resulting simulated data patterns to the ones observed in our Compustat sample enables us to validate our model. We can also measure how the propensity of model firms to use financing asset sales relates to firm and business cycle characteristics. This analysis helps us to confirm our explanations for the empirical regression results on the relation between investments and financing asset sales, and to derive new predictions on the impact of time-varying business cycle conditions on the dynamic time serial patterns of financing asset sales.

Second, the analysis of a typical (average) firm does not consider the time evolution of the cross sectional distribution of real firm characteristics. As investment, financing, and default rates are nonlinear in firm characteristics, however, it is crucial to measure these rates for simulated samples of firms that match the empirical cross sectional distribution of real firm characteristics. Only the dynamic features of the average rates in these simulated matched samples should then be compared to the empirical average behavior of real firms, and be used to derive new predictions.

6.1. Details on the Simulation

For each simulation we generate an economy of model firms. We set up a grid of different firms, each featuring a unique combination of coupon, scale parameter, and equity issuance cost. Coupons range from two to the largest possible value such that no firm defaults immediately. The step size takes a value of two. Scale parameters for firms with a less cyclical growth opportunity range from 0.79 in the good state and 0.73 in the bad state, and for firms with a more cyclical growth opportunity from 0.80 in the good state and 0.71 in the bad state to the largest possible value such that the option is not exercised immediately, with a step size of 0.3. Equity issuance costs range from 0.04 to 0.09 in the good state, with a step size of 0.005. The equity issuance cost parameter in the bad state is obtained by adding 0.02 to the corresponding value in the good state. The remaining parameters are equal to those of the baseline firm.[14] The grid contains 849 different firm types. The earnings path of each firm type is then simulated forward 25 times over 10 years. The initial state of the simulated economy is selected according to the long-run historical distribution of the states. Firms are exposed to the same macroeconomic shocks, but experience different idiosyncratic shocks, resulting in a model-implied economy populated by more than $20,000$ different firms. This model-implied economy has a broad range of leverage ratios, growth opportunities, and equity issuance costs at the last simulated date.

Next, we calculate the average leverage, Tobin's q, and equity issuance cost for each firm in our Compustat sample to match the model-implied economy to the cross sectional distribution of real firms (we define all empirical variables in Appendix B). We consider a total of 1352 Compustat firms for which we obtain all three measures. Firms with a q value below 1.15, and above 2.15 are winsorized because our model-implied economies hardly contain firms with extremely low or high values of the growth option.[15]

[14] We have verified in simulations for various alternative grids (available upon request) that the results are qualitatively identical.

[15] Firms with a growth option that accounts for less than 13% of the total firm value almost never exercise their option, and firms with a growth option that accounts for more than 54% of the total firm value almost immediately exercise their option.

To match the model-implied economy with their empirical counterpart we select for each observation in the Compustat sample the firm at the last date of the simulated economy that has the minimal Euclidean distance with respect to leverage, q, and the equity issuance cost. The matching is accurate, with an average Euclidean distance of 0.0226. The procedure allows us to construct a cross sectional distribution of model-implied matched firms that closely reflects its empirical counterpart. These matched firms are quarterly simulated forward over 60 years under the historical probability measure. The equityholders of each firm behave optimally conditional on current earnings and on the current business cycle: If current earnings are below the corresponding regime-dependent default boundary, they default immediately; if current earnings are above the corresponding regime-depending option exercise threshold, they exercise the expansion option and select the optimal funding source for the option exercise cost; otherwise, equityholders take no action. To maintain a balanced sample of firms when we simulate the matched firms over time, we exogenously introduce new firms. In particular, we replace each defaulted or exercised firm by a new firm whose growth option is still intact. Replaced firms have the same initial parameter values as the corresponding defaulted or exercised firm at initiation. To ensure robustness of our results, the entire simulation is repeated 100 times. We record and analyze the simulated matched samples.

6.2. Results for the Simulated Matched Samples

In this section, we first show that a typical simulated matched sample exhibits realistic properties to validate our model approach. We then provide additional support for the ability of our model to explain the empirical patterns that we observe in the Compustat data, and discuss novel predictions for financing asset sales.

Table 3 reports averages over all simulations of the mean values, as well as the standard deviations of these mean values, for important variables of the simulated matched samples. Besides the results for the full sample, we also provide statistics that condition on the bad and good states, respectively.

INSERT TABLE 3 NEAR HERE

The results in Table 3 show that, while firms in a simulated sample are only initially matched statically to the average leverage, Tobin's q, and equity issuance cost of Compustat firms, our model generates many key dynamic properties that we observe in our empirical sample (see Appendix B, Table B1). That is, firms in the simulated samples exhibit, on average, procyclical asset values, q values, coverage ratios, and equity values. The average corporate leverage is countercyclical. The simulated samples also resemble several other dynamic features of the Compustat data. For instance, as in our Compustat sample, high q firms have on average a lower leverage and invest more than low q firms.

Next, we investigate in more detail the dynamics of equity financing and investments predicted by a typical simulation of a matched sample. Figure 6 shows the time series of the relative amount of firms that issue equity in the typical sample. The shaded areas represent bad states. Our model firms exhibit procyclical aggregate equity issuance patterns that correspond to well established findings in the empirical literature (e.g., Choe, Masulis, and Nanda 1993, Bayless and Caplinsky 1996).

INSERT FIGURE 6 NEAR HERE

Figure 7 depicts the time series of the investment rate, which is the fraction of firms that exercise their expansion option. The aggregate investment pattern is procyclical (see for corresponding evidence in the empirical investment literature e.g. Barro 1990, Cooper, Haltiwanger, and Power 1999).

INSERT FIGURE 7 NEAR HERE

Figure 8 shows the time series of the aggregate default rate of the typical simulated sample. Aggregate defaults are countercyclical, and often spike in the beginning of a bad state. This pattern is consistent with empirical observations (see e.g. Duffie, Saita, and Wang 2007, Das, Duffie, Kapadia, and Saita 2007).

INSERT FIGURE 8 NEAR HERE

After verifying that the model features realistic sample properties, we now analyze the model's predictions with regard to the cyclical nature of financing asset sales. Figure 9 depicts the time series of the relative number of firms that sell assets to finance the exercise cost of the option in the typical simulated sample. Financing asset sales are generally procyclical, mainly because there is more financing demand from investments during good states. Yang (2008) shows that the procyclicality of asset sales can be due to higher efficiency gains or lower financing costs during good states. Maksimovic and Phillips (2001) argue that more assets are sold in good business cycles due to firms' refocusing in boom. Our results, though, suggest that financing needs also contribute to the procyclical nature of asset sales.

Figure 9 also implies that a pronounced financing asset sales activity can occur in the very beginning of a bad state. This pattern is mainly driven by firms with a less cyclical growth option that have a relatively low investment threshold during bad states. As earnings still tend to be high in the very beginning of a bad state when the economy just left a good state, such firms may benefit from the reduction in the investment cost. These investments lead to clustered financing needs that are partially covered by financing asset sales. The clustering levels off when earnings start to decline with the duration of a recession.[16]

INSERT FIGURE 9 NEAR HERE

Figure 10 compares the time series of the investment rate (solid line) to the one of the financing asset sales rate (dashed line). The distance between the dashed and solid lines decreases during bad states, which indicates that asset sales are a relatively more important funding source for firms' investment activities during bad states. Hence, Figure 10 illustrates that the aggregate dynamics of asset sales and investment across states generated by our model are consistent with our finding in the Compustat data that the correlation between asset sales and investment is significantly higher during bad business cycle states.

INSERT FIGURE 10 NEAR HERE

In Table 4 we summarize additional features of the aggregate simulated model dynamics of financing asset sales that corroborate our predictions from a typical firm at initiation. The conditional asset sale ratio is the percentage of firms in the simulated matched samples that, upon investment, finance the exercise cost of the option by selling assets. As the sources of uncertainty are well defined in our model, we do not run regressions on the simulated samples.[17] Instead, we directly use this conditional ratio in the simulated samples to show that the wealth transfer motive of our model generates financing asset sales patterns that are consistent with the stylized facts in the Compustat sample of Table 1.

Overall, 42% of the investments in the simulated samples are financed with asset sales. If we only consider firms that are in the highest leverage tercile, this ratio increases to 64%. For firms in the lowest leverage tercile, the ratio decreases to 35%. The result that highly leveraged firms in the simulated matched samples have a higher tendency to use financing asset sales upon investment provides supports Prediction 1 and the stylized fact that the correlation between asset sales and investment increases in leverage.

INSERT TABLE 4 NEAR HERE

In bad states, the conditional asset sale ratio increases to 54%, and amounts to 38% in good states. This finding confirms Prediction 2 that firms have a higher propensity to sell assets upon investment during bad

[16] Decreasing the proportion of firms with a less cyclical growth option reduces the clustering, and, hence, the investment rate during bad states. It does, however, not affect the relative propensity of firms to use financing asset sales during bad states.

[17] There no are control variables required for financing asset sales in our simulated samples. From an econometric point of view, it is even problematic to apply regression techniques on simulated samples because most model-firm variables are highly collinear.

states, which is also corroborated by the increased correlation between asset sales and investment during bad business cycle states in the real data. Finally, we analyze which firms drive the high conditional asset sale ratio during bad times in our simulated samples. The last four rows in Table 4 report the asset sale ratios for firms in the simulated samples with a relatively low (L) and high (H) cyclicality of the expansion option during good and bad states, respectively. Consistent with Prediction 3, firms with a low cyclicality have the highest ratio during bad business cycle states, which explains the stylized fact of an increased correlation between asset sales and investments for firms with less cyclical growth options in bad states.

7. Conclusion

In this paper, we analyze the decision of firms to sell assets to fund investments (financing asset sales). We begin by documenting novel empirical patterns of financing asset sales for a sample of U.S. Compustat firms. We find that the correlation between asset sales and investment is significantly higher (i) for firms with higher leverage, (ii) in bad business cycle states, and (iii) for firms with a low cyclicality of the expansion option in bad business cycle states. These stylized facts cannot be explained by traditional motives for asset sales, such as financial distress or external financing constraints.

Against the backdrop of these stylized facts, we study a structural model with time-varying business cycle conditions, embedded inside a representative agent consumption-based asset pricing framework, that endogenizes the choice between asset sales and equity issuance to fund capital expenditures. Notably, equity issuance cost, asset liquidity, and the growth option are subject to cyclicality. Recognizing the impact of cyclicality on growth options, financing decisions, and asset liquidity helps understanding financing asset sales.

Our model shows that at investment, the decrease in the asset volatility and the increase in earnings make debt less risky and, hence, transfer value from equityholders to debtholders. This mechanism leads to the well-known wealth transfer problem that induces underinvestment (Myers 1977). On the other hand, selling assets upon investment increases leverage, which makes debt riskier. The corresponding wealth transfer from debtholders to equityholders ameliorates the wealth transfer problem. Notably, we show that the trade-off between the cost of selling assets, the cost of issuing equity, and the magnitude of the wealth transfer problem explains the empirical patterns we observe for financing asset sales in our Compustat sample.

We also simulate model-implied economies over time that are structurally similar to our Compustat sample. The simulations generate dynamic patterns of investment, financing, default, and financing asset sales that are similar to the ones observed empirically. We additionally find that financing asset sales are procyclical, and can cluster at those points in time at which the economy switches from a bad to a good state.

Overall, we contribute to identifying empirical and theoretical determinants of financing asset sales. The observation that the wealth transfer problem can drive financing asset sales adds to our understanding of corporate financing decisions. Our analysis of the equityholders' choice between issuing equity and selling assets upon investment is a first step towards highlighting and studying financing asset sales. Future work could additionally explore to what extent internal cash and debt capacity affect this choice. Moreover, and in particular from an empirical perspective, the relationship between the wealth transfer problem and financing asset sales should also help explaining debt covenant structures observed in corporate practice.

Appendix A. Derivations

Appendix A.1. The stochastic discount factor, risk-free rates, and market prices of risk

Suppose the continuous-time analog of Epstein-Zin-Weil preferences of stochastic differential utility type (e.g., Duffie and Epstein 1992a, Duffie and Epstein 1992b). The utility index U_t over a consumption process C_s solves

$$U_t = \mathbb{E}^{\mathbb{P}}\left[\int_t^{\infty} \frac{\rho}{1-\delta} \frac{C_s^{1-\delta} - ((1-\gamma)U_s)^{\frac{1-\delta}{1-\gamma}}}{((1-\gamma)U_s)^{\frac{1-\delta}{1-\gamma}} - 1} \mathrm{d}s \,|\, \mathscr{F}_t\right], \tag{A.1}$$

in which ρ determines the rate of time preference, γ is the coefficient of relative risk aversion for a timeless gamble, and $\Psi := \frac{1}{\delta}$ is the elasticity of intertemporal substitution for deterministic consumption paths. Incorporating the separability of time and state preferences and assuming that $\Psi > 1$, i.e., that agents have a preference for early resolution of uncertainty and require expected returns that increase in the uncertainty about future consumption, are necessary to capture the impact of aggregate risk on corporate security values.

Bhamra, Kuehn, and Strebulaev (2010) and Chen (2010) show that solving the Bellman equation associated with the consumption problem of the representative agent yields that the stochastic discount factor m_t follows the dynamics

$$\frac{\mathrm{d}m_t}{m_t} = -r_i \mathrm{d}t - \eta_i \mathrm{d}W_t^C + (e^{\kappa_i} - 1)\,\mathrm{d}M_t, \tag{A.2}$$

in which M_t determines the compensated process associated with the Markov chain. r_i are the regime-dependent risk-free interest rates. The parameters η_i denote the risk prices for systematic Brownian shocks affecting aggregate output. The market prices of consumption risk η_i increase in the agents' risk aversion and consumption volatility. κ_i are the relative jump sizes of the discount factor when the Markov chain leaves state i, i.e., they are the market prices of discount factor jump risk.

Risk-free rates, and the market prices of consumption and jump risk are defined as

$$r_i = \bar{r}_i + \lambda_i \left[\frac{\gamma-\delta}{\gamma-1}\left(w^{-\frac{\gamma-1}{\gamma-\delta}} - 1\right) - \left(w^{-1} - 1\right)\right], \tag{A.3}$$

$$\eta_i = \gamma \sigma_i^C, \tag{A.4}$$

$$\kappa_i = (\delta - \gamma)\log\left(\frac{h_j}{h_i}\right), \tag{A.5}$$

with $i, j = G, B, i \neq j$. The parameters h_G, h_B solve the following non-linear system of equations (e.g. Bhamra, Kuehn, and Strebulaev 2010):

$$0 = \rho\left(\frac{1-\gamma}{1-\delta}\right) h_i^{\delta-\gamma} + \left((1-\gamma)\,\theta_i - \tfrac{1}{2}\gamma(1-\gamma)\left(\sigma_i^C\right)^2 - \rho\frac{1-\gamma}{1-\delta}\right) h_i^{1-\gamma} + \lambda_i\left(h_j^{1-\gamma} - h_i^{1-\gamma}\right) \tag{A.6}$$

The risk-free rates r_i consist of the interest rate if the economy stayed in regime i forever, $\bar{r}_i$, plus a second term adjusting for possible regime switches. The no-jump part of the interest rates, $\bar{r}_i$, is given by

$$\bar{r}_i = \rho + \delta\theta_i - \frac{1}{2}\gamma(1+\delta)\left(\sigma_i^C\right)^2, \tag{A.7}$$

and

$$w := e^{\kappa_B} = e^{-\kappa_G} \tag{A.8}$$

measures the size of the jump in the real-state price density when the economy shifts from bad states to good states (see for example Proposition 1 in Bhamra, Kuehn, and Strebulaev 2010).

Appendix A.2. Derivation of the values of corporate securities after investment

The valuation of corporate debt. Our valuation of corporate debt of a firm that consists of only invested assets in a two regime setting follows (Hackbarth, Miao, and Morellec 2006). We consider the case in which the default boundary in good states is lower than the one in bad states, i.e., $\hat{D}_G < \hat{D}_B$. If the firm defaults, debtholders receive a fraction $\Lambda_i\alpha_i$ of the unleveraged after tax asset value $(1-\tau)Xy_i$. A debt investor requires an instantaneous return equal to the risk-free rate r_i. The instantaneous debt return corresponds to the realized rate of return plus the coupon proceeds from debt. Therefore, an application of Ito's lemma with regime switches shows that debt satisfies the following system of ODEs.

For $0 \leq X \leq \hat{D}_G$:

$$\begin{cases} \hat{d}_G(X) &= \alpha_G\Lambda_G(1-\tau)Xy_G \\ \hat{d}_B(X) &= \alpha_B\Lambda_B(1-\tau)Xy_B. \end{cases} \tag{A.9}$$

For $\hat{D}_G < X \leq \hat{D}_B$:

$$\begin{cases} r_G\hat{d}_G(X) &= c + \tilde{\mu}_G X\hat{d}_G'(X) + \frac{1}{2}\tilde{\sigma}_G^2X^2\hat{d}_G''(X) + \tilde{\lambda}_G\left(\alpha_B\Lambda_B(1-\tau)Xy_B - \hat{d}_G(X)\right) \\ \hat{d}_B(X) &= \alpha_B\Lambda_B(1-\tau)Xy_B. \end{cases} \tag{A.10}$$

For $X > \hat{D}_B$:

$$\begin{cases} r_G\hat{d}_G(X) &= c + \tilde{\mu}_G X\hat{d}_G'(X) + \frac{1}{2}\tilde{\sigma}_G^2X^2\hat{d}_G''(X) + \tilde{\lambda}_G\left(\hat{d}_B(X) - \hat{d}_G(X)\right) \\ r_B\hat{d}_B(X) &= c + \tilde{\mu}_B X\hat{d}_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2X^2\hat{d}_B''(X) + \tilde{\lambda}_B\left(\hat{d}_G(X) - \hat{d}_B(X)\right). \end{cases} \tag{A.11}$$

The boundary conditions read

$$\lim_{X\to\infty}\frac{\hat{d}_i(X)}{X}<\infty,\quad i=G,B, \tag{A.12}$$

$$\lim_{X\searrow\hat{D}_B}\hat{d}_G(X)=\lim_{X\nearrow\hat{D}_B}\hat{d}_G(X), \tag{A.13}$$

$$\lim_{X\searrow\hat{D}_B}\hat{d}'_G(X)=\lim_{X\nearrow\hat{D}_B}\hat{d}'_G(X), \tag{A.14}$$

$$\lim_{X\searrow\hat{D}_G}\hat{d}_G(X)=\alpha_G\Lambda_G(1-\tau)D_Gy_G, \tag{A.15}$$

and

$$\lim_{X\searrow\hat{D}_G}\hat{d}_B(X)=\alpha_B\Lambda_B(1-\tau)D_By_B. \tag{A.16}$$

Condition (A.12) expresses the no-bubbles condition. The remaining boundary conditions are the value-matching conditions (A.13), (A.15), and (A.16), and the smooth-pasting condition at the higher default threshold $\hat{D}_B$ for the debt function in the good state $\hat{d}_G(\cdot)$, Eq. (A.14). The functional form of the solution is

$$\hat{d}_i(X)=\begin{cases}\alpha_i\Lambda_i(1-\tau)Xy_i & X\le\hat{D}_i \quad\quad i=G,B\\ \hat{C}_1X^{\beta_1^G}+\hat{C}_2X^{\beta_2^G}+C_3X+C_4 & \hat{D}_G<X\le\hat{D}_B,\quad i=G\\ \hat{A}_{i1}X^{\gamma_1}+\hat{A}_{i2}X^{\gamma_2}+A_{i5} & X>\hat{D}_B,\quad\quad i=G,B,\end{cases} \tag{A.17}$$

in which $\hat{A}_{G1},\hat{A}_{G2},\hat{A}_{B1},\hat{A}_{B2},A_{G5},A_{B5},\hat{C}_1,\hat{C}_2,C_3,C_4,\gamma_1,\gamma_2,\beta_1^G$, and β_2^G are real-valued parameters to be determined.

First, consider the region $X>\hat{D}_B$. We start by using the standard approach of plugging the functional form $\hat{d}_i(X)=\hat{A}_{i1}X^{\gamma_1}+\hat{A}_{i2}X^{\gamma_2}+A_{i5}$ into both equations of (A.11). Comparing coefficients and solving the resulting two-dimensional system of equations for A_{i5}, we find that

$$A_{i5}=\frac{c\,(r_j+\tilde{\lambda}_i+\tilde{\lambda}_j)}{r_ir_j+r_j\tilde{\lambda}_i+r_i\tilde{\lambda}_j}=\frac{c}{r_i^p}, \tag{A.18}$$

and that $\hat{A}_{Gk}$ is always a multiple of $\hat{A}_{Bk}$, $k=1,2$, with the factor $l_k:=\frac{1}{\tilde{\lambda}_G}(r_G+\tilde{\lambda}_G-\tilde{\mu}_G\gamma_k-\frac{1}{2}\tilde{\sigma}_G^2\gamma_k(\gamma_k-1))$, i.e., $\hat{A}_{Bk}=l_k\hat{A}_{Gk}$. Using these results when comparing coefficients again, it can be shown that γ_1 and γ_2 are the negative roots of the quadratic equation

$$(\tilde{\mu}_B\gamma+\tfrac{1}{2}\tilde{\sigma}_B^2\gamma(\gamma-1)-\tilde{\lambda}_B-r_B)(\tilde{\mu}_G\gamma+\tfrac{1}{2}\tilde{\sigma}_G^2\gamma(\gamma-1)-\tilde{\lambda}_G-r_G)=\tilde{\lambda}_B\tilde{\lambda}_G. \tag{A.19}$$

Due to the no-bubbles condition for debt stated in Eq. (A.12), we take the negative roots.

Next, we solve the region $\hat{D}_G\le X\le\hat{D}_B$. Plugging the functional form $d_G(X)=\hat{C}_1X^{\beta_1^G}+\hat{C}_2X^{\beta_2^G}+$

$C_3X + C_4$ into the first equation of (A.10), we find by comparison of coefficients that

$$\begin{aligned}
\beta_{1,2}^G &= \frac{1}{2} - \frac{\tilde{\mu}_G}{\tilde{\sigma}_G^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_G}{\tilde{\sigma}_G^2}\right)^2 + \frac{2(r_G + \tilde{\lambda}_G)}{\tilde{\sigma}_G^2}} \\
C_3 &= \frac{\tilde{\lambda}_G \alpha_B \Lambda_B (1-\tau) y_B}{r_G + \tilde{\lambda}_G - \tilde{\mu}_G} \\
C_4 &= \frac{c}{r_G + \tilde{\lambda}_G}.
\end{aligned} \tag{A.20}$$

We then plug the functional form (A.17) into conditions (A.13)–(A.16), and obtain a four-dimensional linear system in the remaining four unknown parameters $\hat{A}_{G1}, \hat{A}_{G2}, \hat{C}_1$, and $\hat{C}_2$:

$$\begin{aligned}
\hat{A}_{G1}\hat{D}_B^{\gamma_1} + \hat{A}_{G2}\hat{D}_B^{\gamma_2} + A_{G5} &= \hat{C}_1\hat{D}_G^{\beta_1^G} + \hat{C}_2\hat{D}_B^{\beta_2^G} + C_3\hat{D}_B + C_4 \\
\hat{A}_{G1}\gamma_1\hat{D}_B^{\gamma_1} + \hat{A}_{G2}\gamma_2\hat{D}_B^{\gamma_2} &= \hat{C}_1\beta_1^G\hat{D}_B^{\beta_1^G} + \hat{C}_2\beta_2^G\hat{D}_B^{\beta_2^G} + C_3\hat{D}_B \\
\alpha_G\Lambda_G(1-\tau)\hat{D}_G y_G &= \hat{C}_1\hat{D}_B^{\beta_1^G} + \hat{C}_2\hat{D}_B^{\beta_2^G} + C_3\hat{D}_B + C_4 \\
l_1\hat{A}_{G1}\hat{D}_B^{\gamma_1} + l_2\hat{A}_{G2}\hat{D}_B^{\gamma_2} + A_{B5} &= \alpha_B\Lambda_B(1-\tau)\hat{D}_B y_B.
\end{aligned} \tag{A.21}$$

Define the matrices

$$\hat{M} := \begin{bmatrix} \hat{D}_B^{\gamma_1} & \hat{D}_B^{\gamma_2} & -\hat{D}_B^{\beta_1^G} & -\hat{D}_B^{\beta_2^G} \\ \gamma_1\hat{D}_B^{\gamma_1} & \gamma_2\hat{D}_B^{\gamma_2} & -\beta_1^G\hat{D}_B^{\beta_1^G} & -\beta_2^G\hat{D}_B^{\beta_2^G} \\ 0 & 0 & \hat{D}_B^{\beta_1^G} & \hat{D}_B^{\beta_2^G} \\ l_1\hat{D}_B^{\gamma_1} & l_2\hat{D}_B^{\gamma_2} & 0 & 0 \end{bmatrix} \tag{A.22}$$

and

$$\hat{b} := \begin{bmatrix} C_3\hat{D}_B + C_4 - A_{G5} \\ C_3\hat{D}_B \\ \alpha_G\Lambda_G(1-\tau)\hat{D}_G y_G - C_3\hat{D}_B - C_4 \\ \alpha_B\Lambda_B(1-\tau)\hat{D}_B y_B - A_{B5} \end{bmatrix}, \tag{A.23}$$

such that $\hat{M}\begin{bmatrix}\hat{A}_{G1} & \hat{A}_{G2} & \hat{C}_1 & \hat{C}_2\end{bmatrix}^T = \hat{b}$. The solution for the unknown parameters is given by

$$\begin{bmatrix}\hat{A}_{G1} & \hat{A}_{G2} & \hat{C}_1 & \hat{C}_2\end{bmatrix}^T = \hat{M}^{-1}\hat{b}. \tag{A.24}$$

The value of the tax shield can be calculated by the formula for the value of debt, in which c is replaced by τc, and α is equal to zero. The value of bankruptcy costs is simply obtained by replacing c by zero, and α by $1-\alpha$.

Default policy. The value of equity corresponds to the firm value minus the value of debt. The firm value is given by the value of assets in place plus the value of the option and the tax shield minus default costs.

Once debt has been issued, managers select the ex post default policy that maximizes the value of equity. Formally, the default policy is determined by equating the first derivative of the equity value to zero at the corresponding default boundary:

$$\begin{cases} \hat{e}'_G(\hat{D}^*_G) &= 0 \\ \hat{e}'_B(\hat{D}^*_B) &= 0. \end{cases} \tag{A.25}$$

We solve this problem numerically.

For a firm that receives scaled earnings after investment, the value of corporate securities is solved similarly by replacing X with the scaled level of earnings. For example, if the firm exercises the option in the good state, and finances the exercise cost by issuing equity, the scaled earnings correspond to $(s_G+1)X$. The default boundaries $\hat{D}^*_G$ and $\hat{D}^*_B$ are then expressed in terms of the scaled earnings levels.

Appendix A.3. Derivation of the value of the growth option

The case in which $X_G < X_B$:

We present the derivation of the value of the growth option for a firm that finances the option exercise by issuing equity in good states and selling assets in bad states. The value of the growth option for a firm with an alternative financing strategy can be derived similarly. For each regime i, the option is exercised immediately whenever $X \geq X_i$ (option exercise region); otherwise, it is optimal to wait (option continuation region). This structure results in the following system of ODEs for the value function.

For $0 \leq X < X_G$:

$$\begin{cases} r_G G_G(X) &= \tilde{\mu}_G X G'_G(X) + \frac{1}{2}\tilde{\sigma}^2_G X^2 G''_G(X) + \tilde{\lambda}_G \left(G_B(X) - G_G(X)\right) \\ r_B G_B(X) &= \tilde{\mu}_B X G'_B(X) + \frac{1}{2}\tilde{\sigma}^2_B X^2 G''_B(X) + \tilde{\lambda}_B \left(G_G(X) - G_B(X)\right). \end{cases} \tag{A.26}$$

For $X_G \leq X < X_B$:

$$\begin{cases} G_G(X) &= (1-\tau)s_G X y_G - K_G(1+\Upsilon_G) \\ r_B G_B(X) &= \tilde{\mu}_B X G'_B(X) + \frac{1}{2}\tilde{\sigma}^2_B X^2 G''_B(X) + \tilde{\lambda}_B \left((1-\tau)s_G X y_G - K_B(1+\Upsilon_B) - G_B(X)\right). \end{cases} \tag{A.27}$$

For $X \geq X_B$:

$$\begin{cases} G_G(X) &= (1-\tau)s_G X y_G - K_G(1+\Upsilon_G) \\ G_B(X) &= (1-\tau)s_B X y_B - K_B/\Lambda_B. \end{cases} \tag{A.28}$$

Whenever the process X is in the option continuation region, which corresponds to system (A.26) and the second equation of (A.27), the required rate of return r_i (left-hand side) must be equal to the realized rate of return (right-hand side). The realized rate of return is calculated by applying Ito's lemma for regime switches. In this region, the last term captures the possible jump in the value of the growth option due to a regime switch. It can be expressed as the instantaneous probability of a regime shift, $\tilde{\lambda}_G$ or $\tilde{\lambda}_B$, times the associated change in the value of the option. The first equation of (A.27) and the system (A.28) state the payoff of the option at exercise. The process is in the option exercise region in these cases. The boundary conditions are given by

$$\lim_{X \searrow 0} G_i(X) = 0, \quad i = G, B, \tag{A.29}$$

$$\lim_{X \searrow X_G} G_B(X) = \lim_{X \nearrow X_G} G_B(X), \tag{A.30}$$

$$\lim_{X \searrow X_G} G_B'(X) = \lim_{X \nearrow X_G} G_B'(X), \tag{A.31}$$

$$\lim_{X \nearrow X_B} G_B(X) = (1-\tau) s_B X_B y_B - K_B/\Lambda_B, \tag{A.32}$$

and

$$\lim_{X \nearrow X_G} G_G(X) = (1-\tau) s_G X_G y_G - K_G(1+\Upsilon_G). \tag{A.33}$$

Condition (A.29) ensures that the option value goes to zero as earnings approach zero. Conditions (A.30) and (A.31) are the value-matching and smooth-pasting conditions of the value function in bad times at the exercise boundary in good times. The remaining conditions (A.32)–(A.33) are the value-matching conditions at the exercise boundaries in a good state and a bad state, respectively.

The functional form of the solution is given by

$$G_i(X) = \begin{cases} \bar{A}_{i3} X^{\gamma_3} + \bar{A}_{i4} X^{\gamma_4} & 0 \le X < X_G, \quad i = G, B \\ \bar{C}_1 X^{\beta_1^B} + \bar{C}_2 X^{\beta_2^B} + \bar{C}_3 X + \bar{C}_4 & X_G \le X < X_B, \quad i = B \\ (1-\tau) s_B X y_B - K_B/\Lambda_B & X \ge X_B \quad\quad\quad i = B \\ (1-\tau) s_G X y_G - K_G(1+\Upsilon_G) & X \ge X_G \quad\quad\quad i = G \end{cases} \tag{A.34}$$

in which $\bar{A}_{G3}, \bar{A}_{G4}, \bar{A}_{B1}, \bar{A}_{B2}, \bar{C}_1, \bar{C}_2, \bar{C}_3, \bar{C}_4, \gamma_3, \gamma_4, \beta_1^B$, and β_2^B are real-valued parameters to be determined.

First, consider the region $0 \le X < X_G$, and plug the functional form $G_i(X) = \bar{A}_{i3} X^{\gamma_3} + \bar{A}_{i4} X^{\gamma_4}$ into both equations of (A.26). Comparison of coefficients shows that $\bar{A}_{Gk}$ is a multiple of $\bar{A}_{Bk}$, $k = 3, 4$, with the factor $\bar{l}_k := \frac{1}{\tilde{\lambda}_G}(r_G + \tilde{\lambda}_G - \tilde{\mu}_G \gamma_k - \frac{1}{2}\tilde{\sigma}_G^2 \gamma_k(\gamma_k - 1))$, i.e., $\bar{A}_{Bk} = \bar{l}_k \bar{A}_{Gk}$. Using this result when comparing coefficients, we find that γ_3 and γ_4 correspond to the positive roots of the quadratic equation

$$(\tilde{\mu}_B \gamma + \frac{1}{2}\tilde{\sigma}_B^2 \gamma(\gamma - 1) - \tilde{\lambda}_B - r_B)(\tilde{\mu}_G \gamma + \frac{1}{2}\tilde{\sigma}_G^2 \gamma(\gamma - 1) - \tilde{\lambda}_G - r_G) = \tilde{\lambda}_B \tilde{\lambda}_G. \tag{A.35}$$

The reason for taking the positive roots is given by boundary condition (A.29).

Next, consider the region $X_G \le X < X_B$. Plugging the functional form $G_B(X) = \bar{C}_1 X^{\beta_1} + \bar{C}_2 X^{\beta_2} + \bar{C}_3 X +$

$\bar{C}_4$ into the second equation of (A.27), we find by comparison of coefficients that

$$
\begin{aligned}
\beta_{1,2}^B &= \frac{1}{2}-\frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2} \pm \sqrt{\left(\frac{1}{2}-\frac{\tilde{\mu}_B}{\tilde{\sigma}_B^2}\right)^2 + \frac{2(r_B+\tilde{\lambda}_B)}{\tilde{\sigma}_B^2}}, \\
\bar{C}_3 &= \tilde{\lambda}_B \frac{(1-\tau)s_G y_G}{r_B-\tilde{\mu}_B+\tilde{\lambda}_B}, \\
\bar{C}_4 &= -\tilde{\lambda}_B \frac{K_B/\Lambda_B}{r_B+\tilde{\lambda}_B}.
\end{aligned} \tag{A.36}
$$

The remaining unknown parameters are $\bar{A}_{G3}, \bar{A}_{G4}, \bar{C}_1$, and $\bar{C}_2$. Plugging the functional form (A.34) into conditions (A.30)–(A.33) yields

$$\bar{C}_1 X_G^{\beta_1^B} + \bar{C}_2 X_G^{\beta_2^B} + \bar{C}_3 X_G + \bar{C}_4 = \bar{l}_3 \bar{A}_{G3} X_G^{\gamma_3} + \bar{l}_4 \bar{A}_{G4} X_G^{\gamma_4}, \tag{A.37}$$

$$\bar{C}_1 \beta_1^B X_G^{\beta_1^B} + \bar{C}_2 \beta_2^B X_G^{\beta_2^B} + \bar{C}_3 X_G = \bar{l}_3 \bar{A}_{G3} \gamma_3 X_G^{\gamma_3} + \bar{l}_4 \gamma_4 \bar{A}_{G4} X_G^{\gamma_4}, \tag{A.38}$$

$$\bar{C}_1 X_B^{\beta_1^B} + \bar{C}_2 X_B^{\beta_2^B} + \bar{C}_3 X_B + \bar{C}_4 = (1-\tau)s_B y_B X_B - K_B/\Lambda_B, \tag{A.39}$$

and

$$\bar{A}_{G3} X_G^{\gamma_3} + \bar{A}_{G4} X_G^{\gamma_4} = (1-\tau)s_G y_G X_G - K_G(1+\Upsilon_G). \tag{A.40}$$

This four-dimensional system is linear in its four unknowns $\bar{A}_{G3}, \bar{A}_{G4}, \bar{C}_1$ and $\bar{C}_2$. We define the matrices

$$\bar{M} := \begin{bmatrix} \bar{l}_3 X_G^{\gamma_3} & \bar{l}_4 X_G^{\gamma_4} & -X_G^{\beta_1^B} & -X_G^{\beta_2^B} \\ \bar{l}_3 \gamma_3 X_G^{\gamma_3} & \bar{l}_4 \gamma_4 X_G^{\gamma_4} & -\beta_1^B X_G^{\beta_1^B} & -\beta_2^B X_G^{\beta_2^B} \\ 0 & 0 & X_B^{\beta_1^B} & X_B^{\beta_2^B} \\ X_G^{\gamma_3} & X_G^{\gamma_4} & 0 & 0 \end{bmatrix}, \tag{A.41}$$

and

$$\bar{b} := \begin{bmatrix} \bar{C}_3 X_G + \bar{C}_4 \\ \bar{C}_3 X_G \\ -\bar{C}_3 X_B - \bar{C}_4 + (1-\tau)s_B y_B X_B - K_B/\Lambda_B \\ (1-\tau)s_G y_G X_G - K_G(1+\Upsilon_G) \end{bmatrix}, \tag{A.42}$$

such that $\bar{M}\begin{bmatrix}\bar{A}_{G3} & \bar{A}_{G4} & \bar{C}_1 & \bar{C}_2\end{bmatrix}^T = \bar{b}$. The solution to the remaining four unknowns is given by

$$\begin{bmatrix}\bar{A}_{G3} & \bar{A}_{G4} & \bar{C}_1 & \bar{C}_2\end{bmatrix}^T = \bar{M}^{-1}\bar{b}. \tag{A.43}$$

□

The unleveraged value of the growth option. The unleveraged value of the growth option is calculated by additionally imposing the smooth-pasting boundary conditions at option exercise:

$$\lim_{X \nearrow X_B^{unlev}} G_B^{unlev'}(X) = (1-\tau)s_B y_B \tag{A.44}$$

and

$$\lim_{X \nearrow X_G^{unlev}} G_G^{unlev'}(X) = (1-\tau)s_G y_G. \tag{A.45}$$

The solution method is analog to the one for the leveraged option value up to and including Eq. (A.36). The system of equations (A.37)–(A.40) is augmented by the two equations corresponding to the additional smooth-pasting boundary conditions:

$$\bar{C}_1^{unlev}\beta_1^B\left(X_B^{unlev}\right)^{\beta_1^B-1} + \bar{C}_2^{unlev}\beta_2^B\left(X_B^{unlev}\right)^{\beta_2^B-1} + \bar{C}_3 = (1-\tau)s_B y_B \tag{A.46}$$

and

$$\bar{A}_{G3}^{unlev}\gamma_3\left(X_G^{unlev}\right)^{\gamma_3-1} + \bar{A}_{G4}^{unlev}\gamma_4\left(X_G^{unlev}\right)^{\gamma_4-1} = (1-\tau)s_G y_G. \tag{A.47}$$

The full system is six-dimensional with the six unknowns $\bar{A}_{G3}^{unlev}, \bar{A}_{G4}^{unlev}, \bar{C}_1^{unlev}$, $\bar{C}_2^{unlev}$ X_G^{unlev}, and X_B^{unlev}, linear in the first four unknowns and nonlinear in the last two unknowns. It is solved numerically.

The case in which $X_G \geq X_B$:

The solution of the case $X_G \geq X_B$ can be obtained immediately by renaming regimes in the solution of the presented case for $X_G < X_B$.

Appendix A.4. Firms with invested assets and an expansion option

We first present a proof for the valuation of corporate debt in the case in which $D_G < D_B$, $\hat{D}_G < \hat{D}_B$, and $X_B > X_G$.

Proof of Proposition 2. An investor requires an instantaneous return equal to the risk-free rate r_i for holding corporate debt. The application of Ito's lemma with regime switches shows that debt must, consequently, satisfy the following system of ODEs.

For $0 \leq X \leq D_G$:

$$\begin{cases} d_G(X) &= \alpha_G \Lambda_G \left((1-\tau) X y_G + G_G^{unlev}(X) \right) \\ d_B(X) &= \alpha_B \Lambda_B \left((1-\tau) X y_B + G_B^{unlev}(X) \right). \end{cases} \tag{A.48}$$

For $D_G < X \leq D_B$:

$$\begin{cases} r_G d_G(X) &= c + \tilde{\mu}_G X d_G'(X) + \frac{1}{2} \tilde{\sigma}_G^2 X^2 d_G''(X) \\ & \quad + \tilde{\lambda}_G \left(\alpha_B \Upsilon_B \left((1-\tau) X y_B + G_B^{unlev}(X) \right) - d_G(X) \right) \\ d_B(X) &= \alpha_B \Lambda_B \left((1-\tau) X y_B + G_B^{unlev}(X) \right). \end{cases} \tag{A.49}$$

For $D_B < X < X_G$:

$$\begin{cases} r_G d_G(X) &= c + \tilde{\mu}_G X d_G'(X) + \frac{1}{2} \tilde{\sigma}_G^2 X^2 d_G''(X) + \tilde{\lambda}_G \left(d_B(X) - d_G(X) \right) \\ r_B d_B(X) &= c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2} \tilde{\sigma}_B^2 X^2 d_B''(X) + \tilde{\lambda}_B \left(d_G(X) - d_B(X) \right). \end{cases} \tag{A.50}$$

For $X_G \leq X < X_B$:

$$\begin{cases} d_G(X) &= \hat{d}_G\left((s_G + 1) X \right) \\ r_B d_B(X) &= c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2} \tilde{\sigma}_B^2 X^2 d_B''(X) + \tilde{\lambda}_B \left(\hat{d}_G\left((s_G + 1) X \right) - d_B(X) \right). \end{cases} \tag{A.51}$$

For $X \geq X_B$:

$$\begin{cases} d_G(X) &= \hat{d}_G\left((s_G + 1) X \right) \\ d_B(X) &= \hat{d}_B\left((s_B + 1 - \frac{K_B/\Lambda_B}{(1-\tau) X_{\bar{i}} y_B}) X \right). \end{cases} \tag{A.52}$$

In system (A.48), the firm is in the default region in both good states and bad times. In this region, debtholders receive $\alpha_i \Lambda_i \left((1-\tau) X y_i + G_i^{unlev}(X) \right)$ at default. The firm is in the continuation region in good state, and in the default region in bad states in system (A.49). For the continuation region in good states , the left-hand side of the first equation is the rate of return required by investors for holding corporate debt for one unit of time. The right-hand side is the realized rate of return, computed by Ito's lemma as the expected change in the value of debt plus the coupon payment c. The last term expresses the possible jump in the value of debt in case of a regime switch, that triggers immediate default. Eqs. (A.50) describe the case in which the firm is in the continuation region in both good and bad states. The next system, (A.51), treats the case in which the firm is in the exercise region in good states and in the continuation region in bad states. After exercising the option, the firm owns total assets in place with value $(1-\tau) X y_i + (1-\tau) s_{\bar{i}} X y_i$, reflecting the notion that the exercise cost of the growth option can be financed by issuing equity in good states. The value of debt must then be equal to the value of debt of a firm with only invested assets, i.e., $d_G(X) = \hat{d}_G((s_G + 1) X)$, which is the first equation in (A.51). The second equation in this case is obtained by the same approach as in (A.50). The last term captures the notion that a regime switch from bad states to good states triggers immediate exercise of the expansion option with equity financing. Finally, (A.52) describes the case in which the firm is in the exercise region in both good and bad states. In good states, the earnings of the firm are scaled by

s_G+1. In bad states, the exercise cost K_B is financed by selling $\frac{K_B/\Lambda_B}{(1-\tau)X_{\bar{i}}y_B}$ of the assets in place, such that the earnings of the firm are scaled by $(s_B+1-\frac{K_B/\Lambda_B}{(1-\tau)X_{\bar{i}}y_B})$.

The system is subject to the following boundary conditions.

$$\lim_{X\searrow D_B} d_G(X) = \lim_{X\nearrow D_B} d_G(X), \tag{A.53}$$

$$\lim_{X\searrow D_B} d'_G(X) = \lim_{X\nearrow D_B} d'_G(X), \tag{A.54}$$

$$\lim_{X\searrow D_G} d_G(X) = \alpha_G\Lambda_G\left((1-\tau)D_G y_G + G_G^{unlev}(D_G)\right), \tag{A.55}$$

$$\lim_{X\searrow D_B} d_B(X) = \alpha_B\Lambda_B\left((1-\tau)D_B y_B + G_B^{unlev}(D_B)\right), \tag{A.56}$$

$$\lim_{X\searrow X_G} d_B(X) = \lim_{X\nearrow X_G} d_B(X), \tag{A.57}$$

$$\lim_{X\searrow X_G} d'_B(X) = \lim_{X\nearrow X_G} d'_B(X), \tag{A.58}$$

$$\lim_{X\nearrow X_G} d_G(X) = \hat{d}_G((s_G+1)X_G), \tag{A.59}$$

and

$$\lim_{X\nearrow X_B} d_B(X_B) = \hat{d}_B\left((s_B+1-\frac{K_B/\Lambda_B}{(1-\tau)X_{\bar{i}}y_B})X_B\right). \tag{A.60}$$

Eqs. (A.53) and (A.54) are the value-matching and smooth-pasting conditions for the debt value in the good state at the default boundary of the bad state. Eqs. (A.57) and (A.58) are the corresponding conditions for the debt value in the bad state at the option exercise boundary of the good state. Eqs. (A.55) and (A.56) show the value-matching conditions at the default thresholds, and Eqs. (A.59) and (A.60) are the value-matching conditions at the option exercise boundaries. The default thresholds and option exercise boundaries are chosen by equityholders. Hence, we do not have the corresponding smooth-pasting conditions for debt.

To solve this system, we start with the functional form of the solution in which $A_{G1}, A_{G2}, A_{B1}, A_{B2}, C_1, C_2, C_3, C_4, C_5, C_6, B_1, B_2, B_4, \beta_1^G, \beta_2^G, \beta_1^B, \beta_2^B, \gamma_1, \gamma_2, \gamma_3$, and γ_4 are real-valued parameters to be determined (or to be confirmed).

We first consider the region $D_B < X \leq X_G$. Plugging the functional form $d_i(X) = A_{i1}X^{\gamma_1} + A_{i2}X^{\gamma_2} + A_{i3}X^{\gamma_3} + A_{i4}X^{\gamma_4} + A_{i5}$ into both equations of (A.50) and comparing coefficients, we find that

$$A_{i5} = \frac{c(r_j+\tilde{\lambda}_i+\tilde{\lambda}_j)}{r_i r_j + r_j\tilde{\lambda}_i + r_i\tilde{\lambda}_j} = \frac{c}{r_i^p}. \tag{A.61}$$

As in Appendix A.2, A_{Gk} is always a multiple of A_{Bk}, $k=1,\ldots,4$, with the factor $l_k := \frac{1}{\tilde{\lambda}_G}(r_G+\tilde{\lambda}_G - \tilde{\mu}_G\gamma_k - \frac{1}{2}\tilde{\sigma}_G^2\gamma_k(\gamma_k-1))$, i.e., $A_{Bk} = l_k A_{Gk}$. Using this relation and comparing coefficients, it can be shown that $\gamma_1, \gamma_2, \gamma_3$, and γ_4 correspond to the roots of the quadratic equation

$$(\tilde{\mu}_B\gamma + \tfrac{1}{2}\tilde{\sigma}_B^2\gamma(\gamma-1) - \tilde{\lambda}_B - r_B)(\tilde{\mu}_G\gamma + \tfrac{1}{2}\tilde{\sigma}_G^2\gamma(\gamma-1) - \tilde{\lambda}_G - r_G) = \tilde{\lambda}_B\tilde{\lambda}_G. \tag{A.62}$$

According to Guo (2001), this quadratic equation always has two negative and two positive distinct real roots. The value of debt in both regimes is subject to boundary conditions from below (default) and above (exercise of expansion option). To meet all boundary conditions, we use four terms with the corresponding factors A_{ik} as well as the exponents γ_k, which requires the usage of all four roots of Eq. (A.62). The no-bubbles condition is not considered again because it is already implemented in the value function $\hat{d}_i$ of a firm with only invested assets. The unknown parameters for this region are $A_{Gk},\ k = 1, \ldots, 4$.

Next, we examine the region $D_G \leq X \leq D_B$. Plugging the functional form $d_G(X) = C_1X^{\beta_1^G} + C_2X^{\beta_2^G} + C_3X + C_4 + C_5X^{\gamma_3} + C_6X^{\gamma_4}$ into the second equation of (A.49), we find by comparison of coefficients that

$$\beta_{1,2}^G = \frac{1}{2} - \frac{\tilde{\mu}_G}{\tilde{\sigma}_G^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\tilde{\mu}_G}{\tilde{\sigma}_G^2}\right)^2 + \frac{2(r_G + \tilde{\lambda}_G)}{\tilde{\sigma}_G^2}}, \tag{A.63}$$

$$C_3 = \tilde{\lambda}_G \frac{\alpha_B\Lambda_B(1-\tau)y_B}{r_G + \tilde{\lambda}_G - \tilde{\mu}_G}, \tag{A.64}$$

$$C_4 = \frac{c}{r_G + \tilde{\lambda}_G}, \tag{A.65}$$

$$C_5 = \alpha_B\Lambda_B \frac{\bar{l}_3}{l_3}\bar{A}_{G3}^{unlev}, \tag{A.66}$$

and

$$C_6 = \alpha_B\Lambda_B \frac{\bar{l}_4}{l_4}\bar{A}_{G4}^{unlev}. \tag{A.67}$$

The unknown parameters remaining in this region are C_1 and C_2.

Finally, we consider the region $X_G < X \leq X_B$. Plugging the functional form $B_1X^{\beta_1^B} + B_2X^{\beta_2^B} + Z(X) + \tilde{\lambda}_B \frac{c}{r_i^P(r_B+\tilde{\lambda}_B)} + \frac{c}{r_B+\tilde{\lambda}_B}$ into the second equation of (A.51) and comparing coefficients, we find that

$$Z(X) = \tilde{\lambda}_B B_5 X^{\gamma_1} + \tilde{\lambda}_B B_6 X^{\gamma_2}. \tag{A.68}$$

$$\tag{A.69}$$

The parameters B_5 and B_6 are given by

$$B_5 = \frac{(s_B+1)^{\gamma_1}\hat{A}_{G1}}{r_B - \tilde{\mu}_B\gamma_1 - \frac{1}{2}\tilde{\sigma}_B^2\gamma_1(\gamma_1 - 1) + \tilde{\lambda}_B}, \tag{A.70}$$

and

$$B_6 = \frac{(s_B+1)^{\gamma_2}\hat{A}_{G2}}{r_B - \tilde{\mu}_B\gamma_2 - \frac{1}{2}\tilde{\sigma}_B^2\gamma_2(\gamma_2-1)+\tilde{\lambda}_B}. \tag{A.71}$$

The unknown parameters remaining in this region are B_1 and B_2.

To solve for the unknown parameters $A_{G1}, A_{G2}, A_{G3}, A_{G4}, C_1, C_2, B_1$, and B_2, we plug the functional form (16) into the system of boundary conditions (A.53)–(A.60):

$$\begin{aligned}
\sum_{k=1}^{4} A_{Gk}D_B^{\gamma_k} + A_{G5} &= C_1 D_B^{\beta_1^G} + C_2 D_B^{\beta_2^G} + C_3 X + C_4 + C_5 X^{\gamma_3} + C_6 X^{\gamma_4} \\
\sum_{k=1}^{4} A_{Gk}\gamma_k D_B^{\gamma_k} &= C_1\beta_1^G D_B^{\beta_1^G} + C_2\beta_2^G D_B^{\beta_2^G} + C_3 X + C_5\gamma_3 X^{\gamma_3} + C_6\gamma_4 X^{\gamma_4} \\
\alpha_G\Lambda_G\left((1+\tau)D_G y_G + G_G^{unlev}(D_G)\right) &= C_1 D_G^{\beta_1^G} + C_2 D_G^{\beta_2^G} + C_3 D_G + C_4 + C_5 D_G^{\gamma_3} + C_6 D_G^{\gamma_4} \\
\sum_{k=1}^{4} l_k A_{Gk} D_B^{\gamma_k} + A_{B5} &= \alpha_B\Lambda_B\left((1+\tau)D_B y_B + G_B^{unlev}(D_B)\right) \\
\sum_{k=1}^{4} l_k A_{Gk} X_G^{\gamma_k} + A_{B5} &= B_1 X_G^{\beta_1^B} + B_2 X_G^{\beta_2^B} + Z(X_G) + B_4 \\
\sum_{k=1}^{4} l_k A_{Gk}\gamma_k X_G^{\gamma_k} &= B_1\beta_1^B X_G^{\beta_1^B} + B_2\beta_2^B X_G^{\beta_2^B} + X_G Z'(X_G) \\
\sum_{k=1}^{4} A_{Gk} X_G^{\gamma_k} + A_{G5} &= \hat{d}_G\left((s_G+1)X_G\right) \\
B_1 X_B^{\beta_1^B} + B_2 X_B^{\beta_2^B} + Z(X_B) + B_4 &= \hat{d}_B\left((s_B + 1 - \frac{K_B/\Lambda_B}{(1-\tau)X_{\bar{t}}y_B})X_B\right).
\end{aligned} \tag{A.72}$$

Using matrix notation, we can write

$$M := \begin{bmatrix}
D_B^{\gamma_1} & D_B^{\gamma_2} & D_B^{\gamma_3} & D_B^{\gamma_4} & -D_B^{\beta_1^G} & -D_B^{\beta_2^G} & 0 & 0 \\
\gamma_1 D_B^{\gamma_1} & \gamma_2 D_B^{\gamma_2} & \gamma_3 D_B^{\gamma_3} & \gamma_4 D_B^{\gamma_4} & -\beta_1^G D_B^{\beta_1^G} & -\beta_2^G D_B^{\beta_2^G} & 0 & 0 \\
0 & 0 & 0 & 0 & D_G^{\beta_1^G} & D_G^{\beta_2^G} & 0 & 0 \\
l_1 D_B^{\gamma_1} & l_2 D_B^{\gamma_2} & l_3 D_B^{\gamma_3} & l_4 D_B^{\gamma_4} & 0 & 0 & 0 & 0 \\
l_1 X_G^{\gamma_1} & l_2 X_G^{\gamma_2} & l_3 X_G^{\gamma_3} & l_4 X_G^{\gamma_4} & 0 & 0 & -X_G^{\beta_1^B} & -X_G^{\beta_2^B} \\
l_1\gamma_1 X_G^{\gamma_1} & l_2\gamma_2 X_G^{\gamma_2} & l_3\gamma_3 X_G^{\gamma_3} & l_4\gamma_4 X_G^{\gamma_4} & 0 & 0 & -\beta_1^B X_G^{\beta_1^B} & -\beta_2^B X_G^{\beta_2^B} \\
X_G^{\gamma_1} & X_G^{\gamma_2} & X_G^{\gamma_3} & X_G^{\gamma_4} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & X_B^{\beta_1^B} & X_B^{\beta_2^B}
\end{bmatrix} \tag{A.73}$$

and

$$
b := \begin{bmatrix} -A_{G5} + C_3 D_B + C_4 + C_5 D_B^{\gamma_1} + C_6 D_B^{\gamma_2} \\ C_3 D_B + \gamma_1 C_5 D_B^{\gamma_1} + \gamma_2 C_6 D_B^{\gamma_2} \\ -C_3 D_G - C_4 - C_5 D_G^{\gamma_3} - C_6 D_G^{\gamma_4} + \alpha_G \Lambda_G \left((1-\tau) D_G y_G + G_G^{unlev}(D_G)\right) \\ -A_{B5} + \alpha_B \Lambda_B \left((1-\tau) D_B y_B + G_B^{unlev}(D_B)\right) \\ -A_{B5} + Z(X_G) + B_4 \\ X_G Z'(X_G) \\ -A_{G5} + \hat{d}_G \left((s_G + 1) X_G\right) \\ -Z(X_B) + B_4 + \hat{d}_B \left((s_B + 1 - \frac{K_B/\Lambda_B}{(1-\tau) X_{\bar{t}} y_B}) X_B\right) \end{bmatrix}. \tag{A.74}
$$

The solution to the remaining unknowns is now given by

$$
\begin{bmatrix} A_{G1} & A_{G2} & A_{G3} & A_{G4} & C_1 & C_2 & B_1 & B_2 \end{bmatrix}^T = M^{-1} b. \tag{A.75}
$$

□

The case in which $D_G < D_B$, $\hat{D}_G < \hat{D}_B$, and $X_G > X_B$:

Going through the same steps as in the previous case gives us

$$
M := \begin{bmatrix} D_B^{\gamma_1} & D_B^{\gamma_2} & D_B^{\gamma_3} & D_B^{\gamma_4} & -D_B^{\beta_1^G} & -D_B^{\beta_2^G} & 0 & 0 \\ \gamma_1 D_B^{\gamma_1} & \gamma_2 D_B^{\gamma_2} & \gamma_3 D_B^{\gamma_3} & \gamma_4 D_B^{\gamma_4} & -\beta_1^G D_B^{\beta_1^G} & -\beta_2^G D_B^{\beta_2^G} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_G^{\beta_1^G} & D_G^{\beta_2^G} & 0 & 0 \\ l_1 D_B^{\gamma_1} & l_2 D_G^{\gamma_2} & l_3 D_B^{\gamma_3} & l_4 D_B^{\gamma_4} & 0 & 0 & 0 & 0 \\ X_B^{\gamma_1} & X_B^{\gamma_2} & X_B^{\gamma_3} & X_B^{\gamma_4} & 0 & 0 & -X_B^{\beta_1^G} & -X_B^{\beta_2^G} \\ \gamma_1 X_B^{\gamma_1} & \gamma_2 X_B^{\gamma_2} & \gamma_3 X_B^{\gamma_3} & \gamma_4 X_B^{\gamma_4} & 0 & 0 & -\beta_1^G X_B^{\beta_1^G} & -\beta_2^G X_B^{\beta_2^G} \\ l1 X_B^{\gamma_1} & l2 X_B^{\gamma_2} & l3 X_B^{\gamma_3} & l4 X_B^{\gamma_4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & X_G^{\beta_1^G} & X_G^{\beta_2^G} \end{bmatrix} \tag{A.76}
$$

and

$$
b := \begin{bmatrix} -A_{G5} + C_3 D_B + C_4 + C_5 D_B^{\gamma_1} + C_6 D_B^{\gamma_2} \\ C_3 D_B + \gamma_1 C_5 D_B^{\gamma_1} + \gamma_2 C_6 D_B^{\gamma_2} \\ -C_3 D_G - C_4 - C_5 D_G^{\gamma_3} - C_6 D_G^{\gamma_4} + \alpha_G \Lambda_G \left((1-\tau) D_G y_G + G_G^{unlev}(D_G)\right) \\ -A_{B5} + \alpha_B \Lambda_B \left((1-\tau) D_B y_B + G_B^{unlev}(D_B)\right) \\ -A_{G5} + Z(X_B) + B_4 \\ X_B Z'(X_B) \\ -A_{B5} + \hat{d}_B \left((s_B + 1 - \frac{K_B/\Lambda_B}{(1-\tau) X_{\bar{t}} y_B}) X_B\right) \\ -Z(X_G) + B_4 + \hat{d}_G \left((s_G + 1) X_G\right) \end{bmatrix}. \tag{A.77}
$$

The solution to the unknowns is again given by

$$\begin{bmatrix} A_{G1} & A_{G2} & A_{G3} & A_{G4} & C_1 & C_2 & B_1 & B_2 \end{bmatrix}^T = M^{-1}b. \tag{A.78}$$

Appendix A.5. Bankruptcy costs

For the calculation of bankruptcy costs, the ODEs are given by the following system:

For $0 \leq X \leq D_G$:

$$\begin{cases} b_G(X) &=& (1-\alpha_G\Lambda_G)(1-\tau)Xy_G + G_G(X) - \alpha_G\Lambda_G G_G^{unlev}(X) \\ b_B(X) &=& (1-\alpha_B\Lambda_B)(1-\tau)Xy_B + G_B(X) - \alpha_B\Lambda_B B_B^{unlev}(X). \end{cases} \tag{A.79}$$

For $D_G < X \leq D_B$:

$$\begin{cases} r_G b_G(X) &=& \tilde{\mu}_G X b_G'(X) + \frac{1}{2}\tilde{\sigma}_G^2 X^2 b_G''(X) \\ && +\tilde{\lambda}_G\left((1-\alpha_B\Lambda_B)(1-\tau)Xy_B + G_B(X) - \alpha_B\Lambda_B G_B^{unlev}(X) - b_G(X)\right) \\ b_B(X) &=& (1-\alpha_B\Lambda_B)(1-\tau)Xy_B + G_B(X) - \alpha_B\Lambda_B G_B^{unlev}(X). \end{cases} \tag{A.80}$$

For $D_B < X < X_G$:

$$\begin{cases} r_G d_G(X) &=& c + \tilde{\mu}_G X b_G'(X) + \frac{1}{2}\tilde{\sigma}_G^2 X^2 b_G''(X) + \tilde{\lambda}_G(b_B(X) - b_G(X)) \\ r_B d_B(X) &=& c + \tilde{\mu}_B X d_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 b_B''(X) + \tilde{\lambda}_B(b_G(X) - b_B(X)). \end{cases} \tag{A.81}$$

For $X_G \leq X < X_B$:

$$\begin{cases} b_G(X) &=& \hat{d}_G((s_G+1)X) \\ r d_B(X) &=& c + \tilde{\mu}_B X b_B'(X) + \frac{1}{2}\tilde{\sigma}_B^2 X^2 b_B''(X) + \tilde{\lambda}_B\left(\hat{d}_G((s_G+1)X) - b_B(X)\right). \end{cases} \tag{A.82}$$

For $X \geq X_B$:

$$\begin{cases} b_G(X) &=& \hat{b}_G((s_G+1)X) \\ b_B(X) &=& \hat{b}_B\left((s_B + 1 - \frac{K_B/\Lambda_B}{(1-\tau)X_{\bar{t}}y_B})X\right). \end{cases} \tag{A.83}$$

The boundary conditions are as follows:

$$\lim_{X \searrow D_B} b_G(X) = \lim_{X \nearrow D_B} b_G(X), \tag{A.84}$$

$$\lim_{X \searrow D_B} b'_G(X) = \lim_{X \nearrow D_B} b'_G(X), \tag{A.85}$$

$$\lim_{X \searrow D_G} b_G(X) = (1-\alpha_G \Lambda_B)(1-\tau) D_G y_G + G_G(D_G) - \alpha_G \Lambda_G G_G^{unlev}(D_G), \tag{A.86}$$

$$\lim_{X \searrow D_B} b_B(X) = (1-\alpha_G \Lambda_G)(1-\tau) D_B y_B + G_B(D_B) - \alpha_B G_B^{unlev}(D_B), \tag{A.87}$$

$$\lim_{X \searrow X_G} b_B(X) = \lim_{X \nearrow X_G} b_B(X), \tag{A.88}$$

$$\lim_{X \searrow X_G} b'_B(X) = \lim_{X \nearrow X_G} b'_B(X), \tag{A.89}$$

$$\lim_{X \nearrow X_G} b_G(X) = \hat{b}_G((s_G+1)X_G), \tag{A.90}$$

and

$$\lim_{X \nearrow X_B} b_B(X_B) = \hat{b}_B\left((s_B + 1 - \frac{K_B/\Lambda_B}{(1-\tau)X_{\bar{t}} y_B})X_B\right). \tag{A.91}$$

Eqs. (A.84) and (A.85) are the value-matching and smooth-pasting conditions for bankruptcy costs in good states at the default boundary in bad states. Similarly, Eqs. (A.88) and (A.89) are the corresponding conditions for bankruptcy costs in bad states at the option exercise boundary in good states. Eqs. (A.86) and (A.87) are the value-matching conditions at the default thresholds. They incorporate the fact that upon default, the value of the leveraged growth option switches to the value of the unleveraged growth option. Eqs. (A.90) and (A.91) are the value-matching conditions at the option exercise boundaries.

To solve for the unknown parameters, we plug the functional form

$$b_i(X) = \begin{cases} (1-\alpha_i\Lambda_i)(1-\tau)Xy_i - \alpha_i\Lambda_i G_i^{unlev}(X) + G_i(X) & X \le D_i, & i = G,B \\ C_1 X^{\beta_1^B} + C_2 X^{\beta_2^B} + C_5 X^{\gamma_3} + C_6 X^{\gamma_4} & & \\ \quad + \tilde{\lambda}_G \frac{\alpha_B \Lambda_B y_B (1-\tau)}{r_G - \tilde{\mu}_G + \tilde{\lambda}_G} X + \frac{c}{r_G + \tilde{\lambda}_G} & D_G < X \le D_B, & i = G \\ A_{i1} X^{\gamma_1} + A_{i2} X^{\gamma_2} + A_{i3} X^{\gamma_3} + A_{i4} X^{\gamma_4} + \frac{c}{r_i^P} & D_B < X \le X_G, & i = G,B \\ B_1 X^{\beta_1^B} + B_2 X^{\beta_2^B} + Z(X) + \tilde{\lambda}_B \frac{c}{r_i^P (r_B + \tilde{\lambda}_B)} + \frac{c}{r_B + \tilde{\lambda}_B} & X_G < X \le X_B, & i = B \\ \hat{b}_G((s_G+1)X) & X > X_G, & i = G \\ \hat{b}_B\left((s_B + 1 - \frac{K_B/\Lambda_B}{(1-\tau)X_{\bar{t}} y_B})X\right) & X > X_B, & i = B \end{cases} \tag{A.92}$$

into the system of boundary conditions (A.84)-(A.91). The solution to the unknowns is given by

$$\begin{bmatrix} A_{G1} & A_{G2} & A_{G3} & A_{G4} & C_1 & C_2 & B_1 & B_2 \end{bmatrix}^T = M^{-1} b, \tag{A.93}$$

where

$$M := \begin{bmatrix} D_B^{\gamma_1} & D_B^{\gamma_2} & D_B^{\gamma_3} & D_B^{\gamma_4} & -D_B^{\beta_1^G} & -D_B^{\beta_2^G} & 0 & 0 \\ \gamma_1 D_B^{\gamma_1} & \gamma_2 D_B^{\gamma_2} & \gamma_3 D_B^{\gamma_3} & \gamma_4 D_B^{\gamma_4} & -\beta_1^G D_B^{\beta_1^G} & -\beta_2^G D_B^{\beta_2^G} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_G^{\beta_1^G} & D_G^{\beta_2^G} & 0 & 0 \\ l_1 D_B^{\gamma_1} & l_2 D_B^{\gamma_2} & l_3 D_B^{\gamma_3} & l_4 D_B^{\gamma_4} & 0 & 0 & 0 & 0 \\ l_1 X_G^{\gamma_1} & l_2 X_G^{\gamma_2} & l_3 X_G^{\gamma_3} & l_4 X_G^{\gamma_4} & 0 & 0 & -X_G^{\beta_1^B} & -X_G^{\beta_2^B} \\ l_1\gamma_1 X_G^{\gamma_1} & l_2\gamma_2 X_G^{\gamma_2} & l_3\gamma_3 X_G^{\gamma_3} & l_4\gamma_4 X_G^{\gamma_4} & 0 & 0 & -\beta_1^B X_G^{\beta_1^B} & -\beta_2^B X_G^{\beta_2^B} \\ X_G^{\gamma_1} & X_G^{\gamma_2} & X_G^{\gamma_3} & X_G^{\gamma_4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & X_B^{\beta_1^B} & X_B^{\beta_2^B} \end{bmatrix}, \tag{A.94}$$

$$b := \begin{bmatrix} -A_{G5} + C_3 D_B + C_4 + C_5 D_B^{\gamma_1} + C_6 D_B^{\gamma_2} \\ C_3 D_B + \gamma_1 C_5 D_B^{\gamma_1} + \gamma_2 C_6 D_B^{\gamma_2} \\ -C_3 D_G - C_4 - C_5 D_G^{\gamma_3} - C_6 D_G^{\gamma_4} + (1-\alpha_G\Lambda_G)\left((1-\tau)D_G y_G - \alpha_G\Lambda_G G_G^{unlev}(D_G)\right) + G_G(D_G) \\ -A_{B5} + (1-\alpha_B\Lambda_B)\left((1-\tau)D_B y_B - \alpha_B\Lambda_B G_B^{unlev}(D_B)\right) + G_B(D_B) \\ -A_{B5} + Z\left(X_G\right) + B_4 \\ X_G Z'\left(X_G\right) \\ -A_{G5} + \hat{d}_G\left((s_G+1)X_G\right) \\ -Z\left(X_B\right) + B_4 + \hat{d}_B\left((s_B + 1 - \frac{K_B/\Lambda_B}{(1-\tau)X_{\bar{t}} y_B})X_B\right) \end{bmatrix}, \tag{A.95}$$

$$C_5 = \frac{\bar{l}_3}{l_3}\left(\bar{A}_{G3}^{lev} - \alpha_B\Lambda_B\bar{A}_{G3}^{unlev}\right), \tag{A.96}$$

and

$$C_6 = \frac{\bar{l}_4}{l_4}\left(\bar{A}_{G4}^{lev} - \alpha_B\Lambda_B\bar{A}_{G4}^{unlev}\right). \tag{A.97}$$

The case in which $D_G < D_B$, $\hat{D}_G < \hat{D}_B$, and $X_G > X_B$:

This case can be solved similarly.

Appendix B. Data and Variables

Our sample includes all U.S. manufacturing firms (SIC codes between 2000 and 3999) as provided in the Compustat annual research file for the 1971–2010 period. All variables are deflated to 1982 dollars using the CPI. Only firms with at least 24 consecutive months of data remain in the sample. Furthermore, we winsorize the sample with regard to the book-to-market ratio, market equity, age, investment, asset sale, and stock returns at the 99% and 1% level. In addition, we exclude firms that have a q below zero or above ten to address issues of investment opportunity measurement in the data. We also require firms to hold at least 5 million dollars in fixed assets to eliminate very small firms. The final sample contains of 3,022 firms.

We consider the following firm individual variables: F_t are the net fixed assets (PPENT) at the beginning of the period t, and $Total\ Assets$ are the book values of the assets (AT). $Asset\ Sale$ is equal to the cash proceeds received from the sale of fixed assets (SPPE), and *Investment* is obtained from the Compustat item capital expenditures (CAPX). Both variables are scaled by F_t. We compute the firm individual sales growth as first difference of the Compustat item SALE. We standardize the firm individual sales growth by subtracting the mean and scaling it with its standard deviation. To compute the sample aggregate sales growth we compute then for each year the value-weighted mean sales growth across all sample firms. *Age* is the number of years a firm has been listed at the NYSE/AMEX/NASDAQ, i.e., the current year minus the first year of a firm's stock price entry in the merged CRSP/Compustat file. Using $Total\ Assets$ and *Age*, we construct the SA-index as measure of financial constraints following Hadlock and Pierce (2010) as:

$$-0.737 * Total\ Assets + 0.043 * (Total\ Assets)^2 - 0.04 * Age. \tag{B.1}$$

Since the SA-index is a combination of total assets, squared total assets and age, its values are substantially higher than our dependent variable in the regression analysis, i.e., asset sales, which is a variable that is scaled with F_t. Therefore, we scale the SA-index by 10×10^7. q is a proxy for growth opportunities and calculated as the sum of total debt and market equity divided by the book value of total assets (cf., Hovakimian and Titman 2006). $Financial\ Slack$ corresponds to the sum of cash and short-term investments (CHE) scaled by F_t. We define $Total\ Debt$ as the sum of total liabilities (LT) and total preferred stock (PSTK) excluding deferred taxes (TXDB) and convertible debt (DCVT) scaled by $Total\ Assets$. As a proxy for $Cash\ Flow$ we use the sum of income before extraordinary items, depreciation and amortization (IB + DP) scaled by F_t. *Cov. Ratio* is EBITDA divided by interest expenses (XINT). We adopt an iterative procedure to calculate $Asset\ Volatility$, following the steps in Vassalou and Xing (2004). In particular, we estimate the volatility of equity with daily equity values over the past 12 month for each firm-year observation. This volatility serves as a starting guess for the estimation of the asset volatility. Applying the Black-Scholes formula, we then compute daily asset values over the past 12 month using the daily equity values, total liabilities, the starting guess for the asset volatility, and the risk free interest rate from CRSP. Next, the standard deviation of those asset values can be calculated. This standard deviation is used as the volatility of assets for the next iteration. We repeat this procedure until the asset volatilities from two consecutive iterations converge to a tolerance level of $10E-4$. The Altman (1968) Z-score is a widely used measure of financial distress. It is computed for each firm as:

$$Z = 1.2 * \frac{ACT - LCT}{AT} + 1.4 * \frac{RE}{AT} + 3.3 * \frac{NI + XINT + TXT}{AT} + 0.6 * \frac{ME}{LT} + 0.999 * \frac{SALE}{AT}. \tag{B.2}$$

A higher value >2.99 indicates that the firm is not financially distressed. We compute the equity issuance costs for our sample firms according to the cost function estimated in Hennessy and Whited (2007). In their paper, Hennessy and Whited (2007) provide estimates for the equity issuance cost function for small, large and all firms in their sample. At the end of each year, we sort firms according to their size (ME) into tercile portfolios. (Using the SA-index instead of size as sorting variable does not change the quality of our results.) We then compute the equity issuance cost for the firms in each portfolio for the subsequent year according to the amount of equity that a firm issues in the corresponding year (SSTK). For the firms in the lowest size portfolio, we use the estimation results of Hennessy and Whited (2007) for small firms, for the highest size tercile the estimations for large firms, and for the medium size tercile the estimation results that Hennessy and Whited (2007) obtain for the full sample. We winsorize the estimated equity issuance costs at the 90% level to control for outliers.

In Table B1, we report some basic sample characteristics. The table reports the mean, the standard deviation (Std), the median, the 25 percent (Q25) and the 75 percent quantiles (Q75). Panel A provides summary statistics for different sample variables of the full sample. In Panel B and Panel C, the table reports the same summary statistics but for bad and good states, respectively. We define an aggregate downturn of our firm economy as years in which the sample aggregate sales growth and the annual return across sample firms are in the bottom 25% across all years. We choose this definition of a business cycle downturn mainly because sales growth combined with market based downturn measures are a direct measure of the propagation of positive and negative shocks from the aggregate economy onto the corporate level (see also the downturn definitions in e.g. Opler and Titman 1994, Gilson, John, and Lang 1990). All other years are defined as good state.

Table B1
Compustat Sample Summary Statistics

The table provides summary statistics for different sample variables in Panel A. In Panel B and Panel C, the table reports summary statistics for bad (Panel B) and good (Panel C) states. We define an aggregate downturn of our firm economy as years in which the sample aggregate sales growth and the average annual equity return across sample firms are, simultaneously, in the bottom 25% of all years. All other years are considered as a good state. The table reports the mean, the standard deviation (Std), the median, the 25 percent (Q25), and the 75 percent quantile (Q75). $Total\ Assets$ (AT) and $Fixed\ Assets$ (F) are in million dollars, measured at the beginning of each year. q is the sum of the book value of total debt and the market value of equity divided by the book value of total assets. *Investment* is equal to capital expenditures. *Asset Sale* are the cash proceeds from sale of fixed capital. $Cash\ Flow$ is the sum of income before extraordinary items and depreciation and amortization. $Fin.\ Slack$ is the sum of cash and short-term investments. *Investment*, *Asset Sale*, $Cash\ Flow$, and $Fin.\ Slack$ are scaled by the book value of the beginning-of-period net fixed assets. $Asset\ Volatility$ is the estimated volatility of a firms' assets. *Total debt* is (LT+PSTK-TXDB-DCVT). $Market\ Equity$ is computed as the CRSP monthly share price (PRC) multiplied with the number of outstanding shares (SHROUT). The variable *Cov. Ratio* is computed by dividing $EBITDA$ with the interest expenses. The sample period is 1971 to 2010. The sample consists of 3,022 U.S. manufacturing firms.

Panel A: Summary Statistics – Full Sample Period		
Variable	Mean	Std
$Total\ Assets\ (TA)$	1140.98	3857.31
$Fixed\ Assets\ (F)$	347.59	1135.2331
q	1.3397	1.4996
$Investment/F$	0.2104	0.1145
$Asset\ Sales/F$	0.0169	0.0347
$Cash\ Flow/F$	0.3413	0.7816
$Fin.\ Slack/F$	0.7583	1.6365
$Asset\ Volatility$	0.3951	0.5606
$Total\ Debt/TA$	0.4384	0.1798
$Market\ Equity$	1162.1369	3292.8338
Cov. Ratio	54.6242	735.2741
Panel B: Summary Statistics – Bad Business Cycle States		
Variable	Mean	Std
$Total\ Assets\ (TA)$	968.2125	2496.1238
$Fixed\ Assets\ (F)$	310.3733	730.6911
q	0.881	1.4479
$Investment/F$	0.2226	0.1175
$Asset\ Sales/F$	0.0171	0.04
$Cash\ Flow/F$	0.366	0.6669
$Fin.\ Slack/F$	0.4752	1.2302
$Asset\ Volatility$	0.5313	0.8914
$Total\ Debt/TA$	0.4654	0.1669
$Market\ Equity$	602.085	2514.0054
Cov. Ratio	27.9017	172.8118
Panel C: Summary Statistics – Good Business Cycle States		
Variable	Mean	Std
$Total\ Assets\ (TA)$	1156.1355	3954.1097
$Fixed\ Assets\ (F)$	350.8547	1163.9899
q	1.38	1.4974
$Investment/F$	0.2194	0.1142
$Asset\ Sales/F$	0.0168	0.0392
$Cash\ Flow/F$	0.3391	0.7909
$Fin.\ Slack/F$	0.7832	1.6652
$Asset\ Volatility$	0.3831	0.5120
$Total\ Debt/TA$	0.436	0.1807
$Market\ Equity$	1194.6936	3325.6946
Cov. Ratio	57.0005	765.4944

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Figure 1. Optimal Investment Thresholds. This figure shows the earnings levels at which equityholders optimally exercise the growth option for a range of initial leverage ratios. The lower and upper solid lines are the optimal investment thresholds for a firm that finances the exercise cost of the option by issuing equity in good states and bad states, respectively. The lower and upper dashed lines are the corresponding investment thresholds for a firm that finances the exercise cost of the option by selling assets.

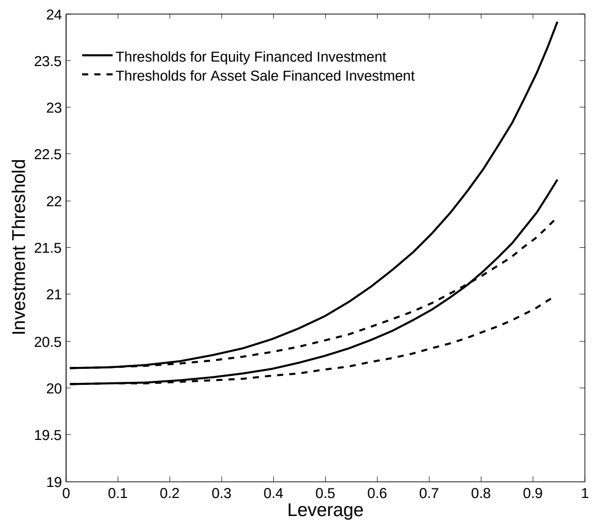

Figure 2. Financing Asset Sales and Firm Value. This figure illustrates the impact of financing asset sales on the value of firms for three initial leverage ratios. The solid line shows the relationship between the increase in the value of a firm from admitting financing asset sales and the scale parameter s_G for high leverage firms. High leverage firms have an initial leverage ratio of 0.75. Leverage is defined as debt value divided by the value of the firm. The dashed and dotted lines plot the relationship for medium leverage firms with an initial leverage of 0.5 and for low leverage firms with an initial leverage of 0.35, respectively.

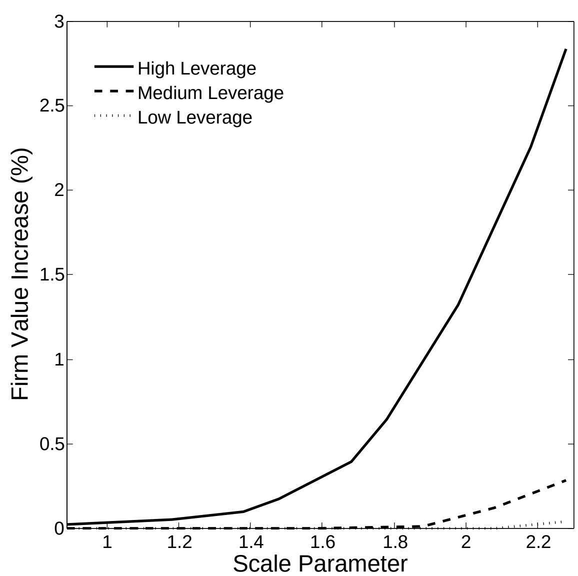

Figure 3. Leverage at Investment. This figure shows the leverage ratios upon investment of a firm that selects initially an optimal leverage ratio and optimally finances the exercise cost of the option in good states (solid line) and bad states (dashed line) as a function of equity issuance costs.

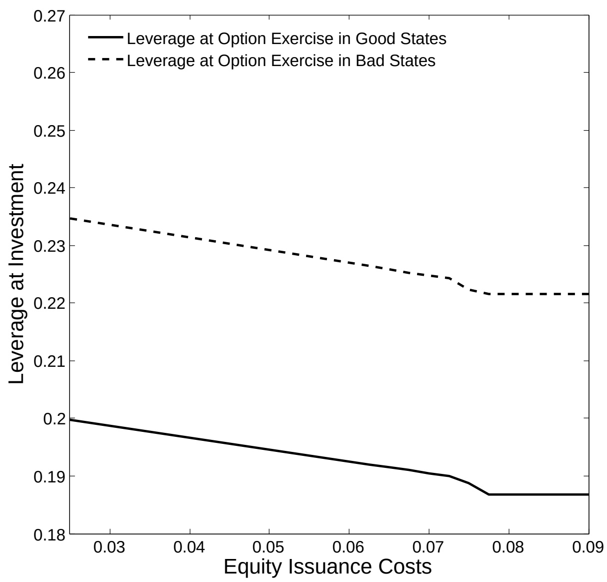

Figure 4. Leverage at Investment and Cyclicality of the Growth Option. This figure shows the leverage ratios upon investment of a firm that selects initially an optimal leverage ratio and optimally finances the exercise cost of the option in good states (solid line) and bad states (dashed line) as a function of equity issuance costs. The dashed and the dashed-dotted lines are the corresponding leverage ratios upon investment of a firm with a more cyclical growth option than the baseline firm.

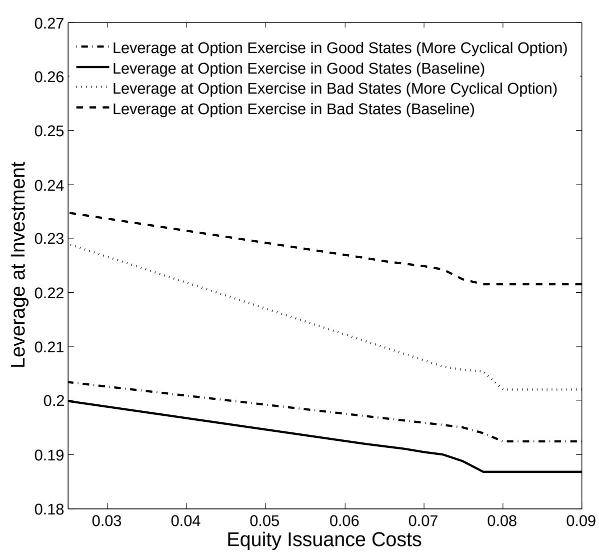

Figure 5. Optimal Financing Choice. This figure depicts equityholders' optimal financing choice. In the region to the right of the dashed line, they select asset sales in good states and bad states to finance the exercise cost of the option. In the region to the left of the solid line, they issue equity in good states and bad states. Between the dashed and the solid lines, equityholders issue equity in good states, and sell assets in bad states to finance the exercise cost.

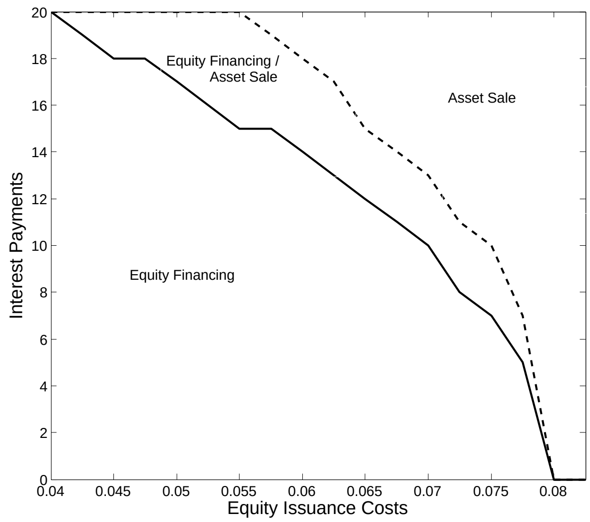

Figure 6. Aggregate Equity Financing. This figure plots the aggregate quarterly ratio of firms in a typical simulated economy that issue equity over time. The shaded regions are bad states, and the white regions are good states.

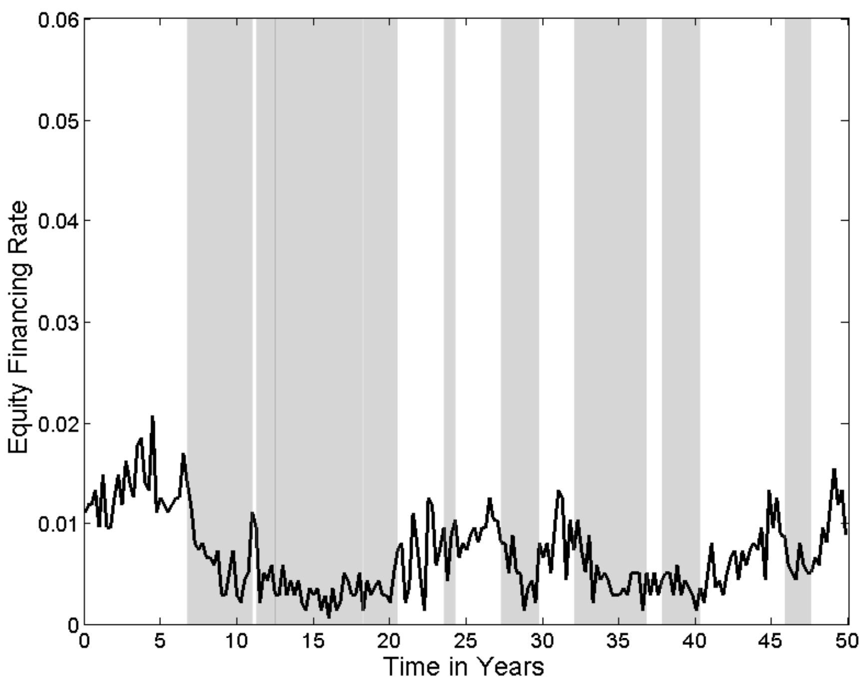

Figure 7. Aggregate Investment. This figure plots the aggregate quarterly ratio of firms in a typical simulated economy that invest over time. The shaded regions are bad states, and the white regions are good states.

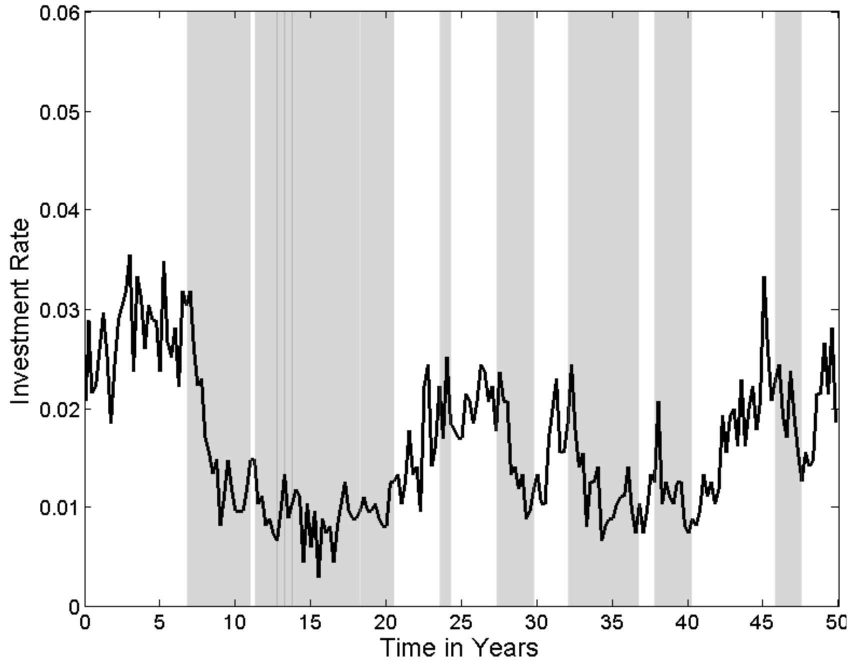

Figure 8. Aggregate Default. This figure plots the aggregate quarterly ratio of firms in a typical simulated economy that default over time. The shaded regions are bad states, and the white regions are good states.

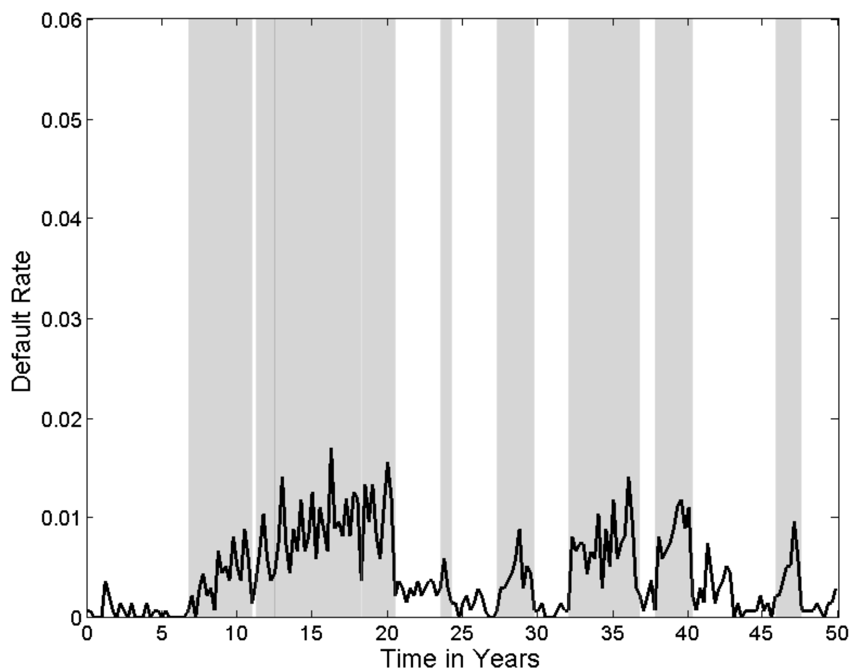

Figure 9. Aggregate Financing Asset Sales. This figure plots the aggregate quarterly ratio of firms in a typical simulated economy that sell assets over time. The shaded regions are bad states, and the white regions are good states.

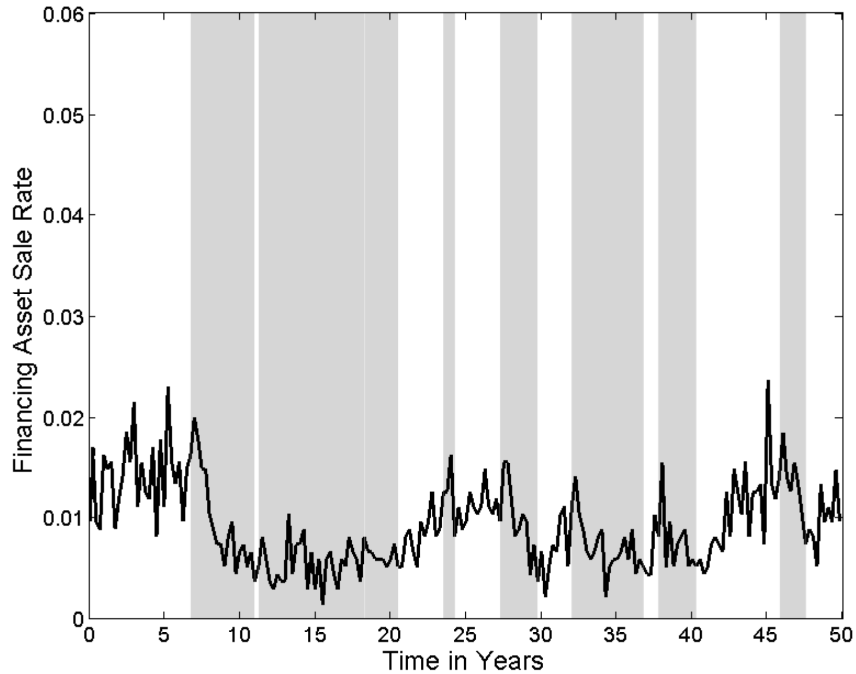

Figure 10. Aggregate Investment and Financing Asset Sales. This figure plots the aggregate quarterly ratio of firms in a typical simulated economy that invest (solid line), and the aggregate ratio of firms that sell assets (dashed line) over time. The shaded regions are bad states, and the white regions are good states.

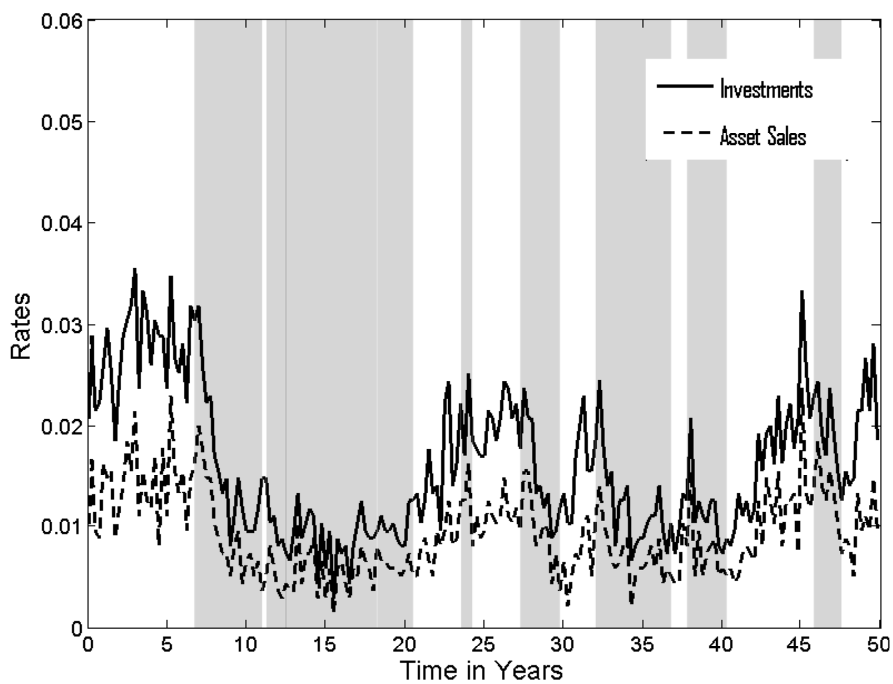

Table 1
Compustat Sample Asset Sale Determinants

The table reports regression coefficients for linear regressions with industry fixed effects and industry clustered autocorrelation robust t-statistics (in parentheses) with *Asset Sale* as dependent variable. *Asset Sale* are the cash proceeds from the sale of fixed capital. *Investment* is equal to capital expenditures. *Cash flow* is the first lag of the sum of income before extraordinary items and depreciation and amortization. q is the first lag of the sum of the book value of total debt and the market value of equity divided by the book value of total assets. *Financial Slack* is the first lag of the sum of cash and short-term investments. *Investment*, *Cash Flow*, *Asset Sale*, and *Financial Slack* are scaled by the book value of the beginning-of-period net fixed assets. The variable *Cov. Ratio* is the first lag of the ratio of *EBITDA* divided by the interest expenses. *Asset Volatility* is the estimated volatility of a firms' assets. *Leverage* is the first lag of (LT+PSTK-TXDB-DCVT) scaled by *Total Assets*. *Bad State* is a dummy that is one if the aggregate sales growth and the average annual equity return across all firms in the sample are, simultaneously, in the bottom 25% of all years. $Corr(q, Salesgr.)$ is the firm individual 5-year rolling correlation of the firm's q with the aggregate annual sales growth across all firms. SA-Index is the financial constraints measure of Hadlock and Pierce (2010). $I_{Low\ Z}$ is a dummy that is one if a firm has a Z-Score (see Equation (B.2)) value below 3. The sample period is 1971 to 2010. N is the number of observations in the corresponding regression. The full sample consists of an unbalanced sample of 3,022 U.S. manufacturing firms.

Dependent variable: Asset sale	(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)
Investment	0.024	0.005	0.004	0.003	0.023	0.005	0.023	0.007
	(5.20)	(0.57)	(0.44)	(0.34)	(5.17)	(0.60)	(4.93)	(1.01)
Cash Flow	-0.002	-0.002	-0.002	-0.004	-0.002	-0.002	-0.002	-0.002
	(-7.16)	(-7.48)	(-7.45)	(-7.01)	(-7.18)	(-7.47)	(-6.76)	(-6.85)
q	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003	-0.003
	(-14.59)	(-13.26)	(-13.46)	(-11.86)	(-14.59)	(-13.2)	(-13.95)	(-12.49)
Financial Slack	-0.000	-0.000	-0.000	0.000	-0.000	-0.000	-0.000	-0.000
	(-2.71)	(-1.12)	(-1.10)	(0.97)	(-2.81)	(-1.21)	(-3.19)	(-1.96)
Cov. Ratio	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
	(-1.40)	(-1.29)	(-1.28)	(-0.51)	(-1.41)	(-1.30)	(-4.52)	(-5.16)
Asset Volatility	-0.001	-0.001	-0.001	-0.004	-0.001	-0.001	-0.001	-0.001
	(-3.02)	(-3.07)	(-2.82)	(-3.86)	(-3.00)	(-3.06)	(-3.07)	(-3.10)
Leverage	0.012	0.004	0.004	0.002	0.012	0.004	0.012	0.003
	(2.89)	(0.73)	(0.77)	(0.40)	(2.84)	(0.75)	(2.98)	(0.56)
Lever. x Invest.		0.044	0.044	0.052		0.043		0.040
		(2.42)	(2.38)	(2.58)		(2.40)		(2.59)
Bad State x Invest.			0.019	0.016				
			(3.23)	(1.98)				
Bad State			-0.006	-0.005				
			(-5.15)	(-3.65)				
Corr(q,Salesgr.)				0.001				
				(1.41)				
Invest. x Corr(q,Salesgr.)				-0.001				
				(-0.25)				
Bad State x Corr(q,Salesgr.)				0.005				
				(2.25)				
Invest. x Bad State x Corr(q,Salesgr.)				-0.024				
				(-2.34)				
SA-Index					0.007	0.003		
					(6.33)	(0.43)		
Invest. x SA-Index						0.013		
						(0.48)		
$I_{Low\ Z}$							0.000	0.000
							(2.39)	(2.04)
Invest. x $I_{Low\ Z}$							0.007	-0.002
							(1.21)	(-0.22)
Lever. x $I_{Low\ Z}$								0.003
								(0.94)
Invest. x Lever. x $I_{Low\ Z}$								0.007
								(0.26)
Industry-Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adj. R^2	0.033	0.034	0.034	0.034	0.033	0.033	0.033	0.034
No. of Obs.	17468	17468	17468	14514	17468	17468	17257	17257

Table 2
Baseline parameter choice

This table summarizes our baseline parameter choice. Panel A lists the annualized parameters of a typical Compustat firm. Panels B and C report our parameter choice for the expansion option and the macroeconomy, respectively.

Parameter	Parameter value	
Panel A: Firm Characteristics	*Good State (G)*	*Bad State (B)*
Initial earnings level (X)	10	10
Tax advantage of debt (τ)	0.15	0.15
Earnings growth rate (μ_i)	0.0782	-0.0401
Systematic earnings volatility ($\sigma_i^{X,C}$)	0.0834	0.1334
Idiosyncratic earnings volatility ($\sigma^{X,id}$)	0.168	0.168
Equity issuance cost (Υ_i)	0.08	0.1
Asset Liquidity (Λ_1)	0.9259	0.9091
Recovery rate (α_i)	0.63	0.57
Panel B: Expansion Option Parameters of a Typical Firm		
Exercise price (K_i)	183.13	160
Scale parameter (s_i)	1.0925	1.03
Panel C: Economy		
Rate of leaving regime i (λ_i)	0.2718	0.4928
Consumption growth rate (θ_i)	0.0420	0.0141
Consumption growth volatility (σ_i^C)	0.0094	0.0114
Rate of time preference (ρ)	0.015	0.015
Relative risk aversion (γ)	10	10
Elasticity of intertemporal substitution (Ψ)	1.5	1.5

Table 3
Simulated Sample Results

The table provides summary statistics for the simulated matched samples over the full sample period, bad states, and good states. The sample period is 50 years with simulated quarterly observations. Each simulated sample consists of 1352 firms that are matched to our Compustat sample. Firms are replaced in case of investment or default. We report the mean of the mean values of 100 simulated samples, and the standard deviation (Std) of the mean across simulations. $Total\ Assets$ (TA) is the total value of firm assets. $Investment$, $Asset\ Sale$, and $Equity\ Finance$ are the annualized percentage number of firms that invest, sell assets, or issue equity, respectively. The q of model firms is obtained by dividing the value of the firm by the value of its invested assets. The variable *Cov. Ratio* corresponds to firm earnings divided by coupon payments. *Leverage* is the market value of debt divided by the market value of the firm. $Equity\ Value/TA$ is the market value of equity scaled by total the total firm value.

	All States		Bad State		Good State	
Variable	Mean	Std	Mean	Std	Mean	Std
$Total\ Assets\ (TA)$	194.52	12.98	161.37	9.37	215.33	9.46
$Investment$	0.081	0.009	0.059	0.007	0.095	0.01
$Asset\ Sales$	0.034	0.012	0.031	0.01	0.036	0.014
$Equity\ Finance$	0.047	0.013	0.028	0.01	0.059	0.015
q	1.45	0.024	1.38	0.018	1.50	0.018
Cov. Ratio	1.83	0.164	1.75	0.146	1.88	0.171
Leverage	0.43	0.027	0.48	0.025	0.39	0.022
$Equity\ Value/TA$	0.576	0.027	0.518	0.025	0.612	0.023

Table 4
Conditional Asset Sale Ratios

The table provides summary statistics for conditional asset sale ratios from the simulated samples. Asset sale and investment are both dummy variables that are equal to one in case of an asset sale or an investment, respectively. To calculate conditional asset sale ratios, we aggregate over all simulations the asset sale and investment observations for the sample that we consider, and divide the sum of asset sale observations by the sum of investment observations. We compute this ratio for all firms, for firms in the highest and the lowest leverage terciles with resorting in every period, during bad and good states, and for firms with a more (H) or less (L) cyclical growth option. For details on the simulation see Section 6. L_{bad} and L_{good} are asset sale ratios of firms with a low cyclicality of the expansion option during bad and good states, respectively. H_{bad} and H_{good} indicate the ratios for firms with a high cyclicality in the two states.

Asset Sale Conditional on Investment	
Total Asset Sales	42.13%
High Leverage Firms	64.31%
Low Leverage Firms	34.69%
Bad States	53.72%
Good States	38.25%
L_{bad}	48.75%
L_{good}	41.22%
H_{bad}	46.12%
H_{good}	41.79%