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CRASH SENSITIVITY AND THE CROSS-SECTION OF EXPECTED STOCK RETURNS

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Abstract

This paper examines whether investors receive compensation for holding crash-sensitive stocks. We capture the crash sensitivity of stocks by their lower tail dependence (LTD) with the market based on copulas. We find that stocks with weak LTD serve as a hedge during crises, but, overall, stocks with strong LTD have higher average future returns. This effect cannot be explained by traditional risk factors and is different from the impact of beta, downside beta, coskewness, and cokurtosis. Our findings are consistent with results from the empirical option pricing literature and support the notion that investors are crash-averse.

Keywords: asset pricing, asymmetric dependence, copulas, crash aversion, downside risk, tail risk

JEL Classification Numbers: C12, G01, G11, G12, G17.

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Abstract

This paper examines whether investors receive compensation for holding crash-sensitive stocks. We capture the crash sensitivity of stocks by their lower tail dependence (LTD) with the market based on copulas. We find that stocks with weak LTD serve as a hedge during crises, but, overall, stocks with strong LTD have higher average future returns. This effect cannot be explained by traditional risk factors and is different from the impact of beta, downside beta, coskewness, and cokurtosis. Our findings are consistent with results from the empirical option pricing literature and support the notion that investors are crash-averse.

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1 Introduction

The empirical option pricing literature shows that deep out-of-the-money puts, i.e., instruments that offer protection against extreme market downturns, have a very high implied volatility, meaning that they are very expensive.¹ This pattern is typically explained by investors being crash-averse or showing signs of ‘crash-o-phobia’ (Rubinstein (1994), Bates (2008)). However, the potential impact of crash aversion on the pricing of the cross-section of individual stocks has caught surprisingly little attention in the literature.² If investors are crash-averse, they derive disproportionately large disutility from large drops in their wealth. Then, crash-sensitive stocks should bear a premium, because stocks that do particularly badly when the market performs very badly, i.e., when aggregate wealth is low, are unattractive assets to hold for such investors. In this paper, we document that crash-sensitive stocks indeed deliver higher returns than crash-insensitive stocks.

To measure crash sensitivity at the individual asset level, we need a dependence concept that allows us to focus on joint extreme events. Standard asset pricing models since Sharpe (1964) and Lintner (1965) argue that the joint distribution of individual stock returns and the market portfolio return determines the cross-section of expected stock returns. According to the empirical interpretation of the traditional CAPM, a stock’s expected return only depends on its beta—its scaled linear correlation with the market—without any focus on tail events. However, the correlation alone cannot characterize the full dependence structure of non-normally distributed random variables such as realized stock returns (Embrechts, McNeil, and Straumann (2002)). Particularly, it cannot capture clustering in the lower tail of the bivariate return distribution between individual securities and the market, which is important if investors are crash-averse. Thus, we develop a novel proxy for stock-individual crash sensitivity using copula methods based on extreme value theory. Specifically, we capture stock individual crash sensitivity based on the extreme dependence between individual stock returns and market returns in the lower-left tail of their joint distribution (also called lower tail dependence, LTD) and investigate its influence on the cross-section of individual stock returns.³

Based on a rolling window estimation using daily return data for US stocks from 1963 to 2012, we calculate copula-based LTD coefficients for each stock and month. We find that stocks with

¹See, e.g., Aït-Sahalia and Lo (2000), Bates (2000), Broadie, Chernov, and Johannes (2009), Jackwerth and Rubinstein (1996), Jackwerth (2000), Rosenberg and Engle (2002), Rubinstein (1994). Garleanu, Pedersen, and Poteshman (2009) show that this effect is driven by high demand for out-of-the-money puts.

²Notable exceptions are the papers by Kelly and Jiang (2014) and Cholette and Lu (2011), which we will discuss in more detail below.

³A positive influence of LTD on returns is expected (but not empirically shown) in Poon, Rockinger, and Tawn (2004): “If tail events are systematic as well, one might expect the extremal dependence between the asset returns and the market factor returns to also command a risk premium.” (p. 586).

previously weak LTD, i.e., stocks that displayed only weak LTD or no LTD with the market at all in the previous 12 months, have significantly higher returns than strong LTD stocks during extreme market downturns. Hence, weak LTD stocks indeed offer some protection against market crashes.

In our main asset pricing tests, we relate individual LTD to average returns in a predictive setting. Our empirical results using portfolio sorts and multivariate regression analysis on the individual firm level show a strong positive impact of LTD in month t on future excess returns in month $t+1$. A value-weighted portfolio consisting of stocks with the strongest LTD delivers higher average future returns of 0.360% per month than a portfolio of stocks with the weakest LTD. This amounts to an annualized spread of 4.32%. Similar results are obtained after controlling for the exposure to systematic risk factors and the impact of various firm characteristics.

The impact of LTD has to be distinguished from the impact of downside beta documented in Ang, Chen, and Xing (2006) as well as from the impact of other higher co-moments. Downside beta focuses on individual securities' exposure to market returns conditional on below-average market returns and is motivated by disappointment aversion (Gul (1992)). LTD is conceptually different from downside beta, as the latter places no particular emphasis on tail events.⁴ Consequently, downside beta captures general downside risk (or disappointment) aversion rather than crash aversion. In contrast, LTD captures the dependence in the extreme left tail of return distributions; it focuses on how individual securities behave during the worst market return realizations within a given period. We find a strong impact of LTD after controlling for the impact of the Ang, Chen, and Xing (2006) downside beta as well as alternative definitions of downside beta as discussed in Post, van Vliet, and Lansdorp (2012). We can also show that the risk premium associated with LTD is not explained by coskewness (Harvey and Siddique (2000)), cokurtosis (Fang and Lai (1997)), idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang (2006)), or a stock's lottery characteristics (Bali, Cakici, and Whitelaw (2011)) and holds after controlling for other systematic risk factors suggested in the literature.

Furthermore, we document that the risk premium for LTD is higher following large stock market declines. This result is consistent with the theoretical predictions of Chen, Joslin, and Tran (2012). They show that disaster risk premia can increase substantially when the risk-sharing capacity of the "optimists" in their model is reduced, and they argue that this is likely to be the case in the aftermath of a crisis. Our findings also are in line with an argument recently made by Gennaioli, Shleifer, and Vishny (2015) that investors fear a future crash more when there was a recent crash that they still remember.

⁴Downside betas conditional on very low market returns (instead of just below mean market returns, as in Ang, Chen, and Xing (2006)) are intuitively more closely related to LTD. However, they cannot be estimated reliably, as we show in Section 3.4.1.

To motivate our use of copula-based LTD coefficients as a relevant risk measure for investors in a stringent way, in the appendix, we introduce a theoretical asset pricing model. We show that some simple regularity assumptions on the representative investor’s utility function are sufficient for LTD to be priced in a stochastic discount factor framework. The main assumptions necessary for this result are that the first four derivatives of the utility function have altering signs, i.e., investors show non-satiation, they are risk-averse, their absolute risk-aversion is decreasing (which is equivalent to investors liking skewness), and they are “temperate” (which is equivalent to investors disliking kurtosis; see Eeckhoudt and Schlesinger (2006)). While our theoretical framework illustrates that these assumptions are sufficient to generate a risk premium for strong LTD stocks, we expect this finding to be reinforced if one would enrich our basic theoretical model with additional behavioral aspects.⁵

Our theoretical model also predicts a negative return premium for the upper tail dependence (UTD) between an individual stock’s return and the market return. Our empirical analysis shows that high UTD stocks indeed earn a negative return premium, but—as also predicted by the model—the effect is smaller in absolute terms than the impact of LTD and not statistically significant at the 10% level in most asset pricing tests.

Our study contributes to several strands of the asset pricing literature. First, it is related to the literature on rare-disaster risk that has caught a lot of attention in the economics and finance literature in recent years (e.g., Barro (2006), (2009), and Pindyck and Wang (2013)). Bollerslev and Todorov (2011) find that much of the aggregate equity risk premium is a compensation for the risk of extreme events, and Gabaix (2012) shows that time-varying rare-disaster risk can explain the equity premium puzzle (as well as several other puzzles in macro finance). Similarly, there is now a small number of recent papers that examine the time-series relationship between tail risk and *aggregate* stock market returns (e.g., Bali, Demirtas, and Levy (2009), Bollerslev and Todorov (2011), and Kelly and Jiang (2014)). They find that proxies for tail risk can predict aggregate market returns.

Second, our study is related to the theoretical and empirical literature on downside risk and loss aversion. Downside risk aversion is already discussed in Roy (1952), who argues that investors display “safety first” preferences, and in Markowitz (1959), who suggests using the semi-variance as a measure of risk. Many subsequent contributions analyze the impact of higher co-moments

⁵For example, the accumulated evidence from experiments designed to verify the cumulative prospect theory by Kahneman and Tversky (1979) shows that individuals are loss-averse and distort the probabilities of low-probability outcomes (like market crashes) heavily upwards (e.g., Abdellaoui (2002)). More recently, Polkovnichenko and Zhao (2013) confirm this pattern using market data from traded financial options to derive empirical probability weighting functions. He and Zhou (2013) show that this can have important implications for investors’ optimal portfolio choice and leads to demand for portfolio insurance.

on expected returns.⁶ Kahneman and Tversky (1979) argue that individuals evaluate outcomes relative to reference points and show that individuals are loss-averse. Although aversion to losses and downside risk aversion are discussed extensively in the literature, only a few papers investigate the effect of loss or disappointment aversion on expected asset returns (Barberis and Huang (2001), Benartzi and Thaler (1995), Barberis, Huang, and Santos (2001), Ang, Bekaert, and Liu (2005), and Lettau, Maggiori, and Weber (2014)). However, these papers (as well as the study by Ang, Chen, and Xing (2006) discussed above) are concerned with general downside risk aversion rather than crash aversion.

Crash aversion has still caught relatively little attention in the cross-sectional asset pricing literature. One exception is Berkman, Jacobsen, and Lee (2011), who examine whether industries that are sensitive to a real crisis index deliver higher returns and find some evidence for this to be the case. The only other papers we are aware of that document an impact of crash sensitivity (or tail risk exposure) on the cross-section of individual stock returns are the papers by Kelly and Jiang (2014) and Cholette and Lu (2011).⁷ These two papers predict aggregate tail risk by applying the tail risk estimator of Hill (1975) to the cross-section of all daily stock returns in a given month. Consistent with our results, Kelly and Jiang (2014) also document that a long-short portfolio that is based on individual stocks' exposure to an aggregate tail risk factor that hedges against tail events delivers significantly negative returns. Similar results are obtained by Cholette and Lu (2011). Our paper differs from these two papers conceptually: we capture crash sensitivity using lower tail dependence between a stock and the market. Thus, our proxy for crash sensitivity of an individual stock has the advantage of being directly based on the joint distribution of its return and the market return. An additional practical advantage of our approach is that we only need a time series of an individual asset's return and the market return (while Kelly and Jiang's implementation of the Hill (1975) estimator requires data on the whole cross-section of all individual daily stock returns at each point in time). Finally, unlike these papers, our paper develops an explicit cross-sectional asset pricing theory and thereby theoretically motivates our focus on crash aversion and the use of LTD to capture crash sensitivity in our empirical analysis. Our paper is also unique in documenting a higher return premium after crises periods, thus providing the first empirical support for the theoretical models of Chen, Joslin, and Tran (2012) and Gennaioli, Shleifer, and Vishny (2015).

Finally, we contribute to the literature on the application of extreme value theory and copulas in

⁶Extensions of the basic CAPM that allow for preferences for skewness and lower partial moments of security and market returns are developed by Kraus and Litzenberger (1976) and Bawa and Lindenberg (1977). Kraus and Litzenberger (1976), Friend and Westerfield (1980), and Harvey and Siddique (2000) document that investors dislike a stock's negative coskewness with the market return. Fang and Lai (1997) and Dittmar (2002) show that stocks with high cokurtosis earn high average returns.

⁷Van Oordt and Zhou (2013) measure tail risk of stocks based on tail betas but do not find evidence of higher returns associated with high tail risk.

finance. Despite its long history in statistics, multivariate extreme value theory has been applied to the analysis of financial markets only recently.⁸ It is mainly used to describe dependence patterns across different markets and assets (Longin and Solnik (2001), Patton (2004), and Elkamhi and Stefanova (2015)). However, to the best of our knowledge, ours is the first paper to investigate extreme dependence structures in the bivariate distribution of individual and market returns based on copulas. Our application details how to fit flexible combinations of basic parametric copulas to this bivariate distribution and how to derive the corresponding tail dependence coefficients. The copula approach has the advantage that extreme dependence is not estimated based on a small number of observations in the tail exclusively, but that information from the whole joint distribution can be used. Furthermore, our approach updates dependence estimates frequently, thus allowing to capture the potentially dynamic nature of tail dependence in an extremely flexible framework.

2 Tail Dependence and Copula Methodology

Most of the standard empirical asset pricing literature focuses on risk factors based on linear correlation coefficients. However, this measure of stochastic dependence is not typically able to completely characterize the dependence structure of non-normally distributed random variables (Embrechts, McNeil, and Straumann (2002)). It is widely recognized since a long time that many financial time series, including stock returns, are non-normally distributed (see, e.g., Mandelbrot (1963) and Fama (1965)). For example, they are often characterized by leptokurtosis. This is problematic because when we are dealing with a fat-tailed bivariate distribution $F(x_1, x_2)$ of two random variables X_1 and X_2 , the linear correlation—and consequently the standard beta estimate—fails to capture the dependence structure in the extreme lower-left and upper-right tail. As an example, consider the following illustrations of 2,000 simulated bivariate realizations based on different dependence structures between (X_1, X_2) shown in Figure 1.

[Insert Figure 1 about here]

In all models, X_1 and X_2 have standard normal marginal distributions and a linear correlation of 0.8, but other aspects of the dependence structure are clearly different. In Panel A we first show an example in which we did not allow for clustering in either tail of the distribution. Panels

⁸Longin and Solnik (2001) use extreme value theory to model the bivariate return distributions between different international equity markets. Ané and Kharoubi (2003) propose to model the dependence structure of international stock index returns via parametric copulas while Poon, Rockinger, and Tawn (2004) present a general framework for identifying joint-tail distributions based on multivariate extreme value theory. Patton (2004) uses copula theory to model time-varying dependence structures of stock returns and Patton (2009) applies copula functions to assess different definitions of market neutrality for hedge funds. Finally, Elkamhi and Stefanova (2015) use copulas to show that accounting for extreme asset comovements is important for portfolio hedging.

B to D show examples of increased dependence in the upper-right tail, in the lower-left tail, and symmetric increased dependence in both tails, respectively. Still, all of these bivariate distributions are characterized by a linear correlation coefficient of 0.8. These examples show that it is often not possible to describe the dependence structure by the linear correlation alone.

Now, we characterize two measures of dependence as limiting cases of conditional probabilities: Consider two bivariate returns (X_1, X_2) , where X_1 is the return of an individual stock and X_2 is the market return with corresponding marginal cumulative distributions F_{X_1} and F_{X_2} . We define

$$P_l(q) = \Pr \left[X_1 < F_{X_1}^{-1}(q) \mid X_2 < F_{X_2}^{-1}(q) \right] \quad (1)$$

as a tail dependence measure in the left tail. X_1 and X_2 are said to be asymptotically independent (dependent) in the left tail if $P_l(q)$ has a limit that is equal (not equal) to zero as q approaches 0 from the right. We define lower tail dependence (LTD) as:

$$\text{LTD} \equiv \lim_{q \rightarrow 0^+} P_l(q). \quad (2)$$

Similarly, we define

$$P_r(q) = \Pr \left[X_1 > F_{X_1}^{-1}(q) \mid X_2 > F_{X_2}^{-1}(q) \right] \quad (3)$$

and

$$\text{UTD} \equiv \lim_{q \rightarrow 1^-} P_r(q) \quad (4)$$

as our measure of upper tail dependence (UTD).

In the following (Section 2.1), we detail how we estimate measures of tail dependence based on copulas.⁹ Then, we describe the development of aggregate tail dependence over time (Section 2.2) and finally assess whether our suggested tail dependence coefficients really measure how stocks do during market crashes and whether they are useful for hedging against extreme outcomes (Section 2.3).

2.1 Copula-Based Estimation of Tail Dependence Coefficients

The main idea of our estimation framework is to model the *whole* dependence structure between individual stock returns and the market return using copulas. We first estimate the marginal

⁹As copula concepts are not yet regularly used in standard asset pricing applications, we provide a short intuitive introduction into the concept in Section A of the Internet Appendix. For a more detailed overview on the use of copulas in econometrics and finance, see Fan and Patton (2014).

distributions of an individual stock return and the market return non-parametrically by their scaled empirical distribution functions. Then, we estimate parameters of different copulas to compute coefficients of tail dependence (i.e., LTD and UTD) based on closed-form solutions.¹⁰

Unfortunately, most basic copulas do not allow to model LTD and UTD simultaneously. Hence, we work with flexible convex combinations of copulas.¹¹ Specifically, we use combinations of simple parametric copulas that either exhibit no tail dependence (the Gauss-, the Frank-, the FGM-, and the Plackett-copula), lower tail dependence (the Clayton-, the Rotated Gumbel-, the Rotated Joe-, and the Rotated Galambos-copula), or upper tail dependence (the Gumbel-, the Joe-, the Galambos-, and the Rotated Clayton-copula). To allow for maximum flexibility in modeling dependence structures, we consider all $4 \times 4 \times 4 = 64$ possible convex combinations that consist of one copula that allows for asymptotic dependence in the lower tail, C_{LTD} , one copula that is asymptotically independent, C_{NTD} , and one copula that allows for asymptotic dependence in the upper tail, C_{UTD} :

$$C(u_1, u_2, \Theta) = w_1 \cdot C_{\text{LTD}}(u_1, u_2; \theta_1) + w_2 \cdot C_{\text{NTD}}(u_1, u_2; \theta_2) + (1 - w_1 - w_2) \cdot C_{\text{UTD}}(u_1, u_2; \theta_3), \quad (5)$$

where Θ denotes the set of the basic copula parameters θ_i , $i = 1, 2, 3$ and the weights w_1 and w_2 .¹²

Our estimation approach for LTD and UTD then follows a three-step procedure. First, based on daily return data for the market and each stock, we estimate a set of copula parameters Θ_j for $j = 1, \dots, 64$ different copulas $C_j(\cdot, \cdot; \Theta_j)$ between the respective marginal distribution of an individual stock return r_i and the market return r_m for each month based on a rolling window of 12 months (Section 2.1.1). We explicitly use a short time horizon of 12 months in the estimation of the copula parameters to account for time-varying dependence in the bivariate distribution of r_i and r_m .

¹⁰Table IA.I. of the Internet Appendix shows the parametric forms and related tail dependencies of the basic copulas used in this study. Alternatives to this approach include a purely non-parametric approach (as suggested in Poon, Rockinger, and Tawn (2004) which relies only on observations from the tail) and a fully parametric approach. Non-parametric test procedures enable us to test for the existence of tail dependence – in the Internet Appendix (Table IA.II) we show that the existence of LTD cannot be rejected for more than 60% of the firm-month observations in our sample using the bottom 1% daily return observations as a cutoff. However, purely non-parametric point estimates for tail dependence coefficients are not very precise (Frahm, Junker, and Schmidt (2005)). Hence, in the following, we rely on an estimation framework using non-parametric margins and parametric copula functions to obtain more precise estimates for LTD and UTD. To avoid the risk of model mis-specification, we consider 64 different copula models in our selection and estimation process.

¹¹Tawn (1988) shows that every convex combination of existing copula functions is again a copula. Thus, if $C_1(u_1, u_2), C_2(u_1, u_2), \dots, C_n(u_1, u_2)$ are bivariate copula functions, then $C(u_1, u_2) = w_1 \cdot C_1(u_1, u_2) + w_2 \cdot C_2(u_1, u_2) + \dots + w_n \cdot C_n(u_1, u_2)$ is again a copula for $w_i \geq 0$ and $\sum_{i=1}^n w_i = 1$.

¹²These convex combinations are similar to other copulas such as the BB1 to BB7 copulas suggested in Joe (1997), but they offer more flexibility. Particularly, as our convex combinations also contain one copula that is asymptotically independent, ours is an extremely flexible and efficient way to model dependence structures and is less prone to model misspecification.

An alternative approach to account for time-variation in dependence would be to specify a dynamic (conditional) copula model similar to Patton (2006) and Jondeau and Rockinger (2006). In this setting the functional form of the copula remains fixed over time whereas the copula parameters vary according to some pre-specified stochastic process. However, while this is a valid approach to capture the dynamic nature of the copula parameters, the approach uses a fixed functional form of the copula function itself. We decided in favor of our more flexible (unconditional) copula framework because it enables us to choose the best parametric copula, i.e., the parametric copula that minimizes the distance to the empirical copula, at each point in time.

Second, we follow Ané and Kharoubi (2003) and select the appropriate parametric copula $C^*(\cdot, \cdot; \Theta^*)$ by minimizing the distance between the different estimated parametric copulas $C_j(\cdot, \cdot; \hat{\Theta}_j)$ and the empirical copula \hat{C} based on the Integrated Anderson-Darling distance (see Section 2.1.2). Third, we compute the tail dependence coefficients LTD and UTD implied by the estimated parameters Θ^* of the selected copula $C^*(\cdot, \cdot; \Theta^*)$ (Section 2.1.3).

2.1.1 Estimation of the Marginal Distribution and the Copula Parameters

The estimation of the set of copula parameters Θ_j for the different copula combinations $C_j(\cdot, \cdot; \Theta_j)$ is performed as follows: Let $\{r_{i,k}, r_{m,k}\}_{k=1}^n$ be a random sample from the bivariate distribution $F(r_i, r_m) = C(F_i(r_i), F_m(r_m))$ between an individual stock return r_i and the market return r_m , where n denotes the number of daily return observations in a given period.¹³ We estimate the marginal distributions F_i and F_m of an individual stock return r_i and the market return r_m non-parametrically by their scaled empirical distribution functions

$$\hat{F}_i(x) = \frac{1}{n+1} \sum_{k=1}^n \mathbb{1}_{r_{i,k} \leq x} \quad \text{and} \quad \hat{F}_m(x) = \frac{1}{n+1} \sum_{k=1}^n \mathbb{1}_{r_{m,k} \leq x}. \quad (6)$$

The estimation of F_i and F_m by their empirical counterparts avoids an incorrect specification of the marginal distributions (see Fermanian and Scaillet (2005) and Charpentier, Fermanian, and Scaillet (2007)). We then estimate the set of copula parameters Θ_j . Each convex combination requires the estimation of five parameters: one parameter θ_i ($i = 1, 2, 3$) for each of the three basic copulas and the two weights w_1 and w_2 . Since we assume a parametric form of the copula functions, the parameters Θ_j can be estimated via the canonical maximum likelihood procedure (Genest, Ghoudi, and Rivest (1995)):

¹³In computing the market return r_m , we exclude stock i , so the market return r_m is slightly different for each stock's time-series regression. This removes potential endogeneity problems when calculating LTD and UTD coefficients for each stock.

$$\hat{\Theta}_j = \operatorname{argmax}_{\Theta_j} L_j(\Theta_j) \quad \text{with} \quad L_j(\Theta_j) = \sum_{k=1}^n \log(c_j(\hat{F}_{i,r_{i,k}}, \hat{F}_{m,r_{m,k}}; \Theta_j)), \quad (7)$$

where $L_j(\Theta_j)$ denotes the log-likelihood function and $c_j(\cdot, \cdot; \Theta_j)$ the copula density. Assuming that $\{r_{i,k}, r_{m,k}\}_{k=1}^n$ is an i.i.d. random sample, $\hat{\Theta}$ is a consistent and asymptotic normal estimate of the set of copula parameters Θ under standard regularity conditions (e.g., Genest, Ghoudi, and Rivest (1995)).¹⁴

2.1.2 How to Select the Right Copula

So far we have pointed out an estimation procedure under the assumption that the copula $C_j(\cdot, \cdot; \Theta_j)$ is known up to a set of parameters Θ_j . The choice of the copula $C^*(\cdot, \cdot; \Theta^*)$ obviously affects the resulting bivariate distribution and the resulting tail dependence coefficients LTD and UTD. However, most applications presented in the literature do not discuss this issue and rely on an arbitrary choice of the copula. To avoid this problem, we follow Ané and Kharoubi (2003) and use the empirical copula function introduced by Deheuvels (1981) to evaluate the fit of different parametric copulas. We proceed as follows:

Let $\{R_{i,k}, R_{m,k}\}_{k=1}^n$ denote the rank statistic of $\{r_{i,k}, r_{m,k}\}_{k=1}^n$, i.e., the smallest individual stock (market) return observation of $r_{i,k}$ ($r_{m,k}$) has rank $R_{i,k} = 1$ ($R_{m,k} = 1$) and the largest individual stock (market) return observation of $r_{i,k}$ ($r_{m,k}$) has rank $R_{i,k} = n$ ($R_{m,k} = n$).

Deheuvels (1981) introduces the empirical copula $\hat{C}_{(n)}$ on the lattice

$$L = \left[\left(\frac{t_i}{n}, \frac{t_m}{n} \right), t_i = 0, 1, \dots, n, t_m = 0, 1, \dots, n \right]$$

by the following equation:

$$\hat{C}_{(n)} \left(\frac{t_i}{n}, \frac{t_m}{n} \right) = \frac{1}{n} \sum_{k=1}^n \mathbb{1}_{R_{i,k} \leq t_i} \cdot \mathbb{1}_{R_{m,k} \leq t_m}. \quad (8)$$

We compute Integrated Anderson-Darling distances $D_{j,IAD}$ between the parametric copulas $C_j(\cdot, \cdot; \hat{\Theta}_j)$ and the empirical copula $\hat{C}_{(n)}$ via

¹⁴Obviously, daily return data often violates the assumption of an i.i.d. random sample. Thus, an alternative approach to the problem of non-i.i.d. data due to serial correlation in the first and the second moment of the time series would be to specify, e.g., GARCH models for the univariate processes, and analyze the dependence structure of the residuals. In Section 3.4.4 we check this alternative approach: results for the LTD and UTD coefficients based on filtered residuals and subsequent asset pricing implications are very similar to our main results using unfiltered data.

$$D_{j,IAD} = \sum_{t_i=1}^n \sum_{t_m=1}^n \frac{\left(\widehat{C}_{(n)}\left(\frac{t_i}{n}, \frac{t_m}{n}\right) - C_j\left(\frac{t_i}{n}, \frac{t_m}{n}; \widehat{\Theta}_j\right) \right)^2}{C_j\left(\frac{t_i}{n}, \frac{t_m}{n}; \widehat{\Theta}_j\right) \cdot \left(1 - C_j\left(\frac{t_i}{n}, \frac{t_m}{n}; \widehat{\Theta}_j\right)\right)}. \quad (9)$$

Hence, we calculate the distance between the predicted value of the parametric copulas $C_j(\cdot, \cdot; \widehat{\Theta}_j)$ and the empirical copula $\widehat{C}_{(n)}$ for every grid point on the lattice L . The estimation of the tail dependence coefficients LTD and UTD is based on the estimated parameters Θ^* of the copula combination $C^*(\cdot, \cdot; \Theta^*)$, which minimizes $D_{j,IAD}$.

The result of our empirical implementation of this procedure shows that all combinations are chosen regularly and no specific copula clearly dominates, which highlights the advantage of picking the copula function that describes the data best, rather than just using one specific ad-hoc copula. The respective frequencies are summarized in Table IA.III in the Internet Appendix. The three copula combinations that are most often selected are the Clayton-Gauss-Galambos-copula (5.96%), the Clayton-Gauss-Rotated Clayton-copula (5.75%), and the Rotated Galambos-Gauss-Rotated Clayton-copula (5.73%).¹⁵

2.1.3 Computation of Tail Dependence Coefficients Based on Convex Combinations of Copulas

Finally, we compute the tail dependence coefficients LTD and UTD implied by the estimated parameters Θ^* of the selected copula $C^*(\cdot, \cdot; \Theta^*)$. The computation of LTD and UTD is straightforward for the basic copulas used in our study (the respective closed form solutions for tail dependence coefficients are shown in Table IA.I in the Internet Appendix). The lower and upper tail dependence coefficient of the convex combination are calculated as the weighted sum of the LTD and UTD coefficients from the individual copulas, respectively, where the weights from (5) are used, i.e., $LTD^* = w_1^* \cdot LTD(\theta_1^*)$ and $UTD^* = (1 - w_1^* - w_2^*) \cdot UTD(\theta_3^*)$. As this procedure is repeated for each stock and month based on an annual estimation horizon, we end up with a panel of tail dependence coefficients $LTD_{i,t}^*$ and $UTD_{i,t}^*$ at the month-firm level.

Our empirical approach to estimate LTD has three advantages: (i) it uses the whole body of data of the bivariate distribution of individual and market returns (thus avoiding the imprecision of tail dependence estimates relying on non-parametric methods that only focus on tail observations). (ii) it is very flexible by picking the convex copula combination that best describes the data (in

¹⁵In a robustness check, we select the best parametric copula based on estimated log-likelihood values instead of Integrated Anderson-Darling distances. We confirm that the copula combinations most frequently picked are the Clayton-Gauss-Galambos-copula (5.91%), the Clayton-Gauss-Rotated Clayton-copula (5.78%), and the Rotated Galambos-Gauss-Rotated Clayton-copula (5.75%) using this alternative selection criterion. Asset pricing results are also very similar (see Table 10).

contrast to approaches using a pre-defined specific functional form for the dependence structure) and by allowing for asymmetric tail dependencies in the upper and lower tail. (iii) it does capture the dynamic nature of the dependence relationship by frequently updating the copula fitting and parameter selection procedure.

2.2 Data, Summary Statistics and the Evolution of Aggregate Tail Dependence

Our sample consists of all common stocks (CRSP share codes 10 and 11) from CRSP trading on the NYSE, AMEX, and NASDAQ between January 1, 1963, through December 31, 2012. So that our results are not driven by very small stocks, we exclude return data from firms that are in the bottom 1% of market capitalization of all stocks in the previous year. Furthermore, we require at least 100 valid daily return observations per year. Overall, there are 2,613,440 firm-month observations after we apply these filters. The number of firms in each month over our sample period ranges from 1,904 to 6,778. Summary statistics are provided in Table 1.

[Insert Table 1 about here]

The first four columns show the mean as well as the 25%, the 50%, and the 75% quantiles for key variables (pooled over all stocks and months). The mean (median) yearly excess return over the risk-free rate of all stocks in our sample is 0.67% (-0.13%), and the mean (median) LTD coefficient is 0.10 (0.07). We also observe considerable variation in LTD, with an interquartile range of nearly 0.15. The mean (median) of UTD is 0.07 (0.04) and is significantly lower than the mean (median) of LTD. The general tendency for stronger asymptotic dependence in the left tail than in the right tail of the distributions is consistent with the well-documented finding that return correlations generally increase in down markets.¹⁶ The rest of the table provides information on the summary statistics regarding other firm characteristics and return patterns that we later use in our empirical analysis. All variable definitions are contained in the Appendix.

The last three columns of Table 1 show the average characteristics of stocks with, respectively, above and below values of LTD in a given month, as well as the difference between the two. Excess returns over the risk-free rate for high LTD stocks are 0.80% p.a., while they are significantly lower at 0.57% p.a. for low LTD stocks. The difference amounts to 0.23% p.a. and is statistically significant at the 1% level. At the same time, high LTD stocks also have significantly higher regular betas (β) and downside betas (β^-), tend to be somewhat larger and more liquid, and have lower book-to-market ratios.

¹⁶In the Internet Appendix, we also look at five year subperiods. We find that UTD is significantly weaker than LTD in each period (Table IA.IV). Increased extreme dependence among international markets during bear markets is also documented in Longin and Solnik (2001) and Poon, Rockinger, and Tawn (2004).

Cross-correlations between the independent variables used in our later analysis are shown in Table 2 and confirm these patterns.

[Insert Table 2 about here]

The correlation between LTD and UTD is relatively moderate, at 0.15, which shows that firms with strong tail dependence in one tail of the distribution do not necessarily exhibit strong tail dependence in the other tail. This finding also justifies our flexible modeling approach for tail dependence, which allows for asymmetric tail dependence in the upper and lower tail. LTD is correlated with downside beta with a correlation coefficient of 0.38 and with regular beta with a correlation coefficient of 0.34. LTD is also related to other co-moments as can be seen from the strongly positive (negative) correlation with cokurtosis (coskewness). We carefully take into account the impact of these correlations in our later analysis.

To get some idea about the temporal variation of tail dependence, we investigate the time series of *aggregate* LTD. We define aggregate LTD, $LTD_{m,t}$, as the yearly cross-sectional, value-weighted, average of $LTD_{i,t}$ over all stocks i in our sample. In Figure 2, we plot the time series of $LTD_{m,t}$.

[Insert Figure 2 about here]

There is no particular time trend in $LTD_{m,t}$.¹⁷ However, the graph does exhibit occasional spikes in $LTD_{m,t}$ that roughly correspond to worldwide financial market crises. The highest values in $LTD_{m,t}$ correspond to 1987, the year of “Black Monday,” with the largest one-day percentage decline in U.S. stock market history, and to the years 2007 through 2011, the years of the recent worldwide financial crisis. This pattern suggests that $LTD_{m,t}$ —similar to return correlations—increases in times of financial crises. Consistent with this argument, the time-series correlation between LTD and the market return is -0.08 , and the time-series correlation between LTD and market volatility is 0.32 . Figure 2 also plots aggregate UTD, $UTD_{m,t}$, defined as the yearly cross-sectional, value-weighted, average of $UTD_{i,t}$. The time series of $LTD_{m,t}$ and $UTD_{m,t}$ are not significantly correlated. We find that in 33 of 49 years of our sample $LTD_{m,t}$ exceeds $UTD_{m,t}$.

2.3 Returns of LTD-sorted Portfolios During Crises

To check whether investors can use weak LTD stocks to hedge against market crashes from an ex-ante point of view, we analyze the relation between past LTD and future returns on some of the most relevant financial crisis days in our sample period. We examine “Black Monday” (October

¹⁷Performing an augmented Dickey-Fuller test rejects the null hypothesis that $LTD_{m,t}$ contains a unit root with a p-value smaller than 2%.

19, 1987), the Asian Crisis (October 27, 1997), the burst of the dot-com bubble (April 14, 2000), and the recent Lehman crises (October 15, 2008). If LTD really captures crash sensitivity, we should see an underperformance of strong LTD stocks as compared to weak LTD stocks on these days. Specifically, we sort stocks into five quintile portfolios at the beginning of each month that contains the respective crisis day based on LTD estimated over the previous twelve months. Then we examine future value-weighted returns of these portfolios during each of the financial crisis days. Results are presented in Table 3.

[Insert Table 3 about here]

As expected (and opposite of what we expect in the overall sample and will later show in Table 4), strong LTD stocks strongly *underperform* weak LTD stocks on each of these individual crisis days. The differences are economically large: the daily return of the weak LTD portfolio is from 1.98% to 5.12% higher than that of the strong LTD portfolio. To assess statistical significance, we also jointly analyze the ten worst return days in our sample. Results are presented in the last column of Table 3 and show that the weak LTD portfolio outperforms the strong LTD portfolio by 2.90% per day on those days. The effect is statistically significant at the 5% level. Note, that we do not claim to predict crises, but that investors could predict which stocks will be particularly badly hit conditional on a crises occurring. Our findings show that crash-sensitive investors can reduce their crises exposure by investing in weak LTD stocks.¹⁸

3 Crash Sensitivity and Future Returns

In the main part of the empirical analysis we look at the relationship between tail dependence coefficients and future monthly security returns. Our asset pricing tests are completely out-of-sample and hence avoid in-sample problems such as overfitting or data mining. We estimate a stock's monthly individual LTD and UTD coefficients based on a rolling window over a period of 12 months. Using this horizon trades off two concerns: First, we need a sufficiently large number of observations to get reliable estimates for our tail dependence coefficients. Second, motivated by the fact that several studies document that risk exposures (like regular beta) are non-stable (see, e.g., Fama and French (1992), and Ang and Chen (2007)), we need to account for time-varying extreme dependence risk by using an estimation window that is not too long. To avoid the impact of autocorrelation and heteroscedasticity in our models, we determine statistical significance in portfolio sorts and multivariate regressions using Newey and West (1987) standard errors.

¹⁸Our results are in line with Agarwal, Ruenzi, and Weigert (2015) who document that highly sophisticated investors (such as hedge funds) actively and successfully time crash risk exposure of their equity holdings.

3.1 Portfolio Sorts

3.1.1 Univariate Sorts

To examine whether tail dependence in the form of LTD and UTD has an impact on the cross-section of future stock returns, we first look at simple univariate portfolio sorts. For each month t we sort stocks into five quintile portfolios based on their LTD and UTD estimated over the previous 12 months. Panel A of Table 4 reports the results of value-weighted sorts based on LTD.¹⁹

[Insert Table 4 about here]

The first column shows considerable cross-sectional variation in LTD: average LTD ranges from 0.00 in the weakest LTD quintile up to 0.27 in the strongest LTD quintile. In the second column, we report the monthly future value-weighted average excess return over the risk-free rate of these portfolios as well as differences in average excess returns between quintile portfolio 5 (strong LTD) and quintile portfolio 1 (weak LTD) in month $t + 1$. We find that stocks with strong LTD have significantly higher average future returns than stocks with weak LTD. Stocks in the quintile with the weakest (strongest) LTD earn an monthly average excess return of 0.296% (0.656%). The return spread between quintile portfolio 1 and 5 is 0.360% (4.32% p.a.), which is statistically significant at the 1% level. These results are consistent with investors' being crash-averse and requiring a premium for holding stocks with strong LTD. However, our findings hitherto are only univariate, and LTD is correlated with several other variables that are related to returns such as regular beta or size (see Table 2). Thus, we also compute the monthly alphas generated by the quintile as well as the difference portfolios based on the one-factor CAPM, the three-factor Fama and French (1993), and the four-factor Carhart (1997) model. Results presented in the last three columns show that alphas always increase monotonically from the weakest to the strongest LTD quintile portfolios. The CAPM-alpha (three-factor alpha, four-factor alpha) of the difference portfolio is economically large, amounting to 0.302% (0.437%, 0.237%) per month, and is always statistically significant at least at the 5% level. The return series of the 5-1 difference portfolio loads significantly positively on the market factor and the UMD momentum factor, while it has a significantly negative exposure to the SMB size factor and the HML book-to-market factor.

In Panel B we report the results of value-weighted sorts based on UTD. We find that stocks with strong UTD have lower average future returns than stocks with weak UTD. The return spread between quintile portfolio 1 and 5 is -0.134% per month, which is not statistically different from zero at the 10% level. When computing alphas, we find that the monthly return spread of the difference

¹⁹Equal-weighted portfolio results are discussed in Section 3.4.4.

portfolio further shrinks (in absolute terms) to -0.091% (for the CAPM-alpha), -0.072% (for the three-factor alpha), and 0.006% (for the four-factor alpha), respectively, and is not significantly different from zero in either case.²⁰ Hence, in the remainder of the paper, we focus on the impact of LTD on the cross-section of average future stock returns.

Alternative Factor Models. We now evaluate whether the return spread due to LTD is explained by alternative factor models. For this purpose, we regress the returns of the (5-1) difference portfolio (consisting of going long stocks with strong LTD and going short stocks with weak LTD) on various sets of asset pricing factors recently proposed in the literature. Results are presented in Table 5.

[Insert Table 5 about here]

First, we include the Pastor and Stambaugh (2003) traded liquidity risk factor in regression (1). In regression (2), we replace the Pastor and Stambaugh (2003) liquidity factor with the Sadka (2006) liquidity factor that is based on the permanent (variable) component of the price impact function. In regression (3) we include the Bali, Cakici, and Whitelaw (2011) factor to control for exposure of our strategy to lottery-type stocks, in regression (4) we include the Baker and Wurgler (2006) sentiment index, orthogonalized with respect to a set of macroeconomic conditions, and in regression (5) the Frazzini and Pedersen (2013) betting-against-beta factor is included.²¹ In regression (6) we replace the momentum factor with the Fama-French short- and long-term reversal factors. Finally, in regressions (7) and (8) we control for exposures to the Fama and French (2014) 5-factor model (extended by a profitability factor and an investment factor) and the Hou, Xue, and Zhang (2015) 4-factor model consisting of the market factor, a size factor, an investment factor, and a profitability factor. In each case, we document a statistically significant and economically meaningful positive regression alpha ranging from 0.22% up to 0.53% per month, showing that alternative factor model specifications cannot explain the return spread associated with LTD.

Overall, the findings from this section suggest that it is possible to create an abnormal future return spread based on information about LTD which is not explained by common asset pricing models.²²

²⁰A negative but (in absolute terms) weaker return premium for UTD as compared to LTD is also predicted by our theoretical model (see Appendix B).

²¹The lottery factor is provided by Nusret Cakici (<http://www.bnet.fordham.edu/cakici/>), the time series of the sentiment factor is taken from <http://people.stern.nyu.edu/jwurgler/>, and the betting-against-beta factor is obtained from Andrea Frazzini's homepage (<http://www.econ.yale.edu/~af227/>).

²²However, these results are only indicative, as we do not take into account any trading costs and other limits of arbitrage. Both are likely to be relevant here, because our trading strategy requires frequent rebalancing and we short stocks with weak LTD (which tend to be small and low β stocks, see Table 1).

3.1.2 Bivariate Sorts

Our univariate result of higher future risk-adjusted returns of strong LTD stocks could be driven by differences in beta,²³ downside beta or differences with respect to other related return characteristics. Thus, as a next step, we conduct double-sorts based on LTD as well as regular beta, downside beta, coskewness, and cokurtosis. We focus on these variables because they are the ones that are most strongly correlated with LTD (see Table 2).

[Insert Table 6 about here]

We first form quintile portfolios sorted on β . Then, within each β quintile, we sort stocks into five portfolios based on LTD. Panel A of Table 6 reports value-weighted future monthly portfolio excess returns over the risk-free rate of the $\beta \times$ LTD portfolios. Within each β quintile we find that the return of the strong LTD portfolio is larger than the return of the weak LTD portfolio. The return differences are all economically large and statistically significant at least at the 5% level. The average spread in excess returns amounts to 0.404% per month and is statistically significant at the 1% level.

LTD is also related to downside beta (β^-) which is defined in Ang, Chen, and Xing (2006) as the stock's β conditional on the market return being below its mean. Thus, in Panel B we report value-weighted future monthly excess returns of $\beta^- \times$ LTD portfolios. We find that stocks in the weak LTD portfolios have an average (across all β^- quintiles) future excess return of 0.204% per month, while stocks in the strong LTD portfolios have an average future excess return of 0.619%. The spread is significant at the 1% level. Amounting to 0.415% per month (4.98% p.a.), it is also economically large. Hence, the impact of LTD on returns is not driven by β^- , either.²⁴

Harvey and Siddique (2000) show that lower coskewness (coskew) is associated with higher expected returns and Fang and Lai (1997) and Dittmar (2002) document that higher cokurtosis (cokurt) is associated with higher expected returns. Thus, in Panel C (D), we show value-weighted average future excess returns of coskew \times LTD (cokurt \times LTD) portfolios. We find that controlling for coskewness in Panel C slightly reduces the impact of LTD. However, LTD still remains a positive and statistically significant predictor of average future returns in both cases.

To summarize, based on bivariate portfolio sorts we provide strong evidence that the risk associated with LTD is related but clearly different from risks associated with regular market beta,

²³Although we already control for linear beta exposure of our portfolios by looking at the CAPM alphas above, we now also analyze dependent portfolio double-sorts on LTD and regular β , which allows us to also control for a possible nonlinear impact of β .

²⁴We show later that our results can also not be explained by alternative definitions of downside beta (see Section 3.4.1).

downside beta, coskewness, and cokurtosis. Double-sorts offer the advantage that they allow us to control for any potential nonlinear impact. However, in double-sorts we can only control for one return characteristic at a time. Thus, we now turn to a multivariate approach that allows us to examine the joint impact of different return and other characteristics of the firm that might have an impact on the cross-section of average future stock returns.

3.2 Multivariate Evidence

We run Fama-MacBeth (1973) regressions on the individual firm level in the period from 1963 to 2012.²⁵ Table 7 presents the regression results of monthly future excess returns on realized LTD and various combinations of control variables in the first five columns.

[Insert Table 7 about here]

In regression (1), we include LTD as the only explanatory variable. It has a positive and highly statistically significant impact with a coefficient estimate of 0.0123 (statistically significant at the 1%-level). In regression (2), we add the stock's UTD coefficient. It shows a significantly negative impact on returns, but the economic magnitude is again much smaller than that of the impact of LTD. In the following regressions, we expand regression model (2) and add β , as well as other firm characteristics such as size, book-to-market, and several other return characteristics that might have an impact on returns.²⁶ Specifically, in regression (3) we add coskewness (coskew), and the Amihud (2002) illiquidity ratio (illiq) as a liquidity proxy, while regression (4) additionally includes previous year returns, idiosyncratic return volatility, cokurtosis of individual returns with the market return, and a stock's lottery features captured by the maximum daily return over the past year, max, similar to that of Bali, Cakici, and Whitelaw (2011). Results show that the impact of LTD on future returns is stable and is even slightly increasing in statistical and economic terms after the inclusion of the control variables. LTD exhibits one of the strongest influence of all variables in terms of statistical power (t-statistics of 5.15 and 4.40, respectively).

Several of the control variables have a significant impact on returns, also, that confirm findings from the existing literature: Firm size (book-to-market ratio) has a negative (positive) impact (e.g., Fama and French (1993)), illiquidity (Amihud (2002)) and cokurtosis (Fang and Lai (1997))

²⁵This econometric procedure has the disadvantage that risk factors are estimated less precisely in comparison to using portfolios as test assets. However, Ang, Liu and Schwarz (2010) show that creating portfolios leads to smaller standard errors of risk factor estimates but does *not* lead to smaller standard errors of cross-sectional coefficient estimates. Creating portfolios destroys information by shrinking the dispersion of risk factors and leads to larger standard errors.

²⁶We winsorize all realizations of our independent variables at the 1% and 99% level in order to avoid outliers driving our results. Our results do not hinge on this winsorization (see Section 3.4).

and Dittmar (2002)) have a positive impact, while max (Bali, Cakici, and Whitelaw (2011)) has a negative impact. We do not find a consistent statistically significant influence of coskewness (Harvey and Siddique (2000)) and idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang (2006) and (2009)).²⁷

In regression (5) we replace β by β^- and β^+ and in regressions (6) and (7), we use 2-month and 3-month ahead returns as the independent variable, respectively. In all cases, our earlier findings are confirmed: there is a very strong positive impact of LTD on average future returns, and the t-statistic for the impact of LTD is always above 3. For longer period returns, we find only weaker effects, showing that LTD is time-varying and highlighting the advantage of our approach using frequently updated LTD estimates. As in Ang, Chen, and Xing (2006), we do not find a significant impact of downside beta on future stock returns.²⁸ Interestingly, including downside beta in our regressions does also not reduce the impact of LTD, showing that the measures capture distinctively different aspect of the dependence structure.

The last column presents the annualized economic significance based on a one standard deviation change of each explanatory variable based on the results from regression (5): a one standard deviation increase of LTD leads to an economically meaningful increase in future returns of 2.64% p.a. This is the fourth-largest effect in terms of economic magnitude of all dependent variables after well-known predictors such as past yearly return (+6.42%), book-to-market (+6.24%), and illiquidity (+4.62%), but still larger than the return effects of higher order co-moments (coskewness: -2.10%; cokurtosis: +2.52%).

3.3 Time-Varying Crash Fears of Investors

In the option pricing literature it is sometimes argued that investors became crash-o-phobic after the experience of the 1987 crash (Rubinstein (1994), Bates (2000)).²⁹ Furthermore, Chen, Joslin, and Tran (2012) argue that the risk premium for disaster risk is typically small, but it increases substantially after a disaster (because then the wealth share of pessimists rises). In a similar vein, Gennaioli, Shleifer, and Vishny (2015) propose a theoretical model where investors overstate the fear of a future market crash when they can remember the occurrence of a black swan event.

Thus, to check whether the occurrence of a financial crisis increases the LTD premium, we split

²⁷Our results are thus in line with Bali, Cakici, and Whitelaw (2011) who document that the inclusion of the max variable drives out the impact of idiosyncratic volatility when predicting future stock returns.

²⁸Ang, Chen, and Xing (2006) document a strong relationship between downside beta and contemporaneous stock returns. However, they do not find a significant relationship between downside beta and future returns (as long as they do not exclude stocks that display the highest standard deviation in the sample.)

²⁹However, this finding is not without any doubt, as studies that find no strong “crash-fear” effect prior to 1987 typically rely on very short pre-1987 samples, due to the lack of option data availability for earlier years.

our data set into two subsamples: The “post market crash” subsample containing the five years after an extreme market downturn has occurred and the “remaining years” subsample.³⁰ Table 8 repeats regression (5) from Table 7 for both subsamples.

[Insert Table 8 about here]

Our findings indicate that the impact of LTD on returns is indeed much stronger in years subsequent to a market crash. The impact of LTD on returns is almosts twice as high in the “post market crash” subsample (with a coefficient for the impact of LTD of 0.0246 in contrast to a coefficient of 0.0136 for the remaining years). Overall, this result implies that investors care about the crash sensitivity of stocks and require a high premium for taking that risk, in particular when a market crash has occurred in the recent past.

3.4 Robustness

In this section we summarize the results from a battery of additional robustness tests. We investigate whether our results hold when we control for downside beta defined in alternative ways (Section 3.4.1), examine alternative estimation procedures of tail dependence coefficients (Section 3.4.2), analyze the contemporaneous relationship between returns and LTD (Section 3.4.3), and summarize results from additional analyses and a battery of stability checks (Section 3.4.4).

3.4.1 Alternative Downside Beta Definitions vs. LTD

Results in Panel B of Table 6 show that our results are not driven by the downside beta (β^-) as defined in Ang, Chen, and Xing (2006). Although the concepts of LTD and β^- seem related, this result is actually not surprising, because the latter focuses on all market returns below the mean, while the former explicitly focuses on extreme events. However, one could argue that alternative definitions of β^- that focus more on the left tail of the market return distribution capture effects more similar to LTD. Hence, to analyze this idea more closely, we repeat our $\beta^- \times$ LTD double-sorts from Table 6 for alternative β^- definitions. Specifically, we calculate downside betas as betas conditional on the market return being below its 10%, 5%, 2%, and 1% quantiles, respectively (rather than being below the mean, as before and as in Ang, Chen, and Xing (2006)).

Moreover, Post, van Vliet, and Lansdorp (2012) and Artavanis (2014) argue that the downside beta estimation framework of Ang, Chen, and Xing (2006) is not in line with economic principles and leads to violations of conditions for coherent risk measures. Thus, we also compute the alternative

³⁰As in Section 2.3 we define “extreme market downturns” as the ten worst return days in our sample. These “exteme market downturns” occurred in 1987, 1997, 1998, 2000, 2008, and 2011.

downside betas investigated in their papers, i.e., the Hogan and Warren (1974) downside beta (β_{HW}^-), the Estrada (2004) downside beta (β_{EST}^-), and the asymmetric response beta (β_{AR}^-) of Harlow and Rao (1989). We calculate these alternative downside betas based on their original definitions (see the variable definitions in the Appendix), as well as for a more restrictive cutoff conditional on the daily market return being below its 10% quantile in the respective year. Table 9 shows the results.

[Insert Table 9 about here]

We report results on the returns of the strong minus weak LTD portfolios within each downside beta quintile in the first five columns, as well as the average of this difference portfolio return across all downside beta quintiles in the last column (as in the last row of Panel B in Table 6) for all alternative β^- definitions. The average monthly difference return ranges from 0.40% (for the β^- definition based on the 10% quantile) up to 0.55% (for the β_{EST}^- definition based on the 10% quantile) and is significant at the 1% level in each case.

Thus, our results on the impact of LTD not only hold after adjusting for various β^- alternatives, they frequently get stronger if we look at more restrictive β^- definitions. At first glance, this pattern might seem unexpected, as more restrictive betas (that focus more on extremely bad market returns) should be more closely related to our LTD measure. However, the reason we find even stronger results for LTD if we use alternative β^- 's with low cutoffs is that they are actually not able to reliably capture dependence in the tails because they are estimated based on a very small number of observations (e.g., only about 12 daily return observations per year for the 5% quintile β^-) and are thus very noisy.³¹

These results also illustrate the advantage of the copula approach in estimating extreme dependence: in estimating the whole dependence structure between individual and market returns using our semi-parametric approach, we make use of all available daily return observations within a year, which allows for a relatively more precise estimation of the dependence structure. Thus, the computation of LTD as described in Section 2.1 is much less noisy and more informative about the true crash sensitivity of a stock.

3.4.2 Alternative Tail Dependence Estimation Procedures

We now investigate whether our results are sensitive to alternative tail dependence estimation procedures. First, instead of selecting the appropriate parametric copula by minimizing the dis-

³¹Correlations between the β^- alternatives and LTD actually decrease from 0.42 for the standard Ang, Chen, and Xing (2006) downside beta, to 0.24, 0.13, 0.04, and 0.00, respectively, for the correlation between LTD and β^- based on the 10%, 5%, 2%, and 1% quintile.

tance between 64 different convex copula combinations and the empirical copula, we ex-ante choose various fixed convex copula combinations. As our ad-hoc fixed copula combinations, we consider the Clayton-Gauss-Galambos-copula, the Clayton-Gauss-Rotated Clayton-copula, the Rotated Galambos-Gauss-Rotated Clayton-copula, the Rotated Gumbel-Frank-Gumbel-copula, the Rotated Gumbel-FGM-Gumbel-copula, and the Rotated Gumbel-Frank-Joe-copula. The first (later) three are the copula combinations most (least) often selected in the estimation procedure (see Table IA.III in the Internet Appendix). We perform Fama-Macbeth (1973) regressions of excess returns on LTD (estimated based on the fixed copula combinations) as well as the full set of control variables (as in regression (5) of Table 7). Results on the coefficient estimates for the influence of LTD are displayed in the first six lines in Table 10.

[Insert Table 10 about here]

Second, we present results for LTD estimated using a convex combination of only two copulas (2-Cop), one that allows for asymptotic dependence in the lower tail, C_{LTD} , and one copula that allows for asymptotic dependence in the upper tail, C_{UTD} :

$$C_{2Cop}(u_1, u_2, \Theta) = w_1 \cdot C_{LTD}(u_1, u_2; \theta_1) + (1 - w_1) \cdot C_{UTD}(u_1, u_2; \theta_2),$$

with $0 \leq w_1 \leq 1$.³²

Third, we show results for LTD when we use estimated log-likelihood values instead of Integrated Anderson-Darling distances when selecting the best copula combination. Finally, we report results where we estimate LTD based on a 24 months and 36 months estimation horizon (instead of our standard 12 month horizon), respectively. Results are shown in the last three rows of Table 10.

We find that LTD remains a significantly positive explanatory factor for the cross-section of average future stocks returns in all cases and remains stable across different specifications. Nevertheless, the slightly weaker results for estimation procedures based on a fixed copula and a combination of only two copulas show that there is additional value in carefully fitting the dependence structure and that our highly flexible 3-copula approach seems to be the most appropriate in our setting. Furthermore, the fact that we find somewhat weaker results based on a 2- and 3-year estimation horizons again shows that LTD is time-varying and supports our approach of using relatively short estimation windows for LTD.

³²As in Section 2.1.2, the estimation of LTD is based on the estimated parameter Θ^* of the copula $C_{2Cop}^*(u_1, u_2, \Theta)$, which minimizes the Integrated Anderson-Darling distance.

3.4.3 Contemporaneous Results

Our main analysis in this paper focuses on the predictive relationship between LTD and future returns. Alternatively, we now also analyze the contemporaneous relationship between realized LTD and stocks returns over the same time period using non-overlapping intervals of one year. Looking at the contemporaneous relationship closely follows papers like Ang, Chen, and Xing (2006) and Lewellen and Nagel (2006), and implicitly assumes that realized returns are on average a good proxy for expected returns. This procedure is mainly motivated by the fact that several studies document that risk exposures (like regular beta) are time-varying (see, e.g., Fama and French (1992) or Ang and Chen (2007)).

Table 11 reports the results of univariate portfolio sorts (similar to Table 4), bivariate portfolio sorts (similar to Table 6), as well as Fama-MacBeth (1973) regressions (similar to Table 7) for the contemporaneous setup.

[Insert Table 11 about here]

We find that all our previous results hold – and are even substantially stronger – in a contemporaneous setting. For example, the contemporaneous value-weighted quintile portfolio spread between stocks with strong LTD and stocks with weak LTD is 10.80% p.a., whereas it is 4.32% p.a. in the predictive setting (see Table 4). Looking at the results for the economic significance from the Fama-MacBeth (1973) regressions in the last column of Panel C, we find that LTD now bears the second largest return premium, only second to the size effect. These findings show that our results from the predictive analysis are probably a lower bound for the actual return premium associated with LTD and suggest that the smaller return spread in the predictive analysis is explained by limited predictive power of realized LTD on future LTD.³³

3.4.4 Temporal Stability & Additional Robustness Tests

In this section we shortly summarize the results from a large number of additional analyses and stability checks that we conduct to analyze whether our main results from Sections 3.1 and 3.2 above are stable.³⁴ First, we show that the results from value-weighted univariate sorts and multivariate regressions are stable over time (Table IA.V). Strong LTD stocks have higher average future returns than weak LTD stocks in every 10-year subperiod between 1963 and 2012 as well as

³³Regressing future LTD (in year $t + 1$) on realized LTD (in year t) in a univariate model delivers a positive and highly statistically significant coefficient estimate with a t-statistic of 11.46. However, the coefficient estimate is only 0.208, i.e., it is far below one, and the model has a relatively low R^2 of about 5.3%.

³⁴All results tables mentioned in this section are presented in the Internet Appendix.

in the earlier period, from 1927 to 1963. The differences are statistically significant in three out of six periods for the portfolio sorts, and in five out of six periods for the multivariate regressions.

Second, we show that our main results from the Fama and MacBeth (1973) regressions are robust if we adjust raw returns based on Fama-French 12 and 48 or SIC 2-digit, 3-digit, and 4-digit industry classifications (left side of Table IA.VI) as well as for return adjustments based on the 125 Daniel, Grinblatt, Titman, and Wermers (1997) characteristic-based benchmarks (right side of Table IA.VI). Our multivariate results also obtain if (a) we do not use Newey-West standard errors in the second stage of the Fama-MacBeth (1973) regressions to determine statistical significance, (b) we do not winsorize the independent variables, (c) we perform a pooled OLS regression with time-fixed effects and standard errors clustered by stock, and (d) we run a pooled OLS regression with time-fixed effects and standard errors clustered by industry.

Third, a possible concern for our analysis is that time-varying volatility impacts the estimation of LTD coefficients. To account for time-varying volatility of stock returns, we fit different volatility time series models to the daily individual stock returns and the market return. We can show that our results are stable if we account for time-varying volatility by first filtering daily return time series using an ARCH(1) model, an GARCH(1,1) model, or an EGARCH(1,1) model before using them in our LTD computation. The impact of LTD estimated from time series residuals is very similar to the impact from LTD estimated using the actual return series (Table IA.VII).

Finally, we repeat our univariate and bivariate portfolio sorts based on equal-weighted returns instead of value-weighted returns (Table IA.VIII). We find that an equal-weighted portfolio formation typically leads to a stronger impact compared to a value-weighted portfolio formation. We again confirm our earlier findings of a statistically and economically important impact of LTD on average future stock returns.

4 Conclusion

The cross-section of expected stock returns reflects a premium for crash sensitivity as measured by a stock return's lower tail dependence, LTD, with the market return. Stocks that are characterized by strong LTD earn significantly higher average future returns than stocks with weak LTD. A value-weighted (equal-weighted) portfolio consisting of stocks with the strongest LTD delivers higher average future returns of 4.3% (4.8%) p.a. than a portfolio of stocks with the weakest LTD. The high average returns earned by strong LTD stocks are not explained by alternative cross-sectional effects, including market beta, size, book-to-market, momentum, liquidity, coskewness, cokurtosis, idiosyncratic volatility, and downside beta.

In contrast, if we focus exclusively on periods of heavy market downturns, we find that stocks with weak LTD outperform stocks with strong LTD. As stocks with weak LTD thus essentially offer an insurance against extreme negative portfolio returns, our results are consistent with the view that investors are willing to pay higher prices and eventually accept lower returns for such stocks. The conjecture that the higher returns of stocks with strong LTD is a reflection of higher equilibrium returns in the presence of crash-averse investors is consistent with findings from the empirical literature on option prices (e.g., Rubinstein (1994) and Polkovnichenko and Zhao (2013)).

On a broader level, we think that the fact that investors can earn a premium for bearing LTD risk has serious implications for financial stability: If financial institutions do not have to bear the expected costs of a severe market downturn (e.g., because regulatory capital requirements do not take into account LTD or because they expect to be bailed out in a severe crisis), they have incentives to invest in exactly those securities that are characterized by strong lower tail dependence with the market in order to earn the associated premium. Such incentives would make those institutions heavily exposed to market crises and could lead to systemic instability. Whether financial institutions are heavily invested in strong LTD assets is an interesting open question for future research. Some suggestive evidence along these lines is again provided in the empirical option market literature.³⁵

³⁵Garleanu, Pedersen, and Poteshman (2009) document that dealers on aggregate hold short positions in out-of-the-money puts—that also offer protection against downturns—while end-users (defined as customers of brokers), seem to hold long positions, i.e., they insure against extreme downside risk.

A Appendix: Definitions and Data Sources of Main Variables

This table briefly defines the main variables used in the empirical analysis. The data sources are: (i) CRSP: CRSP Stocks Database, (ii) KF: Kenneth French Data Library, (iii) CS: Compustat. EST indicates that the variable is estimated or computed based on original variables from the respective data sources.

Panel A: Return-Based Variables		
Variable Name	Description	Source
Return (return)	Monthly raw excess return of a portfolio (stock) over the risk-free rate. As risk-free rate the 1-month T-Bill rate is used.	CRSP, KF, EST
CAPM-Alpha, FF-Alpha, CAR-Alpha	CAPM one-factor, Fama and French (1993) three-factor, and Carhart (1997) four-factor performance alpha of a portfolio over the sample period. We use monthly portfolio returns to estimate the alphas.	CRSP, KF, EST
LTD	Lower tail dependence coefficient of a stock. Estimated based on daily data from one year as detailed in Section 2.2.	CRSP, EST
UTD	Upper tail dependence coefficient of a stock. Estimated based on daily data from one year as detailed in Section 2.2.	CRSP, EST
β	Factor loading on the market factor from a CAPM one-factor regression estimated based on daily data from one year: $\beta = \frac{\text{COV}(r_i, r_m)}{\text{VAR}(r_m)}$.	CRSP, EST
β^-	Downside beta estimated based on daily return data from one year as defined in Ang, Chen, and Xing (2006): $\beta^- = \frac{\text{COV}(r_i, r_m r_m < \mu_m)}{\text{VAR}(r_m r_m < \mu_m)}$, where μ_m is the mean of the daily market return.	CRSP, EST
β^+	Upside beta estimated based on daily return data from one year as defined in Ang, Chen, and Xing (2006): $\beta^+ = \frac{\text{COV}(r_i, r_m r_m > \mu_m)}{\text{VAR}(r_m r_m > \mu_m)}$.	CRSP, EST

Variable Name	Description	Source
β_{HW}^-	Downside Beta beta estimated based on daily return data from one year as defined in Hogan and Warren (1974): $\beta_{HW}^- = \frac{E(r_i \cdot r_m r_m \leq \mu_m)}{E(r_m^2 r_m \leq \mu_m)}$.	CRSP, EST
β_{EST}^-	Downside Beta beta estimated based on daily return data from one year as defined in Estrada (2004): $\beta_{EST}^- = \frac{\sum_{t=1}^T (\min[0, (r_{it} - \mu_m)] \cdot \min[0, (r_{mt} - \mu_m)])}{\sum_{t=1}^T (\min[0, (r_{mt} - \mu_m)])^2}$	CRSP, EST
β_{AR}^-	Downside Beta beta estimated based on daily return data from one year as defined in Harlow and Rao (1989): $\beta_{AR}^- = \frac{E(Xr_i) - E(X)E(r_i)}{E(X^2) - E(X)^2}$ with $X = (r_m \cdot 1_{r_m \leq \mu_m} + E(r_m r_m > \mu_m) \cdot 1_{r_m > \mu_m})$.	CRSP, EST
<i>illiq</i>	The Amihud (2002) illiquidity ratio defined as: $illiq_{i,t} = \frac{1}{Days_t^i} \cdot \sum_{d=1}^{Days_t^i} \frac{ r_{i,d} }{Vol_{i,d}}$, where $Vol_{i,d}$ is security i 's trading volume in dollars on day d and $Days_t^i$ is the number of trading days in year t .	CRSP, EST
idio vola	A stock's idiosyncratic volatility, defined as the standard deviation of the CAPM-residuals of its daily returns.	CRSP, EST
coskew	The co-skewness of a stock's daily returns with the market: $coskew = \frac{E[(r_i - \mu_i)(r_m - \mu_m)^2]}{\sqrt{VAR(r_i)VAR(r_m)}}$	CRSP, EST
cokurt	The co-kurtosis of a stock's daily returns with the market: $cokurt = \frac{E[(r_i - \mu_i)(r_m - \mu_m)^3]}{\sqrt{VAR(r_i)VAR(r_m)^3}^{3/2}}$	CRSP, EST
max	The maximum daily return over the last year or month, respectively.	CRSP

Panel B: Other Firm Characteristics

Variable Name	Description	Source
size	The natural logarithm of a firm's equity market capitalization in million USD.	CS
bookmarket	A firm's book-to-market ratio computed as the ratio of CS book value of equity per share (that is, book value of common equity less liquidation value (CEQL) divided by common share outstanding (CSHO)) to share price (that is, market value of equity per share).	CS

B Theoretical Motivation

This section motivates our empirical approach theoretically. Specifically, we show that copula-based tail dependence coefficients determine discount rates in asset pricing models. We consider a simple theoretical model for illustration in which the representative agent with utility function $u[\cdot]$ maximizes her expected utility under standard regularity conditions. Besides these regularity conditions, we do not make any assumptions about the specific form of the utility function. Thus, our model's results hold for a wide class of possible preferences (e.g. CRRA preferences). We use the stochastic discount factor (SDF) implied from this simple model to show that tail-based co-moment risks determine the risk premium on risky assets. We then show that lower tail (LTD) and upper tail (UTD) are related to tail-based co-moment risks, implying that lower and upper dependence measures determine the risk premium on risky assets.

Regularity conditions We assume $u'[\cdot] > 0$, $u''[\cdot] < 0$, $u'''[\cdot] > 0$, and $u''''[\cdot] < 0$. Arrow (1965) and Pratt (1964) show that the representative investor's utility function exhibits non-satiation ($u'[\cdot] > 0$), risk aversion ($u''[\cdot] < 0$) and a decreasing absolute risk aversion ($u'''[\cdot] > 0$). The restriction on the third derivative of the utility function is related to the concept of prudence in Kimball (1990, 1993). Kimball (1990) shows that the concept of prudence is analogous to the precautionary-saving motive. He shows that absolute prudence must be a decreasing function of the representative investor's wealth. A decreasing absolute prudence and a concave utility restrict the sign of the kurtosis preference to $u''''[\cdot] < 0$. The restriction on the fourth derivative of the utility function is also defined as "temperance" in Eeckhoudt and Schlesinger (2006).

Several papers (Harvey and Siddique (2000), Dittmar (2002), Chabi-Yo (2012), Vanden (2006), among others) have shown that Kimball's concept of prudence plays a key role in determining the price of risk of higher co-moments, such as co-skewness, and higher moments, such as skewness. We will show that the same concept plays a key role in determining the price of risk of tail dependence measures.

A Simple Investor's Problem Consider a one-period $[t, t + 1]$ economy with $t = 0$. In this economy, we assume that there are n risky assets and one risk-free asset. Denote by $R_{f,t}$ the return on the risk-free asset and by R_{t+1} a vector of returns of risky assets. Without loss of generality, the representative investor has an endowed wealth of $W_t = 1$ at time t . She maximizes her expected utility

$$\max_{\omega} E_t(u[W_{t+1}]) \quad (10)$$

subject to the budget constraint

$$W_{t+1} = W_t \left(R_{f,t} + \omega' (R_{t+1} - R_{f,t}) \right), \quad (11)$$

where W_{t+1} is the investor's terminal wealth and ω is a vector of portfolio weights.

Stochastic Discount Factor The Euler equation derived from the first-order conditions of (10) is used to show that the SDFs has the form

$$M_{t+1} = \frac{u' [W_{t+1}^*]}{R_{f,t} E_t (u' [W_{t+1}^*])}, \quad (12)$$

where $W_{t+1}^* = W_t \left(R_{f,t} + \omega^{*'} (R_{t+1} - R_{f,t}) \right)$, and ω^* is the optimal portfolio weight with $\omega^{*'} \mathbf{1} = 1$. Here, $\mathbf{1}$ is an unity vector. Since $W_t = 1$, $W_{t+1}^*/W_t = \omega^{*'} R_{t+1}$ can be interpreted as the return on aggregate wealth. Using the market return $R_{M,t+1}$ as a proxy for the return on aggregate wealth, $R_{M,t+1} = W_{t+1}^*/W_t$.

We denote by S_{t+1} the price of the market index at time $t + 1$ and express the market return as $R_{M,t+1} = S_{t+1}/S_t$. We can, therefore, express the SDF in (12) as

$$M_{t+1} = \frac{1}{R_{f,t}} \frac{u' [S_{t+1}/S_t]}{E_t (u' [S_{t+1}/S_t])}. \quad (13)$$

We now state the following lemma which is from Carr and Madan (2001):

Lemma 1. *Carr and Madan (2001, Eq(1), page 23): Any twice differentiable payoff function with bounded expectation can be spanned by a continuum of OTM European calls and puts. In order words, a collection of twice differentiable functions $H[S]$ can be spanned algebraically as³⁶*

$$H[S] = H[\bar{S}] + (S - \bar{S}) H_S[\bar{S}] + \int_{\bar{S}}^{\infty} H_{SS}[K] (S - K)^+ dK + \int_0^{\bar{S}} H_{SS}[K] (K - S)^+ dK,$$

where $H_S[.]$ ($H_{SS}[.]$) represents the first (second) order derivative of the payoff function $H[.]$ evaluated at \bar{S} .

We exploit Lemma 1 and show in Theorem 1 that the SDF is a linear combination of the market return and a collection of payoff on call and put options. As will be seen shortly, Theorem 1 plays a key role in showing that the tail dependence measures determine the discount rate.

³⁶Bakshi and Madan (2000, Theorem 1, page 212) and Bakshi, Kapadia, and Madan (2003, Theorem 1, page 107) use Lemma 1 to provide economic foundations for valuing derivative securities and study new insights into the economic sources of skewness.

Theorem 1. Assume that the first, second, third, and fourth derivatives of the utility function exist, then Lemma (1) can be used to express the SDF (13) as

$$M_{t+1} = \frac{u' [1]}{u' [a]} \frac{1}{R_{f,t}} + (R_{M,t+1} - 1) \frac{u'' [1]}{u' [a]} \frac{1}{R_{f,t}} + \int_1^{k_{\max}} \frac{u''' [k]}{u' [a]} \frac{1}{R_{f,t}} (R_{M,t+1} - k)^+ dk + \int_0^1 \frac{u''' [k]}{u' [a]} \frac{1}{R_{f,t}} (k - R_{M,t+1})^+ dk, \quad (14)$$

where $k = \frac{K}{S_t}$, k_{\max} is the maximum value of the gross return (whose distribution we assume to have a bounded support), and $a = u'^{-1} [E_t (u' [S_{t+1}/S_t])]$. u'^{-1} is the inverse function of u' .³⁷

Proof. See the Internet Appendix. □

Expected Excess Return Decomposition For the sake of simplicity we drop the time-subscript t and use the the SDF in (14) to express the Euler equation as

$$E [MR_i] = 1. \quad (15)$$

For characterizations to follow, define the expected values

$$\mu_M^u [k] = E [(R_M - k)^+] \quad \text{and} \quad \mu_M^d [k] = E [(k - R_M)^+], \quad (16)$$

and the price of the market risk and the beta of the risky asset as

$$\lambda = -\frac{u'' [1]}{u' [a]} \text{Var} [R_M] \quad \text{and} \quad \beta_i = \frac{\text{Cov}(R_i, R_M)}{\text{Var}[R_M]}. \quad (17)$$

We also define the following tail-based co-moment risks

$$\delta_i^{uu} [k] = \frac{\text{Cov}((R_i - k)^+, (R_M - k)^+)}{\text{Var}((R_M - k)^+)}, \quad \text{and} \quad \delta_i^{ud} [k] = \frac{\text{Cov}((R_i - k)^+, (k - R_M)^+)}{\text{Cov}((k - R_M)^+, (R_M - k)^+)}, \quad (18)$$

$$\delta_i^{du} [k] = \frac{\text{Cov}((k - R_i)^+, (R_M - k)^+)}{\text{Cov}((k - R_M)^+, (R_M - k)^+)}, \quad \text{and} \quad \delta_i^{dd} [k] = \frac{\text{Cov}((k - R_i)^+, (k - R_M)^+)}{\text{Var}((k - R_M)^+)}, \quad (19)$$

and their prices of risks, respectively:

$$\lambda^{uu} [k] = -\frac{u''' [k]}{u' [a]} \text{Var} [(R_M - k)^+], \quad \text{and} \quad \lambda^{ud} [k] = \frac{u''' [k]}{u' [a]} \mu_M^u [k] \mu_M^d [k], \quad (20)$$

$$\lambda^{du} [k] = -\frac{u''' [k]}{u' [a]} \mu_M^u [k] \mu_M^d [k], \quad \text{and} \quad \lambda^{dd} [k] = \frac{u''' [k]}{u' [a]} \text{Var} [(k - R_M)^+]. \quad (21)$$

³⁷We discuss how this theorem relates to recent equilibrium models in which the SDF is a function of nonlinear payoffs such as option payoffs (Vanden (2004)) or in which investor preferences overweight lower-tail outcomes relative to expected utility like the generalized disappointment aversion (GDA) model of Routledge and Zin (2010) in Section C of the Internet Appendix.

From (20) and (21), we observe that $\lambda^{uu}[k] < 0$ and $\lambda^{du}[k] < 0$ while $\lambda^{dd}[k] > 0$ and $\lambda^{ud}[k] > 0$. $\delta_i^{uu}[k]$ and $\delta_i^{dd}[k]$ are upper and lower tail-based co-moment risks. $\delta_i^{du}[k]$ and $\delta_i^{ud}[k]$ are mixed tail-based co-moment risks. Similarly to the market beta, when asset i coincides with the market portfolio, the tail-based co-moment risks defined in (18) and (19) are equal to one. We now expand the Euler equation in terms of covariance and express the expected excess return of the risky asset in terms of tail-based co-moment risks and their market prices in Theorem 2.

Theorem 2. *The expected excess return on any risky asset can be expressed as*

$$\begin{aligned} E[R_i] - R_f &= \lambda\beta_i + \int_1^{k_{\max}} \lambda^{uu}[k] \delta_i^{uu}[k] dk + \int_1^{k_{\max}} \lambda^{du}[k] \delta_i^{du}[k] dk \\ &\quad + \int_0^1 \lambda^{ud}[k] \delta_i^{ud}[k] dk + \int_0^1 \lambda^{dd}[k] \delta_i^{dd}[k] dk, \end{aligned} \quad (22)$$

where $\lambda > 0$, $\lambda^{uu}[\cdot] < 0$, $\lambda^{dd}[\cdot] > 0$, $\lambda^{du}[\cdot] < 0$ and $\lambda^{ud}[\cdot] > 0$.

Proof. See the Internet Appendix. □

Expression (22) shows that beta as well as the tail-based co-moment risks determine the discount rate. The main implication of Theorem 2 is that $\delta_i^{uu}[k]$ and $\delta_i^{du}[k]$ are negatively related to expected excess returns while $\delta_i^{ud}[k]$ and $\delta_i^{dd}[k]$ are positively related to expected excess returns.

The key here is the third derivative of the utility function u , which characterizes investor preferences for skewness. The third derivatives $u'''[k]$ may amplify or reduce the contribution of tail dependence measures on the expected excess return depending on k .³⁸ When our regularity conditions are satisfied, $u'''[k]$ is a decreasing function of k because $u''''[\cdot] < 0$. The magnitude of $u'''[k]$ is extremely large when k is small. In such a case, the tail-based co-moment risks have a significant impact of expected excess returns when k is small. The most interesting tail-based co-moment risks in our context are $\lim_{\varepsilon \rightarrow 0^+} \delta_i^{dd}[\varepsilon]$ and $\lim_{\varepsilon \rightarrow k_{\max}^-} \delta_i^{uu}[\varepsilon]$. We call these risk measures, limiting tail-based co-moment risks.

Impact of the limiting tail-based co-moment risks on expected excess returns Under our regularity conditions, the expected excess return increases disproportionately with $\delta_i^{dd}[\varepsilon]$ in the limiting case $\varepsilon \rightarrow 0^+$. The increase in the expected excess return depends on the shape of $u'''[\varepsilon]$ when ε goes to zero. In other words, the increase is due to the fact that $u''''[\cdot] < 0$.³⁹ Similarly,

³⁸In the special case of quadratic utility, $u'''[k] = 0$ for any k and $\lambda^{uu}[\cdot] = \lambda^{dd}[\cdot] = \lambda^{du}[\cdot] = \lambda^{ud}[\cdot] = 0$. In such a case only the asset's beta determines the expected excess return on risky assets and (22) reduces to the CAPM, i.e. $E[R_i] - R_f = \lambda\beta_i$. Consequently, tail-based co-moment risks contribute to the risk premium on risky assets when the utility is not quadratic.

³⁹It is important to notice that if $u''''[\cdot] = 0$, the expected excess return will not increase disproportionately in the limiting case $\varepsilon \rightarrow 0^+$.

the expected excess return decreases disproportionately with $\delta_i^{uu}[\varepsilon]$ in the limiting case $\varepsilon \rightarrow k_{\max}^-$. The proof is given in the Internet Appendix.

Does Lower Tail Dependence determine discount rates? We now show that the lower (LTD) and upper (UTD) tail dependence measures are positively related to the limiting tail-based co-moment risks. To connect the tail-based co-moment risks to lower and upper tail dependence measures used in Sections 2 through 3, we first recall that

$$\text{LTD}_i = \lim_{q \rightarrow 0^+} P[r_i < F_i^{-1}(q) | r_M < F_M^{-1}(q)], \quad (23)$$

$$\text{UTD}_i = \lim_{q \rightarrow 1^-} P[r_i > F_i^{-1}(q) | r_M > F_M^{-1}(q)]. \quad (24)$$

For characterizations to follow, we define the limiting tail-based co-moment risks $\delta_i^{uu}[\varepsilon]$, $\delta_i^{ud}[\varepsilon]$, $\delta_i^{du}[\varepsilon]$, and $\delta_i^{dd}[\varepsilon]$ as the limit of $\delta_i^\cdot[\varepsilon]$ for ε approaching 1, 0, or k_{\max} , respectively (e.g., $\delta_i^{uu}[1] = \lim_{\varepsilon \rightarrow 1^+} \delta_i^{uu}[\varepsilon]$). The associated prices of risk are defined as $\lambda_i^{uu}[\varepsilon]$, $\lambda_i^{ud}[\varepsilon]$, $\lambda_i^{du}[\varepsilon]$, and $\lambda_i^{dd}[\varepsilon]$, respectively.⁴⁰ We then apply Lemma 2 (see the Internet Appendix) to the integrals in the RHS of (22) and write the expected excess return of the risky asset in (25).

Theorem 3. *A discretization of integrals in the RHS of (22) allows to express the expected excess return as*

$$\begin{aligned} E[R_i] - R_f &= \lambda\beta_i + \frac{1}{2}\lambda^{dd}[0]\delta_i^{dd}[0] + \frac{1}{2}\lambda^{ud}[0]\delta_i^{ud}[0] \\ &+ \frac{1}{2}(k_{\max} - 1)\lambda^{uu}[k_{\max}]\delta_i^{uu}[k_{\max}] + \frac{1}{2}(k_{\max} - 1)\lambda^{du}[k_{\max}]\delta_i^{du}[k_{\max}] \\ &+ \frac{1}{2}(k_{\max} - 1)\lambda^{du}[1]\delta_i^{du}[1] + \frac{1}{2}(k_{\max} - 1)\lambda^{uu}[1]\delta_i^{uu}[1] \\ &+ \frac{1}{2}\lambda^{ud}[1]\delta_i^{ud}[1] + \frac{1}{2}\lambda^{dd}[1]\delta_i^{dd}[1], \end{aligned} \quad (25)$$

where $\lambda > 0$, $\lambda^{uu}[\cdot] < 0$, $\lambda^{dd}[\cdot] > 0$, $\lambda^{du}[\cdot] < 0$ and $\lambda^{ud}[\cdot] > 0$. Further,

- (i) the relevant limiting tail-based co-moment risks $\delta_i^{dd}[0]$ and $\delta_i^{uu}[k_{\max}]$ are linear functions of the lower and upper tail dependence measures LTD_i and UTD_i , respectively,
- (ii) the limiting tail-based co-moment risks $\delta_i^{dd}[0]$ and $\delta_i^{uu}[k_{\max}]$ are increasing functions of the lower and upper, respectively, tail dependence measures:

$$\frac{\partial \delta_i^{dd}[0]}{\partial \text{LTD}_i} > 0, \quad \text{and} \quad \frac{\partial \delta_i^{uu}[k_{\max}]}{\partial \text{UTD}_i} > 0, \quad (26)$$

⁴⁰The detailed definitions for the δ 's and λ 's are shown in the Internet Appendix.

(iii) the tail dependence measures LTD and UTD determine the expected excess return on risky assets, and

$$\frac{\partial(E[R_i]-R_f)}{\partial LTD_i} > 0 \quad \text{and} \quad \frac{\partial(E[R_i]-R_f)}{\partial UTD_i} < 0. \quad (27)$$

Proof. See the Internet Appendix. □

Three key implications emerge from Theorem 3. First, the magnitude of the prices of risks $\lambda^{uu}[\cdot]$, $\lambda^{ud}[\cdot]$, $\lambda^{du}[\cdot]$, and $\lambda^{dd}[\cdot]$ as shown in (20) and (21) are related to the shape of the utility function. Second, (27) shows that LTD (UTD) is positively (negatively) related to the risk premium on risky assets. All else being equal, assets with strong LTD (UTD) have high tail-based co-moment risk and hence have higher (lower) returns on average than assets with weak or zero LTD (UTD). Third, the magnitude of expected excess returns due to LTD is related to $\lim_{\varepsilon \rightarrow 0^+} u'''[\varepsilon]$, while the magnitude of expected excess returns due to UTD is related to $\lim_{\varepsilon \rightarrow k_{\max}^-} u'''[\varepsilon]$. Consequently, the impact of tail dependence on the magnitude of the risk premium depends on the concavity of the second derivative of the utility function $u''[\cdot]$. Since $u''''[\cdot] < 0$, $u'''[\varepsilon]$ is a decreasing function of ε . As ε approaches zero from the right, $u'''[\varepsilon]$ will be large. As ε approaches k_{\max} from the left, $u'''[\varepsilon]$ will be small. Thus, all else being equal, LTD has a stronger impact on the expected excess return than UTD.

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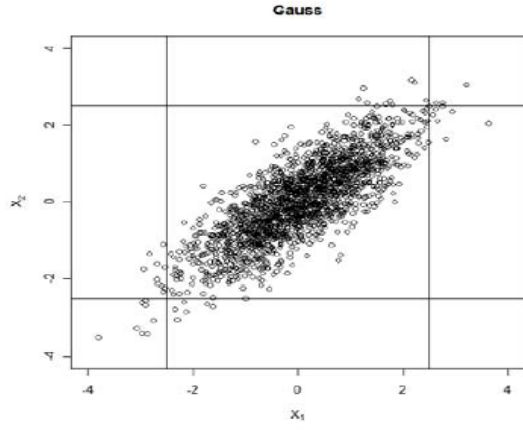
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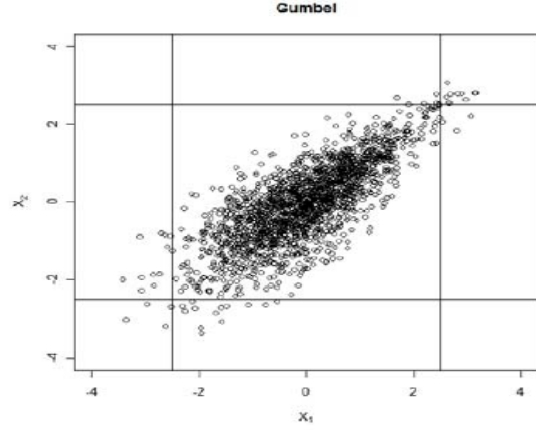
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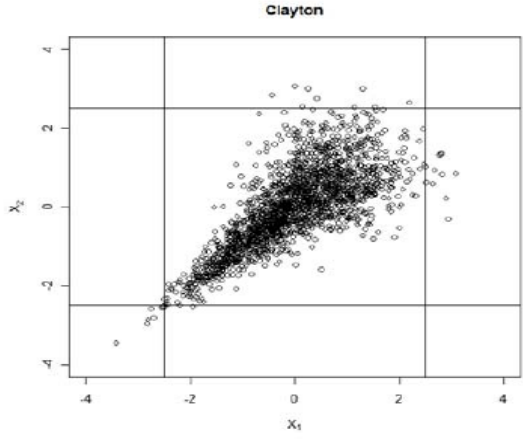
Figure 1: Different Copula Dependence Structures



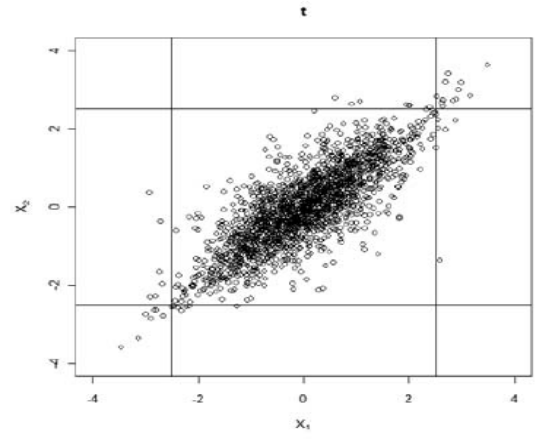
(a) Panel A: Gauss-copula structure



(b) Panel B: Gumbel-copula structure



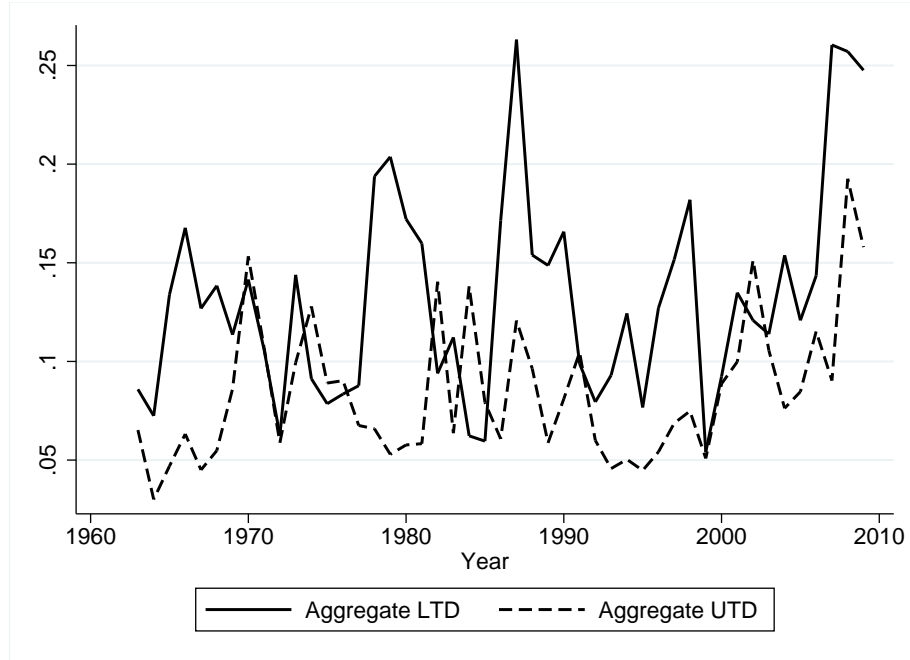
(c) Panel C: Clayton-copula structure



(d) Panel D: Student t-copula structure

This figure displays 2,000 random variates from four bivariate distributions with standard normal marginal distributions. The Gauss-copula (Panel A), the Gumbel-copula (Panel B), the Clayton-copula (Panel C), and the Student t-copula (Panel D) determine the dependence structure. These copulas are defined in Table IA.I in the Internet Appendix. In each case, the linear correlation is set to 0.8.

Figure 2: Aggregate Tail Dependence over Time



This figure displays the evolution of aggregate lower tail dependence, LTD, and aggregate upper tail dependence, UTD, over time. Aggregate LTD (UTD) is defined as the yearly cross-sectional, equal-weighted, average of the individual lower tail dependence coefficients, $LTD_{i,t}$ (upper tail dependence coefficients, $UTD_{i,t}$) between stock returns and market returns over all stocks i in year t in our sample. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012.

Table 1: Summary Statistics

	Mean	25%- Quantile	Median	75%- Quantile	Standard Deviation	Above LTD Median	Below LTD Median	Above – Below
return	0.67%	–7.20%	–0.13%	6.78%	0.214	0.80%	0.57%	0.23%***
LTD	0.100	0.011	0.069	0.158	0.104	0.180	0.019	0.161***
UTD	0.065	0.000	0.036	0.103	0.082	0.075	0.056	0.019***
β	0.737	0.278	0.658	1.120	0.635	0.904	0.556	0.348***
β^-	0.854	0.313	0.785	1.325	0.871	1.141	0.542	0.599***
β^+	0.635	0.081	0.571	1.155	0.951	0.733	0.527	0.206***
size	11.26	9.67	11.08	12.69	2.151	11.66	10.79	0.870***
bookmarket	0.827	0.337	0.625	1.069	0.865	0.756	0.908	–0.152***
illiq	0.175	0.021	0.252	0.300	0.132	0.151	0.206	–0.055***
idio vola	0.541	0.285	0.432	0.666	0.387	0.507	0.577	–0.070***
coskew	–0.092	–0.172	–0.056	0.045	0.294	–0.170	–0.008	–0.162***
cokurt	1.474	0.329	0.913	1.769	2.723	2.118	0.776	1.342***
max	17.41%	8.19%	12.90%	20.98%	0.154	16.94%	17.95%	–1.01%***

This table presents summary statistics for the main variables used in this study (pooled over all stocks and months). The first five columns show the mean, the 25%-quantile, the 50%-quantile (median), the 75%-quantile, and the standard deviation of each variable. The last three columns display mean values of the variables conditional on LTD being above (below) its 50%-quantile as well as the difference with corresponding statistical significance level. We present results for yearly excess stock returns over the 1-month T-Bill rate (return), lower tail dependence (LTD), upper tail dependence (UTD), beta (β), downside beta (β^-), upside beta (β^+), the log of market capitalization (size), book-to-market value (bookmarket), illiquidity (illiq), idiosyncratic volatility (idio vola), coskewness (coskew), cokurtosis (cokurt), and the maximum daily return over the past one year (max). A detailed description of the computation of these variables is given in the main text and in the Appendix. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 2: Correlations

	LTD	UTD	β	β^-	β^+	size	bookmarket	illiq	past return	idio vola	coskew	cokurt	max
LTD	1.00	-	-	-	-	-	-	-	-	-	-	-	-
UTD	0.15	1.00	-	-	-	-	-	-	-	-	-	-	-
β	0.34	0.28	1.00	-	-	-	-	-	-	-	-	-	-
β^-	0.38	0.05	0.72	1.00	-	-	-	-	-	-	-	-	-
β^+	0.16	0.40	0.71	0.37	1.00	-	-	-	-	-	-	-	-
size	0.33	0.30	0.29	0.12	0.24	1.00	-	-	-	-	-	-	-
bookmarket	-0.09	-0.04	-0.12	-0.09	-0.06	-0.32	1.00	-	-	-	-	-	-
illiq	-0.33	-0.29	-0.37	-0.18	-0.29	-0.82	0.24	1.00	-	-	-	-	-
past return	0.08	-0.00	0.15	0.14	0.09	0.11	-0.19	-0.02	1.00	-	-	-	-
idio vola	-0.18	-0.16	-0.03	0.05	-0.07	-0.50	0.07	0.35	-0.10	1.00	-	-	-
coskew	-0.36	0.16	-0.00	-0.18	0.15	-0.05	0.07	0.05	-0.05	0.06	1.00	-	-
cokurt	0.37	0.22	0.23	0.17	0.22	0.22	-0.08	-0.22	0.02	-0.15	-0.77	1.00	-
max	-0.11	-0.13	0.02	0.06	-0.03	-0.39	0.04	0.28	0.10	0.62	0.06	-0.12	1.00

This table presents linear correlations between select variables used in this study: lower tail dependence (LTD), upper tail dependence (UTD), beta (β), downside beta (β^-), upside beta (β^+), the log of market capitalization (size), book-to-market value (bookmarket), illiquidity (illiq), past return, idiosyncratic volatility (idio vola), coskewness (coskew), cokurtosis (cokurt), and the maximum daily return over the past one year (max). A detailed description of the computation of these variables is given in the main text and in the Appendix. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012.

Table 3: Excess Returns of LTD-Portfolios During Financial Crises

Daily Returns — Portfolios Sorted By Past LTD					
Portfolio	“Black Monday”	Asia Crisis	Dot-Com Bubble Burst	Lehman Crisis	Worst 10 days
1 Weak LTD	−14.03%	−5.50%	−3.82%	−7.93%	−6.35%
2	−17.03%	−6.25%	−4.77%	−8.75%	−6.55%
3	−17.34%	−6.69%	−6.83%	−8.14%	−6.89%
4	−16.79%	−6.86%	−6.23%	−8.51%	−7.82%
5 Strong LTD	−19.15%	−7.48%	−7.73%	−10.80%	−9.25%
Strong − Weak	−5.12%	−1.98%	−3.89%	−2.86%	−2.90%** (−2.45)

This table reports value-weighted daily excess returns of stocks sorted by past LTD during days of severe financial crises. Each month we rank stocks based on LTD estimated over the previous twelve months. We investigate future value-weighted returns of these portfolios during “Black Monday” (October 19, 1987), the Asian Crisis (October 27, 1997), the burst of the dot-com bubble (April 14, 2000), and the recent Lehman crisis (October 15, 2008). The last column reports average daily returns in excess of the 1-month T-bill rate of the portfolios on the ten worst return days in our sample. The row labeled “Strong − Weak” reports the difference between the returns of portfolio 5 and portfolio 1 with corresponding statistical significance levels (only last column). The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 4: Univariate Value-Weighted Portfolio Sorts**Panel A: Lower Tail Dependence (LTD)**

Portfolio	LTD	Return	CAPM-Alpha	FF-Alpha	CAR-Alpha
1 Weak LTD	0.00	0.296%	−0.129%*	−0.250%***	−0.108%
2	0.03	0.315%	−0.141%*	−0.157%**	−0.074%
3	0.08	0.380%	−0.075%*	−0.123%*	−0.067%
4	0.15	0.508%	+0.045%	+0.025%	+0.038%
5 Strong LTD	0.27	0.656%	+0.172%**	+0.187%**	+0.129%
Strong – Weak	0.27***	0.360%*** (3.68)	+0.302%*** (3.15)	+0.437%*** (4.72)	+0.237%** (2.35)

Panel B: Upper Tail Dependence (UTD)

Portfolio	UTD	Return	CAPM-Alpha	FF-Alpha	CAR-Alpha
1 Weak UTD	0.00	0.561%	+0.054%	+0.032%	+0.023%
2	0.01	0.526%	+0.055%	+0.019%	+0.007%
3	0.04	0.540%	+0.074%	+0.041%	+0.021%
4	0.09	0.488%	+0.070%	+0.032%	−0.010%
5 Strong UTD	0.23	0.427%	−0.037%	−0.040%	+0.029%
Strong – Weak	0.23***	−0.134% (−1.26)	−0.091% (−0.99)	−0.072% (−0.76)	0.006% (0.09)

This table reports results from univariate portfolio sorts based on LTD (Panel A) and UTD (Panel B). In each month, we rank stocks into quintiles (1-5) and form value-weighted portfolios based on the respective tail dependence measure. The column labeled “Return” reports the future average monthly return in excess of the one-month T-bill rate of the portfolios. The column labeled “CAPM-Alpha” (“FF-Alpha”, “CAR-Alpha”) reports the future average monthly alpha with regard to Sharpe (1964)’s capital asset pricing model (Fama and French (1993)’s three-factor model, Carhart (1997)’s four-factor model). The row labeled “Strong – Weak” reports the difference between the returns of portfolio 5 and portfolio 1, with corresponding statistical significance levels. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 5: Trading Strategy – Alternative Factor Models

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	trad strat	trad strat	trad strat	trad strat	trad strat	trad strat	trad strat	trad strat
marketrf	0.127*** (6.09)	0.108*** (3.75)	0.110*** (4.89)	0.134*** (6.31)	0.137*** (6.81)	0.094*** (4.16)	0.058** (2.51)	0.085*** (3.47)
smb	-0.092*** (-3.13)	-0.196*** (-4.86)	-0.126*** (-3.66)	-0.066** (-2.27)	-0.066** (-2.39)	-0.026 (-0.77)	-0.088*** (-2.67)	-0.060 (-1.65)
hml	-0.203*** (-6.33)	-0.289*** (-6.62)	-0.128*** (-3.69)	-0.185*** (-5.75)	-0.131*** (-3.85)	-0.208*** (-5.39)	-0.141*** (-3.14)	
mom	0.214*** (10.46)	0.239*** (9.12)	0.240*** (11.57)	0.221*** (10.75)	0.242*** (11.89)			
ps liqui	0.043* (1.71)							
sadka tf		-0.722 (-1.25)						
lot max vw			0.097*** (3.71)					
sent orth				-0.003*** (-3.19)				
bab					-0.128*** (-4.45)			
rev short						-0.023 (-0.77)		
rev long						-0.119*** (-2.76)		
rmw							-0.112** (-2.33)	
cma							-0.277*** (-4.05)	
invest								-0.440*** (-7.44)
roe								0.135*** (3.20)
alpha	0.0022** (2.35)	0.0026** (2.06)	0.0027*** (2.98)	0.0024** (2.56)	0.0030*** (3.39)	0.0045*** (4.88)	0.0053*** (5.58)	0.0046*** (4.11)
R^2	0.303	0.361	0.316	0.312	0.317	0.158	0.172	0.178

This table shows regression results of the (5-1) difference portfolio returns (consisting going long stocks with strong LTD and going short stocks with weak LTD) on various combinations of systematic risk factors from various asset pricing models. Regression (1) includes factors of the Carhart (1997) model (smb, hml, mom) plus the Pastor and Stambaugh (2003) traded liquidity risk factor (ps liqui). In regression (2), the Pastor and Stambaugh (2003) liquidity factor is replaced by the Sadka (2006) liquidity factor (sadka tf). In regressions (3) to (5) we include the Bali, Cakici, and Whitelaw (2011) lottery factor (lot max vw), the Baker and Wurgler (2006) sentiment index, orthogonalized with respect to a set of macroeconomic conditions (sent orth), and the Frazzini and Pedersen (2013) betting-against-beta factor (bab), respectively. In regression (6), we replace the Carhart (1997) momentum factor by the Fama-French short- and long-term reversal factors (rev short and rev long). Finally, in regressions (7) and (8) we control for exposures to the Fama and French (2014) 5-factor model and the Hou, Xue, and Zhang (2015) 4-factor model. The alpha of the strategies is shown in the second-to-last row. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 6: Dependent Bivariate Portfolio Sorts**Panel A: Beta (β) and Lower Tail Dependence (LTD)**

Portfolio	1 Low β	2	3	4	5 High β	Average
1 Weak LTD	0.333%	0.342%	0.419%	0.105%	-0.024%	0.235%
2	0.356%	0.310%	0.431%	0.418%	0.041%	0.311%
3	0.512%	0.289%	0.467%	0.341%	0.219%	0.366%
4	0.410%	0.461%	0.489%	0.431%	0.451%	0.446%
5 Strong LTD	0.679%	0.557%	0.711%	0.634%	0.612%	0.639%
Strong – Weak	0.346%*** (3.31)	0.215%** (2.25)	0.291%*** (3.15)	0.529%*** (4.54)	0.637%*** (4.52)	0.404%*** (3.62)

Panel B: Downside Beta (β^-) and Lower Tail Dependence (LTD)

Portfolio	1 Low β^-	2	3	4	5 High β^-	Average
1 Weak LTD	0.312%	0.253%	0.338%	0.132%	-0.017%	0.204%
2	0.291%	0.400%	0.510%	0.345%	0.210%	0.351%
3	0.357%	0.398%	0.451%	0.410%	0.410%	0.405%
4	0.463%	0.524%	0.461%	0.567%	0.601%	0.523%
5 Strong LTD	0.520%	0.678%	0.630%	0.730%	0.537%	0.619%
Strong – Weak	0.208%* (1.82)	0.426%** (4.09)	0.291%** (2.52)	0.598%*** (4.57)	0.554%*** (3.58)	0.415%*** (3.31)

Panel C: Coskewness (coskew) and Lower Tail Dependence (LTD)

Portfolio	1 Low coskew	2	3	4	5 High coskew	Average
1 Weak LTD	0.421%	0.489%	0.535%	0.229%	0.184%	0.372%
2	0.561%	0.310%	0.562%	0.236%	0.104%	0.355%
3	0.510%	0.451%	0.530%	0.468%	0.247%	0.441%
4	0.612%	0.561%	0.618%	0.532%	0.341%	0.533%
5 Strong LTD	0.579%	0.730%	0.772%	0.615%	0.429%	0.625%
Strong – Weak	0.159% (1.17)	0.241%* (1.86)	0.236%* (1.79)	0.386%*** (2.80)	0.245%* (1.85)	0.253%* (1.89)

Table 6: Continued

Panel D: Cokurtosis (cokurt) and Lower Tail Dependence (LTD)

Portfolio	1 Low cokurt	2	3	4	5 High cokurt	Average
1 Weak LTD	0.244%	0.249%	0.341%	0.282%	0.262%	0.276%
2	0.412%	0.453%	0.610%	0.230%	0.120%	0.365%
3	0.567%	0.561%	0.450%	0.451%	0.356%	0.477%
4	0.619%	0.600%	0.510%	0.613%	0.561%	0.581%
5 Strong LTD	0.838%	0.838%	0.801%	0.821%	0.538%	0.767%
Strong – Weak	0.594%*** (4.36)	0.589%** (4.54)	0.460%*** (3.67)	0.539%** (4.88)	0.276%*** (2.61)	0.492%*** (4.02)

This table reports value-weighted future average returns and risk characteristics of 25 portfolios double-sorted on LTD and beta (Panel A), downside beta (Panel B), coskewness (Panel C), and cokurtosis (Panel D), respectively. First, we form quintile portfolios based on 1-year beta, 1-year downside beta, 1-year coskewness, and 1-year cokurtosis, respectively. Then, within each quintile, we sort stocks into five quintile portfolios based on their 1-year LTD. The last column shows the average of the future excess returns of the respective LTD quintile portfolios across all beta, downside beta, coskewness, and cokurtosis quintiles, respectively. The row labeled “Strong – Weak” reports the difference between the future returns of portfolio 5 and portfolio 1 in each beta, downside beta, coskewness, and cokurtosis quintile, respectively, with corresponding statistical significance levels. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 7: Multivariate Regression Results

	(1) return _{t+1}	(2) return _{t+1}	(3) return _{t+1}	(4) return _{t+1}	(5) return _{t+1}	(6) return _{t+2}	(7) return _{t+3}	annualized economic significance based on (5)
LTD	0.0123*** (2.63)	0.0127*** (2.78)	0.0170*** (5.15)	0.0159*** (4.40)	0.0146*** (4.48)	0.0188*** (3.53)	0.0171*** (3.44)	+2.64%
UTD		-0.00815** (-2.14)	0.00291 (1.26)	0.00323 (1.44)	0.00311 (1.27)	0.00504 (1.46)	0.00604 (1.38)	+0.31%
β^-					-0.00255 (-1.35)	-0.00369 (-1.58)	-0.00475 (-1.40)	-1.63%
β^+					0.000239 (0.31)	0.000765 (0.69)	0.000361 (0.27)	+0.27%
β			-0.000373 (-0.21)	-0.00250 (-1.24)				
size			-0.000190 (-0.42)	-0.000454* (-1.84)	-0.000514* (-1.78)	-0.000718* (-1.79)	-0.00107* (-1.92)	-1.31%
bookmarket			0.00987*** (4.90)	0.0105*** (5.47)	0.0103*** (5.48)	0.0203*** (5.69)	0.0293*** (6.22)	+6.24%
coskew			0.0000173 (0.01)	0.00353 (1.33)	-0.00551* (-1.89)	-0.0141*** (-3.43)	-0.0210*** (-4.24)	-2.10%
illiq			0.000567*** (5.03)	0.000506*** (4.36)	0.000538*** (5.18)	0.00104*** (5.55)	0.00145*** (6.21)	+4.62%
past return				0.00745*** (6.07)	0.00752*** (6.06)	0.0149*** (6.92)	0.0212*** (7.07)	+6.43%
idio vola				0.0136 (0.21)	0.0104 (0.15)	-0.00764 (-0.07)	0.00426 (0.03)	0.28%
cokurt				0.00184*** (3.04)	0.00147*** (2.17)	0.00302** (2.95)	0.00463** (3.46)	2.52%
max				-0.0105*** (-2.70)	-0.0103*** (-2.63)	-0.0230*** (-4.21)	-0.0368*** (-5.11)	-1.92%
constant	0.00574*** (2.53)	0.00625*** (2.72)	-0.00192 (-0.30)	-0.000350 (-0.09)	-0.000265 (-0.01)	-0.000629 (-0.11)	0.000996 (0.13)	
R^2	0.004	0.006	0.054	0.071	0.070	0.082	0.088	

This table presents the results of multivariate Fama-MacBeth (1973) regressions of future excess returns over the risk-free rate in month $t+1$ (columns (1) to (5)), month $t+2$ (column (6)), and month $t+3$ (column (7)) on LTD, UTD, downside beta (β^-), upside beta (β^+), beta (β), the log of market capitalization (size), the book-to-market ratio (bookmarket), coskewness (coskew), the Amihud Illiquidity Ratio (illiq), the past 12-month excess returns (past return), idiosyncratic volatility (idio vola), cokurtosis (cokurt), and the maximum daily return over the past one year (max). All risk characteristics (LTD, UTD, β^- , β^+ , β , coskew, idio vola, cokurt) as well as size, bookmarket, and illiq are calculated based on data until month t . The last column displays the change in annualized excess returns for a one standard deviation increase in the respective independent variable based on regression (9). The independent variables are winsorized at the 1% level and at the 99% level. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 8: Time-Varying Crash Fears of Investors

	Post – Market Crash	Remaining Years
LTD	0.0246*** (3.83)	0.0136*** (4.72)
UTD	0.00555 (1.66)	0.00286 (1.27)
β^-	-0.00275 (-1.44)	-0.00218 (-1.35)
β^+	0.000630 (0.35)	0.0000963 (0.12)
size	-0.00104** (-2.03)	-0.000321 (-0.93)
bookmarket	0.0113*** (5.37)	0.0103*** (5.91)
coskew	-0.00722 (-1.22)	-0.00489 (-1.46)
illiq	0.0000283 (0.15)	0.00074 (6.01)
past return	0.00214 (1.50)	0.00784*** (6.74)
idio vola	0.203** (2.32)	-0.0959 (-1.18)
cokurt	0.00164 (1.40)	0.00141* (1.72)
max	-0.0102* (-1.71)	-0.0105** (-2.12)
constant	0.00388 (0.55)	-0.00145 (-0.32)
R^2	0.053	0.076

This table presents the results of multivariate Fama-MacBeth (1973) regressions of monthly future excess returns over the risk-free rate on LTD and other control variables as in regression (5) of Table 7. We provide results for two subsamples: the “Post - Market Crash” subsample containing the five subsequent years after an extreme market downturn has occurred and the “Remaining Years” subsample. We define “extreme market downturns” as the ten worst market return days in our sample. Such extreme market downturns occurred in 1987, 1997, 1998, 2000, 2008, and 2011. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 9: Dependent Portfolio Sorts: Downside Beta Variants vs. LTD

Downside Beta	Cutoff	LTD Portfolio	1 Low β^-	2	3	4	5 High β^-	Average
β^-	$r_m < \mu_m$	S - W	0.21% (1.82)	0.43% (4.09)	0.29% (2.52)	0.59% (4.57)	0.55% (3.58)	0.42% (3.31)
β^-	10% Quantile	S - W	0.47% (2.94)	0.17% (1.36)	0.34% (2.92)	0.55% (4.26)	0.48% (2.77)	0.40% (2.85)
β^-	5% Quantile	S - W	0.45% (2.78)	0.12% (1.02)	0.38% (2.82)	0.60% (4.32)	0.51% (3.56)	0.41% (2.93)
β^-	2% Quantile	S - W	0.49% (3.02)	0.31% (1.59)	0.39% (3.22)	0.51% (3.95)	0.55% (3.31)	0.45% (3.04)
β^-	1% Quantile	S - W	0.51% (3.23)	0.25% (1.40)	0.43% (3.76)	0.63% (4.98)	0.52% (3.57)	0.47% (3.57)
β_{HW}^-	$r_m < \mu_m$	S - W	0.40% (3.41)	0.25% (2.10)	0.39% (3.98)	0.45% (3.56)	0.65% (4.16)	0.43% (3.50)
β_{HW}^-	10% Quantile	S - W	0.34% (2.95)	0.28% (2.18)	0.48% (4.20)	0.49% (3.87)	0.65% (4.04)	0.45% (3.55)
β_{EST}^-	$r_m < \mu_m$	S - W	0.28% (2.91)	0.40% (3.57)	0.35% (2.36)	0.61% (3.77)	1.07% (6.11)	0.54% (3.82)
β_{EST}^-	10% Quantile	S - W	0.25% (2.29)	0.33% (2.66)	0.72% (4.96)	0.40% (2.35)	1.08% (6.60)	0.55% (3.71)
β_{AR}^-	$r_m < \mu_m$	S - W	0.30% (2.87)	0.28% (2.80)	0.35% (3.50)	0.51% (4.26)	0.59% (4.07)	0.41% (3.26)
β_{AR}^-	10% Quantile	S - W	0.24% (2.08)	0.37% (3.58)	0.47% (4.10)	0.49% (3.90)	0.55% (3.57)	0.42% (3.51)

This table reports results of value-weighted portfolios double-sorted on LTD and different alternative definitions of downside betas. First, we form quintile portfolios sorted on downside beta and then, within each of those quintiles, we sort stocks into quintile portfolios based on LTD. As alternative downside beta definitions, we calculate downside betas as betas conditional on the market return being below its 10%, 5%, 2%, and 1% quantile. Furthermore, we report results from double-sorts based on the Hogan and Warren (1974) downside beta (β_{HW}^-), the Estrada (2004) downside beta (β_{EST}^-), and the asymmetric response beta (β_{AR}^-) of Harlow and Rao (1989). We only report results on the future monthly returns of the strong minus weak LTD portfolios within each downside beta quintile in the first five columns, as well as the average of this future difference portfolio return across all downside beta quintiles in the last column (as in the last row of Panel B in Table 6) for all alternative downside beta definitions. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 10: Alternative Tail Dependence Estimation Procedures

Estimation Procedure	LTD (t-stat)	R^2	Econ Sign
(1-A-III)	0.0137*** (4.10)	0.070	+2.43%
(1-A-IV)	0.0123*** (3.02)	0.070	+2.21%
(4-A-IV)	0.0131*** (3.65)	0.070	+2.28%
(2-B-II)	0.0094* (1.79)	0.069	+1.86%
(2-D-II)	0.0104** (2.51)	0.070	+2.00%
(2-B-I)	0.0101** (2.21)	0.070	+1.96%
(2-Cop)	0.0092* (1.82)	0.069	+1.83%
(MLE)	0.0145*** (4.49)	0.070	+2.64%
(24 months)	0.0121*** (3.01)	0.069	+2.19%
(36 months)	0.0109** (2.37)	0.069	+2.04%

This table shows results for the LTD-estimate from Fama-MacBeth (1973) regressions of monthly future excess returns over the risk-free rate on LTD and the full set of controls as in regression (5) from Table 7 (included in the regression but coefficient estimates suppressed in the table) in the first three columns. LTD coefficients are calculated based on the Clayton/Gauss/Galambos (1-A-III) copula, the Clayton/Gauss/Rotated Clayton (1-A-IV) copula, the Rotated Galambos/Gauss/Rotated Clayton (4-A-IV) copula, the Rotated Gumbel/Frank/Gumbel (2-B-II) copula, the Rotated Gumbel/FGM/Gumbel (2-D-II) copula, and the Rotated Gumbel/Frank/Joe (2-B-I) copula (first six rows). In the last four rows, we present results where we estimate LTD based on a convex combination of two copulas (2-Cop), when we use estimated log-likelihood values instead of Integrated Anderson-Darling distances when selecting the best copula combination (MLE), and when we estimate LTD based on a rolling window of 24 months and 36 months, respectively. The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 11: Contemporaneous Results**Panel A: Univariate Value-Weighted Portfolio Sorts**

Portfolio	LTD	Return	CAPM-Alpha	FF-Alpha	CAR-Alpha
1 Weak LTD	0.00	-0.85%	-5.70%***	-8.34%***	-6.92%***
2	0.03	+2.22%	-3.21%**	-4.05%***	-4.86%***
3	0.08	+3.30%	-2.15%*	-2.45%*	-2.51%*
4	0.15	+5.53%	-0.09%	-0.68%	-0.65%
5 Strong LTD	0.27	+9.95%	+3.59%**	+4.35%***	+3.32%**
Strong – Weak	0.27***	+10.80%*** (5.95)	+9.29%*** (5.13)	+12.69%*** (7.30)	+10.25%*** (4.81)

Panel B: Dependent Bivariate Portfolio Sorts**Beta (β) and LTD**

Portfolio	1 Low β	2	3	4	5 High β	Average
1 Weak LTD	-0.95%	2.15%	2.84%	0.04%	-0.39%	0.74%
5 Strong LTD	5.58%	7.06%	7.86%	9.75%	14.77%	9.00%
Strong – Weak	6.54%*** (4.33)	4.90%*** (3.89)	5.03%*** (3.83)	9.72%*** (6.52)	15.16%*** (7.00)	8.27%*** (5.01)

Downside Beta (β^-) and LTD

Portfolio	1 Low β^-	2	3	4	5 High β^-	Average
1 Weak LTD	1.12%	1.41%	1.34%	0.82%	3.19%	1.58%
5 Strong LTD	6.42%	5.92%	7.93%	10.14%	16.23%	9.33%
Strong – Weak	5.30%** (2.38)	4.51%*** (3.08)	6.59%*** (4.97)	9.32%*** (6.10)	13.04%*** (5.12)	7.75%*** (4.42)

Coskewness (coskew) and LTD

Portfolio	1 Low coskew	2	3	4	5 High coskew	Average
1 Weak LTD	2.86%	0.77%	0.24%	-2.22%	-0.85%	0.16%
5 Strong LTD	13.12%	10.46%	8.05%	6.70%	3.96%	8.46%
Strong – Weak	10.26%*** (4.42)	9.70%*** (4.08)	7.81%*** (4.21)	8.92%*** (4.73)	4.81%** (2.44)	8.30%*** (3.99)

Cokurtosis (cokurt) and LTD

Portfolio	1 Low cokurt	2	3	4	5 High cokurt	Average
1 Weak LTD	-2.90%	-2.79%	-3.15%	-1.91%	4.19%	-1.31%
5 Strong LTD	1.75%	3.18%	5.53%	8.84%	12.27%	6.31%
Strong – Weak	4.65%** (2.12)	5.97%** (2.53)	8.68%*** (5.60)	10.75%*** (5.51)	8.07%*** (5.21)	7.62%*** (4.16)

Table 11: continued

Panel C: Multivariate Regression Results

	(1) return	(2) return	(3) return	(4) return	(5) return	Economic Sign. based on (5)
LTD	0.692*** (6.18)	0.712*** (6.40)	0.650*** (9.05)	0.517*** (8.25)	0.540*** (7.96)	+5.78%
UTD		-0.266*** (-3.20)	-0.218*** (-3.25)	-0.325*** (-7.53)	-0.356*** (-7.41)	-2.81%
β^-					0.024 (1.18)	+1.89%
β^+					0.011 (0.91)	+1.12%
β			0.107*** (2.99)	0.118*** (3.74)		
size			-0.017* (-1.77)	-0.046*** (-7.43)	-0.047*** (-7.39)	-6.82%
bookmarket			0.052*** (5.06)	0.034*** (3.49)	0.032*** (3.20)	+3.26%
coskew			0.071 (1.17)	0.062 (1.25)	0.149** (2.26)	+3.20%
illiq			0.309*** (4.98)	0.267*** (3.67)	0.243*** (3.55)	+3.66%
past return				-0.032 (-1.65)	-0.028 (-1.44)	-1.98%
idio vola				-0.192* (-1.98)	-0.145 (-1.52)	-4.22%
cokurt				0.063*** (4.34)	0.117*** (6.08)	+4.85%
max				0.029 (0.33)	0.038 (0.46)	+0.72%
constant	0.049 (1.33)	0.065 (1.67)	0.065 (0.49)	0.477*** (5.55)	0.481*** (5.64)	
R^2	0.014	0.017	0.099	0.153	0.149	

This table reports results of the contemporaneous empirical analysis where we relate realized tail dependence coefficients to portfolio and individual security returns over the same period. Panel A reports results from univariate portfolio sorts based on realized LTD. In each year, we rank stocks into quintiles (1-5) and form value-weighted portfolios at the beginning of each annual period. The column labeled “Return” reports the average annual return in excess of the one-month T-bill rate of the portfolios. The column labeled “CAPM-Alpha” (“FF-Alpha”, “CAR-Alpha”) reports the yearly alpha with regard to Sharpe (1964)’s capital asset pricing model (Fama and French (1993)’s three-factor model, Carhart (1997)’s four-factor model). The row labeled “Strong – Weak” reports the difference between the returns of portfolio 5 and portfolio 1, with corresponding statistical significance levels. Panel B reports value-weighted average annual excess returns over the one-month T-Bill rate of portfolios double-sorted on realized LTD and realized beta, realized downside beta, realized coskewness, and realized cokurtosis, respectively. The row labeled “Strong – Weak” reports the difference between the returns of portfolio 5 and portfolio 1 in each beta, downside beta, coskewness, or cokurtosis quintile with corresponding statistical significance levels. Due to space limitation we only show the results for portfolio 5 and portfolio 1. Finally, Panel C presents the results of multivariate Fama-MacBeth (1973) regressions of yearly stock-level excess returns over the risk-free rate on LTD, UTD, downside beta (β^-), upside beta (β^+), beta (β), the log of market capitalization (size), the book-to-market ratio (bookmarket), coskewness (coskew), the Amihud Illiquidity Ratio (illiq), the past 12-month excess returns (past return), idiosyncratic volatility (idio vola), cokurtosis (cokurt), and the maximum daily return over the past one year (max). All risk characteristics (LTD, UTD, β^- , β^+ , β , coskew, idio vola, cokurt) are calculated contemporaneously. Size, bookmarket, and illiq for year t are calculated using data from (the end of) year $t - 1$. The last column displays the change in annualized excess returns for a one standard deviation increase in the respective independent variable based on regression (5). The sample covers all U.S. common stocks traded on the NYSE/AMEX/NASDAQ, and the sample period is from January 1963 to December 2012. T-statistics are in parentheses and are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.