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WORKING PAPER ON FINANCE NO. 2015/12

SWISS INSTITUTE OF BANKING AND FINANCE (S/BF – HSG)

THIS DRAFT: JUNE 2015



Electricity Market Coupling and the Pricing of Transmission Rights: An Option-based Approach

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This version: June 2015

Abstract

In liberalized electricity wholesale markets, transmission rights valuation and market design have traditionally been closely intertwined. In Europe, where transmission rights are mainly related to cross-border transactions between adjacent markets, the recent roll-out of market coupling mechanisms has even further strengthened this mutual dependence: due to the resulting improved price convergence across coupled markets, spread dynamics have become more intricate and complex to model, which cannot generally be achieved with classic reduced-form approaches commonly used for transmission rights valuation. In this paper, we instead propose a fundamental model for electricity prices in two coupled markets, which adequately reflects the current institutional framework for cross-border trade in Europe. Based on this setting, we analyze in detail how transmission rights can be valued as spread options on the spot prices derived from this framework, and illustrate how related pricing implications compare against the standard Margrabe benchmark.

JEL classification: G12, G13, Q4, Q41

Keywords: Transmission Rights Valuation; Fundamental Model; Multi-Market Modeling; Derivatives Pricing; Energy Market Coupling.

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I Introduction

In network industries, such as wholesale electricity or natural gas, transactions between adjacent market areas are critically dependent on the institutional framework that coordinates access to the (oftentimes congested) interconnection lines linking the respective networks. The design of this mechanism, in turn, not only must adapt to the specificities of the traded commodity – such as the non-storability of electricity preventing real-time scheduling of flows – but also adequately capture the functional context of why said flow goods shall be exchanged across market borders: be it of limited scope in the case of markets relying on the principle of self-sufficiency, or be it vital for those that primarily serve as a transit hub linking their neighboring markets.

With respect to cross-border trade of *electricity*, its overall economic relevance has gradually changed over time – especially for European markets. Starting in early pre-liberalization times as an “emergency back-up” to help balance unexpected (net) supply shortages between the then primarily isolated European electricity markets, cross-border transactions have since become a crucial instrument to realize the idea of a single pan-European electricity market, as promoted by the European Union (EU). In this context, as one of the milestones for fostering more competitive and liquid cross-border trading activity, the EU electricity market directives expressly stipulate third-party access to interconnectors according to a “*non-discriminatory market-based*” allocation mechanism for cross-border capacities.¹

Amongst a variety of such allocation mechanisms that comply with this requirement, explicit ex-ante auctions of physical transmission rights (PTRs) have turned out to become the most widely implemented congestion management method,² which is primarily due to the fact that they only require a low level of institutional standardization between the respective interconnected markets. Hence, rather than embarking on the arduous task of harmonizing the design of European electricity markets, explicit ex-ante auctions help to avoid this problem by instead separating the markets for cross-border transmission

¹See, e.g., Regulations (EC) No. 714/2009 (especially Article 12(2)) and its predecessor, No. 1228/2003.

²Given that in Europe, most interconnectors at that time used to be (and still are) congested, we will use the terms “capacity allocation mechanism” and “congestion management method” interchangeably.

capacity and electricity. Thus, the auction process for PTRs and/or the nomination of corresponding physical cross-border flows are scheduled prior to the clearing of electricity spot markets, so that in general, interconnector regulation does not directly interfere with their individual operational timescales.

However, as has extensively been analyzed in the literature,³ explicit ex-ante mechanisms are generally prone to producing economically inefficient interconnector flows, given that the above timing disconnect forces cross-border traders to submit bids in the capacity market based on the *expected* rather than realized price differential between the respective two markets. As an alternative, implicit auction mechanisms, such as market coupling, integrate the auction of cross-border capacity into the spot auctions of electricity, thus always yielding economically efficient interconnector flows from the cheaper to the more expensive market. Ultimately, perfect price convergence across all markets involved in the coupling mechanism can be reached when the cross-border flows required to equalize prices are operationally feasible and not constraint by insufficient transmission capacities on the interconnection lines.⁴ Since generic transmission contracts, in their broadest sense, can generally be interpreted as a derivative written on the spread between electricity prices in the two interconnected markets, their value must hence be even more sensitive to a change in the respective “regime” for cross-border trade. Clearly, for a transmission *right*, the value of the corresponding underlying spread *option* must be different under a market coupling mechanism (targeting perfect price convergence between markets) as compared to under explicit ex-ante auctioning and scheduling (causing inefficient flows that help sustain price differentials).

In extant literature, however, these institutional details are only rarely taken into account when it comes to the pricing of transmission rights or obligations. Generally,

³See, e.g., Ehrenmann and Smeers (2005), Turvey (2006), Zachmann (2008), or Bunn and Zachmann (2010).

⁴Note that although prevalent in Europe, market coupling mechanisms are currently planned to also be implemented, e.g., in the US electricity markets where – similar to the European situation – policymakers are trying to foster market integration and eliminate “seams inefficiencies” between adjacent networks. As an example, the Tres Amigas Superstation project has been set up in order to unite the three major regional US electricity grids (the Eastern Interconnection, the Western Electricity Coordinating Council (WECC), and the Electric Reliability Council of Texas (ERCOT)); for the management of cross-border flows across the interconnectors at this trading hub, a market coupling mechanism is currently considered. See EPEX-Spot (2012).

most studies focus on the pricing of PTRs in the context of an explicit ex-ante auction regime,⁵ and propose to value the rights as European options on the spread between spot prices in the respective two electricity markets. Important institutional details that characterize the explicit ex-ante auction regime, such as the timing disconnect between the markets for transmission capacity and spot electricity, are left out in most of these analyses, however. For instance, Wobben et al. (2012) model PTRs as options on the spot spread, yet admit that “*in fact these contracts are options on the expected spot prices, because nomination takes place 4h before day-ahead market clearing.*” As such, by assuming that option exercise and determination of its payoff occur simultaneously, most empirical research on the pricing of PTRs – even if only to simplify matters – has thus relied on a framework that, strictly speaking, applies to implicitly auctioned transmission rights rather than to explicit auctions: as will be shown in this paper, it is only for the former type of auction mechanism that the option payoff (i.e., the spot spread) can be considered realized upon option exercise.

Janssen et al. (2011) follow this line of reasoning and compare the value of PTRs under the two regimes by reducing the problem to an analysis of different times of option exercise. More precisely, given that in the case of explicitly auctioned PTRs, option exercise virtually coincides with the ex-ante scheduling of flows, they propose to value such contract as an option on the *expected* spot price spread between the respective two markets, whereas under market coupling, the transmission right shall be valued as an option that is directly written on the same, yet *realized* spread. However, modeling the spot price spread as a univariate asset fitted to historical data, they do not consider that the spread dynamics under a market coupling regime are essentially different as compared to an explicit ex-ante regime, given that the mechanics of the coupling mechanism will lead to increased spot price convergence between the two markets.⁶ Therefore, when comparing the two different types of contracts, it is obvious that an option directly written on the spot spread itself will be worth more than if it was written on an expectation of the spread at maturity and had to be exercised upon nomination – as long as both spreads (erroneously)

⁵The pricing of explicitly auctioned PTRs is examined by, e.g., Marckhoff (2009), Marckhoff and Muck (2009), Bunn and Martoccia (2010), Wobben et al. (2012), and McInerney and Bunn (2013).

⁶See Füss et al. (2015b) for further information.

obey the same dynamics. Nevertheless, the fact that under market coupling, a zero price spread will be obtained more frequently (thus, by contrast, driving down the value of the option on the realized spread) has thus far not yet been considered in literature – neither theoretically, nor empirically.

Hence, it is not only the aspect of market design that makes transmission right valuation under market coupling a challenging task; finding the best way to adequately capture the intricate dependence structure of the price spread on (i) the most important power price determinants in each of the coupled markets, but also on (ii) other variables such as the interconnector capacity, even further increases the complexity of the required modeling set-up – and clearly questions the widespread use of classic reduced-form approaches in this context: a frequently mentioned example relates to the high share of renewables in the German generation mix. For instance, during times of significant wind energy feed-in, this not only lowers German day-ahead prices but may also spill over into adjacent markets via interconnector flows. This effect tends to be strong for the case of market coupling (see, e.g., Heren, 2011; Carr, 2012a), and often results in primarily negative cross-border spreads (with respect to the German market). By contrast, if the fundamentals in the German and neighboring electricity markets are not too far apart, price spreads can be exactly zero for certain hours of the day and for several days in a row. Obviously, these scenarios cannot be reproduced when using a reduced-form modeling approach – be it univariate to model the spread directly, or be it bivariate when modeling the spot price time series for each market separately.

In this paper, we address these challenges and investigate the pricing of transmission rights under market coupling, taking the currently prevailing European market design for coupled electricity markets as a starting point. As such, we contribute to the literature in the following ways: first, we outline the institutional setup for cross-border trading under market coupling compared to explicit ex-ante auctioning and provide empirical evidence that recent changes in market design have clearly affected both pricing levels and exercise behavior for PTRs. In view of the continued implementation of market coupling throughout Europe, spread options between coupled pricing areas are a crucial

tool for hedging and, thus, are seeing considerable demand by both physical as well as financial electricity traders. Using data for the French–German interconnector, we see that the apparent undervaluation of PTRs compared to the corresponding futures spreads disappears when changing from an explicit auction set-up with ex-ante scheduling of flows to an implicit set-up such as market coupling.

Second, to the best of our knowledge, we are the first to derive an analytic pricing formula for a spread option on spot electricity prices in two *coupled* markets. Based on the fundamental framework for modeling two interconnected electricity markets proposed by Füss et al. (2015b), we not only take into account the above mentioned institutional details for cross-border trade but also yield a model that adequately reproduces the distinct spread dynamics that are characteristic for a market coupling regime. More generally, the availability of analytic pricing formulae helps to retain tractability and ease of use for practitioners, but also will facilitate calibration to observed PTR prices in case of an empirical implementation.

Finally, we conduct sensitivity analyses for the key input parameters to our spread option formula in order to analyze in detail the main determinants of value for a transmission right under market coupling. Here, it is important to see that since we model electricity prices as a function of fundamental factors, the relationship between option prices and the current value of the underlying spot spread is no longer unique, but must be disaggregated further to the level of underlying state variables. Importantly, differentials in the levels of the individual state variables – that would otherwise result in a non-zero spread between spot prices in each market – can be mitigated and balanced by the market coupling mechanism, which in turn significantly impacts option prices as compared to the situation when the two markets are isolated and not interconnected.

The remainder of this paper is structured as follows: the next section outlines the workings of the explicit and implicit scheme in more detail and provides further institutional background for cross-border trading of electricity. It also discusses the two allocation schemes from an empirical point of view by examining the price dynamics of several coupled and non-coupled markets in Europe and the US. Section III provides

additional background on the institutional framework for cross-border transmission rights under market coupling. Section IV outlines the general modeling framework. Section V derives the spread option formula for two coupled markets and provides sensitivity analyses to further illustrate the intricacies of spread option pricing under market coupling. Section VI concludes.

II Institutional Background

The organization of cross-border trade of electricity, by its nature, is characterized by a variety of institutional specificities and operational complexities. In the following, to lay the grounds for a detailed analysis of the above mentioned allocation mechanisms and their impact on electricity price dynamics, we will therefore only focus on the key conceptual ideas of each regime, but leave aside other institutional details and technicalities that are not necessary to motivate our modeling framework. Subsequently, we analyze empirically the price dynamics for the allocation regimes in Europe and the US.

A. Explicit Allocation Schemes

The generally widespread implementation of explicit allocation mechanisms is mainly due to the fact that they can quickly be set up without requiring a particularly high level of institutional harmonization between adjacent electricity markets. Importantly, as is characteristic for explicit schemes, physical transmission capacity markets and electricity (spot) markets are separated from each other, with the first usually clearing ahead of the latter. Hence, enabling cross-border trade in this way only requires a low degree of integration and joint coordination between both transmission system operators (TSOs) and power exchanges in adjacent markets. For instance, this avoids the necessity for a common trading platform and uniform exchange rules including a simultaneous closing of day-ahead auctions for (spot) electricity across markets. However, given that day-ahead prices in both markets are not yet known by the time when traders have to submit their

bids for transmission capacity, this generally leads to an inefficient market design.⁷ These inefficiencies manifest in interconnectors frequently not being used up to their thermal capacity limits or in adverse physical flows where electricity is directed from a more expensive market into a cheaper connected market. In such a situation, cross-border traders have failed to correctly anticipate the price spread *ex-ante*, and hence end up having acquired transmission capacity for the “wrong” direction.

From an operational point of view, for most interconnectors, explicitly allocated transmission rights are physical in nature and are assigned for a pre-specified direction of flow (e.g., $F \rightarrow \text{GER}$) and for different timeframes. For instance, prior to the introduction of market coupling at the French–German border, physical transmission rights (PTRs) could be acquired via explicit annual, monthly, and daily auctions. Depending on the timeframe, allocated transmission rights would hence entitle a trader to nominate and physically exchange electricity in a given direction from one market to the other during any hour of the year (month or day, respectively). Figure 1 illustrates in more detail the relevant steps to be taken in order to set up such a cross-border transaction, taking as example the explicit *ex-ante* auction scheme at the French–German border before it was replaced by market coupling in 2010.⁸ When the holder of a periodic (i.e., monthly or yearly) transmission right wanted to engage in a cross-border transaction on day t , the TSO had to be notified by 08:15am on day $t-1$ and provided with a schedule detailing the respective capacities to be nominated during every hour on day t .⁹ In case these capacity holders did not want to trade, their non-nominated capacities originally “earmarked” for periodic transactions were implicitly transferred and added to those capacities to be allocated in the daily market. In that case, according to the “use it or sell it” (UIOSI) principle, traders with monthly or yearly rights received as financial compensation for every megawatt (MW) of non-nominated capacity the price as determined during the daily auction for the corresponding hour.¹⁰ Based on the nominations received by the

⁷This is a well-documented fact in the literature. Inefficient cross-border trade across various European markets has been analyzed by, e.g., Turvey (2006), Kristiansen (2007), Marckhoff and Muck (2009), Bunn and Zachmann (2010), Bunn and Martoccia (2010), or McNerney and Bunn (2013).

⁸Although auction rules may differ across markets, the overall structure and timeline of the allocation procedure described here can generally be seen as representative for other explicit *ex-ante* schemes.

⁹See RTE (2009a), p. 17, for transactions between Germany and France.

¹⁰See RTE (2009b), p. 20.

holders of monthly and yearly transmission rights, the TSO then published an update of the available transmission capacity (ATC) that was to be offered on the daily market by 08:45am on day $t - 1$; bids for the daily auction then had to be submitted by 09:30am at the latest, and results were released half an hour later. As already mentioned, this is well ahead of the close of electricity spot (i.e., day-ahead) markets at around noon: 11:00am for French exchange Powernext and 12:00pm (noon) for its German counterpart EEX at that time. Finally, having been awarded a transmission right in the daily market, traders had to notify the TSO by 02:00pm whether they were going to use it and set up a cross-border transaction on the following day t . As opposed to the UIOSI principle, for holders of daily transmission rights, however, the “use it or lose it” (UIOLI) principle applied instead. From this it follows that in the case of non-nomination, there was no financial compensation; these unused rights were then, in turn, transferred to the intraday market.

In the US, the institutional design for cross-border (i.e., market-to-market) transactions is less harmonized than in Europe and a variety of different allocation methods coexists, sometimes even at the same border. While a detailed description of these is beyond the scope of this paper, we can focus on the following key issues: in view of the nodal pricing system in the US, combining paths of fixed point-to-point transactions would generally allow a trader to ship electricity across states with corresponding path reservations being allocated on a “first-come, first-serve” basis. Given the inherent inefficiency of these fixed allocations, many markets have replaced the physical reservations with a financial transmission rights framework and offer-based export/import scheduling (Spees and Pfeifenberger, 2012). Alternatively, several *merchant* lines have also been awarded transmission rights which are then re-allocated via long-term contracts or shorter-term auctions.¹¹ Nevertheless, irrespective of the exact institutional setting, bid selection and pricing by the independent system operator (ISO) and the physical scheduling of a market-to-market transaction are still two clearly distinct procedures for most electricity markets in the US. Hence, whereas other non-auction based allocation

¹¹Examples include the Hudson and Linden VFT interconnectors at the PJM (Pennsylvania-New Jersey-Maryland Interconnection) - NYISO (New York Independent System Operator) border.

mechanisms may be more common in the US, these mechanisms share a crucial feature of explicit schemes, i.e. of requiring traders to schedule their transactions ex-ante. Even though there are increasing efforts to move the physical scheduling process closer to real-time,¹² traders still need to rely on an expectation of the price spread between markets, which is – roughly speaking – comparable to the explicit allocation schemes used in Europe.

B. Implicit Allocation Schemes

By contrast, the concept of market coupling avoids the above timing problem for cross-border traders by clearing capacity and electricity markets simultaneously, i.e., by implicitly allocating transmission rights within the spot auction of electricity in each market. Conceptually, a pricing mechanism that is closely related to market coupling has already since long been implemented in the Nordic market, Nord Pool, the common electricity market for Norway, Denmark, Sweden, Finland, Estonia, and Lithuania. Here, in a first step, the market clearing system price is determined by equating total demand and supply, as aggregated across each national market, thereby ignoring any potential constraints in transmission capacity between bidding areas.¹³ Hence, in case of no congestion, there is a single price for all bidding areas. Otherwise, depending on the location of the transmission bottleneck(s), the market is “split” into several pricing zones that are assigned different area prices, and that comprise one or more bidding areas.

The results of the “market splitting” mechanism are in general similar to what is achieved by the variant of market coupling that is currently implemented between major Western European markets, and that is examined in this study. Yet, in contrast to Nord Pool, it is not a single power exchange that determines (in a “top-down” way)

¹²For instance, the PJM and NYISO markets have recently proposed “Coordinated Transaction Scheduling” (CTS) where traders have the option to submit a price spread at which they are willing to set up a market-to-market transaction rather than placing individual bids in each market. While supposed to enhance economically efficient interconnector use, transactions still need to be scheduled based on an expectation of the spread.

¹³In the Nordic market, national markets are further subdivided into one or more *bidding areas*, reflecting transmission bottlenecks both within and across national markets. For instance, the Danish market is subdivided into two bidding areas (Western and Eastern Denmark) which are linked by a 600 MW interconnector that is frequently congested.

whether to subdivide or retain a single pricing zone across markets. Instead, market coupling is characterized by a more de-centralized (“bottom-up”) approach where the joint coordination effort between all involved national TSOs and power exchanges determines which national markets are “coupled” with each other to form a single pricing zone.

Specifically, for every day and each interconnector, TSOs determine the amount of available transmission capacity (ATC) for which the corresponding transmission rights are no longer allocated explicitly but instead assigned to the respective power exchanges in the adjacent markets. In order to achieve spot price convergence, a joint optimization algorithm considers the net import/export positions of each participating exchange for a given clearing price and thus determines the optimal cross-border flows. As such, a buy-order in a higher-priced market can be matched with a sell-order in a cheaper adjacent market, thus mitigating the price differential with any resulting cross-border flows implicitly covered by the transferred capacity rights. As shown by Meeus et al. (2009) or Weber et al. (2010), the underlying optimization problem can be re-formulated as maximizing overall welfare, constrained by interconnector capacities and other real-time limitations. If one of these constraints is binding – e.g., if flows necessary to equalize prices exceed the ATC –, then prices cannot fully converge and the resulting congestion rent is collected by the owner of the interconnector.

The previously outlined explicit setup at the French–German border was modified when Germany joined the already existing market coupling between France, the Netherlands, and Belgium (the so-called “Trilateral Coupling”) on 09-Nov-2010, marking the starting point for the Central Western Europe (CWE) day-ahead market coupling. As a major change to the pre-existing setting, an explicit allocation of transmission rights for interconnector access is henceforth only held for monthly and yearly contracts, whereas on a day-ahead stage, all cross-border transmission capacities are available to cover the market coupling flows.¹⁴ Following the harmonization of day-ahead gate closure times

¹⁴Note that the transmission capacity of an interconnector is usually “sliced” and allocated for different timescales. Thus, off-exchange transactions, such as customized OTC cross-border trades, can still be executed by traders acquiring monthly or yearly transmission rights, whereas all remaining interconnector capacity can be used by power exchanges for market coupling purposes. Importantly, as pointed out by Meeus (2011), retaining such a split helps to avoid a potential monopolization of the entire organization of cross-border trade of electricity by the power exchanges participating in market coupling.

(12:00pm noon) at all involved power exchanges, the market coupling algorithm then starts to determine the optimal cross-border flows at around 12:05pm. Finally, the resulting day-ahead prices are published by the CWE power exchanges by 12:55pm.¹⁵

C. Ex-Ante Scheduled Transactions in Europe and the US

The general economic inefficiency of explicit ex-ante schemes is analyzed in Figure 2 where the LHS graph plots the spread between German and French day-ahead power prices against corresponding net transit nominations as percentage of available interconnector capacity. Clearly, not only are interconnector capacities oftentimes left (partially) unused – although a non-zero price spread in an efficient market setting would call for additional cross-border flow until the price spread is fully exploited or the maximum interconnector capacity is reached. Even worse, out of all nominations during the observation period (26-Oct-2006 to 09-Nov-2010), approx. 32% led to flows in the wrong direction altogether.

On a more detailed level, the data also reveal that during both peak- and off-peak hours, French day-ahead power prices exceed German prices more frequently than vice versa – or, more precisely, during approx. 58% of all hourly periods.¹⁶ This pattern, in turn, also seems to impact the nomination behavior by cross-border traders: conditional on the spread being negative, i.e. for French day-ahead prices exceeding German prices, net interconnector flows in the economically correct direction (i.e., to the higher-priced market) can be observed during approx. 82% of all hourly peak- and off-peak periods.¹⁷ By contrast, given a positive day-ahead spread (due to German prices being higher than their French counterparts), the share of net interconnector flows in the right direction is disproportionately lower: correct flows can only be observed during approx. 49% of all hours. Hence, market participants tend to bet on a negative spread, with a positive spread not only being less frequently observed but apparently also harder to predict correctly.

This diverse picture also manifests as regards the degree of *net* capacity utilization:

¹⁵See <http://www.marketcoupling.com/about-emcc/daily-operations>.

¹⁶Also note that the French–German interconnection, as a series of multiple alternating current (AC) links, has a higher capacity for GER→F than for F→GER.

¹⁷Note that in practice, traders may schedule more flows for those hours where they can be sure to properly anticipate the price differential between the markets. Hence, on a volume-weighted basis and conditional on a negative spread, even 93% of all exchanged MWs have flown in the correct direction.

On an overall basis, interconnectors are used at slightly less than 50% of their thermal capacity when French power prices exceed German power prices. For the opposite case, average *net* capacity utilization even gets driven down to less than 10% since, as mentioned above, adverse flows are observed more frequently. Disaggregating further, we can observe that for a price spread of up to 5 EUR/MWh (in either direction) and excluding all adverse flows,¹⁸ French–German interconnectors are still used at only slightly more than 40% of their thermal capacity. For the complementary case where only price spreads larger than 5 EUR/MWh are considered, “gross” capacity utilization for flows in the economically correct direction increases to 69%. Clearly, a comparably lower (absolute-level) spread between day-ahead prices increases the likelihood for traders of having predicted it wrong during the capacity auction and of hence having to engage in an unprofitable trade, which ultimately reduces both gross and net capacity utilization.

Similar results can be found when analyzing interconnector flows between the PJM and NYISO markets in the US during the first half of 2013, as shown in the RHS graph of Figure 2.¹⁹ Here, we see that economically inefficient interconnector flows are not only present in day-ahead markets but also when gate closure is moved closer to real-time (currently 75 minutes prior to the operational hour): in 44% of all 5-minute intervals, inefficient flows from the more expensive into the cheaper market could be observed. As regards (net) capacity utilization, the RHS graph clearly shows that the system operator includes a safety margin of approx. 10% of transfer capacity (which may vary over time and which our data is not adjusted for).²⁰ However, even abstracting from this, capacity utilization seems to be less endogenously driven by price differentials than in the LHS graph. In fact, imports into NYISO prevail (81% of all flows) although positive price spreads (economically justifying an import) were only observed in 51% of all cases.

¹⁸By contrast, the previous two examples related to *net* capacity utilization where adverse flows are included: I.e., for the theoretical case of an interconnector being used up to its full thermal capacity in both directions, resulting net capacity utilization would be 0%.

¹⁹For the actual real-time price spreads, we use the “NY Keystone” (proxy bus) and “PJM NYIS” prices available via Bloomberg. Five-minute scheduled interconnector flows relate to the PJM/NYISO interface (i.e., excluding the merchant Linden VFT and Hudson lines as well as the Neptune Underwater Transmission Line). Data shown for the period from 01-Jan-2013 until 30-Jun-2013. See www.monitoringanalytics.com for further information.

²⁰In order to guarantee operational security, a fraction of the interconnector capacity is usually reserved in order to be able to quickly react to contingencies.

D. Day-Ahead Price Spreads under CWE Market Coupling

The start of the CWE market coupling has fundamentally changed the dynamics of the day-ahead price spreads between Germany–France and Germany–Netherlands, respectively, as Figure 3 clearly illustrates for hours 9 and 18 of the day: for both hourly periods, with few exceptions at mid-November 2010, the spreads are mainly negative or zero, providing clear evidence for interconnector flows henceforth taking economically correct directions – i.e., from the then mainly lower-priced German market to the higher-priced Dutch and French day-ahead markets. The data show that until year-end 2010, *exact* price convergence was reached in 81% of all hourly periods for the German–Dutch day-ahead spread and in approx. 52% of all hours for the German–French spread. Note however, that for specific hours of the day, these percentages can vary significantly. For instance, for the German–French spread over the same period, German day-ahead prices during hour 24 of the day tend to clearly fall below French prices, so that price convergence could only be reached during less than 23% of all hours. By contrast, for hour 17 (18), a zero price spread was observed during 80% (74%) of all periods.

These patterns also manifest when analyzing the *long-term* behavior of the day-ahead spread between the German and the Dutch electricity market, as illustrated in Figure 4. For the spread between the prices of electricity to be delivered in both markets during hour 12 (11:00am - 12:00pm/noon) on the next day, Figure 4 shows that since the introduction of the CWE market coupling, positive price spreads were no longer observed at all – with very few, notable exceptions: On 27 March 2011, after the close of the day-ahead auction for the following day, the market coupling optimization algorithm could not be run due to a bug in the system which was ultimately caused by the change to daylight-saving time on that day. Consequently, as a fallback mechanism in the case of such a de-coupling, *explicit* (“shadow”) auctions had to be organized in order to allocate the available daily transmission capacity.²¹ For the remaining days, however, the spread was either zero or negative, implying that whenever available transmission capacity is insufficient in order to reach price convergence between the two markets, this applies to flows for the direction

²¹See, e.g., EPEX-Spot (2011) or CREG (2011). A positive spread could also be observed on 29-Oct-2011, which, interestingly, again coincides with the clock change back to winter time.

GER→NL, but not vice versa. More recently, negative spreads have been slightly more prevalent, thus driving down the rate of price convergence, which can be attributed to the steadily growing renewables feed-in in Germany, combined with continued strong levels of natural gas prices affecting the gas-based Dutch electricity market.²²

Table 1 summarizes the above qualitative observations by providing descriptive statistics for the German–Dutch day-ahead spread during selected hours of the day and for the corresponding baseload spread, i.e. the average of all 24 hourly spreads. As the key characteristic of market coupling, price convergence – driving down the spread to zero – is reflected in several of the statistics. For instance, note that a zero baseload spread could be observed for approx. 30% of all days since 09-Nov-2010, implying that price convergence was not only reached for several hours, but even for all 24 hours on these days. On a more detailed level, we also see that conditional on the spread being negative, both mean and standard deviation tend to be lower. Consequently, since the introduction of market coupling, not only can negative spreads $s^i < 0$ be observed less frequently, their spikiness is also reduced in terms of absolute size and variation. This change in the dynamics of the spread and, hence, of its individual components, can be explained by the fact that under market coupling, supply and demand shocks occurring in the interconnected markets can be mitigated more easily – to the extent that these shocks are non-synchronous: The coordination of cross-border flows between coupled markets always helps to reduce the economic scarcity (abundance) of *net* supplies that is signaled by electricity prices in one market spiking upwards (downwards).²³

In Section III, we will revisit these aspects from a theoretical point of view and further outline the institutional setup for cross-border trading of transmission rights under market coupling and provide empirical evidence on how changes in market design, and thus pricing

²²The role of natural gas as marginal fuel in the Dutch market has been further strengthened since the commissioning of the BritNed subsea cable that links the Dutch with the British market. However, note that there may also be more subtle, technical aspects to be considered: Especially for highly meshed grids, loop flows may lead to physical flows that differ from corresponding commercial point-to-point transactions. See Weber et al. (2010) for an illustrative example. In practice, for instance, high wind generation in the northern part of Germany may lead to unexpected loop flows of electricity into the neighboring Dutch, Czech, and Polish markets, from where it flows back to higher-demand areas in Southern Germany. TSOs need to reflect potential loop flows as contingencies within the safety margins of their interconnectors, which, in turn, reduces capacity available for market coupling flows.

²³See De Jonghe et al. (2008) and Huisman and Kiliç (2013) for an analysis of such volatility reduction potential in the context of the Trilateral Market Coupling between France, Belgium, and the Netherlands.

levels, have affected the exercise behavior for PTR's.

III Transmission Rights and Market Coupling

A. Physical and Financial Transmission Contracts

Being a network-bound commodity, the transfer of electricity across markets may be subject to transmission congestion on the respective interconnection line, which prevents price convergence and has traders face locational price risk instead. In order to hedge such transmission risk, instruments and contracts that are available in the areas of cross-border trading and congestion management can generally be categorized according to two criteria: *(i)* enforceability of the contract (rights vs. obligations), and *(ii)* the nature of the settlement mechanism (financial vs. physical).

Transmission *rights* are characterized by a convex payoff profile based on their inherent optionality for the option holder to collect the price difference between two connected markets, to the extent it is non-negative for a pre-specified direction of flow and given hour of the day. For instance, if a generator in market 1 who is contractually obliged to supply electricity to customers in neighboring market 2 hedges his cross-zonal position with a transmission right for the direction $1 \rightarrow 2$, he will be entitled to receive the price spread for every transmitted megawatt hour (MWh) of electricity if market 2 is more expensive than market 1 – but not vice versa (since the right is only uni-directional). Transmission *obligations*, by contrast, require their holders to also bear the downside risk of a negative price spread for a given direction of flow – as implied either from the (mandatory) physical scheduling of flows or the payoff from a financially settled transmission obligation.

Regarding the second criterion, *financial* transmission contracts are not directly tied to the scheduling of physical flows of electricity across a possibly congested interconnection line, but instead replicate the same payoff (upon financial settlement) as if the corresponding physical positions had been entered into. By contrast, *physical* transmission contracts are more operational in nature, and are primarily intended to establish an exchange of electricity – be it mandatory or optional – between two connected

markets. As such, while both physical and financial contracts are ultimately linked to the available transmission capacity of the underlying interconnector,²⁴ only the former class of contracts provide factual network access to the holder. Instead, with an exclusively FTR-based set-up, all cross-border transactions are essentially handled through the involved power exchanges or, e.g., a market coupling facilitator. In this case, and returning to the above example, a generator in market 1 cannot directly serve the customers in market 2 via physically transferring electricity via the interconnector, but can replicate the payoff from such transaction by *(i)* selling its own generation in market 1, *(ii)* sourcing the owed volumes in market 2, and *(iii)* hedging his cross-zonal position with a financial transmission contract.

Out of this classification, physical contracts are prevalent in Europe and are mainly established as rights (i.e., PTRs) which are often also referred to as a “carve-out” of cross-border transmission capacity (Duthaler and Finger, 2008). However, an important contractual feature, the so-called “Use-It-or-Sell-It” (UIOSI) property, also allows these rights to be used as financial transmission rights (FTRs). For instance, given a monthly (yearly) PTR,²⁵ its holder is entitled to schedule physical flows of electricity across the respective interconnector during any individual hour of that month (year). In case the trader does not want to exercise his right for a certain hour (e.g., because it might be uncertain as to how the price differential between the markets will turn out, thus risking an economically inefficient trade by transferring electricity from a more expensive market

²⁴In the case of financial contracts, options or obligations issued by the transmission system operator (TSO) need to be funded by the congestion rent that is collected for the respective interconnector concerned, and that depends on (physical) transmission constraints such as the capacity of the line. Hence, given the integrated nature of meshed electricity networks, revenue adequacy (between issued contracts and congestion rents to fund them) is generally ensured by carrying out a simultaneous feasibility test that determines the optimal number of contracts to be issued for all interconnectors within a given network at the same time. Note that in general, a lower number of financial transmission options can be released for a given interconnector than in the case of obligations due to the inability of netting opposite flow directions. See, e.g., ETSO (2006) or, more generally, Kristiansen and Rosellón (2013) for further information.

²⁵In Europe, PTRs are usually auctioned off for different timescales, i.e., a certain fraction of the overall interconnector capacity is allocated via yearly and monthly auctions; the remaining capacity either is also auctioned off via the day-ahead capacity market (where transmission rights for all 24 hours of the subsequent day are allocated individually) or, in the case of market coupling, is made available to the power exchanges and/or market coupling office. Hence, in the latter case, OTC traders can still set-up tailor-made, non-exchange based cross-border transactions by acquiring monthly or yearly transmission rights (instead of the daily rights no longer offered to the market).

into a cheaper market), the TSO needs to get notified one day in advance, usually in the morning hours well ahead of the close of the day-ahead electricity markets.²⁶

As a financial compensation in case of non-exercise, the trader then receives the market value of the forfeited option(s) relating to those hour(s) during which he does not want to schedule any cross-border flows. Hence, in the case of explicit auctions, the unused capacity (that was originally allocated to holders of monthly/yearly transmission rights) is returned to the market and added to those capacities that are up for allocation on the day-ahead stage. The prices achieved for the corresponding hours during that auction then yield the financial compensation for traders who chose not to exercise. Yet in the case of market coupling, there are no auctions of *short-term* transmission rights that would help to establish the financial compensation in a similar manner: on a day-ahead stage, these capacities are not allocated to the market but directly passed on to the power exchanges that schedule the market coupling flows intended to achieve price convergence. In this case, the financial compensation instead is defined as the actual spread between the prices resulting from the day-ahead auctions of electricity for the respective coupled markets.

Importantly, this mechanism thus not only always ensures revenue adequacy (Duthaler and Finger, 2008), but also allows to perfectly hedge price differences between adjacent *coupled* markets in Europe by using monthly/yearly PTRs with UIOSI property and not exercising them. As such, this combination of physical and financial features could also be seen as a first step towards transitioning to a fully financial setting comparable to the US, especially given that FTRs were also recently proposed as an alternative to the current physical set-up in Europe by the Agency for the Cooperation of Energy Regulators (ACER).²⁷

B. PTRs and the Impact of the CWE Market Coupling

The implications of the above theoretical concepts and specificities of market design can best be illustrated when we examine again the introduction of the Central Western Europe

²⁶See, e.g., RTE (2009a) and RTE (2009a) where this process is outlined for cross-border trading at the French–German border.

²⁷For further information, see ACER (2011) and de Maere d’Aertrycke and Smeers (2013).

(CWE) market coupling and how it changed the valuation of physical transmission rights. As already mentioned in Section II, prior to the implementation of CWE market coupling in November 2010, when Germany joined the then already existing “Trilateral Coupling” of the French, Belgian and Dutch electricity markets, PTRs with UIOSI property for the French–German and Dutch–German borders were allocated via explicit ex-ante auctions on a yearly, monthly, and day-ahead basis. Under the coupling mechanism, monthly and yearly PTRs are still auctioned explicitly, yet as will be seen, both previous valuation approaches and patterns of exercise for these rights have fundamentally changed.

In the academic literature cited in the introduction section, PTRs are generally proposed to be valued as a portfolio of hourly spread options or exchange options (Margrabe, 1978) with payoff $(P_{1,T} - P_{2,T})^+$ at maturity T – as implied by the physical positions taken by a cross-border trader who is long in market 2 and short in market 1 whenever electricity is cheaper in the former market than in the latter. Following this line of reasoning, and abstracting from other contractual details for the time being, prices paid for monthly transmission rights should reflect an inherent optionality and, hence, exceed the corresponding spread of 1-month ahead futures prices. Taking the French–German border as an example, as can be seen in the top panel of Figure 5, the auction prices for monthly PTRs more or less closely track the spread in 1-month ahead futures contracts when the spread is positive, or are bounded at zero when the spread is negative for a given direction of flow. However, at several instances prior to the start of the CWE market coupling, prices paid were clearly lower than the corresponding futures spread, which contradicts the above valuation argument. This is again illustrated in the middle panel of Figure 5, where the absolute-value premia of PTR prices vs. futures spreads are shown for the direction of flow that implies a positive futures spread.²⁸

This apparent undervaluation of PTRs especially during the years 2006-09, i.e., *prior* to the German market joining the market coupling mechanism, is due to several reasons. Clearly, cross-border trading in Europe had previously suffered from rather low levels

²⁸Specifically, at every auction date, we obtain a PTR price for each direction that could be compared with the corresponding futures spread. However, we refrain from comparing the PTR price for the direction for which the futures spread is negative since the concept of a PTR premium over a negative futures spread might be ill-defined in this context.

of investor interest and trading activity so that the observed negative price differences between PTRs and futures spreads could merely be due to market participants demanding a premium for the illiquidity of the transmission rights. Importantly, however, the above interpretation of a PTR with payoff $(P_{1,T} - P_{2,T})^+$ at maturity essentially ignores the fact that under the previous regime, “option exercise” actually had to take place before the spread $P_{1,T} - P_{2,T}$ was determined: Recall that holders of longer-term transmission rights either had to notify their intended transactions to the TSO in the morning or, alternatively, could opt to receive as financial compensation the PTR price at which their rights were re-sold in the daily auction. In either case, however, the inefficiencies of the explicit auction setting would affect both the option payoff in case of physical exercise or the price of the re-sold daily PTRs and, hence, of the alternative financial compensation.

Since the start of the CWE market coupling, however, this setting has changed. As outlined above, although *daily* transmission capacity is no longer offered to the market at all, this does not apply to longer-term rights for monthly and yearly transmission capacity which are still allocated explicitly. In addition, according to the UIOSI feature, the financial compensation for non-exercised long-term rights is now directly tied to the actually realized spot price difference during the respective hourly period (to the extent it is positive).²⁹ Hence, (automatic) exercise of the option takes place whenever the day-ahead spread between the two relevant markets implies a positive intrinsic value of the instrument. Therefore, it is only under market coupling that the underlying to the PTR with a UIOSI feature coincides with the *realized* spread (and not the *expected* spread), and only in this case should valuation of such a contract be based on the payoff profile $(P_{1,T} - P_{2,T})^+$.

Moreover, this change to the specifications of the UIOSI principle under market coupling has clearly changed the overall characteristics of these instruments, which may also help to further promote liquidity in both primary and secondary markets. On the one hand, physical traders such as generators now benefit from an increased hedge

²⁹See RTE (2009b), pg. 57: “(...) a price equal to (...) the price difference between the relevant day-ahead spot markets in Belgium, France, the Netherlands, and Germany for the considered Hourly Period (which might be zero (0) EUR/MWh), as far as the direction of the Programming Authorization equals the direction of flow resulting from relevant power exchange prices (...).”

effectiveness that also allows them to increasingly resort to non-domestic markets in order to hedge forward their production. For instance, recently observed declines in trading volume for Dutch and Belgian futures contracts were attributed to several Dutch and Belgian generators henceforth hedging by using structures based on the more liquid German futures contracts (Carr, 2012b), which can then be complemented with PTRs to account for the remaining spread risk. On the other hand, the possibility to use the PTRs with UIOSI feature as a (purely) financial transmission right is not only appealing to generators seeking effective hedges, but may also help to spark interest from other investor groups. Thus, especially for speculators and other players without physical assets, previous barriers to take part in European cross-border trade may be lowered, which more generally opens up further perspectives for trades to be financially motivated rather than physically. These developments are also reflected in Figure 5 where in the mid panel, we can qualitatively observe that the undervaluation of PTRs (compared to futures spreads) has clearly been mitigated, given that the pricing of these instruments obviously has become more competitive in general since 09-Nov-2010. Also, the bottom panel indicates that since the start of market coupling, actually nominated capacities have strongly decreased, which provides clear evidence for the above mentioned shift in applicability of PTRs towards hedging (or even speculation) rather than for physical shipping.

IV Modeling Two Coupled Electricity Markets

A. Reduced-Form Modeling Approaches

Although popular, reduced-form approaches to electricity price modeling increasingly struggle to keep pace with recent developments and regulatory changes in electricity markets, such as the introduction of negative electricity prices at some power exchanges, or the impact of carbon emission allowances as additional price driver.³⁰ The interconnectivity of European electricity markets adds to these complexities and poses new challenges: unlike under the explicit ex-ante scheduling of cross-border flows that

³⁰See, e.g., Fanone et al. (2013) and Carmona et al. (2012), respectively, for further information.

caused the typical oscillating behavior of the day-ahead price spreads between adjacent markets, spread dynamics under market coupling are now fundamentally different, given that prices in coupled markets will usually coincide much more frequently.

In the case of spread options, the dynamics of the underlying price spread have previously most often been modeled either indirectly as the difference between individual spot price time series in a bivariate reduced-form setting (see, e.g., Marckhoff, 2009; or Wobben et al., 2012) or directly as a univariate asset (see, e.g., Cartea and Gonzalez-Pedraz, 2012; or Marckhoff and Muck, 2009). Both approaches, however, will clearly yield a mis-specified model if to be applied to coupled electricity markets, which is due to both structural and practical reasons.

More precisely, not only do reduced-form settings fail, by their inherent nature, to make use of additional information about fundamental factors that strongly drive electricity price dynamics, such as generating fuels, domestic electricity demand, or available generation capacity. Extending a pricing framework to include this kind of information, especially when combined with forecasts for some of these factors, can help increase pricing performance significantly (Füss et al., 2015a). Consequently, reduced-form models thus also leave aside the same type of information for adjacent markets, which, however, is crucial in view of the increasing interconnectivity of European electricity markets.

Furthermore, against this background, calibration of these models is also problematic and may yield unreliable estimation results. Especially for electricity markets, structural breaks due to a change in market design, such as a first-time roll-out of market coupling or also the case of other national markets joining an existing coupling, may occur relatively often, so that proper and sound model calibration directly based on historic spot (or forward) price data is almost impossible. Note that these problems can be mitigated when instead inferring price dynamics from underlying state variables that may be unaffected

by such changes in market design.³¹

In addition, the transmission capacity of an interconnector has a crucial impact on the behavior of the spread between two adjacent markets. Although limited by an upper (thermal) bound, the actually available capacity can fluctuate a lot over time, which need not only be due to varying safety margins as imposed by TSOs, but also due to frequent maintenance and unexpected outages. Kristiansen (2007), for instance, examines cross-border electricity flows between Denmark and Germany during the years 2003-2004, and reports that the interconnector was not operating at all for 23% of the time due to scheduled maintenance. Therefore, when calibrating a reduced-form model – be it univariate to model the spread directly or bivariate for its components – to such *historic* data, the resulting price dynamics would implicitly be driven by an *average* interconnector capacity that may well be too low.³² Equivalently, the same problem arises when a *future* outage of the interconnector is announced so that the dynamics of the spread would need to be adjusted for this event.

Note that when alternatively implementing a regime-switching approach with the regimes being defined according to available transmission capacity of the interconnector, these problems could potentially be mitigated. However, as can empirically be observed, not only do available capacities fluctuate considerably for many European interconnectors, which might require to factually distinguish more regimes than is technically sensible from a modeling point of view; but also may special cases (e.g., where the interconnector can temporarily be used in one direction only) lead to additional complexities. In any case, the resulting model dynamics would still not be driven by the *economic causality* of cross-border exchange under market coupling, which, however, is prominently incorporated in the fundamental modeling approach that we present for transmission rights valuation further below.

³¹This is especially the case for the state variables that will be used to model electricity prices in our fundamental framework presented further below – i.e., for electricity demand and the generating fuel. Unlike demand and fuel prices, available generation capacity – that is also often used to model electricity prices in a fundamental framework (see, e.g., Cartea and Villaplana, 2008; Aïd et al., 2013; Füss et al., 2015a) – might nevertheless be affected by a change in market design since this may impact the possibly strategic bidding behavior of the generators in each of the interconnected markets.

³²This is due to the averaging over periods of both full use and outages at the same time.

B. Fundamental Modeling Approaches

In order to address the shortcomings of the reduced-form approaches when it comes to transmission rights valuation, we propose to use the modeling framework for two coupled markets that is originally developed in Füss et al. (2015b). This set-up belongs to the class of fundamental (or structural/hybrid) electricity pricing models,³³ and models electricity prices for each market i as a function of underlying state variables, i.e., domestic electricity demand $D_{i,t}$ and the price for the domestic generating fuel $q_{i,t}$. Having established the dynamics for these fundamental price drivers, we then derive electricity spot prices $P_{i,t}$ for each market based on an exogenous functional specification that mimics the typical price formation process in electricity spot markets, based on a supply-demand equilibrium with totally inelastic demand and a steeply increasing merit-order curve. Finally, the interconnector flow that links the two markets is introduced into the setting and derived by (i) equating spot prices in each market, and (ii) considering the arising non-linearity in the flow due to the upper/lower bounds to interconnector capacity K . We briefly re-state the model in the following and refer to Skantze et al. (2004), Coulon (2013), and Füss et al. (2015a, 2015b) for further information.

Regarding the first state variable, the dynamics of $D_{i,t}$ are modeled based on a mean-reverting Ornstein-Uhlenbeck (OU) process and a deterministic function in order to reflect the distinct seasonalities that can generally be observed for electricity demand (see, e.g., Cartea and Villaplana, 2008; or Füss et al., 2015a). On a filtered probability space $(\Omega, \mathcal{F}^D, \mathbb{F}^D = (\mathcal{F}^D)_{t \in [0, T^*]}, \mathbb{P})$, demand $D_{i,t}$ is hence assumed to be governed by the following dynamics:

$$D_{i,t} = q_{i,t} + s_{D_i}(t), \quad (1)$$

$$dq_{i,t} = -\kappa_{q_i} q_{i,t} dt + \eta_{q_i} dB_{i,t}, \quad (2)$$

where $q_{i,t}$ is an OU-process for market i with mean-reversion parameter κ_{q_i} , $B_{i,t}$ a

³³See, e.g., Pilipovic (1998), Eydeland and Wolyniec (2002), or Benth et al. (2008b) for a general overview on different electricity pricing models, and Carmona and Coulon (2012) for an exhaustive summary of the approaches subsumed under the class of fundamental models.

standard Brownian motion, and $s_{D_i}(t)$ a deterministic seasonality function. Note that when modeling two geographically neighboring markets, it is natural to assume that $q_{1,t}$ and $q_{2,t}$ are correlated, i.e., we allow for $dB_{1,t}dB_{2,t} = \varrho_q dt$.

Regarding the price process for the generating fuel, we again follow Füss et al. (2015b) and employ the mean-reverting one-factor model by Schwartz (1997) along with a deterministic component; hence, on a filtered probability space $(\Omega, \mathcal{F}^g, \mathbb{F}^g = (\mathcal{F}^g)_{t \in [0, T^*]}, \mathbb{P})$, the log fuel price, $\ln g_t$, shall be driven by the following dynamics:

$$\ln g_{i,t} = X_{i,t} + s_{g_i}(t), \quad (3)$$

$$dX_{i,t} = -\kappa_{X_i} X_{i,t} dt + \eta_{X_i} dW_{i,t}, \quad (4)$$

where $X_{i,t}$ is an OU-process for market i with mean-reversion parameter κ_{X_i} , $W_{i,t}$ a standard Brownian motion, and $s_{g_i}(t)$ a deterministic seasonality function. Again, potential correlation between the marginal fuel price processes in the two markets is considered by allowing for $dW_{1,t}dW_{2,t} = \varrho_X dt$.

For pricing derivatives, such as spread options in the context of transmission rights valuation, we have to transfer the above $\mathbb{P}$ -dynamics for $D_{i,t}$ and $g_{i,t}$ to a risk-neutral measure $\mathbb{Q}$ by introducing market prices of demand and fuel price risk, $\Lambda_{q_i}(t) = \int_0^t \lambda_{q_i,s} ds$ and $\Lambda_{X_i}(t) = \int_0^t \lambda_{X_i,s} ds$.³⁴ Note that in the case of our market setting, however, there is no unique equivalent martingale measure given that non-traded risk factors (such as electricity demand $D_{i,t}$) cannot be hedged and, hence, render the market incomplete. Following, e.g., Carmona et al. (2013) or, more generally, Bjork (2009), we thus assume that the market selects a risk-neutral pricing measure $\mathbb{Q}$ which fulfills $\mathbb{Q} \in \{\mathbb{Q} \sim \mathbb{P}: \text{the discounted price of each tradable asset is a local } \mathbb{Q}\text{-martingale}\}$.

Finally, we thus have the following four-variate Gaussian setting where conditional

³⁴When pricing derivatives in electricity markets, the market price of risk has often empirically been observed to change signs and/or exhibit term structure-like patterns, which can be explained by, e.g., different degrees of hedging pressure of consumers and producers depending on their time horizon for risk diversification (see, e.g., Benth et al., 2008a; Weron, 2008; or, more generally, Bessembinder and Lemmon, 2002). Given that, e.g., in the case of yearly transmission rights, the underlying portfolio of hourly spread options comprises a wide range of maturities, we propose to use time-varying market prices of risk, thus yielding a more flexible parametrization for our class of risk-neutral measures $\mathbb{Q}$.

on time t and under $\mathbb{Q}$, $q_{1,T}$, $q_{2,T}$, $X_{1,T}$, and $X_{2,T}$ are distributed as follows:

$$\begin{bmatrix} q_{1,T} \\ q_{2,T} \\ X_{1,T} \\ X_{2,T} \end{bmatrix} \stackrel{\mathbb{Q}}{\sim} N \left(\begin{bmatrix} \mu_{q_1} \\ \mu_{q_2} \\ \mu_{X_1} \\ \mu_{X_2} \end{bmatrix}, \begin{bmatrix} \sigma_{q_1}^2 & \rho_q \sigma_{q_1} \sigma_{q_2} & 0 & 0 \\ \rho_q \sigma_{q_1} \sigma_{q_2} & \sigma_{q_2}^2 & 0 & 0 \\ 0 & 0 & \sigma_{X_1}^2 & \rho_X \sigma_{X_1} \sigma_{X_2} \\ 0 & 0 & \rho_X \sigma_{X_1} \sigma_{X_2} & \sigma_{X_2}^2 \end{bmatrix} \right), \quad (5)$$

with

$$\begin{aligned} \mu_{q_i}(t, T) &= q_{i,t} e^{-\kappa_{q_i}(T-t)} + \eta_{q_i} \int_t^T e^{-\kappa_{q_i}(T-s)} \lambda_{q_i,s} ds, \\ \sigma_{q_i}^2(t, T) &= \frac{\eta_{q_i}^2}{2\kappa_{q_i}} (1 - e^{-2\kappa_{q_i}(T-t)}), \\ \mu_{X_i}(t, T) &= X_{i,t} e^{-\kappa_{X_i}(T-t)} + \eta_{X_i} \int_t^T e^{-\kappa_{X_i}(T-s)} \lambda_{X_i,s} ds, \\ \sigma_{X_i}^2(t, T) &= \frac{\eta_{X_i}^2}{2\kappa_{X_i}} (1 - e^{-2\kappa_{X_i}(T-t)}), \\ \rho_q(t, T) &= \frac{1}{\sigma_{q_1} \sigma_{q_2}} \frac{\varrho_q \eta_{q_1} \eta_{q_2}}{\kappa_{q_1} + \kappa_{q_2}} (1 - e^{-(\kappa_{q_1} + \kappa_{q_2})(T-t)}), \\ \rho_X(t, T) &= \frac{1}{\sigma_{X_1} \sigma_{X_2}} \frac{\varrho_X \eta_{X_1} \eta_{X_2}}{\kappa_{X_1} + \kappa_{X_2}} (1 - e^{-(\kappa_{X_1} + \kappa_{X_2})(T-t)}). \end{aligned}$$

For ease of notation, we introduce the shorthand notation that $\mu_{(\cdot)}$, $\sigma_{(\cdot)}$, and $\rho_{(\cdot)}$ refer to $\mu_{(\cdot)}(t, T)$, $\sigma_{(\cdot)}(t, T)$, and $\rho_{(\cdot)}(t, T)$.

In order to link state variables and electricity spot prices, we invoke a continuous-time equilibrium by intersecting inelastic demand $D_{i,t}$ with an exponential supply curve in each market, thereby considering the role of marginal fuel prices $g_{i,t}$ as primary cost driver when (conventionally) generating electricity. Additionally, the two electricity markets are interconnected where the flow on the corresponding transmission line is denoted by $J(t)$. Electricity spot prices $P_{i,t}$ in market i with $i = \{1, 2\}$ are then defined as follows:

$$P_{1,t} = \alpha_1 g_{1,t}^{\delta_1} \exp(\beta_1 D_{1,t} - \gamma_1 J(t)), \quad (6)$$

$$P_{2,t} = \alpha_2 g_{2,t}^{\delta_2} \exp(\beta_2 D_{2,t} + \gamma_2 J(t)), \quad (7)$$

where the flow $J(t)$ on the interconnector results from a simplified allocation rule that is to reproduce the economically efficient allocation of cross-border capacities under market coupling and the resulting price convergence.³⁵ Hence, in a first step, the (unconstrained) interconnector flow $\tilde{J}(t)$ is determined by equating the spot prices in the coupled markets:

$$\begin{aligned} P_{1,t} &\stackrel{!}{=} P_{2,t} \\ \alpha_1 g_{1,t}^{\delta_1} \exp\left(\beta_1 D_{1,t} - \gamma_1 \tilde{J}(t)\right) &= \alpha_2 g_{2,t}^{\delta_2} \exp\left(\beta_2 D_{2,t} + \gamma_2 \tilde{J}(t)\right). \end{aligned} \quad (8)$$

Solving for $\tilde{J}(t)$ then yields:

$$\tilde{J}(t) = \frac{1}{\gamma_1 + \gamma_2} [\ln \alpha_1 - \ln \alpha_2 + \delta_1 \ln g_{1,t} - \delta_2 \ln g_{2,t} + \beta_1 D_{1,t} - \beta_2 D_{2,t}]. \quad (9)$$

However, given that flows on the interconnector are limited by its thermal capacity K , we require for operationally feasible flows $J(t)$ that $-K \leq J(t) \leq K$ and, hence, define $J(t) = \max\left(\min\left(\tilde{J}(t), K\right), -K\right)$.³⁶ Consequently, electricity spot prices in the two markets are piecewise defined according to three different scenarios as implied by the above non-linearity in $J(t)$:

$$P_{1,t} = P_{1,t}^{ex} \mathbb{I}_{\{\tilde{J}(t) \leq -K\}} + P_{1,t}^{un} \mathbb{I}_{\{-K < \tilde{J}(t) < K\}} + P_{1,t}^{im} \mathbb{I}_{\{\tilde{J}(t) \geq K\}}, \quad (10)$$

$$P_{2,t} = P_{2,t}^{im} \mathbb{I}_{\{\tilde{J}(t) \leq -K\}} + P_{2,t}^{un} \mathbb{I}_{\{-K < \tilde{J}(t) < K\}} + P_{2,t}^{ex} \mathbb{I}_{\{\tilde{J}(t) \geq K\}}, \quad (11)$$

where $P_{1,t}^{ex}$ ($P_{2,t}^{ex}$) is the spot price at time t in market 1 (market 2) if it is exporting electricity to market 2 (market 1). Physical interconnector flows are then constrained by the capacity of the transmission line and an amount of K gigawatt (GW) units is exported from one market to the other. $P_{1,t}^{un}$ ($P_{2,t}^{un}$) is the time- t spot price if the interconnection line between the two markets is *un*-constrained. In such case, there is no congestion and market 1 (market 2) may either be exporting or importing at below the capacity limit K . Correspondingly, $P_{1,t}^{im}$ ($P_{2,t}^{im}$) is the spot price of electricity in market 1 (market 2) if

³⁵Again, we refer to Füss et al. (2015b) for further details.

³⁶Note that we could easily accommodate the case of an interconnection line with different levels of transmission capacity for each direction, i.e., $K_{1 \rightarrow 2} \neq K_{2 \rightarrow 1}$, yet refrain from doing so to simplify our analysis.

it is in import-state and $J(t)$ has reached its capacity limit. More explicitly, we have for market 1 (prices for market 2 are defined analogously):

$$P_{1,t}^{ex} = \alpha_1 g_{1,t}^{\delta_1} \exp(\beta_1 D_{1,t} - \gamma_1(-K)), \quad (12)$$

$$P_{1,t}^{un} = \alpha_1 g_{1,t}^{\delta_1} \exp(\beta_1 D_{1,t} - \gamma_1 \tilde{J}(t)), \quad (13)$$

$$P_{1,t}^{im} = \alpha_1 g_{1,t}^{\delta_1} \exp(\beta_1 D_{1,t} - \gamma_1 K). \quad (14)$$

V Valuation and Analysis of Transmission Rights

A. Valuation as Spread Option

Let $V_t(T)$ denote the value of a transmission right providing access to an interconnector between two coupled markets at time t and with maturity T . As mentioned above, under market coupling, $V_t(T)$ can be determined by interpreting the right as an option written on the spread between spot electricity in the two markets, with payoff $(P_{1,T} - P_{2,T})^+ = \max(P_{1,T} - P_{2,T}, 0)$. $V_t(T)$ can then be derived as follows:

$$\begin{aligned} V_t(T) = & e^{-r(T-t)} \mathbb{E}_t^{\mathbb{Q}} [(P_{1,T}^{ex} - P_{2,T}^{im})^+ \mathbb{I}_{\{\tilde{J}(T) \leq -K\}} + (P_{1,T}^{un} - P_{2,T}^{un})^+ \mathbb{I}_{\{-K \leq \tilde{J}(T) \leq K\}} \\ & + (P_{1,T}^{im} - P_{2,T}^{ex})^+ \mathbb{I}_{\{\tilde{J}(T) \geq K\}}]. \end{aligned}$$

Given that in the first two states ($\tilde{J}(T) \leq -K$ and $-K \leq \tilde{J}(T) \leq K$), the option will expire worthless, whereas it will always be in the money for $\tilde{J}(T) \geq K$, we now yield:

$$\begin{aligned} V_t(T) = & e^{-r(T-t)} \mathbb{E}_t^{\mathbb{Q}} [(P_{1,T}^{im} - P_{2,T}^{ex}) \mathbb{I}_{\{\tilde{J}(T) \geq K\}}] \\ = & e^{-r(T-t)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{ub}^{\infty} \left(\alpha_1 g_{1,T}^{\delta_1} \exp(\beta_1 D_{1,T} - \gamma_1 K) - \right. \\ & \left. \alpha_2 g_{2,T}^{\delta_2} \exp(\beta_2 D_{2,T} + \gamma_2 K) \right) \\ & \cdot \phi(q_{1,T}|q_{2,T}) \phi(q_{2,T}) \phi(X_{1,T}|X_{2,T}) \phi(X_{2,T}) dq_{1,T} dq_{2,T} dX_{1,T} dX_{2,T}, \quad (15) \end{aligned}$$

where the bound ub is derived from the inequation $\tilde{J}(T) \geq K$ (with $\tilde{J}(T)$, in turn, similarly defined as in Equation (9)) and solving for $q_{1,T}$. Furthermore, in order to yield an analytic

pricing formula for the value of a transmission right, we need to compute several integrals over Gaussian densities, which can be simplified by relying on the following standard result:³⁷

$$\int_{-\infty}^l e^{cx} \Phi\left(\frac{a+bx}{d}\right) \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} dx = e^{\frac{1}{2}c^2} \Phi_2\left(l-c, \frac{a+bc}{\sqrt{b^2+d^2}}; \frac{-b}{\sqrt{b^2+d^2}}\right), \quad (16)$$

where a, b, c, d , and l are constants, $\Phi(\cdot)$ and $\Phi_2(\cdot, \cdot; \rho)$ are the cumulative distribution functions of the univariate and bivariate (correlation ρ) standard normal distribution.

Applying Equation (16) and leaving out the maths in between, we can derive the following closed-form solution for $V_t(T)$:³⁸

$$V_t(T) = e^{-r(T-t)} \left[\mathbb{E}_t^{\mathbb{Q}} [P_{1,T}^{im}] \Phi\left(-\frac{\mathcal{A}_X + \mathcal{A}_q + \bar{\mathcal{K}}}{\sqrt{\mathcal{C}_X + \mathcal{C}_q}}\right) - \mathbb{E}_t^{\mathbb{Q}} [P_{2,T}^{ex}] \Phi\left(-\frac{\mathcal{B}_X + \mathcal{B}_q + \bar{\mathcal{K}}}{\sqrt{\mathcal{C}_X + \mathcal{C}_q}}\right) \right], \quad (17)$$

with

$$\begin{aligned} \mathcal{A}_X &= \frac{\delta_2}{\beta_1} \mu_{X_2} - \frac{\delta_1}{\beta_1} \mu_{X_1} - \frac{\delta_1^2}{\beta_1} \sigma_{X_1}^2 + \frac{\delta_1 \delta_2}{\beta_1} \rho_X \sigma_{X_1} \sigma_{X_2}, & \mathcal{A}_q &= \frac{\beta_2}{\beta_1} \mu_{q_2} - \mu_{q_1} - \beta_1 \sigma_{q_1}^2 + \beta_2 \rho_q \sigma_{q_1} \sigma_{q_2}, \\ \mathcal{B}_X &= \frac{\delta_2}{\beta_1} \mu_{X_2} - \frac{\delta_1}{\beta_1} \mu_{X_1} + \frac{\delta_2^2}{\beta_1} \sigma_{X_2}^2 - \frac{\delta_1 \delta_2}{\beta_1} \rho_X \sigma_{X_1} \sigma_{X_2}, & \mathcal{B}_q &= \frac{\beta_2}{\beta_1} \mu_{q_2} - \mu_{q_1} + \frac{\beta_2^2}{\beta_1} \sigma_{q_2}^2 - \beta_2 \rho_q \sigma_{q_1} \sigma_{q_2}, \\ \mathcal{C}_X &= \left(\frac{\delta_1}{\beta_1}\right)^2 \sigma_{X_1}^2 - 2 \frac{\delta_1 \delta_2}{\beta_1^2} \rho_X \sigma_{X_1} \sigma_{X_2} + \left(\frac{\delta_2}{\beta_1}\right)^2 \sigma_{X_2}^2, & \mathcal{C}_q &= \sigma_{q_1}^2 - 2 \frac{\beta_2}{\beta_1} \rho_q \sigma_{q_1} \sigma_{q_2} + \left(\frac{\beta_2}{\beta_1}\right)^2 \sigma_{q_2}^2, \\ \bar{\mathcal{K}} &= \frac{\gamma_1 + \gamma_2}{\beta_1} K - \frac{1}{\beta_1} \mathcal{S}, \\ \mathcal{S} &= \ln \alpha_1 - \ln \alpha_2 + \delta_1 s_{g_1}(T) - \delta_2 s_{g_2}(T) + \beta_1 s_{D_1}(T) - \beta_2 s_{D_2}(T). \end{aligned}$$

We remark that in the case of spread options on electricity spot prices, the underlyings can obviously not be used to set up a hedging portfolio and replicate the option payoff; also, although we model electricity prices as derived from a continuous-time equilibrium between underlying state variables, we do not employ a fully agent-based optimization setting, so that alternatively resorting to utility indifference pricing only for

³⁷For further information on the result in Equation (16) (which ultimately is based on a change-of-variables transformation), see Geske (1979) and also Carmona and Coulon (2012).

³⁸Note that in below Equation (17), we omit to state $\mathbb{E}_t^{\mathbb{Q}} [P_{1,T}^{im}]$ and $\mathbb{E}_t^{\mathbb{Q}} [P_{2,T}^{ex}]$ explicitly – both of which, however, can easily derived based on Equations (12) and (14), respectively, and the properties of the lognormal distribution.

valuing derivatives (but not for determining the underlying price processes as equilibrium outcomes) may be seen as inconsistent.³⁹ However, even for an incomplete market setting, derivative prices have to be consistent within the set of all traded contracts, as required by the (albeit weaker) no-arbitrage arguments that we invoke here for spread option pricing: even though derivative prices will not be unique but depend on the respective equivalent martingale measure chosen, they will exclude the possibility for arbitrage profits based on the set of all other instruments that are actually traded. Note in this context that although modeling electricity prices as a function of underlying state variables increases the number of risk-neutral parameters to be extracted, it can help at the same time to enlarge the set of traded contracts that can be used for calibration purposes – especially when considering electricity derivatives as not being written on spot electricity, but instead as derivatives on underlying electricity demand and the generating fuel. Thus, in an empirical implementation, depending on the scope and number of contracts to be considered in order to extract the parametrization for the risk-neutral measure $\mathbb{Q}$, derivatives on only one of these underlying risk factors, such as natural gas options/futures, could additionally be included and allow for an even more robust calibration.

B. Sensitivity Analysis for Transmission Rights

In order to examine the key determinants of value for transmission rights, Figures 6 and 7 illustrate payoff profiles $(P_{1,T} - P_{2,T})^+$ and option values $V_t(T)$ for given interconnector capacities K of 0 GW, 2 GW, and 5 GW.⁴⁰ Note, however, that unlike for a “classic” payoff profile of a plain vanilla spread option, we here cannot directly plot the option payoff $(P_{1,T} - P_{2,T})^+$ against the underlying spot spread as measured on the abscissa. Recalling that for $K > 0$, the spot spread consists of the components in Equations (10)

³⁹See, e.g., Kluge (2006) for an introduction to utility indifference pricing in the context of electricity markets and Henderson (2002) for a general overview.

⁴⁰We assume two identically parametrized markets where the main input parameters to our model are set as follows: $s_{D_i} = 40$ GW, $s_{g_i} = 0.5$ (thus leaving out any seasonalities in electricity demand or fuel prices as a matter of simplification), $\kappa_{q_i} = 0.5$, $\kappa_{X_i} = 0.001$, $\eta_{q_i} = 1.0$, $\eta_{X_i} = 0.02$, $\beta_i = 0.1$, and $\delta_i = 0.5$. Furthermore, even though from an empirical point of view, the case of $\rho_q > 0$ and $\rho_X > 0$ may be more relevant, we assume state variables to be uncorrelated when analyzing their individual impact on option prices (instead of having to separate “overlapping effects” arising from non-zero correlations). Finally, we also assume $\lambda_{q_i,t} = \lambda_{X_i,t} = 0$ and $r = 0$.

and (11), we see that plotting option values $V_t(T)$ obtained for different interconnector capacities K against the same spot spread obviously would lead to distortions: notably, the probability of observing a given level of the spread in the market will not be the same across different capacities K . Instead, in the below analysis, we vary the fundamental state variables that are at the origin of changes in the spot spread, which not only avoids the above problem, but also allows for a more detailed picture on how $V_t(T)$ is impacted differently by movements in electricity demand and fuel prices.

In Figure 6, we analyze how both payoff and value of the transmission right, $V_t(T)$, depend on deseasonalized demand in market 1, $q_{1,t}$ (thereby keeping the initial values of the other state variables fixed at zero, i.e., $q_{2,t} = X_{i,t} = 0$). Interestingly, the kink in the payoff profiles (indicating where the option starts to be in the money) is shifting to the right for higher levels of interconnector capacity K , which is due to the increased price convergence through the market coupling mechanism: for $K > 0$, the interconnector helps to meet higher demand in market 1 (as indicated by $q_{1,t}$) with additional supplies from the neighboring market and thus creates a “deadband” of differentials in the underlying state variables that can still be balanced in order to maintain a zero price spread between electricity markets. In the LHS graph, we furthermore see that irrespective of the specific level of K , all transmission right values for an assumed time-to-maturity of $\tau = T - t = 10$ days are virtually straight horizontal lines and insensitive to the initial value chosen for $q_{1,t}$. Hence, if today’s variation in electricity demand does not impact the final distribution of the option payoff upon maturity, all demand shocks away from the long-term (seasonal) mean must reverse by then, as is ensured by a correspondingly high speed of mean reversion for our OU-process (as specified in Equations (1) and (2)): In fact, having set $\kappa_{q_i} = 0.5$ implies a half-life for demand shocks of only slightly more than one day, as is characteristic for electricity demand.⁴¹ Consequently, $q_{1,t}$ will not impact the option value $V_t(T)$ but for very short-term maturities. By contrast, when instead varying the deseasonalized log-fuel price $X_{1,t}$ in Figure 7, it is obvious that for the same maturity, the generally lower speed of mean reversion clearly causes option values to be

⁴¹Note that the parameter values used for our sensitivity analyses are in line with the empirical values as estimated in Füss et al. (2015a). High levels for the speed of mean reversion for electricity demand are also confirmed by, e.g., Pirrong and Jermakyan (2008) or Coulon et al. (2013).

more sensitive to variation in fuel prices. Since positive (negative) deviations of $X_{1,t}$ from the long-term mean will decay only very slowly, they will still be persistent at maturity and, compared to the horizontal lines in the LHS graph, will lead to a relative increase (decrease) in option value.

With respect to the question how interconnector capacity K affects option prices, it is obvious to see from both figures that higher levels of K facilitate price convergence, thus strongly reducing the likelihood of reaching the export state $\tilde{J}(T) \geq K$. Therefore, taking the case of $K = 5$ GW in Figure 7 as an example, and with all state variables expected to mean-revert to their unconditional moments by maturity at time T , $V_t(T = t + 10)$ is already close to zero and coincides with the horizontal axis. At the same time, this underlines the degree of model risk if transmission rights under market coupling were (erroneously) to be valued as spread options within a classic reduced-form setting. Given that interconnectivity and ensuing frequent price convergence between the markets would be ignored (i.e., $K = 0$), option values would generally be overstated: for instance, $V_t(T = t + 10)$ for $K = 0$ is almost four times as high as with $K = 2$ – and in fact, we can show that $V_t(T)$ for $K = 0$ will always serve as an upper bound to option prices.

More precisely, for the case of two isolated markets with no possibilities to import/export electricity from each other, Equation (17) above entails Margrabe's formula (see Margrabe, 1978) as a special case and reduces to the well-known case of an exchange option on two lognormally-distributed assets, $\ln(P_{1,T}^{iso})$ and $\ln(P_{2,T}^{iso})$ with mean μ_{P_i} , variance $\sigma_{P_i}^2$, and correlation ρ_{P_1,P_2} . As is straightforward to show, for $K = 0$, our option pricing formula then can be summarized as follows:

$$V_t(T) = e^{-r(T-t)} [F_{1,t}^{iso}(T)\Phi(\mathcal{D}_1) - F_{2,t}^{iso}(T)\Phi(\mathcal{D}_2)], \quad (18)$$

with

$$\mathcal{D}_1 = \frac{\ln\left(\frac{F_{1,T}^{iso}}{F_{2,T}^{iso}}\right) + \frac{1}{2}\sigma_{P_1,P_2}^2}{\sigma_{P_1,P_2}}, \quad \mathcal{D}_2 = \frac{\ln\left(\frac{F_{1,T}^{iso}}{F_{2,T}^{iso}}\right) - \frac{1}{2}\sigma_{P_1,P_2}^2}{\sigma_{P_1,P_2}},$$

where $F_{i,t}^{iso}(T) = \mathbb{E}_t^{\mathbb{Q}}[P_{i,T}^{iso}]$ and $\sigma_{P_1,P_2}^2 = \sigma_{P_1}^2 - 2\rho_{P_1,P_2}\sigma_{P_1}\sigma_{P_2} + \sigma_{P_2}^2$. Even though it may

be intuitively clear that $V_t(T)$ must increase when interconnector capacity gets smaller, it is helpful to realize how this effect is reproduced by the mechanics of our model: notably, for $K \rightarrow 0$, we can already see that the arguments to the cumulative distribution functions (cdf) in Equation (17) converge towards $\mathcal{D}_1$ and $\mathcal{D}_2$, respectively. For identically parametrized markets, we then have $F_{1,T}^{iso} = F_{2,T}^{iso}$ and $\mathcal{D}_1 = -\mathcal{D}_2$. This, in turn, implies that for $K = 0$, the *difference* between the “weightings” $\Phi(\mathcal{D}_1)$ and $\Phi(\mathcal{D}_2)$ will be highest and, given the curvature of the standard normal cdf, will start to become smaller with increasing K (as a consequence of both functional arguments to the cdf in Equation (17) becoming smaller themselves). At the same time, it is only for $K = 0$ that $\mathbb{E}_t^{\mathbb{Q}}[P_{1,T}^{im}]$ and $\mathbb{E}_t^{\mathbb{Q}}[P_{2,T}^{ex}]$ coincide, with the latter term otherwise exceeding the former; consequently, for non-zero K , not only will the probability weightings converge to each other, but also will $\mathbb{E}_t^{\mathbb{Q}}[P_{1,T}^{im}]$ and $\mathbb{E}_t^{\mathbb{Q}}[P_{2,T}^{ex}]$ move in opposite directions, causing $V_t(T)$ in Equation (17) to become even smaller and, hence, establishing the Margrabe formula in Equation (18) as an upper bound.

Finally, Figure 8 shows additional sensitivities of the option value $V_t(T)$ when varying (i) the interconnector capacity K , and (ii) the ratio of unconditional (long-term) demand between the two markets. Additionally, the probability of the spread option being “in the money” is shown in the panels on the right, where we have computed the risk-neutral probability $\mathbb{Q}(A)$ with $\mathbb{Q}(A) = \mathbb{E}_t^{\mathbb{Q}}[\mathbb{I}_A]$ (where A refers to the event $\tilde{J}(T) \geq K$).

As can be seen in the top row of Figure 8, the option value is very sensitive to the interconnector capacity K given that an increase in K strongly facilitates price convergence between the two markets, in which case the option will expire worthless. In fact, for our example, an interconnector capacity of 5 GW is already sufficient to drive both option value and probability of being in the money down to zero. Also note that for $K = 0$, i.e., for above mentioned Margrabe case, price convergence cannot be reached (unless by coincidence); thus, given that all other parameters were chosen to be equal in the two markets, the probability of exercising the option is exactly 1/2.

Varying the ratio of long-term demand in the two markets μ_{q_1}/μ_{q_2} for $T \gg t$, the option value is zero for ratios less than 1.0: In this case, higher demand in market 2 always induces higher electricity prices $P_{2,T}$ at maturity, so that $(P_{1,T} - P_{2,T})^+$ will be

zero. On the other hand, a small increase of the ratio beyond 1.0 implies *ceteris paribus* a strong increase in option value. Finally, note that although the two markets are again identically parametrized, the probability $\mathbb{Q}(A)$ for a long-term demand ratio of exactly 1.0 is now below $1/2$. For a fixed interconnector capacity (which here has been set to $K = 2$), price convergence between the two markets can now be reached more easily than in the previous case outlined above, which must be compensated by a higher demand ratio in order to reach a probability of $1/2$.

VI Conclusion

Driven by continuing efforts to foster electricity market integration across Europe, cross-border trading has gradually evolved out of a primarily operational into a more liquid setting where transactions are increasingly becoming financial rather than physical. Following the recent roll-out of market coupling across the central European markets, and accompanied by a small but important change with respect to the UIOSI feature of physical transmission rights, longer-term PTRs now benefit from increased hedge effectiveness – given that the common inefficiencies when scheduling flows ex-ante no longer prevail. At the same time, the possibility to not exercise PTRs (but use them as FTRs instead) has sparked further investor interest, with financial players such as the trading arms of major investment banks increasingly taking part in PTR auctions. As preliminary empirical evidence confirms, these changes have resulted in more competitive auction bids for PTRs that entail a premium above the corresponding futures spread, which in turn is in accordance with the common spread option approach to transmission rights valuation.

However, the intricate price dynamics of the spot spread between two coupled markets render previously used reduced-form approaches unreliable and call for more granular modeling frameworks: the fundamental setting presented in this paper incorporates a simplified representation of a market coupling mechanism and, thus, adequately reproduces the stylized facts of electricity prices in coupled markets – which hence also applies to the corresponding spread dynamics. Our log-normal setting furthermore allows

us to derive a closed-form pricing formula for transmission rights by interpreting the right as an option on the spot spread between the two markets, which is important both for computing the related option Greeks and other risk management measures, but even more so when it comes to an empirical implementation of the model.

Aspects for further research are abundant and, hence, our paper could be taken as a starting point for further research centering on the analysis of transmission rights valuation in a fundamental modeling context: First of all, our setting could be extended to include additional coupled markets, which – albeit at the cost of increasing complexity – will clearly enhance its applicability and scope of use given the continued spreading (and linking) of both new and existing coupling initiatives. In this context, the availability of analytic pricing formulae will be crucial: not only because it usually allows for a better understanding of the model mechanics, but also since computational speed becomes even more important the more markets are to be modeled at the same time. Otherwise, simulations may likely suffer from the curse of dimensionality, making calibration to observed PTR or other derivatives prices – which is a frequently used calibration method for the case of an incomplete market setting – even less viable. Also, given the disproportionately high influence of renewable generation sources on spot prices in Germany, the value of transmission rights for neighboring markets could start to become sensitive to the potential occurrence of negative prices (e.g., at least for few selected hours of the day during which negative prices are most likely to occur). Hence, it would be interesting to extend our model to allow for negative-price regimes, as could be achieved with, e.g., a regime-switching set-up.⁴²

From an empirical point of view, it would be important to test the pricing performance of our fundamental setting, using publicly available PTR auction prices for one or several borders at the same time. Although the small sample size – PTRs are auctioned off only once a month/year – may affect the results, this also poses new challenging questions when it comes to calibrating our model. In order to yield stable and robust calibration results, trade prices of other electricity spot and/or derivatives

⁴²See, e.g., Coulon et al. (2013) for an application in the context of fundamental electricity pricing models.

contracts may need to be considered, too, and it would be interesting to see whether our setting is sufficiently flexible to adequately price all these contracts at the same time. Finally, note that although not discussed explicitly in this paper, the *market splitting* mechanism used in the Nordic electricity market, NordPool, is conceptually similar to the market coupling mechanism analyzed in this paper. Within NordPool, transmission risk between pricing areas is not managed via transmission rights but instead with Contracts for Difference (CfD), which represents an entirely financial-market based solution.⁴³ Given that our model could easily be adapted to this slightly different institutional framework, CfDs could be valued as the price of a forward spread between the local area price and the system price, thereby again taking into account that the market splitting mechanism facilitates prices convergence.

⁴³See, e.g., Kristiansen (2004) or Marckhoff and Wimschulte (2009) for further information.

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Table 1: **Descriptive Statistics: GER–NL Day-Ahead Spread**

This table reports summary statistics for the spread s^i between day-ahead electricity prices in Germany and the Netherlands for selected hours of the day ($i = \{6, 9, 12, 15, 18, 21\}$). The baseload spread (s^{base} ; “Base”) is defined as the (unweighted) arithmetic average of all 24 hourly spreads on a given day. The “Pre-Market Coupling” sample covers the period from 01-Jan-2002 until 08-Nov-2009. The “Post-Market Coupling” sample covers the period thereafter until 31-Dec-2012. $Mean^+$ and $Std. Dev.^+$ are defined as $\mu^+ = \mathbb{E}[s^i | s^i > 0]$ and $\sigma^+ = \sqrt{\mathbb{E}[(s^i - \mu^+)^2 | s^i > 0]}$, respectively. Note that spreads beyond a threshold of 500 EUR/MWh were excluded from the samples in order to avoid distortions. All data is sourced from Bloomberg.

Hour of Day		6	9	12	15	18	21	Base
Pre-Market Coupling	n	3234	3234	3225	3229	3205	3234	3234
	$s^i < 0$	33.6%	52.3%	69.2%	73.1%	65.5%	61.9%	68.6%
	$s^i = 0$	0.4%	0.1%	0.2%	0.2%	0.1%	0.3%	0.0%
	Mean	2.01	-2.45	-15.09	-9.10	-12.07	-4.44	-4.84
	Mean ⁺	5.34	6.58	7.89	5.64	6.29	3.83	3.30
	Mean [−]	-4.50	-10.66	-25.31	-14.51	-21.75	-9.51	-8.57
	Median	1.81	-0.32	-3.90	-3.02	-2.00	-1.55	-1.83
	Min	-129.47	-464.17	-481.07	-477.09	-480.13	-376.49	-154.98
	Max	43.11	256.84	345.05	430.09	412.31	70.14	74.63
	Std. Dev.	7.13	21.97	46.34	33.84	47.07	15.31	13.47
	Std. Dev. ⁺	4.94	13.45	19.82	18.96	19.79	4.92	6.10
	Std. Dev. [−]	6.25	24.83	50.90	36.43	53.93	17.21	14.27
	Skewness	-2.03	-6.83	-4.58	-6.15	-4.97	-8.08	-4.17
	Kurtosis	39.29	141.51	35.15	79.64	40.01	138.19	32.41
Post-Market Coupling	n	784	784	784	784	784	784	784
	$s^i < 0$	14.7%	15.1%	35.1%	38.5%	24.9%	24.1%	60.5%
	$s^i = 0$	83.0%	82.7%	64.5%	61.4%	73.5%	74.4%	30.2%
	Mean	-1.77	-1.17	-4.22	-4.92	-3.00	-2.70	-3.11
	Mean ⁺	2.38	12.15	2.92	0.75	9.10	5.07	0.77
	Mean [−]	-12.47	-9.64	-12.05	-12.78	-12.67	-11.53	-5.25
	Median	0.00	0.00	0.00	0.00	0.00	0.00	-0.50
	Min	-217.23	-40.15	-76.90	-139.76	-57.39	-57.04	-89.43
	Max	7.02	30.01	5.41	0.75	28.58	9.86	8.23
	Std. Dev.	11.58	5.47	8.39	9.63	7.82	6.88	6.67
	Std. Dev. ⁺	2.03	10.30	2.25	n.a.	7.26	3.33	1.38
	Std. Dev. [−]	28.01	8.80	10.31	11.85	10.63	9.57	7.85
	Skewness	-15.45	-2.17	-2.92	-4.83	-2.89	-3.16	-6.43
	Kurtosis	273.80	17.34	12.17	50.54	11.09	12.47	69.76



Figure 1: **Timeline for the Explicit Ex-Ante Auction Scheme**

This figure illustrates the timing sequence of the market for cross-border transmission capacity between France and Germany as well as of the day-ahead electricity markets in these two countries. Note that the nomination procedure, the setup of the capacity auction, and the gate closure times of the power exchanges relate to the explicit ex-ante auction scheme which was in place prior to the implementation of the CWE (Central Western Europe) market coupling as per 09-Nov-2010. See RTE (2009a) and RTE (2009b) for further information.

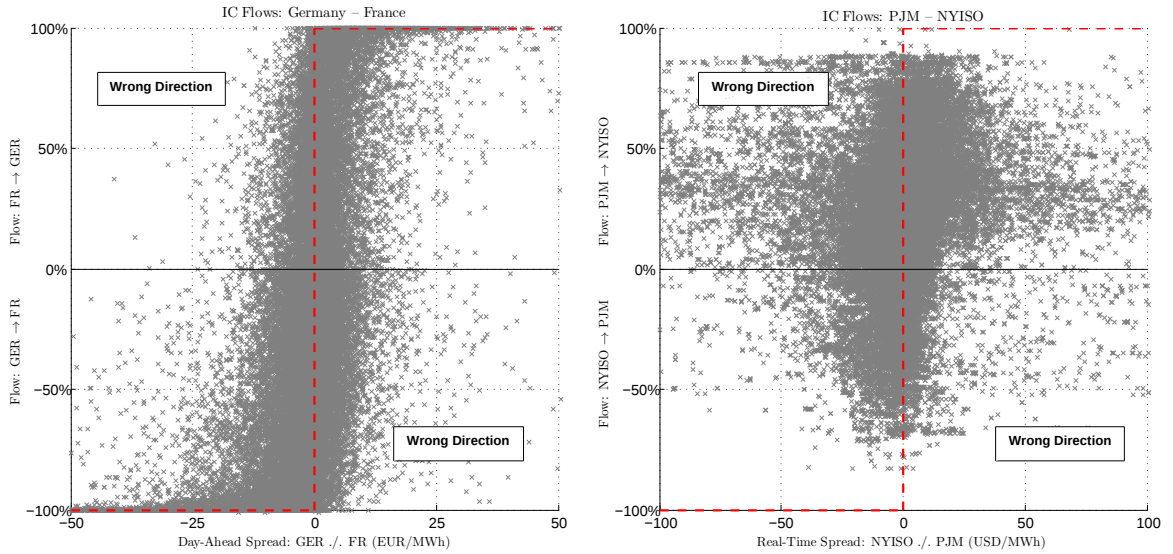


Figure 2: Interconnector Flows: Germany – France and PJM – NYISO

This figure illustrates inefficient interconnector use in both Europe and the US. The LHS graph plots the day-ahead price spread between Germany and France versus net transit nominations (as percentage of interconnector capacity) for the same hours. Data shown for the period from 26-Oct-2006 to 09-Nov-2010. The RHS graph plots the real-time price spread between the PJM and NYISO markets versus 5-minute scheduled flows (as percentage of interconnector flow limits). Data shown for the period from 01-Jan-2013 to 30-Jun-2013. The red dashed line indicates economically efficient flows (as would have occurred under market coupling) such that there is no flow in the wrong direction, nor less-than-optimal flow in the right direction. Source: RTE (<http://clients.rte-france.com>), NYISO (<http://www.nyiso.com>), and Bloomberg.

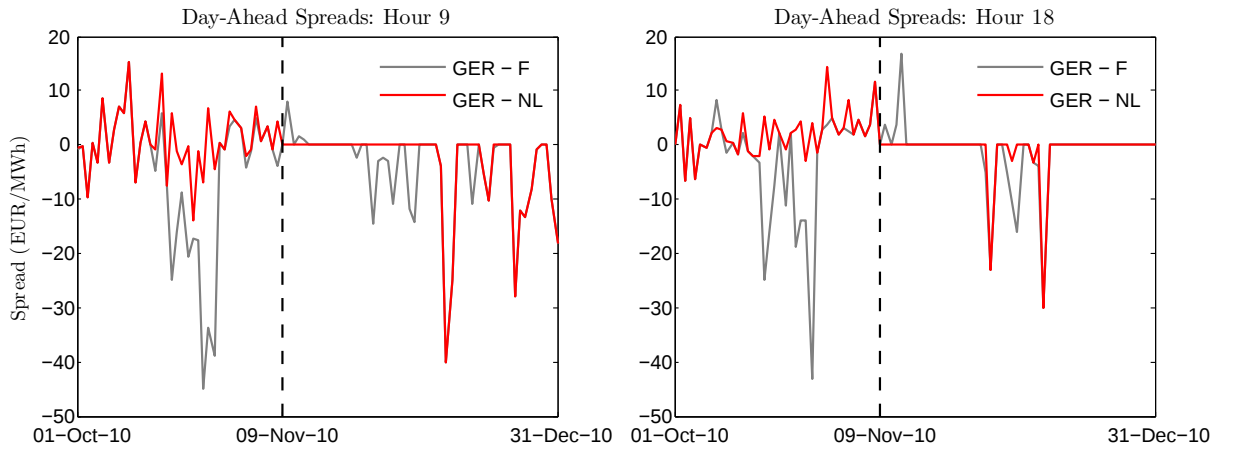


Figure 3: Spread in Day-Ahead Prices: GER – F and GER – NL

This figure shows the spread in day-ahead electricity prices between Germany and France as well as Germany and the Netherlands. The LHS graph displays day-ahead prices for hour 9 of the day (8:00–9:00am), whereas the RHS graph relates to hour 18 (5:00–6:00pm). The vertical dashed line marks the start of the CWE (Central Western Europe) market coupling between Germany, France, the Netherlands, and Belgium as per 09-Nov-2010. All data sourced from Bloomberg.

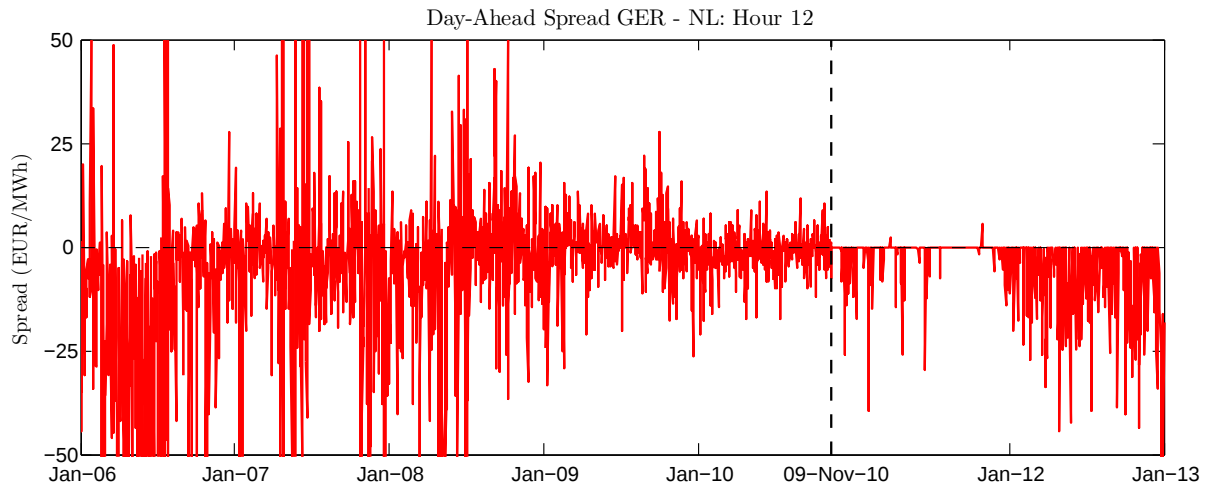


Figure 4: Spread in Day-Ahead Prices: Germany – Netherlands

This figure shows the spread in day-ahead electricity prices between Germany and the Netherlands during the period from 01-Jan-2006 to 31-Dec-2012. Underlying day-ahead prices relate to hour 12 of the day (11:00am–12:00pm). The vertical dashed line marks the start of the CWE (Central Western Europe) market coupling between Germany, France, the Netherlands, and Belgium as per 09-Nov-2010. All data sourced from Bloomberg.

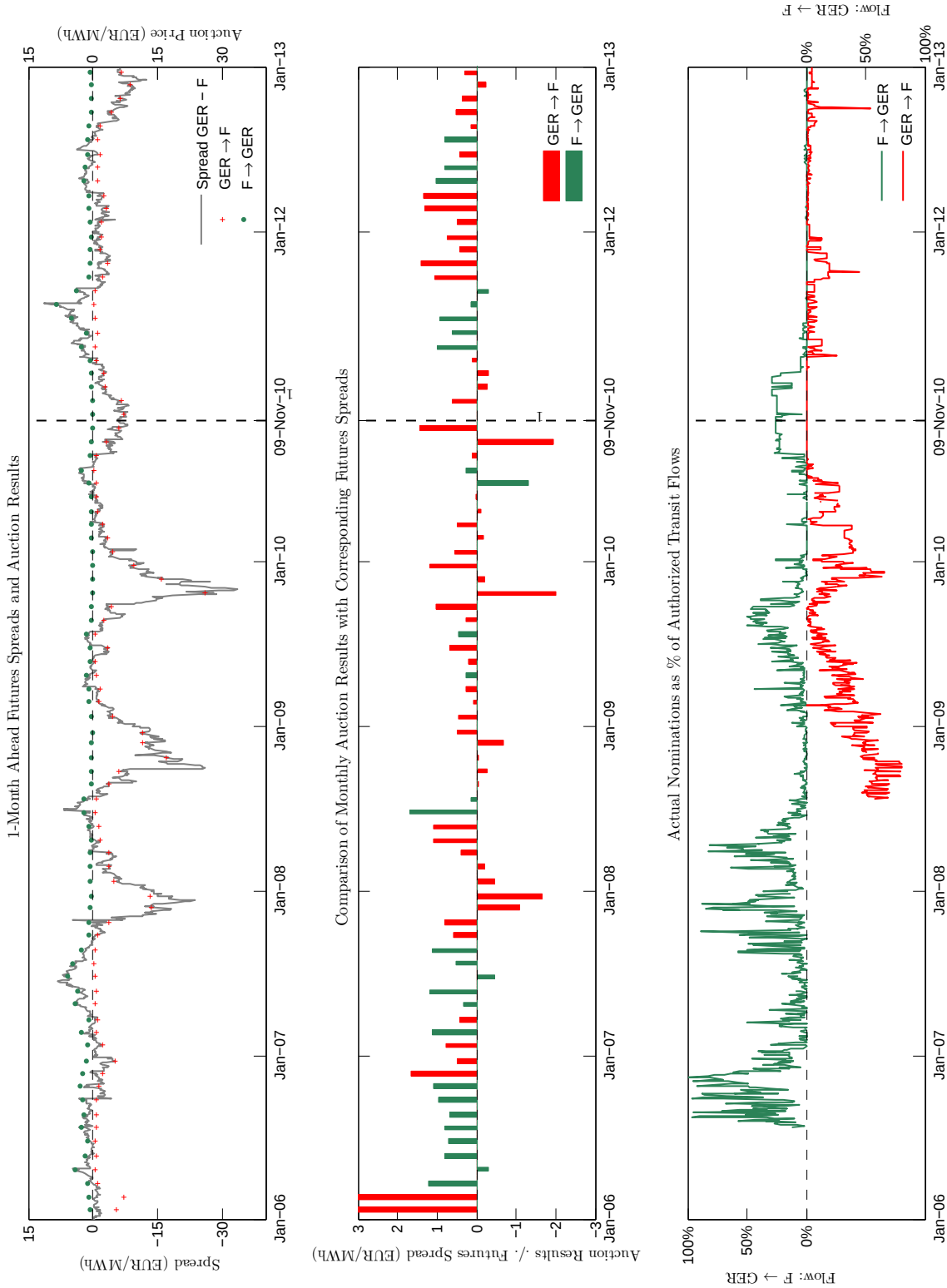


Figure 5: Monthly Auction Results and Nominations at the French – German Interconnector

The top panel shows the spread in (generic) 1-month ahead futures contracts between Germany and France as well as corresponding results for monthly auctions of interconnector capacity for $F \rightarrow GER$ (positive range of secondary y-axis) and $GER \rightarrow F$ (negative range) during the period from 01-Jan-2006 to 31-Dec-2012. The middle panel illustrates the premium of auction prices over the corresponding futures spread. The bottom panel shows the development of actually nominated transit flows as percentage of authorized flows. The vertical dashed line marks the start of the CWE (Central Western Europe) market coupling between Germany, France, the Netherlands, and Belgium as per 09-Nov-2010. All data sourced from Bloomberg, Amprion (<http://www.amprion.net>), and RTE (<http://clients.rte-france.com>).

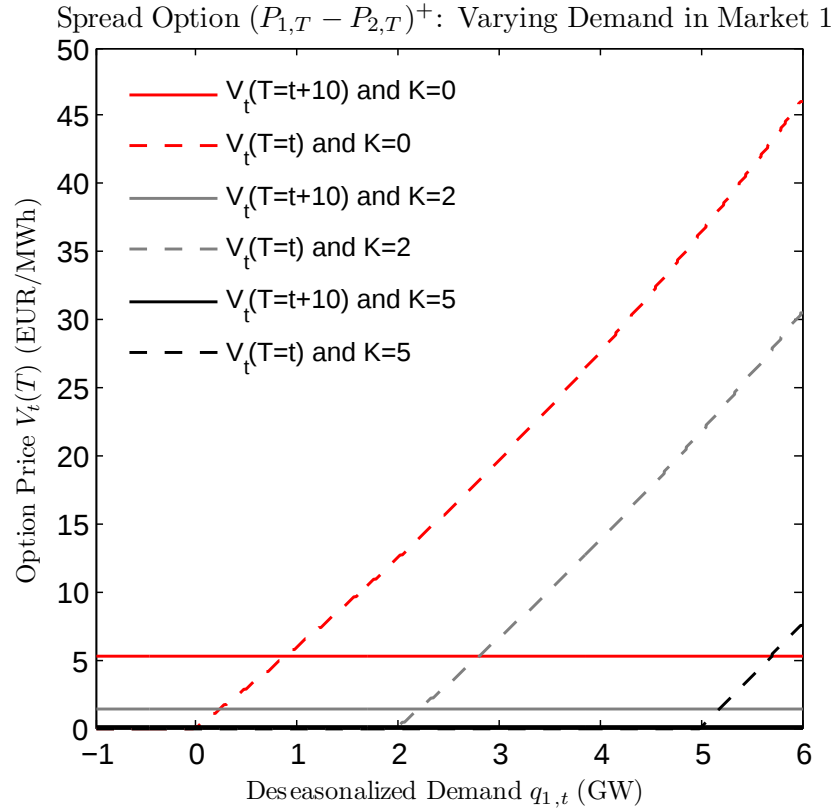


Figure 6: Transmission Right Values and Payoff Profiles: Varying Demand

This figure shows the payoff profiles for an option written on the spot spread between two coupled electricity markets, $(P_{1,T} - P_{2,T})^+$, and assuming given interconnector capacities K of 0 GW, 2 GW, and 5 GW. Additionally, the spread option value $V_t(T)$ is plotted for a time-to-maturity of $\tau = T - t = 10$ days. Note that since the underlying of the option, i.e., the spot spread $P_{1,t} - P_{2,t}$, is itself sensitive to the interconnector capacity K , we instead vary de-seasonalized demand $q_{1,t}$ in market 1 in order to avoid distortions (thereby keeping the other state variables in market 1 and 2 fixed at zero).

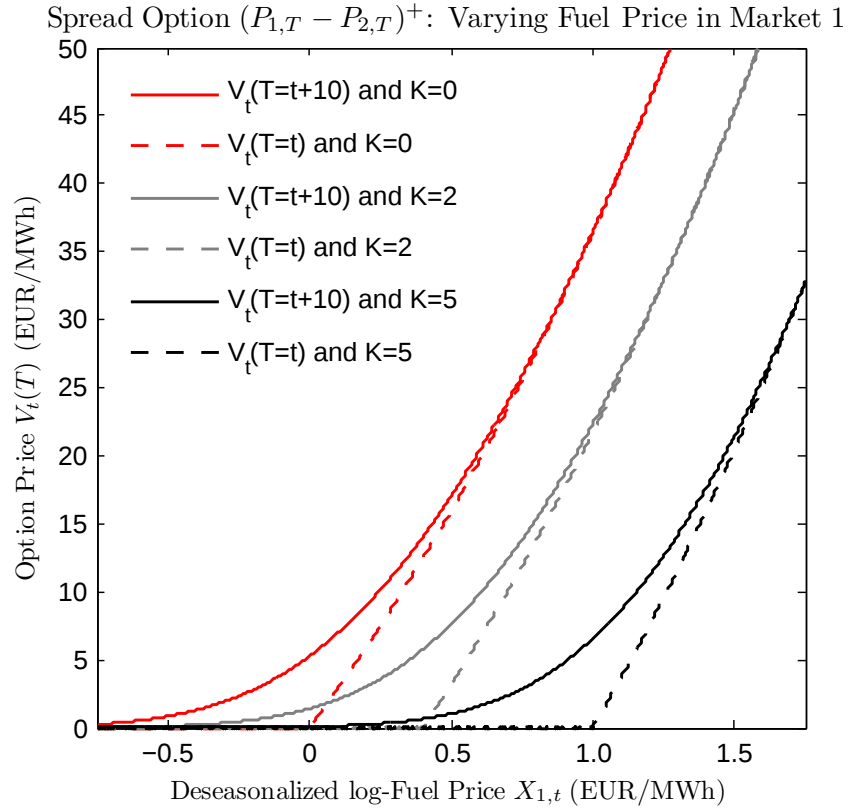


Figure 7: Transmission Right Values and Payoff Profiles: Varying log-Fuel Price

This figure shows the payoff profiles for an option written on the spot spread between two coupled electricity markets, $(P_{1,T} - P_{2,T})^+$, and assuming given interconnector capacities K of 0 GW, 2 GW, and 5 GW. Additionally, the spread option value $V_t(T)$ is plotted for a time-to-maturity of $\tau = T - t = 10$ days. Note that since the underlying of the option, i.e., the spot spread $P_{1,t} - P_{2,t}$, is itself sensitive to the interconnector capacity K , we instead vary the log-fuel price $X_{1,t}$ in market 1 in order to avoid distortions (thereby keeping the other state variables in market 1 and 2 fixed at zero).

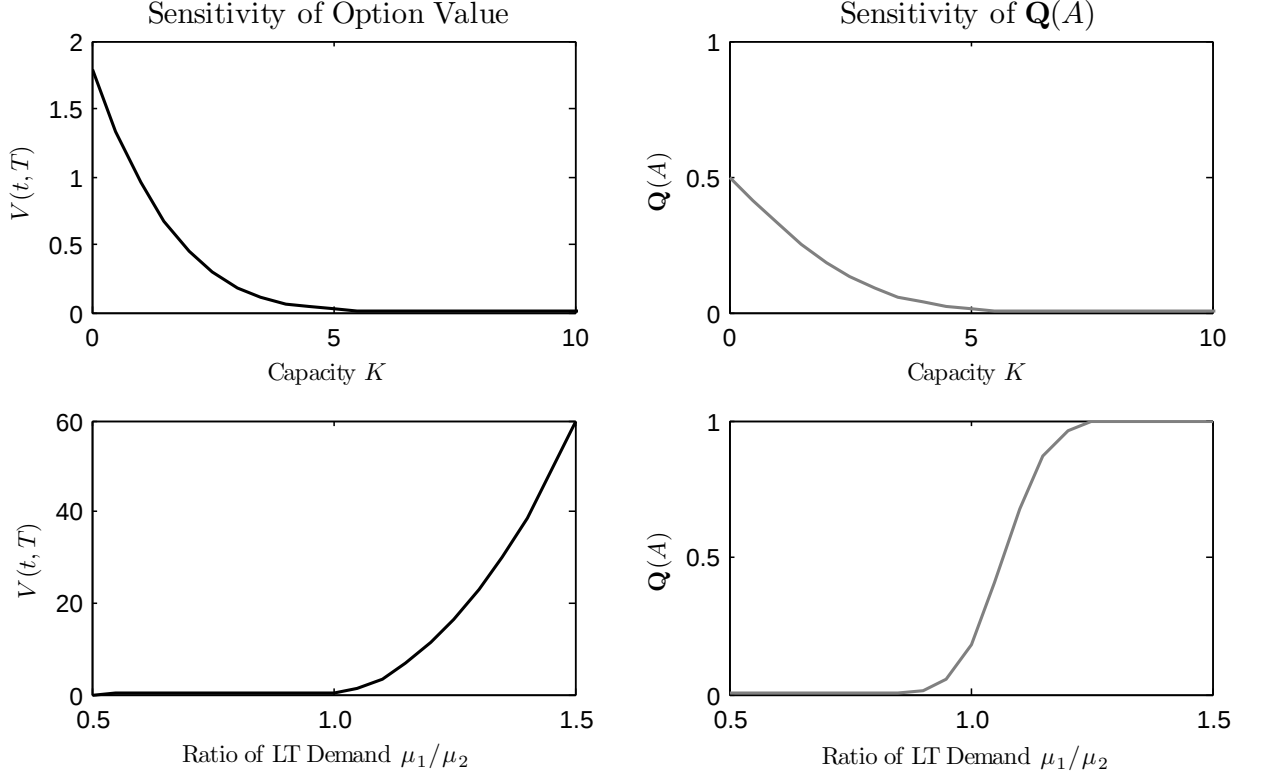


Figure 8: **Transmission Right Sensitivities**

This figure shows sensitivities of the transmission right value $V_t(T)$ (with payoff $(P_{1,T} - P_{2,T})^+$ at time T) as well as of the risk-neutral probability $Q(A)$ (where $Q(A) = \mathbb{E}_t^Q [\mathbb{I}_A]$ with A referring to the event $\tilde{J}(T) \geq K$), when varying key determinants: (i) interconnector capacity K , and (ii) the ratio of unconditional (long-term) demand between the two markets.