THE VALUE OF CREDITOR GOVERNANCE: DEBT RENEGOTIATIONS IN AND OUTSIDE DISTRESS

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ABSTRACT

This paper analyzes a structural model of a levered firm that can renegotiate debt outside and in distress. The firm renegotiates outside distress to waive its financing covenant when raising investment funds and renegotiates in distress to avoid bankruptcy costs. Incorporating the ability to renegotiate both outside and in distress is crucial to explaining timing patterns of debt renegotiations. Capturing realistic incentives and timing patterns of renegotiations allows us to quantify the value of creditor governance. We explain a rich set of empirical facts in terms of influence of renegotiable debt on security values and corporate policies.

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1. Introduction

The traditional governance approach presumes that managers and equity holders determine firm policies and debt holders are passive unless a firm is in distress. Recent empirical studies challenge this view with evidence that creditors influence corporate decisions through debt renegotiations. Specifically, such renegotiations occur frequently, are mostly not due to distress, and have a profound impact on corporate policies (Roberts and Sufi, 2009b; Nini, Smith, and Sufi, 2012; Denis and Wang, 2014; Roberts, 2015). Yet, the theoretical literature lacks a model that rationalizes the timing when debt renegotiations occur. Capturing the timing patterns, however, is crucial to assess the quantitative impact of creditors’ influence on firms.

This paper develops a model to quantify the impact of firms’ ability to renegotiate debt. The novelty of this model is that firms can renegotiate their debt not only in distress, but also outside distress when new funds are raised. We call this renegotiation model the “R-model”. The R-model is based on a structural corporate finance framework of a levered firm in the spirit of Mello and Parsons (1992). To incorporate cross sectional differences in expected financing needs, we consider an investment opportunity as modeled in Hackbarth and Mauer (2012). Following Fan and Sundaresan (2000), firms in distress renegotiate debt with a debt-equity swap (distressed reorganization) to avoid bankruptcy costs. We also allow firms to include a covenant that restricts subsequent debt financing in the initial debt contract. Firms’ aspiration to raise new debt funds at investment leads to debt renegotiation outside distress. Firms implement the covenant ex-ante because renegotiation with the initial debt holders at investment disciplines the ex-post investment and financing decisions. This mechanism to mitigate the agency costs of debt is consistent with the recent empirical literature showing that non-distressed covenant renegotiation constitutes an important governance channel through which debt holders influence firm policies such as investment, operating, or financing decisions (e.g., Roberts, 2015).

The R-model provides several key implications for creditor governance. Specifically, it explains the impact of renegotiable debt on (1) timing patterns, (2) credit spreads, (3) firm value, and (4) corporate policies. First, we show that incorporating renegotiation both outside and in distress is crucial to rationalize the empirically observed timing patterns of renegotiation. Second, the R-model explains the impact of debt renegotiation on credit spreads. It is consistent with the average empirical credit spreads and improves the ability of structural models to match the cross section of observed debt prices. Third,
having a model with realistic motives and timing patterns of renegotiations allows us to quantify the impact of the ability to renegotiate debt on firms. We find that debt renegotiation constitutes an economically important governance channel that increases baseline firm value by 3.8%. Fourth, the R-model explains empirical facts and generates novel predictions on covenant structures and corporate financial policies. We proceed by explaining each of these key findings in more detail.

First, we investigate to what extent the R-model rationalizes the timing patterns of debt renegotiations. As empirical benchmark, we use the sample of private credit agreements in Nini, Smith, and Sufi (2009), for which Denis and Wang (2014) and Roberts and Sufi (2009b) report stylized facts of debt renegotiation patterns. We generate simulated samples of model-implied firms that are structurally similar to this real firms sample. In addition to the R-model, we use a model that considers only distressed reorganization; we label this model the “FSI-model.” Finally, we compare timing patterns of debt renegotiations in the simulated samples to those in the real firms sample. Empirically, the average time elapsed to a renegotiation is around 59% of the initially stated maturity. The R-model closely reflects this statistic by implying a duration of 60.6%. Conversely, the FSI-model predicts a duration of 96.2%, which is well beyond the empirical counterpart. We also show that by including non-distressed renegotiation, we can improve the matching of many additional empirical renegotiation timing patterns.

Second, we use the R-model to explain how renegotiable debt affects credit spreads and debt control premiums. The R-model reflects the average observed credit spreads. It also explains the empirical impact reported in Davydenko and Strebulaev (2007) of the ability to renegotiate debt, bargaining power, and liquidation costs on credit spreads. Additionally, the R-model generates a debt control premium, which is discovered in the recent empirical work of Feldhütter, Hotchkiss, and Karakas (forthcoming). While existing structural models are quite successful in replicating the average observed credit spreads (Chen, 2010; Bhamra, Kuehn, and Strebulaev, 2010), they still struggle to explain the cross-firm variation in credit spreads (Eom, Helwege, and Huang, 2004; Zhang, Zhou, and Zhu, 2009). We show that incorporating debt renegotiation provides a crucial step towards quantitatively rationalizing the cross section of observed credit spreads. The improvement, however, arises only when we consider that debt renegotiations can occur both in distress and outside distress. Our debt pricing results are also of interest for practitioners. They imply that covenant structure, bargaining power, and renegotiation
frictions are quantitatively important to estimate the credit spread of a firm, in addition to the parameters traditionally associated with default risk.

Third, we investigate the quantitative impact of the ability to renegotiate debt on firm value. We start by quantifying the benefit of installing a renegotiable financing covenant. This benefit arises from a mitigation in the agency cost of debt. Specifically, the issuance of new debt to finance the investment cost dilutes the initial debt claim and, hence, transfers wealth from initial debt to equity holders. Therefore, firms without covenant protection acting in the interest of equity holders expropriate initial debt holders through overlevering at investment and overinvestment compared to policies that maximize firm value (Hackbarth and Mauer, 2012). A covenant prohibiting the issuance of new debt protects initial debt holders from this ex-post claim dilution. In case the covenant is enforced, equity holders have to finance the investment cost by issuing additional equity. To increase the value of their claim by the additional tax shield of new debt, equity holders prefer to renegotiate the covenant with initial debt holders at investment. We model this renegotiation as a bargaining game and solve for the Nash solution. This solution allows us to assign different levels of bargaining power to the two parties. Equity holders offer initial debt holders an increase in promised interest rate in exchange for waiving the covenant. This offer reflects that debt holders can negotiate a change in debt terms that improves the value of their claim in many real cases of non-distressed covenant renegotiations (Kahan and Rock, 2009; Feldhütter, Hotchkiss, and Karakas, forthcoming). The increase in promised interest rate and amount of new debt are determined as the Nash bargaining solution. We show that this solution to the renegotiation game leads to the firm value maximizing leverage at investment. Covenant renegotiation also mitigates the overinvestment problem because equity holders do not expropriate initial debt holders at investment by issuing too much new debt. Thus, the possibility of debt covenant renegotiation outside distress reduces the agency costs of debt due to overleverage and overinvestment.

Installing a renegotiable covenant also carries a cost. Without this covenant, firms have a relatively strong incentive to avoid distressed reorganization because they lose the opportunity to expropriate debt holders ex-post. With this covenant, firms cannot expropriate debt holders at investment. Hence, they reorganize earlier. A higher tendency to distressed reorganization, however, is costly owing to the lost tax shield. This mechanism emphasizes the importance of modeling the dependence between distressed and non-distressed amendments to investigate the impact of renegotiable debt on firms.
The benefit of a renegotiable covenant dominates its cost such that firm value increases when incorporating a covenant. Non-distressed covenant renegotiation is crucial to quantify the total value of the ability to renegotiate debt. Specifically, considering solely distressed reorganization increases firm value by only 1.87%. Additionally incorporating covenant renegotiation more than doubles this increase, namely, to 3.76%. This number shows that debt renegotiation is an economically crucial governance channel that considerably increases firm value. We also find that the bargaining power of equity holders is more important for firm value with both distressed and non-distressed renegotiation than when incorporating only distressed reorganization.

Fourth, and finally, we explore the predictions of the R-model on corporate financial policies. The finding that a renegotiable covenant considerably increases firm value provides a rationale for the frequent use of financing covenants. Roberts and Sufi (2009a), for example, show that almost 90% of credit agreements contain a restriction on the borrower’s total debt. By analyzing the impact of firm parameter variation on the benefit and cost of installing a financing covenant, we can derive cross sectional covenant predictions. For instance, firms with stronger bargaining power of equity holders or higher default cost should have a lower tendency to restrict additional debt issuance with covenants. We also show that the ability to renegotiate outside distress increases the optimal market leverage. Additionally, it reduces the sensitivity of market leverage to the value of the investment opportunity. The latter finding suggests that the empirical finding of Billett, King, and Mauer (2007) that financing covenant restrictions attenuate the negative impact of growth opportunities on leverage for public bonds applies to renegotiable private debt as well. Our analysis indicates that this pattern should be even more pronounced for private debt than for public debt. With respect to the timing of investment, the R-model implies that a financing covenant delays investment mainly because equity holders cannot expropriate debt holders upon investment.

Our paper addresses different veins of the literature. Most importantly, we contribute to recent empirical studies suggesting that covenants are primarily renegotiated not to avoid default, but rather to allow creditors intervene in firm policies subject to conflicts of interest (Dichev and Skinner, 2002; Chava and Roberts, 2008; Denis and Wang, 2014; Bradley and Roberts, 2015). Roberts and Sufi (2009b), Wang (2013), and Roberts (2015) support this argument by showing that most debt renegotiations have little to do with corporate distress or default. Even covenant violations rarely lead to firm liquidation or acceleration of a loan but rather entail renegotiation resulting in stronger contractual
restrictions (Smith, Jr. and Warner, 1979; Nini, Smith, and Sufi, 2009; Gopalakrishnan and Parkash, 1995). This literature concludes that debt renegotiation constitutes a debt governance channel for creditors. We include this debt governance aspect by showing that the ability to renegotiate debt reduces both the agency cost of debt and the expected default costs. By exploring a model capturing the empirical timing patterns of debt renegotiations, we provide the first quantitative analysis of the impact of renegotiation as a debt governance channel on firms. Our insights complement those of Gamba and Triantis (2014) who investigate how non-renegotiable debt covenants mitigate investment and financing distortions.

The existing theoretical literature features several models incorporating either distressed reorganization or non-distressed renegotiation. The main idea behind the distressed reorganization models in Giammarino (1989), Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Mella-Barral (1999), Fan and Sundaresan (2000), and Sundaresan and Wang (2007) is that equity holders can reduce contractual debt obligations due to costly bankruptcy threats when firm performance deteriorates. Models with non-distressed renegotiation, to the best of our knowledge, consider only static discrete time approaches. The motives for renegotiation in these studies are related to investment. Bergman and Callen (1991) show that when firm profit is low, equity holders can credibly threaten debt holders to adapt a suboptimal investment policy that saps firm value. This threat allows them to renegotiate debt to their advantage. In Berlin and Mester (1992), the firm manager has an incentive to underinvest in safe activities and overinvest in growth activities. Covenants oblige to a minimum required investment in safe activities. The value of the option to renegotiate stems from the ability to invest less in safe activities and, consequently, more in growth activities when all parties realize that an unfavorable outcome is unlikely. Gorton and Kahn (2000) examine the moral hazard problem between the borrower and lender to analyze the design, pricing, and renegotiation of loan contracts. The borrower can undertake a costly, risk-increasing action (asset substitution), whereas the lender can demand collateral upon the arrival of bad news. The role of the initial debt contract is to impact the bargaining in the subsequent contract renegotiation. As in the R-model, lenders extract some surplus from equity holders’ investment decision by increasing the loan interest rates. Dessein (2005) argues that good borrowers are willing to shift formal control rights over investment to the less informed investor because they want to signal congruent preferences. Bad borrowers find such concessions too costly. Garleanu and Zwiebel (2009) show that since the borrower
is better informed about the potential of future wealth transfer, lenders receive strong ex-ante decision rights. When information is revealed, renegotiation occurs to transfer some control rights back to the borrower.

We contribute to this theoretical literature in three ways. First, we show that one needs to incorporate distressed and non-distressed renegotiation simultaneously to explain empirically observed renegotiation timing patterns. Second, we emphasize that considering distressed and non-distressed renegotiations in isolation neglects the inherent link between them. In particular, the anticipated outcome of non-distressed renegotiation is a main driver of the distressed reorganization risk. Third, we provide a model capturing the empirically observed motives and timing patterns of debt amendments, thereby allowing us to quantify the impact of the ability to renegotiate debt and of bargaining power on the value of corporate securities and firm policies.

Our paper also complements existing quantitative studies on the pricing of debt (Fan and Sundaresan, 2000; Huang and Huang, 2012; Hackbarth, Miao, and Morellec, 2006; Almeida and Philippon, 2007; Chen, Collin-Dufresne, and Goldstein, 2009; Bhamra, Kuehn, and Strebulaev, 2010; Chen, 2010; Arnold, Wagner, and Westermann, 2013). We show that the R-model explains the quantitative impact of debt renegotiation and bargaining power on credit spreads. We, thereby, contribute to the literature by rationalizing that not simply leverage, but also debt structure and debt ownership are important to explain observed debt prices (Datta, Datta-Iskandar, and Patel, 1999; Davydenko and Strebulaev, 2007; Lin, Ma, Malatesta, and Xuan, 2011; He and Xiong, 2012). Additionally, we address the notion that debt prices reflect the surplus extracted by lenders, as formalized by Rajan (1992) in a discrete time model. The corresponding control premium has recently been examined in the empirical bond pricing literature (Feldhütter, Hotchkiss, and Karakas, forthcoming). To the best of our knowledge, we are the first to assess the control premium quantitatively in a debt pricing model.

Finally, we speak to the literature on the impact of financing frictions on investment (Whited, 1992; Hennessy, 2004; Childs, Mauer, and Ott, 2005; Chava and Roberts, 2008; Hackbarth and Mauer, 2012). By modeling debt covenant renegotiation as a specific channel through which financing frictions affect investment, we derive new quantitative insights on the impact of the financing link on investment.

The remainder of this paper is organized as follows. In Section 2, we present the R-model, which is solved in Section 3. Section 4 analyzes the R-model. In Section 5, we
use the R-model to explain empirically observed renegotiation patterns, credit spreads, covenant structures, and financial policies. Finally, Section 6 concludes.

2. The model

Our structural framework is in the spirit of Mello and Parsons (1992) and the extension for investment by Hackbarth and Mauer (2012). The novel feature of the model is that we incorporate not only the possibility of distressed reorganization as suggested by Fan and Sundareshan (2000), but also the ability to renegotiate a covenant outside distress.

In the following, we present the R-model. We first describe the firm’s assets in place and the investment opportunity. Next, we discuss the covenant, debt covenant renegotiation, and the financing of investment. Finally, we present distressed reorganization.

2.1. Assumptions

Assets are continuously traded in complete and arbitrage-free markets. Investors may lend and borrow at the risk-free rate $r$. Corporate taxes are paid at a constant rate $\tau$ on operating cash flows, and full offsets of corporate losses are allowed. Firms act in the best interest of equity holders and choose the investment policy to maximize equity value.

The firm’s assets in place and investment opportunity. We consider an infinitely lived firm with assets in place and an investment opportunity. At each time $t$, assets in place generate a cash flow $X_t$. The cash flow $X_t$ constitutes the exogenous state variable. Following Grossman and Hart (1986) and Bolton and Scharfstein (1996), we assume that $X_t$ is observable, but not verifiable by courts or other outside parties. Hence, an ex-ante contract specifying, for example, financing or investment policies contingent on cash flows is not feasible because courts cannot enforce it. This assumption is standard in the debt overhang literature (e.g., Bhattacharya and Faure-Grimaud, 2001; Favara, Morellec, Schroth, and Valta, forthcoming). The cash flow $X_t$ of the firm follows a geometric Brownian motion under the risk-neutral probability measure $\mathbb{Q}$

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad X_0 > 0, \quad (1)$$

in which $\mu$ is the drift, $\sigma$ the volatility, and $W_t$ a Brownian motion under $\mathbb{Q}$. 

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In practice, investment financing is an important motive of covenant renegotiations (Roberts, 2015). Hence, we incorporate an investment that generates funding needs. The investment opportunity is modeled as an American call option on the firm’s cash flow, analogous to Hackbarth and Mauer (2012). Specifically, if equity holders decide to invest at time $\bar{t}$, an investment cost $I$ must be paid to receive an additional future cash flow of $(s - 1)X_t$ for some factor $s > 1$ for all future times $t \geq \bar{t}$. After investment, the firm consists of only invested assets. The investment decision is irreversible. In reality, many investment opportunities are not perfectly correlated to existing assets or may occur and disappear randomly. Our investment specification is a simplification to investigate the impact of investment funding needs on debt renegotiation in a tractable fashion.

Initially, the firm is financed by issuing equity and private debt of infinite maturity. Private debt is one of the largest if not the largest source of funds for U.S. corporations (Krishnaswami, Spindt, and Subramaniam, 1999; Denis and Mihov, 2003). While in practice renegotiations occur also with public bonds, they are more costly, have a higher exposure to free-rider problems, and are more difficult to coordinate than renegotiations with private debt holders (Rajan, 1992; Krishnaswami, Spindt, and Subramaniam, 1999). After initial debt is issued, the firm pays a total coupon rate $c_A^0$ to initial debt holders (A). The firm also pays corporate taxes at a constant rate $\tau$. If the required debt service exceeds the cash flow, shareholders can inject funds to finance the coupon. Alternatively, shareholders have the possibility to default on their debt obligations (Leland, 1998), in which case the firm is immediately liquidated. Debt holders enjoy absolute priority of their claims. As in Hackbarth and Mauer (2012), debt holders obtain the unlevered value of assets in place times the recovery rate $\alpha$, whereas the investment option is lost in default.

Debt covenant renegotiation and the financing of investment. We allow the firm to install a covenant in the initial debt contract that prevents the issuance of additional debt. Covenants restricting new debt financing are ubiquitous in debt contracts of real firms (e.g., Smith, Jr. and Warner, 1979, Bradley and Roberts, 2015). A covenant specifying future debt financing contingent on cash flows is not feasible because $X_t$ is not

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1Changes to covenants, for example, must be approved by bondholders representing no less than two-thirds of the total principal. Such a majority approval is difficult in the case of diffusely held public bonds because the Trust Indenture Act of 1939 gives the trustees in public bond issues only limited discretion during renegotiation outside bankruptcy.

2Alternatively, we can assume that only a fraction of the investment opportunity is lost in default and that the remaining fraction of the investment opportunity may or may not be levered up in the future. These alternative assumptions have only minor impact on the results.
verifiable by courts. Therefore, we consider a covenant that restricts the issuance of any additional debt. Empirical and theoretical evidence shows that debt restructuring occurs commonly upon lumpy investment (Dudley, 2012; Hackbarth and Mauer, 2012) and that firms initiate covenant modifications primarily to alter investment, operating, or financing policies (Roberts, 2015). Hence, we incorporate that equity holders can renegotiate the covenant with debt holders upon investment to finance the investment cost through both additional equity and new debt. Specifically, debt holders agree to waive the covenant in exchange for a compensation offered by equity holders. The total compensation required by debt holders includes both the full compensation for the dilution of initial debt due to the issuance of new debt and part of the renegotiation surplus associated with the additional tax shield of new debt. The compensation to initial debt holders consists of an additional compensation coupon such that the total coupon (after investment) to initial debt holders is $c^A_{01} = c^A_0 + c^A_1 \geq c^A_0$. Changes in interest rate at renegotiations are common in practice. Roberts and Sufi (2009b), for example, find that 55% of renegotiations entail alteration of the coupon, with an average change of 64 bps in interest rate spread, or 40% of the initial spread. Several contract terms such as amount and maturity are usually renegotiated along with the interest rate (Denis and Wang, 2014). In real non-distressed covenant renegotiations, debt holders can typically negotiate a change in contract terms that improves the value of their claim (Kahan and Rock, 2009; Feldhüttner, Hotchkiss, and Karakas, forthcoming). Our compensation coupon captures this observation in a tractable fashion. In Section 3.3, we show that the mean of the compensation does not alter the model’s solution and results and that the assumption of compensation via an interest rate alteration is therefore without loss of generality. The coupon to new debt holders (B) after renegotiation is denoted by $c^B_0$. The sum of the values of new debt and new equity corresponds to the investment cost $I$.

Debt covenant renegotiation constitutes a simple approach through which firms can relax the financing friction of covenants in place. While several alternative channels can remove a covenant, they are costly to firms. Repaying callable or short term debt, for example, entails restructuring costs (e.g., Christensen, Flor, and Miltersen, 2014) and search frictions to obtain new financing (e.g., Hugonnier, Malamud, and Morellec, 2015). Violating a covenant without the consent of the creditors imposes serious consequences on firms’ investment and financing policies, collateral requirements, monitoring and reporting frequencies, ratings, CEO turnover, and interest rate spreads (e.g., Chava and Roberts, 2008; Nini, Smith, and Sufi, 2009). These consequences could explain why
covenant renegotiations occur more frequently than covenant violations (Denis and Wang, 2014). For tractability, we assume that the alternative channels to remove a covenant are prohibitively costly.

When equity holders decide to invest, the new capital structure is determined as the Nash solution to the renegotiation game. The renegotiation game is characterized as follows. Debt holders can enforce the prevailing covenant and prevent equity holders from issuing additional debt, which constitutes the outside option or disagreement point of the renegotiation game. We assume that the outside option of equity holders is to immediately invest and finance the cost by issuing equity only. The Nash solution determines the sharing rule between equity and debt holders for the surplus from financing the investment cost by issuing a mix of debt and equity as opposed to financing with an equity issuance only. While equity holders receive surplus through the issue proceeds of new debt, the initial debt holder’s surplus is obtained through the compensation coupon. Hence, the sharing rule corresponds to the combination of coupons \( \{ c_A^1, c_B^1 \} \) given the initial coupon \( c_A^0 \). Equity holders’ bargaining power is denoted by \( \eta \), and, hence, debt holders’ bargaining power is given by \( 1 - \eta \).

Reorganization: Debt renegotiation in corporate distress. We model a distressed reorganization as a debt-equity swap following Fan and Sundaresan (2000). In particular, if cash flows deteriorate, equity holders offer debt holders to swap their original debt against equity. A disagreement triggers immediate default as described above, inducing a loss of the investment opportunity and a fraction \( 1 - \alpha \) of the unlevered value of assets in place. Thus, the firm’s claim holders have an incentive to reorganize to avoid these default costs. The fraction of equity offered to debt holders in exchange for their debt, denoted by \( 1 - \theta \), corresponds to the Nash solution. A distressed debt-equity swap implies that the firm becomes an all-equity firm, and, in particular, it loses its tax shield. Hence, while corporate taxes encourage debt financing by shielding part of a firm’s cash flow from taxation, distressed reorganization limits the incentive to issue debt.

Figure 1 presents the timeline of the model. Reorganization can occur before or after investment. We assume that the option to invest is preserved in case of previous reorganization. That is, if equity holders first reorganize after firm initiation (“initial reorganization”), they may still invest later. If equity holders invest subsequently (“investment after initial reorganization”), they issue new debt to finance investment. Equity
holders can again decide to reorganize this new debt ("reorganization after investment following reorganization") in case of distress.

If reorganization occurs after covenant renegotiation at investment ("reorganization after investment"), the fraction $1 - \theta$ of the unlevered firm value is offered jointly to initial and new debt holders. We assume that this fraction $1 - \theta$ is shared between initial and new debt holders in proportion to the value of their respective coupons. That is, initial debt holders receive a fraction $(1 - \theta_1) \frac{c_0}{c_0 + c_1}$ and new debt holders receive a fraction $(1 - \theta_1) \frac{c_1}{c_0 + c_1}$ of the unlevered firm value. This sharing rule is motivated by equal priority of debt claims. Section 3 shows that value functions and corporate policies are not affected by this sharing rule.

3. Model solution

Figure 1 illustrates the use of subscripts in the model timeline. Value functions and parameters after firm initiation but before covenant renegotiation at investment or initial reorganization carry a subscript 0. After covenant renegotiation at investment or after initial reorganization, we use a subscript 1. To distinguish between these two cases, the additional subscript $l$ ($l$ for low) labels value functions after initial reorganization, while the subscript $h$ ($h$ for high) labels value functions after covenant renegotiation at investment. Value functions and parameters after reorganization following investment and after investment following reorganization carry a subscript 2. Finally, a subscript 3 indicates the case in which a firm has reorganized twice (reorganization after investment following initial reorganization).

The model is solved by backward induction. First, Section 3.1 calculates the value functions after the initial reorganization. These calculations include the solution to the final reorganization game and the investment decision after initial reorganization. Next, we derive the value functions and reorganization policy after covenant renegotiation at investment (Section 3.2). Subsequently, Section 3.3 defines and solves the bargaining game of covenant renegotiation determining the compensation coupon to initial debt holders and the coupon of new debt. Finally, we solve for the value functions of corporate
securities before covenant renegotiation or distressed reorganization (Section 3.4), and
determine the initial reorganization threshold, the investment boundary at which the
covenant is renegotiated, and the optimal initial capital structure (Section 3.5).

### 3.1. Initial reorganization and the value functions thereafter

The value functions after investment following reorganization correspond to the case ana-
lyzed in Fan and Sundaresan (2000). An overview of the final reorganization is presented
in the following, while further details can be found in Appendix A.

We start by considering the reorganization after investment following reorganization. Denote this reorganization boundary by $S_2$. In this reorganization game, the outside option is that equity holders default on their debt obligation and debt holders receive the unlevered after-tax asset value net of default costs. The unlevered after-tax asset value is calculated as

$$v_3(X) = \frac{1 - \tau}{r - \mu}X.$$  

Following Fan and Sundaresan (2000), the sharing rule $1 - \theta_2$, i.e., the fraction of the unlevered assets offered to debt holders in exchange for their claim, is determined as the Nash solution to the final reorganization game

$$\theta_2 = \arg \max \left\{ \tilde{\theta}_2 v_3(S_2) - 0 \right\} \eta \left\{ \left( 1 - \tilde{\theta}_2 \right) v_3(S_2) - \alpha v_3(S_2) \right\}^{1-\eta},$$

and

$$= \eta (1 - \alpha).$$

The final reorganization boundary as well as the value functions for debt and equity after investment following reorganization are presented in closed form in Appendix A.

Next, we consider the period after initial reorganization but before investment. Denote the corresponding investment threshold by $U_1$. In this case, the firm is all-equity financed with the option to simultaneously invest and relever.

Using the closed-form solution for the firm value after investment following reorganization (see Appendix A), we calculate the investment boundary $U_1$ in closed form as

$$U_1 = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu) I}{(s - 1) (1 - \tau) + s \tau (1 - \beta_2)^{1/\mu} (1 - \theta)},$$

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in which
\[ \beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left( \frac{1}{2} - \frac{\mu}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}}. \]  
(6)

The value of equity, \( e_{1l}(X) \), corresponds to the firm value calculated in closed form in Appendix A.

Next, we consider the initial reorganization. Denote the initial reorganization boundary by \( S_0 \). The outside option is that equity holders default on their debt obligation, and debt holders receive the unlevered value of assets in place net of default costs. That is, the outside option is given as a fraction \( \alpha \) of the unlevered asset value \( v_1(X) = \frac{1 - \tau}{r - \mu} X \). The sharing rule \( 1 - \theta_0 \), i.e., the fraction of the unlevered asset value offered to debt holders in exchange for their claim in the initial reorganization, is determined as the Nash solution to the initial reorganization game

\[
\theta_0 = \arg \max_{\tilde{\theta}_0} \left\{ \tilde{\theta}_0 e_{1l}(S_0) - 0 \right\}^\eta \left\{ \left( 1 - \tilde{\theta}_0 \right) e_{1l}(S_0) - \alpha \frac{1 - \tau}{r - \mu} S_0 \right\}^{1-\eta}. \]  
(7)

In Appendix A, we show that the sharing rule to this reorganization game is given as

\[
\theta_0 = \eta \left( 1 - \alpha \frac{v_1(S_0)}{e_{1l}(S_0)} \right). \]  
(8)

Eq. (8) reveals that the fraction \( 1 - \theta_0 \) offered to debt holders in exchange for their claim depends on the reorganization boundary \( S_0 \). The functional form of the solution to \( \theta_0 \) differs from the functional form of the solution to the reorganization game after investment following reorganization \( \theta_2 \), which is a constant determined by Eq. (4). The reason for this difference is that the outside option of initial reorganization determining Eq. (7) implies that the investment opportunity and a fraction of the unlevered assets are lost. Conversely, the outside option of the final reorganization determining Eq. (3) reflects that only a fraction of the unlevered asset value is lost.
3.2. Value functions after covenant renegotiation at investment

Reorganization after covenant renegotiation is analogous to the final reorganization after investment following reorganization. Hence, the Nash solution is

$$\theta_1 = \eta (1 - \alpha),$$

as in Fan and Sundaresan (2000). Let $d_{1h} (\cdot; c^A_{01})$, $d_{1h} (\cdot; c^B_1)$, and $e_{1h} (\cdot; c^A_{01} + c^B_1)$ denote the value of initial debt, new debt, and equity, respectively, after covenant renegotiation at investment. Because of the existence of two creditors at this stage, we explicitly write the dependence of the coupon. $S_1$ is the reorganization boundary for the debt-equity swap after covenant renegotiation at investment. The value functions of total debt and equity are calculated in Fan and Sundaresan (2000). They are analogous to the case of investment after initial reorganization (see Section 3.1). We provide the full solution to these value functions in Appendix B.

3.3. Debt covenant renegotiation and investment

The threshold triggering covenant renegotiation at investment is denoted by $U_0$. We define the sharing rule as $\{ c^A_{01}, c^B_1 \}$, i.e., as the total coupon to initial debt holders and the coupon of new debt whose issue proceeds accrue to equity holders. The surplus to equity from covenant renegotiation is the difference between the values of equity in renegotiation and in disagreement. The value of equity at covenant renegotiation is the total value of equity given the cash flow after investment and the total new coupon plus the issue proceeds from new debt less the investment cost. The value of equity in disagreement corresponds to the total value of equity given the cash flow after investment, but a coupon equal to the initial coupon less the investment cost. Hence, the surplus to equity holders at covenant renegotiation, $SE(U_0; c^A_{01}, c^B_1)$, is

$$SE(U_0; c^A_{01}, c^B_1) = e_{1h} \left( sU_0; c^A_{01} + c^B_1 \right) + d_{1h} \left( sU_0; c^B_1 \right) - I - e_{1h} \left( sU_0; c^A_0 \right) - I$$

Similarly, we calculate the surplus to debt holders as the difference between the value of debt at covenant renegotiation and the value of debt in disagreement. Upon covenant renegotiation, equity holders promise initial debt holders an additional compensation.
coupon of \(c^A_{01} - c^A_0 > 0\). In disagreement, the coupon to debt holders remains unchanged at the initial coupon. Hence, the surplus to debt holders, \(SD \left(U_0; c^A_{01}, c^B_1\right)\), is
\[
SD \left(U_0; c^A_{01}, c^B_1\right) = d_{1h} \left(sU_0; c^A_{01}\right) - d_{1h} \left(sU_0; c^A_0\right).
\]
(11)

\(\{c^A_{01}, c^B_1\}\) is determined as the Nash solution to the covenant renegotiation game:
\[
\{c^A_{01}, c^B_1\} = \max_{\{c^A_{01}, c^B_1\}} \left\{ SE \left(U_0; \tilde{c}^A_{01}, \tilde{c}^B_1\right) \right\}^{\eta} \left\{ SD \left(U_0; \tilde{c}^A_{01}, \tilde{c}^B_1\right) \right\}^{1-\eta}
\]
\[
= \max_{\{c^A_{01}, c^B_1\}} \left\{ e_{1h} \left(sU_0; \tilde{c}^A_{01} + \tilde{c}^B_1\right) + d^B_{1h} \left(sU_0; \tilde{c}^B_1\right) - e_{1h} \left(sU_0; c^A_0\right) \right\}^{\eta}
\]
\[
\cdot \left\{ d_{1h} \left(sU_0; \tilde{c}^A_{01}\right) - d_{1h} \left(sU_0; c^A_0\right) \right\}^{1-\eta},
\]
(12)
in which we used Eqs. (10) and (11). The total surplus, \(ST \left(U_0; c^A_{01}, c^B_1\right)\), is calculated as
\[
ST \left(U_0; c^A_{01}, c^B_1\right) = SE \left(U_0; c^A_{01}, c^B_1\right) + SD \left(U_0; c^A_{01}, c^B_1\right)
\]
(13)
\[
= e_{1h} \left(sU_0; c^A_1\right) + d^B_{1h} \left(sU_0; c^B_1\right) + d_{1h} \left(sU_0; c^A_1\right)
\]
\[
- e_{1h} \left(sU_0; c^A_0\right) - d_{1h} \left(sU_0; c^A_0\right)
\]
\[
= f_{1h} \left(sU_0; c_1\right) - e_{1h} \left(sU_0; c^A_0\right) - d_{1h} \left(sU_0; c^A_0\right)
\]
(15)
in which \(f_{1h} (\cdot)\) denotes the firm value after covenant renegotiation at investment. The following Proposition 1 presents the basic properties of the Nash solution to the covenant renegotiation game. All proofs can be found in the Appendix.

**Proposition 1.** If initial debt carries a renegotiable covenant preventing the issuance of new debt, the Nash solution to the covenant renegotiation game
\[
\{c^A_{01}, c^B_1\} = \max_{\{c^A_{01}, c^B_1\}} \left\{ e_{1h} \left(sU_0; \tilde{c}^A_{01} + \tilde{c}^B_1\right) + d^B_{1h} \left(sU_0; \tilde{c}^B_1\right) - e_{1h} \left(sU_0; c^A_0\right) \right\}^{\eta}
\]
\[
\cdot \left\{ d_{1h} \left(sU_0; \tilde{c}^A_{01}\right) - d_{1h} \left(sU_0; c^A_0\right) \right\}^{1-\eta},
\]
(16)

exhibits the following properties:

(i) The total coupon \(c_1 = c^A_{01} + c^B_1\) determined by the Nash solution corresponds to the first-best coupon \(c^{fb}_1\) maximizing the value of the firm, i.e.,
\[
c_1 = c^{fb}_1 = \frac{r}{r - \mu} \frac{\beta_2 - 1}{\beta_2} (1 - \beta_2) \frac{1}{\sqrt{2}} (1 - \theta) sU_0.
\]
(17)
Further, the total surplus from covenant renegotiation, \( ST(U_0; c^A, c^B) \), depends on the coupons \( c^A_{01} \) and \( c^B_1 \) only through \( c_1 \), i.e., the sum of the coupons to initial and new debt holders. It is given by

\[
ST(U_0; c^A, c^B) = ST(U_0; c_1) = f_{1h}'(sU_0) - e_{1h}(sU_0; c^A) - d_{1h}(sU_0; c^A),
\]

in which \( f_{1h}'(sU_0) \) denotes the first-best firm value at the cash flow level \( sU_0 \).

(ii) \( \{c^A_{01}, c^B_1\} \) is such that the surplus from covenant renegotiation to initial debt holders, \( SD(U_0; c^A, c^B) \), and the surplus from covenant renegotiation to equity holders, \( SE(U_0; c^A_{01}, c^B_1) \), satisfy

\[
SE(U_0; c^A_{01}, c^B_1) = \eta ST(U_0; c_1) \quad (19)
\]

\[
SD(U_0; c^A, c^B) = (1 - \eta) ST(U_0; c_1), \quad (20)
\]

i.e., the two parties receive a fraction of the total surplus corresponding to their respective bargaining power.

Proposition 1 is intuitive. Property (i) states that covenant renegotiation leads to first-best leverage at investment. The reason is Pareto efficiency of the Nash solution. Property (ii) shows that the total renegotiation surplus is shared according to the bargaining power of the two parties.

The properties of the Nash solution to the covenant renegotiation game are robust to the assumptions on the sharing rule of equity in reorganization between initial and new debt holders as well as to the mean of compensation to initial debt holders upon renegotiation. Specifically, the presented model assumes that at reorganization after investment, equity is assigned to debt holders according to the fraction of their respective coupon. That is, initial [new] debt holders obtain a fraction \( (1 - \theta_1) \frac{c^A_{01}}{c^A_{01} + c^B_1} \) of \( (1 - \theta_1) \frac{c^B_1}{c^A_{01} + c^B_1} \) of total equity. Further, we assume that initial debt holders are compensated with an additional coupon \( c^A_{01} - c^A_0 \). Proposition 1 remains valid with alternative assumptions. For example, the assumption that initial debt holders are compensated with a one-time payment at covenant renegotiation still yields first-best leverage at investment and a sharing of the renegotiation surplus proportional to the claimants’ bargaining power.
3.4. Value functions before covenant renegotiation or initial reorganization

In this section, we present the value functions before covenant renegotiation or initial reorganization for equity and corporate debt, denoted by \( e_0(X) \) and \( d_0(X) \), respectively. The reorganization threshold is denoted by \( S_0 \), and the boundary for covenant renegotiation at investment is denoted by \( U_0 \).

**Proposition 2.** (i) The value of equity in the continuation region \( S_0 \leq X \leq U_0 \) is given by

\[
e_0(X) = A_0^{e_0} + A_1^{e_0} X + A_2^{e_0} X^\beta_1 + A_3^{e_0} X^\beta_2,
\]

in which

\[
\beta_{1,2} = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left( \frac{1}{2} - \frac{\mu}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}},
\]

\[
A_0^{e_0} = -\frac{(1 - \tau) c_0^A}{r}, \quad A_1^{e_0} = \frac{1 - \tau}{r - \mu}.
\]

\( A_2^{e_0}, A_3^{e_0} \) jointly solve the system

\[
M \begin{bmatrix} A_2^{e_0} \\ A_3^{e_0} \end{bmatrix}^T = b^{e_0},
\]

in which

\[
M = \begin{bmatrix} S_0^{\beta_1} & S_0^{\beta_2} \\ U_0^{\beta_1} & U_0^{\beta_2} \end{bmatrix},
\]

\[
b^{e_0} = \begin{bmatrix} -A_0^{e_0} - A_1^{e_0} S_0 + \theta_0 e_{1I} (S_0) \\ \eta F_{1h} s U_0 + (1 - \eta) e_{1h} (s U_0; c_0^A) - \eta d_{1h} (s U_0; c_0^A) - A_0^{e_0} - A_1^{e_0} U_0 \end{bmatrix}.
\]

\( F_{1h} \) is the factor to calculate the first-best firm value \((f_{1h}(X) = F_{1h} X)\), i.e.,

\[
F_{1h} = \left[ 1 - \tau + \tau (1 - \beta_2) \frac{1}{\beta_2} (1 - \theta) \right] \frac{1}{r - \mu},
\]

and \( e_{1I} (\cdot) \) is given in closed form in Appendix B, Eqs. (58)-(60).

(ii) The value of corporate debt in the continuation region \( S_0 \leq X \leq U_0 \) is given by

\[
d_0(X) = A_0^{d_0} + A_2^{d_0} X^\beta_1 + A_3^{d_0} X^\beta_2,
\]
in which $\beta_{1,2}$ are defined in Eq. (22) and

$$A_0^d = \frac{c_0^A}{r}.$$  \hfill (29)

$A_2^d, A_3^d$ jointly solve the system

$$M \begin{bmatrix} A_2^d & A_3^d \end{bmatrix}^T = b^d,$$  \hfill (30)

in which

$$M = \begin{bmatrix} S_0^{\beta_1} & S_0^{\beta_2} \\ U_0^{\beta_1} & U_0^{\beta_2} \end{bmatrix},$$  \hfill (31)

$$b^d = \begin{bmatrix} -A_0^d + (1 - \theta_0) e_{11} (S_0) \\ (1 - \eta) F_{1h} sU_0 + (1 - \eta) e_{1h} (sU_0; c_0^A) - \eta d_{1h} (sU_0; c_0^A) - A_0^d \end{bmatrix}.$$  \hfill (32)

$F_{1h}$ is as in Eq. (27) and Appendix B, Eqs. (58)-(60) states $e_{11} (\cdot)$ in closed form.

The values of initial debt and equity are independent of our assumptions on the equity sharing rule in reorganization between initial and new debt holders as well as of the mean of compensation to initial debt holders upon covenant renegotiation. The reason is that the properties of the Nash solution are robust to alternative assumptions. For example, a sharing rule in reorganization less favorable for initial debt holders entails an increase in compensation coupon. However, the Nash solution requires that the increase in compensation coupon lead to the same surplus as with a proportional sharing rule. Therefore, the values of equity and initial debt remain constant.

### 3.5. Reorganization threshold, renegotiation boundary, and capital structure

For a given initial coupon $c_0^A$, equity holders solve

$$\{ S_0, U_0 \} = \arg \max_{S_0, U_0} e_0 (X).$$  \hfill (33)
The boundary conditions for equity are

\[ e_0 (S_0) = \theta_0 e_{1l} (S_0) \]  \hspace{1cm} (34)
\[ e_0 (U_0) = e_{1h} (sU_0; c_1) - \left( I - d_{1h} (sU_0; c_1^B) \right) . \]  \hspace{1cm} (35)

Hence, the smooth-pasting conditions read

\[ \frac{\partial}{\partial X} e_0 (X) |_{X=S_0} = \eta \frac{\partial}{\partial X} e_{1l} (X) |_{X=S_0} - \eta \alpha \frac{1 - \tau}{r - \mu} \]  \hspace{1cm} (36)
\[ \frac{\partial}{\partial X} e_0 (X) |_{X=U_0} = \eta f_{1h} s + (1 - \eta) \frac{\partial}{\partial X} e_{1h} (sX; c_A^0) |_{X=U_0} - \eta \frac{\partial}{\partial X} d_{1h} (sX; c_A^0) |_{X=U_0} \]  \hspace{1cm} (37)

in which \( \theta_0 \) is inserted as given in Eq. (7). Recall that \( e_{1l} (\cdot), e_{1h} (\cdot; c_A^0), \) and \( d_{1h} (\cdot; c_A^0) \) as well as their partial derivatives are available in closed form. Finally, equity holders choose the initial capital structure by maximizing the value of their objective function ex-ante. Hence, equity holders solve

\[ c_A^0 = \arg \max_{c_A^0} \{ e_0 (X_0) + d_0 (X_0) \} , \]  \hspace{1cm} (38)

subject to Eqs. (36)-(37). A closed-form solution does not exist. We use numerical procedures and verify the optimality of the renegotiation and reorganization boundaries numerically.

4. Model analysis

In this section, we derive the main implications of the covenant renegotiation feature for corporate policies and firm value.

4.1. Baseline parameter selection

We use baseline parameter values to reflect a typical S&P 500 firm. The risk free interest rate is \( r = 5\% \), the risk-neutral growth rate of the cash flows \( \mu = 3\% \), the volatility of the cash flow \( \sigma = 23\% \), the tax advantage of debt \( \tau = 15\% \), and the recovery rate \( \alpha = 60\% \). The initial cash flow is normalized to \( X_0 = 1 \). The bargaining power of equity holders is set to \( \eta = 0.5 \), as in the base case of Fan and Sundaresan (2000).
For the investment opportunity, we choose an investment cost of $I = 20$. A scale parameter $s = 1.8$ implies an initial market-to-book ratio of 1.62 for our baseline firm, which closely reflects the average in our empirical sample of firms with private credit agreements from Nini, Smith, and Sufi (2009). The initial market-to-book ratio of a model firm is calculated by dividing the market value of the firm by the value of invested assets.

### 4.2. Impact of a renegotiable covenant

Table I compares corporate policies and firm values in different models and for various parameters. The NR-model has no debt renegotiation or covenant (cf. Hackbarth and Mauer, 2012). The FSI-model is the distressed reorganization model of Fan and Sundaresan (2000) augmented for investment as in Hackbarth and Mauer (2012). Equity holders choose investment, financing, and distressed reorganization policies that maximize the ex-post value of their claim. The FSI-model is similar to the model of Sundaresan and Wang (2007), the only difference being that parties in the latter model renegotiate contractual coupon payments instead of a debt-equity swap. The FSI-first-best corresponds to the FSI-model but applies investment and financing policies to maximize firm value. Equity holders choose the distressed reorganization policy. Details of the FSI-first-best model are presented in Appendix F. In the FSI-model, equity holders issue excessive new debt at investment (overleverage): the leverage at investment is 72% in the FSI-model, whereas it is only 63% in the FSI-first-best. The reason is that equity holders do not internalize the impact of the increased distressed reorganization risk associated with issuing new debt on the value of initial debt, but fully benefit from the additional tax shield. Because the investment and restructuring benefit at investment accrues to equity holders, and due to the transfer of wealth from initial debt to equity holders (expropriation) owing to the large issuance of new debt, equity holders also invest too soon (overinvestment). In particular, the renegotiation (investment) boundary declines from 2.76 in the FSI-first-best to 1.94 in the FSI-model. The FSI-model firm implements a lower initial coupon and, hence, a smaller tax shield than the FSI-first-best firm because the overleverage at investment and overinvestment problems render initial debt more expensive. Due to these agency costs of debt, the value of the FSI-first-best firm is 2.15% above that of the FSI-model. Qualitatively, the presented agency costs also arise without distressed reorganization, as shown in Hackbarth and Mauer (2012). In the framework
with distressed reorganization, however, they are larger, primarily because firms with the possibility of distressed reorganization choose a higher initial coupon ex-ante owing to their ability to avoid costly default through reorganization. This higher coupon enlarges the agency cost of debt.

The gross benefit to firms of installing a renegotiable financing covenant in the initial debt contract consists of mitigating the agency costs of debt. In particular, a renegotiable covenant solves overleverage at investment, as shown in Proposition 1. It also mitigates the large investment distortion (overinvestment) because renegotiation prevents expropriation of initial debt holders. Without overleverage at investment and smaller investment distortion, a firm implements a larger initial coupon. We measure the benefit from the mitigation of total agency costs of debt as the difference between the FSI-first-best and FSI-model firm values. As shown in column $GB$, this benefit corresponds to 2.15%.

Including a financing covenant also entails a cost. In the FSI-models, equity holders capture the investment benefit, which includes the value of exercising the investment opportunity plus the additional tax shield of new debt less the investment costs. Further, the issuance of new debt at investment transfers wealth from initial debt to equity holders. If equity holders reorganize with a debt-equity swap before investment, they must share the future investment benefit with initial debt holders who obtain part of the equity in this swap. Furthermore, equity holders lose the ex-post wealth transfer facility because the firm is all equity financed at investment. To maintain the opportunity to capture the investment benefit and transfer wealth, equity holders’ incentive to reorganize before investment is relatively weak. In the R-model, equity holders need to share the renegotiation surplus with initial debt holders and, hence, cannot transfer wealth at investment. Therefore, if equity holders reorganize before investment, the value of the opportunity they lose is smaller than that in the FSI-model. Hence, the reorganization risk in the R-model is larger than that in the FSI-model. Higher distressed reorganization risk reduces firm value as the tax shield earned between reorganization and investment is lost in case of reorganization and because firms install a lower initial coupon.

To quantify the value loss of the higher distressed reorganization risk caused by the ability of covenant renegotiation, we calculate the “R-first-best” in the fourth line of Panel A. The R-first best corresponds to the R-model but applies firm-value maximizing investment and financing policies. Equity holders, however, choose the distressed reorganization policy. Details of the R-first best are presented in Appendix F. The difference
Table I
The impact of a renegotiable financing covenant

This table shows the impact of a renegotiable financing covenant on firm value and policies for different parameters. Panel A displays the baseline results. In Panel B–G, one parameter is changed at a time. \( \eta \) is the bargaining power of equity holders, \( s \) the scale parameter of the investment opportunity, \( \tau \) the corporate tax rate, \( \alpha \) the recovery rate at default, \( \sigma \) the volatility of cash flow, and \( \mu \) the cash flow drift. The NR-model is a model without debt renegotiation but with investment as presented in Hackbarth and Mauer (2012). The FSI-model is the distressed reorganization model of Fan and Sundaresan (2000) augmented for investment. Equity holders choose the financing, investment, and reorganization policies to maximize the value of their claim in this model. The FSI-first-best corresponds to the FSI-model but applies firm-value maximizing investment and financing policies. The R-model considers a firm with distressed reorganization and a renegotiable covenant. The R-first-best corresponds to the R-model but applies firm-value maximizing investment and financing policies; however, equity holders choose the distressed reorganization policy. \( S_0 \) is the reorganization boundary, \( U_0 \) the investment boundary initiating covenant renegotiation in the R-models, and \( c_0^A \) the initial coupon. \( lev_U \) and \( lev_v \) are leverage at investment and initial leverage, respectively, and \( f_0 \) is the initial firm value. \( GB \) is the gross benefit and \( C \) the cost of a renegotiable covenant. Gross benefit is defined as the total agency costs of debt in the FSI model, and cost is the value loss due to increased reorganization risk and underinvestment in the R-model. The net benefit of covenant renegotiation, \( NB \), is defined as the percentage firm value increase of the R-model compared to the FSI-model, i.e., \( NB = 100 \cdot \left( 1 - \frac{\frac{f_0}{f_0^{FSI}}}{\frac{f_0}{f_0^{R}}} \right) \).

<table>
<thead>
<tr>
<th>Panel A: Baseline parameters, optimal initial coupon</th>
<th>( S_0 )</th>
<th>( U_0 )</th>
<th>( c_0^A )</th>
<th>( lev_U )</th>
<th>( lev_v )</th>
<th>( f_0 )</th>
<th>GB</th>
<th>C</th>
<th>NB</th>
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<tbody>
<tr>
<td>NR-model</td>
<td>0.21</td>
<td>2.28</td>
<td>1.19</td>
<td>0.57</td>
<td>0.31</td>
<td>66.49</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>FSI-model</td>
<td>0.25</td>
<td>1.94</td>
<td>1.08</td>
<td>0.72</td>
<td>0.26</td>
<td>67.73</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FSI-first-best</td>
<td>0.53</td>
<td>2.76</td>
<td>2.40</td>
<td>0.63</td>
<td>0.53</td>
<td>69.19</td>
<td>2.15</td>
<td></td>
<td></td>
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<tr>
<td>R-first-best</td>
<td>0.53</td>
<td>2.70</td>
<td>2.19</td>
<td>0.63</td>
<td>0.54</td>
<td>68.99</td>
<td>2.15</td>
<td>-0.29</td>
<td></td>
</tr>
<tr>
<td>R-model</td>
<td>0.52</td>
<td>2.78</td>
<td>2.19</td>
<td>0.63</td>
<td>0.53</td>
<td>68.99</td>
<td>2.15</td>
<td>-0.29</td>
<td>1.86</td>
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<tr>
<th>Panel B: Higher bargaining power (( \eta = 0.75 ))</th>
<th>( S_0 )</th>
<th>( U_0 )</th>
<th>( c_0^A )</th>
<th>( lev_U )</th>
<th>( lev_v )</th>
<th>( f_0 )</th>
<th>GB</th>
<th>C</th>
<th>NB</th>
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<tr>
<td>FSI-model</td>
<td>0.29</td>
<td>2.00</td>
<td>1.00</td>
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<td>0.24</td>
<td>67.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-model</td>
<td>0.51</td>
<td>2.75</td>
<td>1.68</td>
<td>0.56</td>
<td>0.42</td>
<td>67.89</td>
<td>1.44</td>
<td>-0.21</td>
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<tr>
<th>Panel C: Higher scale parameter (( s = 2.2 ))</th>
<th>( S_0 )</th>
<th>( U_0 )</th>
<th>( c_0^A )</th>
<th>( lev_U )</th>
<th>( lev_v )</th>
<th>( f_0 )</th>
<th>GB</th>
<th>C</th>
<th>NB</th>
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<tr>
<td>FSI-model</td>
<td>0.13</td>
<td>1.40</td>
<td>0.61</td>
<td>0.69</td>
<td>0.12</td>
<td>82.39</td>
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<tr>
<td>R-model</td>
<td>0.50</td>
<td>1.98</td>
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<td>0.63</td>
<td>0.49</td>
<td>83.73</td>
<td>1.91</td>
<td>-0.29</td>
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<th>Panel D: Higher tax rate (( \tau = 0.2 ))</th>
<th>( S_0 )</th>
<th>( U_0 )</th>
<th>( c_0^A )</th>
<th>( lev_U )</th>
<th>( lev_v )</th>
<th>( f_0 )</th>
<th>GB</th>
<th>C</th>
<th>NB</th>
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<tr>
<td>FSI-model</td>
<td>0.35</td>
<td>2.01</td>
<td>1.55</td>
<td>0.73</td>
<td>0.37</td>
<td>66.15</td>
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<tr>
<td>R-model</td>
<td>0.53</td>
<td>2.96</td>
<td>2.18</td>
<td>0.65</td>
<td>0.55</td>
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<td>2.24</td>
<td>-0.49</td>
<td>1.75</td>
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<tr>
<th>Panel E: Higher recovery rate (( \alpha = 0.9 ))</th>
<th>( S_0 )</th>
<th>( U_0 )</th>
<th>( c_0^A )</th>
<th>( lev_U )</th>
<th>( lev_v )</th>
<th>( f_0 )</th>
<th>GB</th>
<th>C</th>
<th>NB</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSI-model</td>
<td>0.24</td>
<td>1.88</td>
<td>1.18</td>
<td>0.84</td>
<td>0.28</td>
<td>68.68</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-model</td>
<td>0.52</td>
<td>2.82</td>
<td>2.51</td>
<td>0.74</td>
<td>0.60</td>
<td>70.19</td>
<td>2.55</td>
<td>-0.34</td>
<td>2.20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel F: Higher volatility of cash flows (( \sigma = 0.25 ))</th>
<th>( S_0 )</th>
<th>( U_0 )</th>
<th>( c_0^A )</th>
<th>( lev_U )</th>
<th>( lev_v )</th>
<th>( f_0 )</th>
<th>GB</th>
<th>C</th>
<th>NB</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSI-model</td>
<td>0.23</td>
<td>2.03</td>
<td>1.06</td>
<td>0.71</td>
<td>0.25</td>
<td>68.03</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-model</td>
<td>0.51</td>
<td>2.99</td>
<td>2.28</td>
<td>0.63</td>
<td>0.53</td>
<td>69.32</td>
<td>2.21</td>
<td>-0.31</td>
<td>1.90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel G: Lower cash flow drift (( \mu = 0.02 ))</th>
<th>( S_0 )</th>
<th>( U_0 )</th>
<th>( c_0^A )</th>
<th>( lev_U )</th>
<th>( lev_v )</th>
<th>( f_0 )</th>
<th>GB</th>
<th>C</th>
<th>NB</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSI-model</td>
<td>0.30</td>
<td>2.08</td>
<td>0.86</td>
<td>0.73</td>
<td>0.33</td>
<td>40.97</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-model</td>
<td>0.51</td>
<td>2.79</td>
<td>1.40</td>
<td>0.63</td>
<td>0.54</td>
<td>41.60</td>
<td>1.82</td>
<td>-0.30</td>
<td>1.53</td>
</tr>
</tbody>
</table>

22
between this firm value and the FSI-first-best firm value quantifies the impact of the R-model’s larger reorganization risk. With baseline parameters, it is –0.29%, as shown in column C.

Corporate policies with a renegotiable covenant in the R-model are shown in the last line of Panel A. The difference between the R-first-best and R-model is that equity holders choose all corporate policies to maximize the value of their claim in the latter case. Because initial debt holders capture part of the renegotiation surplus at investment, equity holders invest somewhat too late compared to a policy maximizing firm value (underinvestment). The quantitative impact of this investment timing friction in the R-model, however, is very limited. In particular, firm values rounded to two decimals are equal in the R-first-best and R-model in the base case.

The benefit of a renegotiable covenant dominates the cost such that the firm value increases by 1.86% (2.15%–0.29%) in the R-model compared to the FSI-model. This net benefit measures the quantitative importance to firms of including a renegotiable covenant. We tabulate this net benefit in the last column \(NB\) of Table I.

We also calculate the total value of the ability to renegotiate debt to firms. In the NR-model without any renegotiation in line one of Panel A of Table I, we obtain a firm value of 66.49. Adding distressed reorganization increases firm value by 1.87% to 67.73 (FSI-model in Panel A), mainly because reorganization avoids default costs. With an additional renegotiable covenant in the R-model, the firm value increases by 3.76% to 68.99. In this case, the possibility to renegotiate debt avoids default costs and reduces the agency costs of debt. These numbers show that incorporating non-distressed renegotiation more than doubles the impact of the possibility to renegotiate debt on firm value. Hence, the neglect of covenant renegotiation severely underestimates the value of firms’ ability to renegotiate debt.

Firm parameters influence the magnitude of the benefit and cost of a renegotiable covenant and, hence, the value to firms of installing a covenant. To explore this influence, we change one parameter in each Panel B–G of Table I and report the resulting corporate policies and firm values.

We start by analyzing how bargaining power affects our results in Panel B of Table I. The response of firms to a larger \(\eta\) is to reduce the initial coupon. With a lower initial coupon, the agency costs of debt become smaller. Thus, the benefit of a renegotiable
covenant declines compared to Panel A (column GB). Reduction in initial coupon also mitigates the distressed reorganization risk and, therefore, the cost of a renegotiable covenant (column C). The decline in benefit is stronger than that of the cost. Therefore, the net benefit of a renegotiable covenant is smaller than that in Panel A.

Table I also shows that firm value is more sensitive to bargaining power in the R- than the FSI-model. For instance, Panels A and B imply that the FSI-model’s firm value declines by 0.99% when \( \eta \) raises from 0.5 to 0.75 from the increase in distressed reorganization risk. The R-model’s firm value decreases by 1.46%. This decline is stronger than that in the FSI-model mainly because the benefit of a renegotiable covenant diminishes with \( \eta \).

Similar to Panel B, a larger scale parameter of the investment opportunity in Panel C reduces the optimal initial coupon, which decreases both the benefit and cost of a renegotiable covenant. The decline in benefit is stronger than that of the cost such that the net benefit of a renegotiable covenant is smaller than that in Panel A.

We also analyze the impact of an increase in tax rate in Panel D. A higher tax rate increases the cost of a renegotiable covenant for two reasons. First, the investment benefit of issuing new debt rises, which increases the difference between the FSI- and R-models’ distressed reorganization risk. Second, the expected loss in tax shield due to higher reorganization risk in the R-model has a larger value when the tax rate increases. As the benefit of a renegotiable covenant changes only slightly with a higher tax rate, the increase in cost dominates. Thus, the value of a renegotiable covenant is smaller in Panel D than in Panel A, as shown in column NB.

The larger recovery rate in Panel E raises the optimal initial coupon compared to Panel A, which increases both the benefit and cost of a renegotiable covenant. Further, equity holders have a stronger incentive to avoid distressed reorganization in the FSI-model when the recovery rate is larger because the fraction of equity that debt holders obtain at distressed reorganization is higher. Thus, the difference in reorganization risk between the R- and FSI-models increases, enlarging the cost of a renegotiable covenant. Overall, the increase in benefit dominates the increase in cost, such that having a renegotiable covenant is more important than in Panel A. Finally, Panels F and G in Table I show that the value of a renegotiable covenant to firms increases with volatility and declines with the cash flow drift.
5. Results and empirical predictions

The recognition that firms renegotiate both at distress and upon investment allows us to explain empirically observed renegotiation patterns, credit spreads, covenant structures, and financial policies. It also yields novel testable predictions.

5.1. Waiting times to renegotiation

We first investigate the model-implied expected waiting time from initiation to any renegotiation (i.e., covenant renegotiation or distressed reorganization). To this end, we calculate the expected waiting time in closed form (see Appendix G). The expected waiting time to renegotiation for a baseline firm is 2.66 years.

As shown in Table I of Appendix G, firms with a more valuable investment opportunity renegotiate the financing covenant at a lower threshold than firms with a less valuable investment opportunity. Thus, the expected waiting time to covenant renegotiation decreases when the scale parameter $s$ increases. Firms with a more valuable investment opportunity, however, reorganize at a lower boundary than firms with a less valuable investment opportunity (see Table I of the Appendix). Hence, the expected waiting time to distressed reorganization increases with the scale parameter $s$. Thus, the question of whether the expected waiting time to any renegotiation (covenant renegotiation or distressed reorganization) decreases or increases with $s$ is not trivial.

Figure 2 plots the expected waiting time to any renegotiation against the market-to-book ratio, which is determined by the scale parameter $s$. It shows that the expected waiting time is decreasing in the market-to-book ratio. The reason is that the negative impact of $s$ on the expected waiting time to covenant renegotiation dominates.

Since both the reorganization threshold and covenant renegotiation boundary are decreasing in $s$, the conditional probability of covenant renegotiation, given any renegotiation, is increasing in $s$. To assess the importance of covenant renegotiation relative to distressed reorganization, we calculate the conditional probability in closed form (see
Appendix G). As Figure 3 shows, the conditional probability of covenant renegotiation is 41.4% in the baseline firm with a market-to-book ratio of 1.62.

With a market-to-book ratio of 2.4, the conditional probability of covenant renegotiation is 71.4% and, thus, much higher than that of distressed reorganization. Even for low market-to-book ratios, covenant renegotiations remain important. For example, a market-to-book ratio of 1.2 still implies a conditional probability of covenant renegotiation of about one quarter (25.6%).

Overall, the analysis suggests that covenant renegotiation contributes substantially to the realized waiting times to renegotiation. This insight naturally leads to the question of whether the R-model explains empirically observed debt renegotiation timing patterns. We address this question in the next section.

5.2. Explaining empirical renegotiation patterns

We investigate the R-model’s ability to explain the renegotiation timing patterns reported in the empirical literature. To this end, we adopt a simulation approach in the spirit of Strebulaev (2007). The reasons for the simulation approach are twofold. First, the expected waiting times derived in the previous section are for optimally financed firms at initiation. Firms in the real world, however, issue renegotiable debt at times other than initiation. Thus, when a firm issues renegotiable debt, its capital structure, distance to reorganization, and distance to covenant renegotiation typically differ from those of a firm at initiation. These deviations can be represented in a simulation-based approach, in which cash flows fluctuate over time. Second, Figures 2 and 3 show that the expected waiting time to renegotiation as well as the conditional probability of covenant renegotiation are non-linear in the market-to-book ratio. Due to these non-linearities, deviations from the average market-to-book ratio do not translate linearly into deviations from the average waiting time. Hence, it would be misleading to predict renegotiation patterns in the cross section of empirical firms based on an average representative model-implied firm.

To conduct the simulation, we first construct an empirical sample of a cross section of firms. We collect the 3720 private credit agreements covered in Nini, Smith, and Sufi
(2009) from their home page. This sample has been analyzed by several papers with regard to renegotiations (e.g., Roberts and Sufi, 2009b; Wang, 2013; Denis and Wang, 2014). It consists of all private loans in the Loan Pricing Corporation Dealscan database extended to nonfinancial public firms from 1996 to 2005 with available Compustat information for which the original credit agreements are available in SEC filings. We merge this set with Compustat Data, which leaves us with 3688 observations, and we calculate the average market-to-book and market leverage ratios for each observation over the four previous quarters. The market-to-book ratio is constructed as Total Assets (item 44) minus Book Equity plus Market Equity, scaled by Total Assets. Book Equity is Total Assets minus Total Liabilities (item 54) minus Preferred Stock (item 55) plus Deferred Taxes (item 52). Market Equity is the Equity Price (item 14) times Common Shares Outstanding (item 61). The market leverage ratio is Book Debt, divided by Total Assets minus Book Equity plus Market Equity. After dropping the firms with any missing data over the four previous quarters, our final empirical sample consists of 3,070 private credit agreements (“true cross section”) characterized by the market-to-book and leverage ratios of the corresponding firm. Market leverage and market-to-book ratios are Winsorized at the upper and lower 2.5 percentiles.

Next, we generate a model-implied sample of firms reflecting the true cross section, following the simulation-based approach of Strebulaev (2007) as extended by Arnold, Wagner, and Westermann (2013). For this purpose, we generate a large universe of initially optimally financed model-implied firms. Specifically, we consider the firms for which the initial scale parameter ranges from $s = 1$ to $s = 2.95$ using a step size of 0.05. Firms with a scale parameter of $s = 2.95$ invest at a threshold of $X_t = 1.13$. Larger scale parameters imply a high likelihood of almost immediate investment after initiation. For each scale parameter $s$, we consider 100 initially identical firms. Thus, our universe consists of 41,000 firms. This universe is simulated forward for ten years with a quarterly frequency. We denote this procedure “pre-simulation.” Firms that reorganize or invest are replaced by an optimally financed firm with an identical investment opportunity that is not yet exercised. For each model-implied firm, the market-to-book ratio is the sum of the book value of debt plus the market value of equity divided by the value of invested assets, and the quasi-market leverage is given by the book value of debt divided by the sum of the book value of debt plus the market value of equity. The book value of debt is defined as the value of debt at initiation.
We now match the 3,070 observations in the true cross section with simulated, model-implied firms. Specifically, for each observation in the true cross section, we select the simulated firm at the end of the pre-simulation with the smallest squared sum of distances with respect to leverage and market-to-book ratios. Thus, we obtain a sample of 3,070 model-implied matched firms.\(^3\)

Using this model-implied matched sample, we investigate the timing patterns of debt renegotiation. To this end, we simulate the matched sample forward with a quarterly frequency. To maintain a balanced sample over time, we replace a reorganized or invested firm by the corresponding firm at matching. That is, each replacement uses a firm with an identical investment opportunity and cash flow as at the time of matching. We reflect the average contract maturity of 46.7 months in Denis and Wang (2014) by choosing a horizon of 48 months. The procedure is denoted as “post-simulation.” We report the statistics of renegotiation patterns in this post-simulation.

We conduct the pre-simulation 100 times. For each pre-simulation, we consider 100 post-simulations, resulting in a total of 10,000 simulations.

In general, our matching procedure is accurate. At matching, the square root of the sum of squared distances between real firms and simulated sample firms with respect to leverage and market-to-book ratios is 0.15 with an average cross sectional standard deviation of 0.32. Table II shows the summary statistics for the true cross section from Nini, Smith, and Sufi (2009), and for our simulated samples at matching. The matched samples are structurally similar to the empirical sample. The main differences are due to observations in the true cross section with very large market-to-book ratios that do not match closely with the R-model.

Denis and Wang (2014) and Roberts and Sufi (2009b) report stylized facts about the empirical renegotiation patterns in large, randomly selected subsamples of the sample of Nini, Smith, and Sufi (2009). The statistics are summarized in columns one and two of Table III. On average, slightly over 60\% of contracts are renegotiated at least once, and only around 18\% of debt renegotiations are associated with corporate distress.\(^4\) The reported time to the first renegotiation of contracts that are subsequently renegotiated is

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\(^3\)We also repeat the procedure by matching the book leverage and market-to-book ratios. The impact on our results is minor. Further, we verify that the matching outcome does not depend on the distribution of scale parameters \(s\) in the pre-simulation.

\(^4\)Roberts (2015) and Wang (2013) confirm in their sample that most renegotiations are initiated by borrowers in response to changing conditions, rather than from lender interventions due to close default.
This table shows the summary statistics for the true cross section from Nini, Smith, and Sufi (2009), and for the simulated model-implied samples at matching. The market-to-book ratio in the simulated samples is calculated as the sum of the book value of debt plus the market value of equity divided by the value of invested assets, and the quasi-market leverage by the book value of debt divided by the sum of the book value of debt plus the market value of equity.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>True cross section</th>
<th>Simulated samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market-to-book ratio mean</td>
<td>1.69</td>
<td>1.65</td>
</tr>
<tr>
<td>Market-to-book ratio median</td>
<td>1.39</td>
<td>1.39</td>
</tr>
<tr>
<td>Market-to-book ratio standard deviation</td>
<td>0.90</td>
<td>0.67</td>
</tr>
<tr>
<td>Market leverage mean</td>
<td>0.41</td>
<td>0.45</td>
</tr>
<tr>
<td>Market leverage median</td>
<td>0.40</td>
<td>0.43</td>
</tr>
<tr>
<td>Market leverage standard deviation</td>
<td>0.22</td>
<td>0.18</td>
</tr>
</tbody>
</table>

around 1.4 years. We calculate the average effective duration of an initial contract, i.e., the mean waiting time to the first renegotiation or final maturity (in case of contracts not renegotiated) from the results presented in the papers mentioned above. The average effective duration is 2.0 years in Denis and Wang (2014) and 1.5 years in Roberts and Sufi (2009b).\(^5\) We obtain the effective duration and the duration in percentage of the initially stated maturity from the results presented in Table 2 of Denis and Wang (2014). The former measure is the average time to the first renegotiation or maturity expressed in terms of initially stated maturity. The latter measure is the average time to any renegotiation or maturity divided by the initially stated maturity, including renegotiations after the first renegotiation. Roberts and Sufi (2009b) report this measure for their sample. In both papers, the durations are around 60% of the initially stated maturity. We additionally depict statistics on the renegotiation frequency illustrated in Denis and Wang (2014). Conditional on being renegotiated, the average contract is renegotiated 2.7 times. Figure 1 in their study also shows that around 37% of contracts are renegotiated just once, 22% twice, and close to 13% three times. About 53% of contracts are renegotiated between two and five times during their lifespan.

Next, we present the renegotiation patterns in the post-simulations of model-implied samples. To assess the marginal contribution of covenant renegotiations, we first run our

\(^5\)Roberts and Sufi (2009b) report a relatively large fraction of loans in their sample to disappear or reach the end of the sample without a renegotiation. They are, on average, characterized by long stated maturities, which could explain part of the difference between the average effective durations in Denis and Wang (2014) and Roberts and Sufi (2009b).
Table III
Empirical and model-implied timing patterns of debt renegotiation

Empirical and model-implied timing patterns of debt renegotiation. The first line shows the portion of debt contracts renegotiated before maturity. % distressed renegotiations is the percentage of total debt renegotiations associated with corporate distress. Time to first renegotiation is the average time in years to the first renegotiation of contracts that are renegotiated. Effective duration is the mean waiting time to the first renegotiation or final maturity in years. Effective duration % of stated maturity is the average time to the first renegotiation or maturity, expressed in terms of initially stated maturity. Duration % of stated maturity is the average time to any renegotiation or to maturity divided by the initially stated maturity, including renegotiations occurring after the first renegotiation. Average renegotiation frequency is the mean number of renegotiation rounds of contracts renegotiated. The last four lines summarize the frequencies of renegotiation rounds. The empirical samples are the loan contract samples of Denis and Wang (2014), and Roberts and Sufi (2009b). In the simulated samples of the FSI-model, firms only reorganize to avoid default but have an investment option in the spirit of Hackbarth and Mauer (2012). In the simulated samples of the R-model, firms can renegotiate at investment and reorganize to avoid default.

<table>
<thead>
<tr>
<th></th>
<th>Empirical DW 2014</th>
<th>Empirical RS 2009b</th>
<th>Simulated samples FSI-model</th>
<th>Simulated samples R-model</th>
</tr>
</thead>
<tbody>
<tr>
<td>% renegotiated</td>
<td>60.6</td>
<td>64.5</td>
<td>6.3</td>
<td>33.0</td>
</tr>
<tr>
<td>% distressed renegotiations</td>
<td>-</td>
<td>18</td>
<td>100</td>
<td>26.9</td>
</tr>
<tr>
<td>Time to first renegotiation</td>
<td>1.3</td>
<td>1.5</td>
<td>2.3</td>
<td>1.3</td>
</tr>
<tr>
<td>Effective duration</td>
<td>2.0</td>
<td>1.5</td>
<td>3.8</td>
<td>2.4</td>
</tr>
<tr>
<td>Eff. duration % of stated maturity</td>
<td>58</td>
<td>-</td>
<td>97.4</td>
<td>77.8</td>
</tr>
<tr>
<td>Duration % of stated maturity</td>
<td>61</td>
<td>57</td>
<td>96.2</td>
<td>60.6</td>
</tr>
<tr>
<td>Average renegotiation frequency</td>
<td>2.7</td>
<td>-</td>
<td>1.33</td>
<td>2.7</td>
</tr>
<tr>
<td>% renegotiated once</td>
<td>37</td>
<td>-</td>
<td>84.2</td>
<td>43.3</td>
</tr>
<tr>
<td>% renegotiated twice</td>
<td>22</td>
<td>-</td>
<td>8.7</td>
<td>17.3</td>
</tr>
<tr>
<td>% renegotiated three times</td>
<td>13</td>
<td>-</td>
<td>3.0</td>
<td>12.0</td>
</tr>
<tr>
<td>% renegotiated two to five times</td>
<td>53</td>
<td>-</td>
<td>14.4</td>
<td>44.8</td>
</tr>
</tbody>
</table>

The third column of Table III summarizes the results for the simulated samples. Only 6.3% of contracts are on average renegotiated. By construction, all renegotiations are due to corporate distress. The durations are much longer than their empirical counterparts, and the frequency pattern of the renegotiation rounds per contract fails to reflect that in the empirical samples. For example, only few contracts are renegotiated more than once.

The final column of Table III shows the renegotiation patterns implied by the R-model including both distressed reorganization and covenant renegotiation. 33% of contracts are renegotiated. This number suggests that some renegotiations in the empirical sample are triggered by motives beyond those reflected in the R-model. We can, however, match the firm conditions in which renegotiations occur quite well. Specifically, 26.9% of debt
renegotiations are distressed reorganizations compared to 18% in the empirical sample of Roberts and Sufi (2009b). The effective duration is 2.4 years, the average time to first renegotiation 1.3 years, the effective duration 77.8%, and the duration 60.6%. A comparison of these measures to those in columns one and two shows that the R-model explains the duration patterns in the empirical sample well. Moreover, the R-model improves the matching of all empirical timing and duration measures considerably compared to the FSI-model. We also investigate the renegotiation frequencies in the R-model. 17.3% of contracts are renegotiated twice and 12% three times. These numbers are much closer to the empirical fractions of 22% and 13% compared to 8.7% and 3% of the FSI-model.

5.3. Credit spreads

Table IV presents credit spreads in various models. The NRC-model is the model in Hackbarth and Mauer (2012) with covenant protected debt, in which debt can be renegotiated neither at investment nor in distress. Hence, investment has to be financed with new equity. We consider the NRC-model to distinguish the impact of the ability to renegotiate from that of having a covenant on credit spreads. For the NRC- and FSI-models, credit spreads are calculated by first dividing the initial coupon by the initial market value of debt and then subtracting the risk free interest rate. In the model with both distressed reorganization and covenant renegotiation (R-Model), the spread between the debt yield and risk free rate is an insufficient measure of credit risk compensation for two reasons. First, the initial market value of debt reflects also the present value of the surplus debt holders obtain at covenant renegotiation. The surplus to debt holders in renegotiation is defined in Eq. (11). Second, the initial coupon is relatively low because debt holders anticipate that they obtain part of the renegotiation surplus at covenant renegotiation. For comparability, we first subtract the present value of the surplus to debt holders from the initial debt value to obtain the net debt value. Next, we solve for the fixed coupon over the entire debt maturity that implies the same debt value as the net debt value. Finally, we calculate credit spreads as this fixed coupon divided by the initial net debt value and then subtract the risk free interest rate. This credit spread corresponds to the credit default swap (CDS) spread an external investor would require to bear the same credit risk as the debt investor in the R-model. The CDS investor would obtain a constant CDS spread and no surplus at renegotiation. To compare our results to the observed credit spreads, we use average parameters consistent with Davydenko.
and Strebulaev (2007), who empirically investigate the determinants of credit spreads associated with renegotiation. Specifically, the market-to-book ratio is 1.86, the leverage 32.2%, the asset volatility 23.9%, and the risk free interest rate 5.93%. The remaining parameters correspond to those of our baseline firm.

### Table IV
Debt Renegotiation and Credit Spreads

This table depicts the impact of renegotiable debt on corporate credit spreads. In the first column, we calculate these spreads in a model without any renegotiation (NRC-Model), in which initial debt is covenant protected. The second column uses the distressed reorganization framework of Fan and Sundaresan (2000) augmented for investment (FSI-model). The credit spreads in these two models are calculated by dividing the initial coupon by the market value of debt, and subtracting the risk free interest rate. The third column shows the credit spreads in the model with both distressed reorganization and covenant renegotiation at investment (R-model). Credit spreads in this model are calculated by first subtracting the present value of the surplus from the initial debt market value to obtain the net debt value. We then solve for the constant coupon on debt that implies the same value as the net debt value. Finally, we calculate the credit spreads by dividing this coupon with the initial net debt value and subtracting the risk free interest rate. The control premium is the present value of the surplus that debt holders extract at covenant renegotiation, expressed as a percentage of the initial debt value. Baseline parameters are consistent with Davydenko and Strebulaev (2007), given a bargaining power of 0.5.

<table>
<thead>
<tr>
<th></th>
<th>NRC-model</th>
<th>FSI-model</th>
<th>R-model</th>
<th>Control premium %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline parameters</td>
<td>49</td>
<td>214</td>
<td>80</td>
<td>7.88</td>
</tr>
<tr>
<td>Leverage = 0.372</td>
<td>63</td>
<td>264</td>
<td>98</td>
<td>6.23</td>
</tr>
<tr>
<td>Volatility $\sigma = 0.289$</td>
<td>86</td>
<td>277</td>
<td>117</td>
<td>7.98</td>
</tr>
<tr>
<td>Market-to-book=1.91</td>
<td>47</td>
<td>221</td>
<td>80</td>
<td>8.14</td>
</tr>
<tr>
<td>Recovery rate $\alpha = 0.65$</td>
<td>48</td>
<td>201</td>
<td>76</td>
<td>8.18</td>
</tr>
<tr>
<td>Bargaining power $\eta = 0.25$</td>
<td>49</td>
<td>193</td>
<td>54</td>
<td>12.97</td>
</tr>
<tr>
<td>Bargaining power $\eta = 0.75$</td>
<td>49</td>
<td>250</td>
<td>112</td>
<td>3.46</td>
</tr>
</tbody>
</table>

Column 1 of Table IV shows that the credit spread in the NRC-model is 49 basis points (bps) in the baseline firm. This credit spread fails to reflect both the mean empirical spread of 109 bps and the positive dependence of the spread on proxies for bargaining power reported in Davydenko and Strebulaev (2007).

The R-model reflects three features of empirically observed credit spreads both qualitatively and quantitatively: average spread levels, the impact of the possibility to renegotiate debt on credit spreads, and cross sectional credit spread patterns. First, Column 3 of Table IV shows that an average baseline firm has a credit spread of 80 bps, which repli-

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6While Davydenko and Strebulaev (2007) investigate the credit spreads of corporate bonds, these spreads also reflect the reduction in agency costs of overleverage and overinvestment when the same firms have some covenant protected private debt.
cates the actually observed mean [median] of 109 [85] bps quite well. The R-model closely matches the observed credit spreads for a bargaining power of approximately $\eta = 0.72$.

Second, a comparison of the R-model to the NRC-model allows us to predict the impact of the possibility to renegotiate debt on firms. For the baseline firm, the possibility to renegotiate debt increases the credit spreads by 31 bps (80–49 bps). The empirical counterpart to a NRC-firm (without the possibility to renegotiate debt) is a firm with large renegotiation frictions. Davydenko and Strebulaev (2007) use the amount of public debt as a proxy for renegotiation friction because it is difficult to renegotiate public debt in practice. They show that going from the 5th to the 95th percentile of this proxy increases the credit spreads empirically by 23 bps, which is close to the 31 bps difference between the NRC- and R-models. Without covenant renegotiation, however, a structural model cannot replicate this pattern. To confirm this point, we augment the NRC-model with distressed reorganization. The resulting credit spread of the baseline firm is 50 bps (not in the table), implying that the possibility to renegotiate in distress increases the spread by only 1 bps (50–49). The primary reason for the stronger impact of renegotiation in the R-model is that the renegotiable covenant implies a larger leverage at investment, which increases the current credit spreads.

Third, the R-model implies a positive dependence of credit spreads on the bargaining power of equity holders. Column 3 in Table IV shows that an increase from $\eta = 0.25$ to $\eta = 0.75$ augments the credit spreads by 58 bps (112–54 bps). The dependence declines when bankruptcy costs become smaller. With $\alpha = 0.8$, for example, the credit spreads rise by only 45 bps when the bargaining power increases from 0.25 to 0.75 (not in the table). This pattern reflects the empirical relation between bargaining power and credit spreads. Specifically, Davydenko and Strebulaev (2007) show that credit spreads augment with proxies for bargaining power and that this dependence declines with proxies for liquidation costs. In quantitative terms, they illustrate that going from the 5% to the 95% percentiles of the (significant) bargaining power proxies increases the credit spreads by around 13 bps. The authors, however, argue that the quantitative impact of bargaining power may actually be higher than their empirical estimate because proxies are noisy measures of bargaining power. Additionally, they show that the impact of bargaining power is stronger for firms that can renegotiate debt more easily.

We calculate the credit spreads of the FSI-model in Column 2 of Table IV. This model neglects the fact that a renegotiable covenant can mitigate agency costs of debt.
from overleverage at investment and from overinvestment. The FSI-model severely over-
estimates credit spreads by 105 bps (214–109 bps) compared to its empirical counterpart.
Even a considerable variation in bargaining power does not help to drive the spread into
a reasonable range. The FSI-model also predicts a quantitative impact of the possibility
to renegotiate debt that is too large. Specifically, omitting the possibility of distressed
reorganization in the FSI-model leads to the NR-model generating a credit spread of 108
bps. We compare the FSI- and NR-models because both have no covenant thus allowing
us to isolate the pure predicted impact of the ability to renegotiate on credit spreads. The
FSI-model predicts an increases of 106 bps (214–108 bps) in credit spreads, well beyond
the empirical estimation of 23 bps in Davydenko and Strebulaev (2007). Additionally,
Column 2 shows that the FSI-model estimates the impact on credit spreads of a five per-
centage point increase in leverage to 50 bps, in volatility to 63 bps, in market-to-book to
7 bps, and in recovery rate to -13 bps. As illustrated empirically by Davydenko and Stre-
bulaev (2007), a five percentage point increase in these firm characteristics changes the
credit spreads by only 8, 6, 0, and -0.5 bps, respectively. Column 3 shows that incorpo-
rating non-distressed renegotiation in the R-model also helps to mitigate the FSI-model’s
overestimation of the credit spread sensitivities to standard firm parameters.

Overall, we show that incorporating debt renegotiation is important to improve the
ability of structural models to explain both the average and cross section of empirically
observed credit spreads. However, the improvement emerges only if we consider that debt
renegotiation can occur both at distress and outside distress.

The R-model has further implications for debt pricing besides explaining the impact
of debt renegotiation on credit spreads. In particular, the importance of non-distressed
covenant renegotiation for the timing patterns of debt renegotiations described in Section
5.2 and creditors’ ability to extract part of the renegotiation surplus suggest that real
debt prices should reflect a sizeable creditor control premium. Indeed, the recent empirical
study of Feldhütter, Hotchkiss, and Karakas (forthcoming) shows that creditor control
generates a premium in bond prices. The study measures this control premium in the
covenant violation sample of Nini, Smith, and Sufi (2012) by calculating the percentage
premium of public bonds over otherwise identical credit default swap (CDS) implied bond
prices. The resulting average control premium is 1.089%. The R-model’s counterpart to
this premium is the present value of the renegotiation surplus to debt holders, expressed as
a percentage of the initial debt value. The control premium of the R-model is summarized
in the last column of Table IV. Consistent with Feldhütter, Hotchkiss, and Karakas

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(forthcoming), the R-model’s premium spikes around covenant renegotiation events upon which control rights are shifted to creditors (not in the table) and increases with the recovery rate. A novel prediction is that the control premium should be larger for firms with a higher market-to-book ratio. We also predict a larger size for the control premium than that in the study of Feldhütter, Hotchkiss, and Karakas (forthcoming) for two reasons. First, the latter measures the premium of public bonds, whereas we model the premium of private debt that is easier for creditors to renegotiate. Second, the bonds in Feldhütter, Hotchkiss, and Karakas (forthcoming) do not coincide with the bank loans in Nini, Smith, and Sufi (2012) for which a covenant is actually violated and, hence, need to be renegotiated. Feldhütter, Hotchkiss, and Karakas (forthcoming) argue that such bonds can still indirectly profit from a general corporate restructuring involving all creditors after a violation. For the private loans that we model, however, creditors directly renegotiate the terms of their contract with the firm. In the base case, the R-model’s control premium is 7.88%. This premium strongly depends on bargaining power. With \( \eta = 0.75 \), for example, the control premium is only 3.46%.

5.4. Covenant structures

Section 4.2 shows that including a renegotiable financing covenant increases firm value. However, in the NR-model without renegotiation, we find that a financing covenant reduces the baseline firm value from 66.49 to 64.84 due to reduction in the firm’s financial flexibility to upstructure capital. These results are consistent with the empirical observation that financing covenants are ubiquitous in debt contracts (Bradley and Roberts, 2015) and private debt is more likely to have restrictive covenants than public bonds (Kahan and Tuckman, 1993; Kwan and Carleton, 2010). We also derive cross sectional predictions in terms of firms’ incentives to install a financing covenant in private debt. Specifically, the firm value differences between the FSI- and R-models in Table I of Section 4.2 imply that firms with higher bargaining power, larger growth opportunities, higher tax rate, lower recovery rate, smaller volatility, and lower cash flow drift should have a weaker incentive to implement a financing covenant.

Our cross sectional results are consistent with the empirical results in Bradley and Roberts (2015) that private debt covenants are more likely for higher cash flow volatility. However, according to the same study, high market-to-book firms have a larger propensity toward private debt covenants. Demiroglu and James (2010) relativize this empirical
result by showing that banks include more but looser financial covenants in loans of growth firms.

5.5. Financial and investment policies

Figure 4 plots the optimal initial leverage of a baseline firm with a renegotiable financing covenant (solid line) and that of a FSI-model firm (dashed line) against the bargaining power of equity holders.

As covenant renegotiation reduces the agency costs of debt, the optimal initial leverage of the baseline firm with renegotiation is larger than that of the FSI-model firm. This difference is particularly pronounced for a lower $\eta$ primarily because the net benefit of a financing covenant declines with bargaining power (see Section 4.2). The analysis implies that the choice of capital structure strongly depends on whether a financing covenant is included. A novel empirical prediction is that the impact of covenants on capital structure should be particularly pronounced for firms in which equity holders have weak bargaining power. Additionally, we predict that bargaining power is more important to the leverage choice of firms with a financing covenant.

Figure 5 plots the market-to-book ratio against optimal leverage for the R-model (solid line), FSI-model (dashed line), NRC-model with a covenant (dashed-dotted line), and NR-model without a covenant (dotted line). In the R- and FSI-models, we set $\eta = 0.5$. Billett, King, and Mauer (2007) find that the covenant protection of public bonds reduces the negative relation between leverage and growth opportunities. The dashed-dotted and dotted lines are consistent with this empirical finding. The agency costs of debt increase with the importance of the investment opportunity, thus explaining the decline of the dotted line in the NR-model. As a covenant mitigates the agency costs of debt, the decline of the dashed-dotted line in the NRC-model is less pronounced. With distressed reorganization, debt becomes riskier, thus raising the agency costs of debt. Therefore, the difference in the decline between the dashed and solid lines of the models with debt renegotiation is more pronounced than that between the dotted and dashed-dotted lines. A novel prediction of this comparison is that the covenant protection of
renegotiable private debt should reduce the negative relation between leverage and growth opportunities more than the covenant protection of public debt.

In Figure 6, we investigate the impact of covenants and bargaining power on investment timing in the baseline firm. The solid line depicts the optimal investment threshold of the R-model and the dashed line that of the FSI-model firm. Both lines are plotted for an optimal initial coupon.

A comparison of the solid and the dashed lines yields the empirical prediction that installing a financing covenant delays investment. The reason is that equity holders obtain a lower surplus at investment and cannot expropriate debt holders in the R-model compared to the FSI-model. We also find that this investment delay should be more distinct for low levels of η for which the surplus reduction is particularly pronounced and the initial leverage is larger, thus increasing the wealth expropriation at investment in the FSI-model.7

6. Conclusion

Existing corporate finance models cannot explain the quantitative impact of the possibility to renegotiate debt on the value of corporate securities. We argue that this gap is due to the lack of these models’ ability to reflect that debt renegotiations generally occur very frequently and, specifically, mostly arise outside distress (see, e.g., Roberts and Sufi, 2009b). In this paper, we develop a model in which equity holders can renegotiate debt with creditors either upon investment to waive a financing covenant or at corporate distress to reorganize the capital structure. Renegotiation at investment mitigates the agency costs of debt and distressed reorganization avoids the expected bankruptcy costs. We find that incorporating renegotiation both outside and in corporate distress is crucial to explain the empirically observed patterns of the timing of debt renegotiation.

7Both empirical predictions hold with a fixed instead of optimal initial coupon.
A model capturing a realistic timing of debt renegotiations provides a fundamental step towards understanding the quantitative impact of creditors through the possibility of renegotiating their claims on firms. The presented model explains a rich set of cross-sectional empirical results in terms of the impact of firm characteristics on credit spreads, covenant structures, and corporate financial policies. A key insight is that a model capturing the timing patterns of renegotiations implies a first order effect of creditor governance on firm value. Further, the bargaining power of equity holders and creditors is a quantitatively important determinant of firm policies and of the value of corporate securities. A recent literature investigates the impact of creditor rights on the access to finance, leverage, investment, risk taking, and demand for debt of distressed firms (Porta, Lopez-de-Silanes, and Shleifer, 2008; Acharya, Sundaram, and John, 2011; Vig, 2013; Favara, Morellec, Schroth, and Valta, forthcoming). Our results on the importance of covenant renegotiation motivate the consideration of creditor rights also in future empirical studies of non-distressed firms. To stimulate further research in this direction, we generate several novel testable predictions on the impact of bargaining power on corporations. We also suggest that private debt should entail a sizeable control premium beyond that observed in public bonds, and that this premium should strongly depend on bargaining power. This hypothesis has not been explored in the empirical literature.

We only analyze the renegotiation of a financing covenant. Considering alternative covenants could generate supplementary insights on the impact of creditor governance on firms. We also believe that non-distressed covenant renegotiation should help rationalize observed corporate debt structures, such as the choice between private and public debt or firms’ debt ownership structure. We look forward to future research addressing these extensions.
References


Figure 1. **Timeline of the potential occurrence of debt renegotiation.** This figure shows the sequence of the potential occurrence of covenant renegotiation, distressed reorganization, and investment over time. The notation for the corresponding thresholds are in parenthesis. The subscripts apply to the corresponding value functions.
Figure 2. Expected waiting time to renegotiation. This figure plots the relation between the market-to-book ratio of a R-model firm and the expected waiting time to any renegotiation.

Figure 3. Probability of covenant renegotiation conditional on any renegotiation. This figure shows the relation between the market-to-book ratio of a R-model firm and the probability of covenant renegotiation given any renegotiation occurs.
Figure 4. Optimal initial leverage choice. This figure shows the relation between optimal initial leverage and bargaining power of equity holders for the R-model firm (solid line) and for the FSI-model firm (dashed line).

Figure 5. Relation between market-to-book ratio and leverage. This figure plots the relation between market-to-book and market leverage for a baseline firm in the R-model (solid line) and in the FSI-model (dashed line). The dashed-dotted line plots this relation for a NR-model firm without renegotiation and the dotted line that for a NRC-model firm without renegotiation but with a financing covenant.
Figure 6. **Investment timing.** This figure shows the relation between the optimal investment threshold and bargaining power of equity holders in the R-model (solid line) and in the FSI-model (dashed line). We use the optimal initial coupon for each firm.

**Appendix**

**A. Initial reorganization, value functions thereafter**

$S_0$ and $U_0$ denote the initial reorganization and investment boundary, respectively. Define $S_0^{\nu}$ as the first time the firm reaches the initial reorganization threshold, i.e.,

$$S_0^{\nu} := \inf \{ t \geq 0 : X_t \leq S_0 | X_0 = X \} ,$$

and let $U_0^{\nu}$ denote the first time it reaches the investment threshold, i.e.,

$$U_0^{\nu} := \inf \{ t \geq 0 : X_t \geq U_0 | X_0 = X \} .$$

$T_0$ is the first time the firm reaches initial reorganization or investment, i.e.,

$$T_0 := \inf \{ S_0^{\nu}, U_0^{\nu} \} .$$
This Appendix A considers the case in which $T_0 = S_0^\nu$, i.e., reorganization occurs before investment.

The value functions and corporate policies after investment following initial reorganization are analogous to the case derived in Fan and Sundaresan (2000). We also present them for the sake of completeness.

**Unlevered firm value.** The unlevered firm value after the second (final) reorganization, $v_3(X)$, corresponds to

$$v_3(X) = \frac{1 - \tau}{r - \mu} X. \quad (42)$$

**The valuation of equity after investment following initial reorganization.** The value of equity after investment following initial reorganization, denoted by $e_2(X)$, corresponds to the present value of the expected payoffs to shareholders until reorganization plus the payoff at reorganization, i.e.,

$$e_2(X) = E^Q \left[ \int_{T_0}^{S_2^\nu} e^{-r(u-t)} (1 - \tau) (X_u - c_2) \, du + e^{-rS_2^\nu} \theta_2 v_3(S_2) \big| X_t = X \right], \quad (43)$$

in which $S_2^\nu$ is the first time the firm reaches the reorganization threshold after investment following reorganization, i.e.,

$$S_2^\nu = \inf \{ t \geq T_0 : X_t \leq S_2 \big| T_0 = S_0^\nu \}, \quad (44)$$

and $S_2$ is the firm’s reorganization threshold after investment following initial reorganization. $c_2$ is the coupon of the firm after investment following reorganization. Solving the corresponding Hamilton-Jacobi-Belman equation yields

$$e_2(X) = A_0^{c_2} + A_1^{c_2} X + A_2^{c_2} X^{\beta_2}, \quad (45)$$

in which

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left( \frac{1}{2} - \frac{\mu}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}}, \quad (46)$$

$$A_0^{c_2} = -\frac{1 - \tau}{r} c_2, \quad A_1^{c_2} = \frac{1 - \tau}{r - \mu}, \quad (47)$$

$$A_2^{c_2} = \left( \frac{r - \mu}{r} \right)^{1 - \beta_2} \left( \frac{\beta_2}{\beta_2 - 1} \right)^{-\beta_2} c_2^{-\beta_2} \frac{1 - \tau}{r - \mu} (1 - \theta_2)^{1 - \beta_2} \frac{1}{1 - \beta_2}. \quad (48)$$
The optimal reorganization threshold after investment following initial reorganization is calculated as
\[ S_2 = \frac{\beta_2}{\beta_2 - 1} \frac{r - \mu}{r} \frac{1}{1 - c_2} \tag{49} \]

The valuation of corporate debt after investment following initial reorganization. Similarly, the value of total corporate debt after investment following initial reorganization, denoted by \( d_2 (X) \), corresponds to the present value of the expected payoffs to debt holders until reorganization plus the payoff at reorganization, i.e.,
\[ d_2 (X) = \mathbb{E}^Q \left[ \int_t^{S_2^u} e^{-r(u-t)} c_2 du + e^{-rS_2^u} (1 - \theta_2) v_3 (S_2) | X_t = X \right]. \tag{50} \]

Solving the associated Hamilton-Jacobi-Belman equation, we find that
\[ d_2 (X) = A_{d_2}^0 + A_{d_2}^2 X^{\beta_2}, \tag{51} \]
in which
\[ A_{d_2}^0 = \frac{c_2}{r} \tag{52} \]
\[ A_{d_2}^2 = \left( \frac{r - \mu}{r} \right)^{-\beta_2} \left( \frac{\beta_2}{\beta_2 - 1} \right)^{-\beta_2} c_2^{1 - \beta_2} \frac{1}{r} \frac{1}{1 - \theta_2} \left( 1 - \tau \right)^{\beta_2} \left( \beta_2 - 1 \right)^{-1} \tag{53} \]
and \( \beta_2 \) is defined in Eq. (46).

Capital structure at investment after initial reorganization. The threshold for investment after initial reorganization is denoted by \( U_1 \). At investment after reorganization, equity holders choose the first-best capital structure. The reason is that the firm is an all equity financed firm after initial reorganization and the issue proceeds of new debt accrue to equity holders. The optimal (first-best) capital structure is obtained by maximizing firm value, which yields the coupon
\[ c_2 = c_2^{fb} = \frac{r}{r - \mu} \frac{\beta_2 - 1}{\beta_2} (1 - \beta_2) \frac{1}{\beta_2} (1 - \theta_2) U_1. \tag{54} \]

We calculate the corresponding first-best firm value as
\[ f_2 (U_1) = f_2^{fb} (U_1) = \left[ 1 - \tau + \tau (1 - \beta_2) \frac{1}{\beta_2} (1 - \theta_2) \right] \frac{U_1}{r - \mu}. \tag{55} \]
The firm value can be decomposed into the unlevered asset value and the value of the tax shield, in which the latter also incorporates the loss of tax shield due to the debt-equity-swap in case of final reorganization.

The valuation of equity after initial reorganization. The value of equity after initial reorganization, denoted by \( e_{1l}(X) \), corresponds to the present value of the expected payoffs to shareholders until investment plus the expected value of the payoff at investment, i.e.,

\[
e_{1l}(X) = \mathbb{E}^Q \left[ \int_t^{U_1^\nu} e^{-r(u-t)} (1 - \tau) X_u du + e^{-rU_1^\nu} (f_2(sU_2) - I) |X_t = X \right],
\]

(56)
in which \( U_1^\nu \) is the first time the firm reaches the investment threshold after initial reorganization, i.e.,

\[
S_2^\nu = \inf \{ t \geq T_0 : X_t \geq U_1 | T_0 = S_0^\nu \},
\]

(57)
and \( U_1 \) is the firm's investment threshold after initial reorganization. \( f_2(\cdot) \) is the firm value after investment following initial reorganization as given in closed-form by Eq. (55). Solving the corresponding Hamilton-Jacobi-Belman equation yields

\[
e_{1l}(X) = A_{1l} e_{1l}^1 X + A_{2l} e_{1l}^2 X, \]

(58)
in which

\[
\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2\tau}{\sigma^2}}, \quad A_{1l}^1 = \frac{1 - \tau}{r - \mu},
\]

(59)

\[
A_{2l}^1 = \left(\frac{\beta_1}{\beta_1 - 1}\right)^{-\beta_1} \left(\frac{I(r - \mu)}{(s - 1) (1 - \tau) + s\tau (1 - \beta_2) + (1 - \theta_2)}\right)^{-\beta_1} \frac{I}{\beta_1 - 1}.
\]

(60)

Initial reorganization. The initial reorganization problem reads

\[
\theta_0 = \arg \max_{\tilde{\theta}_0} \left\{ \tilde{\theta}_0 e_{1l}(S_0) - 0 \right\}^{\eta} \left\{ \left(1 - \tilde{\theta}_0\right) e_{1l}(S_0) - \alpha \frac{1 - \tau}{r - \mu} S_0 \right\}^{1 - \eta}.
\]

(61)
This problem is equivalent to

\[
\theta_0 = \arg \max_{\tilde{\theta}_0} \eta \log \left\{ \tilde{\theta}_0 e_{1l}(S_0) \right\} + (1 - \eta) \log \left\{ \left(1 - \tilde{\theta}_0\right) e_{1l}(S_0) - \alpha \frac{1 - \tau}{r - \mu} S_0 \right\}.
\]

(62)
The first order condition with respect to $\theta_0$ is

$$0 = \eta \frac{e_{\Pi}(S_0)}{\theta_0 e_{\Pi}(S_0)} + (1 - \eta) \frac{-e_{\Pi}(S_0)}{(1 - \theta_0) e_{\Pi}(S_0) - \alpha v_1(S_0)}.$$  \hspace{1cm} \text{(63)}

Define the surplus of initial reorganization to equity holders, debt holders, and total, as

$$SE(S_0) = \theta_0 e_{\Pi}(S_0) \hspace{1cm} \text{(64)}$$
$$SD(S_0) = (1 - \theta_0) e_{\Pi}(S_0) - \alpha v_1(S_0) \hspace{1cm} \text{(65)}$$
$$ST(S_0) = e_{\Pi}(S_0) - \alpha v_1(S_0) \hspace{1cm} \text{(66)}$$

respectively. Plugging these definitions into Eq. (63) and solving yields

$$SE(S_0) = \eta ST(S_0) \hspace{1cm} \text{(67)}$$
$$SD(S_0) = (1 - \eta) ST(S_0) \hspace{1cm} \text{(68)}$$

which shows that the surplus of renegotiation is apportioned such that each party receives a fraction of total surplus corresponding to its respective bargaining power. Finally, Eq. (67) implies the solution as stated in Eq. (8) of Section 3.1:

$$\theta_0 = \eta \left(1 - \alpha \frac{v_2(S_0)}{e_{\Pi}(S_0)}\right). \hspace{1cm} \text{(69)}$$

\textbf{B. Value functions after covenant renegotiation at investment}

We now consider the case in which $T_0 = U^\nu_0$, i.e., investment takes place before initial reorganization. The value of equity after covenant renegotiation at investment, $e_{1h}(X)$, is analogous to the case after investment following initial reorganization (Eqs. (45)-(48)). The reorganization threshold after covenant renegotiation at investment exhibits the same functional form as the reorganization threshold after investment following initial reorganization (Eq. (49)):

$$S_1 = \frac{\beta_2}{\beta_2 - 1} \frac{r - \mu}{r} \frac{1}{1 - \theta_1} (c_{01}^A + c_{11}^B). \hspace{1cm} \text{(70)}$$

The value function of total corporate debt is analogous to the value function of corporate debt after investment following initial reorganization (Eqs. (51)-(53)).
The value function of corporate debt of initial debt holders, who have a claim on the coupon $c_{A_{01}}^A$, is given by:

$$d_{1h} (X; c_{A_{01}}^A) = A_{01}^{d_{A_{01}}} + A_{2h}^{d_{A_{02}}} X^{\beta_2},$$

in which

$$A_{01}^{d_{A_{01}}} = \frac{c_{A_{01}}^A}{r}$$

$$A_{2h}^{d_{A_{02}}} = \left( \frac{\beta_2 (r - \mu)}{(\beta_2 - 1) r} \right)^{-\beta_2} \frac{1}{r} c_{A_{01}}^A \left( e_1 (c_1) - \beta_2 \left( \frac{1}{1 - \theta_1} \right)^{1-\beta_2} \left( 1 - \tau \right) \frac{\beta_2}{\beta_2 - 1} - 1 \right).$$

Analogously, the value function of corporate debt of new debt holders, who have a claim on the coupon $c_{B_{1}}^B = c_1 - c_{A_{01}}^A$, can be written as:

$$d_{1h} (X; c_{B_{1}}^B) = A_{01}^{d_{B_{1}}} + A_{2h}^{d_{B_{2}}} X^{\beta_2},$$

in which

$$A_{01}^{d_{B_{1}}} = \frac{c_{B_{1}}^B}{r}$$

$$A_{2h}^{d_{B_{2}}} = \left( \frac{\beta_2 (r - \mu)}{(\beta_2 - 1) r} \right)^{-\beta_2} \frac{1}{r} c_{B_{1}}^B \left( e_1 (c_1) - \beta_2 \left( \frac{1}{1 - \theta_1} \right)^{1-\beta_2} \left( 1 - \tau \right) \frac{\beta_2}{\beta_2 - 1} - 1 \right).$$

C. Covenant renegotiation

Proof of Proposition 1. The covenant renegotiation problem is stated in Eq. (12), Section 3.3 of the main text. A change of variables yields

$$\max_{\{c_{01}^A, c_{01}^B\}} \left( e_{1h} (sU_0; \tilde{c}_{01}^A + \tilde{c}_{1}^B) + d_{1h} (sU_0; \tilde{c}_{1}^B) - e_{1h} (sU_0; c_{01}^A) \right)^{\eta}$$

$$\cdot \left( d_{1h} (sU_0; \tilde{c}_{01}^A) - d_{1h} (sU_0; c_{01}^A) \right)^{1-\eta}$$

$$= \max_{\{c_{01}^A, c_{01}^B\}} \left( e_{1h} (sU_0; \tilde{c}_1) + d_{1h} (sU_0; \tilde{c}_1 - \tilde{c}_{01}^A) - e_{1h} (sU_0; c_{01}^A) \right)^{\eta}$$

$$\cdot \left( d_{1h} (sU_0; \tilde{c}_{01}^A) - d_{1h} (sU_0; c_{01}^A) \right)^{1-\eta}$$

$$= \max_{\{\tilde{c}_1\}} \left[ \max_{\{c_{01}^A\}} \left( e_{1h} (sU_0; \tilde{c}_1) + d_{1h} (sU_0; \tilde{c}_1 - \tilde{c}_{01}^A) - e_{1h} (sU_0; c_{01}^A) \right)^{\eta}$$

$$\cdot \left( d_{1h} (sU_0; \tilde{c}_{01}^A) - d_{1h} (sU_0; c_{01}^A) \right)^{1-\eta} \right].$$
The last equality uses the fact that \( \tilde{c}_1 \) is bounded by the firm’s debt capacity from above and by zero from below; \( \tilde{c}_{01}^A \) is bounded by \( \tilde{c}_1 \) above and by zero from below. Hence, the range of \( \{ c_{01}^A, \tilde{c}_1 \} \) is a convex and compact set in \( \mathbb{R}^2 \). Note that this property also guarantees the existence of the maximum. Therefore, we proceed by solving the following problem for an arbitrary, but fixed, \( \tilde{c}_1 \):

\[
\{ c_{01}^A \} = \arg \max_{\{ c_{01}^A \}} \left( e_1 h(sU_0; \tilde{c}_1) + d_1 h(sU_0; \tilde{c}_1 - \tilde{c}_{01}^A) - e_1 h(sU_0; c_0^A) \right)^\eta \\
\cdot \left( d_1 h(sU_0; \tilde{c}_{01}^A) - d_1 h(sU_0; c_0^A) \right)^{1-\eta}.
\]

(78)

Recall that total surplus from covenant renegotiation is calculated as

\[
ST(U_0; c_{01}^A, c_1) = f_{1h}(sU_0; c_1) - e_{1h}(sU_0; c_0^A) - d_{1h}(sU_0; c_0^A),
\]

cf. Eq. (15). In particular, for any given total coupon \( c_1 \), total firm value and total surplus are independent of the coupon to initial debt holders \( c_{01}^A \), such that we can write

\[
ST(U_0; c_{01}^A, c_1) = ST(U_0; c_1).
\]

(80)

Intuitively, firm value is determined by the total coupon, but not by the allocation of the total coupon to new and existing debt holders. Hence, re-writing Eq. (78), using this insight and the definition of total surplus in Eq. (13), yields that for any given \( \tilde{c}_1 \), \( c_{01}^A \) is the solution to the maximization problem

\[
\{ c_{01}^A \} = \arg \max_{\{ c_{01}^A \}} \left( SE(U_0; \tilde{c}_{01}^A, \tilde{c}_1) \right)^\eta \left( ST(U_0; \tilde{c}_1) - SE(U_0; \tilde{c}_{01}^A, \tilde{c}_1) \right)^{1-\eta}.
\]

(81)

The first order condition with respect to \( c_{01}^A \) reads:

\[
\eta \frac{\partial}{\partial c_{01}^A} SE(U_0; c_{01}^A, \tilde{c}_1) + (1 - \eta) \frac{-\partial}{\partial c_{01}^A} SE(U_0; c_{01}^A, \tilde{c}_1)}{ST(U_0; \tilde{c}_1) - SE(U_0; c_{01}^A, \tilde{c}_1)} = 0,
\]

(82)

which is equivalent to

\[
SE(U_0; c_{01}^A, \tilde{c}_1) = \eta ST(U_0; \tilde{c}_1).
\]

(83)

Consequently,

\[
SD(U_0; c_{01}^A, \tilde{c}_1) = (1 - \eta) ST(U_0; \tilde{c}_1),
\]

(84)
which completes the proof of (ii). Intuitively, the surplus to equity holders is a fraction $\eta$ of total surplus, in which $\eta$ corresponds to equity holders’ bargaining power. In turn, the surplus to debt holders is a fraction $1-\eta$ of total surplus, in which $1-\eta$ corresponds to debt holders’ bargaining power.

To prove (i), consider the maximization problem

$$\{c_1\} = \arg \max_{\{c_1\}} \left( SE\left(U_0; c^A_{01}, \tilde{c}_1\right) \right)^{\eta} \left( ST\left(U_0; \tilde{c}_1\right) - SE\left(U_0; c^A_{01}, \tilde{c}_1\right) \right)^{1-\eta}$$

$$= \arg \max_{\{c_1\}} \left( \eta ST\left(U_0; \tilde{c}_1\right) \right)^{\eta} \left( (1-\eta) ST\left(U_0; \tilde{c}_1\right) \right)^{1-\eta},$$  \hspace{1cm} (85)

in which we use Eqs. (83) and (84). Then, the first order condition with respect to $c_1$ is given by:

$$\eta \frac{\partial}{\partial c_1} ST\left(U_0; c_1\right) + (1-\eta) \frac{\partial}{\partial c_1} ST\left(U_0; c_1\right) = 0,$$  \hspace{1cm} (86)

which is equivalent to

$$\frac{\partial}{\partial c_1} ST\left(U_0; c_1\right) = 0,$$  \hspace{1cm} (87)

which proves part (i) using concavity of the total surplus. In words, the total new coupon from renegotiation is determined such that total surplus is maximized. Because total surplus is maximized at the optimal first-best capital structure at $U_0$, the total coupon is given by

$$c_1 = c^{fb}_1 = \frac{r}{r-\mu} \frac{\beta_2-1}{\beta_2} \left(1-\beta_2\right)^{\frac{1}{\beta_2}} (1-\theta) U_0,$$  \hspace{1cm} (88)

analogous to Appendix A, Eq. (54). Hence,

$$c^B_1 + c^A_{01} = c^{fb}_1.$$  \hspace{1cm} (89)

The corresponding first-best firm value at $U_0$ is analogous to Eq. (55) of Appendix A:

$$f_1\left(U_0\right) = f^{fb}_1 = \left[1-\tau + \tau (1-\beta_2)^{\frac{1}{\beta_2}} (1-\theta)\right] \frac{U_0}{r-\mu}$$  \hspace{1cm} (90)
The valuation of equity. The value of equity before investment or initial reorganization, \( e_0(X) \), corresponds to the present value of expected future cash flows to equity holders, i.e.,

\[
e_0(X) = \mathbb{E}^Q \left[ \int_0^{T_0} e^{-r(u-t)} (1 - \tau) \left( X_u - c_{01}^A \right) du \mid X_t = X \right] + \mathbb{E}^Q \left[ 1_{T_0=S_0^e} e^{-r(S_0^e-t)} \theta_0 e_{1h}(S_0) \mid X_t = X \right] + \mathbb{E}^Q \left[ 1_{T_0<U_0^e} e^{-r(U_0^e-t)} \left( e_{1h}(sU_0; c_1) - (I - d_{1h}(sX; c_{1B})) \right) \mid X_t = X \right],
\]

(91)
in which \( S_0^e, U_0^e, \) and \( T_0 \) are defined in Eqs. (39)-(41), and \( c_{01}^A \) and \( c_n \) are determined by Eq. (12). The first line of Eq. (91) corresponds to the cash flow to equity holders until initial reorganization or covenant renegotiation at investment. In case of distressed reorganization, equity holders obtain a fraction \( \theta_0 \) of the unlevered firm value (second line). The third line shows the value of equity at covenant renegotiation. The associated ordinary differential equation (ODE) reads

\[
\begin{align*}
e_0(X) &= \theta_0 e_{1h}(X) & 0 \leq X \leq S_0 \\
e_0(X) &= (1 - \tau) (X - c_{01}^A) + \mu X e_0'(X) + \frac{1}{2} \sigma^2 X^2 e_0''(X) & S_0 < X < U_0 \\
e_0(X) &= e_{1h}(sX; c_{01}^A + c_{1B}^B) - (I - d_{1h}(sX; c_{1B})) & X \geq U_0,
\end{align*}
\]

(92)
subject to the boundary conditions

\[
\begin{align*}
e_0(S_0) &= \theta_0 e_{1h}(S_0) & (93) \\
e_0(U_0) &= e_{1h}(sU_0; c_1) - (I - d_{1h}(sU_0; c_{1B})) . & (94)
\end{align*}
\]

Standard arguments and plugging Eqs. (18) and (19) into Eq. (94) imply the solution as stated in the main text of Section 3.4, Eqs. (21)-(27).
The valuation of corporate debt. The value of debt before investment or initial reorganization, \( d_0(X) \), corresponds to the present value of expected future cash flows to debt holders, i.e.,

\[
d(X) = \mathbb{E}^Q \left[ \int_0^{T_0} e^{-r(u-t)} c_0^A du \mid X_t = X \right] + \mathbb{E}^Q \left[ \mathbf{1}_{T_0=U_0^0} d_{1h}(sU_0; c_0^A) \right].
\]

The first term in the first line of Eq. (95) shows the value of coupon payments to debt holders until covenant renegotiation at investment or initial reorganization. In case of covenant renegotiation at investment, initial debt holders have a claim on the total coupon \( c_0^A \) determined by the Nash solution, which corresponds to the second term in the first line of Eq. (95). In case of reorganization, debt holders receive a fraction \((1 - \theta_0)\) of the unlevered equity value, line two of Eq. (95). The ODE for the value of debt is given by

\[
\begin{cases}
  d_0(X) = (1 - \theta_0) e_{1l}(X) & 0 \leq X \leq S_0 \\
  rd_0(X) = c_0^A + \mu X d_0(X) + \frac{1}{2} \sigma^2 X^2 d''_0(X) & S_0 < X < U_0 \\
  d_0(X) = d_{1h}(sX; c_0^A) & X \geq U_0,
\end{cases}
\]

subject to the boundary conditions

\[
d_0(S_0) = (1 - \theta_0) e_{1l}(X_S), \quad d_0(U_0) = d_{1h}(sX; c_0^A).
\]  

Standard arguments and plugging Eqs. (18) and (20) into Eq. (97) imply the solution as stated in the main text of Section 3.4, Eqs. (28)–(32).

E. FSI-model

The FSI-model corresponds to the model of Fan and Sundaresan (2000), augmented with lumpy investment as in Hackbarth and Mauer (2012).

There is no covenant after initial reorganization because the firm is all equity financed up to investment. Hence, the value functions and boundaries \( e_{1l}(X), e_2(X), d_2(X), v_3(X), U_1 \), and \( S_2 \) are calculated with the same formulas as in the R-Model.
Similarly, the formulas for reorganization after covenant renegotiation remain unchanged \((e_{1h}(X), \text{total debt } d_{1h}(X), \text{and } S_1)\).

We now derive the value functions before investment or initial reorganization. At the investment boundary \(U_0\), equity holders determine the coupon of new debt by maximizing the value of equity and new debt:

\[
c_{1}^{B} = \underset{c_{1}}{\arg \max} e_{1h} \left( sU_{0}; c_{0}^{A} + c_{1}^{B} \right) + d_{1h} \left( sU_{0}; c_{1}^{B} \right)
\] (98)

The resulting coupon \(c_{1}^{B}\) corresponds to second-best financing of the investment opportunity because equity holders do not incorporate the dilution of initial debt holders’ claim. Hence, equity holders overlever, i.e., the resulting total coupon \(c_{1} = c_{0}^{A} + c_{1}^{B}\) is larger than the first-best coupon (Hackbarth and Mauer, 2012). The value of initial debt incorporates this potential dilution of the debt claim at investment.

This financing choice affects the boundary condition in the ordinary differential equation (ODE) that determines the value of equity:

\[
\begin{align*}
e_{0}(X) &= \theta_{0} e_{1l}(X) & 0 \leq X \leq S_{0} \\
e_{0}(X) &= (1 - \tau) \left( X - c_{0}^{A} \right) + \mu X e_{0}'(X) + \frac{1}{2} \sigma^{2} X^{2} e_{0}''(X) & S_{0} < X < U_{0} \\
e_{0}(X) &= e_{1h} \left( sX; c_{0}^{A} + c_{1}^{B} \right) - (I - d_{1h} \left( sX; c_{1}^{B} \right)) & X \geq U_{0},
\end{align*}
\] (99)

subject to the boundary conditions

\[
\begin{align*}
e_{0}(S_{0}) &= \theta_{0} e_{1l}(S_{0}) \quad (100) \\
e_{0}(U_{0}) &= e_{1h} \left( sU_{0}; c_{1} \right) - (I - d_{1h} \left( sU_{0}; c_{1}^{B} \right)), \quad (101)
\end{align*}
\]

in which \(c_{1}^{B}\) is determined by Eq. (98). The functional form of the value function of equity corresponds to that presented in the R-Model (Proposition 2), but with the vector

\[
\begin{bmatrix}
-A_{0}^{e_{o}} - A_{1}^{1} S_{0} + \theta_{0} e_{1l}(S_{0}) \\
e_{1h} \left( sU_{0}; c_{0}^{A} + c_{1}^{B} \right) + d_{1h} \left( sU_{0}; c_{1}^{B} \right) - I - A_{0}^{e_{o}} - A_{1}^{1} U_{0}
\end{bmatrix}.
\] (102)
The associated smooth-pasting conditions are:
\[
\begin{align*}
\frac{\partial}{\partial X} c_0(X) \big|_{X=S_0} &= \theta \frac{\partial}{\partial X} e_{II}(X) \big|_{X=S_0} \quad (103) \\
\frac{\partial}{\partial X} e_0(X) \big|_{X=U_0} &= \frac{\partial}{\partial X} e_{1h}(sX; c_1) \big|_{X=U_0} + \frac{\partial}{\partial X} d_{1h}(sX; c_1^B) \big|_{X=U_0} \quad (104)
\end{align*}
\]

Finally, equity holders choose the initial capital structure by maximizing the value of their objective function ex-ante. Therefore, equity holders solve
\[
c_{0}^A = \arg \max_{c_{0}^A} \{ e_0(X_0) + d_0(X_0) \}.
\]

Equity holders’ problem consists of solving Eq. (105) subject to Eqs. (103)-(104). A closed-form solution does not exist. We use numerical procedures and verify the optimality of the investment and reorganization boundaries numerically.

Similar to the valuation of equity, the ODE for the value of debt is given by
\[
\begin{align*}
d_0(X) &= (1 - \theta_0) e_{II}(X) \quad 0 \leq X \leq S_0 \\\nrd_0(X) &= c_{0}^A + \muXd_0(X) + \frac{1}{2}\sigma^2X^2d''_0(X) \quad S_0 < X < U_0 \\
d_0(X) &= d_{1h}(sX; c_{0}^A) \quad X \geq U_0,
\end{align*}
\]
subject to the boundary conditions
\[
d_0(S_0) = (1 - \theta_0) e_{II}(X_S), \quad d_0(U_0) = d_{1h}(sX; c_{0}^A). \quad (107)
\]

In particular, in case of investment, the value of initial debt is still determined by initial coupon payments $c_{0}^A$, but incorporates the increase in default risk due to the issuance of new debt. Hence, the functional form of the value function of debt corresponds to that presented in the R-Model (Proposition 2), but with the vector
\[
b^d = \begin{bmatrix} -A_{0}^d + (1 - \theta_0) e_{II}(S_0) \\ d_{1h}(sU_0; c_{0}^A) - A_{0}^d \end{bmatrix}.
\]

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F. Value functions of firms with first-best investment and financing decisions

FSI-first-best. The FSI-first-best considers a firm in the FSI model, but with firm value maximizing investment and financing decisions. Specifically, all value-matching conditions in the valuation of corporate securities correspond to those of a FSI-model firm (see Appendix E). In the FSI-first-best, however, the smooth-pasting condition at the investment boundary and the new coupon at investment are firm value-maximizing instead of equity-value maximizing as in the FSI-model. Specifically, the first-best coupon $c_t^{fb}$ is installed at investment instead of equity holders’ coupon choice determined by condition (98). Further, we replace the smooth-pasting condition (104) by the condition that ensures firm value maximization:

$$\frac{\partial}{\partial X} e_0(X) \bigg|_{X=U_0} + \frac{\partial}{\partial X} d_0(X) \bigg|_{X=U_0} = f_{1h}s. \quad (109)$$

R-first-best. Recall that the R-model implies first-best financing at investment. The R-first-best corresponds to a firm in the R-model, but with firm value maximizing investment decision. Specifically, all value-matching conditions in the valuation of corporate securities correspond to those of a R-model firm (see Appendix D). In the R-first-best, however, the smooth-pasting condition at the investment boundary is firm value-maximizing instead of equity-value maximizing as in the R-model. We replace the smooth-pasting condition (37) by the condition that ensures firm value maximization:

$$\frac{\partial}{\partial X} e_0(X) \bigg|_{X=U_0} + \frac{\partial}{\partial X} d_0(X) \bigg|_{X=U_0} = f_{1h}s. \quad (110)$$

G. Waiting time to debt renegotiation

Standard arguments imply that $M_t := \mathbb{E}[T_0 | \mathcal{F}_t]$ is a martingale, in which $T_0$ corresponds to the waiting time to covenant renegotiation at investment or initial reorganization (defined in Eq. (41)). Further:

$$M_t = \mathbb{E}[T_0 1_{T_0 \leq t} + T_0 1_{T_0 > t} | \mathcal{F}_t] = T_0 1_{T_0 \leq t} + \mathbb{E}[(T_0 - t) 1_{T_0 \leq t} + t 1_{T_0 > t} | \mathcal{F}_t] \quad (111)$$

$$= T_0 1_{T_0 \leq t} + t 1_{T_0 > t} + \mathbb{E}[(T_0 - t) 1_{T_0 > t} | \mathcal{F}_t] = T_0 1_{T_0 \leq t} + 1_{T_0 > t} (t + w(X_t)), \quad 59$$
in which we used the Markov property, and \( w (X_t) = \mathbb{E} [(T_0 - t) \mid \mathcal{F}_t] \) denotes the waiting time to any renegotiation (either distressed or covenant renegotiation) from time \( t \) on. An application of Ito’s lemma shows that for \( T_0 \leq t \), \( w (X_t) \) solves the ODE

\[
\mu X w' (X) + \frac{1}{2} \sigma^2 X^2 w'' (X) + 1 = 0, \tag{112}
\]

subject to the boundary conditions

\[
w (S_0) = 0, \quad w (U_0) = 0. \tag{113}
\]

The solution is given by

\[
w (X) = \frac{2 \sigma^2}{2 \mu - \sigma^2} - \frac{2 \log (X)}{2 \mu - \sigma^2} + 2 \left( 2S_0^{1-2\nu} \mu \log (U_0) - 2U_0^{1-2\nu} \mu \log (S_0) \right.
\]

\[
+ \left. S_0^{2\nu-1} \sigma^2 \exp \left( -\frac{1}{\sigma^2} (2\mu - \sigma^2) \left( \log (S_0) + \log (U_0) \right) \right) \right)
\]

\[
- U_0^{2\nu-1} \sigma^2 \exp \left( -\frac{1}{\sigma^2} (2\mu - \sigma^2) \left( \log (S_0) + \log (U_0) \right) \right)
\]

\[
- S_0^{1-2\nu} \sigma^2 \log (U_0) - U_0^{1-2\nu} \sigma^2 \log (S_0) \right) \left( (2\mu - \sigma^2)^2 \left( S_0^{1-2\nu} - U_0^{1-2\nu} \right) \right)^{-1}
\]

\[
+ (2 \log (U_0) - 2 \log (U_0)) \left( X^{2\nu-1} (2\mu - \sigma^2) S_0^{1-2\nu} - U_0^{1-2\nu} \right)^{-1}. \tag{114}
\]

Similarly, \( N_t := \mathbb{P} (T_i = U_0^t \mid \mathcal{F}_t) \) is a martingale. Using analogous arguments, the ODE for the probability of covenant renegotiation, \( u (X_t) = \mathbb{E} \left[ 1_{T=U_0^t} \mid \mathcal{F}_t \right] \), reads

\[
\mu X u' (X) + \frac{1}{2} \sigma^2 X^2 u'' (X) = 0, \tag{115}
\]

subject to the boundary conditions

\[
u (S_0) = 0, \quad u (U_0) = 1. \tag{116}
\]

The solution is given by

\[
u (X) = \frac{1}{S_0^{2\nu-1} - U_0^{2\nu-1}} \left( \left( \frac{X}{S_0 U_0} \right)^{1-2\nu} - U_0^{2\nu-1} \right). \tag{117}
\]