School of Finance

University of St.Gallen

FUNDING ILLIQUIDITY

MATTHIAS RUPPRECHT
JAN WRAMPELMEYER

WORKING PAPERS ON FINANCE NO. 2016/01

SWISS INSTITUTE OF BANKING AND FINANCE (S/BF – HSG)

JANUARY 2016
THIS VERSION: SEPTEMBER 5, 2019

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Matthias Rupprecht

University of St. Gallen†

Jan Wrampelmeyer

VU Amsterdam

Tinbergen Institute‡

September 5, 2019

*We thank Marc Arnold, Martin Brown, Markus Brunnermeier, Hans Gersbach, Antoine Martin, Vincent Maurin, Cyril Monnet, Erwan Morellec, Kjell Nyborg, Angelo Ranaldo, Jean-Charles Rochet, Paul Söderlind, André C. Silva, Dimitri Vayanos, and Andrea Vedolin, as well as participants of the ECB-IMF Workshop on "Money Markets, Monetary Policy Implementation and Market Infrastructures", of the EFMA, and of research seminars at the Austrian National Bank, Deutsche Bundesbank, Queen Mary University of London, Swiss National Bank, Tilburg University, University of Zurich, and VU Amsterdam for helpful comments. This work was supported by the Sinergia grant 154445 from the Swiss National Science Foundation.

†Matthias Rupprecht, University of St. Gallen, E-mail: matthias.rupprecht@unisg.ch.

‡Jan Wrampelmeyer, Vrije Universiteit Amsterdam, School of Business and Economics, De Boelelaan 1105, 1081HV Amsterdam, The Netherlands. Email: j.wrampelmeyer@vu.nl. Tel: +31 20 59 83012.

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We provide a theoretical model for funding liquidity that extends the literature by allowing financial institutions to raise short-term unsecured funding in addition to secured funding. We identify a new liquidity spiral, a credit limit spiral, for unsecured funding and show how it reinforces the margin and loss spiral for secured funding. Our model brings to light a dual role of margins. While higher margins reduce secured funding, they relax the funding constraint in the unsecured market ceteris paribus. We show that there is commonality and substitution effects in funding illiquidity, and discuss how central bank and regulatory policies can prevent illiqudity.

KEYWORDS: Funding liquidity, unsecured and secured funding, liquidity spirals, monetary policy, regulation

JEL CODES: E43, E58, G01, G12, G21, G28

1. Introduction

Funding liquidity refers to the ability to raise cash on short notice, which is crucial for the well-functioning of the financial system. In times of immediate funding needs, financial institutions borrow in the unsecured money market and obtain funding in the secured (or "repo") market[1] in order to avoid liquidating (long-term) financial assets. If funding liquidity dries up in both markets simultaneously, securities markets become illiquid due to asset (fire-) sales, prompting bank failures and spillover effects throughout the financial system with severe consequences even for the real economy.

The theoretical literature provides several key contributions focusing on the relation between secured funding and market liquidity (e.g., Gromb and Vayanos, 2002, 2018; Brunnermeier and Pedersen, 2009).[2] In these papers, collateral constraints impede arbitrageurs such as, e.g., hedge funds, from providing market liquidity, thereby causing even tighter collateral constraints and market illiquidity. In this paper, we extend the setting of Brunnermeier and Pedersen (2009) to apply to financial institutions more generally, e.g., commercial banks, by allowing them to raise short-term funding in the secured *and* unsecured money market. Our model can explain the intricate dynamics of funding illiquidity across funding markets and the contagion channels with securities market illiquidity, providing a theoretical foundation for empirically observed patterns (Gorton and Metrick, 2012; Nyborg and Östberg, 2014). Our model shows that liquidity spirals arise across unsecured and secured funding, leading to commonality and substitution effects in funding illiquidity, and provides insights about how

[1]A repurchase agreement or "repo" is essentially a collateralized loan based on a simultaneous sale and forward agreement to repurchase securities at the maturity date. Throughout this paper, we use the terms secured funding, collateralized funding, and repo interchangeably.

[2]For a survey of the limits to arbitrage literature and how arbitrageurs financial constraints affect asset prices, see Gromb and Vayanos (2010).

central bank and regulatory policies can prevent illiqudity.

A joint model of unsecured and secured funding illiquidity is important for at least three reasons. First, the unsecured money market is a crucial source of funding that simply cannot be ignored. In Europe, unsecured funding amounted to more than 90% of short-term euro interbank borrowing before the 2007/09 global financial crisis, and the new main euro short-term interest rate benchmark will continue to be based on this market (European Central Bank, 2012, 2018). Moreover, banks actively use both unsecured and secured funding and substitute between the two sources (Di Filippo, Ranaldo, and Wrampelmeyer, 2018). In the United States, the fed funds market remains an important source of short-term funding that was not completely frozen even during the period associated with the Lehman bankruptcy (Afonso, Kovner, and Schoar, 2011). In contrast to previous papers, our model accounts for these empirical observations, in particular shedding light on the substitution between both funding sources and common funding illiquidity. Second, an analysis of funding illiquidity limited to secured funding as in, e.g., Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009), provides only a partial depiction of the financial system and applies only to a restricted set of agents (e.g., hedge funds). By adding unsecured funding, our model also applies to large, systemically important financial institutions, such as commercial banks and investment banks, that play a key role in the financial system and rely heavily on short-term unsecured borrowing. Third, a joint model supports the effort of policy makers to establish successful measures that counteract funding illiquidity while accounting for cross-market effects. Similarly, it helps understand the effects of exit strategies from such policies on funding markets. For example, in response to the recent global financial crisis (see, e.g., French et al., 2010), central banks' unconventional monetary policies included offering refinancing via repos at attractive conditions (e.g., long-term refinancing operations (LTROs) by the ECB), purchasing banks' unsecured commercial papers (e.g., the FED's Commercial Paper Funding Facility), or quantitative easing (QE) via large scale asset purchases (e.g., the FED's and ECB's QE programs), specifically targeting different funding and securities markets. Given possible substitution or contagion across markets, it is important to understand whether a policy measure aimed at, e.g., stabilizing secured funding, could negatively affect unsecured funding, thus undermining its overall effectiveness.

The key feature of our model is that banks can borrow in the secured and unsecured money market. In the secured market, borrowing is subject to margins (haircuts to the value of the collateral security), and in the unsecured market borrowers face a credit limit and pay a (default) risk premium atop the risk-free rate. Our model shows that illiquidity can arise from a small shock (e.g., to borrowers' own capital or a securitys fundamental value) that leads to deteriorating funding volumes and securities prices. We find three novel mechanisms in such an illiquid equilibrium. First, in addition to funding problems in the secured market[3], we identify a new liquidity spiral, namely, a *credit limit* spiral in the unsecured market. A shock increases borrowers' default risk, which induces lenders to reduce credit limits and provide less unsecured funding. Lower credit limits in the unsecured market lead to further downward pressure on prices, higher default risk, and further credit rationing, thus triggering a credit limit spiral.

Second, funding illiquidity in the unsecured and secured market are related and the credit limit spiral in the unsecured market and the margin and loss spirals in the secured market mutually reinforce each other. For instance, when initial losses lead to a decrease in credit

[3]In the secured market, margins tend to increase after initial losses, which may trigger secured liquidity spirals (Gromb and Vayanos, 2002; Brunnermeier and Pedersen, 2009).

limits such that banks have to sell assets, this leads to market illiquidity, further downward price pressure, and thus an increase in margins. In fact, the availability of unsecured funding in addition to secured funding allows borrowers to further increase their asset holdings and thus worsens the loss spiral, i.e., the eroding effect on capital following an asset price shock. These dynamics are summarized in Figure 1, which shows the credit limit spiral in the unsecured money market, the margin spiral in the secured money market, and the combined loss spiral, including feedback effects from both funding markets.

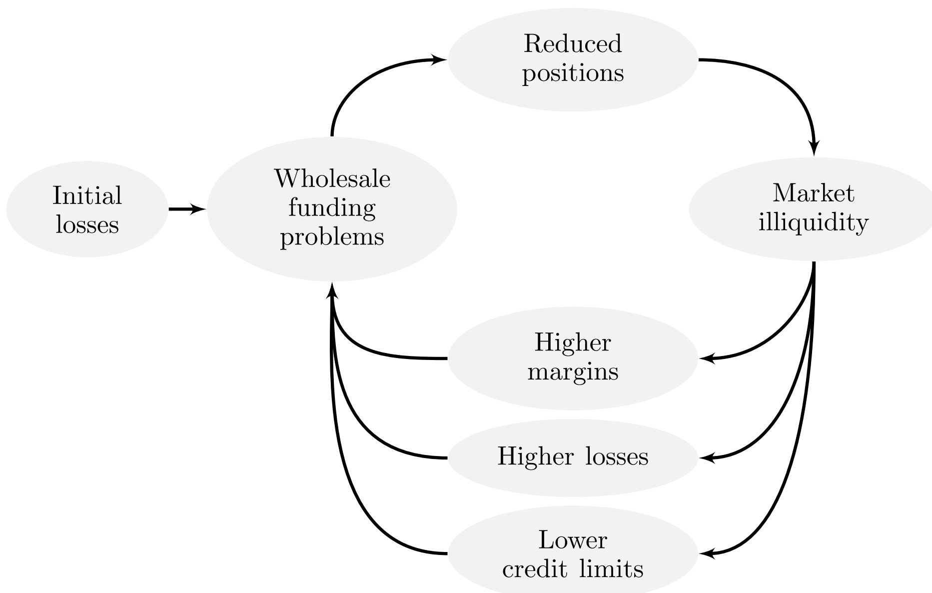

Figure 1. Liquidity Spirals
The figure shows the credit limit spiral in the unsecured money market (outer circle), the margin spiral in the secured money market (Brunnermeier and Pedersen (2009), inner circle), and the combined loss spiral, including feedback effects from both money markets. Spirals start when initial losses lead to funding problems, i.e., banks' funding constraints are binding in both the unsecured and the secured money market at the same time.

Third, our model brings to light a dual role of margins for funding illiquidity. On the one hand, an increase in margins further limits the amount a bank can borrow against a collateral asset, thus tightening the collateral constraint and reducing the availability of secured funding. On the other hand, higher margins relax the funding constraint for unsecured borrowing, ceteris paribus, by reducing the amount of (senior) secured debt in banks' balance sheets, which decreases the probability of a borrower's default on its unsecured loans. All else equal, this relaxation allows for a substitution of liquidity and implies that the share of unsecured funding increases following a shock.

Market fragility and funding illiquidity do not arise when borrowing in one of the funding markets is unconstrained. We call this a liquid equilibrium, and banks substitute a loss of funding liquidity in the constrained market by borrowing more in the unconstrained market. For instance, if banks have sufficient spare collateral, they can offset a loss of unsecured funding in the secured market. Similarly, markets are not fragile if banks can compensate a loss in secured funding by raising more unsecured funds if credit limits are not binding.

After establishing the market dynamics under asymmetric information, we derive the optimal funding provision when lenders have full information about the fundamental value of the borrowers' assets. In this benchmark case, funding and asset markets are liquid, because lenders reduce margins in the secured market and credit limits in the unsecured market when the asset price is below the fundamental value. We show that unconventional monetary policies can prevent fragility and restore liquidity, but only when secured and unsecured funding markets are taken into account jointly. For instance, offering refinancing facilities with lower margins ("haircuts") than in the private market eases funding constraints in the secured market, but at the same time lowers the amount that lenders are willing to provide in the unsecured market. Therefore, a joint perspective is crucial as central bank haircuts need to be low enough to compensate for the crowding out of unsecured funding in order to be effective. In contrast, central bank asset purchases affect both funding markets, as enhanced market liquidity relaxes banks' funding constraints in the secured and unsecured market jointly.

In addition to monetary policy, we examine the effects of the main, regulatory measures (e.g., Dodd-Frank Act and Basel III) that were introduced in the aftermath of the 2007/09 financial crisis, namely countercyclical capital buffers, leverage ratios, and liquidity coverage ratios. Our model delivers two important results for policy makers and regulators: First, regulation should aim to prevent that funding constraints are binding in both the secured and unsecured funding market simultaneously. In fact, as long as financial institutions are able to borrow in one of the two money market segments, they can substitute secured and unsecured funding, thereby preventing funding illiquidity. Second, static measures such as a constant maximum leverage ratio do not adequately address fragility. In contrast, countercyclical measures can preempt the adverse consequences of excess deleveraging and illiquidity in "bad" times.

Our model provides several testable implications, including (i) a link between credit limits and market illiquidity (ii) commonality in secured and unsecured funding illiquidity, (iii) a marginal increase in margins leads to a marginally higher share of unsecured funding, (iv) higher initial leverage causes a stronger reduction in total asset holdings after a shock, and (iv) central bank liquidity provision counteracts asset fire sales and funding illiqudity. We perform a simple empirical analysis using bank-level data and the European sovereign debt crisis as an example of an asset/wealth shock. Despite the limited number of observations, the empirical results support the main predictions of our model. Our model is also consistent with findings in the existing empirical literature. For instance, Nyborg and Östberg (2014) document a link between unsecured funding and asset market illiquity. Gorton and Metrick (2012) show that high funding cost in the unsecured market and increased haircuts in the secured market occur at the same time. Di Filippo, Ranaldo, and Wrampelmeyer (2018) jointly analyze unsecured and secured funding at the individual bank level providing evidence that banks actively substitute funding between the unsecured and secured market.

Our paper contributes to the growing body of theoretical literature on funding markets. Only few papers jointly analyze both unsecured and secured funding. Ahnert et al. (2018) highlight asset encumbrance with long-term secured debt, whereas Wolski and van de Leur (2016) study network structures and lenders' asset allocation towards secured and unsecured loans. Matta and Perotti (2016) model the interaction of repo with demandable debt under asset liquidity risk in a global games approach to bank runs in the spirit of Rochet and Vives (2004) and Goldstein and Pauzner (2005). Auh and Sundaresan (2018) determine borrowers' optimal liability structure and model the interaction between long-term secured debt and short-term unsecured debt in a corporate finance context. de Fiore, Hoerova, and Uhlig (2018) model the implications of recent trends in interbank markets for monetary policy and the broader economy, whereas Hoerova and Monnet (2016) focus on the relation between money markets and central bank lending. We contribute to this literature by highlighting how shocks can lead to funding illiquidity and mutually reinforcing liquidity spirals in both unsecured and secured funding markets.

Most prior theoretical research on the interbank market either focuses on unsecured *or* secured funding, sometimes in connection with central bank liquidity. For the unsecured market, various studies highlight reasons for potential market breakdowns, such as liquidity hoarding (e.g., Acharya and Skeie, 2011; Heider, Hoerova, and Holthausen, 2015) or counterparty credit risk (e.g., Stiglitz and Weiss, 1981; Freixas and Jorge, 2008; Bruche and Suarez, 2010). Our model shows that access to secured funding can reduce the frictions and the resulting externalities described in this literature. On the other hand, secured funding strains can reinforce credit limit and loss spirals in the unsecured market and even trigger a market breakdown.

Assuming that banks only have access to secured funding, Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009) model fire sale mechanisms originating from funding constraints in the secured money market. We show that depending on whether unsecured funding is constrained, these fire sales are either magnified, or mitigated when substitution from secured to unsecured funding can sufficiently stabilize the market. The borrower's capacity to roll over short-term loans (e.g., Acharya and Viswanathan, 2011; Acharya, Gale, and Yorulmazer, 2011) and the market design (Martin, Skeie, and von Thadden, 2014) affect the fragility of secured funding. Our model sheds light on new dynamics of fragility, i.e., secured and unsecured liquidity spirals mutually reinforce each other, inducing commonality in funding and asset market illiquidity.

Finally, our model highlights the interrelations between secured and unsecured funding as well as central bank policies. Other papers examine the role of specific central bank measures such as lender of last resort facilities (Acharya and Tuckman, 2014), interest rate cuts (Freixas, Martin, and Skeie, 2011), liquidity injections (Allen, Carletti, and Gale, 2009), or haircut policies (Ashcraft, Gârleanu, and Pedersen, 2011; Koulischer and Struyven, 2014).[4]

The remainder of this paper is structured as follows. Section 2 presents the model setup and the market equilibrium. In Section 3, we derive margins for secured lending and credit limits for unsecured lending. Section 4 analyzes market fragility and liquidity spirals and highlights the relation between unsecured and secured funding liquidity. In Section 5, we derive the full-information outcome of our model and assess the consequences and efficacy of

[4]Starting with Poole (1968), a number of papers model banks' central bank reserves management. Recent contributions linking central bank policy and the interbank market are, e.g., Afonso and Lagos (2015) and Bech and Monnet (2016).

central bank and regulatory policy for money market liquidity. Section 6 contains a simple empirical exercise. Section 7 concludes.

2. The Model

Our model extends the economy in Brunnermeier and Pedersen (2009) to allow for unsecured funding of a single asset (portfolio) as described below.

2.1. Assets

There is one asset traded at times $t = 0, 1, 2, 3$, which pays off its fundamental value ν at time 3. The risk-free rate is normalized to zero and the asset is in zero aggregate supply. The fundamental value of the asset accumulates $\kappa > 0$ in each period and evolves according to

$$\nu_{t+1} = \nu_t + \kappa + \sigma_{t+1}\varepsilon_{t+1}, \tag{1}$$

where ε_t has a standard normal cumulative distribution function Φ. Fundamental volatility σ_t follows an autoregressive conditional heteroskedasticity (ARCH) process,

$$\sigma_{t+1} = \underline{\sigma} + \theta|\Delta\nu_t - \kappa|, \tag{2}$$

with $\underline{\sigma}, \theta \geq 0$ and $\theta < 1$, implying that volatility increases after a shock and that the process is stationary.[5] We denote the market price of the asset by p_t, and the signed deviation of the

[5]The model can easily by generalized, e.g., using other (G)ARCH dynamics. For simplicity, we choose the dynamics in Equation (2), in line with Brunnermeier and Pedersen (2009).

price from the fundamental value by

$$\Lambda_t = p_t - \nu_t, \tag{3}$$

representing, e.g., the spread between a bond's price and its par value on the yield curve as in Hu, Pan, and Wang (2013). The absolute amount of this deviation, $|\Lambda_t|$, is our measure of market illiquidity.

2.2. Agents

The economy is populated by three groups of agents. Specifically, *customers* trade assets with other agents called *banks*, who borrow from *lenders* in the secured and unsecured money market to fund their positions.[6] Figure 2 schematically shows the model setup. In the following, we

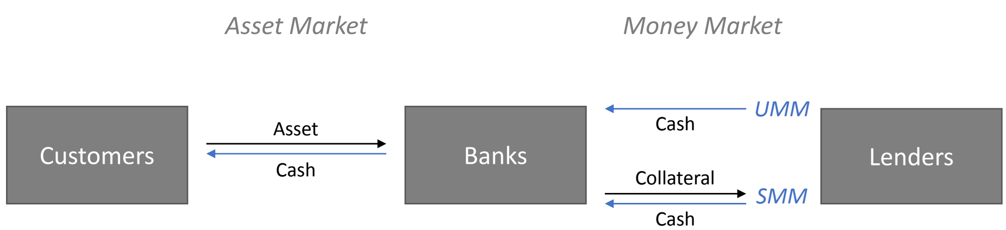

Figure 2. Model setup
This figure schematically shows the model setup. Customers trade with banks in the asset market (left-hand side). Banks obtain funding from lenders in the money market (right-hand side). Money market funding can be unsecured (UMM) and secured (SMM).

present each group of market participants as well as the trade dynamics between these agents.

Customers. There are three risk-averse customers $k = 0, 1, 2$, with initial cash holding $W_0^k > 0$ and zero shares of the asset.[7] At time 0, customers learn that they will experience

[6]The group of customers refers to, e.g., asset managers or hedge funds, banks represent, e.g., large commercials banks or broker-dealers with a liquidity deficit, and lenders are money market funds or (small) commercial banks with a liquidity surplus.
[7]The group of customers is identical to Brunnermeier and Pedersen (2009).

an endowment shock of z^k assets at time 3. These shocks represent binding orders that customers must execute in the market until time 3. All shocks are random and aggregate to zero, $\sum_{k=0}^{2} z^k = 0$. With probability $(1-a)$, all customers arrive in the market place at time 0 and trade their endowment shocks with each other. In the complementary case, with probability a, customer $k=0$ arrives at time 0, customer $k=1$ at time 1, and customer $k=2$ at time 2. Thus, at time 2, it holds that $\sum_{k=0}^{2} z^k = 0$, while at times $t = 0, 1$, aggregate supply is non-zero. We focus on the case $z^0 > 0$, $Z_1 > 0$ and $z^2 < 0$, where $Z_t := \sum_{k=0}^{t} z^k$ is the total demand shock. This means that aggregate supply is positive at times 0 and 1, and negative at time 2, which allows us to concentrate on banks' funding structure and asset holdings at times 0 and 1.

After arriving in the market place, customers choose their position y_t^k in each period to maximize their exponential utility function $U(W_3^k) = -\exp(-\gamma W_3^k)$ over final wealth, i.e.,

$$\max_{y_t} -\mathbb{E}_t \left[e^{-\gamma W_3^k} \right], \tag{4}$$

subject to their wealth dynamics

$$W_{t+1}^k = W_t^k + (\Delta p_{t+1} - \kappa)(y_t^k + z^k). \tag{5}$$

Equation (5) states that customers' wealth increases with the asset's (clean) price and asset holdings $y_t^k + z^k > 0$.

Lenders. Lenders are identical, risk-neutral, and deposit their excess funds in the money market as one-period loans.[8] There are two types of money market loans which differ with respect to collateralization. We refer to collateralized loans as *secured* funding and denote the total secured lending volume by M_t^s. Analogously, we refer to uncollateralized loans as *unsecured* funding and denote the total unsecured lending volume by M_t^u.[9]

Lenders monitor banks' balance sheet as the sum of secured and unsecured money market loans and banks' own capital W_t, hence verifying their asset position $x_t p_t$. We differentiate between the cases of informed and uninformed lenders. If lenders are informed, which will serve as our benchmark scenario in Section 5.1, their information set is $\mathcal{F}_t = \sigma\{z, \nu_0, \ldots, \nu_t, p_0, \ldots, p_t, x_0, \ldots, x_t, \eta_1, \ldots, \eta_t\}$, i.e., lenders know the fundamental value of the asset as well as shock η, and thus the true probability of counterparty default.[10] In contrast, if lenders are uninformed they can only observe market prices, such that their information set is given by $\mathcal{F}_t = \sigma\{p_0, \ldots, p_t, x_0, \ldots, x_t\}$. This friction arises from the basic asymmetric information problem in lending markets, where lenders cannot verify the borrower's exact asset risk (e.g., Stiglitz and Weiss, 1981).

Lenders protect against counterparty credit risk by rationing secured and unsecured funding. To that end, the basic problem of an uninformed lender arises from filtering out the extent to which price volatility is due to a fundamental shock or illiquidity, i.e., his prior assumption about the probability of customers' sequential arrival, a. Since we look at money market

[8] Lenders invest in liquid securities in the spirit of Allen and Gale (2000). In our model, all loans have a maturity of one-period. We thus abstract from a shortening of maturity as in, e.g., Kuo et al. (2014), which could be an additional source of fragility.

[9] Alternatively, one could include two groups of lenders that both observe a bank's balance sheet: one group that lends in the secured market and another group that lends in the unsecured market. Results from this alternative setup are the same.

[10] In line with Brunnermeier and Pedersen (2009), we assume that both informed and uninformed lenders know the parameters of the fundamental value process (κ, $\underline{\sigma}$, and θ).

instruments, which are generally perceived as liquid, we follow Brunnermeier and Pedersen (2009) and let lenders' prior probability of an asynchronous endowment shock be small, i.e., $a \to 0$, so that they find it likely that $p_t = \nu_t$. With this assumption, we can solve lenders' secured and unsecured funding constraints.[11]

For secured funding, lenders protect against collateral risk by setting the margin m_t such that it covers the position's π-value-at-risk (where π is a small number close to zero), i.e.,

$$\pi = \Pr\left(-\Delta p_{t+1} > m_t | \mathcal{F}_t\right). \tag{6}$$

We assume that borrowers use funds from other business units to repay lenders when losses exceed margins. This assumption does not affect the results of our model but simplifies equations as the secured interest rate becomes zero. The corresponding secured funding volume satisfies

$$\overline{M}_t^s = \mathrm{VaR}_t^\pi(x_t p_{t+1}). \tag{7}$$

The key bankruptcy assumption is that lenders receive back secured loans immediately, while there might be a significant repayment delay for unsecured loans exposing lenders to a potential loss in the short run. Lenders acknowledge that secured debt claims enjoy legal seniority over the repayment of unsecured loans in case of a counterparty default. That is, borrowers first and fully settle their secured debt obligations before unsecured creditors can

[11]If lenders assign a very low probability to aggregate shocks, they assume that market prices are affected by fundamentals rather than illiquidity, and increase margins following a price shock. The case of $a > 0$ is discussed in Brunnermeier and Pedersen (2009) and holds equivalently in our model. That is, lenders need to decide to what extent a change in the price is due to fundamentals or due to order imbalances and thus illiquidity. In this case, they attribute a price increase or a modest price decline to a change in fundamentals, and behave similarly to the case of $a = 0$. Conversely, for a large price decline lenders actually increase credit limits as the probability of illiquidity is higher, thus anticipating that banks will profit from investing at lower prices.

receive their funds back (or a residual fraction of them). Hence, unsecured loans bear the risk of severe repayment delay. As is common in the literature (e.g., Acharya and Skeie, 2011), we model this by assigning unsecured loans to defaulted borrowers a zero value in the short run.[12]

We let lenders protect against counterparty default by imposing a credit limit on the amount of unsecured funds banks are able to borrow over one period. In line with common risk management practice in financial institutions and similar in spirit to, e.g., Rochet and Tirole (1996), we model the credit limit by a cap $\overline{M}_t^u$ that is determined by the banks' maximum default probability λ lenders are willing to accept, i.e.,

$$\lambda = \Pr\left(x_t p_{t+1} - M_t^s < \overline{M}_t^u | \mathcal{F}_t\right), \tag{8}$$

so that the corresponding unsecured funding volume satisfies

$$\overline{M}_t^u = \mathrm{VaR}_t^{\lambda}(x_t p_{t+1} - M_t^s). \tag{9}$$

To compensate for the risk of a short-run loss, lenders demand an interest rate $i_t > 0$, which depends on M_t^u and represents a risk premium over the risk-free rate. Given the lenders' funding constraints, we next consider banks and their optimal investment decision.

[12]Our results hold irrespective of the exact value lenders receive as long as secured creditors can expect higher reimbursement than unsecured creditors in case of counterparty default in the short-run. For instance, for European repos traded via a central clearing counterparty (CCP), lenders are protected by several layers of defense undertaken by the clearing party to guarantee repayment (for a detailed description see Mancini, Ranaldo, and Wrampelmeyer (2016)). In contrast, unsecured loans are unavailable in the short-run until the counterparty's bankruptcy case is filed and claims are negotiated.

Banks. Banks are identical, risk-neutral, and accommodate customers' trading needs by taking long positions x_t in the asset at times 0 and 1, i.e., $x_0, x_1 \geq 0$ (and sell to customer $k = 2$ at time 2). They start out with capital $W_0 > 0$ and fund their asset holdings by raising short-term debt from lenders in the money market. As secured loans are collateralized, they are tied to the asset side as shown in Table 1. Specifically, the secured funding volume on the liability side is the asset price net of the margin, $M_t^s = x_t(p_t - m_t)$, where x_t is the number of assets pledged to the lender. The difference between the asset value tied to secured funding and M_t^s is termed *overcollateralization*, and is equal to the total margin value $x_t m_t$.

Balance sheet

Assets	Liabilities
$x_t(p_t - m_t)$	Secured Debt: M_t^s
$x_t m_t$	Unsecured Debt: M_t^u
	Capital: W_t

Table 1. Banks' Balance Sheet.
The table shows banks' asset holdings and liability structure.

As Table 1 shows, the overcollateralization term is funded with banks' own capital and unsecured loans. Let $\overline{M}_t^u$ denote the maximum amount that banks are able to borrow in the unsecured market, we can summarize banks' funding constraint as

$$x_t m_t \leq \overline{M}_t^u + W_t. \tag{10}$$

Since unsecured funding is more expensive than secured funding, i.e., $i_t > 0$, banks always lever asset positions in the secured market and only fund the overcollateralization term using unsecured funds. This also implies that banks in our model never willingly hold unencumbered assets.[13] Banks maximize their expected wealth, subject to their budget constraint (10), where W_t evolves according to

$$W_{t+1} = W_t + (p_{t+1} - p_t)x_t - i_t M_t^u + \eta_t, \tag{11}$$

and η_t is an independent wealth shock arising from, e.g., other business units. While all banks legally have limited liability, we refer to W_t as working capital in the spirit of a bank's short-term financial wealth. If $W_t \leq 0$, banks are bankrupt and must choose $x_t = 0$.[14] In this case, their utility is $\psi_t W_t$, where we let $\psi_t = 1$ capture negative consumption equal to a bank's total loss in line with our bankruptcy assumption discussed above.[15] We define competitive equilibria as follows.

Definition 1. *An equilibrium is a price process p_t, such that (i) x_t maximizes banks expected final wealth subject to the funding constraint* (10)*, (ii) each y_t^k maximizes customer k's expected utility after arriving in the asset market and is zero beforehand, (iii) margins are set according to* (6)*, (iv) the credit limit is set according to* (8)*, and (v) asset and money markets clear, $x_t + \sum_{k=0}^{2} y_t^k = 0$.*

In the next section, we analyze the equilibrium outcome of the economy.

[13] In Section 5.3.3 we discuss regulatory requirements to hold unencumbered assets.

[14] In line with the empirical evidence of Mitchell, Pedersen, and Pulvino (2007), we assume that borrowers cannot raise new capital in the short-run.

[15] This bankruptcy assumption allows us to focus on short-term funding and curbs banks' risk-taking incentives usually associated with a firm's limited liability (see Brunnermeier and Pedersen (2009) on other bankruptcy assumptions which lead to qualitatively the same results).

2.3. Equilibrium

The equilibrium strategies are solved backwards using dynamic programming. Let Γ be a customer's value function and J the value function of a bank. At time 2, customer k maximizes

$$\Gamma_2(W_2^k, p_2, \nu_2) = \max_{y_2^k} \; - e^{-\gamma(\mathbb{E}_2\left[W_3^k\right] - \frac{\gamma}{2}\text{Var}_2\left[W_3^k\right])}. \tag{12}$$

Knowing that the asset pays off at time 3, the solution to this problem is

$$y_2^k = \frac{\nu_2 - p_2}{\gamma(\sigma_3)^2} - z^k. \tag{13}$$

Since all customers are in the market at time 2, aggregate demand is zero and $p_2 = \nu_2$. Thus, we get $\Gamma_2(W_2^k, p_2 = \nu_2, \nu_2) = -e^{-\gamma W_2^k}$ and $J_2(W_2, p_2 = \nu_2, \nu_2) = W_2$. This result holds in every period $t = 0, 1, 2$ if all customers arrive in the market at time 0. We focus on the case of customers' sequential arrival, when customers $k = 0$ and $k = 1$ are in the market at time 1 with demand

$$y_1^k = \frac{\nu_1 - p_1}{\gamma(\sigma_2)^2} - z^k, \tag{14}$$

and value function

$$\Gamma_1(W_1^k, p_1, \nu_1) = -e^{-\gamma\left[W_1^k + \frac{(\nu_1 - p_1)^2}{2\gamma(\sigma_2)^2}\right]}. \tag{15}$$

At time 1, it is optimal for banks to invest up to their funding constraint (10) as long as expected profits are larger than costs, i.e.,

$$x_1(\nu_1 - p_1 + \kappa) > i_1 M_1^u, \tag{16}$$

and are indifferent among all possible positions x_1 if Equation (16) is zero. For a liquid equilibrium to exist, we assume that $x_1 \kappa \geq i_1 M_1^u$, i.e., non-negative carry.[16] The shadow cost of capital, denoted by ϕ_1, is 1 plus the net return on capital as long as banks are not bankrupt, i.e.,

$$\phi_1 = 1 + \frac{x_1(\nu_1 - p_1 + \kappa) - i_1 M_1^u}{W_1}. \tag{17}$$

If a bank is bankrupt and $W_1 < 0$, then $\phi_1 = \psi_1$, so that the value function at time 1 is given by $J_1(W_1, p_1, \nu_1, p_0, \nu_0) = W_1 \phi_1$.

At time 0, only customer $k = 0$ is in the market and maximizes $\mathbb{E}_0\left[\Gamma_1(W_1^k, p_1, \nu_1)\right]$, while banks maximize their expected wealth, $\mathbb{E}_0[W_1 \phi_1]$, subject to the funding constraint (10).

3. Margins and Credit Limits (Time 1)

To determine banks' secured and unsecured funding, we derive lenders' margin requirement and credit limit, respectively. In this section, we show that when lenders are uninformed, margins are an increasing function and the credit limit is a decreasing function of asset volatility and market illiquidity.

Margins are set according to the asset's π-value-at-risk in Equation (6). Uninformed lenders believe that prices change due to a change in the fundamental value, such that

$$\pi = 1 - \Phi\left(\frac{m_1 + \kappa}{\sigma_2}\right). \tag{18}$$

Solving for m_1, we find that margins are increasing in price volatility and market illiquidity,

[16]Negative carry is the holding cost associated with a long position and prevents investors from trading against a mispricing (e.g., Dow and Gorton, 1994; Gromb and Vayanos, 2010). The resulting wedge between price and fundamental value would thus account for costs rather than illiquidity.

which we summarize in Lemma 1.

Lemma 1. *Margins are an increasing function of price volatility and market illiquidity, and are given by*

$$m_1 = \overline{\sigma} + \overline{\theta}|\Delta p_1 - \kappa| - \kappa, \tag{19}$$

$$= \overline{\sigma} + \overline{\theta}|\Delta \nu_1 + \Delta\Lambda_1 - \kappa| - \kappa, \tag{20}$$

where

$$\overline{\sigma} = \underline{\sigma}\Phi^{-1}(1-\pi), \tag{21}$$

$$\overline{\theta} = \theta\Phi^{-1}(1-\pi). \tag{22}$$

Higher price volatility arising from a fundamental shock and/or market illiquidity increases margins because $\theta > 0$. Higher margins imply that lenders require banks to have more "skin in the game", i.e., fund more of the asset's value with own capital or other funds. In relative terms, margins are equivalent to a "haircut" h_1 of the asset price, $h_1 = m_1/p_1$. As shown in Table 1, the debt value of each asset is the market price less the margin, such that the total secured funding volume amounts to $M_1^s = x_1(p_1 - m_1)$.

For unsecured loans, lenders set banks' default probability λ and estimate the credit limit by Equation (8) as

$$\lambda = \Pr\left(-\Delta p_2 > \frac{W_1}{x_1}|\mathcal{F}_1\right), \tag{23}$$

i.e., banks default if $W_1 \leq 0$. Analogous to the computation of margins, this translates to

$$\lambda = 1 - \Phi\left(\frac{\frac{W_1}{x_1} + \kappa}{\sigma_2}\right), \tag{24}$$

which we can solve as

$$\overline{M}_1^u = W_1\left(\frac{p_1}{\overline{\overline{\sigma}} + \overline{\overline{\theta}}|\Delta p_1 - \kappa| - \kappa} - 1\right) - M_1^s, \tag{25}$$

where

$$\overline{\overline{\sigma}} = \underline{\sigma}\Phi^{-1}(1 - \lambda), \tag{26}$$

$$\overline{\overline{\theta}} = \theta\Phi^{-1}(1 - \lambda). \tag{27}$$

Equation (25) relates unsecured funding $\overline{M}_1^u$ to a bank's (short-term) leverage by a term $p_1/(\overline{\overline{\sigma}} + \overline{\overline{\theta}}|\Delta p_1 - \kappa| - \kappa)$. As price volatility increases, lenders cut back on money market loans, thus introducing procyclicality in bank leverage that amplifies funding constraints (Shin, 2009). In fact, procyclicality in funding constraints has been identified as one of the major causes of fragility in the recent financial crisis and is now addressed in the latest Basel III capital requirements. To capture this effect, we let $b_1 \equiv \overline{\overline{\sigma}} + \overline{\overline{\theta}}|\Delta p_1| - \kappa$ denote the minimum capital *buffer* (per unit of an asset) that lenders require banks to hold relative to their short-term liabilities. We can further simplify Equation (25) by substituting $M_1^s = (p_1 - m_1)\frac{W_1 + \overline{M}_1^u}{m_1}$, and arrive at a lender's basic lending decision in the money market.

Proposition 1. *Credit limits for unsecured borrowing are decreasing in price volatility and market illiquidity. For $\lambda > \pi$, we have $m_t > b_t$, and lenders are willing to provide unsecured* *funding up to the constraint, which is equal to:*

$$\overline{M}_1^u = W_t \left(\frac{\overline{\sigma} + \overline{\theta}|\Delta p_1 - \kappa| - \kappa}{\overline{\overline{\sigma}} + \overline{\overline{\theta}}|\Delta p_1 - \kappa| - \kappa} - 1 \right), \tag{28}$$

$$= W_1 \left(\frac{m_1}{b_1} - 1 \right). \tag{29}$$

Lenders demand a premium for the risk of repayment delay, which is priced such that

$$0 = (1 - \lambda) i_1 M_1^u + \lambda(-M_1^u). \tag{30}$$

When banks are credit-constrained, $M_1^u = \overline{M}_1^u$*, the interest rate for unsecured loans is*

$$i_1 = \frac{\lambda}{1 - \lambda}. \tag{31}$$

For $\lambda \leq \pi$*, lenders only provide secured funding and* $\overline{M}_1^u = 0$*.*

The intuition of Proposition 1 is that unsecured lending describes a trade off between earning a higher premium and bearing the risk of late repayment. Proposition 1 also states that the unsecured market is entirely closed when $\lambda \leq \pi$, in which case our model collapses to that of Brunnermeier and Pedersen (2009). Given that banks are largely financed using unsecured funds, in particular before the 2008 financial crisis, we are interested in the case of $\lambda > \pi$. In the next section, we show that banks' exposure to unsecured funding has important implications for the fragility of liquidity, and so we assume that $\lambda > \pi$ from now on.

4. Funding and Market Illiquidity

In this section, we show that liquidity is fragile and liquidity spirals can arise jointly in unsecured and secured funding markets. To that end, we first analyze the impact of illiquidity on banks' funding structure at time 1, and then characterize the role of liquidity risk at time 0.

4.1. Fragility and Liquidity Spirals (Time 1)

Following Brunnermeier and Pedersen (2009), liquidity is fragile when a small shock can lead to a large drop in the equilibrium price. In a stable liquid equilibrium, a small price drop leads to excess demand and prices immediately readjust to the equilibrium level. When liquidity is fragile, a small price drop can lead to an illiquid equilibrium in which prices are much lower. Consequently, fragility arises when the demand curve is not monotonically decreasing in the price.

If either the secured or unsecured market is unconstrained, liquidity is not fragile and prices are equal to fundamental values. For instance, assume that banks fund assets in the secured market until all their capital is deployed, but the credit limit in the unsecured market is still slack. In this scenario, a shock (e.g., to the asset's fundamental value) increases margins, but the bank would absorb the reduction in secured funding by raising the corresponding funds in the unsecured market. Similarly, substitution from unsecured to secured funding occurs if the unsecured credit limit is reached but banks' own capital is abundant. As a result, no fire sales occur and the market price remains equal to the fundamental value, i.e., $p_1 = \nu_1$. Only if banks are constrained in both funding markets simultaneously, liquidity becomes fragile and the equilibrium price is lower than the fundamental value. Lemma 2 defines fragile liquidity when banks borrow in both the secured and unsecured market.

Lemma 2. *Liquidity is fragile at time 1 if at least one of the following conditions holds:*

(i) *Banks' position x_0 is larger than $\underline{x}$.*

(ii) $\overline{\overline{\theta}} = \theta\Phi^{-1}(1-\lambda)$ *is larger than $\underline{\theta}$ and the probability, a, of sequential arrival of customers is smaller than $\underline{a}$.*

The first condition in Lemma 2 states that the slope of banks' demand curve can become positive if losses on existing positions are sufficiently large. Condition (ii) differs from Brunnermeier and Pedersen (2009) as it depends on the default probability that lenders are willing to accept for *unsecured* loans. Given, $\lambda \geq \pi$ and thus $\overline{\overline{\theta}} \leq \overline{\theta}$, fragility is less likely to arise as lenders are willing to provide unsecured funding at higher default probabilities.

When funding constraints are binding, we find three distinct liquidity spirals that interact with each other and mutually deteriorate market and funding liquidity. The first spiral is a *loss* spiral that erodes banks' capital through losses on existing positions. The second liquidity spiral is a *margin* spiral that leads to higher margin requirements as prices decrease. And third, a *credit limit* spiral emerges in the unsecured market as tighter credit lines reduce the available amount of unsecured funding in response to increasing counterparty default risk. To fully understand the dynamics after a shock to banks' wealth, Proposition 2 summarizes these liquidity spirals.

Proposition 2. *In a stable illiquid equilibrium with selling pressure from customers, $Z_1 > 0$, and banks' position $x_0 > 0$,*

(i) *the price sensitivity to a shock* $\eta_1 < 0$ *is given by:*

$$\frac{\partial p_1}{\partial \eta_1} = \frac{1}{m_1 \frac{2}{\gamma(\sigma_2)^2} + \frac{\partial m_1}{\partial p_1} x_1 - x_0 - \frac{\partial \overline{M}_1^u}{\partial p_1}}. \tag{32}$$

(ii) *A credit limit spiral arises when* $\frac{\partial \overline{M}_1^u}{\partial p_1} > 0$, *which happens with positive probability when lenders are uninformed and* a *is small enough.*

(iii) *The credit limit spiral amplifies the margin spiral,* $\frac{\partial m_1}{\partial p_1} < 0$, *and loss spiral arising from previous positions* $x_0 > 0$.

Equation (32) can be written as an infinite series to visualize the spiral dynamics, with each term corresponding to one loop in the spiral. For any $k > 0$ and l such that $|l| < k$, it holds that $\frac{1}{k-l} = \frac{1}{k} + \frac{l}{k^2} + \frac{l^2}{k^3} + \ldots$, where $k = m_1 \frac{2}{\gamma(\sigma_2)^2} > 0$ and $l = \frac{\partial m_1}{\partial p_1} x_1 - x_0 - \frac{\partial \overline{M}_1^u}{\partial p_1}$. Intuitively, spirals initiate from a wealth shock that decreases the price by an amount k. The system is fragile because prices drop further due to the three liquidity spirals: lower prices exert losses on existing positions x_0, which deteriorate banks' mark-to-market capital, increase margins because $\frac{\partial m_1}{\partial p_1} < 0$, and decrease the credit limit in the unsecured market as $\frac{\partial \overline{M}_1^u}{\partial p_1} > 0$. The last two effects capture money market funding illiquidity, which, together with the loss spiral, are mutually reinforcing with an asset's market illiquidity.

Proposition 2 shows that the price sensitivity to a wealth shock is larger when banks borrow both secured *and* unsecured as compared to borrowing only in the secured market. This is because access to unsecured funds allows banks to further lever their capital by increasing their asset holdings, which amplifies the mutual downward pressure on the market price, such that loss, credit limit, and margin spirals keep reinforcing each other until the illiquid equilibrium is reached.

Fragility and liquidity spirals not only arise following a shock to banks' wealth, but also when there is a shock to fundamentals $\Delta\nu_1$, a shock to volatility, or a shock to customer demand Z_1. In these cases, the price sensitivities are multiples of $\frac{\partial p_1}{\partial \eta_1}$ and liquidity spirals amplify the initial price shock as shown above. Using convexity arguments, it can be shown mathematically that the total price decline of any such shock is larger than the sum of the individual effects.

4.2. Commonality and Substitution of Funding Liquidity (Time 1)

Overall, Proposition 2 shows that secured, unsecured, and asset markets are interconnected and can become illiquid at the same time. While the margin spiral increases the need for additional capital, the loss and credit limit spirals reduce banks' (funding) capital for financing higher margins. These adverse feedback loops continue until funding demand and supply equate to an (illiquid) equilibrium. We further examine the equilibrium funding volumes in Proposition 3.

Proposition 3. *In a stable illiquid equilibrium with $Z_1 > 0$, $x_0 > 0$, and $\Delta p_1 < 0$,*

(i) *liquidity spirals reduce secured funding as*

$$\frac{\partial M_1^s}{\partial p_1} = x_1\left(1 - \frac{\partial m_1}{\partial p_1}\right) + \frac{p_1 - m_1}{b_1}\left(\frac{\partial W_1}{\partial p_1} - x_1\frac{\partial b_1}{\partial p_1}\frac{m_1}{b_1}\right) > 0, \tag{33}$$

(ii) *and unsecured funding because*

$$\frac{\partial \overline{M}_1^u}{\partial p_1} = \frac{\partial W_1}{\partial p_1}\left(\frac{m_1}{b_1} - 1\right) + x_1\left(\frac{\partial m_1}{\partial p_1} - \frac{m_1}{b_1}\frac{\partial b_1}{\partial p_1}\right) > 0. \tag{34}$$

(iii) *Since* $\frac{\partial m_1}{\partial p_1} < 0$*, the margin spiral represents a marginal substitution effect from secured to unsecured funding, and total money market funding* MM_1 *decreases by*

$$\frac{\partial MM_1}{\partial p_1} = \frac{p_1 - b_1}{b_1}\left(\frac{\partial W_1}{\partial p_1} - x_1 \frac{\partial b_1}{\partial p_1}\right) > 0. \tag{35}$$

Ultimately, money market funding depends on the loss spiral, $\frac{\partial W_1}{\partial p_1} = x_0$*, and the credit limit spiral, given by* $\frac{\partial b_1}{\partial p_1} = -\bar{\bar{\theta}}$.

Proposition 3 shows that funding liquidity declines jointly, resulting in lower secured and unsecured funding volumes and thus in a decline of total money market funding volume. Part (i) of Proposition 3 shows that all three liquidity spirals lead to a reduction in secured funding M_1^s, whereas part (ii) shows that unsecured funding M_1^u declines because the loss spiral as well as the increase in the buffer b_1 dominate the margin spiral. The latter effect represents substitution from secured to unsecured funding, as higher margins reduce secured funding and, ceteris paribus, increase unsecured funding by the same magnitude. as a consequence, the decline in total money market volume in part (iii) depends no longer on the margin spiral, and occurs because of the loss and credit limit spirals (the latter represented by $\frac{\partial b_1}{\partial p_1}$). This result is intuitive, as banks can fund higher margins using their own capital *and* unsecured funds, such that the availability of unsecured funding (assuming capital is constant in the short run) becomes a crucial component contributing to common funding illiquidity. This finding differs significantly from Brunnermeier and Pedersen (2009), for whom the margin spiral is indeed the only source of funding illiquidity given that arbitrageurs (e.g., hedge funds or other investment firms) typically have no access to unsecured funding. We reaffirm this result for secured funding (part (i)), but find that with access to unsecured funds, i.e., $\lambda > \pi$, the extent of illiquidity is determined by the loss spiral and credit limit spiral, that is, the ease with which banks can obtain funding in the unsecured money market.

Proposition 3 provides an important insight into the relation between secured and unsecured funding by unveiling the contagion and substitution channels leading to fragile liquidity. Before discussing the implications for monetary and regulatory policies, we turn to time 0 and analyze the impact of time-1 liquidity risk on banks' funding structure.

4.3. Liquidity Risk (Time 0)

In this section, we show that the risk of higher margins and lower credit limits matters already at time 0 even when funding constraints are not binding.

In Section 2.2, we discussed that the assumption of $\psi = 1$ implies that banks can have negative wealth, which relates W_1 in our model to banks' short-term wealth rather than overall equity. This assumption curbs banks' risk-taking as they choose not to trade to their constraints already at time 0 due to the disutility from becoming bankrupt at time 1.[17] Thus, banks choose position x_0 by maximizing their expected profits $\mathbb{E}_0[\phi_1(W_0+x_0(p_1-p_0)-i_0M_0^u)]$. Solving for x_0 gives the time-0 price p_0 as summarized in Proposition 4.

Proposition 4. *At time 0, the asset price is given by*

$$p_0 = \mathbb{E}_0[p_1] + \frac{Cov_0(\phi_1, p_1)}{\mathbb{E}_0[\phi_1]} - m_0 i_0 - M_0^u \frac{\partial i_0}{\partial x_0}, \tag{36}$$

which decreases with negative covariance $Cov_0(\phi_1, p_1)$, more unsecured funding M_0^u, higher margin m_0, and higher funding cost i_0.

[17] When banks invest to their constraints already at time 0, the analysis is similar to time 1. Details are provided in the appendix.

Equation (36) states that the price at time 0 is the expected time-1 price, adjusted for by several liquidity risk terms. That is, a negative covariance term captures liquidity risk as the asset payoff is lower when illiquidity, denoted by ϕ_1, is higher. Moreover, higher margins, more unsecured funding and higher interest rates capture the notion of liquidity risk as banks face an increase in funding costs on the entire amount borrowed in the unsecured market once the credit limit becomes binding at time 1.

5. Monetary and Regulatory Policy Implications

In this section, we assess several monetary and regulatory policy measures with respect to their effectiveness in reducing and preventing fragility. For that purpose, we return to the analysis of fragility at time 1 and contrast our results against the benchmark case in which lenders are informed and know the fundamental value of the asset.

5.1. Benchmark Case

An informed lender has information set $\mathcal{F}_t = \sigma\{z, \nu_0, \ldots, \nu_t, p_0, \ldots, p_t, x_0, \ldots, x_t, \eta_1, \ldots, \eta_t\}$ and thus knows the fundamental values and that $p_2 = \nu_2$ in the next period. Proposition 5 describes the unsecured credit limit and margins set by informed lenders.

Proposition 5. *When lenders are informed and know the fundamental value of the asset and that $p_2 = \nu_2$ at time 2, the capital buffer b_1^* and margin m_1^* are set according to*

$$b_1^* = \overline{\overline{\sigma}} + \overline{\overline{\theta}}|\Delta\nu_1 - \kappa| - \kappa + \Lambda_1, \tag{37}$$

$$m_1^* = \overline{\sigma} + \overline{\theta}|\Delta\nu_1 - \kappa| - \kappa + \Lambda_1, \tag{38}$$

such that the total funding volume is characterized by

$$\frac{\partial MM_1^*}{\partial p_1} = (x_0 - x_1)\left(\frac{p_1 - b_1^*}{b_1^*}\right). \tag{39}$$

Thus, the more the price decreases below the fundamental value, i.e., $\Lambda_1 < 0$*, the more informed lenders increase the unsecured credit limit for unsecured funding and decrease margins for secured funding.*

Proposition 5 shows that informed lenders provide stabilizing funding conditions by decreasing margins and increasing credit limits during times of illiquidity. Lenders do so because they know banks will profit when the price returns to the fundamental value at time 2. The difference to Brunnermeier and Pedersen (2009) is that lenders also relax the credit limit knowing that $p_2 = \nu_2$, allowing them to earn more interest on the total unsecured volume lent to banks. Given our previous result that the margin spiral only redistributes funding between the secured and unsecured market (Proposition 3), the optimal funding volume in Equation (39) under full information becomes a function of the optimal capital buffer b_1^*.

In the following, we analyze several central bank and regulatory policies with regard to their effectiveness in stabilizing funding conditions and preventing fragility as shown in Proposition 5.

5.2. Central Bank Monetary Policy

In the recent crises, many central banks conducted unconventional monetary policies in addition to conventional interest rate policy to ease funding strains and price pressure on banks' asset holdings. For example, the Federal Reserve (FED) and European Central Bank (ECB) openly intervened in secondary markets through outright security purchases to provide market liquidity and allow banks to divest illiquid assets. In addition, most central banks created lending facilities for assets that have become highly capital-intense or ineligible as collateral for secured funding in the private market.[18]

In our model, haircut policy can be shown by central banks offering margins m_1^{cb} that are characterized by $m_1^{cb} < m_1$. Unlike informed lenders, central banks have no superior knowledge of an asset's fundamental value and so we analyze haircut policy by margins that are lower than in the private market. Furthermore, we model a central bank's asset purchases by introducing additional demand x_1^{cb} to the market clearing condition, which ultimately reduces customers' supply of assets to $-y_1 - x_1^{cb}$. Proposition 6 provides the main implications of central banks' unconventional policies for fragility.

Proposition 6. *Central bank unconventional policies have the following effects on fragility:*

(i) *For any $b_1^* < m_1^{cb} < m_1$, haircut policy stabilizes secured funding but crowds out unsecured funding and fragility remains the same. Haircut policy can only stabilize funding if margins are set such that $m_1^{cb} = b_1^*$, which ultimately leads to banks only borrowing from the central bank.*

(ii) *For any $x_1^{cb} > 0$, asset purchases reduce fragility by accommodating customers' demand shocks and mitigating liquidity spirals.*

Part (i) of Proposition 6 states that central banks' haircut policy relaxes banks' funding constraints by increasing the amount banks can borrow against pledging their assets as

[18] Some examples of unconventional measures involving haircut policies include the ECB's extension of eligible (riskier) assets for its repo loans in March 2009 and January 2011, or the FED's Term Auction Facility (TAF), Primary Dealer Credit Facility (PDCF), Term Securities Lending Facility (TSLF), and Term Asset-Backed Securities Loan Facility (TALF).

collateral. However, such margins equally reduce the substitution effect, which destabilizes unsecured funding and leads to the same level of fragility as without intervention. The consequence of lower central bank margins is that unsecured funding is crowded out, and only the share of M_1^s on total money market funding increases. As shown by Equation (39), fragility is prevented only if margins are set low enough to accommodate the crowding out effect on unsecured funding, i.e., $m_1^{cb} = b_1^*$, implying that banks fund their entire asset position with the central bank.

By contrast, asset purchases of the central bank always reduce fragility by counteracting customers' excess supply in the market, thereby alleviating funding constraints in both the secured and unsecured market simultaneously and enhancing assets' market liquidity. Clearly, an asset purchase program large enough to absorb the entire excess supply from the market can prevent fragility and eliminate liquidity spirals.

5.3. Regulatory Policy

In the previous section, we have shown that central bank interventions can improve banks' funding conditions during market turmoils. However, such policy measures are not only implemented when funding constraints are already binding, but their scale and efficacy are impossible to determine without central banks possessing full information. This is why it is important to understand whether the current regulatory environment can preserve slack constraints a priori and reduce the possibility of fragility.

We discuss three important policy measures, namely maximum leverage ratio, capital buffer, and liquidity coverage ratio in the context of our model. These measures are part of the Basel III and Dodd-Frank regulatory frameworks and strive to improve banks' resilience to sudden changes in asset values.

5.3.1 Leverage Ratio

Excessive leverage has been one of the key triggers of the recent crises, which led regulators to impose a cap on bank leverage. Intuitively, a maximum leverage ratio is only effective when preventing banks from levering all the way up to their constraints. An important outcome from our model is that (funding) liquidity follows procyclical patterns. This implies that a dynamic rather than static leverage ratio is more effective in preserving slack constraints. Leverage is time-varying and depends on liquidity, meaning that when haircuts are low, banks can strongly lever their balance sheet such that already a small shock can lead to liquidity spirals and fragility. A countercyclical leverage ratio can enhance financial stability by limiting leverage in economically good times, so that banks exhibit slack funding constraints in economically bad times. Moreover, banks unexposed to distressed assets are potential providers of market liquidity in times of stress, and may be inhibited from buying assets if constrained by a static leverage ratio. Thus, our model suggests that a dynamic maximum leverage ratio, which takes into account financial cycles is preferable over a static cap on bank leverage.

5.3.2 Capital Buffers

According to the current regulatory framework, banks are required to hold capital as a percentage of their risk-weighted assets, plus an additional countercyclical capital buffer in good times (if required by national regulators). Such a buffer must be held as additional core capital, which banks can utilize when, e.g., the budget constraint (10) becomes binding. Given that the capital buffer must be held in cash or near-money assets, it adds to a bank's funding capital and can (at least partially) offset an initial shock without banks having to (fire-) sell assets.

5.3.3 Liquidity Coverage Ratio

New regulations also comprise a liquidity coverage ratio (LCR), which requires banks to hold high-quality liquid assets (as a fraction of short-term expected liabilities) to better cope with sudden liquidity needs. These assets must be unencumbered and readily convertible into cash, so that they can be sold or pledged to obtain funding in case of an asset shock. In contrast to, e.g., structured products like asset-backed securities, high-quality liquid assets are more transparent and information on fundamental values is more reliable. Consequently, the capital raised through selling or pledging liquid securities alleviates a bank's funding constraint and compensates for a sudden reduction in short-term liabilities. Yet, a limitation of the LCR is that high-quality liquid assets are defined based on policy makers' pre-established scenario parameters, which may fail to fully reflect the actual funding and market liquidity risks.

6. Empirical Application

As a final step, we perform a simple empirical exercise to assess the main mechanisms of our model. To that end, we take the European sovereign debt crisis in 2010 as a real-world example of an asset shock, including significant losses in value of government bonds of Greece, Ireland, Italy, Portugal, and Spain (GIIPS) in response to the cut in Greece's credit rating to "junk" status in April 2010. The shock was large that it had severe effects on banks'balance sheets and likely let to binding funding constraints. We analyze changes in banks' funding structure following the shock and highlight the role of margins.

6.1. Data

A complete empirical analysis of our theoretical results would require bank-specific data on assets and liabilities, money market and central bank borrowing volume, margins, and interest rates. These data are not publicly available, unfortunately. Instead, we combine data from the richest sources available to construct proxies for the various quantities of interest.

First, we use European government bond holdings from stress test data published regularly by the European Banking Association (EBA) since March 2010. From this list, we take all banks that participated in the stress tests for March and December 2010. We denote total bond holdings of bank j in year t by $B_{j,t}$.[19] Second, for these banks we collect yearly balance sheet data on money market funding volumes, including secured and unsecured borrowing and lending for 2009 through 2010. We compute each bank's yearly secured (unsecured) *net* borrowing volume as the difference between the secured (unsecured) borrowing and lending volumes. To comply with our theoretical analysis, we consider net borrowers in the money market, for which the sum of secured and unsecured net borrowing is positive.[20] For each bank, we compute the share of unsecured funding, denoted by $S_{j,t}$, as the ratio of unsecured net borrowing over total money market net borrowing. Additionally, we compute a bank's leverage ratio, $L_{j,09}$, as total assets over equity in 2009. Third, we construct a proxy for the average margin for banks' bond portfolio. We obtain margin span parameters published by LCH.Clearnet, a major clearing house and provider of risk and collateral management

[19] We use the March 2010 EBA stress test bond holdings for end-of-year 2009, as stress tests were first conducted in March 2010.

[20] Banks borrow and lend in the money market at the same time, such that the difference between borrowing and lending volumes identifies the "net funding demand" for each bank and market. If net funding demand is positive, a bank is a net borrower, and if it is negative, it is a net lender. Thus, we investigate net borrowers, because those banks tap the money market in need for funding and therefore correspond to the borrowers in our model.

services, as a measure of country-specific margins for a range of government bonds .[21] We construct bank-specific margins by the average of margin parameters weighted by each bank's exposure to each country from the EBA stress test data. That is, we multiply each margin with a bank's position in that bond and divide by the total position of that portfolio. We denote the weighted average margin by $m_{j,t}$. Lastly, we obtain data on central bank borrowing from Bruegel, which include a breakdown of Eurosystem liquidity across national European central banks.[22] As a proxy for bank-specific central bank exposure we use the share of their GIIPS holdings relative to all their national peers' GIIPS holdings, taking the full list of banks participating in the stress tests.[23] To measure the reliance on central bank funding, we divide each bank's borrowing volume from the central bank by the bank's total assets. We denote this variable by $CB_{j,t}$. Merging the different data sources results in a total sample of 26 banks. We provide the list of banks and variables that are included in our sample in the Internet Appendix.

6.2. Regression Analysis

To empirically test the main mechanisms of our model, we perform cross-sectional least squares regressions for changes in variables between the two time periods, namely 2009 and 2010. We use 2009 as the reference date prior to the European sovereign debt crisis and 2010 as time of stress in GIIPS bonds. First, we investigate the relation between margins and the share of unsecured funding. According to the model, we expect a positive relation, as higher margins

[21] For instance, in March 2010 the spread on German government bonds was 1.49% and on Greek bonds 7.99%, whereas in October 2010 the respective spreads were 1.27% and 17.75%.

[22] Countries not considered by Bruegel include non-Euro countries such as the U.K. and Sweden for which we collect data manually from the respective national central banks and convert them into Euro. The Dutch central bank does not publish the necessary information, so that no data are available for the Netherlands.

[23] For example, BNP Paribas' share of France's central bank funding volume reported by Bruegel is computed as the ratio of its GIIPS exposure relative to all French banks' GIIPS exposure provided by the EBA.

lead to a substitution from secured to unsecured funding, and thus an increase in $S_{j,t}$. We control for banks' reliance on central bank funding by including the change in $CB_{j,t}$ from 2009 to 2010. In sum, we estimate the following regression:

$$\Delta S_{j,10} = \beta_0 + \beta_1 \Delta m_{j,10} + \beta_2 \Delta CB_{j,10} + \epsilon_j. \tag{40}$$

Second we investigate the relation between banks' deleveraging, i.e., the reduction in their asset positions, from 2009 to 2010 and their pre-crisis leverage as well as the change in their reliance on central bank funding. According to our model, banks with higher initial leverage should be affected more by the shock and thus have to deleverage more and rely more on central bank funding. We estimate the following regression model:

$$\Delta B_{j,10} = \beta_0 + \beta_1 \Delta CB_{j,10} + \beta_2 L_{j,09} + \epsilon_j. \tag{41}$$

Table 2 shows the regression results. Despite the limited number of observations and the simplicity of the empirical exercise, the results support the predictions of our theoretical model. An increase of margins and a decrease of reliance on central bank liquidity is associated with an increase in the share of unsecured borrowing. Moreover, higher leverage in 2009 is associated with a larger deleveraging.

7. Conclusion

This paper provides a comprehensive theoretical model for funding illiquidity. It offers a unified framework to analyze the relation between secured and unsecured funding liquidity,

Table 2
Regression Results for Funding Shares and Bond Holdings

This table shows the results of regressing changes in banks' funding shares (Columns (1) to (3)) and changes in bond holdings (Columns (4) to (6)) on explanatory variables derived from our model. Standard errors are shown in parentheses. The stars ***, **, and * indicate statistical significance at the 1%, 5%, and 10% level, respectively.

	Δ_{09}^{10} Share			Δ_{09}^{10} Bonds		
	(1)	(2)	(3)	(4)	(5)	(6)
Δm	0.081^{**}		0.079^{*}			
	(0.037)		(0.038)			
ΔCB		-0.777	-0.553		145.357^{*}	145.282^{**}
		(1.083)	(1.019)		(76.176)	(63.002)
L_{09}				-1.312^{***}		-1.291^{***}
				(0.386)		(0.363)
const.	-0.103	-0.030	-0.101	10.745	-19.909^{***}	9.887
	(0.063)	(0.061)	(0.067)	(9.747)	(4.377)	(9.112)
R^2	0.17	0.02	0.18	0.32	0.13	0.45
Obs.	26	25	25	26	25	25

and the interaction with assets' market liquidity. Our model shows that markets can be fragile and adverse liquidity spirals can arise, when banks face funding problems in both the secured and the unsecured money market at the same time. In such a scenario, credit limit and loss spirals in the unsecured market, mutually reinforcing with liquidity spirals in the secured market, exert downward pressure on asset prices and deteriorate banks' capital. In contrast, the possibility to substitute unsecured and secured liquidity can alleviate funding constraints and stabilize money markets.

We derive the optimal funding structure under full information and analyze central bank and regulatory policy. We show that unconventional monetary policies can prevent fragility and restore liquidity, but only when secured and unsecured funding markets are taken into account jointly. That is, haircut policy is effective only when margins are reduced such that they can compensate for the resulting crowding out of unsecured funding. On the other hand, central bank asset purchases positively affect both funding markets jointly, as enhanced market liquidity relaxes banks' funding constraints. This finding has implications for monetary policy normalization in the sense that the effect of reversing purchase programs on money markets is stronger than tightening central bank lending conditions.

Regarding financial market regulation, our model suggests that regulation should strive for slack funding constraints across banks and time to prevent mutually reinforcing liquidity spirals across funding markets. It also shows that money market funding liquidity follows procyclical patterns, such that countercyclical maximum leverage ratios and capital buffers enhance banks' resilience to future shocks more adequately than static measures. The recent introduction of the liquidity coverage ratio forces banks to hold unencumbered assets, which can be pledged to obtain secured funding or sold in the market to relax banks' capital constraint during crisis times. An important policy implication from our model is that the most effective regulatory measure is a combination of countercyclical leverage ratio, capital buffers, and unencumbered assets to ease funding strains in times when capital becomes scarce. Jointly, these regulatory measures counteract future fragility and make banks more resilient to adverse market conditions.

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Appendix A: Proofs

Proof of Lemma 1. Margins are set according to the asset's π-value-at-risk (where π is a small number close to zero):

$$\pi = Pr\left(-\Delta p_{t+1} > m_t | \mathcal{F}_t\right) \tag{A.1}$$

Lenders assume price changes are equal to changes in fundamental value, i.e., $\Delta p_t = \Delta \nu_t = \kappa + \sigma_{t+1}\varepsilon_{t+1}$, where $\varepsilon_t \sim N(0,1)$. Therefore,

$$\pi = 1 - \Phi\left(\frac{m_t + \kappa_t}{\sigma_{t+1}}\right). \tag{A.2}$$

Solving for m_t completes the proof. □

Proof of Proposition 1. Using similar arguments as in the proof of Lemma 1, the default probability for unsecured loans is

$$\lambda = 1 - \Phi\left(\frac{\frac{W_t}{x_t} + \kappa_t}{\sigma_{t+1}}\right). \tag{A.3}$$

Banks borrow up to the credit limit and use all capital and unsecured funds to pay margins, such that $x_t = \frac{\overline{M}_t^u + W_t}{m_t}$. Solving for $\overline{M}_t^u$ and substituting the solution for margins from Lemma 1 completes the proof. □

Proof of Lemma 2. In equilibrium, supply equals demand, i.e.,

$$x_1 = -\sum_{k=0}^{k} y_1^k = Z_1 + \frac{2(p_1 - \nu_1)}{\gamma(\sigma_2)^2}. \tag{A.4}$$

Together with banks' funding constraints, $x_1 \leq \frac{\overline{M}_1^u + W_1}{m_1}$, this yields

$$\overline{M}_1^u + W_1 \geq m_1 \left(Z_1 + \frac{2(p_1 - \nu_1)}{\gamma(\sigma_2)^2} \right). \tag{A.5}$$

Next, we plug in banks' wealth dynamics $W_1 = W_0 + x_0 \Delta p_1 - M_0^u i_0^u + \eta_1$ and define function $G(p_1)$:

$$G(p_1) := m_1 \left(Z_1 + \frac{2(p_1 - \nu_1)}{\gamma(\sigma_2)^2} \right) - \overline{M}_1^u - x_0 p_1 - c_0 \leq \eta_1, \tag{A.6}$$

where $c_0 = W_0 - M_0^u i_0^u + x_0 p_0$ summarizes all $t = 0$ terms.

If η_1 is sufficiently high, the inequality holds for $p_1 = \nu_1$ and the economy is an a stable liquid equilibrium. If η_1 is sufficiently low, the bank defaults. Fragility arises if $G(p_1)$ can be decreasing in p_1 for intermediate values of η_1.

We first show that fragility arises for $a = 0$. In this case:

$$G^0(p_1) : = m_1 \left(Z_1 + \frac{2(p_1 - \nu_1)}{\gamma(\sigma_2)^2} \right) - W_1 \left(\frac{m_t - b_t}{b_t} \right) - x_0 p_1 - c_0 \tag{A.7}$$

$$= \left(\overline{\overline{\sigma}} + \overline{\overline{\theta}} |\Delta p_1 - \kappa| - \kappa \right) \left(Z_1 + \frac{2(p_1 - \nu_1)}{\gamma(\sigma_2)^2} \right) - x_0 p_1 - c_0, \tag{A.8}$$

There are two terms that can lead to $G(p_1)$ being a decreasing function of p_1. First, when $x_0 > \underline{x}$, i.e., previous positions are large enough, the whole function can be decreasing. Second, when $p_1 < p_0 + \kappa$, $\overline{\overline{\theta}} |\Delta p_1 - \kappa| = \overline{\overline{\theta}}(p_0 + \kappa - p_1)$ is decreasing in p_1, which can make the whole expression negative if $\overline{\overline{\theta}} = \theta \Phi^{-1}(1 - \lambda)$ is large enough. The final part of the proof is analogous to the fragility proof of Brunnermeier and Pedersen (2009). When $a > 0$ approaches zero, G converges uniformly to G^0 on any compact set of prices, because both margins and credit limit converge to Equations (20) and (29), respectively. Given that the limit function G^0 has a decreasing part, we can choose prices $p_1^a < p_1^b$, such that we obtain $\tau := G^0(p_1^a) - G^0(p_1^b) > 0$.

Choose $\underline{a} > 0$ such that the difference between G and G^0 is at most $\tau/3$ for $a < \underline{a}$ by uniform convergence. Then,

$$G(p_1^a) - G(p_1^b) = G^0(p_1^a) - G^0(p_1^b) + [G^0(p_1^a) - G(p_1^a)] - [G(p_1^b) - G^0(p_1^b)] \tag{A.9}$$

$$\geq \tau - \frac{\tau}{3} - \frac{\tau}{3} = \frac{\tau}{3} > 0, \tag{A.10}$$

which proves that G has a decreasing part. $\square$

Proof of Proposition 2. Part (i) of the proposition builds on the results from the proof of Lemma 2. When funding constraints in both the secured and unsecured market are binding, we have:

$$m_1 \left(Z_1 + \frac{2(p_1 - \nu_1)}{\gamma(\sigma_2)^2} \right) - \overline{M}_1^u - x_0 p_1 - c_0 = \eta_1, \tag{A.11}$$

Taking the total derivative, with respect to η_1, we get:

$$\frac{\partial m_1}{\partial p_1} \frac{\partial p_1}{\partial \eta_1} \left(Z_1 + \frac{2(p_1 - \nu_1)}{\gamma(\sigma_2)^2} \right) + m_1 \frac{2}{\gamma(\sigma_2)^2} \frac{\partial p_1}{\partial \eta_1} - \frac{\partial \overline{M}^u}{\partial p_1} \frac{\partial p_1}{\partial \eta_1} - x_0 \frac{\partial p_1}{\partial \eta_1} = 1 \tag{A.12}$$

Equation (32) follows after rearranging the expression.

Parts (ii) and (iii) follow from analyzing $\frac{\partial \overline{M}_1^u}{\partial p_1}$ and $\frac{\partial m_1}{\partial p_1}$. For $\Delta p_1 < \kappa$ and $\kappa > 0$

$$\frac{\partial \overline{M}_1^u}{\partial p_1} = x_0 \frac{\overline{M}_1^u}{W_1} + W_1 \frac{\left(\overline{\theta} - \overline{\overline{\theta}} \right) \kappa}{\left(\overline{\overline{\sigma}} - \overline{\overline{\theta}} \Delta p_1 + \left(\overline{\overline{\theta}} - 1 \right) \kappa \right)^2} > 0. \tag{A.13}$$

Moreover, as a approaches 0, $\frac{\partial m_1}{\partial p_1}$ approaches $-\overline{\theta} < 0$ for $\Delta p_1 < \kappa$. This means that there are credit limit spirals and margin spirals with positive probability. $\square$

Proof of Proposition 3. We obtain part (i) of Proposition 3 by taking the derivative of $M_1^s = x_1(p_1 - m_1)$ with respect to price p_1, which gives

$$\frac{\partial M_1^s}{\partial p_1} = \frac{m_1 - p_1 \frac{\partial m_1}{\partial p_1}}{m_1^2} \left(\overline{M}_1^u + W_1\right) + \frac{p_1 - m_1}{m_1} \left(\frac{\partial \overline{M}_1^u}{\partial p_1} + \frac{W_1}{\partial p_1}\right) > 0. \tag{A.14}$$

Plugging $\frac{\partial \overline{M}_1^u}{\partial p_1}$ into Equation (A.14) leads to (i) in the proposition.

The results of (ii) and (iii) follow directly from the calculations in the proposition.

Proof of Proposition 4. The banks' first order condition is:

$$\mathbb{E}_0 \left[\phi_1 \left((p_1 - p_0) - \frac{\partial i_0}{\partial x_0} M_0^u - \frac{\partial M_0^u}{\partial x_0} i_0\right)\right] = 0 \tag{A.15}$$

Rearranging gives:

$$p_0 \mathbb{E}_0 [\phi_1] = \mathbb{E}_0 [\phi_1 p_1] - \mathbb{E}_0 [\phi_1] \frac{\partial i_0}{\partial x_0} M_0^u - \mathbb{E}_0 [\phi_1] \frac{\partial M_0^u}{\partial x_0} i_0 \tag{A.16}$$

With $\frac{\partial M_0^u}{\partial x_0} = m_0$:

$$p_0 = \frac{\mathbb{E}_0 [\phi_1, p_1]}{\mathbb{E}_0 [\phi_1]} - \frac{\partial i_0}{\partial x_0} M_0^u - m_0 i_0. \tag{A.17}$$

Using, $Cov_0(\phi_1, p_1) = \mathbb{E}_0 [\phi_1, p_1] - \mathbb{E}_0 [\phi_1] \mathbb{E}_0 [p_1]$ we arrive at the result of the proposition.

When banks are constrained already at time 0, the corresponding shadow cost of capital ϕ_0 is given by:

$$\phi_0 = \mathbb{E}_0[\phi_1] \left[1 + \frac{x_0(\mathbb{E}_0 \left[\frac{\phi_1}{\mathbb{E}_0[\phi_1]} p_1\right] - p_0 + \kappa) - i_0 M_0^u}{W_0}\right]. \tag{A.18}$$

□

Proof of Proposition 5. There is no asymmetric information and lenders know the correct distribution of price changes:

$$\Delta p_2 \sim N\left(\kappa - \Lambda_1, \underline{\sigma} + \theta|\Delta\nu_1 - \kappa|\right) \tag{A.19}$$

Thus, illiquidity does not increase perceived volatility, but instead increases expected returns. Using the true price process together with Equations (6) and (8) gives the unsecured credit limit $\overline{M}_1^u$ and margin m_1. □

Proof of Proposition 6. Part (i) follows directly from the discussion in the text. For part (ii), note that with asset purchases, function G^0 becomes:

$$G^0(p_1) := m_1\left(Z_1 + \frac{2(p_1 - \nu_1)}{\gamma(\sigma_2)^2} - x_1^{cb}\right) - \overline{M}_0^u - x_0 p_1 - c_0. \tag{A.20}$$

Thus, fragility is less likely to arise because customers' demand pressure is smaller. Similarly, the margin spiral is reduced as

$$\frac{\partial p_1}{\partial \eta_1} = \frac{1}{m_1 \frac{2}{\gamma(\sigma_2)^2} + \frac{\partial m_1}{\partial p_1}\left(Z_1 + \frac{2(p_1-\nu_1)}{\gamma(\sigma_2)^2} - x_1^{cb}\right) - x_0 - \frac{\partial \overline{M}_1^u}{\partial p_1}}, \tag{A.21}$$

and the credit limit spiral is lower as well:

$$\frac{\partial \overline{M}_1^u}{\partial p_1} = x_0 \frac{\overline{M}_1^u}{W_1} + \left(Z_1 + \frac{2(p_1 - \nu_1)}{\gamma(\sigma_2)^2} - x_1^{cb}\right) \frac{\left(\overline{\theta} - \overline{\overline{\theta}}\right)\kappa}{\overline{\overline{\sigma}} - \overline{\overline{\theta}}\Delta p_1 + \left(\overline{\overline{\theta}} - 1\right)\kappa}. \tag{A.22}$$

□