

School of Finance



University of St.Gallen

A STRUCTURAL MODEL FOR ELECTRICITY FORWARD PRICES

FRED ESPEN BENTH

FLORENTINA PARASCHIV

WORKING PAPERS ON FINANCE NO. 2016/11

**INSTITUTE OF OPERATIONS RESEARCH AND COMPUTATIONAL FINANCE
(IOR/CF – HSG)**

MAY 2016



A structural model for electricity forward prices

Fred Espen Benth* Florentina Paraschiv*[†]

May 23, 2016

Abstract

Structural models for forward electricity prices are of great relevance nowadays, given the major structural changes in the market due to the increase of renewable energy in the production mix. In this study, we derive a spatio-temporal dynamical model based on the Heath-Jarrow-Morton (HJM) approach under the Musiela parametrization, which ensures an arbitrage-free model for electricity forward prices. The model is fitted to a unique data set of historical price forward curves. As a particular feature of the model, we disentangle the temporal from spatial (maturity) effects on the dynamics of forward prices, and shed light on the statistical properties of risk premia, of the noise volatility term structure and of the spatio-temporal noise correlation structures. We find that the short-term risk premia oscillates around zero, but becomes negative in the long run. We identify the Samuelson effect in the volatility term structure and volatility bumps, explained by market fundamentals. Furthermore we find evidence for coloured noise and correlated residuals, which we model by a Hilbert space-valued normal inverse Gaussian Lévy process with a suitable covariance functional.

JEL Classification: C02, C13, C23

Keywords: spatio-temporal models, price forward curves, term structure volatility, risk premia, electricity markets

*Department of Mathematics, University of Oslo, PO Box 1053 Blindern, N-0316 Oslo, Norway, fredb@math.uio.no, Fax: +47 22 85 43 49.

[†]*Corresponding author: Florentina Paraschiv, University of St. Gallen, Institute for Operations Research and Computational Finance, Bodanstrasse 6, CH-9000 St. Gallen, Switzerland, florentina.paraschiv@unisg.ch, Phone: +41 (0)71 224 30 81.

1 Introduction

There exist two main approaches for modelling forward prices in commodity and energy markets. The classical way goes by specifying a stochastic model for the spot price, and from this model derive the dynamics of forward prices based on no-arbitrage principles (see Lucia and Schwartz (2002), Cartea and Figueroa (2005), Weron and Zator (2014), Benth, Kallsen, and Meyer-Brandis (2007), Barndorff-Nielsen, Benth, and Veraart (2013) and Benth, Klüppelberg, Müller, and Vos (2014)). The alternative is to follow the Heath–Jarrow–Morton approach and to specify the dynamics of the forward prices directly, as it has been done in Roncoroni and Guiotto (2001), Benth and Koekebakker (2008), Weron and Borak (2008) and Kiesel, Schindlmayr, and Boerger (2009). All these studies model the forward prices using multifactor models driven by Brownian motion. However, empirical findings in Koekebakker and Ollmar (2005), Frestad (2008) suggest that there is a substantial amount of variation in forward prices which cannot be explained by a few common factors. Furthermore, the models that directly specify the dynamics of forward contracts ignore the fact that the returns of forward prices in electricity markets are far from being Gaussian distributed and have possible stochastic volatility effects.

Random-field models for forward prices in power markets have been explored statistically and mathematically by Andresen, Koekebakker, and Westgaard (2010). There the authors model electricity forwards returns for different times to maturity using a multivariate normal inverse Gaussian (NIG) distribution to capture the idiosyncratic risk and heavy tails behavior and conclude the superiority of this approach versus Gaussian-based multifactor models in terms of goodness of fit. Their analysis seems to be based on the assumption that forward prices follow an exponential spatio-temporal stochastic process. When modeling forward prices evolving along time *to* maturity rather than time *at* maturity, one must be careful with how the time to maturity affects a price change. Indeed, in this so-called Musiela parametrization context of forward prices an additional drift term must be added to the dynamics to preserve arbitrage-freeness of the model.

In this paper we propose to model the forward price dynamics by a spatio-temporal random field based on the Heath-Jarrow-Morton (HJM) approach under the Musiela parametrization (see Heath, Jarrow, and Morton (1992)), which ensures an arbitrage-free dynamics. After discretizing the model in time and space, we can separate seasonal features from risk premium and random perturbations of the prices, and apply this to obtain information of the probabilistic characteristics of the data. Our model formulation disentangles typical components of forward prices like: the deterministic seasonality pattern and the stochastic component including the market price of risk and the noise. We show the importance of rigourously modeling each component in the context of an empirical application to electricity forward prices, in which a unique panel data set of 2'386 hourly price forward curves is employed for the German electricity index PHELIX. The index is generated each day for a horizon of 5 years, ranging from 01/01/2009 until 15/07/2015. Each day a new price forward curve (PFC) is generated based on the newest information from current futures prices observed at EPEX.¹

The dynamics of price forward curves (PFCs) are modeled with respect to two dimensions: temporal and spatial (the space dimension here refers to *time to maturity* of the forward). In particular, the changes in the level of a PFC for one specific maturity point between consecutive days reflect two features:

Firstly, as time passes, dynamics in time of on-going futures prices with a certain delivery period reflect changes in the market expectation. In particular, maturing futures are replaced by new ones in the market.² Changes in the market expectations reflect updates in weather forecasts, energy policy announcements or expected market structural changes. Germany adopted the Renewable Energy Act (EEG) in 2000, accordingly to which producers of renewable energies (wind, photovoltaic etc.) receive a guaranteed compensation (technology dependent feed-in tariffs). Renewable energies are fed with

¹Electricity for delivery on the next day is traded at the European Power Exchange (EPEX SPOT) in Paris.

²In the German electricity market, weekly, monthly, quarterly or yearly futures are traded.

priority into the grid, replacing thus in production other traditional fuels (oil, gas, coal). Given the difficulty of getting accurate weather forecasts, electricity demand/supply disequilibria became more frequent, which increased the volatility of electricity prices. Furthermore, it has been empirically shown that due to the low marginal production costs of wind and photovoltaic, the general level of electricity prices decreased over time (see Paraschiv, Erni, and Pietsch (2014)), which explains the shift in time of the general level of the analyzed PFCs.

Secondly, as time passes, the time to maturity of one specific product decreases and maturing futures are replaced by new ones in the market. In the German electricity market, weekly, monthly, quarterly and yearly futures are traded. Given the little number of different exchange-traded futures, and thus different maturities, the stochastic component of the (deseasonalized) PFCs shows a typical step-wise pattern. Hence, consecutive changes in time in the level of the PFC for a fixed maturity point on the curve are influenced additionally by changes between two maturity points on the initial curve, which accounts for the time to maturity dimension. Both effects are displayed in Figure 1.

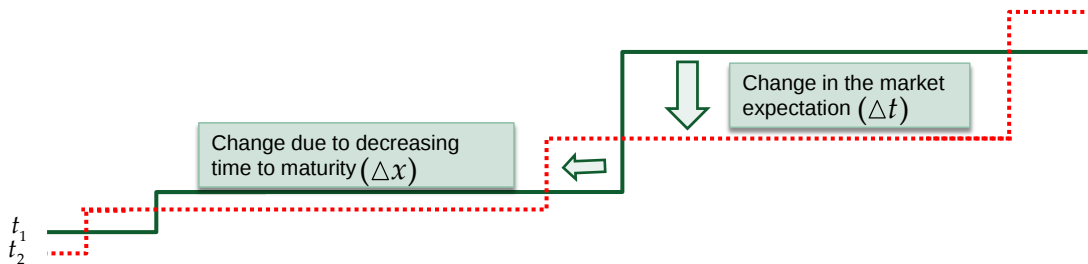


Figure 1: The effect of time and maturity change on the dynamics of forward prices.

Our proposed model is fitted to the observed PFCs. We first perform a deseasonalization of the initial curves, where the seasonal component takes into account typical patterns observed in electricity prices (see Blöchliger (2008), Paraschiv (2013)). We fur-

ther estimate the market price of risk in the deseasonalized curves (stochastic component) and examine, in this context, the distribution of the noise volatility and its spatio-temporal correlations structures. Our results show that the short-term risk premia oscillates around zero, but becomes negative in the long run, which is consistent with the empirical literature (Burger, Graeber, and Schindlmayr (2007)). The descriptive statistics of the noise marginals reveals clear evidence for a coloured-noise with leptokurtic distribution and heavy-tails, which we suggest to model by a normal inverse Gaussian distribution (NIG).³ We further examine the term structure of volatility where we are able to identify the Samuelson effect and volatility bumps. The occurrence of volatility bumps are explained by the trading activity in the market for futures of specific maturities (delivery periods). The spatial correlation structure of the noise is stationary with a fast-decaying pattern: decreasing correlations with increased distance between maturity points along one curve.

Based on the empirical evidence, we further stylize our model and specify a spatio-temporal mathematical formulation for the coloured noise time series. After explaining the Samuelson effect in the volatility term structure, the residuals are modeled by a NIG Lévy process with values in a convenient Hilbert space, which allows for a natural formulation of a covariance functional. We model, in this way, the typical fat tails and fast-decaying pattern of spatial correlations. We bring, thus, several contributions of our modeling approach over Andresen, Koekebakker, and Westgaard (2010): we disentangle the temporal from spatial (maturity) effects on the dynamics of forward prices, and shed light on the statistical properties of risk premia, of the noise volatility term structure and of the spatio-temporal noise correlation structures. In conclusion, we formulate an arbitrage-free random field model for the forward price dynamics in power markets which honours the statistical findings.

The idea of modeling power forward prices with a random field model goes back to

³Similar results can be found in Frestad, Benth, and Koekebakker (2010), who analyzed the distribution of daily log returns of individual forward contracts at Nord Pool and found that the univariate NIG distribution performed best in fitting the return data.

Audet, Heiskanen, Keppo, and Vehviläinen (2004), who studied theoretically a Gaussian model with certain mean-reversion characteristics. A mathematical treatise of the more general random field models of HJM type as we propose in this paper can be found in Benth and Krühner (2014). The issue of pricing derivatives for such random field models is discussed in Benth and Krühner (2015), while Benth and Lempa (2014) analyse portfolio strategies in energy markets with infinite dimensional noise. Our proposed forward price dynamics is thus suitable for further applications to both derivatives pricing and risk management. Efficient numerical approaches for simulation are also available, see Barth and Benth (2014). Ambit fields is an alternative class of random fields which can be used for dynamic modeling of forward prices in power markets, see Barndorff-Nielsen, Benth, and Veraart (2014). In Barndorff-Nielsen, Benth, and Veraart (2015) and Benth and Krühner (2015), infinite-dimensional cross-commodity forward price models are proposed and analysed.

The rest of the paper is organized as follows: In section 2 we present the mathematical formulation of the spatio-temporal random field model. In sections 3 and 4 we describe the data used for the application and present descriptive statistics on the risk premia, volatility, correlations and noise. The estimation results are shown in section 5, and in section 6 we specify a mathematical model for the residuals based on the statistical findings. Finally, section 7 concludes.

2 Spatio-temporal random field modeling of forward prices

The Heath-Jarrow-Morton (HJM) approach (see Heath, Jarrow, and Morton (1992)) has been advocated as an attractive modelling framework for energy and commodity forward prices (see Benth, Šaltytė Benth, and Koekebakker (2008), Benth and Krühner (2014)/(2015), Benth and Koekebakker (2008), Clewlow and Strickland (2000)). If $F_t(T)$

denotes the forward price at time $t \geq 0$ for delivery of a commodity at time $T \geq t$, we introduce the so-called Musiela parametrization $x = T - t$ and let $G_t(x)$ be the forward price for a contract with time to maturity $x \geq 0$. The graphical representation in Figure 2 shows comparatively the difference between thinking in terms of “time at maturity”, T , versus “time to maturity” x . Note that $G_t(x) = F_t(t + x)$. It is known that the stochastic process $t \mapsto G_t(x)$, $t \geq 0$ is the solution of a stochastic partial differential equation (SPDE),

$$dG_t(x) = (\partial_x G_t(x) + \beta(t, x)) dt + dW_t(x) \quad (1)$$

where $\partial_x = \partial/\partial x$ is the differential operator with respect to time to maturity x , β is a spatio-temporal process modelling the market price of risk and finally W is a spatio-temporal random field which describes the randomly evolving residuals in the dynamics.

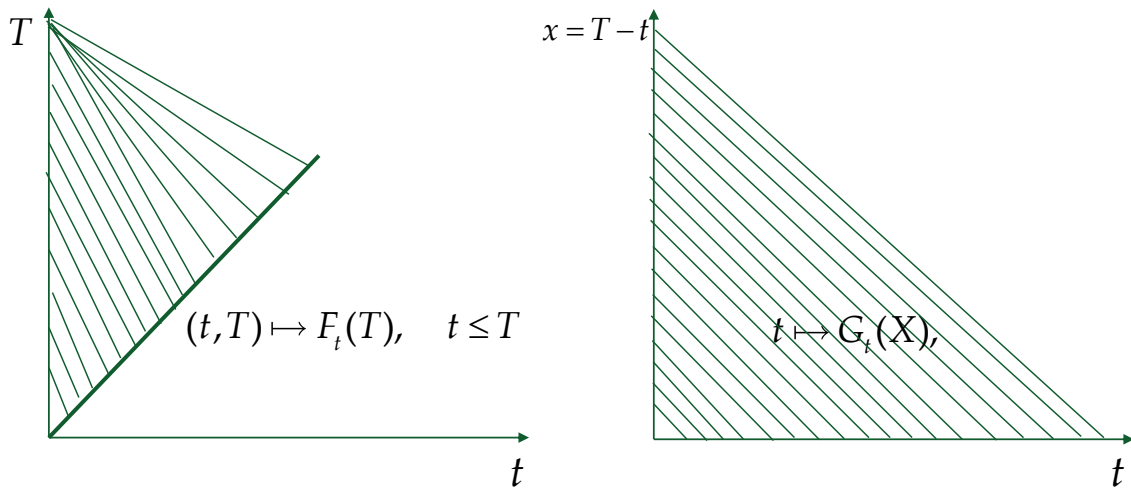


Figure 2: Theoretical model: time at maturity (first graph) versus time to maturity (second graph).

To make the model for the forward price dynamics G rigorous, it has to be formulated as a stochastic process in time, taking values in a space of curves on the positive real

line $\mathbb{R}_+$. Typically, this space of curves are endowed with a Hilbert space structure. Denoting this Hilbert space of curves by $\mathcal{H}$, the SPDE (1) is interpreted as a stochastic differential equation in $\mathcal{H}$. Moreover, the $\mathcal{H}$ -valued process W_t is a martingale, and encodes a correlation structure in space and time for the forward prices, as well as the distribution of price increments at fixed times of maturity x and the term structure of volatility. The latter includes the Samuelson effect, which is predominant in commodity markets where stationarity of prices is an empirical characteristic. We refer to Benth and Krühner (2015) for a rigorous mathematical description and analysis of (1) in the Hilbert space framework, where a specific example of an appropriate space of curves $\mathcal{H}$ suitable for commodity markets is proposed.

In this paper we will analyse a discrete-time version of the process G_t , obtained from an Euler discretization of (1). In particular, our focus will be on an analysis of the seasonal structure, the market price of risk and finally the probabilistic features of the noise component W_t . To this end, suppose that

$$G_t(x) = f_t(x) + s_t(x), \quad (2)$$

where $s_t(x)$ is a deterministic seasonality function. We assume that $\mathbb{R}_+^2 \ni (t, x) \mapsto s_t(x) \in \mathbb{R}$ is a bounded and measurable function, typically being positive. Note that if we construct the seasonality function from a spot price model, then naturally $s_t(x) = s(t+x)$, where s is the seasonality function of the commodity spot price (see Benth, Šaltytė Benth, and Koekebakker (2008)). We furthermore assume that the deseasonalized forward price curve, denoted by $f_t(x)$, has the dynamics

$$df_t(x) = (\partial_x f_t(x) + \theta(x)f_t(x)) dt + dW_t(x), \quad (3)$$

with $\mathbb{R}_+ \ni x \mapsto \theta(x) \in \mathbb{R}$ is a bounded and measurable function modeling the risk

premium. With this definition, we note that

$$\begin{aligned}
dF_t(x) &= df_t(x) + ds_t(x) \\
&= (\partial_x f_t(x) + \theta(x)f_t(x)) dt + \partial_t s_t(x) dt + dW_t(x) \\
&= (\partial_x F_t(x) + (\partial_t s_t(x) - \partial_x s_t(x)) + \theta(x)(F_t(x) - s_t(x))) dt + dW_t(x).
\end{aligned}$$

In the natural case, $\partial_t s_t(x) = \partial_x s_t(x)$, and therefore we see that $F_t(x)$ satisfy (1) with $\beta(t, x) := \theta(x)f_t(x)$, i.e., that the market price of risk is proportional to the deseasonalized forward prices. Note that we have implicitly assumed differentiability of $s_t(x)$ in the above derivation.

Let us next discretize the dynamics of f_t in (3), in order to obtain a time series dynamics of the (deseasonalized) forward price curve. Let $\{x_1, \dots, x_N\}$ be a set of equidistant maturity dates with resolution $\Delta x := x_i - x_{i-1}$ for $i = 2, \dots, N$. At time $t = \Delta t, \dots, M\Delta t$, where $M\Delta t = T$ for some terminal time T , we observe for each maturity date $x \in \{x_1, \dots, x_N\}$ a point on the price-forward curve $F_t(x)$ and a corresponding point on the seasonality curve $s_t(x)$. A standard approximation of the derivative operator ∂_x is

$$\partial_x f_t(x) \approx \frac{f_t(x + \Delta x) - f_t(x)}{\Delta x}$$

Next, after doing an Euler discretization in time of (3), we obtain the time series approximation for $f_t(x)$. With $x \in \{x_1, \dots, x_N\}$ and $t = \Delta t, \dots, (M-1)\Delta t$,

$$f_{t+\Delta t}(x) = (f_t(x) + \frac{\Delta t}{\Delta x}(f_t(x + \Delta x) - f_t(x)) + \theta(x)f_t(x)\Delta t + \epsilon_t(x)) \quad (4)$$

where $\epsilon_t(x) := W_{t+\Delta t}(x) - W_t(x)$. We define the time series $Z_t(x)$ for $x \in \{x_1, \dots, x_N\}$ and $t = \Delta t, \dots, (M-1)\Delta t$,

$$Z_t(x) := f_{t+\Delta t}(x) - f_t(x) - \frac{\Delta t}{\Delta x}(f_t(x + \Delta x) - f_t(x)) \quad (5)$$

which implies

$$Z_t(x) = \theta(x)f_t(x)\Delta t + \epsilon_t(x), \quad (6)$$

where changes between the stochastic components of forward curves incorporate risk premia and changes in the noise. Since we are interested in analysing the properties of the noise volatility, to account for Samuelson effect in forward prices, the model residuals $\epsilon_t(x)$ are further decomposed in:

$$\epsilon_t(x) = \sigma(x)\tilde{\epsilon}_t(x) \quad (7)$$

where $\tilde{\epsilon}_t(x)$ are the standardized residuals.

The time series model (6) will be our point of study in this paper, where we are concerned with inference of the market price of risk proportionality factor $\theta(x)$ and the probabilistic structure of $\tilde{\epsilon}_t(x)$. As our case is power markets, we aim at a (time and space) discrete curve $Z_t(x)$ from forward prices over a delivery period. How to recover data for Z in such markets will be discussed in the next section. We remark here that we will choose a procedure of constructing a seasonal function which provides information on $s_t(x)$ at discrete time and space points. By smooth interpolation, we may assume that $\partial_t s_t(x) = \partial_x s_t(x)$.

3 Generation of Price Forward Curves: theoretical background

In our empirical analysis we employed a unique data set of hourly price forward curves (HPFC) $F_t(x_1), \dots, F_t(x_N)$ generated each day between 01/01/2009 and 15/07/2015 based on the latest information from the observed futures prices for the German electricity Phe-lix price index. In this section we describe how these curves were produced from market prices.

3.1 Construction of the hourly price forward curves

For the derivation of the HPFCs we follow the approach introduced by Fleten and Lemming (2003). At any given time the observed term structure at EEX is based only on a limited number of traded futures/forward products. Hence, a theoretical hourly price curve, representing forwards for individual hours, is very useful but must be constructed using additional information. We model the hourly price curve by combining the information contained in the observed bid and ask prices with information about the shape of the seasonal variation.

Recall that $F_t(x)$ is the price of the forward contract with maturity x , where time is measured in hours, and let $F_t(T_1, T_2)$ be the settlement price at time t of a forward contract with delivery in the interval $[T_1, T_2]$. The forward prices of the derived curve should match the observed settlement price of the traded future product for the corresponding delivery period, that is:

$$\frac{1}{\sum_{\tau=T_1}^{T_2} \exp(-r\tau/a)} \sum_{\tau=T_1}^{T_2} \exp(-r\tau/a) F_t(\tau - t) = F_t(T_1, T_2) \quad (8)$$

where r is the continuously compounded rate for discounting per annum and a is the number of hours per year. A realistic price forward curve should capture information about the hourly seasonality pattern of electricity prices. For the derivation of the seasonality shape of electricity prices we follow Blöchliger (2008) (chapter 6). Basically we fit the HPFC to the seasonality shape by minimizing

$$\min \left[\sum_{x=1}^N (F_t(x) - s_t(x))^2 \right] \quad (9)$$

subject to constraints of the type given in equation (8) for all observed instruments, where s_t is the hourly seasonality curve (we refer to Fleten and Lemming (2003) for details).⁴

⁴In the original model, Fleten and Lemming (2003) applied, for daily time steps, a smoothing factor

To keep the optimization problem feasible, overlapping contracts as well as contracts with delivery periods which are completely overlapped by other contracts with shorter delivery periods, are removed.

3.2 Seasonality shape

For the derivation of the shape s_t we follow the procedure discussed in Blöchlinger (2008), see pp. 133–137, and in Paraschiv (2013). In a first step, we identify the seasonal structure during a year with daily prices. In the second step, the patterns during a day are analyzed using hourly prices. Let us define two factors, the factor-to-year ($f2y$) and the factor-to-day ($f2d$) (following the notation in Blöchlinger (2008)). By $f2y$ we denote the relative weight of an average daily price compared to the annual base of the corresponding year:

$$f2y_d = \frac{S^{day}(d)}{\sum_{k \in \text{year}(d)} S^{day}(k) \frac{1}{K(d)}} \quad (10)$$

$S^{day}(d)$ is the daily spot price in the day d , which is the average price of the hourly electricity prices in that day. $K(d)$ denotes the number of days in the year when $S^{day}(d)$ is observed. The denominator is thus the annual base of the year in which $S(d)$ is observed.

To explain the $f2y$, we use a multiple regression model (similar to Blöchlinger (2008)):

$$f2y_d = \alpha_0 + \sum_{i=1}^6 b_i D_{di} + \sum_{i=1}^{12} c_i M_{di} + \sum_{i=1}^3 d_i CDD_{di} + \sum_{i=1}^3 e_i HDD_{di} + \varepsilon \quad (11)$$

- $f2y_d$: Factor to year, daily-base-price/yearly-base-price
- D_{di} : 6 daily dummy variables (for Mo-Sat)

to prevent large jumps in the forward curve. However, in the case of hourly price forward curves, Blöchlinger (2008) (p. 154) concludes that the higher the relative weight of the smoothing term, the more the hourly structure disappears. We want that our HPFC reflects the hourly pattern of electricity prices and therefore in this study we have set the smoothing term in Fleten and Lemming (2003) to 0.

- M_{di} : 12 monthly dummy variables (for Feb-Dec); August will be subdivided in two parts, due to summer vacation
- CDD_{di} : Cooling degree days for 3 different German cities
- HDD_{di} : Heating degree days for 3 different German cities

where CDD_i/HDD_i are estimated based on the temperature in Berlin, Hannover and Munich.

- Cooling Degree Days (CDD) = $\max(T - 18.3^\circ C, 0)$
- Heating Degree Days (HDD) = $\max(18.3^\circ C - T, 0)$

We transform the series $f2y_d$ from daily to hourly, by considering the same factor-to-year $f2y_d$ for each hour t observed in the day d . In this way we construct hourly $f2y_t$ series, which later enter the shape s_t . The $f2d$, in contrast, indicates the weight of the price of a particular hour compared to the daily base price.

$$f2d_t = \frac{S^{hour}(t)}{\sum_{k \in \text{day}(t)} S^{hour}(k) \frac{1}{24}} \quad (12)$$

with $S^{hour}(t)$ being the hourly spot price at the hour t . We know that there are considerable differences both in the daily profiles of workdays, Saturdays and Sundays, but also between daily profiles during winter and summer season. Thus, following Blöchlinger (2008) we suggest to classify the days by weekdays and seasons and to choose the classification scheme presented in Table 1. The workdays of each month are collected in one class. Saturdays and Sundays are treated separately. In order to obtain still enough observations per class, the profiles for Saturday and Sunday are held constant during three months.

The regression model for each class is built quite similarly to the one for the yearly seasonality. For each profile class $c = \{1, \dots, 20\}$ defined in Table 1, a model of the

Table 1: *The table indicates the assignment of each day to one out of the twenty profile classes. The daily pattern is held constant for the workdays Monday to Friday within a month, and for Saturday and Sunday, respectively, within three months.*

	J	F	M	A	M	J	J	A	S	O	N	D
Week day	1	2	3	4	5	6	7	8	9	10	11	12
Sat	13	13	14	14	14	15	15	15	16	16	16	13
Sun	17	17	18	18	18	19	19	19	20	20	20	17

following type is formulated:

$$f2d_t = a_o^c + \sum_{i=1}^{23} b_i^c H_{t,i} + \varepsilon_t \quad \text{for all } t \in c. \quad (13)$$

where $H_i = \{0, \dots, 23\}$ represents dummy variables for the hours of one day.

The seasonality shape sw_t can be calculated by $sw_t = f2y_t \cdot f2d_t$, and becomes the forecast of the relative hourly weights. It is additionally multiplied by the observed futures prices, in order to align the shape at the prices level. This yields the seasonality shape s_t which is finally used to deseasonalize the price forward curves, as shown in Equation 2.

An alternative approach to extract power forward curves from a discrete set of traded contract using spline interpolation is suggested by Benth, Koekebakker, and Ollmar (2007). Recently, Caldana, Fusai, and Roncoroni (2016) proposed a method combining non-parametric filtering with convex interpolation.

4 Empirical analysis

The original input to our analysis are 2'386 hourly price forward curves for PHELIX, the German electricity index, generated each day between 01/01/2009 and 15/07/2015, for a horizon of 5 years. The curves have been provided by the Institute of Operations Research and Computational Finance, University of St. Gallen. In a first step, we eliminated the

deterministic component of the hourly price forward curves, as shown in Equation (2). To keep the analysis tractable, we chose to work with daily, instead of hourly curves. Thus, the stochastic component of each hourly price forward curve, $f_t(x)$, has been filtered out for hour 12 of each day over a horizon of 2 years.⁵ The choice of hour 12 is intuitive, since it has been empirically shown that over noon electricity prices are more volatile, due to the increase in the infeed from renewable energies over the last years in Germany (Paraschiv, Erni, and Pietsch (2014)). It is interesting, therefore, to analyse the volatility of the noise $\epsilon_t(x)$ (Equation (6)) for this particular trading period.

In this section, we analyse the stochastic component of price forward curves and examine further the market price of risk, the distribution of the noise volatility and its spatio-temporal correlations structures.

4.1 Analysis of the stochastic component of Price Forward Curves

In Table 2 we show the evolution of production sources for electricity generation in Germany between 2009–2014 in yearly percent averages. We observe that between 2010 and 2011 there has been an increase of 30% in the wind infeed, while the solar production source almost doubled. Between 2011–2014 the infeed from wind and photovoltaic increased further, however in smaller steps. The increase of renewable energies in electricity production poses a particular challenge for power generators, since both wind and solar energies are difficult to forecast. Thus, market players must balance out forecasting errors in their power production, which increases the trading activity in the market (as shown in Kiesel and Paraschiv (2015)). Due to high forecasting errors in renewables, it is more difficult to plan accurately the required traditional capacity to cover the forecasted demand for the day-ahead. Demand/supply disequilibria are therefore expected to occur and, in consequence, extremely large price changes, so-called spikes, are observed. Given that

⁵For the generation of PFCs on horizons longer than 2 years, only yearly futures are still observed, so the information about the market expectation becomes more general. We therefore decided to keep the analysis compact and analyse 2 years long truncated curves.

the electricity production in Germany is to a large extent coal based, and that ramping up/down coal power plants can be done only at high costs, shocks in supply of electricity are usually difficult to be balanced out, and thus spikes can occur in clusters, leading to an increased volatility of electricity prices.

While there is empirical evidence for the direct impact of wind and photovoltaic on electricity spot prices (Paraschiv, Erni, and Pietsch (2014)), we investigated further whether the higher volatile infeed from renewable energies, which substituted traditional plants in production, is also reflected in the market expectation of futures prices. In Figures 12–13 we display the pattern of the stochastic component of price forward curves ($f_t(x)$, as shown in Equation (2)) generated at 1st February each year between 2010–2014. We observe that the volatility of the stochastic component is increasing significantly between 2010–2011 and decreases between 2012–2013. The increased volatility of the stochastic component of curves between 2010–2011 can be interpreted as a consequence of the sustainable increase in the infeed from renewable energies, wind and photovoltaic, between these two consecutive years, supported and planned by the energy policy makers in Germany (see Paraschiv, Erni, and Pietsch (2014)).

More renewables in the market lead to more uncertainty around the market expectation of electricity prices and thus, to more volatile futures prices. However, after 2012 the infeed from renewables increased by lower percentages, so the lower volatility in the stochastic component of PFCs show an adaption process: Market participants have a better ease and certainty of valuation and a better understanding of the role of renewables for electricity production.

4.2 Analysis of the risk premium

In the case of storable commodities, arbitrage-based arguments imply that forward prices are equal to (discounted) expected spot prices. However, electricity is non-storable, so this link does not exist here. Therefore, it can be expected that forward prices are formed

	2009	2010	2011	2012	2013	2014
Coal	42.6	41.5	42.8	44	45.2	43.2
Nuclear	22.6	22.2	17.6	15.8	15.4	15.8
Natural Gas	13.6	14.1	14	12.1	10.5	9.5
Oil	1.7	1.4	1.2	1.2	1	1
Renewable energies from which	15.9	16.6	20.2	22.8	23.9	25.9
Wind	6.5	6	8	8.1	8.4	8.9
Hydro power	3.2	3.3	2.9	3.5	3.2	3.3
Biomass	4.4	4.7	5.3	6.3	6.7	7.0
Photovoltaic	1.1	1.8	3.2	4.2	4.7	5.7
Waste-to-energy	0.7	0.7	0.8	0.8	0.8	1
Other	3.6	4.2	4.2	4.1	4	4.3

Table 2: Electricity production in Germany by source (%), as shown in Paraschiv, Erni, and Pietsch (2014).

as the sum of the expected spot price plus a risk premium that is paid by risk-averse market participants for the elimination of price risk. We estimated Equation (6) for each time-series $Z_t(x)$ and $f_t(x)$, $t \in \{1, \dots, T\}$ of each point $x \in \{x_1, \dots, x_N\}$. For taking $\Delta t = 1day$ and $\Delta x = 1day$, the estimated risk premia will be a $(1 \times (N - 1))$ vector. Estimation results are shown in Figure 3.

We observe that the risk premia take values between a minimum of -0.086 and maximum 0.017 . They oscillate around zero and have a higher volatility over the first three quarters of the year along the curve, so for shorter time to maturities. However, on the medium/long-run the risk premia are predominantly negative and the volatility becomes more constant for the second year.

The finding that the short-term risk premia oscillate around zero is consistent with the findings in the literature. For example Pietz (2009) found that the risk premium may be positive or negative, depending on the average risk aversion in the market. It may vary in magnitude and sign throughout the day and between seasons. Furthermore, Paraschiv, Fleten, and Schürle (2015) found that short-term risk premia are positive during the week and decrease or become negative for the weekend. The disentangled pattern of risk premia between seasons, working/weekend days cannot be investigated

here directly, though, since we used for the estimation a time-series of each point along one curve, making use of all generated PFCs used as input. We are in fact interested to examine the evolution of risk premia with increasing time to maturity. In the long-run, the negative risk premia confirm previous findings in the literature (see e.g., Burger, Graeber, and Schindlmayr (2007)): producers accept lower futures prices, as they need to make sure that their investment costs are covered.

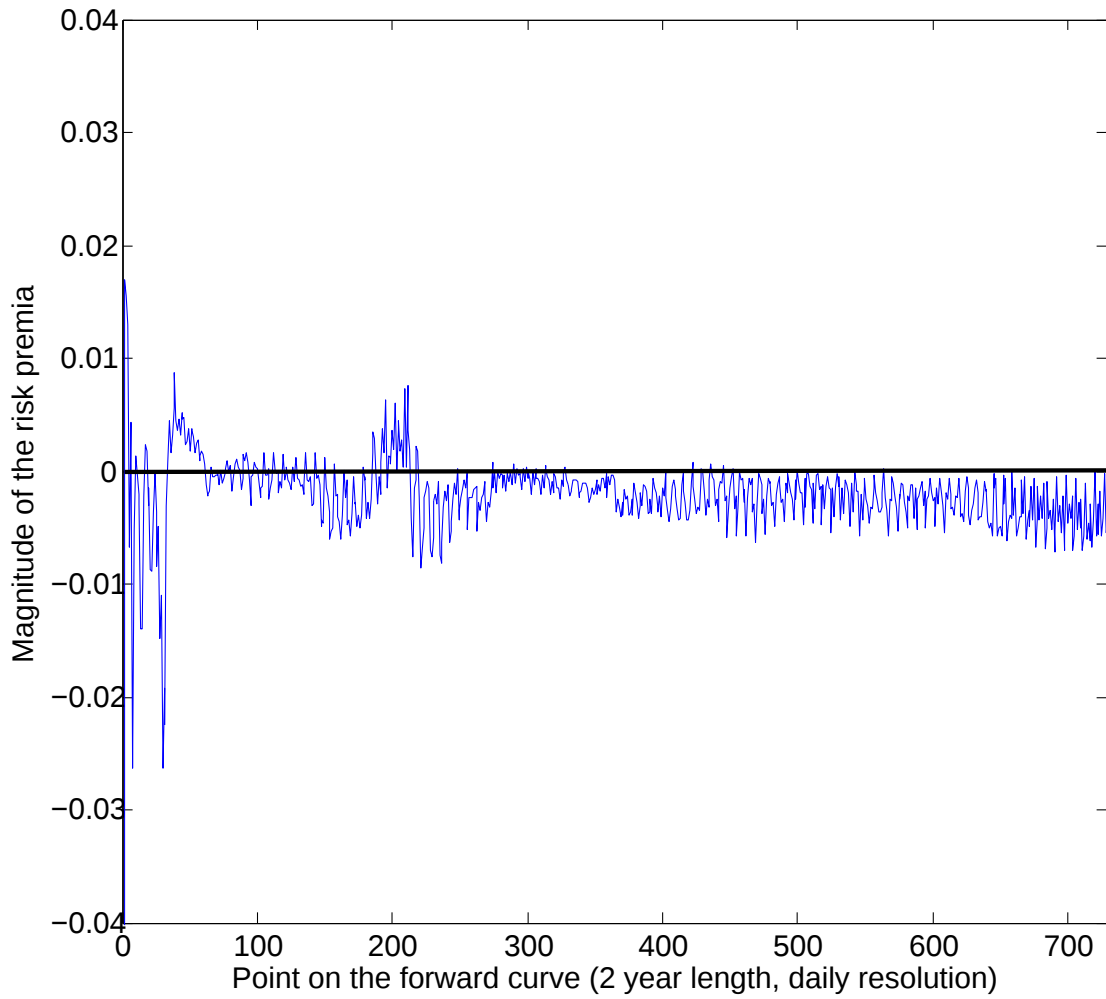


Figure 3: Risk premia along one curve (2-year length, daily resolution)

4.3 Analysis of term structure volatility

In Figure 4 we plot the term structure volatility $\sigma(x)$, for $x \in \{x_1, \dots, x_N\}$, as defined in Equation 7. Overall we observe that the volatility decreases with increasing time to maturity. In particular, it decays faster for shorter time to maturity and it shows a bump around the maturity of 1 month. Around the second (front) quarter the volatility starts increasing again, showing a second bump around the third quarter. The reason is that for time to maturities longer than one month, in most of the cases weekly futures are not available anymore, so the next shortest maturity available in the market is the front month future. That means: if market participants are interested in one sub-delivery period within the second month, there are no weekly futures available to properly price their contracts, but the only available information is from the front month future price. It is known that the volume of trades for this front month future increases, thus inducing a higher volatility of the corresponding forward prices. In Figure 5 we observe that indeed, the front month future has the highest and the most volatile volume of trades over the investigated time period, compared to the other monthly traded contracts. A similar effect is around the front quarter, when monthly futures are not observed anymore, but the information about the level of the (expected) price is given by the corresponding quarterly future contract. In consequence, the volume of trades for the front quarterly future and for the 2nd available quarterly future increases, these being the most traded products in the market, as shown in Figure 6. This explains the increase in the volatility during the front quarter segment of the forward curve and the second bump.

The jigsaw pattern of the volatility curve reflects the weekend effect: the volatility of forwards is lower during weekend versus working days. A similar pattern is observed in the spot price evolution, as shown in Paraschiv, Fleten, and Schürle (2015).

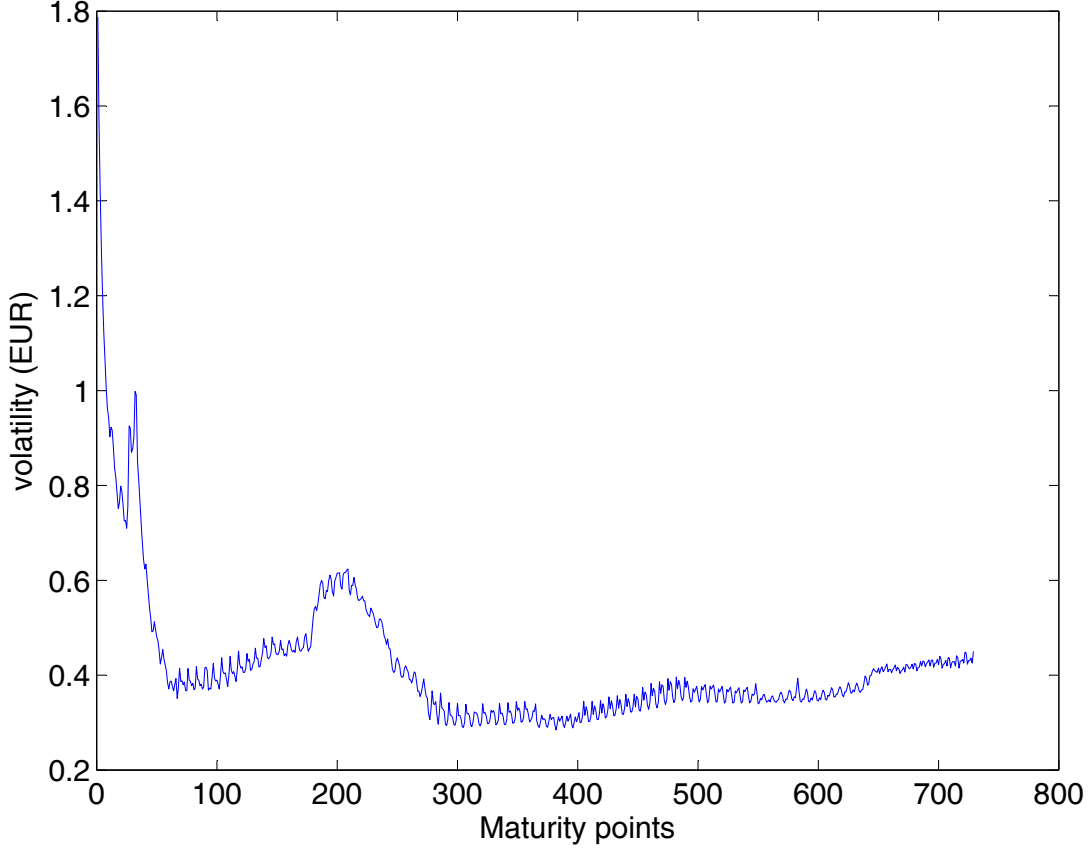


Figure 4: The empirical volatility term structure

4.4 Statistical properties of the noise time series

The analysis of the noise time-series $\tilde{\epsilon}_t$ (see Equation (7)) is twofold: First, we examine the statistical properties of individual time series $\tilde{\epsilon}_t(x_i)$ and in particular we check for stationarity, autocorrelation and ARCH/GARCH effects. Secondly, we examine patterns in the correlation matrix with respect to the time/maturity dimensions. Thus, we are interested in the correlations between $\tilde{\epsilon}_t(x_i)$ and $\tilde{\epsilon}_t(x_j)$, for $i, j \in \{1, \dots, N\}$, $t = 1, \dots, T$ to examine the effect of the time to maturity on the joint dynamics between the noise components. Furthermore we are interested in the correlations between noise curves, with respect to the points in time where these have been generated: correlations between $\tilde{\epsilon}_m(x_1), \dots, \tilde{\epsilon}_m(x_N)$ and $\tilde{\epsilon}_n(x_1), \dots, \tilde{\epsilon}_n(x_N)$, for $m, n \in \{1, \dots, T\}$. The analysis is

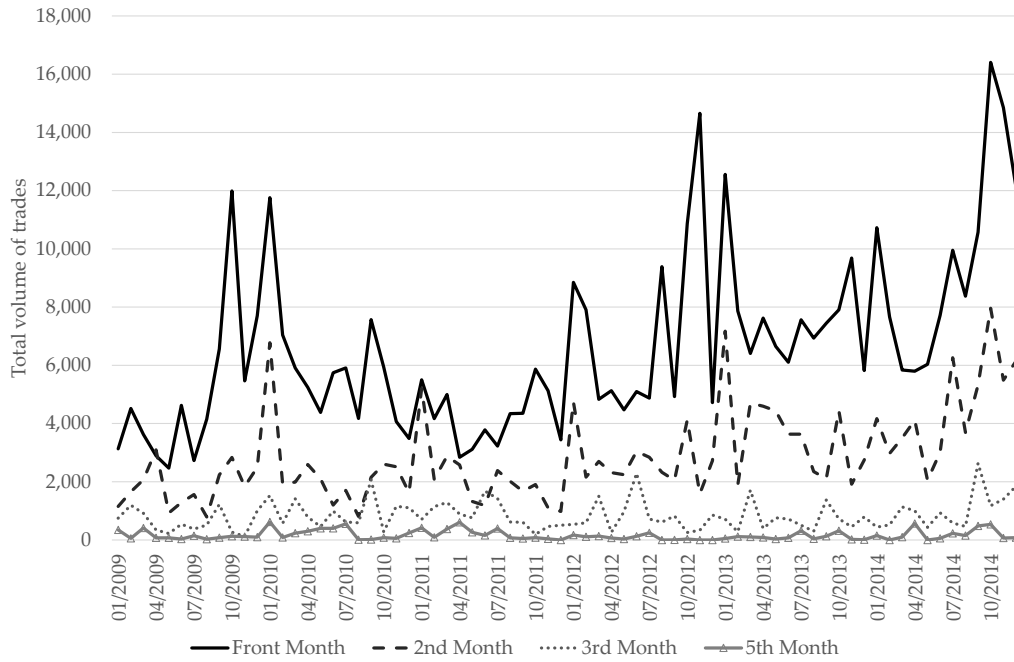


Figure 5: The sum of traded contracts for the monthly futures at EPEX (own calculations, source of data: ems.eex.com).

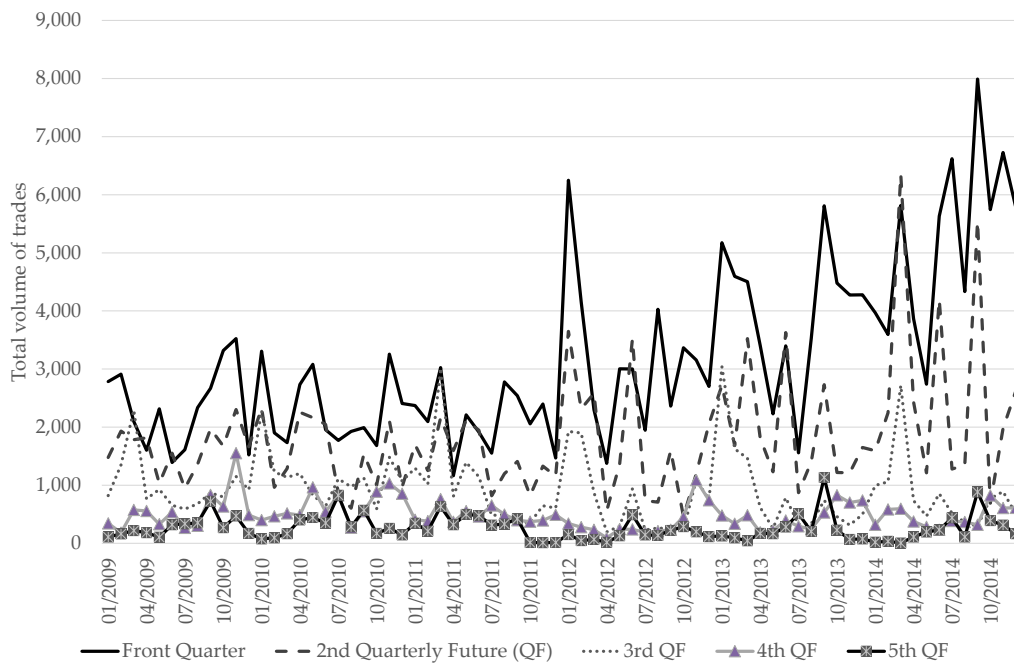


Figure 6: The sum of traded contracts for the quarterly futures at EPEX (own calculations, source of data: ems.eex.com).

performed initially for taking $\Delta x = 1 \text{ day}$ and $\Delta t = 1 \text{ day}$, as defined in Equation (5).

We are further interested to see whether the statistical properties of the noise as well as the correlations between its components change, if we vary the maturity step Δx in Equation (5) (and implicitly Δt). We believe that various maturity steps will lead to slightly different properties of the noise, given the stepwise pattern of the deseasonalized price forward curves $f_t(x)$, as shown in Figures 12 and 13. The stepwise pattern comes from the different level of futures prices of different maturities taken as input for the generation of price forward curves. Futures have different delivery periods, weekly, monthly, quarterly, yearly, and at each point when a new future is observed, the level changes. This pattern is taken over in the stochastic component $f_t(x)$. Furthermore, within one week, we observe the weekend effect: the price level is different between working/weekend days. All these cause sparse matrices in the noise, given the many values of “zero” obtained after differentiating.

To assess the impact of stepwise changes in the stochastic component of price forward curves $f_t(x)$, we replicated the analysis for one additional case study: We further investigated the effect of a change between consecutive weekly futures prices by taking $\Delta x = 7 \text{ days}$. This maturity step accounts further for the impact of a change in the level of the curve when monthly/quarterly products become available (or mature, being replaced by new ones in the market).

4.4.1 Stationarity, Autocorrelation, ARCH/GARCH effects

The stationarity, autocorrelation pattern and ARCH/GARCH effects are computed for each case study of $\Delta x/\Delta t$, namely 1 day and 7 days shifts in maturity (and time). To reduce the complexity, we compute these statistics for time series of equidistant points along the curve’s length: $\tilde{\epsilon}_t(x_k)$, where $k \in \{1, \dots, N\}$. In choosing k we increment over 90 days (approximately one quarter) along one noise curve. To test for stationarity, we applied the Augmented Dickey-Fuller (ADF) and Phillips-Perron tests for a unit root in

each univariate time series $\tilde{\epsilon}_t(x_k)$. Results are confirmed when applying the Kwiatkowski-Phillips-Schmidt-Shin test statistic for stationarity (with intercept, no trend).⁶ Results are available in Tables 3 and 4 for the case studies $\Delta x = 1 \text{ day}$ and $\Delta x = 7 \text{ days}$, respectively. For $h = 0$, we fail to reject the null that series are stationary. Thus, all statistical tests conclude that time series $\tilde{\epsilon}_t(x_k)$ are stationary.

We further tested the hypothesis that the $\tilde{\epsilon}_t(x_k)$ series are autocorrelated. Autocorrelation test results are shown in Tables 3 and 4. We replicated the test for the level of the noise time series and for their squared values (columns 2 and 3, respectively). $h1 = 0$ indicates that there is not enough evidence to suggest that noise time series are autocorrelated. In Figures 7 and 8 we display the autocorrelation function for series $\tilde{\epsilon}_t(x_k)$ for $k \in 90, 180, 270, 360$, for the level and squared residuals, respectively. In the first case, the pattern of the autocorrelation function for the level of residuals shows a typical white noise pattern. Still, as expected, the autocorrelation function shows a slight decaying pattern in the second case, when we look at the squared residuals. The decaying pattern becomes more obvious when we move to the case study two, where the change in maturity (and time) is set to 7 days, as shown in Figure 9. This is not surprising, since an increment of maturity points and time of 7 days leads to less zero increments in the noise time series overall, which allows a more visible pattern of autocorrelation. Results of the autocorrelation test conclude our findings from the visual inspection: if in the basic case study of $\Delta x = 1 \text{ day}$ we did not find evidence for autocorrelation in all time series of the noise (Table 3, second and third columns), there is clear evidence for autocorrelation in all series with increasing maturity step $\Delta x = 7 \text{ days}$.

We further tested the hypothesis that there are significant ARCH effects in the $\tilde{\epsilon}_t(x_k)$ series by employing the Ljung-Box Q-Test. Results are shown in the last columns of Tables 3 and 4. $h2 = 1$ indicates that there are significant ARCH effects in the noise time-series. Independent of the maturity/time step chosen, time series are characterized

⁶see Appendix 8.1 for the detailed test statistics.

by ARCH effects, and thus by a volatility clustering pattern. In Equation 7 we filter the volatility out of the marginal noise $\epsilon_t(x)$. However, the volatility is not time-varying in our model, which explains that there is a (slight) evidence for remaining stochastic volatility (conditional heteroschedasticity) in the standardized residuals $\tilde{\epsilon}_t(x)$.

In the light of the identified ARCH/GARCH effects in the marginals $\tilde{\epsilon}_t(x_k)$, we inspect their tail behavior by plotting the kernel smoothed empirical densities versus normal distribution for series $k \in 1, 90, 180, 270$, as shown in Figure 11. We observe the strong leptokurtic pattern of heavy tailed marginals.

Overall we conclude that the model residuals are coloured noise, with heavy tails (leptokurtic distribution) and with a tendency for conditional volatility.

$\tilde{\epsilon}_t(x_k)$	Stationarity	Autocorrelation $\tilde{\epsilon}_t(x_k)$	Autocorrelation $\tilde{\epsilon}_t(x_k)^2$	ARCH/GARCH
	h	h1	h1	h2
Q0	0	1	1	1
Q1	0	0	0	0
Q2	0	1	1	1
Q3	0	0	1	1
Q4	0	1	0	0
Q5	0	1	1	1
Q6	0	1	1	1
Q7	0	1	1	1

Table 3: The time series are selected by quarterly increments (90 days) along the maturity points on one noise curve. Hypotheses tests results, case study 1: $\Delta x = 1day$. In column 'Stationarity', if $h = 0$ we fail to reject the null that series are stationary. For 'Autocorrelation', $h1 = 0$ indicates that there is not enough evidence to suggest that noise time series are autocorrelated. In the last column, $h2 = 1$ indicates that there are significant ARCH effects in the noise time-series.

4.4.2 Spatial Correlation

In the autocorrelation functions examined above, we show that there are temporal correlations between forward curves produced at different points in time. In addition, we are interested in the spatial correlation structure between $\tilde{\epsilon}_t(x_i)$ and $\tilde{\epsilon}_t(x_j)$, for $i, j \in 1, \dots, N$, to examine how noise correlations change with increasing distance between the matu-

$\tilde{\epsilon}_t(x_k)$	Stationarity	Autocorrelation $\tilde{\epsilon}_t(x_k)$	Autocorrelation $\tilde{\epsilon}_t(x_k)^2$	ARCH/GARCH
	h	h1	h1	h2
Q0	0	1	1	1
Q1	0	1	1	1
Q2	0	1	1	1
Q3	0	1	1	1
Q4	0	1	1	1
Q5	0	1	1	1
Q6	0	1	1	1
Q7	0	1	1	1

Table 4: The time series are selected by quarterly increments (90 days) along the maturity points on one noise curve. Hypotheses tests results, case study 2: $\Delta x = 7days$. In column 'Stationarity', if $h = 0$ we fail to reject the null that series are stationary. For 'Autocorrelation', $h1 = 0$ indicates that there is not enough evidence to suggest that noise time series are autocorrelated. In the last column, $h2 = 1$ indicates that there are significant ARCH effects in the noise time-series.

rity points along one curve. In Figure 10 we observe that correlations oscillate between positive and negative, which is expected, given the nature of the (coloured) noise time series (stationary, oscillating around 0). As expected, spatial correlations between maturity points of up to 1 month (about 30 day) decay fast with increasing distance between them. This reflects the higher interest of market participants for maturing contracts. The correlations between maturity points situated at distances longer than 30 days are very low, oscillating around zero. However, correlations between 1 year distant maturity points slightly increase. This shows that the stochastic component of forward prices is driven by common factors at the same time of the year, which is reflected in a higher correlation between yearly futures products.

5 Modeling approach and estimation of the noise

Given the heavy tails of marginals identified in Figure 11, we model the noise marginals $\tilde{\epsilon}_t(x)$ by a Normal Inverse Gaussian distribution (NIG). The NIG distribution is a special case of the Generalized Hyperbolic Distribution for $\lambda = -1/2$ and its density reads (see

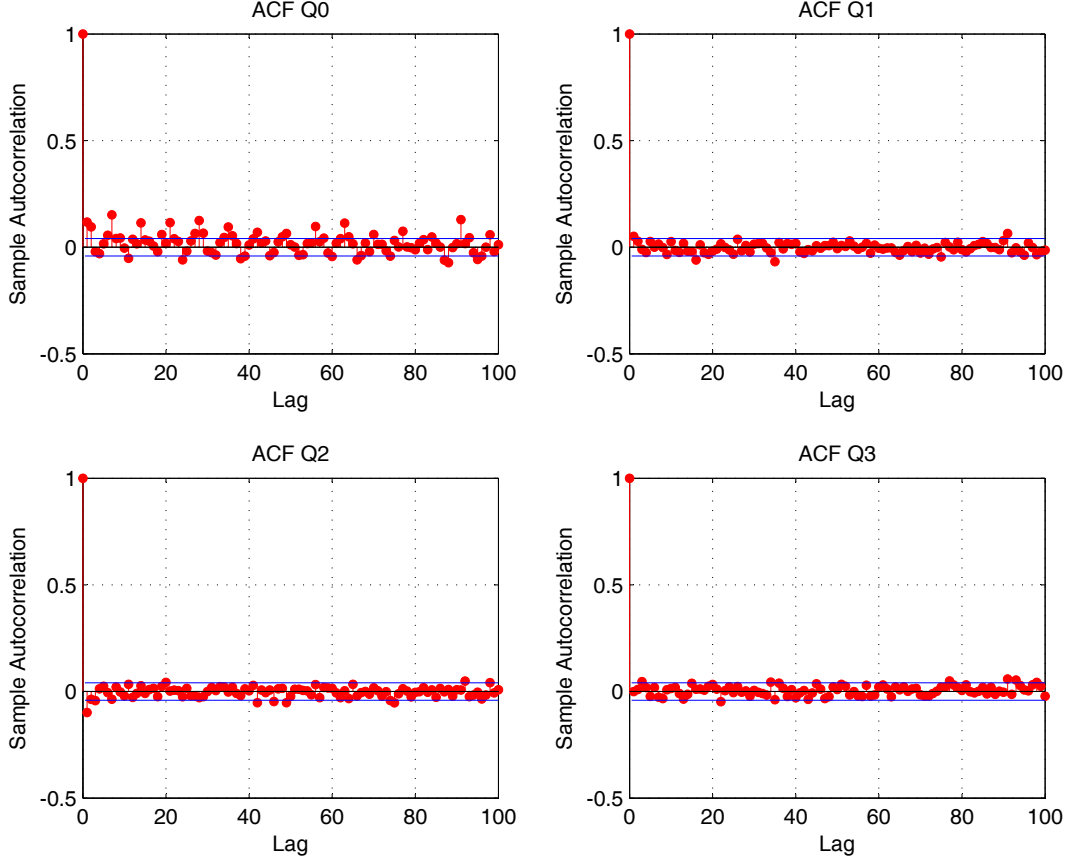


Figure 7: Autocorrelation function in the level of the noise time series $\tilde{\epsilon}_t(x_k)$, by taking $k \in \{1, 90, 180, 270\}$, case study 1: $\Delta x = 1 \text{ day}$.

Benth, Šaltytė Benth, and Koekebakker (2008)):

$$f_{NIG}(x) = \frac{\alpha}{\pi} \exp(\delta \sqrt{\alpha^2 - \beta^2} + \beta(x - \mu)) \frac{K_1(\alpha \delta \sqrt{1 + (\frac{x-\mu}{\delta})^2})}{\sqrt{1 + (\frac{x-\mu}{\delta})^2}} \quad (14)$$

We have firstly fitted a NIG by moment estimators. We observed that the fitted density performs visibly better than a normal distribution in explaining the leptokurtic pattern of time series. In a second step, we estimated NIG by maximum likelihood (ML).

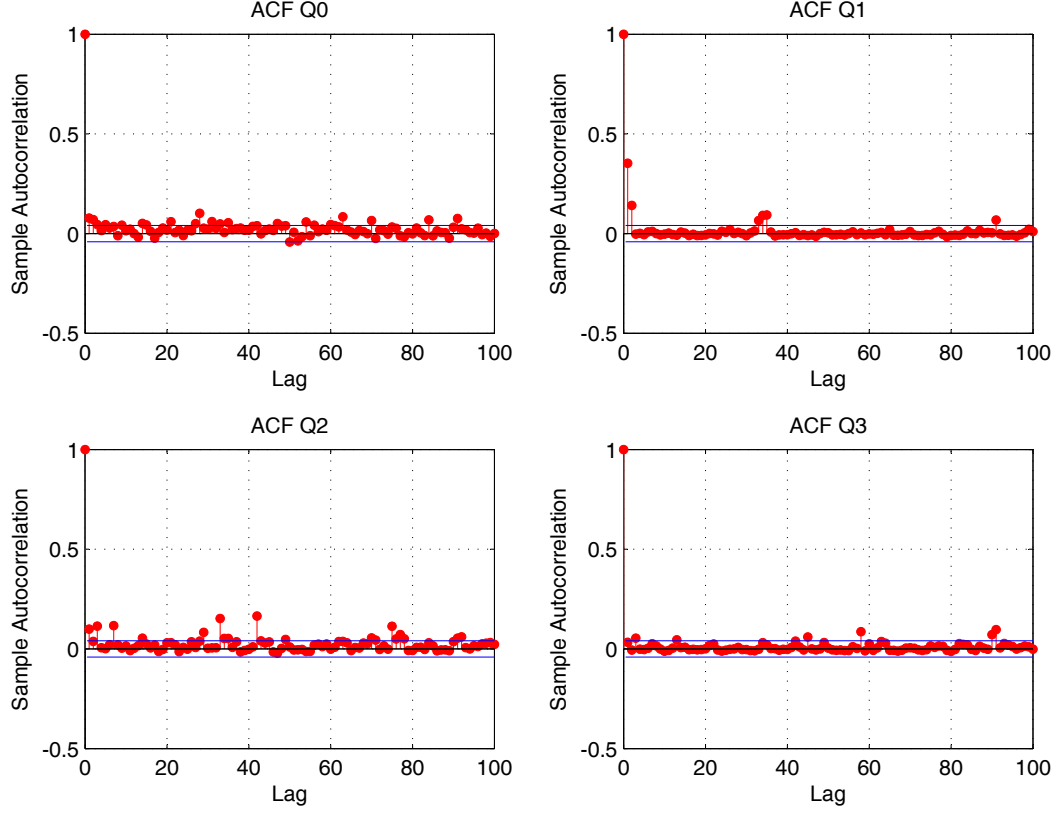


Figure 8: Autocorrelation function in the squared time series of the noise $\tilde{\epsilon}_t(x_k)^2$, by taking $k \in \{1, 90, 180, 270\}$, case study 1: $\Delta x = 1 \text{ day}$.

The mathematical formulation of the likelihood function and related gradients as input to the numerical optimization procedure are given in Appendix 8.2.

The ML estimates improved further the fit of the NIG density. In Table 5 we show the ML estimates for the NIG distribution fitted to $\tilde{\epsilon}_t(x_k)$ by taking $k \in \{1, 90, 180, 270\}$. In Figure 11 we show the kernel density estimates versus normal and the two versions of the NIG estimation. We confirm a realistic performance of the NIG distribution in explaining the heavy tail behavior of noise marginals.

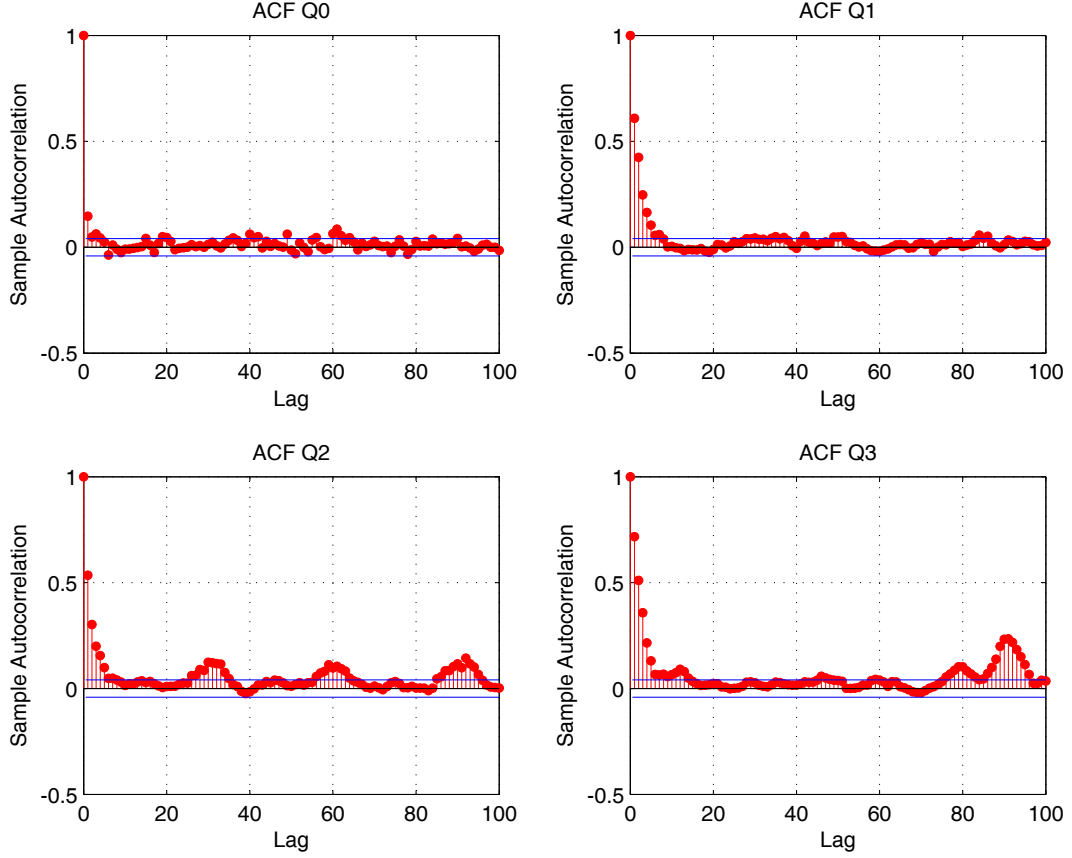


Figure 9: Autocorrelation function in the squared time series of the noise $\tilde{\epsilon}_t(x_k)^2$, by taking $k \in \{1, 90, 180, 270\}$, case study 2: $\Delta x = 7 \text{ days}$.

Parameter	Q0	Q1	Q2	Q3
δ	1.115 (0.102)	0.252 (0.007)	0.193 (0.005)	0.226 (0.006)
α	1.083 (0.152)	0.188 (0.046)	0.116 (0.052)	0.195 (0.037)
β	0.111 (0.054)	-0.012 (0.024)	0.000 (0.022)	-0.001 (0.021)
μ	-0.011 (0.049)	0.000 (0.004)	-0.004 (0.007)	-0.002 (0.007)

Table 5: Maximum likelihood estimates of NIG to $\tilde{\epsilon}_t(x_k)$ by taking $k \in \{1, 90, 180, 270\}$ for Q0,...,Q3, respectively. Standard errors are shown in parentheses.

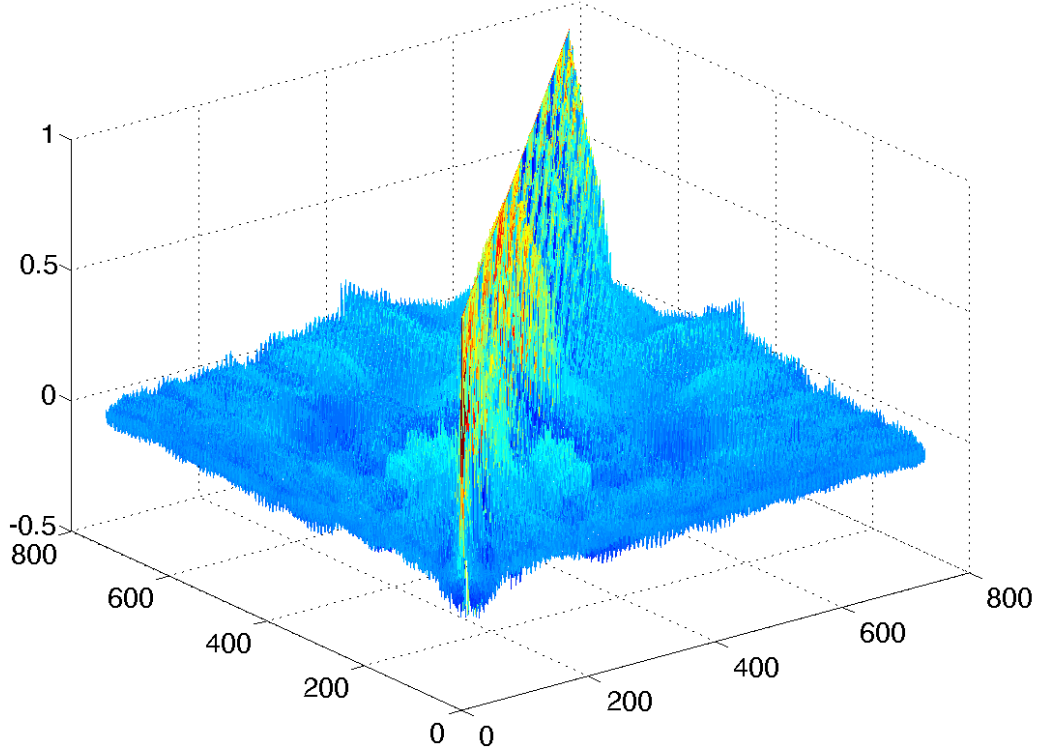


Figure 10: Correlation matrix with respect to different maturity points

6 Revisiting the spatio-temporal model of forward prices

In our empirical analysis of EPEX electricity forward prices, we have made use of a time series discretization of the deseasonalized term structure dynamics $f_t(x)$ defined in (3). We have estimated the parameter of the market price of risk $\theta(x)$, and have analysed empirically the noise residual $dW_t(x)$ expressed as $\epsilon_t(x) = \sigma(x)\tilde{\epsilon}_t(x)$ in a discrete form in (7). The purpose of this Section is to recover an infinite dimensional model for $W_t(x)$ based on our findings.

To this end, we recall that $\mathcal{H}$ is a separable Hilbert space of real-valued function on $\mathbb{R}_+$, where W_t is a martingale process. As suggested by the notation, a first model could simply be to assume that W is a $\mathcal{H}$ -valued Wiener process. However, this would mean that we expect $t \mapsto W_t(x)$ to be a Gaussian process for each $x \geq 0$, which is at stake with our empirical findings showing clear non-Gaussian (or, coloured noise) residuals. After explaining the Samuelson effect, the residuals could be modelled nicely by a NIG distribution.

A potential model of W could be

$$W_t = \int_0^t \Sigma_s dL_s, \quad (15)$$

where $s \mapsto \Sigma_s$ is an $L(\mathcal{U}, \mathcal{H})$ -valued predictable process and L is a $\mathcal{U}$ -valued Lévy process with zero mean and finite variance. We refer to (Peszat and Zabczyk, 2007, Sect. 8.6) for conditions to make the stochastic integral well-defined. As a first case, we can choose $\Sigma_s \equiv \Psi$ time-independent, being an operator mapping elements of the separable Hilbert space $\mathcal{U}$ into $\mathcal{H}$. An increment in W_t can be approximated (based on the definition of the stochastic integral, see (Peszat and Zabczyk, 2007, Ch. 8)) as

$$W_{t+\Delta t} - W_t \approx \Psi(L_{t+\Delta t} - L_t) \quad (16)$$

Choose now $\mathcal{U} = L^2(\mathbb{R})$, the space of square integrable functions on the real line equipped with the Lebesgue measure, and assume Ψ is an integral operator on $L^2(\mathbb{R})$, i.e., for $g \in L^2(\mathbb{R})$, the mapping

$$\mathbb{R}_+ \ni x \mapsto \Psi(g)(x) = \int_{\mathbb{R}} \tilde{\sigma}(x, y) g(y) dy \quad (17)$$

defines an element in $\mathcal{H}$. Furthermore, if $\text{supp } \tilde{\sigma}(x, \cdot)$ is concentrated in a close neighborhood of x , we can further make the approximation $\Psi(g)(x) \approx \tilde{\sigma}(x, x)g(x)$. As a result,

we find

$$W_{t+\Delta t}(x) - W_t(x) \approx \tilde{\sigma}(x, x)(L_{t+\Delta t}(x) - L_t(x)). \quad (18)$$

In view of the definition of $\epsilon_t(x)$ in (7), we can choose $\sigma(x) = \tilde{\sigma}(x, x)$ to be the model for the Samuelson effect that we identified and discussed in Subsect. 4.3, and we let L_t be a NIG Lévy process with values in $L^2(\mathbb{R})$ to model the standardized residuals $\tilde{\epsilon}_t$ (see Benth and Krühner (2015) for a definition of such a process).

Recall from Fig. 10 the spatial correlation structure of $\tilde{\epsilon}_t(x)$. This provides the empirical foundation for defining a covariance functional $\mathcal{Q}$ associated with the Lévy process L . In general, we know that for any $g, h \in L^2(\mathbb{R})$,

$$\mathbb{E}[(L_t, g)_2(L_t, h)_2] = (\mathcal{Q}g, h)_2$$

where $(\cdot, \cdot)_2$ denotes the inner product in $L^2(\mathbb{R})$ (see (Peszat and Zabczyk, 2007, Thm. 4.44)).

The covariance functional will be a symmetric, positive definite trace class operator from $L^2(\mathbb{R})$ into itself. It can be specified as an integral operator on $L^2(\mathbb{R})$ by

$$\mathcal{Q}g(x) = \int_{\mathbb{R}} q(x, y)g(y) dy, \quad (19)$$

for some suitable “kernel-function” q . If q is symmetric, positive definite and continuous function, then it follows from Thm. A.8 in Peszat and Zabczyk (2007) that $\mathcal{Q}$ is a covariance operator of L if we restrict ourselves to $L^2(\mathcal{O})$, where $\mathcal{O}$ is a bounded and closed subset of $\mathbb{R}$. Indeed, we can think of $\mathcal{O}$ as the maximal horizon of the market, in terms of relevant times to maturity (recall that we have truncated the forward curves in our empirical study to a horizon of 2 years).

If we assume $g \in L^2(\mathbb{R})$ to be close to δ_x , the Dirac δ -function, and likewise, $h \in L^2(\mathbb{R})$ being close to δ_y , $(x, y) \in \mathbb{R}^2$, we find approximately

$$\mathbb{E}[L_t(x)L_t(y)] = q(x, y)$$

From the spatial correlation study of $\tilde{\epsilon}_t$, we observe that the correlation is stationary in space in the sense that it only depends on the distance $|x - y|$. Hence, with a slight abuse of notation, we let $q(x, y) = q(|x - y|)$. A simple choice resembling to some degree the fast decaying property in Fig. 10 is $q(|x - y|) = \exp(-\gamma|x - y|)$ for a constant $\gamma > 0$. We further note that from Benth and Krühner (2015), it follows that $t \mapsto (L_t, g)_2$ is a NIG Lévy process on the real line. If $g \approx \delta_x$, then we see that $L_t(x)$ for given x is a real-valued NIG Lévy process. With these considerations, we have established a possible model for W which is, at least approximately, consistent with our empirical findings for ϵ_t .

Let us briefly discuss why we suggest to use $\mathcal{U} = L^2(\mathbb{R})$ and introduce a rather complex integral operator definition of Ψ . As mentioned in Sect. 2, the Hilbert space $\mathcal{H}$ to realize the deseasonalized forward price dynamics f_t should be a function space on $\mathbb{R}_+$. $L^2(\mathbb{R})$ is a space of equivalence classes, and the evaluation operator $\delta_x(g) = g(x)$ is not a continuous linear operator on this space. A natural Hilbert space where indeed δ_x is a linear functional (e.g., continuous linear operator from the Hilbert space to $\mathbb{R}$) is the so-called Filipovic space. The Filipovic space was introduced and studied in the context of interest rate markets by Filipovic (2001), while Benth and Krühner (2014, 2015) have proposed this as a suitable space for energy forward curves. From Benth and Krühner (2014) we have a characterization of the possible covariance operators of Lévy processes in the Filipovic space, which, for example, cannot be stationary in the form suggested for q above. Using $\mathcal{U} = L^2(\mathbb{R})$ opens for a much more flexible specification of the covariance operator, which matches nicely the empirical findings on our electricity data. On the other hand, we need to bring the noise L over to the Filipovic space, since we wish to have dynamics of the term structure in a Hilbert space for which we can evaluate the curve at a point $x \geq 0$, that is, $\delta_x(f_t) = f_t(x)$ makes sense. We recover the actual forward price dynamics $t \mapsto F(t, T)$ in this case by

$$F(t, T) = f(t, T - t) = \delta_{T-t}(f_t).$$

We remark that for elements f in the Filipovic space, $x \mapsto f(x)$ will be continuous, and weakly differentiable. To specify Ψ as an integral operator on $L^2(\mathbb{R})$, we can bring any element of $L^2(\mathbb{R})$ to a smooth function. Indeed, the convolution product of a square integrable function with a smooth function will yield a smooth function (see (Folland, 1984, Prop. 8.10)). This enables us to select "volatility" functions $\tilde{\sigma}$ ensuring that W_t becomes an element of the Filipovic space. Unfortunately, a simple multiplication operator $\Psi(g)(x) = \sigma(x)g(x)$ will not do the job, as this will not be an element of the Filipovic space for general $g \in L^2(\mathbb{R})$. In conclusion, with $\mathcal{H}$ being the Filipovic space, we choose a different space for the noise L to open up for flexibility in modelling the spatial correlation, and an integral operator for Ψ to ensure that we map the noise into the Filipovic space, at the same time modeling the Samuelson effect.

To follow up on the integral operator, we know the function $\tilde{\sigma}(x, y)$ on the diagonal $x = y$, since here we want to match with the observed curve for the Samuelson effect. In a neighborhood around x , we smoothly interpolate to zero such that $\tilde{\sigma}(x, \cdot)$ has a support close to $\{x\}$, and such that the function defines an integral operator being sufficiently regular. One possibility is to define $\tilde{\sigma}(x, y) = \eta(x)\bar{\sigma}(|x - y|)$, where $\bar{\sigma} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is smooth, $\bar{\sigma}(0) = 1$, and $\text{supp } \bar{\sigma}$ is the interval $(-a, a)$ for a small. With this definition, we have that η models the Samuelson effect, the operator Ψ is a convolution product with $\bar{\sigma}$, followed by a multiplication with η . With η being an element of the Filipovic space, we have specified Ψ as desired. By inspection of the curve for the volatility term structure in Figure 4, a first-order approximation of it could be a function $\eta(x) = a \exp(-\zeta t) + b$, for constants a, ζ and b , where $b > 0$ is the long-term level and $\zeta > 0$ measures the exponential decay in the short end. We note that $\eta(0) = a + b$ will be the spot volatility. With such a specification, η will be an element of the Filipovic space since it is smooth and asymptotically constant. As we see, this simple model fails to account for the pronounced bumps in the curve that we have discussed earlier. By a more sophisticated model, one can take these into account as well.

Our empirical analysis also show indications of stochastic volatility effects. We will not discuss possible GARCH/ARCH specifications in continuous time, but briefly just mention that we can choose $\Sigma_s = V_s \Psi$, where $s \mapsto V_s$ is a $\mathbb{R}_+$ -valued stochastic process. For example, we can define V to be the Heston stochastic volatility dynamics (see Heston (1993)) or the BNS stochastic volatility model (see Barndorff-Nielsen and Shephard (2001)). In this case, it would be natural to suppose L to be a Wiener process in $L^2(\mathbb{R})$, since the additional stochastic volatility process V will induce non-Gaussian distributed residuals. We leave the further discussion on stochastic volatility models in infinite dimensional term structure models for future research (see however, Benth, Rüdiger, and Süss (2015) for a Hilbert-valued Ornstein-Uhlenbeck processes with stochastic volatility).

7 Conclusion and future work

In this study, we derived a spatio-temporal dynamical model based on the Heath-Jarrow-Morton (HJM) approach under the Musiela parametrization (see Heath, Jarrow, and Morton (1992)), which ensures an arbitrage-free model for electricity forward prices. A discretized version of the model has been fitted to electricity forward prices to examine the probabilistic characteristics of the data. We disentangled the seasonal pattern from the market price of risk and random perturbations of prices and analysed empirically their statistical properties.

As a special feature of our model, we further disentangled the temporal from spatial (time to maturity) effects on the dynamics of forward prices, which marks one of the main contributions of this study to the academic literature (see Andresen, Koekebakker, and Westgaard (2010)). After filtering out both temporal and spatial effects of price forward curves and the market price of risk, we estimated the term structure volatility. Finally, our model residuals show a white-noise pattern, which validates our modeling assumptions.

The model has been fitted to a unique data set of historical daily PFCs for the

German electricity market. We firstly performed a deseasonalization of the initial curves, where the seasonal component takes into account typical deterministic dynamics observed in the German electricity prices (see Blöchlinger (2008), Paraschiv (2013)). We further estimated the risk premia in the deseasonalized curves (stochastic component) and examined, in this context, the distribution of the noise: term structure volatility and its spatio-temporal correlations structures. Our results show that the short-term risk premia oscillate around zero, but become negative in the long run, which is consistent with the empirical literature (Paraschiv, Fleten, and Schürle (2015), Burger, Graeber, and Schindlmayr (2007)). We found that the noise marginals are coloured-noise with a strong leptokurtic pattern and heavy-tails, which have been successfully modeled by a normal inverse Gaussian distribution (NIG). There were signs of stochastic volatility effects as well. The high performance of the NIG distribution in modeling the noise marginals of forward electricity prices confirms previous findings of Frestad, Benth, and Koekebakker (2010). The term structure of volatility decays overall with increasing time to maturity, a typical Samuelson effect. However, the term structure of volatility in our data set has additionally clear bumps around the maturity of 1 month and third quarter, both being related to an increased activity in the market for the corresponding futures contracts. Our analysis also detects a fast decaying pattern in the spatial correlations as a function of distance between times to maturity.

Our empirical findings mark an additional contribution over existing related literature Andresen, Koekebakker, and Westgaard (2010): we shed light on the statistical properties of risk premia, of the noise, volatility term structure and of the spatio-temporal noise correlation structures. Notably, we look at price residuals where the maturity effect is corrected for, unlike the approach of Andresen, Koekebakker, and Westgaard (2010).

Based on the empirical insights, we revisited the spatio-temporal model of forward prices and derived a mathematical model for the noise. After explaining the Samuelson effect in the volatility term structure, the residuals are modeled by an infinite dimensional

NIG Lévy process, which allows for a natural formulation of a covariance functional. We model, in this way, the typical fat tails and fast-decaying pattern of spatial correlations. Still, our empirical findings show some slight remaining volatility clustering effects in the standardized residuals, which can be described by a stochastic volatility model formulation. However, we will discuss and develop stochastic volatility models in infinite dimensional term structure models in future research.

8 Appendix

8.1 Tests for unit roots

Tests for unit roots in the $\epsilon_t(x_k)$ series, for $k \in \{7, 8, 9, 10\}$.

Test	Null hypothesis	Q0	Q1	Q3	Q4
ADF test	Unit root	-4.476*	-4.701*	-3.504*	-3.600*
PP test	Unit root	-52.550*	-51.755*	-52.623*	-52.720*
KPSS test	Stationarity	0.564	0.399	0.329	0.367

Table 6: *Unit root test results for series $\epsilon_t(x_k)$ for quarterly increments in $k \in \{1, 90, 180, 270\}$. Note: One star denotes significance at the 1% level. ADF refers to Augmented Dickey-Fuller test, PP to the Philips-Peron test and KPSS to the Kwiatkowski-Phillips-Schmidt-Shin test. The lag structure of the ADF test is selected automatically on the basis of the Bayesian Information Criterion (BIC). For PP and KPSS tests the bandwidth parameter is selected according to the approach suggested by Newey and West (1994).*

8.2 Maximum likelihood for NIG

In the mathematical formulation below, $\lambda = \frac{1}{2}$:

$$\begin{aligned}
L(\alpha, \beta, \delta, \mu | \epsilon_1(x), \dots, \epsilon_T(x)) = & T \log a(\lambda, \alpha, \beta, \delta) + \left(\frac{\lambda}{2} - \frac{1}{4} \right) \sum_{i=1}^T \log (\delta^2 + (\epsilon_i(x) - \mu)^2) \\
& + \sum_{i=1}^T \left[\log K_{\lambda-\frac{1}{2}} \left(\alpha \sqrt{\delta^2 + (\epsilon_i(x) - \mu)^2} \right) + \beta (\epsilon_i(x) - \mu) \right]
\end{aligned} \tag{20}$$

where $K_{\lambda-\frac{1}{2}}$ is the modified Bessel function of third kind (see Benth, Šaltytė Benth, and Koekebakker (2008)) and

$$a(\lambda, \alpha, \beta, \delta) = \frac{(\alpha^2 - \beta^2)^{\lambda/2}}{\sqrt{2\pi} \alpha^{\lambda-\frac{1}{2}} \delta^\lambda K_\lambda \left(\delta \sqrt{\alpha^2 - \beta^2} \right)} \tag{21}$$

The derivatives of the likelihood function are,

$$\begin{aligned}
\frac{\partial L(\alpha, \beta, \delta, \mu | \epsilon_1(x), \dots, \epsilon_T(x))}{\partial \alpha} = & T \frac{\delta \alpha}{\sqrt{\alpha^2 - \beta^2}} R_\lambda \left(\delta \sqrt{\alpha^2 - \beta^2} \right) \\
& - \sum_{i=1}^T \sqrt{\delta^2 + (\epsilon_i(x) - \mu)^2} R_{\lambda-\frac{1}{2}}(a)
\end{aligned} \tag{22}$$

$$\frac{\partial L(\alpha, \beta, \delta, \mu | \epsilon_1(x), \dots, \epsilon_T(x))}{\partial \beta} = T \left(-\frac{\delta \beta}{\sqrt{\alpha^2 - \beta^2}} R_\lambda \left(\delta \sqrt{\alpha^2 - \beta^2} \right) - \mu \right) + \sum_{i=1}^T \epsilon_i(x) \tag{23}$$

$$\begin{aligned}
\frac{\partial L(\alpha, \beta, \delta, \mu | \epsilon_1(x), \dots, \epsilon_T(x))}{\partial \delta} = & T \left(-\frac{\lambda}{\delta} + \frac{1}{2} \sqrt{\alpha^2 - \beta^2} \left(R_{-\lambda} \left(\delta \sqrt{\alpha^2 - \beta^2} \right) \right. \right. \\
& \left. \left. + R_\lambda \left(\delta \sqrt{\alpha^2 - \beta^2} \right) \right) \right) + \frac{1}{2} \sum_{i=1}^T \left(\frac{(2\lambda - 1) \delta}{\delta^2 + (\epsilon_i(x) - \mu)^2} \right. \\
& \left. - \frac{\alpha \delta}{\sqrt{\delta^2 + (\epsilon_i(x) - \mu)^2}} \left(R_{-\lambda+\frac{1}{2}}(a) + R_{\lambda-\frac{1}{2}}(a) \right) \right)
\end{aligned} \tag{24}$$

$$\begin{aligned}
\frac{\partial L(\alpha, \beta, \delta, \mu | \epsilon_1(x), \dots, \epsilon_T(x))}{\partial \mu} = & -T\beta + \sum_{i=1}^T \left(-\frac{(\lambda - \frac{1}{2}) (\epsilon_i(x) - \mu)}{\delta^2 + (\epsilon_i(x) - \mu)^2} \right. \\
& \left. + \frac{\frac{1}{2} \alpha (\epsilon_i(x) - \mu)}{\sqrt{\delta^2 + (\epsilon_i(x) - \mu)^2}} \left(R_{-\lambda+\frac{1}{2}}(a) + R_{\lambda-\frac{1}{2}}(a) \right) \right)
\end{aligned} \tag{25}$$

where $R_\lambda(x) := \frac{K_{\lambda+1}(x)}{K_\lambda(x)}$, $x > 0$.

Acknowledgements

Fred Espen Benth acknowledges financial support from the project FINEWSTOCH, funded by the Norwegian Research Council. The authors thank very much Guido Haarbrücker, Claus Liebenberger and Michael Schürle from the Institute for Operations Research and Computational Finance, University of St. Gallen, for the continued support in the data collection step of this work.

References

- ANDRESEN, A., S. KOEKEBAKKER, AND S. WESTGAARD (2010): “Modeling electricity forward prices using the multivariate normal inverse Gaussian distribution,” *The Journal of Energy Markets*, 3(3), 3–25.
- AUDET, A., P. HEISKANEN, J. KEPPO, AND A. VEHVILÄINEN (2004): “Modeling electricity forward curve dynamics in the Nordic market,” in *Modelling Prices in Competitive Electricity Markets*, ed. by D. W. Bunn, pp. 251–265. Wiley, Chichester.
- BARNDORFF-NIELSEN, O. E., F. E. BENTH, AND A. VERAART (2013): “Modelling energy spot prices by volatility modulated Lévy-driven Volterra processes,” *Bernoulli*, 19, 803–845.
- (2014): “Modelling electricity futures by ambit fields,” *Advances in Applied Probability*, 46, 719–745.
- (2015): “Cross-commodity modelling by multivariate ambit fields,” in *Commodities, Energy and Environmental Finance, Fields Institute Communications Series*, ed. by R. Aid, M. Ludkovski, and R. Sircar, pp. 109–148. Springer Verlag.
- BARNDORFF-NIELSEN, O. E., AND N. SHEPHARD (2001): “Non-Gaussian Ornstein-Uhlenbeck-based models and some of their uses in economics,” *Journal of the Royal Statistical Society, Series B*, 63, 167–241.
- BARTH, A., AND F. E. BENTH (2014): “The forward dynamics in energy markets – infinite dimensional modeling and simulation,” *Stochastics*, 86, 932–966.
- BENTH, F.-E., J. KALLSEN, AND T. MEYER-BRANDIS (2007): “A non-Gaussian Ornstein-Uhlenbeck process for electricity spot price modeling and derivatives pricing,” *Applied Mathematical Finance*, 14, 153–169.
- BENTH, F. E., C. KLÜPPELBERG, G. MÜLLER, AND L. VOS (2014): “Futures pricing in electricity markets based on stable CARMA spot models,” *Energy Economics*, 44, 392–406.
- BENTH, F. E., AND S. KOEKEBAKKER (2008): “Stochastic modeling of financial electricity contracts,” *Energy Economics*, 30, 1116–1157.
- BENTH, F. E., S. KOEKEBAKKER, AND F. OLLMAR (2007): “Extracting and applying smooth forward curves from average-based commodity contracts with seasonal variation,” *Journal of Derivatives*, 15, 52–66.

- BENTH, F. E., AND P. KRÜHNER (2014): “Representation of infinite-dimensional forward price models in commodity markets,” *Communications in Mathematics and Statistics*, 2, 47–106.
- (2015): “Derivatives pricing in energy markets: an infinite dimensional approach,” *SIAM Journal on Financial Mathematics*, 6, 825–869.
- BENTH, F. E., AND P. KRÜHNER (2015): “Subordination of Hilbert space valued Lévy processes,” *Stochastics*, 87, 458–476.
- BENTH, F. E., AND J. LEMPA (2014): “Optimal portfolios in commodity markets,” *Finance & Stochastics*, 18, 407–430.
- BENTH, F. E., B. RÜDIGER, AND A. SÜSS (2015): “Ornstein-Uhlenbeck processes in Hilbert space with non-Gaussian stochastic volatility,” *Submitted manuscript. Available on arXiv:1506.07245*.
- BENTH, F. E., J. ŠALTYTĖ BENTH, AND S. KOEKEBAKKER (2008): *Stochastic Modelling of Electricity and Related Markets*. World Scientific, Singapore.
- BLÖCHLINGER, L. (2008): *Power prices – a regime-switching spot/forward price model with Kim filter estimation*, Dissertation of the University of St. Gallen, No. 3442.
- BURGER, M., B. GRAEBER, AND G. SCHINDLMAYR (2007): *Managing Energy Risk – An Integrated View on Power and Other Energy Markets*. Wiley.
- CALDANA, R., G. FUSAI, AND A. RONCORONI (2016): “Energy forward curves with thin granularity,” Available at SSRN: abstract=2777990.
- CARTEA, A., AND M. FIGUEROA (2005): “Pricing in electricity markets: a mean reverting jump diffusion model with seasonality,” *Applied Mathematical Finance*, 12, 313–335.
- CLEWLOW, L., AND C. STRICKLAND (2000): *Energy Derivatives: Pricing and Risk Management*. Lacima Publications, London.
- FILIPOVIC, D. (2001): *Consistency Problems for Heath-Jarrow-Morton Interest Rate Models*. Springer Verlag, Berlin Heidelberg.
- FLETEN, S.-E., AND J. LEMMING (2003): “Constructing forward price curves in electricity markets,” *Energy Economics*, 25, 409–424.
- FOLLAND, G. (1984): *Real Analysis*. John Wiley & Sons, New York.

- FRESTAD, D. (2008): “Common and unique factors influencing daily swap returns in the Nordic electricity market, 1997–2005,” *Energy Economics*, 30, 1081–1097.
- FRESTAD, D., F. BENTH, AND S. KOEKEBAKKER (2010): “Modeling term structure dynamics in the Nordic electricity swap market,” *Energy Journal*, 31, 53–86.
- HEATH, D., R. JARROW, AND A. MORTON (1992): “Bond pricing and the term structure of interest rates: a new methodology for contingent claim valuation,” *Econometrica*, 60, 77–105.
- HESTON, S. (1993): “A closed-form solution for options with stochastic volatility with applications to bond and currency options,” *Review of Financial Studies*, 6, 327–343.
- KIESEL, R., AND F. PARASCHIV (2015): “Econometric Analysis of 15-Minute Intraday Electricity Prices,” *University of St.Gallen, School of Finance Research Paper No. 2015/21*.
- KIESEL, R., G. SCHINDLMAYR, AND R. BOERGER (2009): “A two-factor model for the electricity forward market,” *Quantitative Finance*, 9, 279–287.
- KOEKEBAKKER, S., AND F. OLLMAR (2005): “Forward curve dynamics in the Nordic electricity market,” *Managerial Finance*, 31, 73–94.
- LUCIA, J. J., AND E. S. SCHWARTZ (2002): “Electricity prices and power derivatives: evidence from the Nordic power exchange,” *Review of Derivatives Research*, 5, 5–50.
- PARASCHIV, F. (2013): “Price dynamics in electricity markets,” in *Risk Management in Energy Production and Trading*, ed. by P. G. Kovacevic, R.M., and M. Vespucci, pp. 57–111. Springer.
- PARASCHIV, F., D. ERNI, AND R. PIETSCH (2014): “The impact of renewable energies on EEX day-ahead electricity prices,” *Energy Policy*, 73, 196–210.
- PARASCHIV, F., S.-E. FLETEN, AND M. SCHÜRLE (2015): “A spot-forward model for electricity prices with regime shifts,” *Energy Economics*, 47, 142–153.
- PESZAT, S., AND J. ZABCZYK (2007): *Stochastic Partial Differential Equations with Lévy Noise*. Cambridge University Press, Cambridge.
- PIETZ, M. (2009): “Risk premia in electricity wholesale spot markets empirical evidence from Germany,” *Working Paper, Center for Entrepreneurial and Financial Studies. Technical University of Munich*.

- RONCORONI, A., AND P. GUIOTTO (2001): “Theory and Calibration of HJM with Shape Factors,” *Mathematical Finance, Springer Finance*, pp. 407-426.
- WERON, R., AND S. BORAK (2008): “A semiparametric factor model for electricity forward curve dynamics,” *Journal of Energy Markets*, 1, 3–16.
- WERON, R., AND M. ZATOR (2014): “Revisiting the relationship between spot and futures prices in the Nord Pool electricity market,” *Energy Economics*, 44, 178–190.

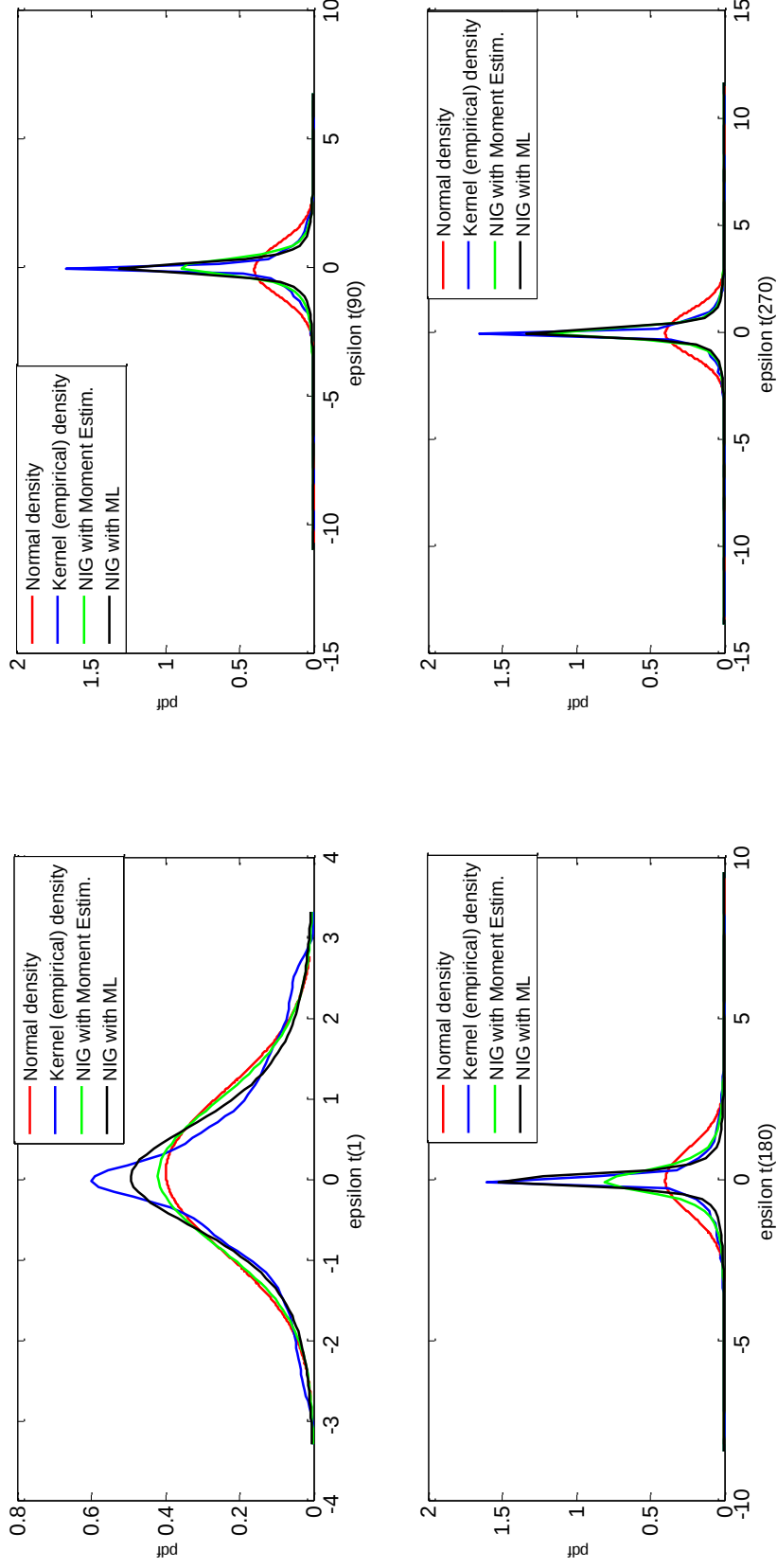


Figure 11: Density plots for $\tilde{\epsilon}_t(x_k)$, where $k \in \{1, 90, 180, 270\}$. The empirical density of the noise time series (blue) is compared to the densities of a normal distribution (red) and to the densities of a NIG distribution fitted to the data based on the moment estimates (green) and maximum likelihood (black)

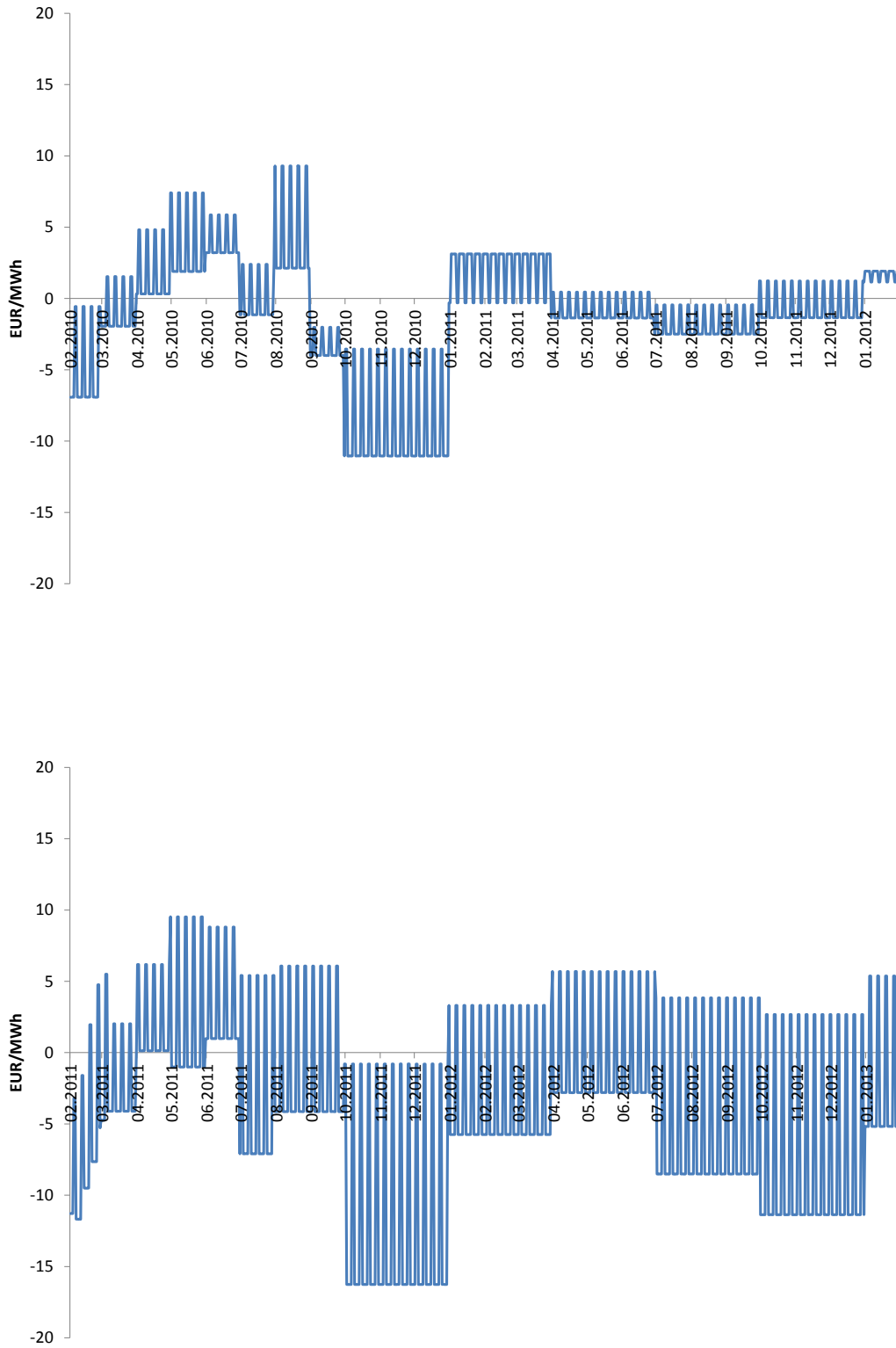


Figure 12: Stochastic component of price forward curves (Equation (2)) generated at 01/02/2010 (above) and 01/02/2011 (below)

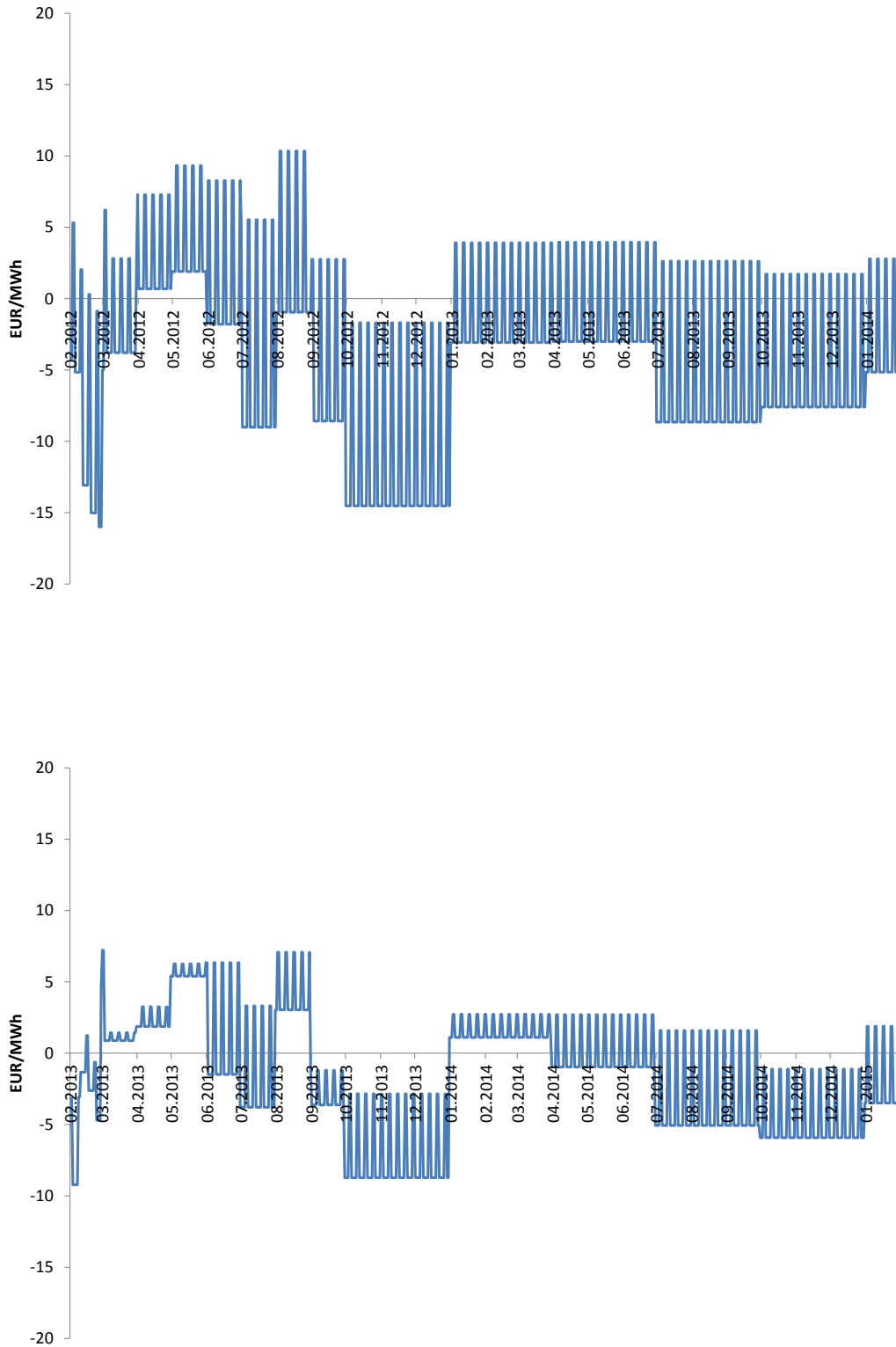


Figure 13: Stochastic component of price forward curves (Equation (2)) generated at 01/02/2012 (above) and 01/02/2013 (below)