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Trading Volume, Illiquidity and Commonalities in FX Markets*

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Abstract

In a regime of floating FX rates and open economies, it is important to understand the way through which FX rates, volatility, and trading volume interrelate. To uncover this, we provide a simple theoretical framework to jointly explore these factors in a multi-currency environment. Through the use of a unique intraday data representative for the global FX market, the empirical analysis validates our theoretical predictions: (i) more disagreement increases FX trading volume, volatility, and illiquidity, (ii) stronger commonalities pertain to more effifficient (arbitrage-free) currencies, and (iii) the Amihud (2002) measure, for which we provide a theoretical underpinning, is effective in measuring FX illiquidity. Not only do these findings support an integrated analysis of FX rate evolution and risk, but our work also offers a straightforward method to measure FX illiquidity and commonality. For investors, these insights should increase the efficiency of trading and risk analysis. For policy makers, our work highlights the developments of FX global volume, volatility, and illiquidity across time and currencies, which can be important for the implementation of monetary policy and financial stability.

Keywords: FX Trading Volume, Volatility, Illiquidity, Commonalities, Arbitrage.

J.E.L. classification: C15, F31, G12, G15

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1 Introduction

Since the demise of the post-war Bretton Woods system in the 1970s, the international financial system has witnessed growing capital mobility and wider movements of foreign exchange (FX) rates. In such a regime of floating FX rates and open economies, those dealing with a currency other than that of the base currency are concerned with the (adverse) evolution of FX rates, their volatility, and market dynamics such as trading volume and illiquidity. It is thus a natural question of how FX rates, volatility, and trading volume interrelate.

In this paper, we provide a simple theoretical framework to jointly explain FX rates, trading volume, and volatility in a multi-currency environment. Linked by triangular no-arbitrage conditions, FX rate movements are determined by common information and differences in traders' reservation prices, or disagreement, that induce trading. In such a unified setting, our model outlines two main drivers within and across currencies: First, investors' disagreement is the common determinant of trading volume and volatility of each FX rate. Second, the no-arbitrage condition is the "glue" across currencies creating commonality in trading volume, volatility, and illiquidity. Our model also provides an intuitive theoretical underpinning for the Amihud illiquidity measure (Amihud, 2002). Using new and unique intraday data representative for the global FX spot market, the empirical analysis validates our main theoretical predictions, that is, (i) more disagreement increases FX trading volume, volatility, and illiquidity, (ii) stronger commonalities pertain to more efficient (arbitrage-free) currencies, and (iii) our illiquidity proxy, that is the Amihud index computed on the basis of high-frequency returns, is effective in measuring FX illiquidity.

The joint analysis of FX volume and volatility is important for at least three reasons. First, the FX market is the world's largest financial market, with a daily traded volumed of USD 6.6 trillion (Bank of International Settlements, 2019). Despite its importance and apparent enormous liquidity, an in-depth understanding of FX volume is still missing. At least two reasons can explain this. On the one hand, FX rates are commonly traded over-the-counter, which is notoriously opaque and fragmented. The microstructure of the FX market is explained in detail in Lyons (2001), and King et al. (2012). The recent developments of the FX markets are discussed in Rime and Schrimpf (2013), and Moore et al. (2016). On the other hand, there has been a lack of comprehensive volume data on a global scale. Second, FX rates are key for pricing many assets, including international stocks, bonds, and derivatives, and for assessing their risk. They are also relevant for policymaking such as conducting (unconventional) monetary policy and FX interventions. A better understanding of whether and how FX volume, volatility and illiquidity determine FX rates can improve all these tasks. Third, distressed markets such as currency crises are characterized by sudden FX rates movements, drops in liquidity, and raises in volatility. It could thus be supportive of financial stability to highlight the sources of volatility and illiquidity, how they reinforce each other, and across currencies.

Our analysis proceeds in two steps: theory and empirics. Our theory builds upon an equilibrium model in which the evolution of the FX rate is driven by the arrival of new information and by the trading activity. The trading volume is induced by the deviation of an individual agent's reservation prices from the observed market price. The continuous-time feature of the model allows us to obtain consistent measurements of the underlying unobservable quantities, such as volatility and illiquidity, and to relate them to the trading volume. Furthermore, agents trade in a multi-currency environment in which direct FX rates are tied to cross rates by triangular no-arbitrage conditions, implying that direct and arbitrage-related (or synthetic) rates must equate in equilibrium, while the trading volume reflects the dependence on the aggregated information flows across FX rates. Thus, trading volume is the driving force processing information and reservation prices in currency values and attracting FX rates to arbitrage-free prices.

Three propositions arise from our theoretical framework: First, trading volume and volatility are driven by traders' disagreement. Second, the combination of volatility and volume provides an intuitive closed-form solution for measuring illiquidity by means of the high-frequency version of the Amihud (2002) index. Third, trading volume, volatility and liquidity across FX rates are linked by no-arbitrage conditions, which lead to the commonalities across FX rates. Since arbitrage passes through the trading activity (volume), more liquid currencies reveal stronger commonalities and price efficiency (in terms of smaller deviations from triangular arbitrage condition). Recalling the market adage that "liquidity begets liquidity" (see, e.g., Foucault et al., 2013), our theory puts forward that liquidity begets liquidity across currencies (commonality) and it also begets price efficiency (no-arbitrage).

Set against this background, we test the main empirical predictions derived from our theory. To do this, we utilize three data sets. First, trading volume data come from CLS Bank International (CLS), which operates the largest payment-versus-payment (PVP) settlement service in the world. Hasbrouck and Levich (2017) provide a very comprehensive description of the CLS institutional setting and Gargano et al. (2019) show that CLS data cover around 50% of the FX global turnover compared to the BIS triennial surveys. Trading volume is measured at the hourly level, across 29 currency pairs over a 5-year period from November 2011 to November 2016. For the same FX panel, we obtain intraday spot rates from Olsen data. For each FX rate and each minute of our sample, we observe the following quotes: ask, bid, low, high, close, and mid price. By merging these two data sets, we can analyze the hourly/daily/weekly/monthly time series of trading volume, volatility, and FX rate and bid-ask spread evolutions. Third, we utilize EBS data as it provides ultra high-frequency order and trade data to validate our illiquidity measure, to compare the broad OTC FX market (featured by CLS data) and with its interdealer segment (EBS), as well as to analyze market dynamics surrounding some representative events.

To test the empirical predictions, we carry out the following analysis. First, we perform a descriptive analysis that uncovers some (new) stylized facts. For instance, we find that FX trading volume and illiquidity follow common intraday patterns and seasonalities indicating market fragmentation across geographical areas and FX rates consistent with the OTC nature of the FX global market. Then, we perform various regressions to test the three above-mentioned theoretical propositions. Three main results emerge: First, trading volume and volatility are linked by a very strong positive relationship both within and across FX rates. To provide more direct evidence that both are governed by disagreement between the agents, we show that volume and volatility increase with heterogeneous beliefs as measured in Beber et al. (2010). Second, we provide evidence that our illiquidity measure is effective in capturing FX illiquidity episodes and correlate with well-accepted high-frequency and low-frequency measures of FX illiquidity. Finally, we perform a comprehensive analysis of commonalities in FX volume, volatility, and illiquidity. To do this, we employ three methodologies: principal component analysis, linear regressions for triangular arbitrage relations, and the connectedness index of Diebold and Yilmaz (2014). All of them deliver a consistent picture that can be summarized in three main findings: First, there is a strong commonality across FX volumes and volatilities. Second, more liquid currencies have stronger commonalities, that is, liquidity begets liquidity across currencies. Third, more liquidity currencies obey more to (triangular) arbitrage conditions, that is, liquidity begets price efficiency. This holds true when the FX market reacts to directional FX movements as measured by co-jumps (Caporin et al., 2017) that presumably capture new common information such as ”big news” and macroeconomic announcements (see, e.g. Bollerslev et al., 2018 and Perraudin and Vitale, 1996) associated with little disagreement.

Our paper contributes to two strands of the literature: First, we contribute to prior research on trading and liquidity in financial markets. While most previous studies on volume have mainly focused on stocks (see, e.g., Vayanos and Wang (2013) for a recent literature survey), there is a growing literature on trading and liquidity in FX markets (see, e.g., Mancini et al., 2013 and Karnaukh et al., 2015). Most previous studies focus on specific aspects of FX liquidity such as transaction costs[1] or order flow, which is as the net of buyer-initiated and seller-initiated orders. Following the seminal paper by Evans and Lyons (2002), order flow has drawn much attention as the main determinant of FX rate formation. Among others, order flow is studied in Bjønnes and Rime (2005), Berger et al. (2008), Frömmel et al. (2008), Breedon and Ranaldo (2013), Evans and Lyons (2002), Evans (2002), Mancini et al. (2013), Payne (2003), and Rime et al. (2010). In contrast, the literature on trading volume is scant due to the paucity of comprehensive data on the FX global volume. Prior research has focused on the interdealer segment in which Electronic Broking Services (EBS) and Reuters are the two predominant platforms. For instance, Evans (2002) uses Reuters D2000-1 data, Payne (2003) analyze data from D2000-2 while Mancini et al. (2013), and Chaboud et al. (2007) utilize data from EBS. Other sources of trading volume data are proprietary data sets from some specific banks (see, e.g., Bjønnes and Rime, 2005; Menkhoff et al., 2016), central banks, or FX futures or forward contracts (see, e.g,. Bjønnes et al., 2003; Galati et al., 2007; Grammatikos and Saunders, 1986; Levich, 2012; Bech, 2012).

[1]Transaction costs are typically measured in terms of bid-ask spreads that tend to increase with volatility. FX transaction costs in spot and future markets are studied in Bessembinder (1994), Bollerslev and Melvin (1994), Christiansen et al. (2011), Ding (1999), Hartmann (1999), Huang and Masulis (1999), Hsieh and Kleidon (1996), and Mancini et al. (2013).

Only with the recent access to CLS data, research on FX global volume at relatively high frequencies (e.g. daily) became possible. Except for CLS, the only source of global FX trading volume is the triennial survey of central banks conducted by the BIS. It provides a snapshot of FX market volume on a given day once every three-years. Fischer and Ranaldo (2011) look at global FX trading around central bank decisions. Hasbrouck and Levich (2017) measure FX illiquidity using volume and volatility data. Gargano et al. (2019) analyze the profitability of FX trading strategies exploiting the predictive ability of FX volume. Theoretically, we add to the extant literature by building a continuous-time model in a multiple-currency setting, which serves the purpose of defining a theoretical foundation for FX price determination in connection to FX volume, volatility, and illiquidity. Although simple and abstracting from market "imperfections", our model provides a closed-form and intuitive expression for our illiquidity measure inspired by Amihud (2002). Empirically, we are the first providing a joint analysis of intraday FX global volume, (realized) volatilities, and illiquidity that supports two empirical predictions from our theory: First, disagreement drives trading volume and volatility; Second, our FX measure in the spirit of Amihud (2002) is effective in measuring FX illiquidity.

Second, we contribute to the literature on commonalities in liquidity, which has extensively studied liquidity co-movements of stocks (see, e.g., Chordia et al., 2000; Hasbrouck and Seppi, 2001; Karolyi et al., 2012). In FX markets, this issue is empirically analyzed in Mancini et al. (2013), and Karnaukh et al. (2015). Prior research has provided some theoretical explanations for liquidity commonality. For instance, when dealers are active in two markets (or assets), they tend to reduce their liquidity supply in case of trading losses (Kyle and Xiong, 2001) or underfunding constraints (Cespa and Foucault, 2014). From an asset pricing perspective, investors require higher expected returns and invest less in assets exposed to liquidity risk (see, e.g., Acharya and Pedersen, 2005); additionally, illiquidity and low asset prices might endogenously result from erosion of arbitrageurs' wealth (Kondor and Vayanos, 2019). We contribute to the literature by modeling arbitrage and disagreement as the sources of commonality in trading *volume*, volatility and illiquidity, and providing empirical evidence that more liquid currencies have stronger commonalities that in turn, maintain FX rates more tied to arbitrage values.

This paper is organized as follows: Section 2 presents the simple theoretical setting for an unified analysis of volatility, volume and illiquidity on the FX rates, and their commonalities. Section 3 introduces the dataset and discusses summary statistics. Section 4 presents the empirical analysis. Finally, Section 5 concludes the paper.

2 A unified model for FX rates, volatility and volume

Let's first consider a world with two currencies, x (base) and y (quote). We assume that the market consists of a finite number $J \geq 2$ of active participants, who trade on the FX rate $x|y$. Within a given trading period of unit length (e.g. an hour, a day, a week), the market for the currency pair $x|y$ passes through a sequence of $i = 1, \ldots I$ equilibria. The evolution of the equilibrium price is motivated by the arrival of new information to the market. At intra-period i, the desired position of the j-th trader ($j = 1, \ldots, J$) on the FX rate $x|y$ is given by

$$q_{i,j}^{x|y} = \xi^{x|y}(p_{i,j}^{x|y;*} - p_i^{x|y}), \quad \xi^{x|y} > 0, \tag{1}$$

where $p_{i,j}^{x|y;*}$ is the reservation price of the j-th trader and $p_i^{x|y}$ is the current market price (both measured in logs). The equilibrium function in (1) is analogous to the baseline equation in the Mixture-of-Distribution Hypothesis (MDH) theory of Clark (1973), and Tauchen and Pitts (1983), which provides a stylized representation of the supply/demand mechanism on the market at the intraday level.[2] The reservation price of each trader might reflect some of the following: individual preferences, liquidity issues, asymmetries in information sets and different expectations about the fundamental values of the FX rate. In general, the reservation price can deviate from the market price because of idiosyncratic reasons inducing the j-th trader to trade.

The term $\xi^{x|y}$ is a positive constant capturing the market depth: The larger $\xi^{x|y}$, the larger quantities of x can be exchanged for y (and viceversa) for a given difference $p_{i,j}^{x|y;*} - p_i^{x|y}$. In other words, $\xi^{x|y}$ measures the capacity of the market to allow large quantities to be exchanged at the intersection between demand and supply, thus recalling the concept of resilience. Figure 1 illustrates the demand/supply mechanism of the j-th trader for the $x|y$ FX rates. If $(p_{i,j}^{x|y;*} - p_i^{x|y}) > 0$, this means that the j-th trader believes that the equilibrium trading price of $x|y$ is too low, i.e. currency x should be more expansive relatively to y, so he will buy x and sell y. On the contrary, if $(p_{i,j}^{x|y;*} - p_i^{x|y}) < 0$, the j-th trader will buy y and sell x. The quantity exchanged for a unit change of $p_{i,j}^{x|y;*} - p_i^{x|y}$ is given by the slope $\xi^{x|y}$.

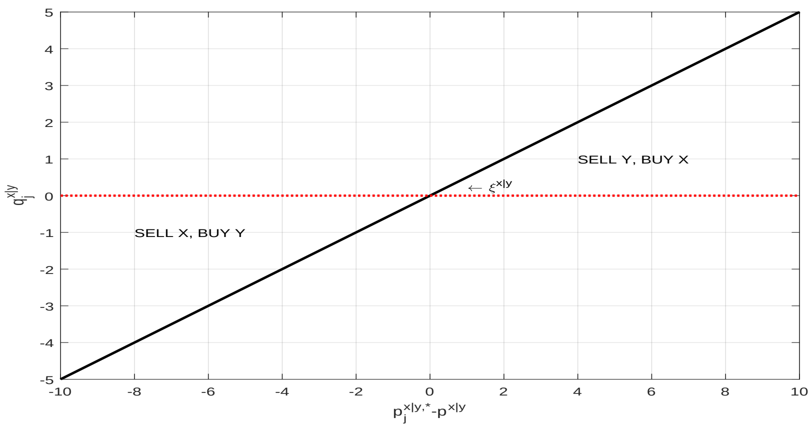


Figure 1: Trading function for the j-th trader for $x|y$ with $\xi^{x|y} = 0.5$.

The baseline assumptions of the model (linearity of the trading function and constant number of active traders) are inevitably very stylized. As for the form of the equilibrium function in (1), note that the trades take place on short intraday intervals of length $\delta = 1/I$ and they are

[2] See also the empirical analysis in Andersen (1996) and the survey in Karpoff (1987). According to Bauwens et al. (2006) only one out of the 19 studies of MDH is on exchange rates.

generally associated with small price variations. Therefore, it is not restrictive to assume the equilibrium function to be linear on small price changes. FX rates can be dispersed and heterogeneous outside the interdealer segment, as emphasized by Evans and Rime (2016). Hence, it can be argued that the assumption of J active traders observing a market price recalls more a centralized market rather than a fragmented one. Since the market clearing condition must hold, that is $\sum_j q_{i,j}^{x|y} = 0$, it follows that the average of the reservation prices clears the market, that is $p_i^{x|y} = \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{x|y;*}$, and the log-return is $r_i^{x|y} := \Delta p_i^{x|y} = p_i^{x|y} - p_{i-1}^{x|y}$. Furthermore, as new information arrives, the traders adjust their reservation prices, $\Delta p_{i,j}^{x|y;*}$, resulting in a change in the market price given by the average of the increments of the reservation prices. The generated trading volume in each i-th sub-interval is therefore

$$v_i^{x|y} = \frac{\xi^{x|y}}{2}\sum_{j=1}^{J} |\Delta p_{i,j}^{x|y;*} - \Delta p_i^{x|y}|,$$

where $\Delta p_{i,j}^{x|y;*} = p_{i,j}^{x|y;*} - p_{i-1,j}^{x|y;*}$. We assume that the reservation price about the $x|y$ FX rate is driven by the following law of motion

$$dp_j^{x|y;*}(t) = \mu_j^{x|y}(t)dt + \sigma_j^{x|y}(t)dW_j^{x|y}(t), \quad j = 1, \dots, J \tag{2}$$

where $W_j(t)$ is a Wiener process that is independent between each trader, i.e. $W_l(t) \perp\!\!\!\perp W_m(t)$ $\forall l \neq m$ and the term $\sigma_j^{x|y}(t) \geq 0$ is the stochastic volatility process of the j-th trader which is assumed to have locally square integrable sample paths. The term $\mu_j(t)$ is a predictable and finite variation drift process, which might represent the long-run expectation of the j-th trader about the FX rate and it could be function of fundamental quantities like interest rates differentials and long-term macroeconomic views.

By allowing $\sigma_j^{x|y}$ to be different across traders, we are implicitly introducing heterogeneity among them. This also reconciles with many realistic features including the evidence of long-memory in volatility that is obtained by the superposition of traders operating at different frequencies, see for instance the heterogeneous autoregressive model of Müller et al. (1997), and Corsi (2009). This setup is coherent with a representation of a frictionless market where each trader participates through its reservation price to the determination of a new equilibrium price by carrying new information. Consider an interval of unit length, e.g. an hour, a day or a month. For ease of exposition and tractability, we assume that trades happen on an equally spaced and uniform grid, $i = 1, 2, \dots I$. On the i-th discrete sub-interval of length $\delta = \frac{1}{I}$, we have that

$$p_{i,j}^{x|y;*} = \int_{\delta(i-1)}^{\delta i} \mu_j(s)ds + \int_{\delta(i-1)}^{\delta i} \sigma_j^{x|y}(s)dW_j^{x|y}(s). \tag{3}$$

Proposition 1. *Over an interval of unit length, the trading volume, $v = \sum_{i=1}^{I} v_i^{x|y}$, and volatility, as measured by the square root of the realized variance, $RV^{x|y} = \sum_{i=1}^{I}\left(r_i^{x|y}\right)^2$, or by the realized* *power variation of Barndorff-Nielsen and Shephard, 2003, $RPV^{x|y} = \sum_{i=1}^{I} |r_i^{x|y}|$, carry information about the investor disagreement on a given FX rate.*

Proof in Appendix A.1.

Resorting to the continuous-time framework outlined in Barndorff-Nielsen and Shephard (2002b,a), we can precisely measure the variability of the FX rates by computing RPV (or RV) over intervals of any length (e.g. hours, days, weeks and months). Furthermore, the equilibrium theory presented above allows us to relate this variability to the aggregated level of disagreement among investors, which in turns leads to the observed trading volume. It should be stressed that the asymptotic results (in the limit for $I \to \infty$) behind Proposition 1 are derived by abstracting from microstructural frictions (namely *microstructure noise*), like transaction costs in the form of bid-ask spread, clearing fees or price discreteness, which are intimately related and endogenous to the trading process; see the recent works of Darolles et al. (2015, 2017) for an extension of a reduced-form version of the MHD with liquidity frictions. In the simplified context provided by the model, the microstructural features linked to the actual cost of trading in the form of bid-ask spread are not explicitly included but they pose essentially empirical issues, which will be appropriately tackled in the empirical analysis. For instance, as it is common in the literature on volatility measurement (see, e.g., Bandi and Russell, 2008; Liu et al., 2015) in the following analysis we will work under the maintained assumption that sampling at 5-minute intervals is sufficient to guarantee that a new equilibrium price is determined. The latter is representative of the aggregated information contained on the demand and supply sides of the market. From a statistical point of view, the microstructure noise dominates over the volatility signal as $I \to \infty$, thus leading to distorted measurements of the variance. However, over moderate sampling frequencies, e.g. 5-minute intervals over 24 hours ($I = 288$), the prices and quantities determined in equilibrium in each sub-interval can be considered (almost) free of microstructure noise contamination, and representative of new equilibria. Rather than microstructural features, our setting emphasizes the role of the aggregated disagreement on fundamentals leading to the definition of a new price equilibrium, in which each trader participates in proportion to the information contained in her new reservation prices. Next section shows how this theory can be successfully adopted as an encompassing framework to characterize the illiquidity and the commonalities in volatility and volume on the global FX markets.

2.1 Measuring FX Illiquidity

In light of Proposition 1 and analogously to the illiquidity proxy in Amihud (2002), we can define a continuous-time version of the illiquidity index, called the *high-frequency (HF) Amihud index* as

$$A^{x|y} := \frac{RPV^{x|y}}{v^{x|y}}, \tag{4}$$

which is a measure of the price impact, that gives the amount of volatility on a unit interval (as measured by $RPV^{x|y} = \sum_{i=1}^{I} |r_i^{x|y}|$) associated with the trading volume $v^{x|y} = \sum_{i=1}^{I} v_i^{x|y}$ generated in the same period. In other words, $A^{x|y}$ measures the amount of FX volatility associated with a unit of trading volume. The following proposition highlights the main determinants of this illiquidity measure.

Proposition 2. *Consider the illiquidity measure defined in (4). In the limit for $I \to \infty$ and under homogeneity of traders, i.e. $\sigma_j^{x|y} = \sigma^{x|y} \quad \forall j = 1, 2, \ldots, J$,*

$$p \lim_{I\to\infty} A^{x|y} = \frac{2}{\xi^{x|y} J \sqrt{(J-1)}}. \tag{5}$$

Proof in Appendix A.2.

Proposition 2 shows that on a generic period of unit length (an hour, a day, a month), $A^{x|y}$ is inversely related to the slope, $\xi^{x|y}$, of the equilibrium function in (1). That is, for a given difference between the reservation price and the market price, $A^{x|y}$ decreases as this slope increases. In particular, for large values of $\xi^{x|y}$ large volume would be associated with small variations between the prevailing price and the reservation price for each trader, thus signaling market depth and liquidity. Instead, when $\xi^{x|y} \to 0^+$, i.e. in the limiting case of a flat equilibrium function in (1), the liquidity is minimal (and $A^{x|y}$ diverges), since no actual trade takes place. Under the assumption of homogeneity of the traders, $\sigma_j^2(t) = \sigma^2(t)\ \forall j = 1, \ldots, J$, Proposition 2 also highlights the inverse relationship between the number of active traders on the market and illiquidity.[3] Thanks to the theory of realized variance coupled with the availability of high-frequency data, we are able to construct a precise measure of illiquidity, which we can relate to the structural parameters of the model.

2.2 Commonalities in FX volume and volatility

In this section, we derive equilibrium relations between returns, trading volumes and volatilities *across* different FX rates. These relations are instrumental to the interpretation of commonalities in trading volumes and volatilities as well as information processing in global FX markets. Let's therefore consider a world with three currencies, x , y and z. The market for the currency pairs $x|y$, $x|z$ and $z|y$ also passes through a sequence of $i = 1, \ldots I$ equilibria and the evolution of the equilibrium price of each currency pair is motivated by the arrival of new information to the market. By the triangular no-arbitrage parity it must hold that

$$p_i^{x|y} = p_i^{x|z} + p_i^{z|y}, \tag{6}$$

where $p_i^{x|z} = \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{x|z;*}$ and $p_i^{z|y} = \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{z|y;*}$. Hence, the *synthetic* return on $x|y$ results to be

$$\widetilde{r}_i^{x|y;z} = \frac{1}{J}\sum_{j=1}^{J} \Delta p_{i,j}^{x|z;*} + \frac{1}{J}\sum_{j=1}^{J} \Delta p_{i,j}^{z|y;*}. \tag{7}$$

[3]Relaxing the assumption of homogeneity would result in the ratio of two aggregated volatility measures, each estimating the weighted average of the variance carried by each trader, see equation (26) in Appendix A.2.

Hence, by imposing the triangular no-arbitrage relation on the FX rates, which prescribes that $r_i^{x|y} = \tilde{r}_i^{x|y;z}$, it follows that

$$\frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{x|y;*} = \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{x|z;*} + \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{z|y;*}. \tag{8}$$

This means that the average of the traders' reservation prices on $x|y$ must be equal to the sum of the average traders' reservation prices of $z|y$ and $x|z$. This also means that each trader can take a direct position on $x|y$ or operate on the synthetic rate by forming independent beliefs on $x|z$ and $z|y$, thus generating trading volume on each individual FX rate.

Proposition 3. *Trading volume, volatility and liquidity across FX rates are linked by no-arbitrage constraints, which lead to the commonalities across FX rates.*

- *The synthetic volatility (as measured by the square root of* $\widetilde{RV}^{x|y;z} = \sum_{i=1}^{I}(\tilde{r}_i^{x|y;z})^2$*, or by the realized power variation,* $\widetilde{RPV}^{x|y;z} = \sum_{i=1}^{I}|\tilde{r}_i^{x|y;z}|$*), and the synthetic volume, defined as*

$$\tilde{v}_i^{x|y;z} = \frac{\xi^{x|y}}{2}\sum_{j=1}^{J}|\Delta p_{i,j}^{x|z;*} - \Delta p_i^{x|z} + \Delta p_{i,j}^{z|y;*} - \Delta p_i^{z|y}|, \tag{9}$$

  are functions of the traders' disagreement on $x|z$ *and* $z|y$ *and they are proportional to the strength of the correlation between these FX rates.*

- *Both the synthetic volume and the synthetic illiquidity, defined as* $\tilde{A}^{x|y;z} = \frac{\widetilde{RPV}^{x|y;z}}{\tilde{v}^{x|y;z}}$*, are positively related to the variability of the arbitrage price violations, as measured by* $RPVE^{x|y;z} = \sum_{i=1}^{I}|pe_i^{x|y;z}|$*, where* $pe_i^{x|y;z} = \tilde{r}_i^{x|y;z} - r_i^{x|y}$*.*

Proof in Appendix A.3.

Proposition 3 introduces the concept of *synthetic volatility* and *synthetic volume*, which are associated with the no-arbitrage equilibrium constraints and depend on the extent of the individual disagreement on the FX rates of $x|z$ and $z|y$. Both synthetic volatility and volume are functions of the aggregated correlation in beliefs between $x|z$ and $z|y$, and hence the expression of the commonalities in the global FX rates. For a given level of traders' disagreement on $x|y$ (leading to trading volume on $x|y$), we can measure the associated synthetic volume on $x|z$ and $z|y$, which is proportional to the correlation between the aggregated reservation prices on $x|z$ and $z|y$. The same is true for the synthetic volatility, as measured by the realized variance of the synthetic return.

3 Data and preliminary analysis

3.1 Data sets

Our empirical analysis relies on three data sets. First, trading volume data come from CLS, which is the largest payment system for the settlement of foreign exchange transactions. Through a payment-versus-payment mechanism, this infrastructure supports FX trading by reducing settlement risk and supporting market efficiency. For each hour of our sample period and each currency pair, we observe the settlement value and the number of settlement instructions. Our data set covers 29 currency pairs (15 currencies) over the period from November 1, 2011 to October 19, 2016, for a total of 1280 trading days and 30720 hourly observations for each FX rate.[4] Following the literature (see, e.g., Mancini et al., 2013), we exclude observations between Friday 10 p.m. and Sunday 10 p.m. since only minimal trading activity is observed during these non-standard hours.[5] In 2017, the core of CLS was composed of 60 settlement members including the top ten FX global dealers, and thousands of third parties (other banks, non-bank financial institutions, multinational corporations and funds), which are customers of settlement members. The total average daily traded volume submitted to CLS was more than USD 1.5 trillion, which is around 30% of the total daily volume recorded in the last available BIS triennial survey (Bank of International Settlements 2016). However, after adjusting for the large fraction of BIS volume originating from interbank trading across desks and double-counted prime brokered "give-up" trades, the CLS data should cover about 50% of the FX market (Gargano et al., 2019; Hasbrouck and Levich, 2017).

In our study, we focus on FX spot transactions. Except for some exceptions such as the Renminbi, the CLS spot FX rates in our sample are highly representative of the entire FX market. For instance, the currency pairs involving USD and EUR cover more than 85% (94%) of the total trading volume of the BIS triennial survey. To the best of our knowledge, only a few papers have analyzed CLS volume data so far. First, Fischer and Ranaldo (2011) study five aggregated currencies (e.g. all CLS-eligible currencies against the U.S. dollar, Euro, Yen, Sterling, and Swiss franc) rather than currency pairs. Hasbrouck and Levich (2017) analyze every CLS settlement instruction during April 2013. Gargano et al. (2019) use the same dataset to perform an asset pricing analysis. Ranaldo and Somogyi (2019) analyze the heterogeneous price impact of CLS order flows decomposed by market participants.

The second data set is obtained from Olsen Financial Technologies, which is the standard source for academic research on intraday FX rates. By compiling historical tick data from the

[4]The full dataset contains data for 18 major currencies and 33 currency pairs. To maintain a balanced panel, we exclude the Hungarian forint (HUF), which enters the dataset only on 07 November 2015. Moreover, we discard USDILS and USDKRW due to very infrequent trades. The remaining 29 currency pairs are: AUDJPY, AUDNZD, AUDUSD, CADJPY, EURAUD, EURCAD, EURCHF, EURDKK, EURGBP, EURJPY, EURNOK, EURSEK, EURUSD, GBPAUD, GBPCAD, GBPCHF, GBPJPY, GBPUSD, NZDUSD, USDCAD, USDCHF, USDDKK, USDHKD, USDJPY, USDMXN, USDNOK, USDSEK, USDSGD, and USDZAR.

[5]In this paper, times are expressed in GMT.

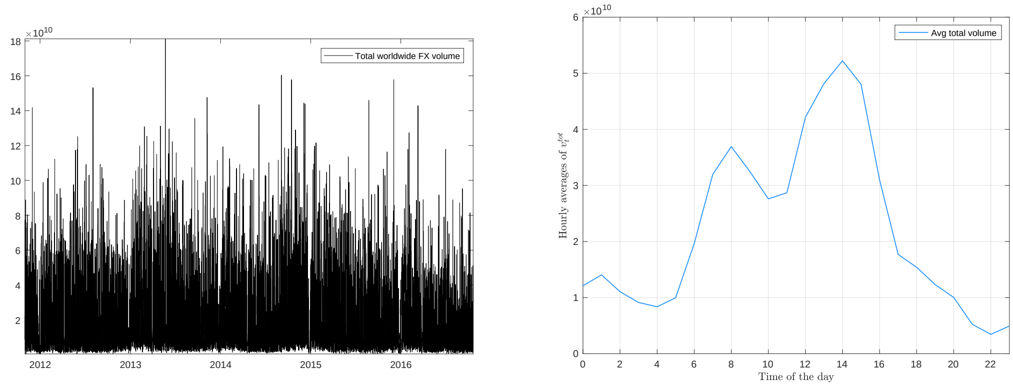
main consolidators such as Reuters, Knight Ridder, GTIS and Tenfore, Olsen data are representative of the entire FX spot market rather than specific segments such as the interdealer FX market dominated by two electronic limit order markets: EBS and Reuters. For each minute of our sample period and each currency pair, we observe the following quotes: bid, ask, high, low, and mid-prices. With these data at hand, we can analyze at least four aspects of FX rates: (i) the FX rate movements at one minute or lower frequencies; (ii) the realized volatility, realized power variation or other measures of return dispersion; (iii) the quoted bid-ask spread as a measure of transaction cost; and (iv) violations of triangular arbitrage conditions.

The third data set is obtained from Electronic Broking Services (EBS), which is the major interdealer trading platform for many currencies including USDEUR, USDJPY, and EURCHF that are studied more extensively in this paper.[6] All our CLS FX rates are also traded in the EBS, which operates an order-driven electronic trading system that unites buyers and sellers of spot FX around the globe on a pre-trade anonymous central limit order book. We access trade and order data for the entire 2016 year. The EBS data set is organized in a time-slice basis, that is, at the end of each 100-millisecond, we observe the total amount of trades, either buys or sells during the time slice interval. Notice that EBS provides the exact identification if a trade is buyer- or seller-initiated. In addition, we trade price and volume (in millions of base currency). About order data, we observe the ten best bid and offer (or ask) quotes capturing the depth of the book.

3.2 Descriptive analysis

In this subsection, we highlight some (new) stylized facts characterizing the times series of volume, volatilities and illiquidity measures associated with the 29 FX rates under investigation. First, we look into intraday patterns, and then we study the daily time series of FX volume, volatility, and illiquidity. To start with the intraday analysis, Figure 2 displays the total hourly volume series, denoted as $v_t^{tot} = \sum_{l=1}^{L} v_t^l$, where v_t^l is the hourly volume on the l-th FX rate. Panel a) of Figure 2 highlights the size and deepness of the FX market, with an average of around 20 billion USD traded every hour. Moreover, the series of total volume is rather persistent and it clearly displays cyclical patterns, which can be associated with strong intradaily seasonality. We explicitly account for the intradaily patterns assuming $\log(v_t) = s_t\beta + \epsilon_t$, where s_t contains hourly and day-of the-week dummies, so that the *filtered* volume is obtained as $v_t^f = \frac{v_t}{e^{s_t\hat{\beta}}}$. Panel b) of Figure 2 reports the estimates of the hourly averages of the total global volume. The plot highlights that the average total volume is higher during the opening hours of the European and American stock markets, while it is very low between 10 p.m. and 12 a.m. as most of the largest stock markets are closed, while it has a relative peak associated with the opening of Tokyo (2 a.m.). Moreover, the total volume is the largest on average between 3 p.m. and 4 p.m., i.e. before the WMR Fix, for which there is a well-documented literature about the large traders

[6] The other main interdealer platform is Thomson Reuters. Some FX rates e.g. involving the British pound are mainly traded on it.



(a) Time series of v_t^{tot}

(b) Hourly averages of v_t^{tot}

Figure 2: Time series of the total global volume and its hourly averages.

submitting a rush of orders before the setting of the daily benchmarks for FX prices (see, e.g., Marsh et al. (2017); Evans (2018)).

Turning the attention to individual FX rates, Figure 3 reports the hourly average share of the total volume of the five most liquid FX rates (by volume size). Firstly, as expected all the most liquid FX rates involve the USD as either base or quote currency. As for the total volume, the trading volume of the most liquid FX rates displays clear (intraday) seasonal patterns. For the individual FX rates, these patterns are suggestive of local effects in given geographical areas, coherent with the OTC segmented nature of FX markets. For instance, USDJPY covers around 30% of the total FX volume between 12 p.m. and 4 p.m., that are the hours in which Far East markets are open. AUDUSD contributes with a 15% in the same hours, while its market share strongly declines to 7% during the central hours of the day. EURUSD is by far the most traded FX rate, with a share above 30% between 7 a.m. to 6 p.m.. A similar pattern characterizes also GBPUSD with an average share ranging between 5% and 10%. Finally, USDCAD is mostly traded at the opening of the business hours in North America, i.e. between 12 p.m. and 10 p.m., with approximately 10% share on the total volume. These five FX rates amount to a share of more than 70% of the total global volume in every hour. In summary, the seasonal patterns are clearly discernible in two dimensions. First, on an intraday scale, the trading volume follows the working time in each country or jurisdiction defining the currency pair. This means that round-the-clock, the trading volume of New Zeeland dollar is the first to increase, followed by Asian, European, and American currencies. Second, official banking holidays clearly reduce trading activity. The seasonalities and calendar effects will be carefully considered in our empirical analysis.

Concerning the relationship between volatility and volume, Figure 4 shows that the averages of hourly RPV and hourly volume for USDEUR and USDJPY follow the same patterns. Two considerations arise: First, at the intraday level when the volatility on the FX rates is high, also the volume is high, which points to a greater variation in traders' reservation prices. Thus,

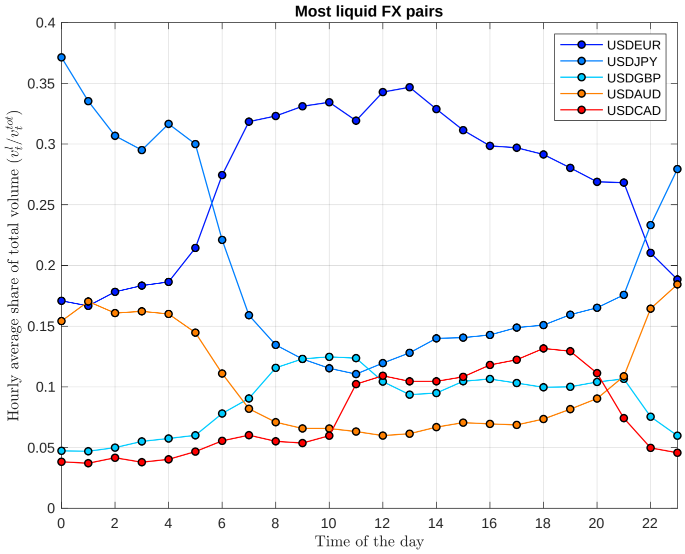


Figure 3: Averages of hourly volume (relative to the total hourly volume) of the five most liquid FX rates, which are (in order) USDEUR, USDJPY, USDGBP, USDAUD, USDCAD.

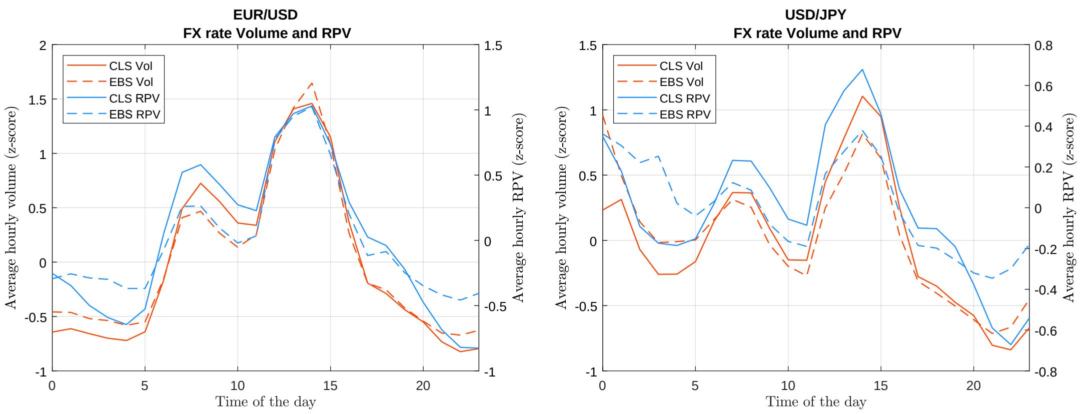


Figure 4: Averages of hourly RPV and hourly trading volume. Hourly RPV and volume are based on the sum of absolute 5-minutes returns and trading volume in each hour, respectively. In Panel a) USDEUR, in Panel b) USDJPY. Intraday patterns based on CLS and EBS data are depicted with continuous and dashed lines, respectively.

Figure 4 provides *prima facie* evidence to Proposition 1 in our theoretical setting: volatility and volume are mostly governed by a common latent factor, which depends on the level of heterogeneous beliefs (disagreement) between the agents. Second, the comparison between CLS data (continuous lines) and EBS data (dashed lines) in Figure 4 suggests that the FX global market and the FX interdealer segment follow the same systematic patterns both in terms of trading volume and volatility.

Before performing the empirical analysis, we examine how daily changes in trading volume correlate with daily changes in volatility (as measured by daily RPV) and other factors that proved to explain FX liquidity in the previous literature (see, e.g., Mancini et al., 2013; Karnaukh et al., 2015) and trading activity in stock markets (see, e.g., Chordia et al., 2001). Although some of these variables are likely to be mutually endogenous, attention is directed towards documenting novel correlation patterns pertaining to FX trading volume rather than causation. To this purpose, the 29 currency pairs are pooled together to examine whether daily FX volume is linked to changes in overall market conditions and its liquidity. More precisely, we consider the following linear regression model for daily trading volume

$$\Delta v_t = \beta_0 + \beta_1' \Delta x_t + \beta_2 \gamma_t + \beta_3' \delta_t + \beta_4 \Delta v_{t-1} + \varepsilon_t, \tag{10}$$

where x_t is a vector of regressors subsuming daily (realized) volatility, relative bid-ask spread (BAS), a widespread proxy of funding strains (TED spread, i.e. the yield spread between the U.S. three-month Libor and T-bills) and a proxy for uncertainty (FX VIX, i.e. the JP Morgan Global FX implied volatility index). Notice that as an uncertainty measure, the FX VIX should covariate positively with disagreement (see, e.g., Söderlind, 2009).[7] Further, γ_t denotes a dollar appreciation dummy and δ_t includes day-of-the-week dummies. The same regression is then estimated for different dependent variables: volatility (RPV), the high-frequency Amihud illiquidity measure, and the relative bid-ask spread (BAS). All regressors, besides the dummy variables, enter in log-differences and all models include the lagged dependent variable.

Some novel patterns emerge from the analysis reported in Table 1. First, FX trading volume increases with RPV and TED spread, whereas it decreases with the BAS. Hence, the common (intraday) volume-volatility pattern showed in Figure 4 also arises in the temporal evolution of trading volume and volatility. Second, trading volume increases with FX implied volatility (FX VIX) supporting the idea that trading volume increases with disagreement.[8] Third, trading volume follows an inverted U-shape relation across weekdays, that is, larger trading volumes tend to occur in the middle of the week. Fourth, as expected volatility increases with bid-ask spreads and tends to be lower when the U.S. dollar appreciates, possibly due to its status as international currency reserve and safe haven against several currencies (see, e.g., Ranaldo and Söderlind, 2010; Maggiori, 2017). Finally, (negative) autocorrelation and weekdays effects are discernible for FX trading volume, volatility, illiquidity, and relative bid-ask spread.

4 Empirical analysis

The theoretical setup in Section 2 offers three main propositions. For each of them, we provide an in-depth empirical analysis in a separate subsection.

[7]See also Buraschi and Jiltsov (2006); Buraschi et al. (2014) for a theoretical and empirical analysis of the linkage between implied volatility and disagreement.

[8]Consistent results hold when replacing FX VIX with other measures of option-implied volatility such as the EURUSD 1-week, 1-month, and 3-month at-the-money implied volatility.

	(1) Δ Volume	(2) Δ RPV	(3) Δ HF Amihud	(4) Δ Relative BAS
Δ Volume	-	0.1374[a]	-	-0.0440[a]
		(98.78)		(-43.36)
Δ RPV	1.301[a]	-	-	0.3995[a]
	(92.12)			(148.7)
Δ Relative BAS	-0.9197[a]	0.9049[a]	0.7961[a]	-
	(-40.9)	(154.3)	(42.50)	
USD	0.0068	-0.0085[a]	-0.0086[a]	-0.0086[a]
	(1.619)	(-6.232)	(-1.964)	(-9.423)
Δ TED	0.1174[b]	0.0301[c]	-0.1746[a]	-0.0754[a]
	(2.347)	(1.844)	(-3.315)	(-6.945)
Δ VXY	0.1956[b]	0.6120[a]	-0.5974[a]	0.4271[a]
	(2.348)	(22.48)	(-6.825)	(23.52)
Monday	-0.3508[a]	0.0324[a]	0.3285[a]	-0.0858[a]
	(-50.68)	(14.57)	(45.26)	(-60.69)
Tuesday	0.0206[a]	-0.0176[a]	-0.1089[a]	-0.0362[a]
	(3.010)	(-8.140)	(-15.51)	(-24.38)
Thursday	-0.1072[a]	0.0005	0.1300[a]	0.0054[a]
	(-16.65)	(0.235)	(19.10)	(3.909)
Friday	-0.0766[a]	-0.1072[a]	0.1932[a]	0.0643[a]
	(-11.34)	(-51.62)	(28.41)	(46.22)
Lagged Dep.	-0.3494[a]	-0.0458[a]	-0.4211[a]	-0.2114[a]
	(-81.07)	(-35.95)	(-91.69)	(-55.48)
Constant	0.0986[a]	0.0231[a]	-0.1034[a]	0.0146[a]
	(19.17)	(14.23)	(-19.32)	(13.52)
R^2	0.418	0.586	0.404	0.573

Table 1: Regressions of volume, volatility (RPV), illiquidity, and bid-ask spread. Volume and RPV are the daily trading volume and daily RPV respectively, the high-frequency Amihud is the ratio between daily RPV and daily volume, and the bid-ask spread is the daily average of one-minute spreads. The *t*-statistics are in parentheses and the error variance are robust to heteroskedasticity and autocorrelation in the residuals. Except for dummy variables, all variables are taken in logs and changes. The superscripts *a*, *b* and *c* indicate significance at 1%, 5% and 10% significance level respectively.

4.1 Determinants of trading volume and volatility

Proposition 1 in Section 2 postulates that volatility and trading volume are proportional to the level of heterogeneity in the reservation prices between agents, that is traders' disagreement about the fundamental asset value. While RPV and RV provide a precise proxy of the unobserved FX volatility given by the availability of high-frequency prices, measuring traders' disagreement is a non-trivial task, and it requires to resort to additional information external to the current dataset. In particular, we follow the approach of Beber et al. (2010) and measure disagreement as heterogeneity in beliefs of market participants by using a detailed data set of currency forecasts made by a large cross-section of professional market participants. More specifically, we collect all Thomson Reuters surveys recorded at the beginning of every month during our sample period and compute measures of cross-sectional dispersion such as the (standardized) standard deviation of FX forecast and the high-low range from the distribution of FX forecasts of on average about 50 market participants.[9] This measure of heterogeneity in beliefs that we call *disagreement* is the main regressor in two-panel regressions in which monthly trading volume and monthly RPV are the dependent variables. In addition to our measure of disagreement, we include a constant, the lagged dependent variable, and FX illiquidity proposed in Karnaukh et al. (2015) as a control. All variables are taken in logs and changes.[10] As shown in column (1) and (2) of the bottom right panel of Table 2, both trading volume and volatility increase with disagreement providing evidence in support to our first empirical prediction. We also conduct various sub-sampling analyses substantiating this finding. For instance, the same result holds regardless for most and least liquid FX rates and also when looking at the currency pairs with USD only. Also, the main results remain qualitatively the same when we include further controls such as the TED and VXY (not tabulated). Moreover, both trading volume and volatility tend to increase with FX illiquidity, consistent with dealers' inventory imbalances and hot potato effects Lyons (1997).

4.2 Does the high-frequency Amihud index measure illiquidity?

Proposition 2 in Section 2 provides a closed-form expression for illiquidity in the spirit of Amihud (2002), i.e. the ratio between volatility and trading volume. The empirical prediction is that illiquidity decreases with market depth and the number of active traders. The visual inspection of Figure 5, representing the intraday development of the high-frequency Amihud measure on EURUSD and USDJPY, suggests that illiquidity tends to decrease when international financial centers are open, that is, when the FX market is deep and populated by active traders. More precisely, it is discernible that FX illiquidity abruptly decreases at the opening of the European markets, and it is minimal when both European and American markets are jointly open. After 8 p.m. the illiquidity grows again, and it is maximal during the night hours. A consistent pattern arises for USDJPY (the right-hand side of Figure 5): market illiquidity reduces at the opening of the main financial markets Tokyo, London and New York and it sensibly increases again after 4 p.m..

[9] The total number of monthly observations included in the regression is 940, which includes the following 26 currency pairs: AUDJPY, AUDNZD, CADJPY, EURAUD, EURCAD, EURCHF, EURGBP, EURJPY, EURNOK, EURSEK, GBPCAD, GBPCHF, GBPJPY, USDAUD, USDCAD, USDCHF, USDEUR, USDGBP, USDHKD, USDJPY, USDMXP, USDNOK, USDNZD, USDSEK, USDSGD and USDZAR. Not for all currency pairs, forecasts are available from November 2011 onwards. The exact number of market participants depends on the currency pair. We report results using standard deviations of FX forecast. Using high-low ranges, we obtain very similar results.

[10] We perform additional analysis including further regressors such as the TED and FX VIX, and the results remain qualitatively the same.

	10 least liquid FX rates				10 most liquid FX rates			
	(1) **Δ Volume**	(2) **Δ RPV**	(3) **Δ HF Amihud**	(4) **Δ Rel. BAS**	(1) **Δ Volume**	(2) **Δ RPV**	(3) **Δ HF Amihud**	(4) **Δ Rel. BAS**
Δ Disagreement	0.0858^a	0.1802^a	0.0045	0.0500^a	0.0195	0.1690^c	0.0705^b	0.0279^c
	(2.98)	(3.56)	(0.15)	(3.66)	(0.55)	(1.93)	(2.48)	(1.76)
Δ Illiquidity	0.0609^b	0.4528^a	0.1658^a	0.1246^a	0.0818^a	0.6149^a	0.2314^a	0.1521^a
	(2.49)	(8.09)	(5.81)	(9.30)	(4.35)	(5.32)	(3.96)	(6.86)
Lagged Dep.	-0.3752^a	-0.3090^a	-0.3635^a	0.0636	-0.3309^a	-0.2273^a	-0.1478^b	0.0241
	(-6.12)	(-3.16)	(-4.71)	(1.22)	(-7.34)	(-3.96)	(-2.43)	(0.60)
Constant	-0.0055	-0.0094	0.0008	-0.0068	-0.0103	-0.0141	0.0031	-0.0073
	(-0.49)	(-0.45)	(0.06)	(-1.22)	(-1.20)	(-0.67)	(0.35)	(-1.48)
R^2	0.165	0.414	0.261	0.366	0.134	0.360	0.301	0.351
N	345	345	345	345	497	497	497	497

	All currencies against USD				All currencies			
	(1) **Δ Volume**	(2) **Δ RPV**	(3) **Δ HF Amihud**	(4) **Δ Rel. BAS**	(1) **Δ Volume**	(2) **Δ RPV**	(3) **Δ HF Amihud**	(4) **Δ Rel. BAS**
Δ Disagreement	0.0591^b	0.2202^a	0.0594^b	0.0494^a	0.0530^b	0.1880^a	0.0445^b	0.0408^a
	(2.56)	(4.26)	(2.56)	(3.58)	(2.39)	(3.74)	(2.24)	(4.00)
Δ Illiquidity	0.0504^a	0.4496^a	0.1785^a	0.1234^a	0.0774^a	0.5408^a	0.1955^a	0.1400^a
	(2.94)	(5.75)	(4.62)	(8.88)	(5.26)	(8.11)	(5.65)	(10.65)
Lagged Dep.	-0.3775^a	-0.2708^a	-0.2412^a	0.0262	-0.3506^a	-0.2616^a	-0.2376^a	0.0298
	(-8.81)	(-3.81)	(-4.70)	(0.61)	(-10.03)	(-5.36)	(-4.35)	(1.00)
Constant	-0.0053	-0.0128	-0.0008	-0.0075^c	-0.0098	-0.0117	0.0040	-0.0068^c
	(-0.72)	(-0.82)	(-0.12)	(-1.73)	(-1.52)	(-0.84)	(0.59)	(-1.94)
R^2	0.140	0.335	0.283	0.281	0.146	0.370	0.254	0.347
N	577	577	577	577	940	940	940	940

Table 2: Monthly regression analysis - disagreement on subsamples. The t-statistics are in parentheses and the error variance are robust to heteroskedasticity and autocorrelation in the residuals. Disagreement is measured as the (standardized) standard deviations of Thomson Reuters forecasts, which are available on a monthly basis. Volume and RPV are the monthly trading volume and monthly RPV respectively, where the latter is computed with the (rescaled) sum of the absolute value of 5-minutes log-returns over monthly horizons. The high frequency (HF) Amihud is the ratio between daily RPV and daily volume, bid-ask spread is the daily average bid-ask spread, and illiquidity is taken from Karnaukh et al. (2015). Except for illiquidity, all variables are taken in logs. The superscripts a, b and c indicate significance at 1%, 5% and 10% significance level respectively. The 10 most liquid currency pairs, as measured by the average monthly HF Amihud, are USDEUR, USDJPY, USDHKD, USDGBP, USDCAD, USDAUD, EURGBP, USDCHF, EURCHF, USDSGD. The 10 least liquid currency pairs, as measured by the average monthly HF Amihud, for which the illiquidity measure as in Karnaukh et al. (2015) is available, are GBPCHF, GBPCAD, EURCAD, USDNOK, USDSEK, EURAUD, GBPJPY, USDZAR, EURNOK, USDNZD. The currencies trading against USD are AUD, CAD, CHF, EUR, GBP, HKD, JPY, MXP, NOK, NZD, SEK, SGD, ZAR. The currencies trading against EUR are AUD, CAD, CHF, GBP, JPY, NOK, SEK, EUR.

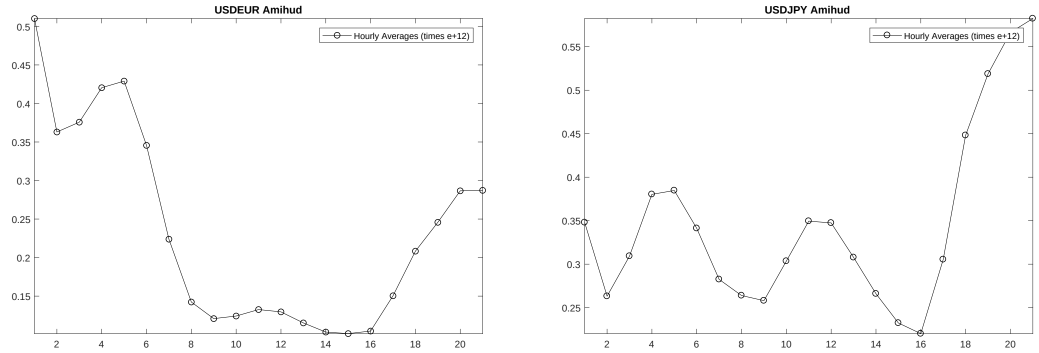


Figure 5: Hourly averages of the HF Amihud measure from 1 a.m. until 9 p.m.. In Panel a) USDEUR, in Panel b) USDJPY.

To shed further light on the measurement ability of the HF Amihud index, we perform various regressions similar to those shown in Tables 1-2. First, we regress changes in the HF Amihud at a daily frequency (i.e. the ratio between daily RPV and daily volume) on daily changes of bid-ask spreads. The results are exhibited in column (3) of Table 1. Second, we regress monthly changes of the HF Amihud on a comprehensive measure of FX illiquidity proposed in Karnaukh et al. (2015) that proved to be highly correlated with precise high-frequency (intraday) data from EBS. The results are presented in column (3) of in Table 2. In both regressions, we include control variables.[11] Overall, we find that the high-frequency Amihud illiquidity measure increases with other well-accepted measures of FX illiquidity.

So far, we have analyzed FX illiquidity on a global scale. Now, we ask the question of whether our FX illiquidity measure is positively correlated with other illiquidity proxies in the FX interdealer segment. We focus on EURUSD since it is primarily traded on the EBS interdealer trading platform.[12] In the same spirit of Hasbrouck (2009), we analyze correlations between illiquidity measures.[13] More specifically, we compute the following proxies: quoted spread (i.e. ask minus bid quotes); relative quoted spread (i.e. quoted spread divided by mid price); effective cost (i.e. the absolute value of the difference between transaction price and mid price);[14] traditional (low-frequency) Amihud measure (i.e. daily return in absolute value over trading volume); cost estimates implied by the Roll model (Roll, 1984), which is computed as the autocovariance between consecutive price changes with negative estimates set to zero; high-low (HL) estimate of the effective spread (Corwin and Schultz, 2012);[15] trade-by-trade order

[11]In addition to daily and monthly time intervals, we have performed the same regressions with weekly data and obtained consistent results.

[12]The analysis of USDJPY provides similar results.

[13]Further details on the construction of the FX illiquidity measure are presented in Appendix B.

[14]The daily (relative) quoted and effective spreads are average values of intraday data snapped at 100-millisecond intervals).

[15]The Corwin & Schultz (CS) spread estimator is computed for each day using hourly maximum, minimum, and closing prices. Hourly closing prices are defined as the last observation available for a given hour. A daily measure is then extracted by averaging out the 23 hourly spread estimates. Negative estimates are set to zero.

flow price impact.[16]

	A_t	BAS_t	EC_t	CS_t	R_t	γ	A_t^*
A_t	1.0000	0.7885	0.6680	0.3877	0.7068	0.9115	-0.0671
BAS_t	0.9080	1.0000	0.7944	0.5247	0.6594	0.7923	-0.0115
EC_t	0.8712	0.9128	1.0000	0.5701	0.8901	0.7455	0.0687
CS_t	0.5460	0.6335	0.6570	1.0000	0.4730	0.4285	-0.0784
R_t	0.6791	0.5696	0.7950	0.4361	1.0000	0.7690	0.0289
γ	0.9041	0.7759	0.8332	0.5583	0.7523	1.0000	-0.0304
A_t^*	0.4005	0.4326	0.3683	-0.0873	0.1933	0.2138	1.0000

Table 3: Correlation matrix for illiquidity measures on a daily basis for the EURUSD rate. Pearson correlations are reported in the lower triangular portion, whereas Spearman correlations are reported in the upper triangular portion of the table. Sample: EBS data from 01-Jan-2016 to 31-Dec-2016. A_t: High frequency Amihud measure, BAS_t: Bid-ask spread, EC_t: effective cost, C_t: Corwin-Schultz measure, R_t: Roll measure, γ: trade-by-trade price impact coefficient, A_t^*; classic low frequency Amihud measure computed with the absolute value of daily log-return.

Table 3 delivers three main messages: First, it clearly shows that our FX illiquidity measure is positively correlated (Pearson) with intraday illiquidity proxies based on EBS data, in particular, the effective cost and order flow price impact. Second, it is also positively correlated with the traditional low-frequency Amihud indicator suggesting that even approximating volatility with daily absolute returns (as in the traditional Amihud indicator) rather than gauging it with more accurate high-frequency measures (RPV), one can obtain a fairly accurate proxy of FX illiquidity. The Spearman rank correlations mostly confirm these results although the magnitudes are somehow lower. Third, our high-frequency Amihud is also correlated (with the expected sign) with other low-frequency estimates of effective spread, in particular, the Roll and HL measures.[17] Overall, we find that our high-frequency illiquidity measure in the spirit of the Amihud indicator covariates positively with other illiquidity proxies suggesting that it is effective in measuring FX illiquidity.

4.2.1 A natural experiment

Another method to assess the validity of the proposed high-frequency illiquidity measure is by means of a meaningful natural experiment. Through the lenses of the theory developed in Section 2, the announcement of the cap removal of the Swiss franc by the Swiss National Bank (SNB) on January 15, 2015, represents an ideal natural experiment. Starting from September 6, 2011, the SNB set a minimum exchange rate of 1.20 francs to the euro (capping franc's appreciation) saying "the value of the franc is a threat to the economy", and that it was "prepared to

[16]The correlation coefficients of the high-frequency Amihud measure with the order flow price impact based on five- or one-minute intervals are similar to the trade-by-trade order flow, although slightly smaller.

[17]The same picture holds when the Roll estimator is augmented with the Gibbs sampling method proposed by Hasbrouck (2009). We also find positive correlations between the high-frequency Amihud indicator and the close-high-low estimate of the effective spread (Abdi and Ranaldo, 2017) based on hourly data, similarly to Corwin & Schultz (CS) spread explained above.

buy foreign currency in unlimited quantities"'. This means that the SNB had a declared binding *cap* on the transaction price that was removed on January 15, 2015.[18])

In terms of our model, the SNB can be considered as the $(J+1)$-th trader. The SNB intervention strategy of selling CHF for EUR in potentially unlimited quantities is implemented if the average of the reservation prices of the J traders falls below the cap, that is if $\frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{*} < \log(1.2)$.[19] Indeed, despite the cap on the transaction price, the reservation prices of individual traders might well be below the 1.20 threshold. For instance, a trader with a reservation price of 1.12 (as the agent with the actual lowest forecast in Thomson Reuters survey before the SNB cap removal) is incline to sell EUR for CHF.[20] In other words, SNB buys (sells) foreign (domestic) currency to guarantee that the transaction price is above the threshold, that is

$$p_i^{EUR|CHF} = \frac{1}{J+1}\sum_{j=1}^{J+1} p_{i,j}^{EUR|CHF,*} \geq \log(1.2), \tag{11}$$

where $p_{i,J+1}^{EUR|CHF,*} = \left(\log(1.2) - \frac{1}{J}\sum_{j=1}^{J} p_{i,j}^{EUR|CHF,*}\right) \times \mathcal{I}\left(\sum_{j=1}^{J} p_{i,j}^{EUR|CHF,*} < \log(1.2)\right)$, where $\mathcal{I}(\cdot)$ is the indicator function. The enforcement of the capping regime by SNB generates extra trading volume. In particular, the trading volume is

$$v_i^{EUR|CHF} = \frac{\xi^{x|y}}{2}\sum_{j=1}^{J} |\Delta p_{i,j}^{EUR|CHF,*} - \frac{1}{J}\sum_{j=1}^{J} \Delta p_{i,j}^{EUR|CHF,*}| + v_i^{SNB}, \tag{12}$$

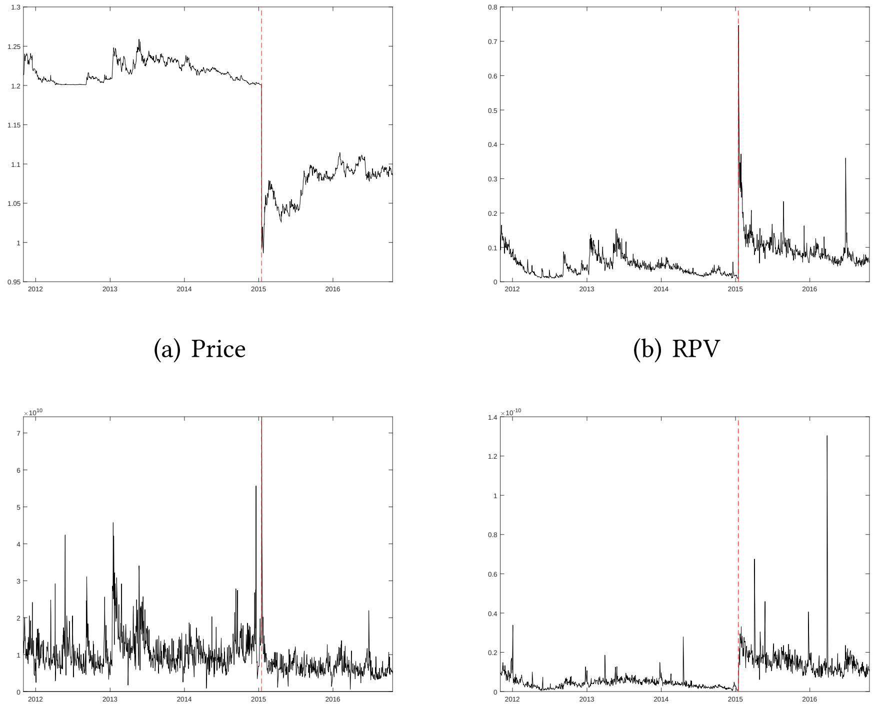
where v_i^{SNB} is the trading volume generated by the central bank to maintain the cap on the FX rate. Hence, the model prescribes a low volatility of the observed returns due to the implicit constraint given by the capping and a larger volume due to FX interventions. This implies that the high-frequency Amihud illiquidity index is lower (higher) before (after) the removal of the FX capping regime.

Figure 6 provides graphical support for the prescriptions of the theoretical model. Indeed, daily volatility (RPV) is relatively low until January 15, 2015, it spikes on the day of the announcement of the un-capping, and it remains high until the end of 2016. The trading volume has the opposite behavior, being relatively high during the capping period and reverting to a lower value after January 15, 2015. Finally, our high-frequency Amihud measure of FX illiquidity displays a clear upward shift after the removal of the Swiss franc cap. To provide statistical support, Table 4 reports the sample average of the main market variables before and after the cap removal. After the announcement, FX volatility significantly increases, trading volume decreases, and liquidity dries up (even discarding the announcement day). Furthermore, the

[18]The SNB announcement was mostly unanticipated by market participants (see, e.g., Jermann (2017); Mirkov et al. (2016).

[19]The SNB actually implemented this strategy by setting a huge ask volume (namely a *wall*) at 1.20, see, e.g., Breedon et al. (2018, Figure 3, p.10).

[20]The Thomson Reuters survey indicates a wide dispersion of the beliefs of professional market participants around 1.20 along most of the capping period.



(a) Price (b) RPV

(c) Volume (d) High frequency Amihud

Figure 6: FX rate (a), Realized power variation (b), Trading Volume (c) and high-frequency Amihud measure of FX illiquidity (d) of EURCHF from 2012 to 2016. The date of the announcement of the cap removal of the Swiss franc by SNB on January 15, 2015 is in vertical red-dashed line.

average trading volume size significantly decreases, suggesting a reduction in market depth. The lack of statistical significance in the change of the dispersion (standard deviations and high-low ranges) in Thomson Reuters survey of forecasts before and after the announcement suggests that market participants do not disagree more (less) after (before) the currency cap removal. This lack of statistical significance also suggests that the liquidity dry-up after the cap removal cannot be explained by a stronger consensus regarding agents' reservation prices.

All in all, the analysis of this natural experiment corroborates the empirical predictions of the theory, that is, the central bank's enforcement of its reservation price leads to lower volatility, larger trading volume, and higher liquidity. By abandoning this regime, opposite patterns arise.

4.3 Commonalities in the FX markets and pricing implications

Proposition 3 in Section 2 is about commonalities in FX trading volume, volatility and liquidity arising from the no-arbitrage condition. The purpose of this subsection is assess this idea empirically. More precisely, we proceed in two steps: First, we analyze commonalities by means of three methods: (i) the factor analysis, (ii) the construction of a FX connectedness index, and (iii) the regression analysis. Second, we study the pricing implications stemming from arbitrage

	Capping	Free floating	Test	p-value	Direction
RPV	4.5480	9.7531	12.71	0.000	⇑
RV	0.0825	0.3508	5.842	0.000	⇑
Volume	1148.8	682.80	-14.34	0.000	⇓
A	0.4255	1.5002	24.21	0.000	⇑
Size	264.84	207.65	-27.55	0.000	⇓
Dis_1	0.0146	0.0166	0.170	0.865	≈
Dis_2	0.0898	0.1015	0.125	0.908	≈

Table 4: Sample averages of realized power variation (*RPV*), realized volatility (RV), trading volume (Volume), high-frequency Amihud illiquidity measure (*A*), and average trade size (Size) during the cappig period (from Nov 1, 2011 to Jan 14, 2015) and after the capping period (from Jan 16, 2015 to Nov 30, 2016 - free floating). The un-capping announcement day (on Jan 15, 2015) is excluded. The variables are measured at daily frequency. To proxy disagreement, we compute the average of the standard deviations (DIS_1) and high-low range (DIS_2) of monthly Thomson Reuters survey of forecasts on the EURCHF rate. The variables have been rescaled. Table also reports a test for the equality of the averages in the two sub-samples, $z = \frac{m_2 - m_1}{\sqrt{v1/n1 + v2/n2}}$, and associated p-values (one-tail) calculated accounting for the auto-correlation in the data.

deviations and commonalities.

4.3.1 Factor analysis and connectedness index

Proposition 3 provides a theoretical underpinning that triangular no-arbitrage relations involving the same currencies can explain the presence of common factor structure on trading volume and volatility across FX rates. Notice that the FX-rate triangular condition can be extended to more than three FX rates. It is generalizable to any number of FX rates tied by triangular relationships. For instance, with four currencies, x, w, z, and y, the log-price is $p_i^{x|y} = p_i^{x|z} + p_i^{z|w} + p_i^{w|y}$, and analogously for the synthetic volume, $\widetilde{v}_i^{x|y}$. This provides support for the existence of a factor structure in cross-sections of FX rates of any order.

To the purpose of studying the commonality in volume, volatility and liquidity across multiple FX rates, we follow the common approach in the literature (see, e.g., Hasbrouck and Seppi, 2001) and apply the principal component analysis (PCA) to the panel of 29 FX rates introduced in Section 3.1. The goal is to identify a common factor structure across the volume, volatility, and illiquidity series of the FX rates and to study the exposure of each rate to it. Table 5 shows that for each individual FX rate, the loadings are positive on the first principal component not only for volume and volatility but also for illiquidity. Notably, the first component explains a large portion of the overall variation of volume, volatility and illiquidity measures of the panel of FX rates, being above 50% in many cases. Moreover, the weight associated with the volume and illiquidity of USDEUR is the highest signaling the leading role of the information on the USDEUR rate in determining the global FX volume. Instead, the loading on RPV for EURDKK

	Hourly			Daily			Weekly		
	Volume	RPV	HF Amihud	Volume	RPV	HF Amihud	Volume	RPV	HF Amihud
AUDJPY	0.1555	0.1884	0.1526	0.1830	0.1986	0.2230	0.1908	0.1963	0.2237
AUDNZD	0.1288	0.1461	0.1418	0.1781	0.1895	0.2195	0.2123	0.1975	0.2322
CADJPY	0.1327	0.1966	0.1045	0.1387	0.1884	0.1153	0.1244	0.1693	0.1034
EURAUD	0.1854	0.1992	0.1852	0.1968	0.2083	0.2277	0.2052	0.2082	0.2217
EURCAD	0.1829	0.2138	0.1709	0.1873	0.2111	0.1835	0.1595	0.2108	0.1899
EURCHF	0.2173	0.1520	0.2065	0.1677	0.1510	0.1871	0.1602	0.1582	0.1868
EURDKK	0.1841	0.0863	0.1819	0.1500	0.0623	0.0111	0.0828	0.0731	0.0038
EURGBP	0.2285	0.2128	0.2419	0.2155	0.2069	0.2508	0.2106	0.2063	0.2545
EURJPY	0.1971	0.1954	0.2110	0.1617	0.1677	0.2401	0.1510	0.1589	0.2385
EURNOK	0.2142	0.1472	0.2217	0.2118	0.1472	0.0612	0.2359	0.1532	0.0543
EURSEK	0.2131	0.1393	0.2179	0.2041	0.1314	0.0535	0.2102	0.1364	0.0391
GBPAUD	0.1599	0.2034	0.1592	0.1865	0.2163	0.2165	0.2033	0.2181	0.2212
GBPCAD	0.1279	0.2045	0.1098	0.1603	0.2067	0.1729	0.1708	0.2089	0.1832
GBPCHF	0.1692	0.2148	0.1595	0.1664	0.2094	0.1882	0.1878	0.2105	0.1977
GBPJPY	0.1823	0.1984	0.1690	0.1463	0.1774	0.1789	0.1480	0.1703	0.1760
USDAUD	0.1839	0.1965	0.1880	0.1951	0.2115	0.2417	0.1484	0.2129	0.2408
USDCAD	0.2070	0.2066	0.2099	0.2106	0.2079	0.2538	0.2109	0.2099	0.2585
USDCHF	0.2236	0.2140	0.2227	0.2184	0.2014	0.2516	0.2216	0.1997	0.2557
USDDKK	0.1573	0.2127	0.1499	0.1394	0.1979	0.1035	0.1280	0.1950	0.1046
USDEUR	0.2320	0.2142	0.2482	0.2011	0.1966	0.2690	0.1499	0.1937	0.2699
USDGBP	0.2291	0.2131	0.2363	0.2235	0.2050	0.2733	0.2368	0.2039	0.2758
USDHKD	0.1555	0.0673	0.1650	0.1515	0.1244	0.0891	0.1429	0.1350	0.0745
USDJPY	0.1748	0.1676	0.1739	0.1912	0.1487	0.2067	0.2359	0.1406	0.2034
USDMXP	0.1459	0.1656	0.1442	0.2211	0.1841	0.1138	0.2491	0.1867	0.0901
USDNOK	0.1909	0.2026	0.1892	0.1611	0.2014	0.0920	0.1238	0.2028	0.0779
USDNZD	0.1669	0.1914	0.1606	0.2003	0.2041	0.2365	0.2126	0.2069	0.2459
USDSEK	0.1939	0.1974	0.1887	0.1737	0.1861	0.0842	0.1509	0.1852	0.0524
USDSGD	0.1528	0.1633	0.1403	0.1671	0.1846	0.0927	0.1326	0.1854	0.0689
USDZAR	0.2179	0.1681	0.2246	0.2233	0.1705	0.0953	0.2427	0.1714	0.0862
EXPL	0.5514	0.6135	0.4519	0.4756	0.6494	0.3771	0.3633	0.6521	0.3939

Table 5: PCA Analysis. The table reports the loadings for each currency pair for trading volume (Volume), volatility (realized power variation, RPV) and illiquidity (high-frequency (HF) Amihud) to the first principal component based on the correlation matrix. The bottom line reports the percentage of explained variance of the first principal component.

is the smallest across all currencies, signaling that the volatility on EURDKK is strongly influenced by the pegging of DKK to EUR. These findings remain qualitatively the same for hourly, daily and weekly time series.

Another way to analyze commonalities is by studying the dynamic interplay between the FX rates across currencies by means of the total connectedness index of Diebold and Yilmaz (2014).[21] The TCI is defined as

$$\text{TCI} = \frac{1}{N} \sum_{i,j=1 i\neq j}^{N} \tilde{d}_{i,j}, \tag{13}$$

[21]See also Greenwood-Nimmo et al. (2016) for an application of the connecteness measure in the context of returns and option-implied moments of FX rates.

where N denotes the number of variables in the system, and $\tilde{d}_{i,j}$ is the i, j entry of the standardized connectedness matrix $\tilde{D}$. The matrix $\tilde{D}$ is defined as $\tilde{d}_{i,j} = \frac{d_{i,j}}{\sum_{j=1}^{N} d_{i,j}}$, with

$$d_{i,j} = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H} (e_i A_h \Sigma e_j)^2}{\sum_{h=0}^{H} (e_i' A_h \Sigma A_h' e_i)}, \quad (14)$$

where A_h is the impulse-response matrix at horizon h associated with a VAR(p) model, Σ is the covariance matrix of the errors, and e_i, e_j are $N \times 1$ selection vectors. By construction, $\sum_{j=1}^{N} \tilde{d}_{i,j} = 1$ and $\sum_{i,j=1}^{N} \tilde{d}_{i,j} = N$. Equation (14) defines the generalized forecast error decomposition, as introduced by Pesaran and Shin (1998). In other words, the TCI measures the average portion over N variables of the forecast error variation of variable i coming from shocks arising from the other $j = 1, \ldots, N-1$ variables of the system. Although less standard in the literature on liquidity commonalities, the TCI approach provides an informative characterization of the connectedness of a system that is richer than the one obtained with a simple linear correlation coefficient. Indeed, the TCI combines information coming from both contemporaneous and dynamic dependence through Σ and A_h, respectively. Moreover, by estimating the VAR model over rolling windows, it is possible to characterize the evolution of the dependence structure between two or more variables by looking at the variations of the TCI over time.

	Hourly				Hourly Seasonally Adjusted				Daily			
	Full	11/14	12/15	13/16	Full	11/14	12/15	13/16	Full	11/14	12/15	13/16
All FX rates												
Volume	0.884	0.880	0.885	0.890	0.726	0.719	0.730	0.731	0.891	0.889	0.891	0.898
RPV	0.910	0.907	0.910	0.916	0.846	0.844	0.850	0.856	0.921	0.920	0.928	0.930
10 Most Liquid												
Volume	0.875	0.873	0.883	0.880	0.709	0.702	0.714	0.722	0.862	0.864	0.863	0.870
RPV	0.893	0.890	0.893	0.904	0.814	0.815	0.818	0.838	0.919	0.920	0.922	0.935
10 Least Liquid												
Volume	0.621	0.607	0.634	0.649	0.275	0.270	0.284	0.289	0.623	0.608	0.617	0.659
RPV	0.808	0.810	0.821	0.819	0.718	0.710	0.728	0.732	0.846	0.829	0.861	0.873

Table 6: Connectedness. The table reports the value of the connectedness index of Diebold and Yilmaz (2014) of trading volume (Volume) and volatility (realized power variation, RPV) for different sampling periods (Full sample, 2011/2014, 2012/2015, and 2013/2016) and for different sets of FX rates. "10 Most Liquid" and "10 Least Liquid" refer to the ten most and least liquid FX rates in terms of total trading volume. The estimates of A_h and Σ are based on a VAR(1) specification.

As shown in Table 6, the connectedness analysis delivers two main findings: First, the overall level of connectedness of volume and volatility is very high and constant over time, being close to 90% for both volatility and volume at an hourly and daily level. The connectedness remains very high also when volume and RPV are filtered from intraday seasonality, being around 70%-80%. This picture corroborates the previous findings obtained from the factor analysis, that is, there is a strong commonality across FX volumes and volatilities. Second, the comparison between the most and least liquid FX rates indicates a stronger connectedness of volume and volatility for the former set of currencies. Indeed, the connectedness on the most liquid FX rates is above, 85% and it remains relatively high for hourly seasonally adjusted series.

On the other hand, the connectedness level sensibly reduces when focusing on the least liquid FX rates. This result is fully consistent with the market adage that "liquidity begets liquidity" (see, e.g., Foucault et al., 2013), in the sense that higher liquidity goes with stronger commonality. It also squares well with the idea that illiquid currency pairs are less (more) exposed to the common (specific) FX-factors as it emerges from the magnitude of the loadings of the first principal component in Table 5. In sum, liquid currencies appear to have stronger cross-currency commonalities than illiquid ones.

4.3.2 Strength of commonality and correlation in FX rates

Common measures of commonality in liquidity are statistical measures such as R^2 or estimated slope coefficient when regressing liquidity of an asset on market liquidity (see, e.g., Chordia et al., 2000). Following the same reasoning but to be consistent with the arbitrage framework theorized in Section 2.2, we measure the strength of the pairwise commonality in volume between $x|y$, $x|z$ and $z|y$ through the following reduced-form model,

$$\log(v_t^{x|y}) = \beta_0 + \beta_1 \log\left(v_t^{x|z} + v_t^{z|y}\right) + \varepsilon_t, \quad t = 1, \ldots, T \tag{15}$$

where $v_t^{x|y}$, $v_t^{x|z}$ and $v_t^{z|y}$ are the log-volume on period t on the FX rates $x|y$, $x|z$ and $z|y$, respectively. In this regression, β_0 reflects the differential in the resiliency levels in the three markets, and β_1 measures the magnitude of commonality in the volume of the three FX rates. The theory outlined in Section 2.2 prescribes that $\beta_1 > 0$. Table 7 reports the estimates of regression (15) for EURUSD, where the aggregate volume combining $v_t^{x|z}$ and $v_t^{z|y}$ is based on the following currencies CHF, GBP, DKK, JPY, AUD, CAD, NOK, SEK.[22]

Overall, it emerges that regression (15) is able to explain a large portion of variability of $v^{EUR|USD}$, and this can be attributed to the portions of common information in $\Delta p_j^{USD|\cdot,*}$ and $\Delta p_j^{EUR|\cdot,*}$, which determine the commonality in volume. At the hourly level, the estimated parameter β_0 reflects the average liquidity differential across currencies, with DKK, SEK and NOK being consistently less liquid than JPY, AUD and GBP. Notably, the parameter β_1 is positive in all cases and it is closer to 1 for the most liquid rates corroborating the idea that liquidity generates commonality. As expected, higher β_1 are associated with higher R^2. When aggregating at the daily or weekly level, the R^2 slightly decreases but the result is qualitatively the same as for the raw hourly volume. The residuals display significant autocorrelation, suggesting that volume imbalances across FX markets are stationary but persistent. These long-lasting disequilibria in volume might be explained by the fragmented OTC structure of the FX market and

[22] Besides these 8 FX rates providing triangular constructions with the EURUSD rate, in our sample the following synthetic FX rates exist: (a) for the USDGBP, via AUD, CAD, CHF, EUR, and JPY; (b) for USDAUD, via EUR, GBP, JPY, and NZD; and (c) for EURCHF, via GBP and USD. We have analyzed all of them obtaining consistent results.

	Hourly			Daily			Weekly		
	β_0^H	β_1^H	R_H^2	β_0^D	β_1^D	R_D^2	β_0^W	β_1^W	R_W^2
Volume									
CHF	6.2791^a	0.7861^a	0.8068	9.0985^a	0.6973^a	0.5305	10.570^a	0.6588^a	0.5035
GBP	3.8371^a	0.8625^a	0.8195	7.8218^a	0.7209^a	0.4134	8.3926^a	0.7163^a	0.3466
DKK	15.592^a	0.2962^a	0.5526	16.073^a	0.4452^a	0.3751	11.644^a	0.6722^a	0.5062
JPY	2.8344^a	0.8799^a	0.4632	15.728^a	0.3963^a	0.2099	18.909^a	0.3145^a	0.1595
AUD	0.7486^a	1.0096^a	0.5051	7.1182^a	0.7599^a	0.5789	7.7954^a	0.7490^a	0.6443
CAD	5.6124^a	0.7936^a	0.7248	10.634^a	0.6176^a	0.3875	1.6341	0.9856^a	0.4167
NOK	13.577^a	0.4635^a	0.6739	14.887^a	0.4782^a	0.2482	15.917^a	0.4704^a	0.1858
SEK	13.388^a	0.4700^a	0.6778	14.220^a	0.5051^a	0.2580	15.300^a	0.4935^a	0.1816
RPV									
CHF	-0.9108^a	0.9189^a	0.7752	-0.9071^a	0.7412^a	0.7091	-0.4929^a	0.7312^a	0.7277
GBP	-0.9777^a	0.9303^a	0.7552	-0.7973^a	0.8884^a	0.6257	-0.6153^a	0.8761^a	0.6337
DKK	0.2941^a	1.094^a	0.932	0.0646^a	1.1223^a	0.9620	-0.1243^a	1.1417^a	0.9774
JPY	-1.2987^a	0.9156^a	0.5717	-1.2604^a	0.6790^a	0.3995	-0.7456^a	0.6775^a	0.3955
AUD	-0.8686^a	1.0431^a	0.5542	-1.0562^a	0.9334^a	0.6033	-0.9423^a	0.9259^a	0.6249
CAD	-0.7850^a	0.9968^a	0.7238	-0.7546^a	0.9889^a	0.6680	-0.7340^a	0.9897^a	0.6872
NOK	-1.7326^a	0.8126^a	0.5366	-1.0991^a	0.8241^a	0.4756	-0.8292^a	0.8538^a	0.4874
SEK	-1.3851^a	0.8718^a	0.5678	-0.7150^a	1.0555^a	0.5693	-0.8242^a	1.1247^a	0.5918
HF Amihud									
CHF	-14.066^a	0.5552^a	0.6956	-14.425^a	0.5417^a	0.7234	-14.631^a	0.5335^a	0.7803
GBP	-12.749^a	0.5985^a	0.7241	-8.7892^a	0.7538^a	0.7489	-7.1268^a	0.8194^a	0.8437
DKK	-24.583^a	0.1506^a	0.2566	-23.235^a	0.2086^a	0.2375	-19.113^a	0.3924^a	0.3665
JPY	-8.8037^a	0.7518^a	0.528	-11.483^a	0.6518^a	0.5832	-11.480^a	0.6520^a	0.6490
AUD	-14.852^a	0.5419^a	0.383	-13.826^a	0.5931^a	0.5401	-11.097^a	0.7082^a	0.6592
CAD	-18.857^a	0.3836^a	0.3733	-13.235^a	0.6319^a	0.5111	-8.4519^a	0.8380^a	0.6493
NOK	-22.516^a	0.2321^a	0.4465	-17.387^a	0.4554^a	0.3539	-13.780^a	0.6116^a	0.4185
SEK	-22.151^a	0.2452^a	0.451	-19.111^a	0.3772^a	0.2138	-18.246^a	0.4144^a	0.1752

Table 7: Commonalities in volume, volatility (realized power variation, RPV) and illiquidity (high frequency Amihud index, HF Amihud). For each currency, the table reports the intercept, slope and R^2 of the regression of the log volume/volatility/Amihud of EURUSD on the log of the sum of volume/volatility/Amihud on the FX rate of the currency indicated in the first column against USD and EUR. The regression is based on data sampled at hourly, daily and weekly level. The superscripts *a*, *b* and *c* indicate significance at 1%, 5% and 10% significance level, respectively.

prolonged time to incorporate agents' heterogeneous priors and (public and private) information into prices, as for conditional volatility (Engle et al., 1990). When replacing volume with volatility (RPV) in equation (15), we note that also volatility displays a large degree of commonality across currencies. The R^2 is generally very well above 50% at both hourly and daily level. Interestingly, the R^2 and the slope coefficient of DKK are almost 1 consistent with the Danish Central Bank policy to keep EURDKK within a very narrow corridor (0.133-0.1346), thus the $Cov(p^{USD|DKK}, p^{USD|EUR}) \approx 1$. In line with the theory, the Danish central bank's intervention to fix the EURDKK rate reduces the commonality in volume and liquidity with the other currencies. Not surprisingly, the high-frequency Amihud illiquidity measure, which combines information on both volatility and volume, also displays an analogous amount of commonality across currencies, being the highest for the most liquid ones.

The theory outlined in Section 2.2 suggests that the commonalities in trading volume across FX rates are driven by the level of correlation among the FX rates, where the synthetic volume is a function of the correlation of the aggregated traders' specific components on different currency pairs, see the the right-hand side of (33) in Appendix A. In other words, the synthetic volume must reveal the strength of the correlation across FX rates. To test this prediction, we consider the following regression

$$v_t^{x|y;z} = \gamma_0 + \gamma_1 \log(\zeta_t) + \gamma_2 v_{t-1}^{x|y;z} + \varepsilon_t, \quad t = 1, \ldots, T, \tag{16}$$

where $v_t^{x|y;z}$ is the fitted log-volume in regression (15), while $\zeta_t = \log(1 + |\hat{\rho}_t|)$ and ρ_t is the realized correlation between returns on $x|z$ and $z|y$, defined as $\hat{\rho}_t = \frac{\sum_{i=1}^{I}(r_{t,i}^{x|y} \cdot r_{t,i}^{z|y})}{\sqrt{RV_t^{x|y}}\sqrt{RV_t^{y|z}}}$. Hence, the term ζ_t measures the strength of the correlation in the FX rates $x|z$ and $z|y$, and the parameter γ_1 is expected to be positive. Table 8 contains the estimates of γ_1 based on regression (16) and on the extended version which controls for liquidity as measured by the bid-ask spreads on $x|z$ and $z|y$. At hourly frequency, the estimates of γ_1 are positive and highly significant in most cases, with the notable exception of DKK. Again, the results suggest that the intervention of the central bank to peg DKK to EUR prevents the trading activity on EURDKK and DKKUSD from fully revealing the correlation structure of the investors' beliefs on EUR and USD. When aggregating over days and weeks, we still obtain generally positive estimates of γ_1 but they are often not significantly different from zero.

4.3.3 Pricing implications

One of our previous results is that liquidity begets liquidity across currencies. As the last step of our study, we address the question of whether liquidity begets price efficiency as well. The rationale of this relationship is again our third theoretical proposition implying that arbitrage keeping FX rates tied to equilibrium relations pass through the trading activity (volume), which in turn it is sustained by liquidity. To do this, we consider the volatility of the no-arbitrage violations (*pricing* errors), defined as $pe^{x|y;z} = \tilde{r}_i^{x|y;z} - r_i^{x|y}$, to investigate whether high liquidity is associated with smaller variability of the pricing errors, where the latter is measured as $\text{RPVE}^{x|y;z} = \sum_{i=1}^{I} |pe_i^{x|y;z}|$.

Empirically, we test the pricing error-liquidity relation in two ways: First, by looking at the systematic relationship between arbitrage deviations and illiquidity; Second, by inspecting whether more liquidity facilitates the price adjustment process. To analyze the systematic pricing errors liquidity relationship, we extend the previous commonality analysis in (15) by

	Hourly		Daily		Weekly	
	γ_0	γ_1	γ_0	γ_1	γ_0	γ_1
Baseline Regression						
CHF	4.2444	0.2141[a]	13.9536	0.1697[a]	8.4427	0.0993[c]
GBP	3.7079	0.2310[a]	18.2764	0.0897[c]	12.1883	0.0485
DKK	6.8630	-0.1447[a]	18.5609	-0.1159	11.2898	0.0615
JPY	6.1447	0.2810[a]	10.9029	0.1511[a]	7.0136	0.0331
AUD	6.8131	0.0642[a]	12.5726	0.0609	8.4234	-0.0696
CAD	4.3665	-0.0764[a]	22.5847	-0.0203	18.2935	0.0079
NOK	6.0960	0.7085[a]	17.2974	0.4519[a]	17.6356	0.1196[c]
SEK	5.6549	0.5923[a]	18.6241	0.1409[a]	15.6949	-0.0160
Control for Liquidity						
CHF	5.3883	0.3172[a]	16.6392	0.4082[a]	12.1837	0.2538[b]
GBP	4.8816	0.2439[a]	18.7933	0.1463[a]	12.9884	0.0658
DKK	7.0838	-0.1499[a]	19.4777	-0.1202[a]	13.8504	0.0317
JPY	7.1983	0.1077[a]	14.6026	-0.0455	10.9355	-0.0353
AUD	7.6219	0.1614[a]	17.4960	0.2028[a]	16.9214	0.0394
CAD	4.5109	-0.0853[a]	22.7906	0.0288	19.6981	0.0878
NOK	8.0024	0.7449[a]	17.3807	0.4529[a]	17.6188	0.1140[c]
SEK	8.2093	0.6576[a]	18.9913	0.1571[a]	16.0763	-0.0129

Table 8: Synthetic volume and correlation. For each currency, the table reports the intercept and the slope of regression (16) for the EURUSD rate with CHF, GBP, DKK, JPY, AUD, CAD, NOK, SEK.

interacting synthetic volume and pricing error variability as follows

$$\log(v_t^{x|y}) = \beta_0 + \beta_1 \log\left(v_t^{x|z} + v_t^{z|y}\right) + \beta_2 \log\left(v_t^{x|z} + v_t^{z|y}\right) \times RPVE_t^{x|y;z} + \varepsilon_t, \quad t = 1,\ldots,T. \quad (17)$$

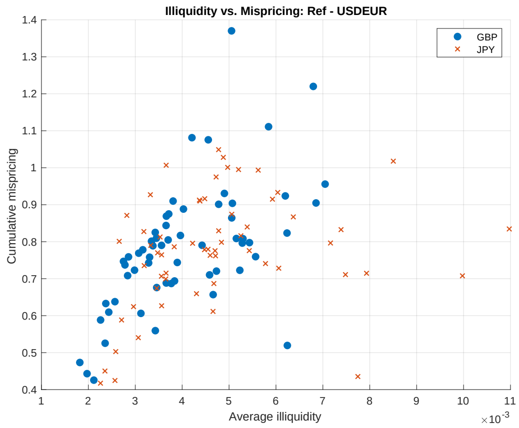
The results are reported in Table 9. As predicted by Proposition 3, we find a positive β_2 for the trading volume indicating that arbitrage deviations attract more trading volume to re-establish price equilibrium. In other words, as the variability of the pricing errors increases, the synthetic volume requires a larger adjustment to match the same level of volume as in the direct FX market. We repeat the analysis to volatility and illiquidity, for which we do not have an empirical prediction about the sign of β_2. In both cases, we find a negative β_2 suggesting that the departure from arbitrage conditions goes with divergent liquidity and volatility patterns across currencies, consistent with the idea that illiquidity hinders the restoration of equilibrium prices.

To study if more liquidity facilitates the price adjustment process we look at the dependence between the variability of pricing errors, as measured by the $\text{RPVE}^{x|y;z}$, and the average illiquidity based on each FX rate allowing for a triangular FX construction, computed as $\overline{A}^{x|y;z} = (A^{x|z} + A^{z|y})/2$. For instance, for EURUSD, EURGBP, and GBPUSD we consider the cumulative absolute deviations between direct (EURUSD) and synthetic rate (via EURGBP, and

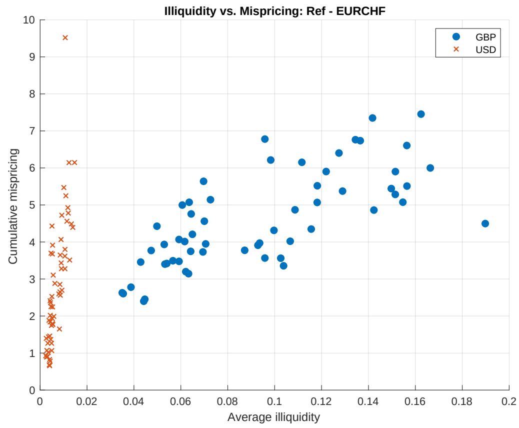
	Volume				RPV				HF Amihud			
	β_0^D	β_1^D	β_2^D	R_D^2	β_0^D	β_1^D	β_2^D	R_D^2	β_0^D	β_1^D	β_2^D	R_D^2
CHF	9.233^a	0.691^a	0.024	0.533	-0.939^a	0.748^a	-0.846^a	0.713	-14.015^a	0.557^a	0.035^c	0.728
GBP	8.237^a	0.697^a	0.206^a	0.434	-1.089^a	0.973^a	-8.080^a	0.689	-9.990^a	0.712^a	-0.149^a	0.758
DKK	16.24^a	0.430^a	0.276^a	0.405	0.107^a	1.115^a	1.052^b	0.965	-23.24^a	0.208^a	-0.007	0.238
JPY	17.18^a	0.328^a	0.304^a	0.247	-1.653^a	0.853^a	-14.24^a	0.532	-12.62^a	0.617^a	-0.281^a	0.619
AUD	7.632^a	0.730^a	0.193^a	0.619	-1.322^a	1.031^a	-8.022^a	0.705	-15.23^a	0.542^a	-0.181^a	0.572
CAD	11.37^a	0.572^a	0.387^a	0.476	-1.153^a	1.043^a	-9.348^a	0.763	-14.57^a	0.583^a	-0.228^a	0.538
NOK	14.89^a	0.478^a	-0.001	0.248	-0.902^a	0.795^a	3.747^a	0.499	-18.24^a	0.421^a	-0.049^b	0.360
SEK	14.22^a	0.505^a	-0.003	0.258	-0.528^a	1.046^a	3.242^c	0.585	-20.32^a	0.330^a	-0.089^b	0.230

Table 9: Commonalities and pricing errors. For each currency, the table reports the intercept, slopes and R^2 of the regression of the log volume/RPV/HF-Amihud of EURUSD on the log of the sum of volume/RPV/HF-Amihud on the FX rate of the currency indicated in the first column against USD and EUR and the interaction term with variability of the pricing errors, as measured by RPVE$^{x|y;z}$. The superscripts a, b and c indicate significance at 1%, 5% and 10% significance level, respectively.

GBPUSD), and the average high-frequency Amihud measures of the two FX rates constituting the triangular arbitrage (EURGBP and GBPUSD). Figure 7 plots the monthly cumulative pricing



(a) Pricing error variation against illiquidity: USDEUR



(b) Pricing error variation against illiquidity: EURCHF

Figure 7: Monthly pricing error variation (RPVE$^{x|y;z}$) against average illiquidity, $\overline{A}_t^{x|y}$.

error variation (RPVE$^{x|y;z}$) against average illiquidity, $\overline{A}_t^{x|y}$, for the USDEUR and EURCHF rates. The figures clearly display a positive relationship between mispricing and illiquidity, which is in line with the prescription of Theorem 3. Also, when liquid currencies are used to construct the triangular arbitrage relation, we observe a steeper dependence between mispricing and illiquidity compared with the illiquid ones (e.g. USD rather than GBP in the right-hand of Figure 7). This suggests that the same amount of additional liquidity is more effective in reducing arbitrage deviations when using liquid currencies rather than illiquid ones. To highlight if this is a systematic feature, we also carry out a statistical analysis performing the following regression

$$RPVE_t^{x|y;z} = \alpha + \beta \overline{A}_t^{x|y;z} + \gamma \overline{BAS}_t^{x|y;z} + \varepsilon_t, \tag{18}$$

where $\overline{A}^{x|y;z}$ denotes the average illiquidity on the FX rate $x|y$ computed with the same currency used to calculate $\text{RPVE}_t^{x|y;z}$. We expect the parameter β to be positive and significant, signaling a positive relation between illiquidity and pricing errors. Analogously, the average bid-ask spread denoted as $\overline{BAS}_t^{x|y;z}$, is also computed in a similar way and it is added to the regression to control for deviations from the pricing equilibrium due to another dimension of illiquidity that is the bid-ask spread. The results of regression (18) are reported Table 10. Overall, the parameter estimates validate the findings observed in the scatter plots in Figure 7. In particular, by regressing monthly RPVE on the average FX illiquidity, we find compelling evidence that liquidity begets price efficiency, i.e. it limits arbitrage deviations. This holds true also when controlling for bid-ask spread differentials, although the significance is reduced for EURUSD when combining with the least liquid currencies (e.g. NOK and SEK).

	EURUSD								EURCHF	
	CHF	GBP	DKK	JPY	AUD	CAD	NOK	SEK	USD	GBP
α	0.36^a	0.50^a	0.49^a	0.62^a	0.51^a	0.26^a	0.51^a	0.72^a	0.25	2.51^a
β	54.77^a	65.12^a	1.01^a	34.06^b	21.54^a	17.37^a	16.15^a	9.07^a	357.9^a	22.13^a
R^2	0.54	0.32	0.14	0.11	0.34	0.40	0.36	0.13	0.65	0.49
α	-0.30^a	0.04	0.59^a	0.14^b	0.13	-0.27^b	-0.10	-0.18^b	-2.48^a	-0.78^c
β	23.21^a	11.15	1.27^a	30.77^a	10.18^b	2.95	2.92	-1.52	227.7^a	16.21^a
γ	30.05^a	31.25^a	-0.98	0.20^a	14.55^a	26.27^a	2.97^a	3.51^a	124.0^a	93.64^a
R^2	0.88	0.64	0.18	0.47	0.46	0.69	0.76	0.71	0.82	0.69

Table 10: RPVE vs. liquidity regression estimation. Table reports the estimates of the linear regression (18) for the FX rates EURUSD and EURCHF, when the triangular no-arbitrage condition is computed with of a third currency, that is CHF, GBP DKK, JPY, AUD, NOK and SEK for EURUSD; USD and GBP for EURCHF. The sample size is $N = 58$ months. The top (bottom) panel reports the estimates when *BAS* is excluded (included) among regressors in (18). The superscripts *a*, *b* and *c* indicate significance at 1%, 5% and 10% significance level respectively.

4.3.4 Detecting directional movements in FX

We now further explore the relationship between liquidity and price adjustments, focusing on the arrival of big news (common to all traders) on a specific currency. We, therefore, extend our baseline theoretical model outlined in Section 2 by assuming that the increments of the reservation log-prices can be disentangled as

$$\Delta p_{i,j}^{x|y;*} = \phi_i^{x|y} + \psi_{i,j}^{x|y}, \quad \text{with} \quad j = 1, \ldots, J,$$

where $\phi_i^{x|y}$ is the common information component about the FX rate $x|y$, stemming from public information events, such as those associated with central banks' announcements. The common term $\phi_i^{x|y}$ could be related to events that trigger common directional expectations among the practitioners about a specific currency. The term $\psi_{i,j}^{x|y}$ represents the investor's specific component about the FX rate between x and y, and it is assumed to follow the diffusive process in (2). Furthermore, the assumption of independence between ϕ_i and $\psi_{i,j}$ and across traders does not allow for reversal or spill-over effects such as those studied in Grossman and Miller (1988) to investigate the mechanics of liquidity provision. The same type of sequential trading behavior has been recently proved to be responsible for crash episodes in Christensen et al. (2016) and associated with changes in the level of investors' disagreement around important news announcements (see, e.g. Bollerslev et al. (2018)).[23] The detection of such informational events needs an accurate identification econometric technique and granular (intraday) data. The recent advances in the literature on *jump* processes come to the aid of this analysis. Similarly to Bollerslev et al. (2018), we rely on a simple setup for the common news component, i.e. the "jumps", to separately identify it from the component of the variations in the FX rates due to the disagreement among traders.[24] Furthermore, we assume that $\phi_i^{x|y}$ can be further decomposed into independent currency-specific variations, that is $\phi_i^{x|y} = \phi_i^x - \phi_i^y$. The terms ϕ_i^x and ϕ_i^y cannot be uniquely identified by looking at a single FX rate since a large variation in the FX rate might be due to good (bad) news on x or bad (good) news on y.

Therefore, we rely on the theory of *co-jumps*, as developed in Caporin et al. (2017), to identify ϕ_i^x (or ϕ_i^y) given a cross-section of FX rates with the same base currency x (or y). In other words, the simultaneous occurrence of a jump in all the FX rates trading with a given base currency x allows us to identify episodes characterized by the realization of a *currency-specific* big news. In turns, this enables us to identify large and sudden directional appreciations or depreciations of one currency against the other currencies associated with no or little disagreement. The test for co-jumps proposed by Caporin et al. (2017) takes the form

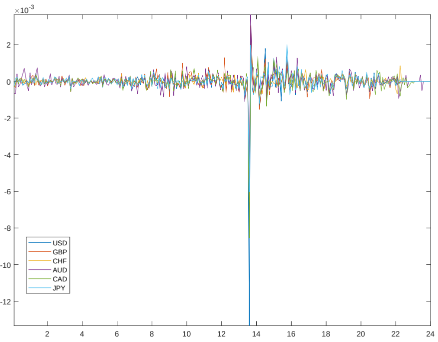
$$\mathcal{CJ} = \frac{1}{\zeta} \sum_{j=1}^{N} \frac{\left(SRV_j - SRV_j^*\right)^2}{SQ_j}, \tag{19}$$

where N denotes the number of currencies trading against a given base currencies, ζ is a design parameter, SRV is the smoothed randomized realized variance of Podolskij and Ziggel (2010), SRV^* is the smoothed version of the truncated realized variance estimator of Mancini (2009) which is robust to jumps, while SQ^* is a smoothed estimator of the quarticity. Under the null hypothesis of absence of co-jumps, $\mathcal{CJ}$ converges to a chi-square distribution with N degrees of freedom. Under the alternative hypothesis of at least one co-jump across all N series, $\mathcal{CJ}$ diverges. Figure 8 illustrates two representative episodes detected with the test for co-jumps developed in Caporin et al. (2017).[25] The left panel reports the log-returns of the FX rates of EUR against the six major currencies, USD, GBP, CHF, AUD, CAD and JPY on November 6, 2015. The

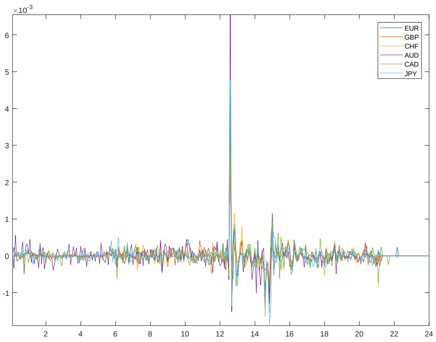
[23] Perraudin and Vitale (1996) also consider jump times as moments at which significant information becomes public knowledge.

[24] Other studies associating large price jumps with news announcements are in Andersen et al. (2007b), Chaboud et al. (2008), and Lee (2011).

[25] We thank the authors for sharing with us their MATLAB code to detect co-jumps.



(a) EUR



(b) USD

Figure 8: Co-jumps analysis. The figures report the five-minute returns on six FX rates on days when the test of co-jumps of Caporin et al. (2017) has detected significant jumps at 0.01% significance level. The left plot reports the returns of the FX rates of USD, GBP, CHF, AUD, CAD and JPY against EUR on November 6, 2015. The right plot reports the returns of the FX rates of EUR, GBP, CHF, AUD, CAD and JPY against USD on May 2, 2014.

sudden depreciation of EUR occurred in reaction to a speech by the President of ECB, Mario Draghi reinforcing traders' belief about the continuation of the Eurosystem's bond purchases (Quantitative Easing) as a stabilization tool to resolve the crisis situations in the financial market. The FX rate reacted with a sudden depreciation of EUR against all other currencies by approximately 1% on an interval of five minutes. The magnitude of such a variation is several times larger than the variation under *normal* market conditions, where the changes in the reservation prices of each individual trader is averaged over J traders. Analogous evidence arises for the appreciation of the USD against all major currencies on May 2, 2014, following the rumors on the beginning of a tapering policy by the Federal Reserve.

We also look at the relation between CLS and EBS volume data when a representative co-jump occurs. Figure 9 reports the standardized hourly EBS and CLS volume on USDEUR on April 27, 2016, for which the test of Caporin et al. (2017) detects significant co-jumps on USD. On that day, the Federal Reserve hosted the Federal Open Market Committee (FOMC), and the market was expecting a rise in the interest rates, which did not take place. Hence, the market reacted with a sharp and immediate depreciation of the U.S. dollar, which was subsequently reabsorbed. Figure 9 displays a number of interesting insights. Fist, EBS and CLS volumes follow similar pattern. Second, trading volume increases for both EBS and CLS during the hour of the announcement, with EBS volume reacting slightly more than CLS during the hour of the announcement (between 6 p.m. and 7 p.m.). Instead, the volume of both EBS and CLS drastically reduces in the subsequent hours, and it remains below the daily average in both cases. This signals that the reaction to the news of the interdealer segment and the remaining FX market was relatively short-lived.

Figure 10 provides evidence on the dynamics of FX prices and volume at a granular level around the time of the announcement by reporting volume and FX price at 15 seconds fre-

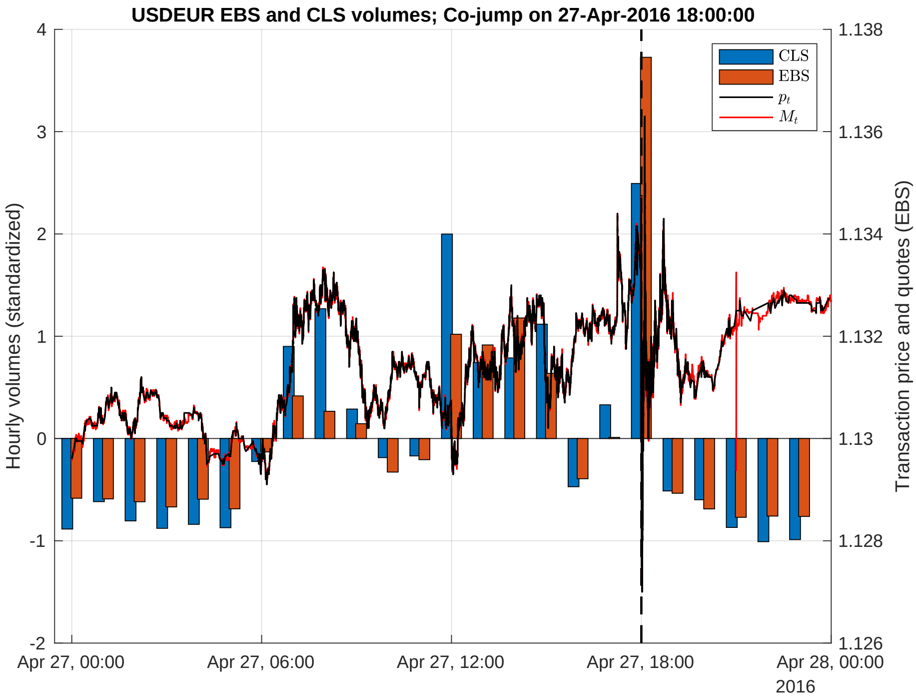


Figure 9: Trading volume and co-jumps on April 27, 2016. The figure reports the hourly EBS volume (red bars) and CLS volume (blue bar) during the day of a detected co-jump on USD. The volume series are those on the USDEUR rate and are reported in deviation from the daily average. The dashed vertical line denotes the hour of the detected co-jump.

quency. At the time of the Fed announcement (at 6:00 p.m.), the FX rate is at 1.33, and the volume has been relatively low since 4 p.m.. Immediately after the announcement, the price drops at 1.30 (-2.2% variation), but the volume is still relatively low, thus indicating a sudden illiquidity episode. In other words, traders have revised their beliefs in the same direction, and this has lead to an (almost) instantaneous determination of a new equilibrium price. In the minutes after the announcement, the volume (and the volatility) of the price have increased enormously until 6:30 p.m., and the FX price moves within a large range of values, between 1.28 and 1.36. This signals the dynamic impact of price jumps on volatility (see, e.g., Andersen et al., 2007a) and hence on volume, which is associated with a large disagreement on the reservation prices of each trader. Finally, after 6:30 p.m., the FX price has returned to fluctuate at its pre-announcement level with relatively low trading volume.

In the following, we look at the pricing implications of the arrival of big news to the FX market, by focusing on the price adjustment mechanism and the synthetic trading volume conditional to a co-jump event.

4.3.5 Co-jumps and price adjustment

To study whether liquidity facilitates price adjustments, we benefit from the identification of large price co-movements captured by the co-jumps. Notably, the synthetic return on $x|y$,

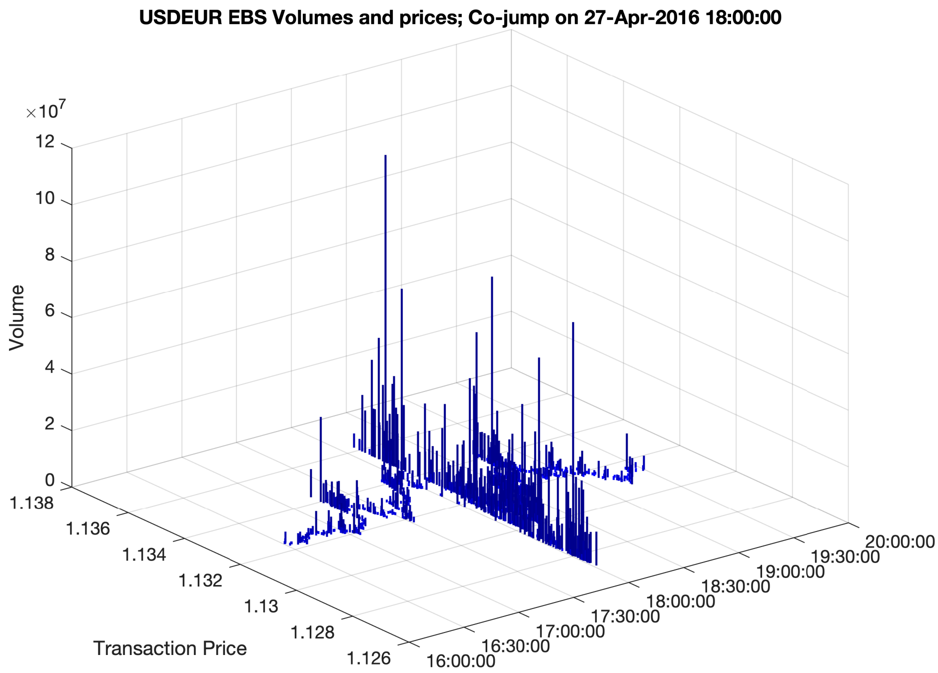


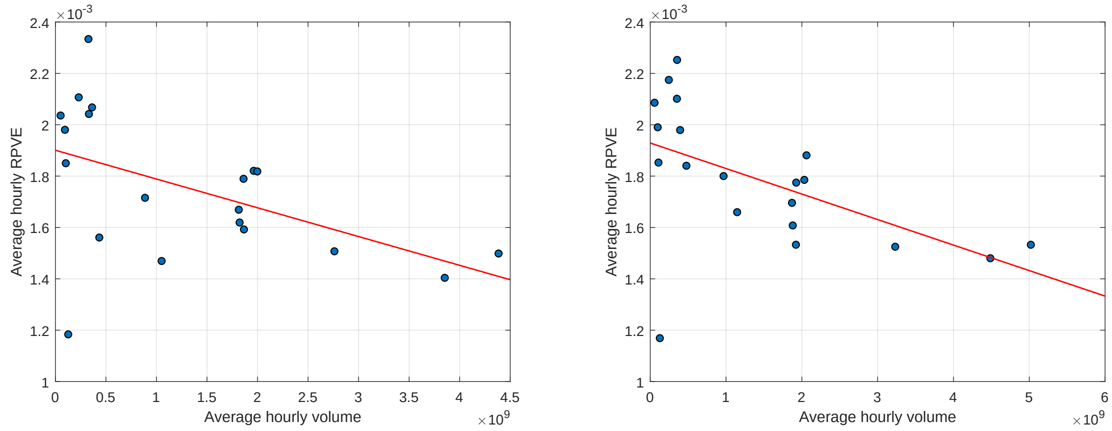
Figure 10: The figure on the right plots the trading volume measured by EBS data (z-axis), the FX price (y-axis) and the time at 15 seconds frequency around the time of the Fed announcement (18:00 GMT).

defined as $\widetilde{r}_i^{x|y;z} := \Delta p_i^{x|z} + \Delta p_i^{z|y}$, is now given by

$$\widetilde{r}_i^{x|y;z} = \phi_i^x - \phi_i^y + \frac{1}{J}\sum_{j=1}^{J}\psi_{i,j}^{x|z} + \frac{1}{J}\sum_{j=1}^{J}\psi_{i,j}^{z|y},$$

where the the term $\phi_i^x - \phi_i^y$ is the same for both $\widetilde{r}_i^{x|y;z}$ and $r_i^{x|y}$. Hence, we look at the dependence between the arbitrage errors and the trading volume when conditioning on the event of common directional news, identified as a co-jump. In particular, the relation between the average volume $\bar{v}_t^{x|y;z} = (v_t^{x|z} + v_t^{z|y})/2$ and the arbitrage error variability, as measured by RVPE$^{x|y;z}$, is depicted in Figure 11. The figure clearly displays a negative cross-sectional relation between the average trading volume on the $x|z$ and $z|y$ FX rates and the no-arbitrage pricing error variability. In other words, the pricing errors are higher when the triangular no-arbitrage returns are constructed via less liquid currencies, such as SEK and NOK. A notable exception is given by DKK, which has a small average mispricing error variability, RVPE$^{EUR|USD,DKK}$, associated with small average trading volume on $v^{EUR|DKK}$ and $v^{EUR|DKK}$. This evidence can be explained again by the fixed exchange rate policy adopted by the Danish central bank, which does not fully reveal the commonality between $v^{EUR|DKK}$ and $v^{EUR|DKK}$.

When conditioning on the arrival of a large common news on the main individual currencies EUR, JPY, USD and GBP (right panel), the average mispricing error on the y-axis increases relatively to the left panel, suggesting that big news arrivals prompt price adjustment processes on individual currencies that can generate larger price dispersion and mispricing errors. However, the negative relation between magnitude of the mispricing and trading volume is maintained. We formally test whether the chances of mispricing are higher for less liquid currencies in re-



(a) Unconditional

(b) Conditional to co-jump

Figure 11: Trading volume and pricing error variability. The figures show the scatter of the average volume on the FX rates (x—z and z—y) constituting the triangular no arbitrage relation (x-axis) versus the average triangular pricing error variability, as measured by RPVE$^{x|y;z}$,(y-axis) for 20 combination of currencies x, y and z. The left panel reports the unconditional relation, while the right panel is conditional to the event of a co-jump on the individual currencies, EUR, JPY, USD and GBP. The line represents the least squares fit.

action to directional FX movements measured by co-jumps. To carry out this test, we consider the following panel regression with fixed effects

$$RPVE_{l,t}^{x|y;z} = \alpha_l + \beta \overline{v}_{l,t}^{x|y;z} + \theta CJ_t + \varphi BA_{l,t} + \gamma_h h_t + \gamma_w w_t + \varepsilon_{l,t}, \tag{20}$$

The term $\overline{v}_{l,t}^{x|y;z}$ is the aggregate volume on the FX rates $x|z_n$ and $z_n|y$ for $n = 1, \ldots N$ currencies. Our sample consists of $N = 10$ currencies and allows us to consider $L = 20$ combinations of x, y and z_n.[26] Table 11 reports the parameter estimates of (20) based on the sample of $L = 20$ combinations of FX rates and $T = 30720$ hours (24×1280 days). The results confirm our prediction, that is, a negative relation between mispricing errors and average volume, which is robust to the inclusion of the relative bid-ask spread as a control for transaction costs (where parameter φ is found significantly positive in all cases). As it also emerges from Figure 11, the co-jumps events are associated with a significant increase in the average level of mispricing ($\theta > 0$). In sum, our results support the idea that liquidity begets price efficiency by systematically reducing pricing errors and facilitating the information processing.

[26]The combinations are: USDAUD/EURAUD, USDSEK/EURSEK, USDNOK/EURNOK, USDCHF/EURCHF, USDCAD/EURCAD, USDJPY/EURJPY, USDGBP/EURGBP, USDDKK/EURDKK, USDAUD/GBPAUD, USDCAD/GBPCAD, USDJPY/GBPJPY, USDCAD/JPYCAD, USDAUD/JPYAUD, EURCAD/GBPCAD, EURJPY/GBPJPY, EURCHF/GBPCHF, EURCAD/JPYCAD, USDAUD/JPYAUD, GBPAUD/JPYAUD, GBPCAD/JPYCAD.

	FE	PO	FE	PO	FE	PO	FE	PO
Volume	0.1164^b	-0.0992^a	-0.3713^a	-0.0923^a	-0.3780^a	-0.0992^a	-0.1818^a	0.0154^a
Bid-Ask	–	–	1.1652^a	0.8694^a	1.1605^a	0.8649^a	0.8379	0.6090^a
CJ	–	–	–	–	0.0003^a	0.0004^a	0.0005^a	0.0006^a
Hourly	no	no	no	no	no	no	yes	yes
Weekly	no	no	no	no	no	no	yes	yes
AR(1)	no	no	no	no	no	no	yes	yes

Table 11: No-arbitrage pricing errors and volume. Panel regression with fixed effect (FE) and pooling (PO). The dependent variable is the triangular pricing error accumulated at the hourly horizon for 20 combinations of FX rates. The regressors are the hourly aggregate trading volume of the two indirect FX rates (Volume) of the triangular arbitrage, the average relative bid-ask spread (Bid-Ask) of the direct FX rate, the dummy variable of the co-jump index on its own (CJ) and interacted with the aggregated trading volume (CJ-Volume) as well as hourly and day-of-the week dummies (Hourly, Weekly respectively). The superscripts a, b and c indicate significance at 1%, 5% and 10% significance level respectively.

5 Conclusion

We provide a unified model for asset prices, trading volume, and volatility. The model is built in continuous-time and allows for multi-asset framework. We apply it to currency markets in which arbitrage conditions tie foreign exchange (FX) rates. Our model outlines new properties of the FX market, including the relationships between trading volume and volatility of direct and arbitrage-related (or synthetic) FX rates. Moreover, it provides a theoretical foundation for common patterns (commonality) of trading volume, volatility, and illiquidity across currencies and time, and an intuitive closed-form solution for measuring illiquidity in the spirit of Amihud (2002).

We test the empirical predictions from our model using new and unique data representative of the global FX spot market. A distinguishing characteristic of our data set is that it includes granular and intraday data representative for the global FX trading volume. As predicted by our model, three main empirical findings arise: First, the difference in market participants' beliefs (disagreement) is the common source of trading volume and volatility. Second, our FX Amihud measure is effective in gauging FX illiquidity. Third, we find strong commonalities in FX volume, volatility, and illiquidity across time and FX rates. Consistent with the market adage that ”liquidity begets liquidity”, we find that more liquid currencies reveal stronger commonality in liquidity. Furthermore, we find that liquidity begets price efficiency, in the sense that more liquid currencies obey more to the triangular arbitrage condition. This finding is also true when ”big news” impacts FX markets, that is, more liquid currencies are less prone to arbitrage deviations.

Several implications emerge from our study. First, by shedding light on the intricate interrelations between FX rates, volume, and volatility, our work should support an integrated analysis of FX rate evolution and risk. Our work also offers a straightforward method to measure FX illiquidity and commonality. For investors, these insights should increase the efficiency of trading and risk analysis. For policymakers, our work highlights the developments of FX global volume, volatility, and illiquidity across time and currencies, which can be important for the implementation of monetary policy and financial stability.

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A Proofs

A.1 Proof of Proposition 1

The log-return and volume at trade i are given by

$$r_i^{x|y} := \Delta p_i^{x|y} = \frac{1}{J}\sum_{j=1}^{J} \Delta p_{i,j}^{x|y,*}, \tag{21}$$

and the volume at i-th trade is

$$v_i^{x|y} = \frac{\xi^{x|y}}{2}\sum_{j=1}^{J} |\Delta p_{i,j}^{x|y,*} - \frac{1}{J}\sum_{j=1}^{J} \Delta p_{i,j}^{x|y,*}|. \tag{22}$$

Based on the return on the i-th interval, we can consider the realized variance, defined as $RV^{x|y} = \sum_{i=1}^{I} (r_i^{x|y})^2$ with $\delta = 1/I > 0$, as introduced by Andersen and Bollerslev (1998). Following Barndorff-Nielsen and Shephard (2002b,a), taking the limit for $\delta \longrightarrow 0$ (that is $I \longrightarrow \infty$), we get

$$p\lim_{I\longrightarrow\infty} RV^{x|y} = \frac{1}{J^2}\mathcal{V}_{x|y}, \tag{23}$$

where $\mathcal{V}_{x|y} = \sum_{j=1}^{J} \mathcal{V}_{x|y,j}$ is the variation of the FX rate on the unit interval generated by the aggregated individual components of $r^{x|y}$. The term $\mathcal{V}_{x|y,j} = \int_0^1 \left(\sigma_j^{x|y}(s)\right)^2 ds$ is the *integrated variance* associated with the j-th trader's specific component. The term $\mu_j(t)$ does not enter in the expression of $V_{x|y,j}$ since the magnitude of the drift, when measured over infinitesimal intervals, is dominated by the diffusive component of $\Delta p_j^{x|y,*}(t)$ driven by the Brownian motion. Following Barndorff-Nielsen and Shephard (2003), for a given $\delta > 0$ we can also define the realized power variation of order one (or realized absolute variation) as $RPV^{x|y} = \sum_{i=1}^{I} |r_i|$. By the properties of the super-position of independent SV processes,[27] the limit for $\delta \longrightarrow 0$ of $RPV^{x|y}$ is

$$p\lim_{I\longrightarrow\infty} \delta^{1/2} RPV^{x|y} = \sqrt{\frac{2}{\pi}}\mathcal{S}_{x|y}, \tag{24}$$

where $\mathcal{S}_{x|y} = \int_0^1 \overline{\sigma}^{x|y}(s)ds$ is the integrated average standard-deviation, where the latter is defined as $\overline{\sigma}^{x|y}(t) = \frac{1}{J}\sqrt{\sum_{j=1}^{J} \sigma_j{}^{x|y^2}(t)}$. Given equation (22), the aggregated volume of $x|y$ on a unit (daily) interval is $v^{x|y} = \sum_{i=1}^{I} v_i^{x|y}$, and letting $I \longrightarrow \infty$, we get

$$p\lim_{I\longrightarrow\infty} \delta^{1/2} v^{x|y} = \frac{\xi^{x|y}}{2}\sqrt{\frac{2}{\pi}}\overline{\mathcal{S}}_{x|y}, \tag{25}$$

[27] Similarly to Barndorff-Nielsen and Shephard (2002b), $\overline{\Delta p}_i^{x|y,*}(t) = \frac{1}{J}\sum_{j=1}^{J} \Delta p_{i,j}^{x|y,*}$ is equivalent in law to $\overline{\Delta p}_i^{x|y,*} = \int_{\delta(i-1)}^{\delta i} \overline{\sigma}^{x|y}(t)dW^{x|y,*}(t)$, where $\overline{\sigma}^{x|y}(t) = \frac{1}{J}\sqrt{\sum_{j=1}^{J} \sigma_j{}^{x|y^2}(t)}$.

with $\overline{\mathcal{S}}_{x|y} = \frac{1}{J}\sum_{j=1}^{J}\int_0^1 \mathcal{J}_j^{x|y}(s)ds$, where $\mathcal{J}_j^{x|y}(t) = \sqrt{(J-1)^2\sigma_j^{x|y^2}(t) + \sum_{s\neq j}\sigma_s^{x|y^2}(t)}$.

A.2 Proof of Proposition 2

Given Proposition 1, we get that

$$p\lim_{I\to\infty} A^{x|y} = \frac{2\mathcal{S}_{x|y}}{\xi^{x|y}\overline{\mathcal{S}}_{x|y}}, \tag{26}$$

which reflects the ratio of the total average standard deviation carried by each trader. Under homogeneity of the traders, i.e. $\sigma_j^{x|y}(t) = \sigma^{x|y}(t)$, we get that

$$\overline{\mathcal{S}}_{x|y} = J\sqrt{J-1}\mathcal{S}_{x|y}, \tag{27}$$

and Proposition 2 follows directly.

In the extreme case of only one observation per trading period $I = \delta = 1$, the illiquidity measure in (4) reduces to the original Amihud index (up to the rescaling by $\sqrt{2/\pi}$),

$$A^{x|y,*} = \frac{|r|^{x|y}}{v^{x|y}}, \tag{28}$$

for which it is not trivial to obtain an expression as a function of the structural parameters analogous to the one in (5). For instance, the expected value of $|r|^{x|y}$ under Gaussianity is proportional to the daily (constant) volatility parameter, i.e. $E\left(|r|^{x|y}\right) = \sigma\sqrt{\frac{2}{\pi}}$, where $\sigma = \sqrt{Var(\Delta p^{x|y})/J}$ in the original MDH theory. In the classic framework, inference on the structural parameters is performed through GMM by relying on the unconditional moments of the observable quantities, which depend on the underlying (unobservable) information flow (see, e.g., Richardson and Smith (1994); Andersen (1996)). Therefore, inference on the structural parameters becomes more precise as we adopt moment conditions based on high-frequency data (see, e.g., Li and Xiu (2016)).

A.3 Proof of Proposition 3

By imposing the no-arbitrage restriction as in Brandt and Diebold (2006), it follows from (7) that the squares of the synthetic returns at the i-th trade can be written as

$$(\widetilde{r}_i^{x|y,z})^2 = (r_i^{x|z} + r_i^{z|y})^2 = (r_i^{x|z})^2 + (r_i^{z|y})^2 + 2r_i^{x|z}r_i^{z|y}.$$

The synthetic return can be expressed as $\widetilde{r}_i^{x|y,z} = \overline{\Delta p_i}^{x|z,*} + \overline{\Delta p_i}^{z|y,*}$, so that we can define the *synthetic* realized variance as $\widetilde{RV}^{x|y,z} = \sum_{i=1}^{I}(\widetilde{r}_i^{x|y,z})^2$. By letting $I \to \infty$, we get

$$p\lim_{I\to\infty}\widetilde{RV}^{x|y,z} = \frac{\mathcal{V}_{x|z} + \mathcal{V}_{z|y} + 2\mathcal{CV}_{x|z,z|y}}{J^2}, \tag{29}$$

where $\mathcal{V}_{x|z} = \sum_{j=1}^{J} \int_0^1 \left(\sigma_j^{x|z}(s)\right)^2 ds$ and $\mathcal{V}_{z|y} = \sum_{j=1}^{J} \int_0^1 \left(\sigma_j^{z|y}(s)\right)^2 ds$ are the components of the return variation generated by the cumulative individual variations of the reservation prices on $x|z$ and $z|y$. The term $C\mathcal{V}_{x|z,z|y}$ is given by

$$C\mathcal{V}_{x|z,z|y} = \sum_{i=1}^{I} \left(\sum_{j=1}^{J} \int_{\delta(i-1)}^{i\delta} \sigma_j^{x|z}(s)\sigma_j^{z|y}(s)\rho_j^{x|z,z|y}(s)ds \right),$$

where $\rho_j^{x|z,z|y}(t) = Corr\left(dW_j^{x|z}(t), dW_j^{z|y}(t)\right)$ is the correlation between the individual components on $x|z$ and $z|y$. All the other covariance terms are zero due to independence. Furthermore,

$$p\lim_{I\to\infty} \delta^{1/2}\widetilde{RPV}^{x|y,z} = \sqrt{\frac{2}{\pi}}\widetilde{\mathcal{S}}_{x|z,z|y}, \tag{30}$$

where $\widetilde{\mathcal{S}}_{x|z,z|y} = \int_0^1 \bar{\tilde{\sigma}}^{x|z,z|y}(s)ds$ is the integrated synthetic average standard-deviation, where the latter is defined as $\bar{\tilde{\sigma}}^{x|z,z|y}(t) = \frac{1}{J}\sqrt{\sum_{j=1}^{J} \tilde{\sigma}_j^{x|z,z|y^2}(t)}$, where

$$\tilde{\sigma}_j^{x|z,z|y^2}(t) = \sigma_j^{x|z^2}(t) + \sigma_j^{z|y^2}(t) + 2\sigma_j^{x|z}(t)\sigma_j^{z|y}(t)\rho_j^{x|z,z|y}(t).$$

For what concerns the trading volume, for the i-th trade on $x|y$ and $x|y$ we have

$$v_i^{x|z} = \frac{\xi^{x|z}}{2}\sum_{j=1}^{J} |\Delta p_{i,j}^{x|z,*} - \Delta p_i^{x|z}|, \quad v_i^{z|y} = \frac{\xi^{z|y}}{2}\sum_{j=1}^{J} |\Delta p_{i,j}^{z|y,*} - \Delta p_i^{z|y}|.$$

Moreover, by the triangular no-arbitrage, $\Delta\tilde{p}_{i,j}^{x|z,*} = \Delta\tilde{p}_{i,j}^{x|z,*} + \Delta\tilde{p}_{i,j}^{z|y,*}$ and $\Delta\tilde{p}_i^{x|z,*} = \Delta\tilde{p}_{i,j}^{x|z,*} + \Delta\tilde{p}_{i,j}^{z|y,*}$, so that the *synthetic* volume of $x|y$ by interaction with z is given by

$$\widetilde{v}_i^{x|y,z} = \frac{\xi^{x|y}}{2}\sum_{j=1}^{J} |\Delta\tilde{p}_{i,j}^{x|z,*} - \overline{\Delta\tilde{p}}_i^{x|z,*} + \Delta\tilde{p}_{i,j}^{z|y} - \overline{\Delta\tilde{p}}_i^{z|y,*}|, \tag{31}$$

which involves quantities that cannot be directly observed. However, by letting $I \to \infty$, we get

$$p\lim_{I\to\infty} \delta^{1/2}\tilde{v}^{x|y,z} = \frac{\xi^{x|y}}{2}\sqrt{\frac{2}{\pi}}\overline{\overline{\mathcal{S}}}_{x|z,z|x}, \tag{32}$$

where $\overline{\overline{\mathcal{S}}}_{x|z,z|y} = \frac{1}{J}\sum_{j=1}^{J} \int_0^1 \tilde{\sigma}_j^{x|z,z|y}(s)ds$, and

$$\tilde{\sigma}_j^{x|z,z|y}(t) = \sqrt{\sigma_j^{x|z^2}(t) + \sigma_j^{z|y^2}(t) + 2\sigma_j^{x|z}(t)\sigma_j^{z|y}(t)\rho_j^{x|z,z|y}(t)}. \tag{33}$$

Equation (33) highlights that the synthetic volume reflects the aggregated trader-specific components on the individual FX rates, $x|z$ and $z|y$, as well as their aggregated correlation as measured by $\rho^{x|z,z|y}$, which reflects the correlation between $\Delta p_j^{x|z,*}$ and $\Delta p_j^{z|y,*}$. It also follows that the synthetic illiquidity, defined as $\tilde{A}^{x|y,z} = \frac{\widetilde{RPV}^{x|y,z}}{\tilde{\nu}^{x|y,z}}$ is such that

$$p\lim_{I\to\infty} \tilde{A}^{x|y,z} = \frac{2\widetilde{\mathcal{S}}_{x|z,z|y}}{\xi^{x|y}\overline{\overline{\mathcal{S}}}_{x|z,z|x}}, \tag{34}$$

which, again reduces to $p\lim_{I\to\infty} \tilde{A}^{x|y,z} = \frac{2}{\xi^{x|y}\sqrt{J(J-1)}}$ under homogeneity of traders, that is $\sigma_j^{x|z}(t) = \sigma^{x|z}(t)$, $\sigma_j^{z|y}(t) = \sigma^{z|y}(t)$ and $\rho_j^{x|z,z|y}(t) = \rho^{x|z,z|y}(t)$.

Finally, by equating $A^{x|y}$ and $\tilde{A}^{x|y,z}$, we get that $\widetilde{\nu}^{x|y,z} = \kappa^{x|y,z}\nu^{x|y}$, where the term $\kappa^{x|y} = \frac{\widetilde{RPV}^{x|y,z}}{RPV^{x|y}}$ represents a proportionality factor, which is equal to 1 in absence of arbitrage violations. Since $\widetilde{r}_i^{x|y,z} = \widetilde{r}_i^{x|y,z} - r_i^{x|y} + r_i^{x|y} = pe_i^{x|y,z} + r_i^{x|y}$, where $pe_i^{x|y,z} = \tilde{r}_i^{x|y,z} - r_i^{x|y}$ is the triangular arbitrage violation (pricing error) at time i, we get that $1 \leq \kappa^{x|y,z} \leq 1 + \frac{RPVE^{x|y,z}}{RPV^{x|y}}$, where $RPVE^{x|y,z} = \sum_{i=1}^{I} |pe_i^{x|y,z}|$ measures the variability of the pricing error. Furthermore, by substituting the expression of $\kappa^{x|y,z}$ in the equation of $\tilde{A}^{x|y,z}$, it follows that

$$A^{x|y} \leq \widetilde{A}^{x|y,z} \leq \frac{1}{\kappa^{x|y,z}}\left(A^{x|y} + \frac{RPVE^{x|y,z}}{\upsilon^{x|y}}\right), \tag{35}$$

B Alternative illiquidity measures

Liquidity measures are computed using the EBS data set from January 1, 2016, to December 31, 2016. In a first step, we separate deals and quotes data, and we exclude data from Friday 10 PM to Sunday 10 PM GMT. Subsequently, the order book quotes series is collapsed such that for each snapshot, we have a single observation. To this end, we select both the best available quotes, that is the highest bid, and lowest ask, and compute volume-weighted ask and bid quotes, so as to keep track of the order book depth. Notice that after the first step, we obtain an irregularly spaced time series. This is because EBS includes new observations only whenever new information become available. So, missing values are replaced by the previous (last available) observation to obtain evenly space time stamps (frequency one tenth of a second).

For each day, we compute the following set of illiquidity measures: high-frequency Amihud, effective cost, bid-ask spread, relative bid-ask spread, Corwin & Schlutz, CHL, Roll estimate, price impact and traditional (low-frequency) Amihud. Intra-day quoted (relative) spread and effective cost are firstly computed at a high frequency. Afterwards, the average for the day is computed. Notice that the effective cost is defined as the absolute distance between the transaction price and the prevailing mid price.

The traditional (low-frequency) Amihud measure is defined as the absolute value of the daily log-return divided by the total trading volume in that day, again using trade data. Roll's measure is computed for each day as the daily autocovariance between consecutive price changes, that is from trade to trade. The measure is computed using high-frequency trade data, which are not generally evenly spaced. Further, no adjustment for the direction has been made.

The Corwin & Schultz (CS) spread estimator, and the CHL measure have been computed for each day using hourly maximum, minimum, and closing prices. Hourly closing prices have been defined as the last observation available for a given hour. A daily measure is then extracted by averaging out the 23 hourly spread estimates. Notice, however, that both measures have been initially proposed as a monthly spread estimator, which is obtained with daily max, min and closing price series. Therefore, it must be taken into account that intra-day seasonalities might potentially affect the measurement. The price impact coefficient, labeled as γ, is computed for each day in a trade-by-trade fashion. We regress the signed square root of the volume against the log-returns, that is

$$r_t = \gamma \cdot sign \cdot \sqrt{vol_t}, \tag{36}$$

where *sign* takes on the value -1 or 1 depending on whether the trade was initiated by a seller or by a buyer, respectively[28]. Notice that this also conveys the definition adopted for order flow. Recall also that time intervals between consecutive deals are not constant.

[28] More precisely, according to EBS data manual for deal records, a 0 indicates that the transaction is seller initiated. Conversely, a 1 corresponds to buyer initiated trades. To compute signed volumes, they are mapped to -1 and 1, respectively.