TRADING VOLUME, ILLIQUIDITY AND COMMONALITIES IN FX MARKETS

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WORKING PAPERS ON FINANCE NO. 2018/23

SWISS INSTITUTE OF BANKING AND FINANCE (S/BF – HSG)

NOVEMBER 15, 2018
Trading Volume, Illiquidity and Commonalities in FX Markets*

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November 15, 2018

Abstract

We provide a unified model for foreign exchange (FX), trading volume, and volatility in a multi-currency environment. Tied by arbitrage conditions, FX rates are determined by common information and trader-specific components generating heterogeneous reservation prices thus inducing trading. Our model outlines new properties including volume-volatility relationships between direct and synthetic FX rates. It also provides a theoretical foundation for commonalities of volume, volatility, and illiquidity across currencies and time, and an intuitive closed-form solution for the price impact measure. Using unique (intraday) data representative for the global FX spot market, the empirical analysis validates our theoretical predictions.

Keywords: FX Trading Volume, Volatility, Illiquidity, MDH, Commonalities, Co-Jumps

J.E.L. classification: C15, F31, G12, G15

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*We are grateful to Tim Bollerslev, Massimiliano Caporin, Federico Carlini, Nina Karnaukh, Lukas Menkhoff, Federico Nucera, Paolo Pasquariello, Mark Podolskij, Roberto Renó, Fabricius Somogyi and Vladyslav Sushko for their relevant remarks on our work. We would also like to thank the participants at 7th Workshop on Financial Determinants of FX Rates Norges Bank, at the 2018 SoFiE conference, at the 2018 DEDA conference and at seminars at City Hong Kong University and University of the Balearic Islands for useful comments.

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1 Introduction

Since the demise of the post-war Bretton Woods system in the 1970s, the international financial system has witnessed a growing capital mobility and wider movements of foreign exchange (FX) rates. In such a regime of floating FX rates and open economies, anyone dealing with a currency other than that of the base currency is concerned with the (adverse) evolution of FX rates, their volatility, and market dynamics such as trading volume and illiquidity. It is thus a natural question how FX rates, volatility, and trading volume interrelate.

In this paper, we provide a simple theoretical framework to jointly explain FX rates, trading volume, and volatility in a multi-currency environment. Tied together by arbitrage conditions, FX rates movements are determined by common information and trader-specific components generating discrepancies in the reservation prices thus inducing trading. In such a unified setting, our model outlines new properties of the FX market including the relationships between trading volume and volatility of direct and arbitrage-related (or synthetic) FX rates. It also provides the theoretical grounding for common patterns (commonality) of trading volume, volatility, and illiquidity across currencies and time, and an intuitive closed-form solution for measuring illiquidity in terms of price impact (Amihud, 2002). Using new and unique intraday data representative for the global FX spot market, the empirical analysis validates our theoretical predictions.

The joint analysis of FX volume and volatility is important for at least three reasons. First, the FX market is the largest financial market in the world with USD 5.1 trillion of daily traded volume (Bank of International Settlements, 2016). Despite its importance and apparent enormous liquidity, an in-depth understanding of FX volume is still missing. This can be explained by at least two reasons. On the one hand, FX rates are commonly traded over-the-counter, which is notoriously opaque and fragmented.\footnote{The microstructure of the FX market is explained in detail in e.g. Lyons (2001) and King et al. (2012). The recent developments of the FX markets are discussed in Rime and Schrmpf (2013) and Moore et al. (2016).} On the other hand, there has been a paucity of comprehensive volume data at a global scale. Second, FX rates are key for pricing many assets including international stocks, bonds, and derivatives, and for assessing their risk. They are also relevant for policy making such as conducting (unconventional) monetary policy and FX interventions. A better understanding of whether and how FX
volume, volatility and illiquidity determine FX rates can improve all these tasks. Third, distressed markets such as currency crises are characterized by sudden FX rates movements, drops in liquidity, and raises in volatility. It could thus be supportive of financial stability to highlight the sources of volatility and illiquidity, how they reinforce each other, and across currencies.

Our analysis proceeds in two steps: theory and empirics. Our theory builds upon the Mixture-of-Distribution Hypothesis (MDH) of Clark (1973) and Tauchen and Pitts (1983), in which the evolution of the equilibrium price is driven by the arrival of new information and by the trading activity, see also the survey in Karpoff (1987) and the empirical analysis in Andersen (1996).\(^2\) The trading volume is induced by the deviation of individual agent’s reservation prices from the observed market price. We extend the MDH theory in three ways. First, we provide a continuous-time version of the model, which allows us to derive consistent measurements of the underlying unobservable quantities, such as volatility. The combination of volatility and volume provides a closed-form intuitive expression for measuring illiquidity in terms of price impact such as the widespread proxy proposed in Amihud (2002). More specifically, illiquidity decreases with market depth and the number of market participants. Second, agents trade in a multi-currency environment in which direct FX rates are tied to cross rates by triangular arbitrage conditions implying that direct and synthetic rates must equate in equilibrium, while the trading volume reflects the dependence on the aggregated information flows across FX rates. This leads us to the definition of synthetic volume, which is a key ingredient to study the commonalities and the degree of integration between the trading volume and volatility on the global FX market. Third, we extend the MDH setting assuming that the common news component is currency-specific and associated with large variations in the FX rates generated by public news, e.g. around various macroeconomic announcements (see e.g. Bollerslev et al., 2016). This component measures a different source of the variability of FX rates compared to those associated with the extent of the disagreement between traders on the reservation price.

Set against the background given by the extended MDH theory, we provide new empirical evidence on the volume-volatility relationship in the FX market and we test the theoretical

\(^2\) As noted by Bauwens et al. (2006), only one out of the 19 studies cited by Karpoff (1987) was on exchange rates.
predictions derived from our model. To do this, we utilize two data sets. First, trading volume data come from CLS Bank International (CLS), which operates the largest payment-versus-payment (PVP) settlement service in the world. Hasbrouck and Levich (2017) provide a very comprehensive description of the CLS institutional setting and Gargano et al. (2017) show that CLS data cover around 50% of the FX global turnover compared to the BIS triennial surveys. Trading volume is measured at the hourly level, across 31 currency pairs over a 5-year period from November 2011 to November 2016.\footnote{The entire set includes 33 currency pairs but the Hungarian forint (HUF) joined the CLS system later. Therefore, EURHUF and USDHUF are available only since 07 November 2015.} For the same FX panel, we obtain intraday spot rates from Olsen data. For each FX rate and each minute of our sample, we observe the following quotes: ask, bid, low, high, close, and midquote. By merging these two data sets, we can analyze the hourly time series of trading volume, realized volatility, and FX rate and bid-ask spread evolutions.

To test the empirical predictions from our theoretical model, we carry out the following analysis. First, we perform a descriptive analysis that uncovers the following (new) stylized facts: (i) FX trading volume and illiquidity follow intraday patterns and seasonalties indicating market fragmentation across geographical areas; (ii) trading volume and volatility are linked by a very strong positive relationship both within and across FX rates, as predicted by our theory. Second, we provide more direct evidence that trading volume and volatility are governed by the heterogeneous beliefs between the agents, measured as in Beber et al. (2010). Third, we examine how FX trading volume and liquidity behave when a central bank determines the FX rate. Specifically, we consider the case of the Swiss National Bank (SNB) and in particular, the regime during which the SNB guaranteed the cap at 1.2 CHF per EUR. Consistent with our theory, we find that both volatility and illiquidity are particularly low in this period, whereas trading volume is high due to the active role of the Swiss central bank. Fourth, we perform a comprehensive time-series analysis to study the systematic relations between FX rate returns, trading volume, and volatility across currencies. As prescribed by our model, we find the following systematic patterns: substantial commonality in trading volume, volatility, and illiquidity and strong co-movements between volatility and volume. Using various methods including the connectedness index of Diebold and Yilmaz (2014), we find that the strength of the commonality increases with the degree of market liquidity of the
FX rate: the volume and volatility of the most liquid currency pairs, e.g. EURUSD, JPYUSD and GBPUSD tend to be characterized by a stronger degree of commonality compared to the more illiquid currency pairs. Finally, we conclude our empirical investigation by studying the tendency of direct and indirect FX rates to deviate from the triangular arbitrage condition, that is, a pricing error proxy. We find that the occurrence of pricing errors decreases with FX trading volume and liquidity. Furthermore, illiquid FX rates are more affected by price misalignment, especially when important public news hit the FX market that we identify using the co-jumps theory proposed in Caporin et al. (2017). This again can be explained by our MDH theory since the size of the information flow on liquid currencies like USD, EUR and JPY is relatively larger and more integrated than that on smaller currencies like NOK and SEK.

Our paper contributes to the recent literature on trading, commonalities and liquidity in FX markets (e.g. Mancini et al., 2013 and Karnaukh et al., 2015). Most previous studies focus on specific aspects of FX liquidity such as transaction costs\(^4\) or order flow.\(^5\) Due to the paucity of comprehensive data on the FX global volume, the literature on trading volume is scant. Prior research has focused on the interdealer segment\(^6\) or proprietary data from some specific banks.\(^7\) Only with the recent access to CLS data, research on FX global volume at relatively high frequencies (e.g. daily) became possible. Fischer and Ranaldo (2011) look at global FX trading around central bank decisions. Hasbrouck and Levich (2017) measure FX illiquidity using volume and volatility data. Gargano et al. (2017) analyze the profitability of FX trading strategies exploiting the predictive ability of FX volume. The extension of the MDH to a continuous-time setting and to a multiple-asset framework serves the purpose of defining a theoretical foundation for (i) FX price determination in connection to FX volume

\(^4\)Transaction costs are typically measured in terms of bid-ask spreads that tend to increase with volatility. FX transaction costs in spot and future markets are studied in Bessembinder (1994), Bollerslev and Melvin (1994), Christiansen et al. (2011), Ding (1999), Hartmann (1999), Huang and Masulis (1999), Hsien and Kleidon (1996), Mancini et al. (2013).

\(^5\)Order flow, which is as the net of buyer-initiated and seller-initiated orders, is analyzed in the seminal paper by Evans and Lyons (2002). Among others, order flow is also studied in Bjønnes and Rime (2005), Berger et al. (2008), Frömmel et al. (2008), Breedon and Ranaldo (2013), Evans and Lyons (2002), Evans (2002), Mancini et al. (2013), Payne (2003), and Rime et al. (2010).

\(^6\)Prior research has studied volume data from the two main trading platforms, i.e. Electronic Broking Services (EBS) and Reuters. For instance, Evans (2002) uses Reuters D2000-1 data, Payne (2003) analyze data from D2000-2 while Mancini et al. (2013) and Chaboud et al. (2007) utilize data from EBS.

\(^7\)See Bjønnes and Rime (2005) and Menkhoff et al. (2016). Other sources of trading volume data are central banks or FX futures or forward contracts, see Bjønnes et al. (2003), Galati et al. (2007), Grammatikos and Saunders (1986), Levich (2012), and Bech (2012).
and illiquidity unconditionally and in reaction to public news, (ii) commonality of trading volume and volatility across currencies and time, (iii) and illiquidity in terms of price impact proxies such as in Amihud (2002). Although abstracting from some market “imperfections” such as liquidity frictions (Darolles et al., 2015 and Darolles et al., 2017), our model provides a straightforward theoretical background to FX price determination and commonalities in trading volume, volatility, and illiquidity within and across assets.

This paper is organized as follows. Section 2 presents the theoretical foundations for an unified analysis of volatility, volume and illiquidity on the FX rates, and their commonalities. Section 3 introduces the dataset and discusses summary statistics. Section 4 presents the empirical analysis of the implications of the model in several directions. Finally, Section 5 concludes the paper.

2 A unified model for FX rates, volatility and volume

We depart from the MDH of Tauchen and Pitts (1983), which provides a stylized representation of the supply/demand mechanism on the market at the intraday level. Let’s first consider a world with two currencies, \( x \) (base) and \( y \) (quote). We assume that the market consists of a finite number \( J \geq 2 \) of active traders, who take long or short positions the FX rate \( x|y \). Within a given trading period of unit length (e.g. an hour, a day, a week), the market for the currency pair \( x|y \) passes through a sequence of \( i = 1, \ldots, I \) equilibria. The evolution of the equilibrium price is motivated by the arrival of new information to the market. At intra-period \( i \), the desired position of the \( j \)-th trader \( (j = 1, \ldots, J) \) on the FX rate \( x|y \) is given by

\[
q_{i,j}^{x|y}(t) = \xi^{x|y}(p_{i,j}^{x|y,*} - p_{i}^{x|y}), \quad \xi^{x|y} > 0
\]  

(1)

where \( p_{i,j}^{x|y,*} \) is the reservation price of the \( j \)-th trader and \( p_{i}^{x|y} \) is the current market price (both measured in logs). The reservation price of each trader might reflect individual preferences, liquidity issues, asymmetries in information sets and/or different expectations about the fundamental values of the FX rate. In general, the reservation price can deviate from the market price because of idiosyncratic reasons inducing the \( j \)-th trader to trade. The term \( \xi^{x|y} \) is a positive constant measuring the resilience of the market: The larger \( \xi^{x|y} \), the larger quantities
of $x$ can be exchanged for $y$ (and vice versa) for a given difference $p_{i,j}^{x|y,*} - p_{i}^{x|y}$. In other words, $\xi_{x|y}$ measures the capacity of the market to allow large quantities to be exchanged on the intersection between the demand and supply side. Figure 1 illustrates the demand/supply mechanism of the $j$-th trader for the $x|y$ FX rates. If $(p_{i,j}^{x|y,*} - p_{i}^{x|y}) > 0$, this means that the $j$-th trader believes that the equilibrium trading price of $x|y$ is too low, i.e. currency $x$ should be more expansive relatively to $y$, so he will buy $x$ and sell $y$. On the contrary, if $(p_{i,j}^{x|y,*} - p_{i}^{x|y}) < 0$, the $j$-th trader will buy $y$ and sell $x$. The amount associated with a unit change of $p_{i,j}^{x|y,*} - p_{i}^{x|y}$ is given by the slope $\xi_{x|y}$.

As new information arrives, the traders adjust their reservation prices, resulting in a change in the market price given by the average of the increments of the reservation prices. This means that the equilibrium condition is $\sum_j q_{i,j}^{x|y} = 0$, this implies that the average of the reservation prices clear the market, so that $p_{i}^{x|y} = \frac{1}{J} \sum_{j=1}^{J} p_{i,j}^{x|y,*}$ and

$$n_{i}^{x|y} = \frac{\xi_{x|y}}{2} \sum_{j=1}^{J} |\Delta p_{i,j}^{x|y,*} - \Delta p_{i}^{x|y}|,$$

where $\Delta p_{i,j}^{x|y,*} = p_{i,j}^{x|y,*} - p_{i-1,j}^{x|y,*}$ and $\Delta p_{i,j}^{x|y} = p_{i,j}^{x|y} - p_{i-1,j}^{x|y}$. The increments of the reservation
log-prices are given by

\[ \Delta p_{i,j}^{x|y} = \phi_i^{x|y} + \psi_{i,j}^{x|y}, \quad \text{with} \quad j = 1, \ldots, J, \]

where \( \phi_i^{x|y} \) is the common information component about the FX rate \( x|y \), stemming from public information events, such as those associated with central banks’ announcements. The common term \( \phi_i^{x|y} \) could also be related to events that trigger common directional expectations among the practitioners about a specific currency. The term \( \psi_{i,j}^{x|y} \) represents the investor’s specific component about the FX rate between \( x \) and \( y \). We assume that \( \phi_i^{x|y} \) and \( \psi_{i,j}^{x|y} \) are independent across \( i \) and \( j \).

The log-return and volume at trade \( i \) are given by

\[ r_i^{x|y} = \Delta p_i^{x|y} = \phi_i^{x|y} + \frac{1}{J} \sum_{j=1}^{J} \psi_{i,j}^{x|y}, \quad (2) \]

and the volume at \( i \)-th trade is

\[ v_i^{x|y} = \frac{e^{x|y}}{2} \sum_{j=1}^{J} |\psi_{i,j}^{x|y} - \bar{\psi}_i^{x|y}|, \quad (3) \]

where \( \bar{\psi}_i^{x|y} = \frac{1}{J} \sum_{j=1}^{J} \psi_{i,j}^{x|y} \). In the following, we will define a continuous-time version of the MDH which will allow us to ex-post measure the variability of the FX rates components in the limit for \( I \to \infty \), and relate it to the observed trading volume and its synthetic counterpart.

### 2.1 The MDH in continuous-time

We assume the following continuous time version of the MDH model to form the basis for volatility measurement, where the dynamics of the the investor-specific component about the FX rate is given by

\[ d\psi_{j}^{x|y}(t) = \sigma_{j}^{x|y}(t) dW_{j}^{x|y}(t), \quad j = 1, \ldots, J \quad (4) \]

where \( W_{j}(t) \) is a Wiener process that is independent between each trader, i.e. \( W_{i}(t) \perp W_{m}(t) \) \( \forall i \neq m \) and the term \( \sigma_{j}^{x|y}(t) \geq 0 \) is the stochastic volatility process of the \( j \)-th trader which is assumed to have locally square integrable sample paths. By allowing \( \sigma_{j}^{x|y} \) to be different across traders, we are implicitly introducing heterogeneity among them. This also reconciles with
the evidence of long-memory in volatility that is obtained by the superposition of different traders operating at different frequencies, see for instance the heterogeneous autoregressive model of Müller et al. (1997) and Corsi (2009). This setup is coherent with a representation of a frictionless market where each trader participates through its reservation price to the price discovery process by carrying new information. It follows that, on the $i$-th discrete sub-interval of length $\Delta = \frac{1}{I}$,

$$\psi_{i,j}^{x|y} = \int_{\Delta(i-1)}^{\Delta i} \sigma_j^{x|y}(s) dW_j^{x|y}(s).$$

(5)

Assume for the moment that the common news term is zero, i.e. $\phi_i^{x|y} = 0$. Based on the return on the $i$-th interval, we can consider the realized variance, defined as $RV_{x|y}^{x|y} = \sum_{i=1}^I (r_{i-1}^{x|y} - r_i^{x|y})^2$ with $\Delta = 1/I > 0$, as introduced by Andersen and Bollerslev (1998). Following Barndorff-Nielsen and Shephard (2002b,a), taking the limit for $\Delta \to 0$ we get

$$p \lim_{I \to \infty} RV_{x|y}^{x|y} = \frac{1}{f^2} V_{\psi_x|y},$$

(6)

where $V_{\psi_x|y} = \sum_{j=1}^J V_{\psi_x|y,j}$ is the variation of the FX rate on the unit interval generated by the aggregated individual components of $r_{x|y}$. The term $V_{\psi_x|y,j} = \int_0^1 \left( \sigma_j^{x|y}(s) \right)^2 ds$ is the integrated variance associated with the $j$-th trader’s specific component. Following Barndorff-Nielsen and Shephard (2003), for a given $\Delta > 0$ we can also define the realized power variation of order one (or realized absolute variation) as

$$RPV_{x|y}^{x|y} = \sum_{i=1}^I |r_i| = \sum_{i=1}^I \left| \frac{1}{J} \sum_{j=1}^J \int_{\Delta(i-1)}^{\Delta i} \sigma_j^{x|y}(s) dW_j^{x|y}(s) \right|.$$  

(7)

By the properties of the super-position of independent SV processes, the limit for $\Delta \to 0$ of $RPV_{x|y}^{x|y}(\Delta)$ is

$$p \lim_{I \to \infty} \Delta^{1/2} RPV_{x|y}^{x|y} = \sqrt{\frac{2}{\pi}} S_{\psi_x|y},$$

(8)

where $S_{\psi_x|y,j} = \int_0^1 \bar{\sigma}^{x|y}(s) ds$ is the integrated average standard-deviation, where the latter is

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8For ease of exposition, we assume that trades happen on an equally spaced and uniform grid, $i = 1, 2, \ldots, I$. This assumption can be relaxed allowing for random trading times.

9Similarly to Barndorff-Nielsen and Shephard (2002b), $\psi_i^{x|y}(t) = \frac{1}{2} \sum_{j=1}^J \psi_{i,j}^{x|y}$ is equivalent in law to $\psi_i^{x|y,*} = \int_{\Delta(i-1)}^{\Delta i} \bar{\sigma}^{x|y}(t) dW^{x|y,*}(t)$, where $\bar{\sigma}^{x|y}(t) = \frac{1}{J} \sqrt{\sum_{j=1}^J \sigma_j^{x|y}(t)}$. 


defined as $\sigma_{x|y}(t) = \frac{1}{J} \sqrt{\sum_{j=1}^{J} \sigma_{j}^{x|y}(t)^{2}}$. Given equation (3), the aggregated volume of $x|y$ on a unit (daily) interval is

$$\nu_{x|y} = \sum_{i=1}^{I} \nu_{i}^{x|y} = \frac{\xi_{x|y}}{2} \sum_{i=1}^{I} \sum_{j=1}^{J} \text{abs} \left( \int_{\Delta(i-1)}^{\Delta i} \sigma_{j}^{x|y}(s) dW_{j}^{x|y}(s) - \frac{1}{J} \sum_{j=1}^{J} \int_{\Delta(i-1)}^{\Delta i} \sigma_{j}^{x|y}(s) dW_{j}^{x|y}(s) \right).$$

Letting $I \to \infty$, we get

$$p \lim_{I \to \infty} \Delta^{1/2} \nu_{x|y} = \frac{\xi_{x|y}}{2} \sqrt{\frac{2}{\pi} \tilde{S}_{\psi_{x|y}}},$$

(9)

with $\tilde{S}_{\psi_{x|y}} = \frac{1}{J} \sum_{j=1}^{J} \int_{0}^{1} \tilde{\sigma}_{j}^{x|y}(s) ds$, where $\tilde{\sigma}_{j}^{x|y}(t) = \sqrt{(J - 1)^{2} \sigma_{j}^{x|y}(t)^{2} + \sum_{s \neq j} \sigma_{s}^{x|y}(t)^{2}}.$

### 2.2 Measuring FX Illiquidity

Analogously to Kyle (1985), we can define a continuous-time version of illiquidity capturing the price impact on $x|y$ as measured in Amihud (2002). The illiquidity measure is

$$A_{x|y} := \frac{RPV_{x|y}}{\nu_{x|y}},$$

(10)

and its limit for $I \to \infty$ is

$$p \lim_{I \to \infty} A_{x|y} = \frac{2S_{\psi_{x|y}}}{\xi_{x|y} \tilde{S}_{\psi_{x|y}}},$$

(11)

which is a positive constant inversely related to the slope of the supply/demand function, $\xi_{x|y}$. For a given difference between the reservation price and the market price, $A_{x|y}$ decreases as the slope of the demand/supply function increases. In particular, for large values of $\xi_{x|y}$ large volume would be associated with small variations between the prevailing price and the reservation price for each trader, thus signaling market depth and liquidity. Instead, when $\xi_{x|y} \to 0^{+}$, i.e. in the limiting case of a flat demand/supply function, the liquidity is minimal, since no actual trade takes place. Moreover, under the assumption of homogeneity of the traders, i.e. $\sigma_{j}^{2}(t) = \sigma^{2}(t) \forall j = 1, \ldots, J$, then the limit of the Amihud reduces to

$$p \lim_{I \to \infty} A_{x|y} = \frac{2}{\xi_{x|y} J \sqrt{(J - 1)}},$$

(12)

which shows how liquidity is inversely related to the number of traders on the market, $J \geq 2$. In the extreme case of only one observation per trading period $I = \Delta = 1$, the continuous-
time Amihud measure in (10) reduces to the original Amihud index (up to the rescaling by $\sqrt{2/\pi}$)

$$A^{x|y,*} = \frac{|r^{x|y}|}{\nu^{x|y}},$$

(13)

for which it is not trivial to obtain an expression as a function of the structural parameters analogous to the one in (12). For instance, the expected value of $|r^{x|y}|$ under Gaussianity is proportional to the daily (constant) volatility parameter in the classic MDH framework, i.e. $E(|r^{x|y}|) = \sigma \sqrt{2\pi}$, where $\sigma = \sqrt{\text{Var}(\phi) + \text{Var}(\psi)} / J$. In the classic MDH framework, inference on the structural parameters is performed through GMM by relying on the unconditional moments of the observable quantities which depend on the underlying (unobservable) information flow, see Richardson and Smith (1994) and Andersen (1996). The availability of high-frequency data coupled with the theory of quadratic variation makes the volatility and consequently the information flow measurable quantities. This means that inference on the structural parameters becomes more precise as we adopt moment conditions based on high-frequency data, see Li and Xiu (2016). Furthermore, well approximating unobservable variables (e.g. volatility) makes it easier to assess if the period-by-period variation in quantities like the observed illiquidity index, $A^{x|y}$, can be attributed to changes the market structure as reflected in breaks in the underlying structural parameters, such as the number of active traders or the liquidity parameter $\xi^{x|y}$, see the analysis in Section 4.2.

### 2.3 Commonalities in FX volume and volatility

Consider now a world with three currencies, $x$, $y$ and $z$. The market for the currency pairs $x|y$, $x|z$ and $z|y$ also passes through a sequence of $i = 1, \ldots, I$ equilibria and the evolution of the equilibrium price of each currency pair is motivated by the arrival of new information to the market. By the triangular no-arbitrage parity it must hold that

$$p_{i}^{x|y} = p_{i}^{x|z} + p_{i}^{z|y},$$

(14)
where $p_{x|z} = \sum_{j=1}^{J} p_{i|z}^{x|z}$ and $p_{z|y} = \sum_{j=1}^{J} p_{i|y}^{z|y}$. In analogy with the assumptions of the MDH, we have that

$$
\Delta p_{i,j}^{x|z,*} = \phi_{i}^{x|z} + \psi_{i,j}^{x|z}, \quad \Delta p_{i,j}^{z|y,*} = \phi_{i}^{z|y} + \psi_{i,j}^{z|y}, \quad j = 1, \ldots, J.
$$

so that the synthetic return on $x|y$ is given by

$$
\hat{r}_{i}^{x|y} = \phi_{i}^{x} + \phi_{i}^{y} + \frac{1}{J} \sum_{j=1}^{J} \psi_{i,j}^{x|z} + \frac{1}{J} \sum_{j=1}^{J} \psi_{i,j}^{z|y}.
$$

Assuming that the common information component on the rate $x|y$, can be disentangled into two currency-specific terms $\phi_{i}^{x}$ and $\phi_{i}^{y}$, with $\phi_{i}^{x|y} = \phi_{i}^{x} - \phi_{i}^{y}$, it follows that

$$
\bar{r}_{i}^{x|y} = \phi_{i}^{x} - \phi_{i}^{y} + \frac{1}{J} \sum_{j=1}^{J} \psi_{i,j}^{x|z} + \frac{1}{J} \sum_{j=1}^{J} \psi_{i,j}^{z|y},
$$

where the common information part of $\bar{r}_{i}^{x|y}$ is the same as for $r_{i}^{x|y}$, that is $\phi_{i}^{x} - \phi_{i}^{y}$. It follows that the MDH coupled with the triangular no-arbitrage relation on the FX rates, i.e. $r_{i}^{x|y} = \bar{r}_{i}^{x|y}$, prescribes that

$$
\frac{1}{J} \sum_{j=1}^{J} \psi_{i,j}^{x|y} = \frac{1}{J} \sum_{j=1}^{J} \psi_{i,j}^{x|z} + \frac{1}{J} \sum_{j=1}^{J} \psi_{i,j}^{z|y}, \quad (15)
$$

which means that, on average with respect to all investors, the individual components on the FX rate $x|y$ must be equal to the sum of the average individual components of the FX rates $z|y$ and $x|z$. By imposing the no-arbitrage restriction as in Brandt and Diebold (2006), it follows that the squares of the synthetic returns at the $i$-th trade can be written as

$$
(\bar{r}_{i}^{x|y})^{2} = (r_{i}^{x|z} + r_{i}^{z|y})^{2} = (r_{i}^{x|z})^{2} + (r_{i}^{z|y})^{2} + 2 r_{i}^{x|z} r_{i}^{z|y}.
$$

\footnote{In Section 4.4 we discuss a strategy to separately identify $\phi_{i}^{x}$ and $\phi_{i}^{y}$ based on a cross section of FX rates and provide an empirical validation of such an assumption.}
In terms of the MDH model of Tauchen and Pitts (1983), under the maintained assumption that $\phi_i^{y,z} = 0$, the synthetic return can be expressed as

$$ (\tilde{r}_i^{x|y})^2 = \left( \frac{1}{J} \sum_{j=1}^{J} \int_{\Delta(t-1)}^{\Delta t} \sigma_j^{x|z}(s) dW_j^{x|z}(s) + \frac{1}{J} \sum_{j=1}^{J} \int_{\Delta(t-1)}^{\Delta t} \sigma_j^{y|z}(s) dW_j^{y|z}(s) \right)^2 $$

so that can also define the synthetic realized variance as $\overline{RV}^{x|y} = \sum_{i=1}^{I} (\tilde{r}_i^{x|y})^2$, which converges to

$$ \lim_{l \to \infty} \overline{RV}^{x|y} = \frac{\nu^{x|z} + \nu^{y|z} + 2\nu^{x|z,y|z}}{J^2}, $$

where $\nu^{x|z} = \sum_{j=1}^{J} \int_{0}^{1} \left( \sigma_j^{x|z}(s) \right)^2 ds$ and $\nu^{y|z} = \sum_{j=1}^{J} \int_{0}^{1} \left( \sigma_j^{y|z}(s) \right)^2 ds$ are the components of the return variation generated by the cumulative individual variations of the reservation prices on $x|z$ and $z|y$. The term $\nu^{x|z,y|z}$ is given by

$$ \nu^{x|z,y|z} = \sum_{i=1}^{I} \left( \sum_{j=1}^{J} \int_{\Delta(t-1)}^{\Delta t} \sigma_j^{x|z}(s) \sigma_j^{y|z}(s) \rho_j^{x|z,y|z}(s) ds \right), $$

where $\rho_j^{x|z,y|z}(t) = Corr (dW_j^{x|z}(t), dW_j^{y|z}(t))$ is the correlation between the individual components on $x|z$ and $z|y$. All the other covariance terms are zero due to independence. For what concerns the trading volume, for the $i$-th trade on $x|y$ and $x|y$

$$ \nu_i^{x|z} = \frac{\xi_i^{x|z}}{2} \sum_{j=1}^{J} \Delta p_i^{x|z} - \Delta p_i^{x|z*}, \quad \nu_i^{y|z} = \frac{\xi_i^{y|z}}{2} \sum_{j=1}^{J} \Delta p_i^{y|z} - \Delta p_i^{y|z*}. $$

Moreover, by the triangular no-arbitrage, $\Delta p_i^{x|z,*} = \phi_i^x - \phi_i^y + \psi_i^{x|z} + \psi_i^{y|z}$ and $\Delta \tilde{p}_i^{x|z} = \phi_i^x - \phi_i^y + \tilde{\psi}_i^{x|z} + \tilde{\psi}_i^{y|z}$, so that the synthetic volume of $x|y$ is given by

$$ \tilde{r}_i^{x|y} = \frac{\xi_i^{x|y}}{2} \sum_{j=1}^{J} |\psi_i^{x|z} - \tilde{\psi}_i^{x|z} + \psi_i^{y|z} - \tilde{\psi}_i^{y|z}|, \quad (17) $$

which depends on the extent of the individual disagreement on the FX rates of $x|z$ and $z|y$. The synthetic volume expression in (17) involves quantities that cannot be directly observed.
However, by letting $I \to \infty$, we get

$$p \lim_{I \to \infty} \Delta^{1/2} \tilde{v}^x|y = \frac{\xi^x|y}{2} \sqrt{\frac{2}{\pi} \mathcal{S}_{\psi^x|z,z|y}}. \quad (18)$$

where $\mathcal{S}_{\psi^x|z,z|y} = \frac{1}{J} \sum_{j=1}^{J} \int_0^1 \sigma_j^x|z,z|y(s) ds$, and

$$\dot{\sigma}_j^x|z,z|y(t) = \sqrt{\sigma_j^x|z|^2(t) + \sigma_j^z|y|^2(t) + 2 \sigma_j^x|z|\sigma_j^z|y| \rho_x|z,z|y(t)}. \quad (19)$$

Equation (19) highlights that the synthetic volume reflects the aggregated trader-specific components on the individual FX rates, $x|z$ and $z|y$, as well as their aggregated correlation as measured by $\rho^{x|z,z|y}$, which reflects the correlation between $\psi^x|z$ and $\psi^z|y$.

2.4 MDH, frictions and reality

The baseline assumptions of the MDH (linearity of the trading function, constant number of active traders, independence and absence of frictions) are inevitably very stylized. As for the form of the equilibrium function in (1), note that the trades take place on short intradaily intervals of length $\Delta = 1/I$ and they are generally associated with small price variations. Therefore, it is not a strong restriction to assume the equilibrium function to be linear on small price changes. Furthermore, the assumption of $J$ active traders is consistent with a centralized market rather than a fragmented one, where the disaggregation across different FX trading venues as emphasized by Evans and Rime (2016). Note that the model can be extended by making $J$ a random variable, i.e. by adopting a probabilistic model for the presence of the $j$-th trader on a given FX rate market in the $i$-th interval, $J_i$. The observed variation of the Amihud illiquidity measure on a period-by-period basis, might indeed reflect the fact that the number of active traders is likely to change over time, together with changes in the ability of the FX market to absorb large orders as measured by the parameter $\xi^{x|y}$, see the empirical results in Section 4.

Furthermore, the assumption of independence between $\phi_i$ and $\psi_{i,j}$ and across traders does not allow for spillovers/feedback/reversal effects such as those studied in Grossman and Miller (1988) and further elaborated in Brunnermeier and Pedersen (2009) to study the mechanics of liquidity provision on financial markets. The same type of sequential trading behavior has
been recently proved to be responsible for crash episodes in Christensen et al. (2016) and associated with changes in the level of investors’ disagreement around important news announcements, see Bollerslev et al. (2016). Analogously, the model is absent of microstructure noise or frictions, like transaction costs in the form of bid-ask spread, clearing fees or price discreteness, which are intimately related and endogenous to the trading process; see the recent works of Darolles et al. (2015, 2017) for an extension of MHD with liquidity frictions.

From a statistical point of view, as $I \to \infty$, the microstructure noise dominates over the volatility signal, thus leading to distorted measurements of the variance. However, over moderate sampling frequencies, e.g. on 5-minute intervals over 24 hours ($I = 288$), the prices and quantities determined in equilibrium in each sub-interval could be considered (almost) free of microstructure noise contamination, and representative of new equilibria on the aggregated supply/demand functions. In other words, the MDH should not be seen as a model for the market microstructural features, but rather as a model for the aggregated disagreement on fundamentals leading to the price discovery process, where each trader participates to the equilibrium price variations in proportion to the information contained in their new reservation prices. As it is common in the literature on volatility measurement see Bandi and Russell (2008) and Liu et al. (2015), in the following analysis we will work under the maintained assumption that sampling at 5-minute intervals is sufficient to guarantee that a new equilibrium price is determined, where the latter is representative of the aggregated information contained on the demand and supply sides of the market. Despite the stylized set of assumptions, the extended MDH theory provided above represents an encompassing framework to model the observed commonalities among volume and volatility on the global FX markets, as illustrated in Section 4.
3 Data and Preliminary Analysis

3.1 Data Sets

Our empirical analysis relies on two data sets covering 29 currency pairs (15 currencies) over the period from November 2011 to November 2016. First, trading volume data come from CLS, which is the largest payment system for the settlement of foreign exchange transactions launched in 2002. By means of a payment-versus-payment mechanism, this infrastructure supports FX trading by removing settlement risk and supporting market efficiency. For each hour of our sample period and each currency pair, we observe the settlement value and number of settlement instructions. Following the literature (e.g. Mancini et al., 2013), we exclude observations between Friday 10PM and Sunday 10PM since only minimal trading activity is observed during these nonstandard hours. In 2017, the core of CLS was composed of 60 settlement members including the top ten FX global dealers, and thousands of third parties (other banks, non-bank financial institutions, multinational corporations and funds), which are customers of settlement members. The total average daily traded volume submitted to CLS was more than USD 1.5 trillion, which is around 30% of the total daily volume recorded in the last available BIS triennial survey (Bank of International Settlements 2016). However, after adjusting for the large fraction of BIS volume originated from interbank trading across desks and double-counted prime brokered ”give-up” trades, the CLS data should cover about 50% of the FX market (Gargano et al., 2017 & Hasbrouck and Levich, 2017). In our study, we focus on FX spot transactions. The CLS spot FX rates in our sample are highly representative of the entire FX market. For instance, the currency pairs involving the USD and EUR cover more than 85% (94%) of the total trading volume of the BIS triennial survey.

To the best of our knowledge, only few papers have analyzed CLS volume data so far. First, Fischer and Ranaldo (2011) study five aggregate currencies (e.g. all CLS-eligible currencies against the U.S. dollar, Euro, Yen, Sterling, and Swiss franc) rather than currency pairs.  

11The full dataset contains data for 18 major currencies and 33 currency pairs. To maintain a balanced panel we exclude the Hungarian forint (HUF), which enters the dataset only on 07 November 2015. Moreover, we discard the USDILS and USDKRW due to very infrequent trades. We obtain very similar results by including them. The remaining 29 currency pairs are: AUDJPY, AUDNZD, AUDUSD, CADJPY, EURAUD, EURCAD, EURCHF, EURDKK, EURGBP, EURJPY, EURNOK, EURSEK, EURUSD, GBPAUD, GBPCAD, GBPCNY, GBPCHF, GBPJPY, GBPNZD, NZDUSD, USDCAD, USDCHF, USDCNY, USDEUR, USDGBP, USDDKK, USDKRW, USDJPY, USDMXN, USDNOK, USDSEK, USDSGD, and USDZAR.

12In this paper, times are expressed in GMT.
Hasbrouck and Levich (2017) analyze every CLS settlement instruction during April 2013. Gargano et al. (2017) use the same data set as in our study to perform an asset pricing analysis and after accounting for internal trades across desks within the same bank and other issues, they show that the CLS data set covers around 50% of the FX global turnover.

The second data set is obtained from Olsen Financial Technologies, which is the standard source for academic research on intraday FX rates. By compiling historical tick data from the main consolidators such as Reuters, Knight Ridder, GTIS and Tenfore, Olsen data are representative of the entire FX spot market rather than specific segments such as the inter-dealer FX market dominated by two electronic limit order markets: EBS and Reuters. For each minute of our sample period and each currency pair, we observe the following quotes: bid, ask, high, low, and midquotes. With these data at hand, we can analyze at least four aspects of FX rates: (i) the FX rate movements at one minute or lower frequencies; (ii) the realized volatility or other measures of return dispersion; (iii) the quoted bid-ask spread as a measure of transaction cost; and (iv) violations of triangular arbitrage conditions.

3.2 Descriptive Analysis

In this section, we highlight and discuss the main (new) stylized facts characterizing the times series of volume, volatilities and illiquidity measures associated with the 29 FX rates under investigation. First, Figure 2 displays the total hourly volume series, denoted as $v_{t}^{\text{tot}} = \sum_{l=1}^{L} v_{l}^{t}$, where $v_{l}^{t}$ is the hourly volume on the $l$-th FX rate. This plot highlights the size

![Figure 2: Time series and auto-correlation function of the total worldwide volume.](image)

and deepness of the FX market, with an average of 23 billions USD dollar traded every
hour. Moreover, the series of total volume is rather persistent and it clearly displays cyclical patterns, which can be associated with strong intradaily seasonality. We explicitly model the intradaily patterns by estimating with OLS the following model

$$\log(\nu_t) = \delta_t \beta + \epsilon_t,$$  \hspace{1cm} (20)

where $\delta_t$ contains hourly and day-of-the-week dummies. We can also obtain the filtered volume as $\nu_t = \frac{\epsilon_t}{\hat{\beta}}$. The hourly average of the total worldwide volume is reported in Figure 3. The plot highlights that the average total volume is higher during the opening hours of the European and American stock markets, while it is very low between 10PM and 12AM as most of the largest stock markets are closed, while it has a relative peak associated with the opening of Tokyo (2 AM). Moreover, the total volume is the largest on average between 3PM and 4PM, i.e. before the WMR Fix, for which there is a well documented literature about the large traders submitting a rush of orders before the setting of the daily benchmarks for FX prices, see e.g. Evans (2018) and Marsh et al. (2017). Finally, Figure 3 shows that the filtering successfully removes the largest part of the seasonal pattern and that the filtered volume displays significant autocorrelation after many periods.

![Figure 3: Hourly average of the total worldwide volume and ACF of the filtered volume.](image)

Turning the attention to individual FX rates, Figure 4 reports the hourly average share of the total volume of the five most liquid FX rates. Firstly, we note that all the most liquid FX rates involve the USD as either base or quote currency, thus indicating that the US dollar is the most traded currency in the world. As for the total volume, the trading volume of the most liquid FX rates displays clear seasonal patterns. For the individual FX rates, these
patterns are indicative of the geographical areas. For instance, USDJPY covers around 30% of the total FX volume between 12PM and 4PM, that are the hours in which Far East markets are open. AUDUSD contributes with a 15% in the same hours, while its market share strongly declines to 7% during the central hours of the day. EURUSD is by far the most traded FX rate, with a share above 30% between 7AM to 6PM. A similar pattern characterizes also GBPUSD with an average share ranging between 5% and 10%. Finally USDCAD is mostly traded at the opening of the business hours in North America, i.e. between 12PM and 10PM, with approximately 10% share on the total volume. Figure 4 also shows that the five main FX rates amount for a share of more than 70% of the total global volume in every hour. Summarizing, the seasonal patterns are clearly discernible in three dimensions. First, on an intraday scale the trading volume follows the working time in each country or jurisdiction defining the currency pair. This means that round-the-clock, the trading volume of New Zealand dollar is the first to increase, followed by Asian, European, and American currencies. Second, the trading volume on Fridays is generally smaller than on the other days. Third, official banking holidays clearly reduce the trading activity. The seasonalties and calendar effects will be carefully considered in our empirical analysis.

Concerning the relationship between volatility and volume, Figure 5 provides prima facie evidence that volatility and volume are mostly governed by a common latent factor, which seen through the lenses of the MDH represents the information flow proportional to the level of heterogeneous beliefs between the agents. Figure 5 shows that the hourly averages of
realized volatility and volume for USDEUR and USDJPY follow the same patterns. At the intradaily level when the volatility on the FX rates is high, also the volume is high, which points to a wider variation of traders' reservation prices. The hourly averages of the Amihud illiquidity indicator reflect similar seasonal patterns. In particular, Figure 5 shows that for the EURUSD rate, the illiquidity quickly decreases at the opening of the European markets and it is minimal when both the European and the American markets are jointly open. After 8PM the illiquidity grows again and it is maximal during the night hours. For the EURUSD rate, market illiquidity reduces at the opening of the main financial markets Tokyo, London and New York and it sensibly increases again after 4PM. This evidence again reconciles with our theoretical model, which prescribes higher liquidity when more traders are active and the market is deeper, that is typically at the opening hours.

Before performing the empirical analysis to test our model predictions, we examine how daily changes in trading volume correlate with daily changes in realized volatility and other
factors that proved to explain FX liquidity in the previous literature (e.g. Mancini et al., 2013 and Karnaukh et al., 2015) and trading activity in stock markets (e.g. Chordia et al., 2001).

<table>
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<th></th>
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<th>(3) Δ Amihud</th>
<th>(4) Δ Relative BAS</th>
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<td></td>
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<td></td>
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<td>(55.59)</td>
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<td>0.7961&lt;sup&gt;a&lt;/sup&gt;</td>
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<td>(0.69)</td>
<td>(-6.26)</td>
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<td>-0.0265&lt;sup&gt;a&lt;/sup&gt;</td>
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<td>0.1932&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.0366&lt;sup&gt;a&lt;/sup&gt;</td>
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<td>(27.46)</td>
<td>(24.03)</td>
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<td>-0.2993&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-0.4211&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-0.2099&lt;sup&gt;a&lt;/sup&gt;</td>
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<td>(-32.25)</td>
<td>(-53.56)</td>
<td>(-33.17)</td>
<td>(-22.64)</td>
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<td>0.0801&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-0.1035&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.0185&lt;sup&gt;a&lt;/sup&gt;</td>
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<td>0.346</td>
<td>0.514</td>
<td>0.285</td>
<td>0.502</td>
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</table>

Table 1: Volatility, Volume and Liquidity Regression. The t-statistics are in parentheses and the error variance are robust to heteroskedasticity and autocorrelation in the residuals. Except for dummy variables, all variables are taken in logs. The superscripts a, b and c indicate significance at 1%, 5% and 10% significance level respectively.

Some of these variables are likely to determine each other endogenously. Rather than causation, the purpose of this analysis is to document some novel correlation patterns pertaining to FX trading volume. More specifically, we perform a panel regression of all currency pairs, in which the FX volume is explained by volatility (i.e. realized volatility), (average intraday)
relative bid-ask spread (BAS), a dummy variable for the dollar appreciation, two common proxies of market stress such as the TED spread (the yield spread between the U.S. 3-month Libor and T-bills) and FX VIX (i.e. the JP Morgan Global FX volatility index), and four weekday dummy variables equal to one if the trading day is on Monday, Tuesday, Thursday, and Friday, respectively. All variables except the dummy variables are taken in logs and changes and all regressions include the lagged dependent variable as additional regressor. For sake of comparison, we repeat similar regressions using the realized volatility, the Amihud measure, as well as the relative bid-ask spread as dependent variables.

Some novel patterns emerge from the figures reported in the Table 1: FX trading volume increases with realized and implied FX volatility as well as TED spread, whereas it decreases with the relative bid-ask spread. Volume on Mondays, Thursdays, and Fridays tends to be lower than on Wednesdays. Other patterns revealed in this table are that (i) the Amihud illiquidity measure increases with the relative bid-ask spread, (ii) realized volatility tends to be lower when the U.S. dollar appreciates, possibly due to its status as international currency reserve and safe haven against several currencies (e.g. Ranaldo and Söderlind, 2010 and Maggiori, 2017), and that (iii) weekdays effects are also discernible for FX volatility, illiquidity, and relative bid-ask spread.

4 Empirical Analysis

The stylized facts outlined in the previous section are in line with the main empirical prediction of the MDH, that is

**Empirical Prediction 1** Volatility and volume are tied by a common latent factor (information flow) which is responsible for their observed positive correlation. Furthermore, illiquidity increases when the market is thin and there are few active agents.

Given the FX institutional setting, intraday volume-volatility co-movements are strong during the most active hours around the world, that is volatility and volume increase when more traders are active on the market and incorporate the news into their reservation prices. Analogously, illiquidity decrease when more traders are actively adjusting their positions with different reservation prices, that is when the major financial centers are open; opposite
patterns when markets are mostly closed.

In the following sections, we provide evidence supporting a number of empirical predictions associated with the MDH theory outlined in Section 2. In particular, the empirical analysis is articulated along four dimensions. First, we empirically assess the relationship between traders’ disagreement, volatility and volume. Second, we look at FX trading volume, volatility and illiquidity in regimes of central bank intervention; Third, we study commonalities in volume, volatility and illiquidity across FX rates. Fourth, we investigate the relation between trading volume, illiquidity and arbitrage, that is, whether the likelihood of arbitrage deviations (disequilibria) increases with lower trading volume, in particular in reaction to important public news arrivals.

4.1 Disagreement

In this paragraph, we will provide empirical evidence supporting the following empirical predictions of MDH.

**Empirical Prediction 2** *Volatility and trading volume are proportional to the level of heterogeneous beliefs between the agents, that is traders’ disagreement about the fundamental value of the FX rates.*

Following Beber et al. (2010), we measure heterogeneity in beliefs of market participants by using a detailed data set of currency forecasts made by a large cross-section of professional market participants that proxies the differences in beliefs. More specifically, we collect all Thomson Reuters surveys recorded at the beginning of every month during our sample period and compute measures of cross-sectional dispersion such as the (standardized) standard deviation of FX forecast and the high-low range from the distribution of FX forecasts of on average about 50 market participants. This measure of heterogeneity in beliefs that we call *disagreement* is the main regressor in four panel regressions in which total trading volume, realized volatility, Amihud measure, and relative bid-ask spread are the dependent variables.13

13The total number of monthly observations included in the regression is 940, which includes the following 26 currency pairs: AUDJPY, AUDNZD, CADJPY, EURAUD, EURCAD, EURCHF, EURGBP, EURJPY, EURNOK, EURSEK, GBPCAD, GBPCHF, GBPJPY, USDAUD, USDCAD, USDCHF, USDEUR, USDGBP, USDHKD, USDJPY, USDMXP, USDNOK, USDNZD, USDSEK, USDSGD and USDZAR. Not for all currency pairs, forecasts are available from November 2011 onwards. The exact number of market participants depends on the currency pair.
In addition to our measure of disagreement, we include a constant, the lagged dependent variable, and FX illiquidity proposed in Karnaukh et al., 2015. All variables are taken in logs and changes.\textsuperscript{14}

As shown in Table 2, three main results emerge from this analysis. First, both trading volume and volatility increase with disagreement providing more direct evidence that supports the empirical prediction of our theory. Second, more disagreement is also associated with more illiquidity in terms of the price impact proxy (Amihud measure) and transaction costs (relative bid-ask spread). Third, both trading volume and volatility tend to increase with FX illiquidity, consistent with dealers’ inventory imbalances and hot potato effects Lyons (1997).

### 4.2 Volume and liquidity under cap: the case of the Swiss franc

On September 6, 2011, when the exchange rate was 1.095 CHF/EUR and appeared to be heading for parity with the euro, the SNB set a minimum exchange rate of 1.20 francs to the euro (capping franc’s appreciation) saying ”the value of the franc is a threat to the economy”, and that it was ”prepared to buy foreign currency in unlimited quantities”. In terms of the

<table>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<td>Δ Disagreement</td>
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<td>0.1736\textsuperscript{a}</td>
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<td></td>
<td>(2.25)</td>
<td>(3.41)</td>
<td>(1.96)</td>
</tr>
<tr>
<td>Δ Illiquidity</td>
<td>0.0779\textsuperscript{a}</td>
<td>0.5431\textsuperscript{a}</td>
<td>0.1961\textsuperscript{a}</td>
</tr>
<tr>
<td></td>
<td>(5.29)</td>
<td>(8.12)</td>
<td>(5.66)</td>
</tr>
<tr>
<td>Lagged Dep.</td>
<td>-0.3499\textsuperscript{a}</td>
<td>-0.2625\textsuperscript{a}</td>
<td>-0.2389\textsuperscript{a}</td>
</tr>
<tr>
<td></td>
<td>(-10.03)</td>
<td>(-5.34)</td>
<td>(-4.38)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0097</td>
<td>-0.0113</td>
<td>0.0041</td>
</tr>
<tr>
<td></td>
<td>(-1.50)</td>
<td>(-0.81)</td>
<td>(0.61)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.146</td>
<td>0.367</td>
<td>0.254</td>
</tr>
</tbody>
</table>

Table 2: Monthly Regression Analysis - Disagreement. The $t$-statistics are in parentheses and the error variance are robust to heteroskedasticity and autocorrelation in the residuals. Disagreement is the standardized Bloomberg FX forecast, and illiquidity is taken from Karnaukh, Ranaldo and Söderlind (2015). Except for illiquidity, all variables are taken in logs.The superscripts $a$, $b$ and $c$ indicate significance at 1%, 5% and 10% significance level respectively.

\textsuperscript{14}We perform additional analyses including further regressors such as the TED and VXY and the results remain qualitatively the same.
MDH, we can consider the Swiss National Bank (SNB) as the \((J+1)\)-th trader and simplify its intervention approach selling CHF for EUR if

\[
\frac{1}{J} \sum_{j=1}^{J} p^*_i,j < \log(1.2),
\]

such that

\[
p_i = \frac{1}{J+1} \sum_{j=1}^{J+1} p^*_i,j \geq \log(1.2),
\]

(21)

where \(p^*_{i,J+1} = (\log(1.2) - \frac{1}{J} \sum_{j=1}^{J} p^*_i,j)I(\sum_{j=1}^{J} p^*_i,j < 1.2)\), where \(I(\cdot)\) is the indicator function.

This means that the central bank adopts a passive strategy, i.e. it buys foreign currency to maintain the floor on the FX rate, but its reservation price does not have an independent source of variation and it is just function of the average reservation prices of the other \(J\) traders. Under the \textit{capping} scheme of the central bank, the trading volume is

\[
\nu_i = \frac{x_{yJ}}{2} \sum_{j=1}^{J} |\psi_{i,j} - \bar{\psi}_{i,j}| + v^C_{i},
\]

(22)

where \(v^C_{i}\) is the trading volume generated by the central bank to guarantee the cap on the FX rate. Hence, the MDH prescribes a low volatility of the observed returns due to the implicit constraint given by the capping and a larger volume due to the intervention of the central bank who buys euros for CHF. This also implies that the Amihud index is lower under the capping period.

**Empirical Prediction 3** The central bank as the \((J+1)\)-th trader intervening with a given reservation price reduces the volatility while increasing the volume, thus increasing the liquidity of an FX rate.

Figure 7 provides graphical support for the prescriptions of the MDH. Indeed, the volatility as measured by RPV is relatively low until January 15, 2015, it spikes on the day of the announcement of the un-capping and it remains high until the end of 2016. The trading volume has the opposite behavior, being relatively high during the capping period and reverting to a lower value after January 15, 2015. Finally, the Amihud measure displays a clear level shift after January 15, 2015, that is the EURCHF rate becomes less liquid after the uncap.
Table 3 provides strong statistical support for the fact that volatility is significantly lower in

![Graphs of Price, RPV, Volume, and Amihud](a) Price, (b) RPV, (c) Volume, (d) Amihud)

the first period, volume is higher and the Amihud measure significantly increases on average in the second part of the sample. To conclude, the active role of the central bank has increased the liquidity of the EURCHF market.

<table>
<thead>
<tr>
<th></th>
<th>Before</th>
<th>After</th>
<th>Test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPV</td>
<td>4.548</td>
<td>9.7531</td>
<td>-12.7093</td>
<td>0.000</td>
</tr>
<tr>
<td>RV</td>
<td>0.0825</td>
<td>0.3508</td>
<td>-5.842</td>
<td>0.000</td>
</tr>
<tr>
<td>VOL</td>
<td>1148.75</td>
<td>682.80</td>
<td>14.34</td>
<td>0.000</td>
</tr>
<tr>
<td>AMIHUD</td>
<td>0.4255</td>
<td>1.5002</td>
<td>-24.21</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 3: Sample averages of realized power variation (RPV), realized volatility (RV), trading volume (Volume) and Amihud measure (Amihud) before (from Nov 1, 2011 to Jan 14, 2015) and after (from Jan 16, 2015 to Nov 30, 2016) announcement of un-capping (on Jan 15, 2015). The variables have been rescaled. Table also reports a test for the equality of the averages in the two sub-samples and associated p-values (one-tail).
4.3 Commonalities

4.3.1 Factor Analysis

Another prediction of the MDH theory is the commonality of volume, volatility and illiquidity across FX rates. Specifically, Section 2.3 provides a theoretical underpinning that trading volume across FX rates are driven by common factors, which are function of the aggregated traders’ specific components on different currency pairs. Notably, the triangular no-arbitrage relation in (26) can be extended to groups of FX rates of any order. For instance, with four currencies, \(x, w, z,\) and \(y\), we have, \(p_x | y = p_x | z + p_z | w + p_w | y\), so that \(\tilde{\nu}_i = \frac{\bar{e}_{xy}}{2} \sum_{j=1}^{J} |\psi_{i,j} - \bar{\psi}_{i,j}|^2\). This provides support for the existence of a factor structure in cross sections of FX rates of any order. In particular,

Empirical Prediction 4 Trading volume, volatility and liquidity across FX rates are linked by no-arbitrage constraints, which are responsible for the commonalities across FX rates.

To the purpose of studying the commonality in volume/volatility and liquidity across multiple FX rates, we apply principal component analysis (PCA) to the panel of 29 FX rates introduced in Section 3.1. The goal is to identify a common factor structure across the volume series of the FX rates and to study the exposure of each rate to it. Table 4 shows that volume, volatility and illiquidity of each individual FX rate load positively on the first principal component in all cases. Notably, the first component explains a large portion of the overall variation of volume, volatility and illiquidity measures of the panel of FX rates, being above 50% in many cases. Moreover, the weight associated with the volume and illiquidity measure of USDEUR is the highest signaling the leading role of the information on the USDEUR rate in determining the global FX volume. Instead, the loading on RPV for EURDKK is the smallest across all currencies, signaling that the volatility on EURDKK is strongly influenced by the pegging of DKK to EUR.

We also study the dynamic interplay between the FX rates across currencies by means of the total connectedness index of Diebold and Yilmaz (2014). The TCI is defined as

\[
TCI = \frac{1}{N} \sum_{i,j=1\neq j}^{N} \tilde{d}_{i,j},
\]  

(23)
<table>
<thead>
<tr>
<th>Currency Pair</th>
<th>Hourly Volume</th>
<th>Hourly RPV</th>
<th>Hourly Amihud</th>
<th>Hourly Seasonally Adjusted Volume</th>
<th>Hourly Seasonally Adjusted RPV</th>
<th>Hourly Seasonally Adjusted Amihud</th>
<th>Daily Volume</th>
<th>Daily RPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUDJPY</td>
<td>0.1555</td>
<td>0.1884</td>
<td>0.1963</td>
<td>0.2031</td>
<td>0.2143</td>
<td>0.1963</td>
<td>0.1830</td>
<td>0.1986</td>
</tr>
<tr>
<td>AUDNZD</td>
<td>0.1288</td>
<td>0.1461</td>
<td>0.1559</td>
<td>0.1539</td>
<td>0.1774</td>
<td>0.1559</td>
<td>0.1781</td>
<td>0.1895</td>
</tr>
<tr>
<td>CADJPY</td>
<td>0.1327</td>
<td>0.1966</td>
<td>0.1133</td>
<td>0.1528</td>
<td>0.2019</td>
<td>0.1133</td>
<td>0.1387</td>
<td>0.1884</td>
</tr>
<tr>
<td>EURAUD</td>
<td>0.1854</td>
<td>0.1992</td>
<td>0.1823</td>
<td>0.1934</td>
<td>0.2144</td>
<td>0.1823</td>
<td>0.1968</td>
<td>0.2083</td>
</tr>
<tr>
<td>EURCAD</td>
<td>0.1829</td>
<td>0.2138</td>
<td>0.1627</td>
<td>0.1711</td>
<td>0.2180</td>
<td>0.1627</td>
<td>0.1873</td>
<td>0.2111</td>
</tr>
<tr>
<td>EURCHF</td>
<td>0.2173</td>
<td>0.1520</td>
<td>0.1852</td>
<td>0.1997</td>
<td>0.1542</td>
<td>0.1852</td>
<td>0.1677</td>
<td>0.1510</td>
</tr>
<tr>
<td>EURDKK</td>
<td>0.1841</td>
<td>0.0863</td>
<td>0.0879</td>
<td>0.0910</td>
<td>0.0495</td>
<td>0.0879</td>
<td>0.1500</td>
<td>0.0623</td>
</tr>
<tr>
<td>EURGBP</td>
<td>0.2285</td>
<td>0.2128</td>
<td>0.2355</td>
<td>0.2284</td>
<td>0.2139</td>
<td>0.2355</td>
<td>0.2155</td>
<td>0.2069</td>
</tr>
<tr>
<td>EURJPY</td>
<td>0.1971</td>
<td>0.1954</td>
<td>0.2367</td>
<td>0.2112</td>
<td>0.1959</td>
<td>0.2367</td>
<td>0.1617</td>
<td>0.1677</td>
</tr>
<tr>
<td>EURNOK</td>
<td>0.2142</td>
<td>0.1472</td>
<td>0.1729</td>
<td>0.1805</td>
<td>0.1203</td>
<td>0.1729</td>
<td>0.2118</td>
<td>0.1472</td>
</tr>
<tr>
<td>EURSEK</td>
<td>0.2131</td>
<td>0.1393</td>
<td>0.1571</td>
<td>0.1764</td>
<td>0.1001</td>
<td>0.1571</td>
<td>0.2041</td>
<td>0.1314</td>
</tr>
<tr>
<td>GBPCHF</td>
<td>0.1692</td>
<td>0.2148</td>
<td>0.1648</td>
<td>0.1488</td>
<td>0.2163</td>
<td>0.1648</td>
<td>0.1664</td>
<td>0.2094</td>
</tr>
<tr>
<td>GBPJPY</td>
<td>0.1823</td>
<td>0.1984</td>
<td>0.1724</td>
<td>0.1754</td>
<td>0.1980</td>
<td>0.1724</td>
<td>0.1463</td>
<td>0.1774</td>
</tr>
<tr>
<td>USDAUD</td>
<td>0.1839</td>
<td>0.1965</td>
<td>0.2361</td>
<td>0.2256</td>
<td>0.2129</td>
<td>0.2361</td>
<td>0.1951</td>
<td>0.2115</td>
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<tr>
<td>USDCAD</td>
<td>0.2070</td>
<td>0.2066</td>
<td>0.2158</td>
<td>0.2144</td>
<td>0.2049</td>
<td>0.2158</td>
<td>0.2106</td>
<td>0.2079</td>
</tr>
<tr>
<td>USDCNH</td>
<td>0.2236</td>
<td>0.2140</td>
<td>0.2321</td>
<td>0.2301</td>
<td>0.2131</td>
<td>0.2321</td>
<td>0.2184</td>
<td>0.2014</td>
</tr>
<tr>
<td>USDDKK</td>
<td>0.1573</td>
<td>0.2127</td>
<td>0.1089</td>
<td>0.0979</td>
<td>0.2129</td>
<td>0.1089</td>
<td>0.1394</td>
<td>0.1979</td>
</tr>
<tr>
<td>USDSEUR</td>
<td>0.2320</td>
<td>0.2142</td>
<td>0.2633</td>
<td>0.2461</td>
<td>0.2155</td>
<td>0.2633</td>
<td>0.2011</td>
<td>0.1966</td>
</tr>
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<td>USDSCHF</td>
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<td>0.2131</td>
<td>0.2575</td>
<td>0.2384</td>
<td>0.2095</td>
<td>0.2575</td>
<td>0.2235</td>
<td>0.2050</td>
</tr>
<tr>
<td>USDCHF</td>
<td>0.1555</td>
<td>0.0673</td>
<td>0.1502</td>
<td>0.1266</td>
<td>0.0917</td>
<td>0.1502</td>
<td>0.1515</td>
<td>0.1244</td>
</tr>
<tr>
<td>USDJNY</td>
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<td>0.1676</td>
<td>0.2116</td>
<td>0.2263</td>
<td>0.1701</td>
<td>0.2116</td>
<td>0.1912</td>
<td>0.1487</td>
</tr>
<tr>
<td>USDMXN</td>
<td>0.1459</td>
<td>0.1656</td>
<td>0.1850</td>
<td>0.1881</td>
<td>0.1538</td>
<td>0.1850</td>
<td>0.2211</td>
<td>0.1841</td>
</tr>
<tr>
<td>USDDNOK</td>
<td>0.1909</td>
<td>0.2026</td>
<td>0.1484</td>
<td>0.1518</td>
<td>0.1852</td>
<td>0.1484</td>
<td>0.1611</td>
<td>0.2014</td>
</tr>
<tr>
<td>USDNZD</td>
<td>0.1669</td>
<td>0.1914</td>
<td>0.1922</td>
<td>0.1908</td>
<td>0.1966</td>
<td>0.1922</td>
<td>0.2003</td>
<td>0.2041</td>
</tr>
<tr>
<td>USDESEK</td>
<td>0.1939</td>
<td>0.1974</td>
<td>0.1457</td>
<td>0.1585</td>
<td>0.1755</td>
<td>0.1457</td>
<td>0.1737</td>
<td>0.1861</td>
</tr>
<tr>
<td>USDSGD</td>
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<td>0.1580</td>
<td>0.1887</td>
<td>0.1677</td>
<td>0.1580</td>
<td>0.1671</td>
<td>0.1846</td>
</tr>
<tr>
<td>USDZAR</td>
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<td>0.1681</td>
<td>0.1915</td>
<td>0.1989</td>
<td>0.1313</td>
<td>0.1915</td>
<td>0.2233</td>
<td>0.1705</td>
</tr>
</tbody>
</table>

| EXPL         | 0.5514       | 0.6135     | 0.3066        | 0.3440                           | 0.5236                        | 0.3066                           | 0.4756       | 0.6494   |

Table 4: PCA Analysis. The table reports the loadings for each currency pair for trading volume (Volume), volatility (realized power variation, RPV) and illiquidity (Amihud) to the first principal component. The bottom line reports the percentage of explained variance of the first principal component.
where $N$ denotes the number of variables in the system, and $\tilde{d}_{i,j}$ is the $i,j$ entry of the standardized connectedness matrix $\tilde{D}$. The matrix $\tilde{D}$ is defined as

$$\tilde{d}_{i,j} = \frac{d_{i,j}}{\sum_{j=1}^{N} d_{i,j}},$$  \hspace{1cm} (24)$$

with

$$d_{i,j} = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H} (e_i A_h \Sigma e_j)^2}{\sum_{h=0}^{H} (e_i' A_h \Sigma A_h' e_i)},$$  \hspace{1cm} (25)$$

where $A_h$ is the impulse-response matrix at horizon $h$ associated with a VAR(p) model, $\Sigma$ is the covariance matrix of the errors, and $e_i, e_j$ are $N \times 1$ selection vectors. By construction, $\sum_{j=1}^{N} \tilde{d}_{i,j} = 1$ and $\sum_{i=1}^{N} \tilde{d}_{i,j} = N$. Equation (25) defines the generalized forecast error decomposition, as introduced by Pesaran and Shin (1998). In other words, the TCI measures the average portion over $N$ variables of the forecast error variation of variable $i$ coming from shocks arising from the other $j = 1, \ldots, N - 1$ variables of the system. The TCI provides a characterization of the connectedness of a system that is richer than the one obtained with a simple linear correlation coefficient. Indeed, the TCI combines information coming from both the contemporaneous and the dynamic dependence structure of the system trough $\Sigma$ and $A_h$, respectively. Moreover, by estimating the VAR model over rolling windows, it is possible to characterize the evolution of the dependence structure between two or more variables by looking at the variations of the TCI over time.

<table>
<thead>
<tr>
<th>All FX rates</th>
<th>Hourly</th>
<th>Hourly Seasonally Adjusted</th>
<th>Daily</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td>0.884</td>
<td>0.880</td>
<td>0.885</td>
</tr>
<tr>
<td>RPV</td>
<td>0.910</td>
<td>0.907</td>
<td>0.910</td>
</tr>
<tr>
<td>10 Most Liquid</td>
<td>Volume</td>
<td>0.875</td>
<td>0.873</td>
</tr>
<tr>
<td></td>
<td>RPV</td>
<td>0.893</td>
<td>0.890</td>
</tr>
<tr>
<td>10 Least Liquid</td>
<td>Volume</td>
<td>0.621</td>
<td>0.607</td>
</tr>
<tr>
<td></td>
<td>RPV</td>
<td>0.808</td>
<td>0.810</td>
</tr>
</tbody>
</table>

Table 5: Connectedness. The table reports the value of the connectedness index of Diebold and Yilmaz (2014) of trading volume (Volume) and volatility (in terms of realized power variation, RPV) for different sampling periods (Full sample, 2011/2014, 2012/2015, and 2013/2016) and for different sets of FX rates (All, Most/Least liquid).

Table 5 shows that the overall level of connectedness of volume and volatility is very high
and constant over time, being close to 90% for both volatility and volume at hourly and daily level. The connectedness remains very high also when volume and RPV are filtered from intradaily seasonality, being around 70% to 80%. Moreover, we note some significant differences when comparing the connectedness among the most liquid rates and the least liquid ones. Indeed, while the connectedness of volume and volatility calculated on the most liquid FX rates is close to the one observed for the full sample, the connectedness level sensibly reduces when focusing on the most illiquid FX rates. Illiquid currency pairs are probably less (more) exposed to the common (specific) FX-factors as it also emerges from the magnitude of the loadings of the first principal component in Table 4. In other words, liquid currencies have stronger cross-currency commonalities than illiquid ones.

To further explore this point, we measure the strength of the pairwise commonality in volume between \(x|y\), \(x|z\) and \(z|y\) through the following reduced-form model,

\[
\log(\nu_{x|y}^t) = \beta_0 + \beta_1 \log(\nu_{x|z}^t + \nu_{z|y}^t) + \varepsilon_t, \quad t = 1, \ldots, T
\]

(26)

where \(\nu_{x|y}^t\), \(\nu_{x|z}^t\) and \(\nu_{z|y}^t\) are the log-volume on period \(t\) on the FX rates \(x|y\), \(x|z\) and \(z|y\), respectively. In this regression, \(\beta_0\) reflects the differential in the resiliency levels in the three markets, that is \(\xi_{x|y}\), \(\xi_{x|z}\) and \(\xi_{z|y}\), while \(\beta_1\) measures the magnitude of commonality in the volume of the three FX rates. The MDH theory outlined in Section 3 prescribes that \(\beta_1 > 0\). The term \(\varepsilon_t\) can also be interpreted as the deviation from the long-run equilibrium between FX volume. Table 6 reports the estimates of regression (26) for the EUR/USD rate, where the synthetic volume is constructed with CHF, GBP, DKK, JPY, AUD, CAD, NOK, SEK. Overall, it emerges that regression (26) is able to explain a large portion of variability of \(\nu^{EUR/USD}\), and this can be attributed to the portions of common information in \(\psi_j^{USD/}\) and \(\psi_j^{EUR/}\), which determine the synthetic volume in equation (19). At the hourly level, the estimated parameter \(\beta_0\) reflects the average liquidity differential across currencies, with DKK, SEK and NOK being consistently less liquid than JPY, AUD and GBP. Notably, the parameter \(\beta_1\) is positive in all cases and it is closer to 1 for the most liquid rates. As expected, higher \(\beta_1\) are associated with higher \(R^2\), which is another measure of commonality (e.g. Chordia et al., 2000). When removing the intradaily seasonality in volume or aggregating at the
Table 6: Commonalities in volume, volatility (measured as realized power variation, RPV) and illiquidity (measured as Amihud index, Amihud). For each currency, the table reports the intercept and the slope of the regression of the log volume/volatility/Amihud proxy of EURUSD on the log of the sum of volume/volatility/Amihud index on the FX rate of this currency with USD and EUR. Table also reports the $R^2$ of the regression and the first ACF of the residuals.

daily level, the $R^2$ slightly decreases but the result is qualitatively the same as for the raw hourly volume. The residuals display significant autocorrelation, suggesting that volume imbalances across FX markets are stationary but persistent. These long-lasting disequilibria in volume might be explained by the fragmented OTC structure of the FX market and prolonged time to incorporate agents’ heterogeneous priors and (public and private) information into prices, as for conditional volatility (Engle et al., 1990). When replacing volume with RPV in equation (26), we note that also volatility displays a large degree of commonality across currencies. The $R^2$ is generally very well above 50% at both hourly and daily level. Interestingly, the $R^2$ of DKK is almost 1 and the slope of the equilibrium is also close to unity. Indeed, DKK is pegged to EUR within a very narrow corridor (0.133-0.1346), thus the
$Cov(p^{USD/DKK}, p^{USD/EUR}) \approx 1$, which is reflected in a slope parameter close to unity. The central bank's intervention to fix an FX rate reduces the commonality in volume and liquidity. Not surprisingly, the Amihud illiquidity measure, which combines information on both volatility and volume, also displays an analogous amount of commonality across currencies, being the highest for the most liquid ones.

### 4.3.2 Synthetic Volume and Correlations

The MDH theory in Section 2.3 suggests that the commonalities in trading volume across FX rates are driven by the level correlation among the FX rates, where the right-hand side of (19) shows that the synthetic volume is a function of the correlation of the aggregated traders’ specific components on different currency pairs. In other words, our theory predicts that

**Empirical Prediction 5** *The synthetic volume reveals the strength of the correlation across FX rates.*

To test this empirical prediction, we consider the following regression

\[
\tilde{\nu}_t^{x|y} = \gamma_0 + \gamma_1 \log(\zeta_t) + \gamma_2 \tilde{\nu}_{t-1}^{x|y} + \varepsilon_t, \quad t = 1, \ldots, T,
\]

(27)

where $\tilde{\nu}_t^{x|y}$ is the synthetic volume which is measured as the fitted log-volume in regression (26), while $\zeta_t = \log(1 + |\rho_t|)$ and $\rho_t$ is the realized correlation between $x|z$ and $z|y$. Hence, the term $\zeta_t$ measures the strength of the correlation in the FX rates $x|z$ and $z|y$, and the parameter $\gamma_1$ is expected to be positive. Table 7 contains the estimates of $\gamma_1$ based on regression (27) and on the extended version which controls for liquidity as measured by the bid-ask spreads on $x|z$ and $z|y$. At hourly frequency, the estimates of $\gamma_1$ are positive and highly significant in most cases, with the notable exception of DKK. Again, the intervention of the central bank to peg DKK to EUR prevents the trading activity on EUR/DKK and DKK/USD from revealing the correlation structure of the investors’ beliefs on EUR and USD. When aggregating over days and weeks, we still obtain generally positive estimates of $\gamma_1$ but they are often non significantly different from zero.
### Baseline Regression

<table>
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<tr>
<th>Currency</th>
<th>Hourly $\gamma_0$</th>
<th>Hourly $\gamma_1$</th>
<th>Daily $\gamma_0$</th>
<th>Daily $\gamma_1$</th>
<th>Weekly $\gamma_0$</th>
<th>Weekly $\gamma_1$</th>
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</thead>
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<tr>
<td>CHF</td>
<td>4.2444</td>
<td>0.2141$^a$</td>
<td>13.9536</td>
<td>0.1697$^a$</td>
<td>8.4427</td>
<td>0.0993$^c$</td>
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<tr>
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<tr>
<td>DKK</td>
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<td>-0.1447$^a$</td>
<td>18.5609</td>
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<tr>
<td>JPY</td>
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<td>0.2810$^a$</td>
<td>10.9029</td>
<td>0.1511$^a$</td>
<td>7.0136</td>
<td>0.0331</td>
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<tr>
<td>AUD</td>
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<td>0.0642$^a$</td>
<td>12.5726</td>
<td>0.0609</td>
<td>8.4234</td>
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<td>CAD</td>
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<td>-0.0764$^a$</td>
<td>22.5847</td>
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<td>NOK</td>
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<td>0.1196$^c$</td>
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<tr>
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<td>-0.0160</td>
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### Control for Liquidity

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<th>Hourly $\gamma_1$</th>
<th>Daily $\gamma_0$</th>
<th>Daily $\gamma_1$</th>
<th>Weekly $\gamma_0$</th>
<th>Weekly $\gamma_1$</th>
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<td>5.3883</td>
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<td>DKK</td>
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<td>JPY</td>
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<td>10.9355</td>
<td>-0.0353</td>
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<td>0.0394</td>
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<tr>
<td>CAD</td>
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<td>0.0878</td>
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<tr>
<td>NOK</td>
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<td>17.3807</td>
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<td>17.6188</td>
<td>0.1140$^c$</td>
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<tr>
<td>SEK</td>
<td>8.2093</td>
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<td>18.9913</td>
<td>0.1571$^a$</td>
<td>16.0763</td>
<td>-0.0129</td>
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</table>

Table 7: Synthetic volume and correlation. For each currency, the table reports the intercept and the slope of the regression of the log synthetic volume of EURUSD on the log of the correlation of the FX rates with USD and EUR.

### 4.4 Co-jumps, trading volume and disequilibria

Let’s consider again a world with two currencies, x (base) and y (quote). Since the common information terms in (2) have the potential to generate large and rapid variations in the FX prices possibly associated with macroeconomic announcements or financial distress in a given country, we assume that they can be formulated as *jump* processes. Similarly to Bollerslev et al. (2016), we rely on a simple setup for the common news component, i.e. the ”jumps”, to separately identify it from the component of the variations in the FX rates due to the disagreement among traders.\(^ {15}\) For instance, $\phi_i^{x|y}$ can be modeled as compound Poisson processes as

$$\phi_i^{x|y} = \sum_{l=1}^{N_i^{x|y}} Z_l^{x|y},$$  \hspace{1cm} (28)

\(^ {15}\)Other studies associating large price jumps with news announcements are in Andersen et al. (2007) and Lee (2011).
where $N_{i}^{x|y}$ is an independent Poisson random variable with intensity $\lambda_{x|y}\Delta$, where $\lambda_{x|y}$ is expressed with respect to the unit scale (e.g. daily). $Z_{i}^{x|y} \sim i.d. D_{x|y}(\theta_{x|y}) \in \mathbb{R}$ where $\theta_{x|y}$ are the parameters associated with the distribution $D_{x|y}$. Furthermore, we assume that $\phi_{i}^{x|y}$ can be further decomposed into currency specific variations, that is $\phi_{i}^{x|y} = \phi_{i}^{x} - \phi_{i}^{y}$. For instance, we can assume that $\phi_{i}^{x} = \sum_{l=1}^{N_{x}^{x}} Z_{l}^{x}$ and $\phi_{i}^{y} = \sum_{l=1}^{N_{y}^{y}} Z_{l}^{y}$. Unfortunately, $\phi_{i}^{x}$ and $\phi_{i}^{y}$ cannot be uniquely identified by looking at a single FX rate since a large variation in the FX rate might be due to good (bad) news on $x$ or bad (good) news on $y$. Therefore, we rely on the theory of co-jumps, as developed in Caporin et al. (2017), to identify $\phi_{i}^{x}$ given a cross section of FX rates with the same base currency $x$. In other words, the simultaneous occurrence of a jump in all the FX rates trading with a given base currency $x$ allows us to identify episodes characterized by the ex-post realization of a currency-specific news common to all traders. In turns, this enables us to determine large and sudden directional variations in the FX rates associated with major common news on a single currency. The test for co-jumps proposed by Caporin et al. (2017) takes the form

$$CJ = \frac{1}{\zeta} \sum_{j=1}^{N} \left( \frac{SRV_{j} - \widetilde{SRV}_{j}}{SQ_{j}} \right)^{2},$$

where $N$ denotes the number of FX rates, $\zeta$ is a design parameter, $SRV$ is the smoothed randomized realized variance of Podolskij and Ziggel (2010), $\widetilde{SRV}$ is the smoothed version of the truncated realized variance estimator of Mancini (2009) which is robust to jumps, while $SQ$ is a smoothed estimator of the quarticity. Under the null hypothesis of absence of co-jumps, $CJ$ converges to a chi-square distribution with $N$ degrees of freedom. Under the alternative hypothesis of at least one co-jump across all $N$ series, $CJ$ diverges.

Figure 8 illustrates two episodes detected with the test for co-jumps developed in Caporin et al. (2017). The left panel reports the log-returns of the FX rates of EUR against the six major currencies, USD, GBP, CHF, AUD, CAD and JPY on November 6, 2015. The sudden depreciation of the Euro follows a speech by the President of ECB, Mario Draghi reinforcing the conviction among traders about the continuation of the Eurosystem’s bond purchases known as quantitative easing as a stabilization tool to resolve the crisis situations.

\[16\] We thank the authors for sharing with us their MATLAB code to detect co-jumps.
in the financial market. The FX rate reacted with a sudden depreciation of EUR against all other currencies by approximately 1% on an interval of five minutes. The magnitude of such a variation is several times larger than the variation under normal market conditions, where the changes in the reservation prices of each individual trader is averaged over $J$ traders. An analogous evidence arises for the appreciation of the USD against all major currencies on May 1, 2014, following the rumors on the beginning of a tapering policy by the Federal reserve.

**Empirical Prediction 6** Directional common news trigger directional common revision of reservation prices and should not generate any extraordinary trading volume.

The MDH prescribes that if new information is common, then nearly no transaction volume should be generated by the trading activity since the traders revise their reservation prices in the same direction. We therefore investigate whether the volume significantly increases when the FX rates are hit by large news. We consider the following panel regression with fixed effects

$$V_{i,t} = \alpha_i + \beta CJ_t + \delta BA_{i,t} + \gamma_h h_t + \gamma_w w_t + \rho V_{i,t-1} + \varepsilon_{i,t},$$

(30)

where $V_{i,t}$ is the log-volume on the $i$-th FX rate trading against a given base currency, $CJ$ is a dummy variable for a significant co-jump on the base currency, $BA_{i,t}$ is the relative
bid-ask spread on the $i$-th FX rate and $h_i$ and $w_i$ are hourly and day-of-the-week dummies capturing seasonal effects. The coefficient $\beta$ captures the increase/reduction in the average trading volume associated with the arrival of a currency-specific news on one of the four main currencies. Regression (30) can be considered the multiple-jumps analogous in the panel setting of the jump regression formalized in Li et al. (2017) and applied in Bollerslev et al. (2016) in the context of macroeconomic announcements. Table 8 reports the estimation results for four different base currencies, EUR, GBP, USD and JPY and 6 FX rates each (including also CHF, AUD and CAD). The coefficient $\beta$ is never significant at 5% level, signaling that the large directional news on the individual base currencies are not coupled with abnormal trading volume on the exchange rates. Furthermore, the theory developed in

<table>
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<th>Baseline</th>
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<th>JPY</th>
</tr>
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<td>PO</td>
<td>FE</td>
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<td>–</td>
<td>–</td>
<td>–</td>
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<table>
<thead>
<tr>
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<th>JPY</th>
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<td>FE</td>
<td>PO</td>
<td>FE</td>
</tr>
<tr>
<td>CJ</td>
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<td>AR(1)</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table 8: Common news and volume. Panel regression with fixed effect (FE) and pooling (PO). The dependent variable is the hourly volume FX rates for six FX rates with different base currency EUR, GBP, USD and JPY. The regressors are the dummy variable for the co-jump (CJ) on the base currency (baseline specification), and a number of controls: the average relative bid-ask spread (BA), and hourly and day-of-the-week dummies and an AR(1) term. The superscripts $a$, $b$ and $c$ indicate significance at 1%, 5% and 10% significance level respectively.

Section 2 prescribes that the equilibrium relation across FX rates passes through the trading activity, thus generating trading volume on different FX rates.

**Empirical Prediction 7** Since the equilibrium relation across FX rates passes through the trading activity (volume), in reaction to a co-jump the chances of mispricing is higher for less liquid currencies
Hence, in the following, we study if no-arbitrage violations across FX rates are associated with a reduced trading activity on less liquid currencies. To answer this question, we consider the following panel regression with fixed effects

\[ E_{i,t} = \alpha_i + \beta \bar{V}_{i,t} (1 + \zeta CJ_t) + \theta \bar{CJ}_{i,t} + \delta BA_{i,t} + \gamma_w w_t + \gamma_h h_t + \epsilon_{i,t}, \]  

where \( E_{i,t} \) is the hourly cumulative no-arbitrage error at time \( t \) for the \( i \)-th synthetic relation defined as

\[ E_{i,t} = \sum_{l=1}^{60} |r_{l,t}^{x,y} - \tilde{r}_{l,t}^{z_n_i}|, \]

where \( r_{l,t}^{x,y} \) is the \textit{direct} one-minute midquote log-return on the FX rate between the currency \( x \) and \( y \), while \( \tilde{r}_{l,t}^{z_n_i} \) is the \textit{synthetic} one-minute log-return on the FX rate \( x|y \) using the currency \( z_{n_i} \). The term \( \bar{V}_{i,t} \) is the total volume generated on the FX rates \( x|z_{n_i} \) and \( z_{n_i}|y \). Our sample consists of \( n = 10 \) currencies and allows us to consider \( I = 20 \) combinations of \( x \), \( y \) and \( z_{n_i} \).

The relation between the average volume \( \bar{V}_{i,t} \) and the average no-arbitrage error \( E_{i,t} \) is

![Figure 9: Trading Volume and pricing errors. The figures show the scatter of the average volume (x-axis) versus the average triangular pricing error (y-axis) for 20 combination of currencies \( x \), \( y \) and \( z \). The left panel reports the unconditional relation, while the right panel is conditional to the event of a co-jump on the individual currencies, EUR, JPY, USD and GBP. The line represents the least squares fit.](image)

The combinations are: USDAUD/EURAUD, USDSEK/EURSEK, USDNOK/EURNOK, USDCAD/EURCAD, USDJPY/EURJPY, USDBGP/EURGBP, USDDKK/EURDKK, USDAUD/GBPUSD, USDCAD/GBPCAD, USDJPY/GBPJPY, USDCAD/JPYCAD, USDAUD/JPYAUD, EUCAD/GBPCAD, EURJPY/GBPJPY, EURCHF/GBPCAD, EURJPY/GBPCAD, EURCHF/GBPCHF, EURCAD/JPYCAD, USDAUD/JPYAUD, GBPAUD/JPYAUD, GBPCAD/JPYCAD.
depicted in Figure 9, which clearly shows a negative relation between the trading volume on the FX rates and the no-arbitrage pricing error. In other words, the pricing errors are higher for less liquid currencies, such as SEK and NOK. A notable exception is given by DKK, which is associated with small trading volume and small pricing errors. However, Danish krona is pegged to the Euro, so we could expect the mispricing on the triangular no-arbitrage relation EURDKK/USDDKK to be of a small order. When conditioning on the arrival of a large common news on the main individual currencies EUR, JPY, USD and GBP (right panel), the average mispricing error on the y-axis increases relatively to the left panel, as the large variations on the FX rates associated with big news on the individual currencies are likely to be associated with large mispricing errors and more dispersion. However, the negative relation between magnitude of the mispricing and trading volume is maintained.

Table 9 reports the parameter estimates of (31) based on the sample of $I = 20$ combination of FX rates and for a sample of $T = 30720$ hours ($24 \times 1280$ days). The results confirm the negative relation between mispricing errors and volume, which is robust to the inclusion of the relative bid-ask spread as a control for the liquidity on the triangular FX rates (where parameter $\delta$ is found significantly positive in all cases). As it also emerges from Figure 9, the co-jumps events are associated with a significant increase in the average level of mispricing ($\theta > 0$), and also with a significantly negative slope of volume ($\zeta < 0$), which signals that large news are embedded into less liquid FX rates at a lower pace than for more liquid ones.

<table>
<thead>
<tr>
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<td>-0.028$^a$</td>
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<td>$-$</td>
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Table 9: No-arbitrage errors and volume. Panel regression with fixed effect (FE) and pooling (PO). The dependent variable is the triangular pricing error accumulated at the hourly horizon for 20 combinations of FX rates. The regressors are the hourly trading volume on the combined FX rate (Volume), the average relative bid-ask spread (Bid-Ask), the dummy variable of the co-jump index on its own (CJ) and interacted with trading volume (CJ-Volume) as well as hourly and weekly dummies. The superscripts $a$, $b$ and $c$ indicate significance at 1%, 5% and 10% significance level respectively. The standard errors are computed with the White (1980) sandwich estimator for panel data models.
5 Conclusion

Building upon the Mixture-of-Distribution Hypothesis (MDH), we provide a unified model for asset prices, trading volume, and volatility. We extend the MDH in a continuous-time and multi-asset framework. We apply it to currency markets in which foreign exchange (FX) rates are tied by arbitrage conditions. Our model outlines new properties of the FX market including the relationships between trading volume and volatility of direct and arbitrage-related (or synthetic) FX rates. It also provides the theoretical foundation for common patterns (commonality) of trading volume, volatility, and illiquidity across currencies and time, and an intuitive closed-form solution for measuring illiquidity in terms of price impact.

We test the empirical predictions from our model using new and unique (intraday) data representative of the global FX spot market. A distinguishing characteristic of our data set is that it includes granular and intraday data on global FX trading volume. As predicted by our model, we provide empirical support that the difference in market participants’ beliefs is the common source of trading volume and volatility determining strong co-movements between them across time and FX rates. We also measure FX illiquidity in terms of price impact. Finally, we show that lower trading volume and illiquidity help explain pricing errors (i.e. triangular arbitrage deviations), especially when currency-specific news events affect all currency pairs.

Several implications emerge from our study. First, by shedding light on the intricate interrelations between FX rates, volume, and volatility, our work should support an integrated analysis of FX rate evolution and risk. Our work also offers a straightforward method to measure FX illiquidity and commonality in terms of price impact. For investors, these insights should increase the efficiency of trading and risk analysis. For policy makers, our work highlights the developments of volatility, volume, and illiquidity across time and currencies, which can be important for the implementation of monetary policy and financial stability.

References


