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WORKING PAPERS ON FINANCE NO. 2019/01

SWISS INSTITUTE OF BANKING AND FINANCE (S/BF – HSG)

FEBRUARY 2019

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This version: February 2019

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Keywords: Asset pricing, Non-linear dependence, Copulas, Crash aversion, Downside risk, Lower tail dependence, Tail risk

JEL classifications: C58, G01, G11, G12, G17.

* We are grateful for helpful comments and suggestions from Marcel Prokopczuk, Stefan Ruenzi, and from seminar participants at the University of Mannheim and the 2018 Annual Meeting of the German Finance Association.

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Abstract: This paper investigates whether multivariate crash risk is priced in the cross-section of expected stock returns. Motivated by a theoretical asset pricing model, we capture the multivariate crash risk of a stock by a combined measure based on its expected shortfall and its multivariate lower tail dependence with the systematic risk factors of the Carhart (1997) model. We find that stocks with a high exposure to joint crashes of the market and the momentum factor bear a risk premium which is not explained by traditional linear factor models or by other downside risk measures. Our results indicate that accounting for the multivariate crash risk of established state variables helps to understand the cross-section of expected stock returns without further expanding the factor zoo.

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1 Introduction

Large stock market downturns are a main concern for both retail and institutional investors in risk management, asset allocation, and portfolio optimization. Starting with Arzac and Bawa (1977) financial economists have recognized that crash risk is also an important factor in the pricing of individual stocks. Assets that exhibit particularly bad returns during market crashes are unattractive because their value deteriorates most at exactly the moment when investor wealth is particularly low. In line with this intuition, Bali et al. (2014), Kelly and Jiang (2014) and Chabi-Yo et al. (2017) find that crash-sensitive assets earn a risk premium in the cross-section of expected stock returns.[1] All of the afore-mentioned studies focus on a stock's univariate crash risk (e.g., its value-at-risk or expected shortfall) or its bivariate crash risk (i.e., joint tail events with the market); to the best of our knowledge, the relationship between a stock's multivariate crash risk (i.e., its exposure to joint crashes of multiple risk factors) and average future stock returns has not been examined yet.

In this paper, we fill this gap and study the effect of multivariate crash risk on the cross-section of expected stock returns. Intuitively, multivariate crash risk is important because it captures exposure to "perfect storm scenarios", i.e., high marginal utility states with low diversification benefits and synchronized declines in multiple systematic factors. To measure multivariate crash risk (MCRASH), we combine an asset's multivariate lower tail dependence (MLTD, i.e., the conditional probability of a tail event for this asset given the occurrence of a multivariate factor crash) and its expected shortfall (ES, i.e., the expected magnitude of a crash scenario for this asset). A stock has high MCRASH if its MLTD and its ES are high, i.e, if (i) the worst returns of the stock cluster with downturns of systematic asset pricing risk factors and (ii) the stock's individual crash risk is substantial.[2]

To obtain a first intuition why multivariate crash risk is distinct from univariate and bivariate crash risk, we provide a stylized example in Figure 1. It shows the return time series of two risk

[1]Van Oordt and Zhou (2016) also test for the presence of a systematic tail risk premium in the cross-section of expected stock returns. They find that historical tail betas predict the future performance of stocks in extreme market downturns and help to assess portfolio tail risks. Atilgan et al. (2017) investigate the impact of a stock's value-at-risk and expected shortfall on the cross-section of expected stock returns. They document a negative relation between left-tail risk and future stock returns.

[2]In the bivariate case, that is, in the case of a joint downturn of the asset and the market return, MCRASH is similar to the tail risk measure of Agarwal et al. (2017) for hedge funds.

factors (A and B) and three assets (1, 2, and 3). There are three crash scenarios for the risk factors: two individual crashes and a joint crash for both factors. We observe that the bivariate crash risk of assets 1 and 2 with the two risk factors is identical in the sense that both assets have one simultaneous crash with each of the factors. However, there are differences in MLTD and thus in MCRASH: while asset 1 is not affected by the joint crash scenario of risk factors A and B, asset 2 realizes a large loss exactly during the time when both factors 1 and 2 crash together. Note that standard bivariate lower tail dependence coefficients (LTD) do not capture this difference.[3] Moreover, the return time series of assets 2 and 3 illustrate the case when two assets have identical MLTD, but differ in ES (with asset 3 having the higher individual crash risk).

[Insert Figure 1 around here]

To formalize these ideas, we analyze the relevance of multivariate crash risk for asset prices in a theoretical framework with a generic projected Stochastic Discount Factor (SDF) that depends on multiple risk factors. In this framework, we propose to combine two Taylor series approximations that relate changes in the underlying risk factors to changes in the discount factor. One of these approximations is focussed on tail events and allows us to link the risk premium to conditional multivariate crash risk. In particular, we can – under weak conditions – show that the asset's expected excess return is an increasing function of both MLTD and ES. Based on these results, we introduce our new systematic risk measure MCRASH, which combines the information in MLTD and ES, and predict that high MCRASH leads to high average future returns of a security.[4]

In our empirical analysis, we investigate the impact of multivariate crash risk on average future returns based on MCRASH-coefficients of the market and the additional risk factors of the Carhart (1997) four-factor model, i.e., the SMB (Small minus Big) size factor, the HML (High minus Low) book-to-market factor, and the UMD (Up minus Down) momentum factor. We choose these four risk factors based on their effectiveness to adequately describe the cross-section of expected stock returns (see Fama and French, 1993, and Carhart, 1997).[5] The measurement of multivariate crash

[3]Numerical examples for this difference between LTD and MLTD are presented in Appendix A. These examples also show that the information in MLTD coefficients is usually different from the information in bivariate LTD coefficients based on linear combinations of risk factors. In particular, we show that a stock's trivariate MLTD with risk factors 1 and 2 is typically not identical to the stock's bivariate LTD with a linear combination of these factors.

[4]Note that if we only consider crashes of a single risk factor, MCRASH simplifies to CRASH which relies on (standard) bivariate LTD and again the asset-specific ES.

[5]Our multivariate crash risk approach can be potentially extended to other risk factors, such as market volatility, market liquidity, or funding liquidity.

risk with these factors is challenging because (i) joint tail events are rarer than individual tail events and (ii) the dependence structure of the Carhart (1997) factors has been shown to vary over time and exhibit large extreme correlations (while unconditional correlations are small or even negative, see Christoffersen and Langlois, 2013). To address both issues, we apply a flexible conditional copula specification for the estimation of MCRASH coefficients. In particular, we rely on a combination of a dynamic skewed-t copula model and GARCH margins following the methodology developed in Christoffersen et al. (2012) and Christoffersen and Langlois (2013).

We employ the 49 value-weighted Fama and French industry portfolios as our test assets. For each month in our sample period from 1970 to 2015 and for each industry, we compute MCRASH coefficients with respect to all factor pairs based on a probability level of 10%.[6] A descriptive analysis of these coefficients reveals that the crash risk for the factor pairs including the market factor (e.g., $\text{MCRASH}^{\text{MKT,UMD}}$) is several times higher than the risk exposure to simultaneous factor crashes without MKT (e.g., $\text{MCRASH}^{\text{SMB,UMD}}$). We also document that all MCRASH coefficients are persistent on the industry level with large positive autocorrelation coefficients up to 48 months.

Given these results, we examine the three multivariate crash risk measures that include the market factor, i.e., $\text{MCRASH}^{\text{MKT,SMB}}$, $\text{MCRASH}^{\text{MKT,HML}}$ and $\text{MCRASH}^{\text{MKT,UMD}}$, in our asset pricing analysis.[7] In our main tests, we relate the MCRASH coefficients of our test assets in month t to average returns in month $t+1$. Univariate portfolio sorts indicate that $\text{MCRASH}^{\text{MKT,SMB}}$, $\text{MCRASH}^{\text{MKT,HML}}$ and $\text{MCRASH}^{\text{MKT,UMD}}$ have a positive impact on average future returns with only $\text{MCRASH}^{\text{MKT,UMD}}$ showing a strongly significant impact. Hence, in the rest of the empirical analysis, we focus on the impact of $\text{MCRASH}^{\text{MKT,UMD}}$ on average future returns. We find that an investment strategy of going long the quintile portfolio with the highest $\text{MCRASH}^{\text{MKT, UMD}}$ coefficients and going short the quintile portfolio with the lowest $\text{MCRASH}^{\text{MKT, UMD}}$ coefficients in month t yields an average return spread of 0.518% per month in $t+1$ with a t-statistic of 2.64. In line with our theoretical prediction, the return spread based on $\text{MCRASH}^{\text{MKT,UMD}}$ is partly due to $\text{MLTD}^{\text{MKT,UMD}}$ (return premium of 0.272%) and ES (return premium of 0.083%).

We check whether the impact of $\text{MCRASH}^{\text{MKT, UMD}}$ on future returns is different from

[6]We also investigate simultaneous crashes of all four factors but find that the extremely low probabilities of such events (see Table 2) impede a reliable computation of the corresponding asset-specific crash risk exposures.

[7]Additional results for univariate CRASH measures and the other combinations are presented in the Appendix.

the impact of other crash risk measures. For this purpose, we conduct portfolio double-sorts based on the four bivariate crash risk coefficients (i.e., $\text{CRASH}^{\text{MKT}}$, $\text{CRASH}^{\text{SMB}}$, $\text{CRASH}^{\text{HML}}$, $\text{CRASH}^{\text{UMD}}$) and MCRASH-coefficients for alternative factor combinations on the one side, as well as $\text{MCRASH}^{\text{MKT, UMD}}$ on the other side. We find that the return effect of $\text{MCRASH}^{\text{MKT, UMD}}$ is not subsumed by any of the other crash risk measures.

We then proceed to investigate the relationship between $\text{MCRASH}^{\text{MKT, UMD}}$ and future returns in a multivariate context controlling for a long array of different industry characteristics, alternative risk measures and factor betas. Our results reveal that the impact of $\text{MCRASH}^{\text{MKT, UMD}}$ on future returns remains strong when we control for an industry's average size (Banz, 1981), average book-to-market value (Basu, 1983), past return (Jegadeesh and Titman, 1993), volatility, coskewness (Harvey and Siddique, 2000), and downside beta (Ang et al., 2006), as well as for linear exposure to MKT, SMB, HML, UMD, the investment and profitability factors from the Fama and French (2015) five-factor model, the Fama and French short-term and long-term reversal factors, the Pástor and Stambaugh (2003) traded liquidity risk factor, the Frazzini and Pedersen (2014) betting-against-beta factor, the Kelly and Jiang (2014) tail risk factor, and the Asness et al. (2017) quality-minus-junk factor. In terms of economic significance, we find that a one standard deviation increase in $\text{MCRASH}^{\text{MKT,UMD}}$ leads to higher annualized average future returns of approximately 3% to 5%. The return effect of $\text{MCRASH}^{\text{MKT, UMD}}$ persists for return horizons up to six months ahead.

We conduct a number of additional tests to confirm the stability of our main result of a positive, statistically significant relationship between $\text{MCRASH}^{\text{MKT,UMD}}$ and future returns. First, we show that this result is not driven by using full-sample parameter estimates for our dynamic copula and GARCH models. In particular, we also find a positive and significant impact of $\text{MCRASH}^{\text{MKT,UMD}}$ on future returns when we only rely on the first half of the sample to estimate the parameters of these models and then use the second half for the asset pricing tests.[8] Second, we confirm that the return spread for $\text{MCRASH}^{\text{MKT, UMD}}$ is generally stable over time, but – in line with the idea of a risk premium – particularly realizes in times when the market return and the UMD momentum return are positive. Third, we show that our results are stable to several choices made in the empirical analysis and hold when we alter the type and frequency of returns in the model estimation,

[8] Hence, based on this empirical setup, investors can implement a real-time trading strategy on $\text{MCRASH}^{\text{MKT, UMD}}$ without any look-ahead bias.

distributional and dependence-specific assumptions, as well as the methodology in the asset pricing tests (among others). Finally, we document that the risk premium for $\text{MCRASH}^{\text{MKT, UMD}}$ is also statistically and economically strong in a different data sample consisting of individual S&P 100 stocks.

Our study contributes to two strands of the literature. First, it is related to the theoretical and empirical asset pricing literature on downside and crash risk. The discussion of downside risk aversion goes back to the ideas in Roy (1952) and Markowitz (1959) on the safety first principle and the use of the semi-variance as a risk measure. Subsequent studies (such as Kraus and Litzenberger, 1976, Friend and Westerfield, 1980, Harvey and Siddique, 2000 and Dittmar, 2002) investigate the impact of higher co-moments on the cross-section of expected stock returns. Ang et al. (2006) show that stocks with higher downside betas as proposed by Bawa and Lindenberg (1977) earn higher average returns and Lettau et al. (2014) document that the downside risk CAPM can jointly rationalize the cross-section of several asset classes.[9] More recent work has been focussed on extreme market downturns: Kelly and Jiang (2014) extract a time-varying tail risk factor from the cross-section of returns using extreme value theory and find that stocks with higher loadings on this factor have higher future returns. Exploiting forward-looking information from S&P 500 index options, Lu and Murray (2018) show that exposure to changes in the ex-ante probability of market crashes explains average future stock returns. Van Oordt and Zhou (2016) measure the sensitivity to systematic tail events based on tail betas and document that these betas predict the future performance of stocks during market crashes, but they do not find evidence for a positive premium associated with this exposure. Chabi-Yo et al. (2017) study the pricing of bivariate LTD with the market portfolio and find that stocks with high bivariate LTD earn significantly higher returns than stocks with low bivariate LTD.[10] Evidence of crash risk for other asset pricing risk factors is scarce and is mostly concerned with momentum crashes. Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) document that the momentum factor experiences infrequent and persistent strings of negative returns and show that volatility-adjusted momentum strategies

[9] Whereas the risk measures in those papers are conditioned on the market return being below a specific threshold, Bali et al. (2014) introduce a "hybrid" measure of stock return tail covariance risk that is conditional on a stock being in a downstate and they document a positive relation between this measure and expected stock returns.

[10] Moreover, the existence of a premium for LTD is also confirmed for other asset classes, e.g., Agarwal et al. (2017) find that hedge funds that load on tail risk earn high future returns. Meine et al. (2016) show that bivariate crash risk is compensated in the cross-section of credit default swaps of banks.

have high Sharpe ratios. Finally, Ruenzi and Weigert (2018) provide a risk-based explanation of the momentum anomaly on equity markets and show that the momentum factor is correlated to market crash risk. We contribute to this strand of literature by extending the analysis of systematic crash risk to a multi-factor setting. Our theoretical results shed new light on the role of univariate and multivariate crash risk for the pricing of financial assets and our empirical analysis reveals that an asset's sensitivity to multivariate crashes of the market and the momentum factor is a key determinant for the cross-section of expected stock returns.

Second, we contribute to the literature on the application of non-linear dependence measurement, extreme value theory, and copulas in finance. Longin and Solnik (2001), Ané and Kharoubi (2003) and Poon et al. (2004) apply extreme value theory and copulas to study extreme dependencies between selected international equity markets. Patton (2004), Jondeau and Rockinger (2006) and Christoffersen et al. (2012) develop dynamic copula models to describe the time-variation in the conditional dependence structure of stock returns. Further applications of copulas in finance include e.g. Patton (2009), who assesses the market neutrality of hedge funds, and Elkamhi and Stefanova (2015), who show that accounting for extreme asset comovements is important for portfolio hedging. Christoffersen and Langlois (2013) use copula methods to investigate the dynamics and the non-linearities in the dependence structure of the four Carhart (1997) equity market factors and their importance for portfolio allocation. Our paper is the first to apply similar copula techniques in multi-factor models to study the asset pricing implications of conditional non-linear dependencies. Our results imply that incorporating such dependence features can improve the performance of asset pricing models without further extending the range of factors.

We proceed as follows. Section 2 provides a theoretical model for the pricing of multivariate crash risk. Section 3 describes our methodology for the measurement of extreme non-linear dependence, which is required for the calculation of our systematic crash risk measures. Section 4 introduces our data sample and presents our estimation results for the crash risk measures. In Section 5, we document the empirical results on the relationship between multivariate crash risk and average future stock returns. Section 6 concludes.

2 Theory

In this section, we first introduce the crash risk measures that we use in our analysis. We then study the theoretical relationship between multivariate crash risk and expected stock returns using a new expansion of the Stochastic Discount Factor. Finally, we illustrate the benefits of our new approach based on a stylized example with specific preferences and distributional assumptions.

2.1 Crash Risk Measures

We first formalize the notion of sensitivity to multivariate crashes by relying on a generalization of bivariate lower tail dependence (LTD) coefficients.[11] Let R_i denote the discrete return of an asset and let $\boldsymbol{X} = (X_1, \ldots, X_N)'$ denote a $N \times 1$ vector of state variables or systematic factors describing changes in the investment opportunity set. Furthermore, let $Q_p[Y] = \inf\{x \in \mathbb{R}; \mathbb{P}[Y \leq x] \geq p\}$ denote the p-quantile of a random variable Y for $p \in (0, 1)$. We define the multivariate lower tail dependence (MLTD) of R_i with the vector $\boldsymbol{X}$ at a (small) probability level p by

$$\text{MLTD}_p^{\boldsymbol{X}}[R_i] := \mathbb{P}[R_i \leq Q_p[R_i] \,|\, X_1 \leq Q_p[X_1], \ldots, X_N \leq Q_p[X_N]]\,. \tag{1}$$

MLTD thus corresponds to the conditional probability that R_i does not exceed its p-quantile given that all state variables are simultaneously at or below their p-quantiles. For small values of p, MLTD measures the exposure of asset i to a multivariate crash or a perfect storm scenario. MLTD can be understood as (crash) beta on the level of probabilities because it can be rewritten as the probability of a joint tail event for asset i and all factors *standardized* by the probability of the corresponding factor crash. $\text{MLTD}_p^{\boldsymbol{X}}[R_i] = 1$ if asset i always crashes simultaneously with the systematic factors. The additional information provided by MLTD compared to bivariate LTD coefficients is illustrated in Appendix A with simple numerical examples.

Due to its quantile-based definition[12], MLTD does not directly account for the severity of crash events. The magnitude of individual crash events is typically captured by univariate tail risk measures such as Expected Shortfall (ES). For a return R_i with a continuous distribution[13], the

[11]See e.g. Poon et al. (2004), Christoffersen et al. (2012) and Chabi-Yo et al. (2017) for applications of bivariate LTD measures in finance.

[12]Note that MLTD is invariant under increasing and continuous transformations of R_i or $X_1, \ldots, X_N$. It is therefore invariant under changes of these marginal distributions. See Appendix B for a derivation of this property.

[13]For general distributions, the Expected Shortfall is defined as $\text{ES}_p[R_i] = -\frac{1}{p}\int_0^p Q_u[R_i]du$.

Expected Shortfall can be defined as the expectation of R_i conditional on R_i not exceeding its p-quantile multiplied by minus one, i.e.

$$\mathrm{ES}_p[R_i] := -\mathbb{E}[R_i \mid R_i \le Q_p[R_i]] \,. \tag{2}$$

In other words, the Expected Shortfall is the (negative) mean of R_i in its p-tail.

Motivated by the theory that we present in the remaining part of this section, we propose MCRASH as a new measure of systematic multivariate crash risk. MCRASH is defined as

$$\mathrm{MCRASH}_p^{\boldsymbol{X}}[R_i] := \left(\mathrm{MLTD}_p^{\boldsymbol{X}}[R_i] - p\right) \mathrm{ES}_p[R_i] \,. \tag{3}$$

It thus combines the information in $\mathrm{MLTD}_p^{\boldsymbol{X}}[R_i]$ on the exposure of asset i to a crash of the systematic factors in $\boldsymbol{X}$ with the information on the specific crash risk of asset i as measured by $\mathrm{ES}_p[R_i]$. For the special case $N = 1$, we obtain the bivariate risk measure

$$\mathrm{CRASH}_p^{X}[R_i] = (\mathrm{LTD}_p^{X}[R_i] - p)\, \mathrm{ES}_p[R_i] \,, \tag{4}$$

which is similar to the systematic tail risk measure proposed by Agarwal et al. (2017).[14]

2.2 Crash Risk and Expected Returns

Our theoretical analysis of the risk premium for an asset i with the discrete return $R_{i,t+1}$ over the period $[t, t+1]$ relies on a nonnegative Stochastic Discount Factor (SDF) M_{t+1}. By its definition, the discount factor satisfies $\mathbb{E}_t[M_{t+1}\,(1 + R_{i,t+1})] = 1$, where $\mathbb{E}_t$ denotes the expectation given the available information in t. The existence of M_{t+1} is guaranteed by no arbitrage (Harrison and Kreps, 1979; Hansen and Richard, 1987). If the return is driven by a set of systematic factors or state variables $\boldsymbol{X}_{t+1}$, the SDF can be replaced by its projection $M_{t+1}^{\boldsymbol{X}} = \mathbb{E}_t[M_{t+1} \mid \boldsymbol{X}_{t+1}]$. More specifically, we assume that $R_{i,t+1} = f_i(\boldsymbol{X}_{t+1}) + \varepsilon_{i,t+1}$, where f_i is an arbitrary function and $\varepsilon_{i,t+1}$ is a zero-mean residual that is not priced conditional on $\boldsymbol{X}_{t+1}$, i.e. $\mathbb{E}_t[\varepsilon_{i,t+1} \mid \boldsymbol{X}_{t+1}] = 0$ and

[14] We will detail the relation to this measure in the following subsection and show that our theory can rationalize a risk premium for this kind of systematic tail risk.

$\text{cov}_t[M_{t+1}, \varepsilon_{i,t+1} \,|\, \boldsymbol{X}_{t+1}] = 0$. Under these assumptions, we obtain

$$\mathbb{E}_t\Big[M^{\boldsymbol{X}}_{t+1}(1 + R_{i,t+1})\Big] = 1. \tag{5}$$

Due to its definition, the projected SDF can be written as $M^{\boldsymbol{X}}_{t+1} = m(\boldsymbol{X}_{t+1})$ with a measurable function $m : \mathbb{R}^N \to \mathbb{R}$. In a standard representative agent framework, this function is given by

$$m(\boldsymbol{x}) = \frac{\delta}{1 + R_{f,t+1}}\, u'(g(\boldsymbol{x})) \qquad \text{with} \qquad \delta := \frac{1}{\mathbb{E}_t[u'(g(\boldsymbol{X}_{t+1}))]}, \tag{6}$$

where $R_{f,t+1}$ is the return of the risk-free asset over $[t, t+1]$, u is the utility function of the representative investor and $g : \mathbb{R}^N \to \mathbb{R}$ is a function that links the state variables in $\boldsymbol{X}_{t+1}$ to the wealth (consumption) of this investor (see p. 166 in Cochrane, 2005 for a similar structure of the SDF). We assume that the joint conditional distribution of $R_{i,t+1}$ and $\boldsymbol{X}_{t+1}$ is continuous. Furthermore, we suppose that the utility function is twice differentiable with $u' > 0$ and $u'' < 0$ (Pratt, 1964). In addition, we require that the partial derivatives of g exist and that the state variables are defined such that $\frac{\partial g}{\partial x_i} > 0$ for $i = 1, \ldots, N$, i.e., the wealth of the representative investor and thus her utility are increasing in each of the state variables. This includes an SDF that depends on a linear combination of the market return, the SMB size factor, the HML book-to-market factor, and the UMD momentum factor with positive weights (see, e.g., Brandt et al., 2009, Kozak et al., 2017).

Since the SDF also prices the risk-free security, we have $\mathbb{E}_t[m(\boldsymbol{X}_{t+1})] = 1/(1 + R_{f,t+1})$, which can be used to rewrite (5) as

$$\mathbb{E}_t[R_{i,t+1} - R_{f,t+1}] = -(1 + R_{f,t+1})\, \text{cov}_t[m(\boldsymbol{X}_{t+1}), R_{i,t+1}]\,. \tag{7}$$

A standard approach to derive a linear factor model from (7) is a first-order Taylor approximation around $\boldsymbol{X}_{t+1} = \boldsymbol{x}_0$ for m. If we choose $\boldsymbol{x}_0$ from the center of the distribution of $\boldsymbol{X}_{t+1}$, the approximation error might be relatively high for extreme realizations of $\boldsymbol{X}_{t+1}$. To address this issue, we propose to use *two* Taylor approximations for m: a first approximation around a value in the center of the distribution and a second approximation around a value in its lower tail. We define

$$T_p^{\boldsymbol{X}} := \{X_{1,t+1} \leq Q_{p,t}[X_{1,t+1}], \ldots, X_{N,t+1} \leq Q_{p,t}[X_{N,t+1}]\} \tag{8}$$

with $Q_{p,t}[X_{i,t+1}]$ as the (conditional) p-quantile of $X_{i,t+1}$ given the information available at time t. Note that $T_p^{\boldsymbol{X}}$ corresponds to the systematic crash event that was used to define MLTD in equation (1). We decompose $m(\boldsymbol{X}_{t+1})$ into

$$m(\boldsymbol{X}_{t+1}) = m(\boldsymbol{X}_{t+1})\, \mathbb{1}(T_p^{\boldsymbol{X}}) + m(\boldsymbol{X}_{t+1})\, \mathbb{1}(\overline{T}_p^{\boldsymbol{X}}), \tag{9}$$

where $\mathbb{1}(A)$ denotes the indicator function of the event A and $\overline{A}$ denotes its complementary event.

For the first summand in (9), we apply a Taylor approximation around a value $\boldsymbol{x}_p$ in the lower tail of the joint distribution. In particular, we rely on the (conditional) Expected Shortfall given the information at time t to choose the elements of $\boldsymbol{x}_p$, i.e. we use $\boldsymbol{x}_p := (-\mathrm{ES}_{p,t}[X_{1,t+1}], \ldots, -\mathrm{ES}_{p,t}[X_{N,t+1}])'$. This choice naturally generalizes the standard approach of a Taylor approximation around the mean to the lower tail of $\boldsymbol{X}_{t+1}$. For the second term in (9), we use a standard approximation around a central value $\boldsymbol{x}_c$ (i.e., $\boldsymbol{x}_c = \boldsymbol{0}$, when we are working with factor returns). In total, we obtain

$$\begin{aligned} m(\boldsymbol{X}_{t+1}) \approx\ & (m(\boldsymbol{x}_p) + \nabla m(\boldsymbol{x}_p) \cdot (\boldsymbol{X}_{t+1} - \boldsymbol{x}_p))\, \mathbb{1}(T_p^{\boldsymbol{X}}) \\ & + (m(\boldsymbol{x}_c) + \nabla m(\boldsymbol{x}_c) \cdot (\boldsymbol{X}_{t+1} - \boldsymbol{x}_c))\, \mathbb{1}(\overline{T}_p^{\boldsymbol{X}}), \end{aligned} \tag{10}$$

where $\nabla m(\boldsymbol{x}_0) := \left(\frac{\partial m}{\partial x_1}(\boldsymbol{x}_0), \ldots, \frac{\partial m}{\partial x_N}(\boldsymbol{x}_0)\right)$. Using (7) and (10), we derive the following decomposition of the expected excess return

$$\mathbb{E}_t[R_{i,t+1} - R_{f,t+1}] \approx \text{no-Tail}_{i,t}^{\boldsymbol{X}} + \text{Tail}_{i,t}^{\boldsymbol{X},0} + \text{Tail}_{i,t}^{\boldsymbol{X},1} \tag{11}$$

in Appendix B.

The component no-Tail$_{i,t}^{\boldsymbol{X}}$ captures the part of the risk premium that is not related to systematic crash events, i.e., when $\mathbb{1}(T_p^{\boldsymbol{X}}) = 0$. It is given by

$$\text{no-Tail}_{i,t}^{\boldsymbol{X}} = -\,(1 + R_{f,t+1})\,\text{cov}_t\Big[(m(\boldsymbol{x}_c) + \nabla m(\boldsymbol{x}_c) \cdot (\boldsymbol{X}_{t+1} - \boldsymbol{x}_c))\,\mathbb{1}(\overline{T}_p^{\boldsymbol{X}}), R_{i,t+1}\Big]\,. \tag{12}$$

This part of the decomposition is structurally similar to the risk premium in standard linear factor models. For small levels of the tail probability p, it holds that $\mathbb{P}_t\Big[\mathbb{1}(\overline{T}_p^{\boldsymbol{X}}) = 1\Big] \approx 1$ and thus

$$\text{no-Tail}_{i,t}^{\boldsymbol{X}} \approx \sum_{j=1}^{N} \beta_{i,t}^{(j)}\ \lambda_t^{(j)} \tag{13}$$

with

$$\lambda_t^{(j)} = -(1 + R_{f,t+1})\,\frac{\partial m}{\partial x_j}(\boldsymbol{x}_c)\,\text{var}_t[X_{j,t+1}] \qquad \text{and} \qquad \beta_{i,t}^{(j)} = \frac{\text{cov}_t[X_{j,t+1}, R_{i,t+1}]}{\text{var}_t[X_{j,t+1}]}. \tag{14}$$

The approximation in equation (13) recovers the well-known representation of the excess return in terms of linear betas $\beta_{i,t}^{(j)}$ and the corresponding prices of risk $\lambda_t^{(j)}$.

$\text{Tail}_{i,t}^{\boldsymbol{X},0}$ and $\text{Tail}_{i,t}^{\boldsymbol{X},1}$ capture the expected excess return due to systematic crashes, i.e., when $\mathbb{1}(T_p^{\boldsymbol{X}}) = 1$. $\text{Tail}_{i,t}^{\boldsymbol{X},0}$ collects the zero-order terms of the tail-focussed Taylor-approximation. As detailed in Appendix B, it is given by

$$\text{Tail}_{i,t}^{\boldsymbol{X},0} = \text{MCRASH}_{p,t}^{\boldsymbol{X}}[R_{i,t+1}]\,\lambda_{p,t}^{\boldsymbol{X}}, \tag{15}$$

where $\text{MCRASH}_{p,t}^{\boldsymbol{X}}[R_{i,t+1}]$ is our new measure of multivariate crash risk as defined in equation (3) conditional on the information available at time t and

$$\lambda_{p,t}^{\boldsymbol{X}} := \delta\,u'(g(\boldsymbol{x}_p))\,\mathbb{P}_t\Big[T_p^{\boldsymbol{X}}\Big]\,. \tag{16}$$

$\lambda_{p,t}^{\boldsymbol{X}}$ collects the terms in $\text{Tail}_{i,t}^{\boldsymbol{X},0}$ that are not asset-specific and can be understood as the price of (zero-order) multivariate crash risk. The magnitude of this price depends on the marginal utility $u'(g(\boldsymbol{x}_p))$ for the joint tail realization $\boldsymbol{X}_{t+1} = \boldsymbol{x}_p$ and the probability $\mathbb{P}_t\Big[T_p^{\boldsymbol{X}}\Big]$ of a multivariate crash for the systematic risk factors. The first component is positive due to $u' > 0$ and it is larger for more severe tail events because of $u'' < 0$ and $\frac{\partial g}{\partial x_i} > 0$, $i = 1, \ldots, N$. $\lambda_{p,t}^{\boldsymbol{X}}$ is thus increasing in the

ES of the systematic factors. The probability $\mathbb{P}_t\left[T_p^{\boldsymbol{X}}\right]$ is also positive and depends on the joint tail behavior of the systematic factors.[15] Due to the positivity of $\lambda_{p,t}^{\boldsymbol{X}}$, the corresponding risk premium is increasing in MCRASH. If MCRASH is positive, i.e. if the lower tail dependence is larger than p and the univariate tail risk as measured by the Expected Shortfall is positive, then $\text{Tail}_{i,t}^{\boldsymbol{X},0}$ will also be positive.

For $N = 1$, (15) simplifies to a relationship between $\text{Tail}_{i,t}^{\boldsymbol{X},0}$ and the bivariate measure $\text{CRASH}_{p,t}^{X_1}[R_{i,t+1}]$, which can be rewritten as

$$\text{Tail}_{i,t}^{\boldsymbol{X},0} = \lambda_{p,t}^{\boldsymbol{X}}\, \text{ES}_{p,t}[X_{1,t+1}]\ \tilde{\beta}_{i,t}^{p} \quad \text{with} \quad \tilde{\beta}_{i,t}^{p} = \frac{\text{CRASH}_{p,t}^{X}[R_{i,t+1}]}{\text{ES}_{p,t}[X_{1,t+1}]}. \tag{17}$$

This rationalizes the systematic tail risk measure introduced by Agarwal et al. (2017) in their analysis of the cross-section of hedge fund returns (using the market return as risk factor, i.e. $X_{1,t+1} = R_{M,t+1}$).[16] In addition, this result provides an alternative explanation for the LTD-premium found by Chabi-Yo et al. (2017).

The premium $\text{Tail}_{i,t}^{\boldsymbol{X},1}$ given by (42) in Appendix B collects the remaining tail-related components of the expected excess-return. It consists of two residual components, which capture deviations of $\boldsymbol{X}_{t+1}$ and $R_{i,t+1}$ from their conditional expectations in the corresponding p-tail, and a third component related to states when the factors realize a systematic tail event but $R_{i,t+1}$ is above its p-quantile. Due to the rarity of joint tail events, these components are hard to estimate exactly. We therefore focus on $\text{Tail}_{i,t}^{\boldsymbol{X},0}$ in our empirical analysis. We are confident that the inclusion of this additional tail term eventually improves the overall approximation quality compared to a standard approach based on a single Taylor approximation, which largely neglects any tail information.

By plugging (13) and (15) into (11), we obtain an asset pricing model of the form

$$\mathbb{E}_t[R_{i,t+1} - R_{f,t+1}] = \alpha_{i,t} + \sum_{j=1}^{N} \beta_{i,t}^{(j)}\, \lambda_t^{(j)} + \text{MCRASH}_{p,t}^{\boldsymbol{X}}[R_{i,t+1}]\, \lambda_{p,t}^{\boldsymbol{X}}. \tag{18}$$

In this specification, the term $\alpha_{i,t}$ captures pricing errors arising from the non-linearity of the function m, the approximation argument in (13) and the terms in $\text{Tail}_{i,t}^{\boldsymbol{X},1}$. $\beta_{i,t}^{(j)}\, \lambda_t^{(j)}$ corre-

[15] Note that this tail behavior also determines the asymptotic price of tail risk in our framework as the limit of $\lambda_{p,t}^{\boldsymbol{X}}$ for $p \to 0$ depends on whether u' grows faster than $\mathbb{P}_t\left[T_p^{\boldsymbol{X}}\right]$ goes to zero.

[16] Agarwal et al. (2017) use an asymptotic notion of LTD ($p \to 0$) for the definition of "Tailrisk". They implement the measure for a probability level of 5% in their baseline analysis.

sponds to the premium for exposure to factor j in a standard linear model, $j = 1, \ldots, N$, and $\lambda_{p,t}^{\boldsymbol{X}}\,\text{MCRASH}_{p,t}^{\boldsymbol{X}}[R_{i,t+1}]$ is the additional crash-related premium.

Based on equation (18) and the positivity of $\lambda_{p,t}^{\boldsymbol{X}}$ in (15), we arrive at our main hypothesis about the cross-sectional pricing of multivariate crash risk.

Hypothesis: The expected excess return of asset i is increasing in its exposure to multivariate crash risk as measured by $\text{MCRASH}_{p,t}^{\boldsymbol{X}}[R_{i,t+1}]$.

We thus expect higher average returns for assets that have a high level of asset-specific (univariate) crash risk *and* a high level of lower tail dependence with the systematic factors $\boldsymbol{X}_{t+1}$. More specifically, in the typical situation[17] $\text{ES}_{p,t}[R_{i,t+1}] > 0$:

- assets with $\text{MLTD}_{p,t}^{\boldsymbol{X}}[R_{i,t+1}] > p$ earn a positive premium compared to the linear factor model, which increases in MLTD and ES,
- assets with $\text{MLTD}_{p,t}^{\boldsymbol{X}}[R_{i,t+1}] < p$ have a lower expected return than predicted by the linear benchmark. Such assets offer diversification against multivariate factor crashes.

Furthermore, the form of $\lambda_{p,t}^{\boldsymbol{X}}$ in equation (16) helps us to identify factor combinations, for which multivariate crash risk can be especially important. In particular, the price of tail risk $\lambda_{p,t}^{\boldsymbol{X}}$ and thus the related risk premium increase...

- in the crash risk of the systematic factors as measured by their Expected Shortfall, i.e. the risk premium is increasing in $\text{ES}_{p,t}[X_{j,t+1}]$ for $j = 1, \ldots, N$,
- and in the probability of a systematic factor crash $\mathbb{P}_t\left[T_p^{\boldsymbol{X}}\right]$.

2.3 A Stylized Example

We now present a stylized theoretical example using specific preferences and distributional assumptions to investigate the potential benefits of our tail-focussed extension of the standard linear model. First, we compare the pricing errors $\alpha_{i,t}$ of our new specification given in (18) with the pricing errors $\alpha_{i,t}^{l}$ of the linear benchmark model

$$\mathbb{E}_t[R_{i,t+1} - R_{f,t+1}] = \alpha_{i,t}^{l} + \sum_{j=1}^{N} \lambda_t^{(j)}\,\beta_{i,t}^{(j)}, \tag{19}$$

[17]For small values of p, the p-quantile of the return is negative so that its Expected Shortfall is positive.

where $\lambda_t^{(j)}$ and $\beta_{i,t}^{(j)}$ are given in (14). Furthermore, we illustrate the magnitude of cross-sectional variation in the pricing errors ($\alpha_{i,t}^l$) of the linear benchmark that our multivariate crash risk measure MCRASH can capture.

We consider the case $N = 2$ and assume a very simple mapping function $g(x_1, x_2) = 0.5\,x_1 + 0.5\,x_2$. As a standard choice for the preferences of the representative investor, we rely on power utility with a risk aversion parameter RRA = 4. We use a flexible parametric model for the conditional distribution of $(R_{i,t+1}, X_{1,t+1}, X_{2,t+1})$, which includes a multivariate normal distribution for the corresponding logarithmic returns as special case.[18] For our baseline specification, the distribution parameters are chosen such that the annualized standard deviations of the factors ($X_{1,t+1}$, $X_{2,t+1}$) are 20% and the standard deviation of $R_{i,t+1}$ is 25%. The pairwise correlations between $X_{1,t+1}$, $X_{2,t+1}$ and $R_{i,t+1}$ are 50%. The marginal distribution of the first factor only exhibits moderate deviations from normality with zero skewness and an excess kurtosis of 1.9, whereas the marginal distribution of $X_{2,t+1}$ is negatively skewed (-1.3) and has a relatively high level of excess kurtosis (5.9). The skewness and the excess kurtosis of the asset return $R_{i,t+1}$ are set to intermediate levels (-0.5 and 3.6).[19] The details of our distributional assumptions and the chosen parameter values are described in Appendix C.

We use $p = 0.01$ as tail probability level and simulate monthly returns. We generate 1,000,000 realizations of the return vector $(R_{i,t+1}, X_{1,t+1}, X_{2,t+1})$ to compute the exact risk premium according to (7) as well as the pricing errors $\alpha_{i,t}$ and $\alpha_{i,t}^L$ of the models presented in (18) and (19). For the baseline specification described above, the exact annualized expected excess return is 12.9%. The corresponding error of the linear model is $\alpha_{i,t}^l = 2.7\%$ and the error of our new approximation is $\alpha_{i,t} = 0.6\%$.

[Insert Figure 2 around here]

Since these errors depend on the distributional assumptions for $(X_{1,t+1}, X_{2,t+1}, R_{i,t+1})$, we repeat our simulations with different parameter sets. We focus on the non-normal factor $X_{2,t+1}$ and vary its volatility parameter, its asymmetry parameter and the heaviness of the univariate and multivariate tails. The resulting approximation errors $\alpha_{i,t}$ and $\alpha_{i,t}^l$ are shown in Figure 2. We find

[18] We simulate the distribution of logarithmic returns and transform the resulting random numbers to discrete returns to avoid problems with returns below minus one.

[19] The location parameters for $X_{1,t+1}$ and $X_{2,t+1}$ are determined using (7) as described in the Appendix C.

that the error of the linear model can reach 3-4% per annum in the presence of pronounced non-normalities or increased levels of volatility. In contrast, the pricing errors of our new SDF expansion are significantly lower with a relative reduction of at least 50% compared to the linear benchmark in the given examples. The new approximation thus captures are large fraction of additional risk premia related to non-normal features of the return and risk factor distribution.

Furthermore, we investigate whether MCRASH can help to explain cross-sectional differences in excess returns. We therefore vary the distributional characteristics of $R_{i,t+1}$ and its dependence structure with the two factors. In particular, we compute MCRASH and the pricing error of the linear model $\alpha^l_{i,t}$ for selected values of the asset's volatility, skewness, tail and dependence parameters. The results are shown in Figure 3. In line with equation (18), we document a positive relation between MCRASH and the alphas of the linear model and observe that the differences in $\alpha^l_{i,t}$ obtained from varying a single characteristic can exceed 200 basis points.

[Insert Figure 3 around here]

3 Econometric Methodology

In this section, we introduce our econometric approach for the measurement of conditional multivariate crash risk. We therefore outline the connection between MLTD coefficients and copulas. Then, we present the dynamic copula model that we use for our empirical analysis and discuss its estimation. Finally, we explain how we estimate an asset's ES and and compute our MCRASH estimates.

3.1 Copulas and MLTD coefficients

We apply copula methods to estimate the MLTD of the asset return R_i with the factor returns $X_1, \dots, X_N$.[20] Our approach relies on Sklar's Theorem, which states that the joint cumulative distribution function (cdf) of a random vector $\boldsymbol{Y} = (Y_1, \dots, Y_N)'$ can be written as

$$F_{Y_1,\dots,Y_N}(y_1, \dots, y_N) = C_{Y_1,\dots,Y_N}(F_{Y_1}(y_1), \dots, F_{Y_N}(y_N)), \tag{20}$$

[20] A copula function is a cumulative distribution function of a $[0, 1]^N$-valued random vector with uniform marginal distributions. For recent reviews on the relevant copula theory, available models and estimation techniques, see Patton (2012) or Fan and Patton (2014).

where $C_{Y_1,\ldots,Y_N}$ is a copula function and F_Y denotes the (marginal) cdf of random variable Y, i.e., $F_Y(y) = \mathbb{P}[Y \leq y]$.

Applying (20) to the random vector $\boldsymbol{Y} = (R_i, X_1, \ldots, X_N)'$ and assuming that its distribution is continuous, it is easy to show that[21]

$$\text{MLTD}_p^{X_1,\ldots,X_N}[R_i] = \frac{C_{R_i,X_1\ldots,X_N}(p,\ldots,p)}{C_{X_1,\ldots,X_N}(p,\ldots,p)}. \tag{21}$$

This generalizes the well known result $\text{LTD}_p^{X_1}[R_i] = C_{R_i,X_1}(p,p)/p$ for bivariate LTD coefficients.

To exploit equation (21), we assume a specific parametric copula family $C^{\boldsymbol{\theta}}$ and estimate its parameter vector as outlined below. Then, LTD and MLTD forecasts are obtained from the plug-in estimator

$$\widehat{\text{MLTD}}_p^{X_1,\ldots,X_N}[R_i] = \frac{C^{\boldsymbol{\theta}}_{R_i,X_1,\ldots,X_N}(p,\, \ldots,\, p;\, \hat{\boldsymbol{\theta}})}{C^{\boldsymbol{\theta}}_{X_1,\ldots,X_N}(p,\, \ldots,\, p;\, \hat{\boldsymbol{\theta}})}. \tag{22}$$

In comparison to a purely non-parametric estimation of MLTD, the parametric approach allows for a dynamic modeling and addresses the rarity of joint tail events.[22]

3.2 A Dynamic Copula Model

To model the conditional dependence structure of the returns and factors, we rely on the skewed t copula introduced by Demarta and McNeil (2007) which can capture extreme tail dependence as well as dependence asymmetries. We use a dynamic implementation of this copula that closely follows the recent applications in Christoffersen et al. (2012), Christoffersen and Langlois (2013), Meine et al. (2016) and Christoffersen et al. (2017).

Our skewed t copula is an implicit copula that is derived from the multivariate generalized hyperbolic skewed t distribution. It is thus available in arbitrary dimensions. A random vector $\boldsymbol{W}$ from the generalized hyperbolic skewed t distribution has the following normal mixture representation $\boldsymbol{W} = \sqrt{V}\,\boldsymbol{Z} + \boldsymbol{\gamma}\,V$, where V and $\boldsymbol{Z}$ are independent random variables and $\boldsymbol{\gamma} \in \mathbb{R}^N$. $\boldsymbol{Z}$ follows an N-dimensional standard normal distribution with the correlation matrix $\boldsymbol{P}$, i.e., $\boldsymbol{Z} \sim \mathcal{N}_N(\boldsymbol{0},\, \boldsymbol{P})$, and V has an inverse Gaussian distribution with $V \sim Ig(\nu/2, \nu/2)$. $\boldsymbol{P}$ determines the level of linear

[21] We derive this result in Appendix B.

[22] To illustrate this problem, suppose that the elements of $(R_i, X_1, \ldots, X_N)'$ are independent. Then, it holds that $C_{R_i,X_1\ldots,X_N}(p,\ldots,p) = p^{N+1}$. For typical values of p such as 0.05 or 0.01, the joint probability in the numerator of (21) thus quickly decreases with the number of factors.

dependence between the elements of $\boldsymbol{W}$. The degree-of-freedom parameter ν calibrates the level of tail dependence and $\boldsymbol{\gamma}$ determines the asymmetry of the distribution. The mixture representation can be used for an efficient simulation from the skewed t distribution and its copula. Further relevant details on the this distribution are summarized in Appendix D.

The skewed t copula is given by

$$C_{st,N}(\boldsymbol{u};\, \boldsymbol{P},\, \nu_c,\, \boldsymbol{\gamma}) = F_{st,N}(q_{st}(u_1;\, \nu_c, \gamma_1), \ldots, q_{st}(u_N;\, \nu_c,\, \gamma_N);\, \boldsymbol{P},\, \nu_c,\, \boldsymbol{\gamma}), \tag{23}$$

where $F_{st,N}(\,\cdot\,; \boldsymbol{P},\, \nu_c,\, \boldsymbol{\lambda}_c)$ denotes the cdf of the N-dimensional skewed distribution with the parameters $\boldsymbol{P}$, ν_c, $\boldsymbol{\gamma}$ and $q_{st}(\,\cdot,\, ; \nu_c,\, \gamma_i)$ denotes the quantile function of a skewed t distribution with the parameters ν_c and γ_i, $i = 1, \ldots, N$. The skewed t copula nests the standard t copula ($\boldsymbol{\gamma} = \boldsymbol{0}$) and the Gaussian copula ($\boldsymbol{\gamma} = \boldsymbol{0}$ and $\nu_c \to \infty$).

Following the techniques applied in Christoffersen et al. (2012), Christoffersen and Langlois (2013), Meine et al. (2016) and Christoffersen et al. (2017), we introduce a DCC-style specification for the correlation matrix $\boldsymbol{P}$ which is driven by the so-called "standardized copula shocks".[23] If $\boldsymbol{U}_t = (U_{1,t}, \ldots, U_{N,t})'$ is a random vector from the skewed t copula, then the corresponding standardized copula shocks $\boldsymbol{Z}_t = (Z_{1,t}, \ldots, Z_{N,t})'$ are obtained from

$$W_{i,t} = q_{st}(U_{i,t};\, \nu_c,\, \gamma_i) \qquad \text{and} \qquad Z_{i,t} = \frac{W_{i,t} - \mathbb{E}[W_{i,t}]}{\sigma[W_{i,t}]} \tag{24}$$

for $i = 1, \ldots, N$.[24] Based on these shocks, we implement a DCC-specification (Engle, 2002) with the modification proposed by Aielli (2013). Specifically, we assume that the constant correlation matrix $\boldsymbol{P}$ is replaced by

$$\boldsymbol{P}_{t+1} = \sqrt{\mathrm{diag}(\boldsymbol{Q}_{t+1})}^{-1} \cdot \boldsymbol{Q}_{t+1} \cdot \sqrt{\mathrm{diag}(\boldsymbol{Q}_{t+1})}^{-1}, \tag{25}$$

[23] The main difference to the econometric specifications used in these studies is that we do not include a time trend.

[24] $(W_{1,t}, \ldots, W_{N,t})'$ follows the multivariate skewed t distribution distribution underlying the implicit copula construction. The additional transformation from $\boldsymbol{W}_t$ to $\boldsymbol{Z}_t$ is simply a standardization. See again Appendix D for the required moments of the generalized hyperbolical skewed t distribution.

where

$$\boldsymbol{Q}_{t+1} = \boldsymbol{S}_c\,(1-\alpha_c-\beta_c) + \alpha_c\,(\bar{\boldsymbol{Z}}_t \cdot \bar{\boldsymbol{Z}}_t') + \beta_c\,\boldsymbol{Q}_t \tag{26}$$

and $\bar{\boldsymbol{Z}}_t = \sqrt{\text{diag}(\boldsymbol{Q}_t)} \cdot \boldsymbol{Z}_t$. $\boldsymbol{S}_c$ is a positive definite $(N \times N)$-matrix, $\alpha_c, \beta_c \in \mathbb{R}$ with $\alpha_c \geq 0$, $\beta_c \geq 0$ and $\alpha_c + \beta_c < 1$.

3.3 Estimation and Marginal Models

To implement our copula-based measurement of MLTD, we need to transform the original sample of asset and factor returns $(\boldsymbol{y}_t)_{t=1,\dots,T}$ with $\boldsymbol{y}_t = (r_t, x_{1,t}, \dots, x_{N,t})'$ into the corresponding copula sample $(\boldsymbol{u}_t)_{t=1,\dots,T}$. Moreover, we have to estimate the parameters of the dynamic copula model.

In a first step, we estimate parametric time series models to obtain a series of conditional cdfs $F_{i,t}$ given the information in t for the marginal distributions, $i = 1, \dots, N+1$ and $t = 0, \dots, T-1$.[25] In line with previous applications of the dynamic skewed t copula, we rely on GARCH models with a flexible residual distribution to capture the volatility dynamics and conditional non-normalities of the univariate return time series. We use standard GARCH(1,1) models with constant mean parameters, i.e., we assume

$$Y_{i,t+1} = \mu_i + \sigma_{i,t+1}\,\tilde{Z}_{i,t+1}, \tag{27}$$

$$\sigma_{i,t+1}^2 = \alpha_{i,0} + \alpha_{i,1}\,(\sigma_{i,t}\,\tilde{Z}_{i,t})^2 + \beta_i\,\sigma_{i,t}^2, \tag{28}$$

where μ_i, $\alpha_{i,0}, \alpha_{i,1}, \beta_i \in \mathbb{R}$, $\alpha_{i,0}, \alpha_{i,1}, \beta_{i,1} > 0$ and $\alpha_{i,1} + \beta_{i,1} < 1$. The time series residual $\tilde{Z}_{i,t+1}$ is assumed to follow Hansen's skewed t distribution that is characterized by a skewness parameter λ_i and a degree of freedom parameter ν_i.[26] The parameter vectors $\boldsymbol{\theta}_i = (\mu_i, \alpha_{i,0}, \alpha_{i,1}, \beta_{i,1}, \nu_i, \lambda_i)$, $i = 1, \dots, N$, for these models are estimated with a quasi-maximum likelihood estimator.

[25] We test the robustness of our asset pricing results in Section 5 based on a non-parametric empirical marginal distribution function. We find that all results are stable. Nevertheless, we prefer the parametric approach for our baseline analysis because it properly accounts for changes in the volatility level (see e.g. Poon et al., 2004 for the importance of heteroscedasticity as a source of "tail dependence").

[26] Note that the skewed t distribution proposed by Hansen (1994) is different from the generalized hyperbolical skewed t distribution that is the basis for our copula analysis. Therefore, the standardized copula shocks $Z_{i,t}$ and the time series residuals $\tilde{Z}_{i,t}$ are not identical.

Using the resulting parameter estimates, we compute

$$u_{i,t+1} = F_{i,t}(y_{i,t+1}) = F_H\left(\frac{y_{i,t+1} - \mu_i}{\sigma_{i,t+1}};\, \nu_i,\, \lambda_i\right) \tag{29}$$

for $i = 1, \dots, N+1$ and $t = 0, \dots, T-1$, where F_H is the cdf of Hansen's skewed t distribution. We thus obtain the copula sample $(\boldsymbol{u}_t)$.

For this sample, we then apply a second stage Maximum Likelihood estimation for the parameters of the dynamic copula model. In particular, we maximize the copula log-likelihood

$$l_{1:T}(\boldsymbol{\theta}_c) = \sum_{t=1}^{T} \log c_{st}(\boldsymbol{u}_t; \boldsymbol{P}_t,\, \nu_c, \boldsymbol{\gamma}) \tag{30}$$

as a function of the parameter vector $\boldsymbol{\theta}_c = (\alpha_c, \beta_c, \nu_c, \boldsymbol{\gamma}')$, where c_{st} denotes the density of the skewed t copula given in Appendix D. The parameter matrix $\boldsymbol{S}_c$ from the DCC dynamics in equation (26) is not included in the parameter vector $\boldsymbol{\theta}_c$ but determined by moment-matching, i.e.,

$$\hat{\boldsymbol{S}} = \frac{1}{T} \sum_{t=1}^{T} \bar{\boldsymbol{z}}_t \cdot \bar{\boldsymbol{z}}_t', \tag{31}$$

where the copula shocks $(\bar{\boldsymbol{z}}_t)$ are computed according to equations (24)-(25) from $(\boldsymbol{u}_t)$.[27]

3.4 Computation of MCRASH coefficients

To compute conditional MCRASH estimates, we combine copula-based conditional MLTD estimates that are obtained from (22) using the copula model described in the previous section and Expected Shortfall estimates that are based on our marginal skewed-t GARCH models outlined in (27)-(28). The MLTD estimates $\widehat{\text{MLTD}}_{p,t}^{\boldsymbol{X}}[R_{i,t+1}]$ are obtained from simulations as explained in Appendix D. The Expected Shortfall estimates $\widehat{\text{ES}}_{p,t}[R_{i,t+1}]$ can be computed analytically given the parameters of the marginal models.[28] For each asset i, we use equation (3) to compute $\widehat{\text{MCRASH}}_{p,t}^{\boldsymbol{X}}[R_{i,t+1}]$ from $\widehat{\text{MLTD}}_{p,t}^{\boldsymbol{X}}[R_{i,t+1}]$ and $\widehat{\text{ES}}_{p,t}[R_{i,t+1}]$.

[27] The exact procedure for evaluating the log-likelihood in equation (30) given a parameter vector $\boldsymbol{\theta}_c$ is summarized in Appendix D.

[28] See e.g. Christoffersen (2012, p. 136) for the relevant results on the skewed-t distribution.

4 Data and Crash Risk Estimates

In this section, we first present our data sample and provide return summary statistics. We proceed with a descriptive analysis of the univariate crash risk of our main risk factors and the probabilities of multivariate crash events. We then investigate the systematic risk exposure of our test assets on these factors based on their CRASH and MCRASH estimates. Finally, we analyze the persistence of these crash risk measures.

4.1 Data and Model Estimation

Our sample consists of the 49 value-weighted Fama and French industries as test assets in the period from 1970 to 2015. We download daily industry returns from the website of Kenneth French and transform these into discrete weekly returns.[29] We analyze the industry portfolios' multivariate crash risk with the four factors of the Carhart (1997) model, i.e., the market factor (MKT), the SMB (Small minus Big) size factor, the HML (High minus Low) book-to-market factor, and the UMD (Up minus Down) momentum factor. Summary statistics of weekly returns for these risk factors and the industries are reported in Table 1.

[Insert Table 1 around here]

In line with previous research, we find that – among the risk factors – the UMD factor shows the highest average weekly return (0.16%) while the SMB factor has an average weekly return of only slightly above zero (0.02%). Among our test assets, and in line with the findings of Hong and Kacperczyk (2009), we observe that sin stock industries (i.e., the "smoking" and "gun" industries) have the highest average weekly returns (0.34% and 0.30%), while stocks from the industries "real estate" and "other" perform the worst (0.11% average weekly returns). Interestingly, all our tests assets fail Jarque-Bera tests at the 1% significance level indicating that the returns are not normally distributed. Moreover, the high kurtosis levels of the return series, with sample estimates exceeding 10 in several cases, show that the return distributions are fat-tailed. The nonparametric ES estimates with $p = 10\%$ range between 3.41% ("Util") and 8.98% ("Coal").[30] Among the factors,

[29] The usage of weekly returns instead of monthly returns allows for a larger sample size, which is important for the estimation of crash events. All our results in the asset pricing tests hold if we use daily instead of weekly returns in the empirical analysis.

[30] This choice is slightly higher than the probability levels $p = 5\%$ or even $p = 1\%$ typically used for VaR and ES. This is due to the focus on joint crashes as we will explain in the following subsection.

MKT has the highest sample-ES with 4.10% followed by the momentum factor UMD with 3.53%. The ES estimates for SMB and HML are only at 2.24% and 2.06%.

The correlation estimates in Table 1 show that the linear dependence between the four risk factors is only modest. The highest (absolute) correlation is -0.24 for MKT and HML. Not surprisingly, the unconditional correlation estimates between the industries and MKT all exceed 50% except for "Precious Metals" with a correlation of 18%. The correlations with SMB are smaller but mostly positive, whereas the sample correlations of the industry returns with HML and UMD are predominantly negative.

To obtain conditional risk and dependence estimates for the given data, we implement the copula-based methodology outlined in Section 3. We first estimate skewed-t GARCH models with weekly factor and industry returns. Then, we estimate copula models for each industry and factor combination. Finally, we calculate univariate crash risk estimates and simulate lower dependence coefficients, CRASH and MCRASH coefficients for each model and each month of our sample period. More specifically, we generate conditional risk estimates for the last week of each month. Each simulation is performed with 1 000 000 multivariate return realizations from the relevant conditional distribution at that time. The results presented in this section are based on the full return sample (in-sample results). For the asset pricing tests in Section 5, we additionally consider selected estimation schemes that use only on a part of the weekly sample or daily data to obtain out-of-sample crash risk forecasts.

4.2 Crash Risk of the Factors

We now investigate the crash risk of the systematic factors. Motivated by our predictions on the magnitude of potential crash risk premia, we present summary statistics on the univariate crash risk of the factors as measured by their Expected Shortfalls and the probabilities of joint factor crashes for selected factor combinations in Table 2.

[Insert Table 2 around here]

Panel A of Table 2 summarizes our results on the Expected Shortfall $\mathrm{ES}_{p,t}$ of MKT, SMB, HML and UMD at the probability level $p = 10\%$. Corroborating the evidence presented in Table 1, we find that the market factor MKT and the momentum factor UMD have the highest average

Expected Shortfall. Furthermore, the momentum factor attains the highest maximum ES over our sample period with a value that is twice as large as the maximum for SMB and HML. We illustrate the evolution of the crash risk for MKT and UMD over time in Panel A of Figure 4, confirming the occurrence of high risk periods for the momentum factor, esp. during the second half of our sample period.

[Insert Figure 4 around here]

Panel B of Table 2 summarizes the probabilities of multivariate crash events for selected factor combinations. We report the probabilities of simultaneous factor crashes for all pairs of MKT, SMB, HML and UMD. Besides, we include probability estimates for a simultaneous crash of all four factors. Multivariate crash events are again defined as realizations of the factor returns that are simultaneously below their respective p-quantiles, i.e., we calculate estimates for the probability of $T_p^{\boldsymbol{X}}$ introduced in equation (8) for various choices of $\boldsymbol{X}$. As a simple benchmark, we report probabilities according to the empirical distribution of our weekly return sample (observed frequencies) and summary statistics on the average conditional probability estimates from our asset-specific dynamic copula models. All estimates are again computed for $p = 10\%$.

The average probabilities of two-dimensional factor crashes range between 1% and 2% for most of the combinations. Only the average probability for joint crashes of MKT and HML (0.58%) is below one percent, which corresponds to the benchmark value under the assumption that the factors are independent. Given the small or even negative unconditional correlations documented in Table 1, this observation is in line with the analysis of Christoffersen and Langlois (2013, p. 1371), who point out that "the extreme correlations are large and positive, so that the linear correlations drastically overstate the benefits of diversification across the factors." Furthermore, the average level of the probabilities explains our choice of $p = 10\%$ for the baseline analysis.[31] The 1%-probability for simultaneous crashes of two factors that we obtain with this choice corresponds to the probability level often used in other (univariate) tail risk studies.[32] Moreover, the magnitudes of the present probabilities explain our focus on bivariate factor crashes throughout most of the

[31] For $p = 5\%$, the average probabilities for simultaneous crashes of two factors range between 0.20% and 0.72% and the empirical frequencies are between 0.21% and 0.96%.

[32] Note that the probability of the factor crashes themselves is an upper bound for the probabilities of joint crashes of the factors and our test assets, which we will analyze in the following section.

analysis. Even using the 10%-quantiles as thresholds, the empirical probability of an all factor crash with weekly returns is only 0.13%,[33] so that we cannot learn much about the pricing implications of such events from historical data.

In addition, Panel B of Table 2 reveals a non-negligible variation in the probability of joint crash events over time. The maximum probabilities for joint factor crashes are typically several times higher than the respective time series averages. The highest levels are reached by the factor pairs including the UMD momentum factor, for which the maxima of joint crash probabilities exceed 5%. We illustrate the time-series of the average model-based joint crash probabilities for the three factor combinations including the market factor (i.e., MKT&SMB, MKT&HML, and MKT&UMD) in Figure 4. Furthermore, we indicate the actual occurrence of multivariate factor crashes for each of these combinations. The graph reveals a non-negligible time-variation in these probabilities, which captures the clustered occurrence of joint crashes. Most notably, joint crashes of MKT and HML almost exclusively occur in the period between 2008 and 2012, which is reflected by a much higher level of the corresponding probability estimates during this period.

Given the theoretical predictions summarized at the end of Section 2.2 and our empirical findings on the crash risk of the factors, we expect a relatively high price of multivariate crash risk for crash events including the MKT and the UMD factor and, in particular, for simultaneous crashes of these two factors.

4.3 Crash Risk of the Industries

Motivated by the theory in Section 2, we now investigate the exposure of our test assets to systematic crash risk. For each industry and each month in our sample, we compute CRASH coefficients with respect to our four main risk factors (i.e., $\text{CRASH}^{\text{MKT}}$, $\text{CRASH}^{\text{SMB}}$, $\text{CRASH}^{\text{HML}}$, and $\text{CRASH}^{\text{UMD}}$) and MCRASH coefficients with respect to all six pairs of these factors (e.g., $\text{MCRASH}^{\text{MKT, SMB}}$).[34] We use time-varying expected shortfall (ES) estimates[35] and lower tail dependence estimates (LTD and MLTD) based on the dynamic copula-methodology described above

[33]The average model-based probability forecast for a simultaneous crash of all four factors is only 0.04%.

[34]We also compute MCRASH coefficients for joint crashes of all factors. However, these estimates are not sufficiently stable because the relevant systematic tail events have extremely low occurrence probabilities for some subperiods of our sample and, therefore, we do not always obtain a positive number of realization of these events, even with 10^7 simulated return vectors.

[35]The time-series averages of the conditional univariate crash risk forecasts are largely comparable to nonparametric estimates presented in Table 1.

to obtain our CRASH and MCRASH estimates.[36] We again use $p = 10\%$ as our tail probability level.[37] Summary statistics for the resulting CRASH and MCRASH coefficients are presented in Table 3.

[Insert Table 3 around here]

Panel A of Table 3 reports descriptive statistics for the time-series averages of the coefficient estimates across the industries. We find that the market is the predominant source of bivariate crash risk. The overall average of $\text{CRASH}^{\text{MKT}}$ (2.42%) is more than four times higher than the average values of $\text{CRASH}^{\text{SMB}}$ (0.54%), $\text{CRASH}^{\text{HML}}$ (-0.05%), and $\text{CRASH}^{\text{UMD}}$ (0.25%).[38] Furthermore, the range of the average crash coefficients across the industries is much higher for $\text{CRASH}^{\text{MKT}}$ than for the other coefficients. This property is inherited from the corresponding LTD-coefficients as shown in Table F.1 in Appendix F.

An important distinction for the characterization of the trivariate MCRASH-coefficients is whether the factor combinations include MKT or not. For the three combinations including this factor, i.e., $\text{MCRASH}^{\text{MKT, SMB}}$, $\text{MCRASH}^{\text{MKT, HML}}$, and $\text{MCRASH}^{\text{MKT, UMD}}$, the average crash risk levels are similar to those of $\text{CRASH}^{\text{MKT}}$ itself and the range of the time-series averages across industries is even slightly larger than for $\text{CRASH}^{\text{MKT}}$. In contrast, we document substantially lower risk exposures for the three factor pairs not including MKT. Furthermore, the non-market combinations exhibit smaller cross-sectional differences.

To gain some insights into the variation of our crash risk measures over time, we present the time series of aggregate ES, aggregate $\text{CRASH}^{\text{MKT}}$, aggregate $\text{CRASH}^{\text{UMD}}$, and aggregate $\text{MCRASH}^{\text{MKT, UMD}}$ in Figure 5. These aggregate measures are defined as the monthly cross-sectional, equal-weighted averages over the corresponding industry coefficients. Furthermore, we include the cross-sectional interquartile range for the selected crash risk measures. The graphs reveal that the time-series behavior of aggregate $\text{CRASH}^{\text{MKT}}$ and aggregate $\text{CRASH}^{\text{MKT, UMD}}$ is largely driven by the dynamics of the aggregate ES, which is by construction similar to the ES

[36] Additional results on the LTD and MLTD estimates are provided in Appendix F.

[37] Note that our asset pricing results do no hinge on the exact value of the tail probability level and our results are very similar for a tail probability level of $p = 5\%$ (see Section 5.4.2).

[38] The negative value for HML reflects that many of the LTD estimates with respect to HML are below $p = 10\%$, which implies a negative CRASH-value according to (4). See Table F.1 in Appendix F for summary statistics on the industries' LTD levels.

of the market factor shown in Panel A of Figure 5. These aggregate crash risk measures thus spike during well-known crisis periods, such as the Black Monday in 1987 and the crisis following the Lehman Bankruptcy in 2008. In contrast, the dynamics of the aggregate exposure to crashes of UMD, as captured by aggregate $\text{CRASH}^{\text{UMD}}$, are somewhat distinct from this pattern, which is caused by the temporal variation of the aggregate LTD^{UMD} estimates as shown in Figure F.1 in Appendix F. For example, aggregate $\text{CRASH}^{\text{UMD}}$ does not go up during 2008 and reaches a peak after the subprime crisis. $\text{MCRASH}^{\text{MKT,UMD}}$ combines features of both lower dimensional measures and exhibits are larger cross-sectional variation.

[Insert Figure 5 around here]

To explore the correlation structure among our CRASH and MCRASH coefficients, we first compute industry-specific correlations between these measures and then take the average over all industries, which we report in Panel B of Table 3. For $\text{MCRASH}^{\text{MKT, UMD}}$, which will be the most important measure in our asset pricing tests, we document a positive average correlation with $\text{CRASH}^{\text{MKT}}$ (0.77) and with $\text{CRASH}^{\text{UMD}}$ (0.30). Interestingly, $\text{CRASH}^{\text{MKT}}$ and $\text{CRASH}^{\text{UMD}}$ themselves have a slightly negative correlation of -0.05. We will carefully control for these correlations when studying the pricing implications of these measures in Section 5.

4.4 Persistence of Multivariate Crash Risk

To hedge against joint risk factor crashes from an ex-ante point of view, investors may be inclined to pay more for stocks with lower levels of systematic crash risk, i.e. for industries with low CRASH and MCRASH estimates. Such strategies are only rational if these features will persist in the future. We therefore analyze the persistence of CRASH and MCRASH and report the corresponding results in Table F.2 in Appendix F.

Panel A of Table F.2 contains average autocorrelations of bivariate CRASH and trivariate MCRASH measures for our test assets. The averages are taken over all individual test assets. We find that all CRASH and MCRASH measures are persistent and display positive autocorrelations up to the 48th month. We also examine the persistence of CRASH and MCRASH using multivariate Fama and MacBeth (1973) regressions on the industry level in Panel B. In particular, we regress CRASH (MCRASH) in month t on CRASH (MCRASH) in $t-12$, $t-24$, $t-36$, and $t-48$. Our results indicate that all lagged variables are positively and, with a single exception, also statistically significantly related to the current level of CRASH (MCRASH).

Finally, we analyze the annual persistence of $\text{MCRASH}^{\text{MKT, UMD}}$ (as a specific example of multivariate crash risk and our main variable in Section 5) using portfolio sorts in Figure 6. In particular, we show the evolution of average equal-weighted $\text{MCRASH}^{\text{MKT, UMD}}$ of five portfolios over time.

[Insert Figure 6 around here]

For this purpose, we sort industries into quintiles based on their $\text{MCRASH}^{\text{MKT, UMD}}$ in year t. Then, the equal-weighted average of $\text{MCRASH}^{\text{MKT, UMD}}$ is computed again over the following four years $t+1$, ...,$t+4$. Our results indicate that $\text{MCRASH}^{\text{MKT, UMD}}$ is persistent and we find that the stock portfolio with the highest (lowest) $\text{MCRASH}^{\text{MKT, UMD}}$ remains to have the highest (lowest) $\text{MCRASH}^{\text{MKT, UMD}}$ also in the following four years.

5 Crash Risk and the Cross-Section of Average Stock Returns

The main part of the empirical analysis examines the relationship between multivariate crash risk and average future stock returns. We employ the 49 value-weighted Fama and French industries as test assets and use 1975 to 2015 as our sample period for the asset pricing tests.[39] To account for the impact of autocorrelation and heteroscedasticity, we determine statistical significance in portfolio sorts and multivariate regressions using Newey and West (1987) standard errors.

5.1 Portfolio Sorts

We start our asset pricing tests with univariate portfolio sorts on the three higher dimensional crash risk measures including the market factor, for which we documented the highest levels of systematic risk and a large cross-sectional variation in Section 4.3. For each month t, we sort our test assets into quintile portfolios based on the multivariate crash coefficients $\text{MCRASH}^{\text{MKT, SMB}}$, $\text{MCRASH}^{\text{MKT, HML}}$, and $\text{MCRASH}^{\text{MKT, UMD}}$. We report average excess returns over the risk-free rate for these quintile portfolios as well as differences in average returns between quintile portfolio 5

[39] We use the first five years to estimate control variables for our multivariate asset pricing tests in Section 5.2, such as, an industry's volatility, coskewness, and different factor betas.

(high MCRASH) and quintile portfolio 1 (low MCRASH) in month $t + 1$. Results are shown in Panel A of Table 4.

[Insert Table 4 around here]

The first three columns of Panel A show that the spreads between the returns of portfolio 5 and portfolio 1 are positive for all three factor combinations but statistically significant only for $\text{MCRASH}^{\text{MKT, UMD}}$. This empirical finding is in line with our theoretical predictions from Section 2.2 because MKT and UMD (i) are the risk factors with the highest expected shortfalls and (ii) display large probabilities of joint crashes (see Figure 4 and Table 2). The return spread based on $\text{MCRASH}^{\text{MKT,UMD}}$ amounts to 0.518% per month with a t-statistic of 2.64. Hence, stocks with high levels of $\text{MCRASH}^{\text{MKT, UMD}}$ earn a premium of 6.22% p.a. compared to stocks with low levels of $\text{MCRASH}^{\text{MKT, UMD}}$. We also observe that future returns are monotonically increasing from the lowest to the highest $\text{MCRASH}^{\text{MKT, UMD}}$ quintile. In the last two columns of Panel A, we show the results of portfolio sorts on the components of $\text{MCRASH}^{\text{MKT, UMD}}$, i.e. we report return spreads for sorts on the assets' individual ES and on the multivariate lower tail dependence $\text{MLTD}^{\text{MKT, UMD}}$. We observe that both components of MCRASH positively contribute to the observed return premium for $\text{MCRASH}^{\text{MKT, UMD}}$ with the larger part of the premium resulting from multivariate lower tail dependence (separate return spread of 0.272% vs. 0.083% for ES).

In contrast to our findings for the multivariate crash risk of MKT and UMD, sorts on $\text{MCRASH}^{\text{MKT, HML}}$ and $\text{MCRASH}^{\text{MKT, SMB}}$ do not generate statistically significant return spreads and do not yield monotonically increasing average future portfolio returns.[40] Given these results, we concentrate on the impact of $\text{MCRASH}^{\text{MKT, UMD}}$ on future returns in the following analyses.

We illustrate our main results from these asset pricing tests in Figure 7. This figure shows the cumulative returns for three long-short investment strategies over time: (i) a trading strategy based on $\text{MCRASH}^{\text{MKT,UMD}}$, (ii) a trading strategy based on $\text{MLTD}^{\text{MKT, UMD}}$, and (iii) a trading strategy based on ES.

[Insert Figure 7 around here]

[40]We also investigate portfolio sorts based on $\text{CRASH}^{\text{MKT}}$, $\text{CRASH}^{\text{SMB}}$, $\text{CRASH}^{\text{HML}}$, $\text{CRASH}^{\text{UMD}}$, non-market MCRASH specifications, and $\text{MCRASH}^{\text{MKT, SMB, HML, UMD}}$ in the Appendix. Our results indicate that – except from $\text{CRASH}^{\text{UMD}}$ – none of these measures generates a positive and statistically significant return spread.

The graph reveals that the cumulative returns of the strategy based on $\text{MCRASH}^{\text{MKT, UMD}}$ are significantly higher than the cumulative returns of the other two strategies. Starting with $100 at the beginning of 1975 (i.e., the beginning of our test period), the final wealth of an investor would amount to $805.95 when pursuing a hypothetical MCRASH(MKT, UMD) trading strategy, which is substantially higher than the final amounts of $265.57 and $94.56 when pursuing the MLTD- or the ES strategy.[41]

We now turn to bivariate asset pricing tests. Although the results of univariate portfolio sorts on the remaining crash risk measures are mostly statistically insignificant, it is possible that the significant return spread based on $\text{MCRASH}^{\text{MKT, UMD}}$ is driven by the test assets' differences in other CRASH and MCRASH measures. Therefore, in the next step, we want to isolate the return premium for $\text{MCRASH}^{\text{MKT, UMD}}$ from the return effects of related crash risk measures and conduct bivariate sorts. For this purpose, for each month t, we sort our test assets into two portfolios (high / low) based on the crash risk coefficients $\text{CRASH}^{\text{MKT}}$, $\text{CRASH}^{\text{SMB}}$, $\text{CRASH}^{\text{HML}}$, $\text{CRASH}^{\text{UMD}}$, $\text{MCRASH}^{\text{MKT, SMB}}$, $\text{MCRASH}^{\text{MKT, HML}}$, $\text{MCRASH}^{\text{SMB, HML}}$, $\text{MCRASH}^{\text{SMB, UMD}}$, and $\text{MCRASH}^{\text{HML, UMD}}$. Then, within each of these portfolios, we sort the test assets into two portfolios (high / low) based on $\text{MCRASH}^{\text{MKT, UMD}}$ and report the average of the excess portfolio returns in month $t+1$. Panel B of Table 4 displays the average high minus low spread of the $\text{MCRASH}^{\text{MKT, UMD}}$ portfolios controlling for each of the other crash risk measures.

Our results reveal that the return effect of $\text{MCRASH}^{\text{MKT, UMD}}$ is not subsumed by the return effects of other CRASH and MCRASH measures. Regardless of which measure we explicitly control for, we obtain positive and significant returns for the average high minus low $\text{MCRASH}^{\text{MKT, UMD}}$ portfolio. These return spreads range from 0.21% to 0.27% per month (i.e., 2.52% to 3.24% p.a.) and are all statistically significant at least at the 10% level.[42] Hence, our empirical analysis provides strong evidence that the return premium based on $\text{MCRASH}^{\text{MKT, UMD}}$ is different from the return impact of other CRASH and MCRASH measures.

[41] Note that the hypothetical MCRASH(MKT, UMD) strategy does not take into account any trading costs and market frictions.

[42] In an unreported test, we also control for the impact of $\text{MCRASH}^{\text{MKT, SMB, HML, UMD}}$. We find that the return spread of $\text{MCRASH}^{\text{MKT, UMD}}$ amounts to 0.28% per month and is statistically significant at the 5% level.

5.2 Multivariate Analysis

Since we can only control for one test asset characteristic at a time in portfolio double sorts, we now turn to a multivariate approach that allows us to examine the impact of $\text{MCRASH}^{\text{MKT, UMD}}$ on future returns controlling for a wide array of industry characteristics, risk measures and factor betas, which have been shown to affect future returns.

As industry characteristics and risk measures, we include an industry's average size (Banz, 1981), average book-to-market value (Basu, 1983), past return (Jegadeesh and Titman, 1993), volatility, coskewness (Harvey and Siddique, 2000), and downside beta (Ang et al., 2006). We also control for industry betas to MKT, SMB, HML, UMD, the investment (RMW) and profitability (CMA) factors from the Fama and French (2015) five-factor model, the Fama and French short-term (STR) and long-term reversal (LTR) factors, the Pástor and Stambaugh (2003) traded liquidity risk factor (PSL), the Frazzini and Pedersen (2014) betting-against-beta factor (BAB), the Kelly and Jiang (2014) tail risk factor (TR), and the Asness et al. (2017) quality-minus-junk factor (QMJ). All industry characteristics and industry betas are defined in the Appendix E; betas are estimated using a monthly rolling window of 60 months.

To investigate the relationship between $\text{MCRASH}^{\text{MKT, UMD}}$ and these control variables, we present the average characteristics and betas of the quintile portfolios sorted on $\text{MCRASH}^{\text{MKT,UMD}}$ in Table 5.

[Insert Table 5 around here]

Panel A documents that $\text{MCRASH}^{\text{MKT, UMD}}$ is significantly positively associated with an industry's past annual return, volatility, and downside beta, whereas it is negatively related to an industry's average size, book-to-market value, and coskewness. The results reported in Panel B show that industries with high $\text{MCRASH}^{\text{MKT, UMD}}$ have strong exposures to the MKT, SMB, UMD, RMW, STR, PSL, and TR factors, whereas they display weak exposures to the HML, CMA, LTR, and QMJ factors. These findings imply that it is important to control for these characteristics and betas in our multivariate asset pricing tests.

After having examined the relationship between $\text{MCRASH}^{\text{MKT, UMD}}$ and different industry characteristics and factor betas, we go on to test our main hypothesis directly in a multivariate framework. To do so, we perform Fama and MacBeth (1973) regressions on the industry test asset level and report the results in Table 6.

[Insert Table 6 around here]

In specifications (1) - (6), we regress excess returns of our test assets in month $t+1$ on $\text{MCRASH}^{\text{MKT, UMD}}$ and different industry characteristics measured in month t. In regression (1), we include $\text{MCRASH}^{\text{MKT, UMD}}$ as the only explanatory variable. Consistent with the results from portfolio sorts, we find a highly statistically significant impact on future returns. Given a standard deviation of 1.82% and a coefficient estimate of 0.186, a one standard deviation increase in $\text{MCRASH}^{\text{MKT, UMD}}$ leads to higher annualized average future returns of approximately 4.74%. In regressions (2) - (6), we add average industry size, average book-to-market value, volatility, coskewness, and downside beta to our model. We find that, in all specifications, the coefficient estimate of $\text{MCRASH}^{\text{MKT,UMD}}$ remains positive and statistically significant at the 1% level. To check how far $\text{MCRASH}^{\text{MKT, UMD}}$ predicts returns in the future, we provide results for regressions with three-months and six-months ahead returns in the specifications (7) and (8). We observe that the impact of $\text{MCRASH}^{\text{MKT, UMD}}$ is positive and strongly statistically significant with t-statistics of 2.15 (three months) and 2.62 (six months).[43]

We next test whether MCRASH coefficients explain future returns after controlling for linear factor exposures and report the multivariate Fama and MacBeth (1973) regression results in Table 7. All specifications regress excess returns of our test assets in month $t+1$ on $\text{MCRASH}^{\text{MKT, UMD}}$ and different industry factor betas measured in month t. They thus implement the MCRASH-based extension of standard linear factor models that we proposed in equation (18).

[Insert Table 7 around here]

In specification (1), we regress the industries' one-month excess returns on the betas of the Carhart (1997) four-factor model. In line with the literature, we find that all factor betas carry a positive coefficient estimate with β_{HML} and β_{UMD} also being strongly statistically significant. In specification (2), we add $\text{MCRASH}^{\text{MKT, UMD}}$ to our model and find that it has a significantly

[43]Note that for the regressions with three-months and six-months ahead returns, t-statistics are computed using Newey and West (1987) standard errors with 6 and 9 monthly lags, respectively, to account for the use of overlapping return data.

positive impact (coefficient estimate of 0.148 with a t-statistic of 2.11). Interestingly, including $\text{MCRASH}^{\text{MKT, UMD}}$ in the regression setup, also drives down the model intercept to a non-significant value. In specifications (3) to (8), we control for linear exposure (betas) to the factors of the Fama and French (2015) five-factor model, the Fama and French (1993) three-factor model extended by the short-term and long-term reversal factor, as well as the Carhart (1997) four-factor model extended with the Pástor and Stambaugh (2003) traded liquidity risk factor, the Frazzini and Pedersen (2014) betting-against-beta factor, the Kelly and Jiang (2014) tail risk factor, and the Asness et al. (2017) quality-minus-junk factor. In all these regressions, we find positive coefficient estimates of $\text{MCRASH}^{\text{MKT, UMD}}$ ranging from 0.124 to 0.170 with corresponding t-statistics between 1.78 and 2.48 and substantial economic significance. For example, we observe that a one standard deviation increase in $\text{MCRASH}^{\text{MKT,UMD}}$ raises annualized average future returns by 3.23% when controlling for exposure to the risk factors of the Carhart (1997) four-factor model. Hence, $\text{MCRASH}^{\text{MKT, UMD}}$ shows a very stable impact on future returns across all our regressions.

In specifications (9) and (10), we regress three-months and six-months ahead excess returns on $\text{MCRASH}^{\text{MKT, UMD}}$ and the betas of the Carhart (1997) model. We again find positive and highly statistically significant coefficient estimates for $\text{MCRASH}^{\text{MKT, UMD}}$ with t-statistics of 2.86 and 3.78.

To sum up, the results in Sections 5.2 and 5.1 provide strong evidence that the impact of $\text{MCRASH}^{\text{MKT, UMD}}$ is not subsumed by other crash risk measures. Moreover, the effect of $\text{MCRASH}^{\text{MKT, UMD}}$ on future returns remains also strong when we explicitly control for different industry characteristics and factor betas in multivariate regressions.

5.3 Out-of-Sample Test

In the main part of our empirical analysis, we estimate the parameters of the GARCH and copula models that are used to calculate CRASH and MCRASH coefficients with weekly data from 1970 to 2015. Although the time-series dynamics of these models are only driven by past returns, this parameter estimation scheme introduces a look-ahead bias, which prevents investors from exploiting the documented abnormal returns associated with $\text{MCRASH}^{\text{MKT, UMD}}$ in a real-time trading strategy. In this section, we change the estimation procedure for the model parameters to provide an out-of-sample analysis of $\text{MCRASH}^{\text{MKT, UMD}}$ and average future returns, which is indeed tradable for investors on a real-time basis.

For this purpose, we divide our sample into two subperiods, namely the periods from (i) 1970 to 1992 and (ii) 1993 to 2015. We estimate the parameters of the marginal models and the copula parameters with information over the first half of our sample, i.e., based on the time series from 1970 to 1992. Subsequently, we run asset pricing tests on the relationship between $\text{MCRASH}^{\text{MKT, UMD}}$ and future returns again controlling for a long list of industry characteristics and factor betas. The results of Fama and MacBeth (1973) regressions for the out-of-sample period from 1993 to 2015 are shown in Table 8. In this table, we repeat specifications (1) to (8) of Table 6 in Panel A and specifications (2) to (10) of Table 7 in Panel B. All respective control variables are included in the regressions, but coefficient estimates are suppressed.

[Insert Table 8 around here]

Our results confirm a strong relationship between $\text{MCRASH}^{\text{MKT, UMD}}$ and future returns for the reduced out-of-sample period. Across all different regression specifications – investigating the relationship between future returns in month $t+1$ and $\text{MCRASH}^{\text{MKT, UMD}}$ measured in month t – we observe positive coefficient estimates ranging from 0.127 to 0.295 with t-statistics between 1.68 and 3.03. Given a standard deviation of 1.72% (for the sample period from 1993 to 2015), it follows that a one standard deviation increase in $\text{MCRASH}^{\text{MKT, UMD}}$ leads to higher annualized average future returns of between 2.62% and 6.09%. In the regressions with cumulative three- and six-months returns, we also obtain positive and statistically significant coefficient estimates for $\text{MCRASH}^{\text{MKT,UMD}}$.

5.4 Additional Results

To further corroborate our results of a significant relationship between multivariate crash risk and average future returns, we conduct a number of additional empirical tests. In particular, we investigate specific time periods in Section 5.4.1, check the stability of our results with a selection of robustness checks in Section 5.4.2, and apply our main analysis on a new data set consisting of individual stocks in Section 5.4.3.

5.4.1 Time Splits

We first analyze the effect of multivariate crash risk on average future returns during two subperiods of our sample and under different market conditions. We therefore rerun specification (2) of Table 7 with selected subsamples and summarize the results in Panel A of Table 9. Factor betas with regard to the Carhart (1997) four-factor model are included in all regressions but suppressed in the table.

[Insert Table 9 around here]

In specifications (1) and (2), we investigate the relationship between $\text{MCRASH}^{\text{MKT, UMD}}$ and future returns in the subperiods from 1975 to 1994 and from 1995 to 2015. We find that the return effect of $\text{MCRASH}^{\text{MKT, UMD}}$ is significant in both subperiods with coefficient estimates of 0.213 and 0.141, respectively, and t-statistics of 2.05 and 1.82. In the remaining specifications of Panel A, we condition the multivariate regressions on periods with positive (negative) market returns and/or positive (negative) realizations of the UMD momentum risk factor. In line with the notion that $\text{MCRASH}^{\text{MKT, UMD}}$ represents a risk premium in the cross-section of expected stock returns, we find that the positive return spread of $\text{MCRASH}^{\text{MKT, UMD}}$ realizes in times of positive market and UMD momentum factor returns. Moreover, we observe that the premium is highest during periods when both the market and the UMD momentum return are positive, while it is negative when both the market and the UMD momentum return realize negative values.

5.4.2 Robustness

We perform a number of robustness checks to show that our results of a positive and statistically significant relation between multivariate crash risk and average future returns are not sensitive to several choices made in our empirical analysis. Specifically, we investigate the stability of our results when we use log returns instead of discrete returns for model estimation, change the frequency in the estimation of $\text{MCRASH}^{\text{MKT, UMD}}$ from weekly to daily, use the empirical distribution and GJR-GARCH models for the univariate margins, apply the "normal" Student-t copula (instead of the asymmetric version) when specifying the multivariate dependence structure, estimate the lower tail dependence coefficients using a 5% cut-off instead of using a 10% cut-off, apply equal-weighted instead of value-weighted industry returns in the empirical analysis, and use a rolling window estimation with 1000 days and daily data to estimate the econometric models. We show the results of the robustness checks in Panel B of Table 9. As in Panel A, we only report coefficient estimates of $\text{MCRASH}^{\text{MKT, UMD}}$ applying specification (2) of Table 7 in the multivariate regression setup. We find that our results are stable across all robustness checks with coefficient estimates for $\text{MCRASH}^{\text{MKT, UMD}}$ that are always statistically significant at least at the 10% level.

5.4.3 Individual Test Assets

This section provides another out-of-sample test of the relationship between $\text{MCRASH}^{\text{MKT, UMD}}$ and future returns. Instead of applying industry portfolios as test assets, we use individual stocks in the following analysis. In particular, we employ the sample of all stocks included in the S&P 100 index between 1976 and 2015.[44] Accounting for the different return periods of the stocks in this sample, we apply a rolling-window estimation scheme, which uses 1250 daily returns to estimate the parameters of our GARCH and copula models. The model parameters are updated yearly in January.[45]

To examine the association between $\text{MCRASH}^{\text{MKT, UMD}}$ and future returns, we again rely on a multivariate regression framework and repeat the specifications (1) to (8) of Table 6 in Panel A and specifications (2) to (10) of Table 7 in Panel B. All respective control variables are included in the regressions, but coefficient estimates are suppressed. Table 10 displays the results.

[Insert Table 10 around here]

We observe that – also in our dataset consisting of individual stocks – there exists a strong relationship between $\text{MCRASH}^{\text{MKT, UMD}}$ and future returns. Across all specifications for the relation between future returns in month $t+1$ and $\text{MCRASH}^{\text{MKT, UMD}}$ measured in month t, we find positive coefficient estimates of $\text{MCRASH}^{\text{MKT, UMD}}$ ranging from 0.435 to 0.559 with corresponding t-statistics between 1.71 and 2.36. Given a standard deviation of 0.69% in this sample, it follows that a one standard deviation increase in $\text{MCRASH}^{\text{MKT, UMD}}$ leads to higher annualized average future returns between 3.60% and 4.95%. In our regressions with three-months and

[44]To prevent our analysis from being affected by a look-ahead selection bias (i.e., stocks that are going to be included in the index tend to be successful stocks), we restrict our sample to months after a stock has been included in the index.

[45]Furthermore, we rely on a slightly simplified version of the dynamic copula model which builds on a standard t- instead of a skewed-t-copula.

six-months ahead excess returns, we also find positive and significant results for the impact of the $\text{MCRASH}^{\text{MKT,UMD}}$ coefficient.

Summarizing our additional tests, we confirm that $\text{MCRASH}^{\text{MKT, UMD}}$ is priced in the subperiods from 1975 to 1994 and 1995 to 2015. We show that the premium for $\text{MCRASH}^{\text{MKT,UMD}}$ survives a large selection of robustness checks and is particularly pronounced during periods of positive market and/or UMD momentum factor returns. Moreover, similar results can be reproduced for a different data set consisting of S&P 100 index stocks.

6 Conclusion

This paper examines the relationship between multivariate crash risk and the cross-section of expected stock returns. Investors that are averse to joint crashes in asset pricing risk factors should require a risk premium for holding assets that have a high sensitivity to such crash scenarios. We propose MCRASH as a combined measure of an asset's expected shortfall and its sensitivity to multivariate risk factor crashes, i.e., states that are typically associated with low diversification benefits and high levels of marginal utility. Using a new expansion of a generic SDF that depends on multiple risk factors, we are able to isolate a tail-related component of an asset's expected return that is increasing in its exposure to multivariate crash risk as measured by MCRASH.

To investigate the validity of this theoretical prediction, we perform an empirical analysis where we employ the 49 value-weighted Fama and French industries as our test assets in the sample period from 1970 to 2015. In line with our theoretical model, we find that the trivariate $\text{MCRASH}^{\text{MKT,UMD}}$ measure shows a significantly positive impact on average future stock returns. Specifically, we find that an investment strategy of going long the quintile portfolio with the highest $\text{MCRASH}^{\text{MKT,UMD}}$ and going short the quintile portfolio with the lowest $\text{MCRASH}^{\text{MKT,UMD}}$ coefficients in month t yields an average return spread of 0.518% in month $t+1$ with a t-statistic of 2.64. Our results are stable when we perform multivariate Fama and MacBeth (1973) regressions between future returns and $\text{MCRASH}^{\text{MKT,UMD}}$ controlling for other industry characteristics and linear factor betas, when we consider different sample periods, and when we employ a selection of individual stocks as test assets.

Our study contributes to the theoretical and empirical literature on downside and crash risk in asset pricing as well as to the growing literature on the application of copulas in finance. We find strong support for the idea that investors care about the multidimensionality of crash risk and thus require a risk premium for it. In addition, our results indicate that capturing additional characteristics of the dependence structure among well-known risk factors helps to improve our understanding of the cross-section of expected stock returns.

A MLTD vs. Bivariate LTD

We present two stylized examples with a simple discrete state space, which illustrate the additional information in MLTD measures beyond standard bivariate LTD coefficients. We also compare MLTD coefficients to bivariate LTD coefficients with respect to a factor portfolio (i.e., LTD with a linear combination of the factors).

We assume that there are four states and two factors X_1 and X_2. Furthermore, we consider the “portfolio factor” $X_p = 0.5\,X_1 + 0.5\,X_2$. X_1 and X_2 have two possible return realizations, which are -20% or $+10\%$. The first state is a “perfect storm scenario”, in which both factors realize -20%. State two and three correspond to individual crash scenarios for X_1 and X_2, respectively. In the fourth state both factors perform well (both $+10\%$). From its definition, it follows that X_p realizes a -20% return in the joint crash state and -5% in the individual crash states. It has a +10% return in the good state.

Our examples include three stocks. Each of the stocks has a return of -20% in two of the four states and a return of $+10\%$ in the remaining two states. Stock A and stock B have negative returns in the joint crash state (state 1). Furthermore, stock A realizes -20% together with factor X_1 in state 2, whereas stock B has its second crash scenario together with factor X_2 in state 3. Stock C hedges against the joint crash scenario (state 1) but it realizes a negative return in both individual crash scenarios (states 2 and 3). This structure is the same for both examples. Our examples only differ with respect to the probabilities of the four states. Panel A in Tables A.1 and A.2 summarizes the probabilities and the return realizations for each state.

Table A.1: Example 1

Panel A: Joint Distribution

State	joint crash	crash X_1	crash X_2	no crash
Probability	3%	3%	3%	91%
X_1	-20%	-20%	10%	10%
X_2	-20%	10%	-20%	10%
X_p	-20%	-5%	-5%	10%
R_A	-20%	-20%	10%	10%
R_B	-20%	10%	-20%	10%
R_C	10%	-20%	-20%	10%

Panel B: LTD and MLTD

	$\text{LTD}_{0.05}^{X_1}$	$\text{LTD}_{0.05}^{X_2}$	$\text{LTD}_{0.05}^{X_p}$	$\text{MLTD}_{0.05}^{X_1,X_2}$
R_A	100%	50%	67%	100%
R_B	50%	100%	67%	100%
R_C	50%	50%	67%	0%

We compute LTD and MLTD at the 5% level. This requires the computation of the 5%-quantiles for all involved marginal distributions. For the stock returns as well as for X_1 and X_2, the 5%-quantiles are -20% in both examples. For the portfolio factor, we have $Q_p[X_p] = -5\%$. We illustrate the computation of the (M)LTD coefficients for stock A and Example 1:

$$\text{LTD}_{0.05}^{X_1}[R_A] = \frac{\mathbb{P}[R_A \leq -0.2,\, X_1 \leq -0.2]}{\mathbb{P}[X_1 \leq -0.2]} = \frac{0.06}{0.06} = 1, \tag{32}$$

$$\text{LTD}_{0.05}^{X_2}[R_A] = \frac{\mathbb{P}[R_A \leq -0.2,\, X_2 \leq -0.2]}{\mathbb{P}[X_2 \leq -0.2]} = \frac{0.03}{0.06} = 0.5, \tag{33}$$

$$\text{LTD}_{0.05}^{X_p}[R_A] = \frac{\mathbb{P}[R_A \leq -0.2,\, X_p \leq -0.05]}{\mathbb{P}[X_p \leq -0.05]} = \frac{0.06}{0.09} = 0.67, \tag{34}$$

$$\text{MLTD}_{0.05}^{X_1,X_2}[R_A] = \frac{\mathbb{P}[R_A \leq -0.2,\, X_1 \leq -0.2,\, X_2 \leq -0.2]}{\mathbb{P}[X_1 \leq -0.2,\, X_2 \leq -0.2]} = \frac{0.03}{0.03} = 1. \tag{35}$$

The resulting LTD coefficients with respect to X_1, X_2 and X_p as well as the multivariate coefficients $\text{MLTD}_{0.05}^{X_1,X_2}$ for all three stocks are shown in Panel B of the Tables A.1 and A.2.

The first example illustrates that the "portfolio" coefficients $\text{LTD}_{0.05}^{X_p}$ do not help to identify the higher exposure to multivariate crash risk of the stocks A and B. All three stocks show the same LTD with respect to the combined factor X_p. In contrast, the $\text{MLTD}_{0.05}^{X_1,X_2}$ clearly shows that stock C is not exposed to the multivariate crash scenario. Our second example additionally illustrates that the "portfolio" coefficients $\text{LTD}_{0.05}^{X_p}$ can even be misleading for the identification of multivariate crash risk. In this case, stock A has the lowest LTD with respect to the factor

Table A.2: Example 2

Panel A: Joint Distribution

State	joint crash	crash X_1	crash X_2	no crash
Probability	4%	4%	40%	52%
X_1	-20%	-20%	10%	10%
X_2	-20%	10%	-20%	10%
X_p	-20%	-5%	-5%	10%
R_A	-20%	-20%	10%	10%
R_B	-20%	10%	-20%	10%
R_C	10%	-20%	-20%	10%

Panel B: LTD and MLTD

	$\text{LTD}_{0.05}^{X_1}$	$\text{LTD}_{0.05}^{X_2}$	$\text{LTD}_{0.05}^{X_p}$	$\text{MLTD}_{0.05}^{X_1,X_2}$
R_A	100%	9%	17%	100%
R_B	50%	100%	92%	100%
R_C	50%	91%	92%	0%

portfolio in spite of its higher exposure to multivariate crash risk compared to stock C. In contrast, $\text{MLTD}_{0.05}^{X_1,X_2}$ clearly identifies the stocks with exposure to the multivariate crash state.

The second example also shows that "average" bivariate LTD coefficients do not contain the same information as our MLTD measure. Despite of its exposure to multivariate crash risk, stock A has the lowest average LTD with X_1 and X_2, i.e. $0.5\,(100\% + 9\%) = 54.5\%$, whereas stock C, which is not exposed to the multivariate crash scenario, has an average LTD to X_1 and X_2 of 70.5%.

These examples show that MLTD provides a clean way of measuring the exposure to multivariate crash risk.

B Proofs

B.1 Decomposition of the Expected Excess Return

Using (7) and (10), we obtain

$$\mathbb{E}_t[R_{i,t+1} - R_{f,t+1}] \approx \text{no-Tail}_{i,t}^{\boldsymbol{X}} + \text{Tail}_{i,t}^{\boldsymbol{X}} \tag{36}$$

with

$$\text{no-Tail}_{i,t}^{\boldsymbol{X}} = -(1+R_{f,t+1})\,\text{cov}_t\Big[(m(\boldsymbol{x}_c) + \nabla m(\boldsymbol{x}_c)\cdot(\boldsymbol{X}_{t+1}-\boldsymbol{x}_c))\,\mathbb{1}(\overline{T}_{\boldsymbol{X}}^p), R_{i,t+1}\Big], \tag{37}$$

$$\text{Tail}_{i,t}^{\boldsymbol{X}} = -(1+R_{f,t+1})\,\text{cov}_t\Big[(m(\boldsymbol{x}_p) + \nabla m(\boldsymbol{x}_p)\cdot(\boldsymbol{X}_{t+1}-\boldsymbol{x}_p))\,\mathbb{1}(T_p^{\boldsymbol{X}}), R_{i,t+1}\Big]. \tag{38}$$

To decompose $\text{Tail}_{i,t}^{\boldsymbol{X}}$ into $\text{Tail}_{i,t}^{\boldsymbol{X},0}$ and $\text{Tail}_{i,t}^{\boldsymbol{X},1}$, we use the following identity

$$R_{i,t+1} = (r_p + (R_{i,t+1} - r_p))\ \mathbb{1}(T_p^i) + R_{i,t+1}\,\mathbb{1}(\overline{T}_i^p), \tag{39}$$

where $r_p := -\text{ES}_{p,t}[R_{i,t+1}]$ is the expected value of $R_{i,t+1}$ in its p-tail and $T_p^i := \{R_{i,t+1} \leq Q_p[R_{i,t+1}]\}$ is an individual tail event for asset i, in which $R_{i,t+1}$ is below its p-quantile. (38) and (39) imply that $\text{Tail}_{i,t}^{\boldsymbol{X}}$ can be expanded as

$$\text{Tail}_{i,t}^{\boldsymbol{X}} = \text{Tail}_{i,t}^{\boldsymbol{X},0} + \text{Tail}_{i,t}^{\boldsymbol{X},1} \tag{40}$$

with

$$\text{Tail}_{i,t}^{\boldsymbol{X},0} = -(1+R_{f,t+1})\,\text{cov}_t\Big[m(\boldsymbol{x}_p)\,\mathbb{1}(T_p^{\boldsymbol{X}}), r_p\,\mathbb{1}(T_p^i)\Big], \tag{41}$$

$$\begin{aligned}\text{Tail}_{i,t}^{\boldsymbol{X},1} = &-(1+R_{f,t+1})\,\text{cov}_t\Big[m(\boldsymbol{x}_p)\,\mathbb{1}(T_p^{\boldsymbol{X}}), (R_{i,t+1}-r_p)\,\mathbb{1}(T_p^i)\Big] \\ &-(1+R_{f,t+1})\,\text{cov}_t\Big[(\nabla m(\boldsymbol{x}_p)\cdot(\boldsymbol{X}_{t+1}-\boldsymbol{x}_p))\,\mathbb{1}(T_p^{\boldsymbol{X}}), R_{i,t+1}\,\mathbb{1}(T_p^i)\Big] \\ &-(1+R_{f,t+1})\,\text{cov}_t\Big[(m(\boldsymbol{x}_p) + \nabla m(\boldsymbol{x}_p)\cdot(\boldsymbol{X}_{t+1}-\boldsymbol{x}_p))\,\mathbb{1}(T_p^{\boldsymbol{X}}), R_{i,t+1}\,\mathbb{1}(\overline{T}_i^p)\Big].\end{aligned} \tag{42}$$

To write $\text{Tail}_{i,t}^{\boldsymbol{X},0}$ in terms of MLTD, we exploit

$$\text{cov}_t\Big[\mathbb{1}(T_p^{\boldsymbol{X}}), \mathbb{1}(T_p^i)\Big] = \mathbb{P}_t\Big[T_p^i \cap T_p^{\boldsymbol{X}}\Big] - \mathbb{P}_t\Big[T_p^i\Big]\ \mathbb{P}_t\Big[T_p^{\boldsymbol{X}}\Big] \tag{43}$$

and $\mathbb{P}_t[T_i^p] = p$. This implies

$$\begin{aligned}\text{Tail}_{i,t}^{\boldsymbol{X},0} =& (1+R_{f,t+1})\,m(\boldsymbol{x}_p)\,\mathbb{P}_t\Big[T_p^{\boldsymbol{X}}\Big]\ \left(\frac{\mathbb{P}_t\Big[T_p^i \cap T_p^{\boldsymbol{X}}\Big]}{\mathbb{P}_t[T_{\boldsymbol{X}}^p]} - p\right)\ \text{ES}_{p,t}[R_{i,t+1}] &(44)\\ =& (1+R_{f,t+1})\,m(\boldsymbol{x}_p)\,\mathbb{P}_t\Big[T_p^{\boldsymbol{X}}\Big]\ \Big(\text{MLTD}_{p,t}^{\boldsymbol{X}}[R_{i,t+1}] - p\Big)\ \text{ES}_{p,t}[R_{i,t+1}], &(45)\end{aligned}$$

where we use that

$$\text{MLTD}^{\boldsymbol{X}}_{p,t}[R_{i,t+1}] = \mathbb{P}_t\left[T^i_p | T^{\boldsymbol{X}}_p\right] = \frac{\mathbb{P}_t\left[T^i_p \cap T^{\boldsymbol{X}}_p\right]}{\mathbb{P}_t\left[T^{\boldsymbol{X}}_p\right]}. \tag{46}$$

B.2 Properties of MLTD

The invariance of MLTD under strictly increasing and continuous transformations follows from the corresponding transformation behavior of quantiles. Let g denote a continuous and increasing function, then it holds that (Dhaene et al., 2002, Theorem 1)

$$Q_p[g(Y)] = g(Q_p[Y]). \tag{47}$$

Let $g_0, \ldots, g_N$ denote strictly increasing and continuous functions, then

$$\mathbb{P}[g_0(R_i) \leq Q_p[g_0(R_i)] \,|\, g_1(X_1) \leq Q_p[g_1(X_1)], \ldots, g_N(X_N) \leq Q_p[g_N(X_N)]] \tag{48}$$

$$= \mathbb{P}[g_0(R_i) \leq g_0(Q_p[R_i]) \,|\, g_1(X_1) \leq g_1(Q_p[X_1]), \ldots, g_N(X_N) \leq g_N(Q_p[X_N])] \tag{49}$$

$$= \mathbb{P}[R_i \leq Q_p[R_i] \,|\, X_1 \leq Q_p[X_1], \ldots, X_N \leq Q_p[X_N]]\,, \tag{50}$$

which implies

$$\text{MLTD}^{g_1(X_1),\ldots,g_N(X_N)}_p[g_0(R_i)] = \text{MLTD}^{X_1,\ldots,X_N}_p[R_i]. \tag{51}$$

The copula representation of MLTD in equation (21) can be derived as follows

$$\text{MLTD}^{X_1,\ldots,X_N}_p[R_i] = \mathbb{P}[R_i \leq Q_p[R_i] \,|\, X_1 \leq Q_p[X_1],\, \ldots,\, X_N \leq Q_p[X_N]] \tag{52}$$

$$= \frac{F_{R_i,X_1,\ldots,X_N}(Q_p[R_i], Q_p[X_1]\, \ldots,\, Q_p[X_N])}{F_{X_1,\ldots,X_N}(Q_p[X_1], \ldots, Q_p[X_N])} \tag{53}$$

$$= \frac{C_{R_i,X_1,\ldots,X_N}\left(F_{R_i}(Q_p[R_i]), F_{X_1}(Q_p[X_1]), \ldots, F_{X_N}(Q_p[X_N])\right)}{C_{X_1,\ldots,X_N}\left(F_{X_1}(Q_p[X_1]), \ldots, F_{X_N}(Q_p[X_N])\right)}. \tag{54}$$

Since the marginal distributions are continuous, we have $F_{R_i}(Q_p[R_i]) = p$ as well as $F_{X_i}(Q_p[X_i]) = p$ and thus

$$\text{MLTD}^{X_1,\ldots,X_N}_p[R_i] = \frac{C_{R_i,X_1\ldots,X_N}(p,\ldots,p)}{C_{X_1,\ldots,X_N}(p,\ldots,p)}. \tag{55}$$

C Assumptions of the Theoretical Example

For the example that we present in Section 2.3, we use a copula specification and asymmetric marginal distributions.

To avoid return realizations below minus one, we simulate from the joint conditional distribution of *logarithmic* returns and transform the obtained random numbers into discrete returns in line with our theory. We use Hansen's Skewed t distribution (Hansen, 1994), which is also used in our empirical analysis (see Section 3.3) for the marginal distributions. For the test asset, we assume $\sigma = 0.26$, $\lambda = -0.3$ and $\nu = 5$, where σ denotes the annualized standard deviation, λ calibrates the asymmetry of the distribution and ν is the degree-of-freedom parameter. We apply $\sigma = 0.2$, $\lambda = -0.1$ and $\nu = 7$ for the first factor and use $\sigma = 0.21$, $\lambda = -0.45$ and $\nu = 4.25$ for the second factor.

To determine the location parameters of $X_{1,t+1}$ and $X_{2,t+1}$, we numerically solve equation (7) for $R_{i,t+1} = X_{1,t+1}$ and $R_{i,t+1} = X_{2,t+1}$ (simultaneously). The location parameter of $R_{i,t+1}$ is irrelevant for the simulations because the covariance underlying all approximations is invariant under deterministic shifts. Of course, this parameter can readily be calculated from the approximations of the excess returns that we obtain from our simulations.

We use the skewed-t-copula introduced in Section 3.2 as dependence model. In the baseline case, the degree-of-freedom parameter of the copula is $\nu_c = 5$ and we assume $\boldsymbol{\gamma} = (-0.3, -0.3, -0.3)'$ for the dependence asymmetry parameter. The copula correlations for the test assets and the two factors are $\rho_{12} = 0.45$ and $\rho_{13} = 0.42$ and the third correlation parameter is $\rho_{13} = 0.45$. These values imply that the simulated discrete returns have correlations of roughly 0.55.

To complete the specification of the SDF, we need the mapping function g, for which we assume $g(x_1, x_2) = 0.5\, x_1 + 0.5x_2$. The utility function of the representative investor is given by

$$u(w) = \frac{w^{1-\eta} - 1}{1 - \eta} \tag{56}$$

in our example with a relative risk aversion (RRA) of $\eta = 4$. Our simulations are based on $R_F = 0.02\%$ for the annualized risk-free rate. We simulate monthly returns, i.e., we use $1/12$ and $\sqrt{1/12}$ to rescale the annualized location and scale parameters for the simulation.

D The Skewed t Copula and its Estimation

The density of the skewed-t copula is given by

$$c_{st,N}(\boldsymbol{u};\, \boldsymbol{P},\, \nu_c,\, \boldsymbol{\gamma}) = \frac{f_{st,N}(q_{st}(u_1;\, \nu_c,\, \gamma_1), \ldots, q_{st}(u_N;\, \nu_c,\, \gamma_N);\, \boldsymbol{P}, \nu_c,\, \boldsymbol{\gamma})}{\prod_{i=1}^{N} f_{st}(q_{st}(u_i;\, \nu_c, \gamma_i);\, \nu_c, \gamma_i)}, \tag{57}$$

where $f_{st,N}(\,\cdot,\,;\boldsymbol{P},\,\nu_c,\,\boldsymbol{\gamma})$ denotes the probability density function of the multivariate skewed distribution. $f_{st}(\,\cdot,\,\nu_c,\gamma_i)$ is the univariate density of the generalized hyperbolic skewed distribution with the parameters ν_c and γ_i. $q_{st}(\,\cdot\,;\nu_c,\gamma_i)$ is the corresponding quantile function (inverse cumulative distribution function). For the multivariate density, it holds that

$$\begin{aligned} f_{st,N}(\boldsymbol{w};\, \boldsymbol{P},\, \nu_c,\, \boldsymbol{\gamma}) = {} & \frac{2^{\frac{2-(\nu_c+N)}{2}}}{\Gamma(\nu_c/2)\,(\pi\,\nu_c)^{N/2}\,\sqrt{\det(\boldsymbol{P})}} \\ & \cdot \frac{K_b\Big(\sqrt{(\nu_c + \boldsymbol{w}'\cdot\boldsymbol{P}^{-1}\cdot\boldsymbol{w})\,\boldsymbol{\gamma}'\cdot\boldsymbol{P}^{-1}\cdot\boldsymbol{\gamma}};\, \frac{\nu_c+N}{2}\Big)\,\exp\Big(\boldsymbol{w}'\cdot\boldsymbol{P}^{-1}\cdot\boldsymbol{\gamma}\Big)}{\sqrt{(\nu_c + \boldsymbol{w}'\cdot\boldsymbol{P}^{-1}\cdot\boldsymbol{w})\cdot\boldsymbol{\gamma}\cdot\boldsymbol{P}^{-1}\cdot\boldsymbol{\gamma}}^{\,-(\nu_c+N)/2}\,\Big(1+\nu_c^{-1}\,(\boldsymbol{w}'\cdot\boldsymbol{P}^{-1}\cdot\boldsymbol{w})\Big)^{(\nu_c+N)/2}}, \end{aligned} \tag{58}$$

where $K_b(\,\cdot,\,;\kappa)$ is a Bessel function of the third kind with parameter κ.[46] For the univariate density this implies

$$f_{st}(w;\nu_c,\,\gamma_i) = \frac{2^{1-(\nu_c+1)/2}}{\Gamma(\nu_c/2)\,\sqrt{\pi\nu_c}}\;\frac{K_b\Big(\sqrt{(\nu_c+w^2)\,\gamma_i^2};\,\frac{\nu_c+1}{2}\Big)\,\exp(w\,\gamma_i)}{\sqrt{(\nu_c+w^2)\,\gamma^2}^{\,-(\nu_c+1)/2}\,(1+\nu_c^{-1}\,w^2)^{(\nu_c+1)/2}}. \tag{59}$$

The cdf F_{st} and the quantile function q_{st} can be approximated by numerical approximation methods or by Monte Carlo simulation based on the stochastic mixture representation presented in Section 3.2. We use the latter methodology with 1,000,000 random numbers for each parameter constellation.[47]

The moments that we need to standardize the copula shocks are given by

$$\mathbb{E}[W_i] = \frac{\nu_c}{\nu_c-2}\,\gamma_i \qquad \text{and} \qquad \sigma[W_i] = \frac{\nu_c}{\nu_c-2} + \frac{2\nu_c^2\,\gamma_i^2}{(\nu_c-2)^2(\nu_c-4)}. \tag{60}$$

[46]See e. g. Demarta and McNeil (2007, p. 120) or Christoffersen et al. (2012, p. 3747) for this characterization. Other authors refer to the relevant function as modified Bessel function of the *second* kind. We use the Matlab function `besselk` for the implementation.

[47]For the estimations and simulations, we fix the seed of the Matlab random generator at 1.

For the computation of the (M)LTD coefficients, we again use Monte Carlo simulations with $M = 1{,}000{,}000$ random vectors. Moreover, we exploit that (M)LTD is invariant under changes of the marginal distributions, so we can directly simulate a random sample $((r_{i,s}, x_{1,s}, x_{2,s})')_{s=1,\dots,M}$ from the multivariate skewed t distribution underlying the copula and then we evaluate

$$\widehat{\text{MLTD}}^{\alpha}_{X_1,X_2}[R_i] = \frac{\sum_{s=1}^{M} 1(r_{i,s} \leq \hat{q}_{R_i}(\alpha),\, x_{1,s} \leq \hat{q}_{X_1}(\alpha),\, x_{2,s} \leq \hat{q}_{X_2}(\alpha))}{\sum_{s=1}^{M} 1(x_{1,s} \leq \hat{q}_{X_1}(\alpha),\, x_{2,s} \leq \hat{q}_{X_2}(\alpha))}, \tag{61}$$

where $\hat{q}_Y(u)$ is a standard quantile estimator.

D.1 Estimation Details

The maximization of the log-likelihood in (30) is performed numerically using the copula density in (57). We use the following parameter bounds: $\alpha_c, \beta_c \in [0.005, 0.995]$, $\nu_c \in [4.0001, 250]$ and $\gamma_i \in [-1, 1]$, $i = 1, \dots, N$.

For given values of α_c, β_c, ν_c and $\boldsymbol{\gamma}$, the calculation of this log-likelihood involves the following steps (Aielli, 2013; Christoffersen et al., 2012):

1. Calculate $\boldsymbol{w}_{t+1}$ and $\boldsymbol{z}_{t+1}$ according to (24) using (60) for $t = 0, \dots, T-1$.
2. Recursively calculate $q_{ii,t+1} = (1-\alpha_c-\beta_c)+\alpha_c\, z_{i,t}^2+\beta_c\, q_{ii,t}$ for $t = 0, \dots, T-1$ and $i = 1, \dots, N$.
3. Calculate $\bar{\boldsymbol{z}}_{t+1} = \sqrt{\text{diag}(\boldsymbol{Q}_{t+1})} \cdot \boldsymbol{z}_{t+1}$.
4. Estimate the unconditional copula correlation $\hat{\boldsymbol{S}}_c$ according to (31).
5. Calculate $\boldsymbol{Q}_{t+1}$ according to (26) for $t = 0, \dots, T-1$.
6. Calculate $\boldsymbol{P}_{t+1}$ according to (25) for $t = 0, \dots, T-1$.
7. Calculate the log-likelihood $l_{1:T}$ from (30) using (57).

E Definitions and Data Sources of Main Variables

This table briefly defines the main variables used in the empirical analysis. The abbreviation KF denotes Kenneth French's Data Library, the abbreviation OP stands for data that is obtained from the authors' homepages of the respective original papers and EST indicates that the variable is estimated or computed based on original variables from the respective data sources.

Panel A: Main Risk Factors

Variable Name	Description	Source
MKT	Value-weighted CRSP market-return in excess of the risk-free rate.	KF
SMB	Small-Minus-Big size factor portfolio return.	KF
HML	High-Minus-Low value factor portfolio return.	KF
UMD	Up-Minus-Down momentum factor portfolio return.	KF

Panel B: Additional Risk Factors

Variable Name	Description	Source
RMW	Fama and French (2015) Robust Minus Weak Factor	KF
CMA	Fama and French (2015) Conservative Minus Aggressive Factor	KF
STR	Short-Term Reversal Factor	KF
LTR	Long-Term Reversal Factor	KF
PSL	Pástor and Stambaugh (2003)'s traded liquidity factor.	OP
BAB	Frazzini and Pedersen (2014)'s betting-against-beta factor.	OP
TR	Kelly and Jiang (2014)'s tail risk factor.	EST
QMJ	Asness et al. (2017)'s quality-minus-junk factor.	OP

Panel C: Tail Dependence Coefficients and Risk Measures		
Variable Name	Description	Source
LTD^{X_1}	Bivariate lower tail dependence between a test asset's return and the risk factor X_1, estimated as detailed in Section 4.	KF, EST
MLTD^{X_1,X_2}	Trivariate lower tail dependence between a test asset's return and the risk factors X_1 and X_2, see equation (1); estimated as detailed in Section 4.	KF, EST
CRASH^{X_1}	Bivariate crash risk of a test asset and the risk factor X_1, see equation (4).	KF, EST
MCRASH^{X_1,X_2}	Trivariate crash risk of a test asset and the risk factors X_1 and X_2, see equation (3).	KF, EST
MCRASH^{X_1,X_2}	Trivariate crash risk of a test asset and the risk factors X_1 and X_2, see equation (3).	KF, EST
ES	(Univariate) Expected Shortfall of a test asset or a risk factor, estimated with the GARCH skewed-t model described in Section 4.	KF, EST
Size	The natural logarithm of an industry's average equity market capitalization in million USD.	KF, EST
Book-To-Market	An industry's average book-to-market ratio computed as the average ratio of CS book value per share to share price.	KF, EST
Past Return	An industry's cumulative past 12-month excess return.	KF, EST
Volatility	Standard deviation estimated with a rolling window of 60 months.	KF, EST
Coskewness	Coskewness of an asset's return and the market return, estimated as in Harvey and Siddique (2000) with a rolling window of 60 months.	KF, EST
Downside Beta	Downside Beta of an asset's return and the market return, estimated as in Ang et al. (2006) with a rolling window of 60 months.	KF, EST

F Additional Empirical Results

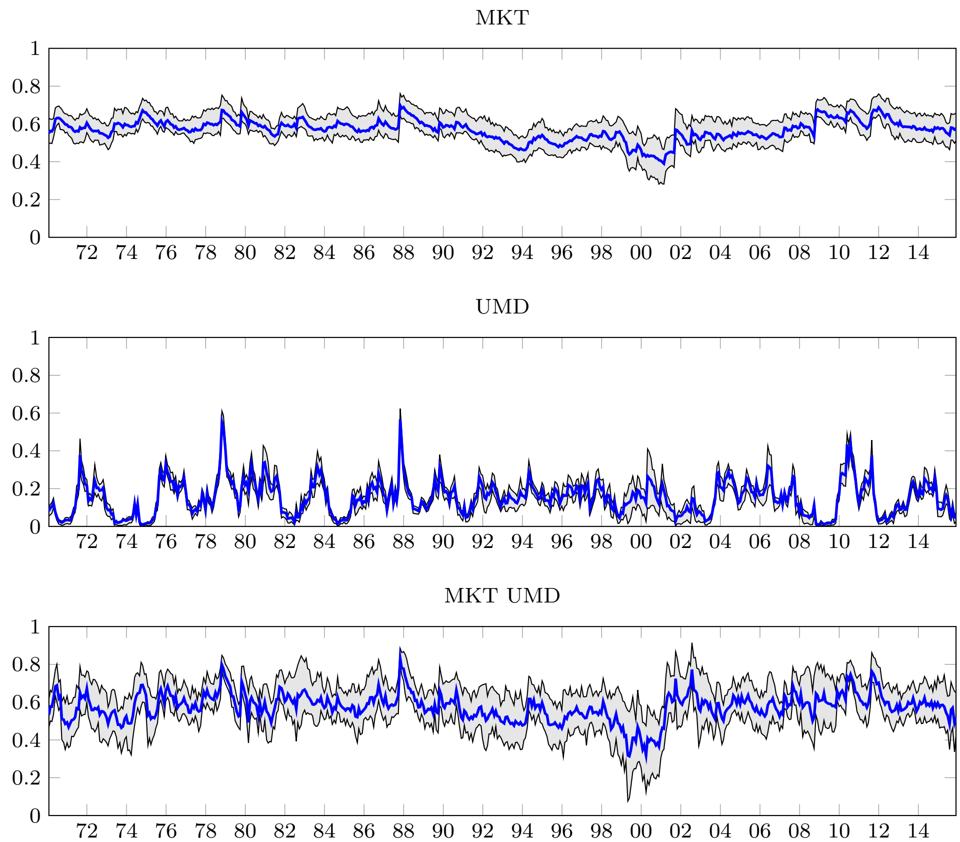
In this appendix, we present additional results from our empirical analysis. Table F.1 and Figure F.1 summarize and illustrate the lower tail dependence of our test assets with our four main risk factors. Table F.2 documents our findings on the persistence of our CRASH and MCRASH coefficients and Table F.3 reports the results of portfolio sorts based on CRASH and MCRASH for additional factors and factor combinations.

Table F.1: Lower Tail Dependence of the Industries

	LTD				MLTD					
	MKT	SMB	HML	UMD	MKT SMB	MKT HML	MKT UMD	SMB HML	SMB UMD	HML UMD
min	0.23	0.11	0.04	0.10	0.33	0.24	0.26	0.05	0.14	0.07
q25	0.50	0.16	0.06	0.14	0.55	0.43	0.51	0.12	0.17	0.11
med	0.57	0.20	0.08	0.15	0.68	0.58	0.59	0.16	0.22	0.13
avg	0.57	0.20	0.08	0.15	0.66	0.56	0.58	0.16	0.22	0.13
q75	0.64	0.23	0.10	0.17	0.77	0.70	0.65	0.20	0.27	0.16
max	0.74	0.28	0.13	0.18	0.92	0.83	0.79	0.24	0.30	0.23

This table shows results on the lower tail dependence of the industry portfolios with respect to our main risk factors (MKT, SMB, HML, and UMD). We report summary statistics on the time-series averages of bivariate and multivariate lower tail dependence estimates across industries. We include LTD with respect to the four factors and MLTD with respect to all factor pairs for $p = 10\%$. We report the minimum (min), the 25%-quantile (q25), the median (med), the average (avg), the 75%-quantile (q75) and the maximum (max) of the time-series averages across industries. The sample period is from January 1970 to December 2015.

Figure F.1: Selected Lower Tail Dependence Measures Over Time



This figure summarizes the evolution of selected aggregate lower tail dependence measures over time. We select the bivariate LTD measures LTD^{MKT} and LTD^{UMD} and the corresponding higher-dimensional measure $MLTD^{MKT,UMD}$. The blue lines correspond to the aggregate measures defined as the equally-weighted average of (M)LTD coefficients at t over all industries in our sample. The shaded areas correspond to the range between the first and the fourth quartile. We use $p = 10\%$, i.e., we select the 10%-quantiles of returns as thresholds for the tail region. All risk measures are based on dynamic skewed-t copula models and univariate GARCH skewed-t models presented in Section 3. The sample period is from January 1970 to December 2015.

Table F.2: Persistence of Crash Risk Coefficients

Panel A: Autocorrelations

Auto-Correlation	CRASH MKT	CRASH SMB	CRASH HML	CRASH UMD	MCRASH MKT SMB	MCRASH MKT HML	MCRASH MKT UMD	MCRASH SMB HML	MCRASH SMB UMD	MCRASH HML UMD
Lag 12	0.43	0.44	0.52	0.32	0.45	0.47	0.30	0.61	0.32	0.18
Lag 24	0.24	0.32	0.21	0.25	0.29	0.27	0.13	0.41	0.12	0.15
Lag 36	0.25	0.36	0.16	0.09	0.32	0.26	0.16	0.38	0.07	0.10
Lag 48	0.13	0.29	0.16	0.10	0.23	0.16	0.12	0.32	0.06	0.15

Panel B: Fama and MacBeth (1973) Regressions

	CRASH MKT	CRASH SMB	CRASH HML	CRASH UMD	MCRASH MKT SMB	MCRASH MKT HML	MCRASH MKT UMD	MCRASH SMB HML	MCRASH SMB UMD	MCRASH HML UMD
Lag 12	0.512***	0.473***	0.505***	0.267***	0.474***	0.435***	0.364***	0.611***	0.292***	0.269***
	(16.62)	(12.75)	(8.21)	(5.81)	(17.06)	(12.30)	(12.61)	(11.62)	(7.99)	(9.41)
Lag 24	0.150***	0.227***	0.141***	0.062*	0.198***	0.178***	0.086***	0.174***	0.135***	0.217***
	(6.00)	(5.70)	(3.41)	(1.77)	(6.07)	(6.48)	(3.40)	(3.77)	(4.87)	(4.53)
Lag 36	0.140***	0.126***	0.053	0.0518*	0.158***	0.138***	0.132***	0.141**	0.194***	0.117***
	(5.51)	(4.05)	(0.93)	(1.69)	(6.37)	(5.57)	(5.65)	(2.33)	(5.58)	(3.07)
Lag 48	0.131***	0.143***	0.137***	0.122***	0.167***	0.120***	0.143***	0.066*	0.257***	0.113***
	(4.34)	(4.63)	(2.76)	(4.41)	(5.51)	(4.93)	(5.83)	(1.87)	(6.61)	(4.55)
Alpha	0.0031***	0.0011***	0.0003	0.0014***	0.0023***	0.0047***	0.0081***	0.0013***	0.0016***	0.0017***
	(5.61)	(4.39)	(1.12)	(4.22)	(3.54)	(9.13)	(8.62)	(3.13)	(4.03)	(4.25)
R^2	0.613	0.661	0.534	0.399	0.698	0.523	0.397	0.6328	0.501	0.405

This table investigates the persistence of our crash risk measures. Panel A reports the autocorrelation coefficients of $CRASH^{MKT}$, $CRASH^{SMB}$, $CRASH^{HML}$, $CRASH^{UMD}$, $MCRASH^{MKT, SMB}$, $MCRASH^{MKT, HML}$, $MCRASH^{MKT, UMD}$, $MCRASH^{SMB, HML}$, $MCRASH^{SMB, UMD}$, and $MCRASH^{HML, UMD}$ with their lagged realizations (up to 48 months into the past). Panel B reports the results of multivariate Fama and MacBeth (1973) regressions of the crash risk coefficients on the corresponding lagged realizations (up to 48 months into the past). The sample period is from January 1970 to December 2015. T-statistics are computed using Newey and West (1987) standard errors with 48 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table F.3: Univariate Portfolio Sorts: Additional CRASH and MCRASH Measures

Panel A: Crash Risk Measures

Portfolio	CRASH MKT	CRASH SMB	CRASH HML	CRASH UMD
1 Weak	0.566%	0.802%	0.743%	0.598%
2	0.709%	0.775%	0.715%	0.743%
3	0.836%	0.696%	0.761%	0.675%
4	0.861%	0.851%	0.865%	0.745%
5 Strong	0.733%	0.567%	0.610%	0.967%
(5) – (1)	0.167%	-0.235%	-0.133%	0.369%*
	(0.77)	(-1.14)	(-0.59)	(1.78)

Panel B: Multivariate Crash Risk Measures

Portfolio	MCRASH SMB HML	MCRASH SMB UMD	MCRASH HML UMD	MCRASH MKT SMB HML UMD
1 Weak	0.718%	0.652%	0.712%	0.645%
2	0.765%	0.802%	0.705%	0.841%
3	0.734%	0.758%	0.692%	0.714%
4	0.873%	0.765%	0.786%	0.727%
5 Strong	0.604%	0.729%	0.815%	0.819%
(5) - (1)	-0.114%	0.077%	0.103%	0.174%
	(-0.49)	(0.39)	(0.51)	(0.95)

This table reports the average future one-month ahead returns of univariate equal-weighted portfolio sorts based on crash risk measures. Panel A describes the results of sorts on the bivariate crash risk measures $\text{CRASH}^{\text{MKT}}$, $\text{CRASH}^{\text{SMB}}$, $\text{CRASH}^{\text{HML}}$, $\text{CRASH}^{\text{UMD}}$. Each month t, we rank our test assets into quintiles (1-5) based on their estimated crash risk coefficients and form equal-weighted portfolios that we hold over the following month $t + 1$. We report average monthly excess returns over the T-Bill. The rows labelled 'Strong - Weak' report differences between the returns of portfolio 5 and portfolio 1 with corresponding T-statistics. Panel B summarizes the results of sorts based on the multivariate crash risk measures $\text{MCRASH}^{\text{SMB, HML}}$, $\text{MCRASH}^{\text{SMB, UMD}}$, $\text{MCRASH}^{\text{HML, UMD}}$, and $\text{MCRASH}^{\text{MKT, SMB, HML, UMD}}$. The sample period is from January 1975 to December 2015. T-statistics are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

T

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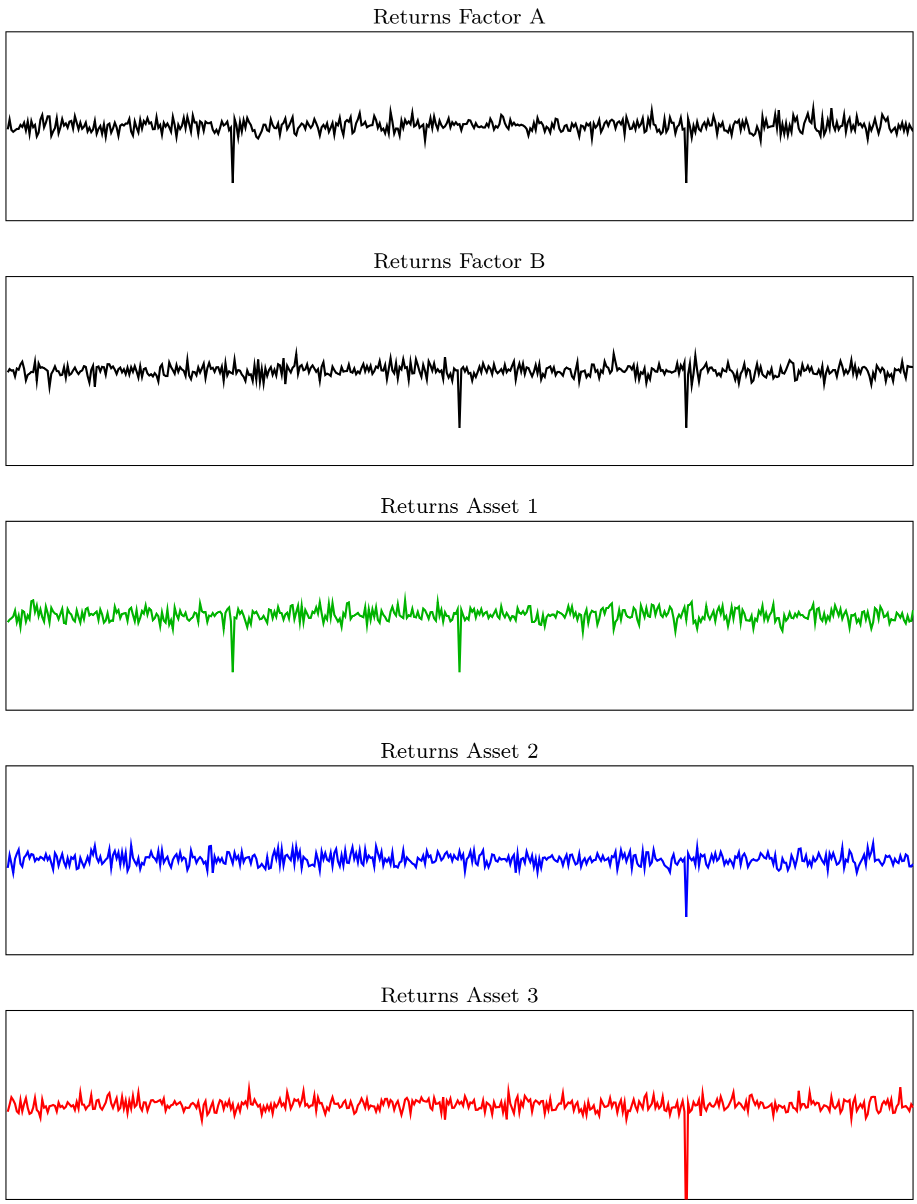
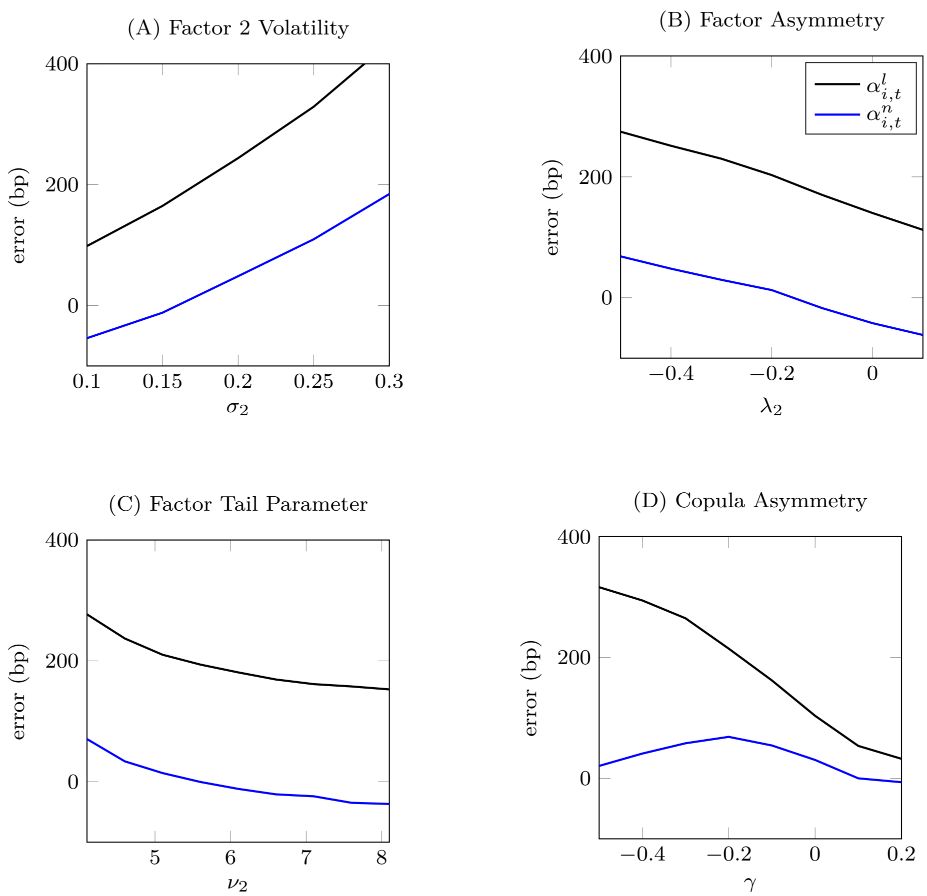
Figure 1: Stylized Example



This figure provides a stylized example of the concept of multivariate crash risk. The first two graphs show the return time-series of two risk factors. Asset 1 (shown in the third graph) is not exposed to the joint multivariate crash of both factors. In contrast, asset 2 (shown in the fourth graph) crashes simultaneously with both factors. The third asset, whose returns are shown in the last graph, is also exposed to the joint factor crash and its individual crash risk is larger than for asset 2.

Figure 2: Pricing Errors

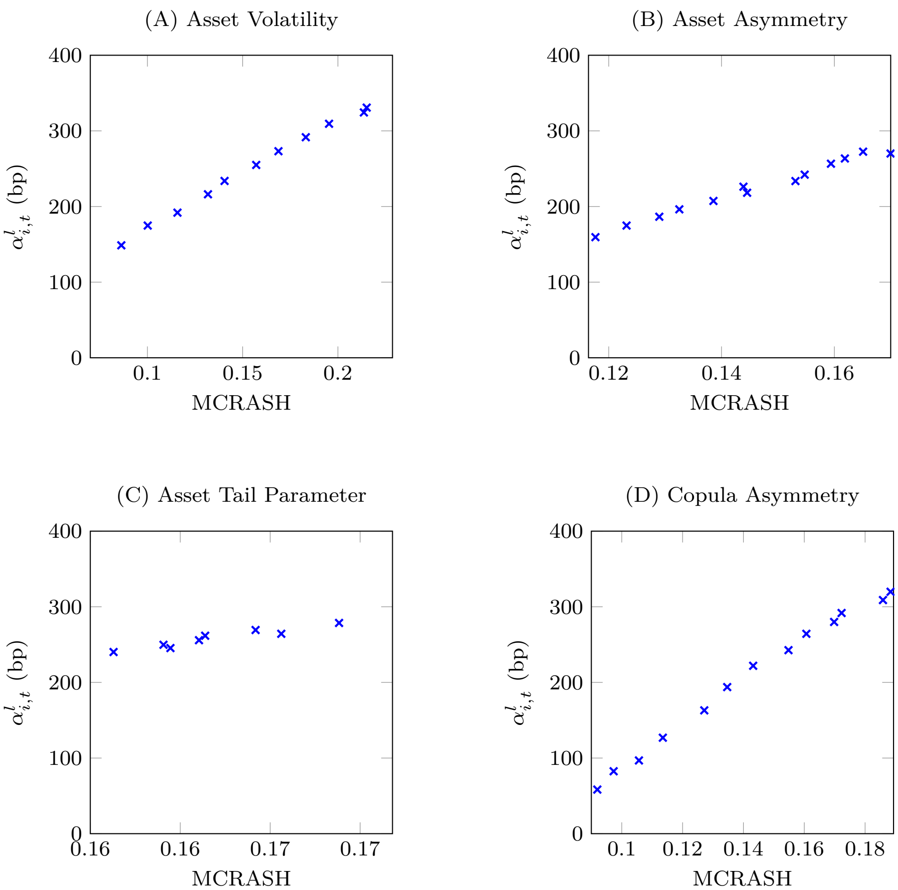


We compare the pricing errors of a standard linear multi-factor model ($\alpha^l_{i,t}$, black lines) with the pricing errors of our new specification including MCRASH ($\alpha^n_{i,t}$, blue lines). These pricing errors are calculated as

$$\alpha^l_{i,t} := \mathbb{E}_t[R_{i,t+1} - R_{f,t+1}] - \left(\beta^{(1)}_{i,t}\,\lambda^{(1)}_t + \beta^{(2)}_{i,t}\,\lambda^{(2)}_t\right),$$
$$\alpha^n_{i,t} := \mathbb{E}_t[R_{i,t+1} - R_{f,t+1}] - \left(\beta^{(1)}_{i,t}\,\lambda^{(1)}_t + \beta^{(2)}_{i,t}\,\lambda^{(2)}_t + \text{MCRASH}^{\boldsymbol{X}}_{p,t}[R_{i,t+1}]\,\lambda^{\boldsymbol{X}}_{p,t}\right)$$

using the preferences and the distributional assumptions described in Section 2.3. We vary selected characteristics of the joint distribution of $(R_{i,t+1}, X_{1,t+1}, X_{2,t+1})$ presented in Appendix C focussing on the second – non-normal – factor $X_{2,t+1}$. The Panels A - C show how changes in the marginal distribution of this factor affect the magnitude of the approximation errors. We vary the volatility parameter σ_2 (Panel A), the skewness parameter λ_2 (Panel B), and the tail parameter ν_2 (Panel C). Panel D illustrates the effect of changing the copula's asymmetry parameter γ. The approximation errors are annualized and reported in basis points.

Figure 3: MCRASH and Alphas



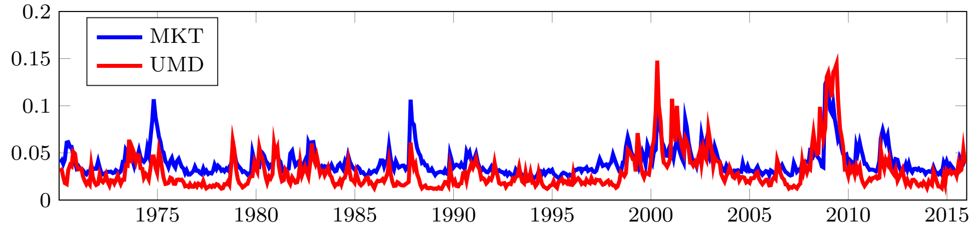
We illustrate the relationship between our new multivariate crash risk measure MCRASH^{X_1,X_2} and the alpha of the linear multi-factor model, i.e.

$$\alpha^l_{i,t} := \mathbb{E}_t[R_{i,t+1} - R_{f,t+1}] - (\beta^{(1)}_{i,t}\,\lambda^{(1)}_t + \beta^{(2)}_{i,t}\,\lambda^{(2)}_t).$$

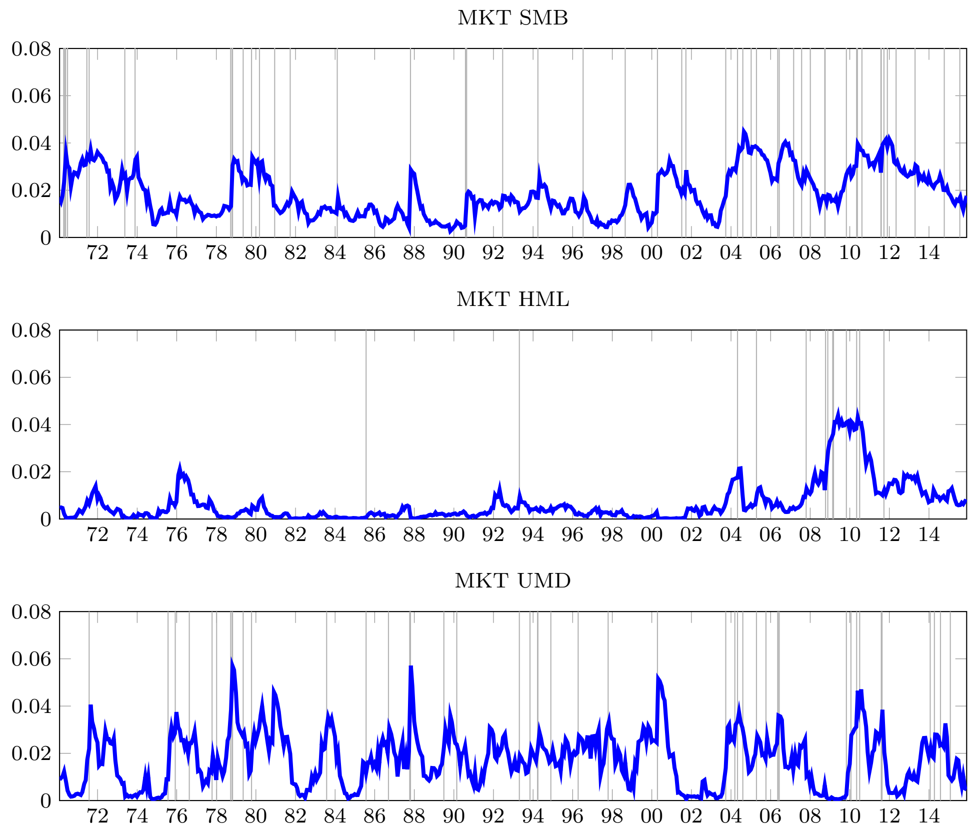
We rely on the preferences and the distributional assumptions described in Section 2.3 and Appendix C. We vary selected characteristics of the asset's marginal return distribution and of the copula describing its dependence structure with the two factors. In particular, we vary the asset's volatility (Panel A), the asymmetry and the tail parameter of the marginal distribution (Panel B and C), and the asymmetry parameter of the copula (Panel D). For each parameter set, we compute MCRASH and the resulting alpha $\alpha^l_{i,t}$. $\alpha^l_{i,t}$ is annualized and reported in basis points. Note that the range of the x-axes differs.

Figure 4: Crash Risk of the Factors

Panel A: Expected Shortfall $p = 10\%$

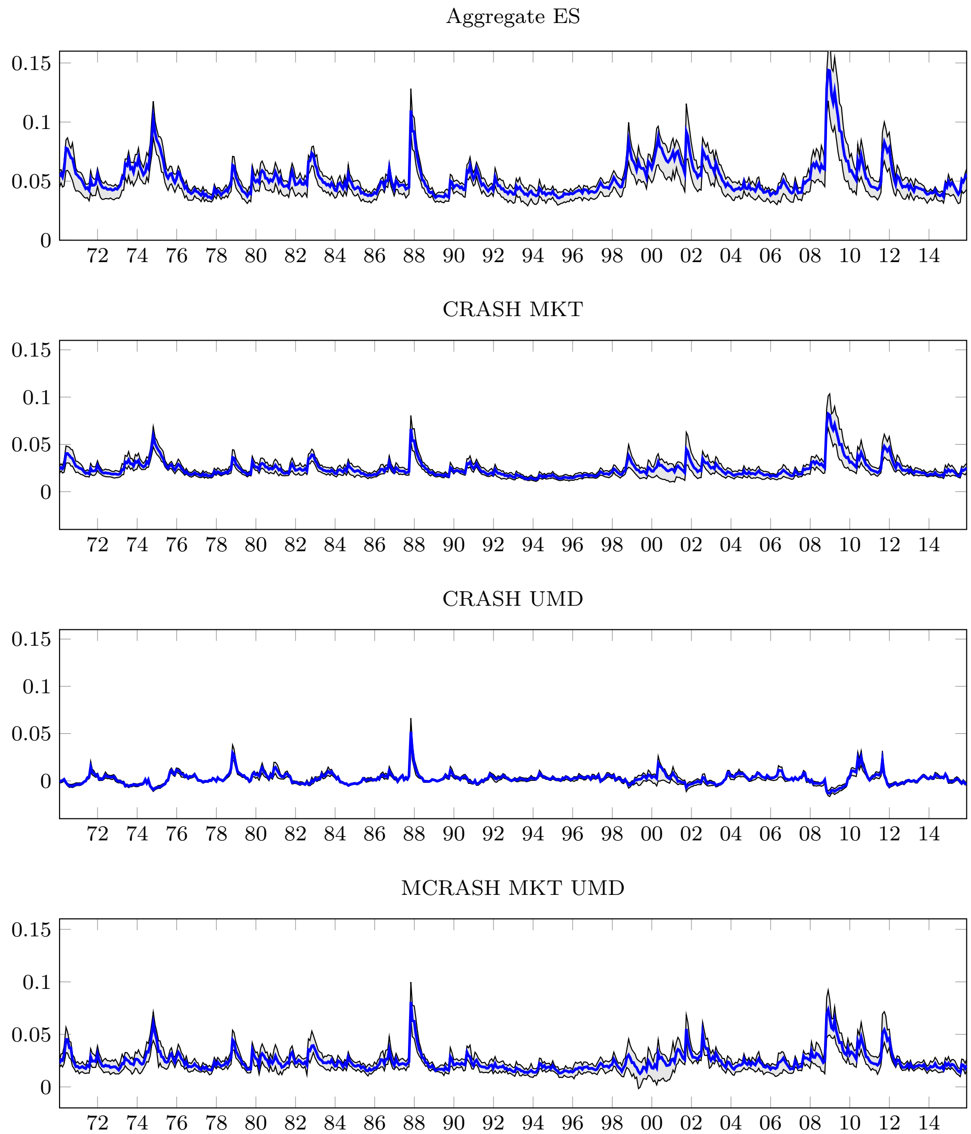


Panel B: Joint Crash Probabilities $p = 10\%$



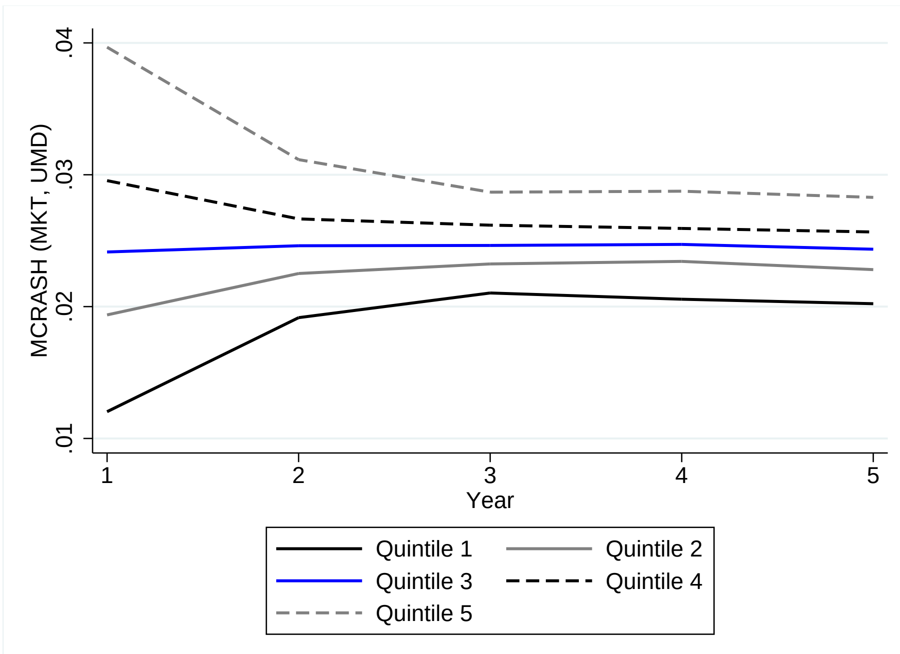
This figure shows the evolution of the crash risk of the selected factors and factor combinations over time. Panel A shows the Expected Shortfall of the market factor (MKT) and the momentum factor (UMD) at the 10% probability level. Panel B displays the probabilities of joint factor crashes and indicates the occurrence of these crashes for selected factor combinations. We present results for two-dimensional crashes involving the market factor (i.e., MKT & SMB, MKT & HML, MKT & UMD). Joint crashes are defined as return realizations that are simultaneously below their respective 10%-quantiles, see the event $T_p^{\boldsymbol{X}}$ defined in equation (8). The blue line corresponds to the average prediction over the asset-specific dynamic copula models that we introduced in Section 4. We apply conditional 10%-quantiles (10%-VaR multiplied by minus one) computed from the skewed-t GARCH models presented in this section as thresholds. The vertical grey lines indicate the occurrence of such crashes in our sample data. The sample period is from January 1970 to December 2015.

Figure 5: Selected Crash Risk Measures Over Time



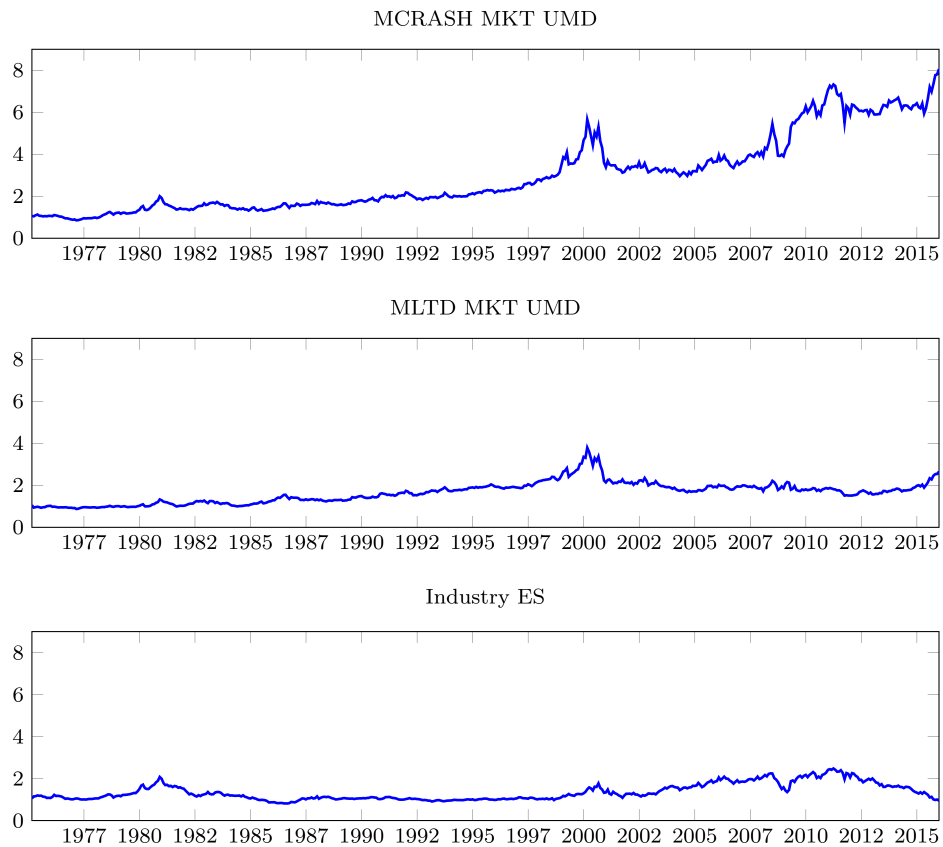
This figure summarizes the evolution of the industry portfolios' individual crash risk and selected systematic crash risk coefficients over time. The first graph shows the Expected Shortfall (ES) of these portfolios. The second and third graph illustrate the bivariate crash risk of the industry portfolios with the momentum factor ($\text{CRASH}^{\text{UMD}}$) and the market factor ($\text{CRASH}^{\text{MKT}}$). The fourth panel summarizes the evolution of the aggregate multivariate crash risk with respect to the market and the momentum factor ($\text{MCRASH}^{\text{MKT, UMD}}$). The blue curves correspond to aggregate crash risk measures, defined as the equal-weighted average of the coefficients at t over all industries in our sample. The shaded areas correspond to the range between the first and the fourth quartile. All risk measures are based on dynamic skewed-t copula models and univariate GARCH skewed-t models presented in Section 3. The sample period is from January 1970 to December 2015. We use the probability level $p = 10\%$.

Figure 6: Persistence of Multivariate Crash Risk



This figure illustrates the persistence of the multivariate crash risk measure with respect to the market and the momentum factor ($\text{MCRASH}^{\text{MKT, UMD}}$). Industries are sorted into quintiles based on their $\text{MCRASH}^{\text{MKT, UMD}}$ in year t. Then, the equal-weighted average of $\text{MCRASH}^{\text{MKT, UMD}}$ of these portfolios is computed again in the following four years $t+1, ..., t+4$. The sample period is from January 1970 to December 2015.

Figure 7: Performance of Selected Crash Risk Trading Strategies



This figure displays the evolution of the cumulative monthly returns for three different long-short investment strategies: (i) a trading strategy based on $\text{MCRASH}^{\text{MKT,UMD}}$, (ii) a trading strategy based on $\text{MLTD}^{\text{MKT,UMD}}$, and (iii) a trading strategy based on ES_p. We use the 20%- and the 80%-quantiles of the conditional crash risk measures as cut-off points and apply monthly rebalancing without accounting for trading costs. We assume an investment of USD 1 at the beginning of 1975. The strategies are implemented with $p = 10\%$. The sample period is from 1975 to 2015.

Table 1: Summary Statistics: Weekly Returns

Panel A: Risk Factors

	Univariate Characteristics									Correlations			
	avg	std	min	max	skew	kurt	JB	VaR	ES	MKT	SMB	HML	UMD
MKT	0.12	2.27	-18.00	13.46	-0.44	8.04	0.1	2.48	4.10	1.00	0.06	-0.24	-0.14
SMB	0.02	1.26	-10.16	6.99	-0.46	9.45	0.1	1.41	2.24	0.06	1.00	-0.14	0.03
HML	0.09	1.28	-9.56	10.64	0.50	11.12	0.1	1.24	2.06	-0.24	-0.14	1.00	-0.24
UMD	0.16	1.89	-16.00	12.65	-1.08	13.05	0.1	1.79	3.53	-0.14	0.03	-0.24	1.00

Panel B: Industries

	Univariate Characteristics									Correlations			
	avg	std	min	max	skew	kurt	JB	VaR	ES	MKT	SMB	HML	UMD
Agric	0.23	3.14	-20.70	23.05	0.10	8.19	0.1	3.28	5.43	0.57	0.18	-0.09	-0.08
Food	0.26	2.06	-15.74	13.53	-0.11	7.33	0.1	2.11	3.43	0.71	-0.10	-0.14	-0.08
Soda	0.26	3.26	-23.64	25.50	0.04	9.41	0.1	3.18	5.51	0.57	-0.07	-0.11	-0.12
Beer	0.27	2.63	-17.35	14.22	-0.26	5.70	0.1	2.82	4.59	0.62	-0.12	-0.17	-0.03
Smoke	0.34	3.14	-17.99	25.18	0.01	7.95	0.1	3.08	5.34	0.51	-0.11	-0.11	-0.04
Toys	0.18	3.39	-29.23	14.06	-0.25	6.48	0.1	3.68	5.89	0.74	0.18	-0.18	-0.17
Fun	0.29	3.81	-23.84	39.24	0.08	12.24	0.1	3.86	6.61	0.75	0.15	-0.11	-0.21
Books	0.21	2.85	-18.62	23.54	0.07	9.48	0.1	2.77	4.97	0.82	0.10	-0.09	-0.19
Hshld	0.20	2.43	-25.07	18.28	-0.56	11.32	0.1	2.46	4.16	0.75	-0.12	-0.22	-0.09
Clths	0.24	3.00	-18.24	19.07	-0.16	6.34	0.1	3.22	5.29	0.78	0.19	-0.09	-0.20
Hlth	0.21	3.67	-23.01	23.85	-0.29	7.41	0.1	3.71	6.56	0.65	0.17	-0.19	-0.07
MedEq	0.23	2.71	-19.15	14.90	-0.40	6.32	0.1	2.91	4.81	0.77	0.05	-0.30	-0.05
Drugs	0.26	2.61	-17.09	20.25	-0.18	6.80	0.1	2.70	4.47	0.75	-0.10	-0.34	-0.03
Chems	0.24	2.86	-18.15	15.54	-0.28	6.95	0.1	2.99	5.02	0.82	0.02	-0.07	-0.19
Rubbr	0.23	2.69	-20.56	19.62	-0.36	8.06	0.1	2.73	4.72	0.78	0.26	-0.08	-0.18
Txtls	0.24	3.32	-23.86	27.74	0.17	12.25	0.1	3.25	5.59	0.70	0.23	0.06	-0.27
BldMt	0.23	2.89	-18.81	22.09	-0.22	8.12	0.1	3.02	5.05	0.83	0.15	-0.03	-0.21
Cnstr	0.21	3.66	-18.92	40.70	0.46	11.55	0.1	3.89	6.30	0.79	0.20	-0.04	-0.19
Steel	0.16	3.69	-26.62	29.53	-0.15	9.44	0.1	3.61	6.48	0.77	0.20	-0.05	-0.20
FabPr	0.14	3.49	-18.70	21.14	-0.18	5.89	0.1	3.96	6.45	0.68	0.29	-0.08	-0.20
Mach	0.21	3.06	-23.09	18.79	-0.35	7.98	0.1	3.18	5.38	0.88	0.21	-0.15	-0.20
ElcEq	0.26	3.14	-19.61	15.30	-0.20	5.84	0.1	3.29	5.43	0.85	0.08	-0.22	-0.14
Autos	0.20	3.32	-21.90	28.98	-0.07	8.84	0.1	3.50	5.75	0.77	0.06	0.05	-0.27
Aero	0.28	3.16	-29.33	16.11	-0.50	8.04	0.1	3.44	5.52	0.77	0.07	-0.11	-0.17
Ships	0.25	3.51	-18.95	15.58	-0.13	5.03	0.1	3.75	6.14	0.63	0.11	-0.05	-0.11
Guns	0.30	3.04	-17.72	17.71	-0.16	6.54	0.1	3.23	5.20	0.56	0.04	-0.10	-0.06
Gold	0.18	4.96	-22.59	35.59	0.52	6.18	0.1	5.48	8.12	0.18	0.15	-0.04	-0.01
Mines	0.21	3.67	-26.93	30.10	-0.02	9.36	0.1	3.75	6.38	0.67	0.16	0.00	-0.21
Coal	0.20	5.20	-28.61	33.68	0.20	7.35	0.1	5.40	8.98	0.54	0.19	-0.04	-0.10
Oil	0.25	2.92	-25.95	13.85	-0.37	7.02	0.1	3.18	5.09	0.68	-0.05	-0.03	-0.09
Util	0.21	2.00	-20.86	13.57	-0.51	11.49	0.1	2.01	3.41	0.65	-0.09	0.07	-0.09
Telcm	0.22	2.39	-21.07	16.45	-0.16	8.31	0.1	2.50	4.07	0.77	-0.05	-0.08	-0.17
PerSv	0.13	3.13	-18.44	22.12	-0.24	6.40	0.1	3.49	5.75	0.76	0.18	-0.18	-0.13
BusSv	0.20	2.69	-16.90	16.02	-0.30	7.11	0.1	2.81	4.80	0.92	0.23	-0.26	-0.14
Hardw	0.20	3.57	-22.50	16.67	-0.21	6.03	0.1	3.82	6.33	0.77	0.12	-0.39	-0.14
Softw	0.21	4.72	-30.15	30.13	-0.05	8.76	0.1	4.90	8.21	0.64	0.23	-0.34	-0.07
Chips	0.23	3.61	-23.48	21.96	-0.16	6.91	0.1	3.78	6.29	0.81	0.20	-0.36	-0.14
LabEq	0.23	3.38	-24.04	18.61	-0.30	6.55	0.1	3.64	6.05	0.83	0.21	-0.34	-0.11
Paper	0.22	2.63	-21.93	15.07	-0.25	7.39	0.1	2.87	4.48	0.80	0.02	-0.06	-0.19
Boxes	0.23	2.88	-17.40	17.95	-0.20	6.55	0.1	3.06	5.07	0.76	0.03	-0.13	-0.15
Trans	0.23	2.89	-22.05	13.65	-0.27	6.77	0.1	3.11	4.98	0.83	0.12	-0.11	-0.18
Whlsl	0.22	2.53	-17.70	11.65	-0.38	6.79	0.1	2.63	4.45	0.86	0.21	-0.20	-0.15
Rtail	0.25	2.70	-16.82	13.96	-0.13	6.06	0.1	2.85	4.63	0.84	0.01	-0.22	-0.13
Meals	0.24	2.92	-15.89	18.87	-0.09	6.14	0.1	3.07	5.13	0.76	0.08	-0.21	-0.14
Banks	0.23	3.21	-22.42	33.11	0.57	15.22	0.1	3.15	5.33	0.80	-0.05	0.13	-0.31
Insur	0.24	2.68	-25.58	21.87	-0.13	11.62	0.1	2.70	4.55	0.82	-0.04	0.01	-0.22
RlEst	0.11	3.45	-21.36	25.81	-0.07	9.54	0.1	3.43	6.22	0.70	0.32	0.01	-0.24
Fin	0.26	3.20	-24.49	27.65	0.16	11.57	0.1	3.15	5.42	0.88	0.10	-0.08	-0.22
Other	0.11	3.16	-20.17	20.05	-0.28	7.51	0.1	3.35	5.76	0.72	0.11	-0.12	-0.20

This table reports summary statistics of risk factor returns (Panel A) and industry returns (Panel B). We display weekly average returns (avg), standard deviations (std), minimum returns (min), maximum returns (max), skewness (skew), kurtosis (kurtosis), p-values of Jarque-Bera tests (JB) as well as non-parametric Value-at-Risk and Expected Shortfall estimates at the $p = 10\%$ probability level. We also show unconditional correlations among the risk factors and unconditional correlations between industry returns and risk factors. The sample period is from January 1970 to December 2015. VaR and ES are in percent.

Table 2: Crash Risk of the Factors

Panel A: Expected Shortfall $p = 10\%$

	MKT	SMB	HML	UMD
min	2.27	1.42	0.90	1.18
q25	3.07	1.79	1.33	1.77
med	3.54	2.00	1.60	2.35
avg	3.98	2.17	1.89	2.98
q75	4.38	2.33	2.12	3.48
max	12.70	8.15	7.53	14.78

Panel B: Probabilities of Joint Factor Crashes $p = 10\%$

	MKT SMB	MKT HML	MKT UMD	SMB HML	SMB UMD	HML UMD	All
emp	2.17	0.58	1.92	1.13	1.71	1.54	0.17
min	0.27	0.01	0.05	0.03	0.11	0.00	0.00
q25	1.06	0.14	0.84	0.69	0.88	0.35	0.00
med	1.60	0.35	1.71	1.02	1.41	0.87	0.00
avg	1.86	0.64	1.72	1.14	1.63	1.28	0.04
q75	2.65	0.75	2.54	1.45	2.12	1.94	0.02
max	4.48	4.38	5.71	3.46	6.62	5.48	1.50

This table summarizes the crash risk of our four main risk factors (MKT, SMB, HML and UMD). Panel A reports summary statistics on the univariate crash risk of these factors as measured by their Expected Shortfall (ES_p) estimates at the probability level $p = 10\%$. We report the minimum (min), the 25%-quantile (q25), the median (med), the average (avg), the 75%-quantile (q75) and the maximum (max) of the forecast series for each of the risk factors. The ES_p-estimates are calculated based on GARCH(1,1)-models with skewed-t margins. In Panel B, we provide summary statistics on the probabilities of multivariate factor crashes. These crashes are defined by all factor returns being smaller than their respective p-quantiles (cp. $T_p^{\boldsymbol{X}}$ defined in equation (8)). We use $p = 10\%$ for setting the quantile thresholds. We report results for all combinations of two factors and for a simultaneous crash of all factors. The first line (emp) corresponds to the empirical frequency of such crashes in our weekly return data. The following summary statistics refer to probability estimates derived from our dynamic copula specification presented in Section 3. For each month in our sample, we first compute equal-weighted averages of the probabilities over each (industry-specific) model. The reported summary statistics refer to the resulting time series of average probability estimates. The sample period is from January 1970 to December 2015. All values in percent.

Table 3: Crash Risk of the Industries

Panel A: CRASH Measures $p = 10\%$

	CRASH				MCRASH					
	MKT	SMB	HML	UMD	MKT SMB	MKT HML	MKT UMD	SMB HML	SMB UMD	HML UMD
min	1.09	0.04	-0.30	-0.01	1.18	1.17	1.36	-0.24	0.15	-0.19
q25	1.96	0.30	-0.18	0.15	2.13	1.67	2.11	0.09	0.30	0.00
med	2.51	0.55	-0.05	0.25	3.08	2.44	2.43	0.32	0.60	0.13
avg	2.42	0.54	-0.05	0.25	2.95	2.39	2.44	0.37	0.63	0.16
q75	2.76	0.77	0.06	0.31	3.76	2.93	2.78	0.62	0.87	0.31
max	3.22	1.11	0.32	0.51	4.71	3.67	3.57	1.33	1.50	0.69

Panel B: Average Correlations CRASH Measures $p = 10\%$

	MKT	SMB	HML	UMD	MKT SMB	MKT HML	MKT UMD	SMB HML	SMB UMD	HML UMD
MKT	1.00	-	-	-	-	-	-	-	-	-
SMB	0.40	1.00	-	-	-	-	-	-	-	-
HML	0.20	0.20	1.00	-	-	-	-	-	-	-
UMD	-0.05	0.16	-0.12	1.00	-	-	-	-	-	-
MKT SMB	0.94	0.53	0.18	-0.05	1.00	-	-	-	-	-
MKT HML	0.68	0.28	0.51	-0.12	0.62	1.00	-	-	-	-
MKT UMD	0.77	0.34	0.10	0.30	0.71	0.52	1.00	-	-	-
SMB HML	0.31	0.53	0.88	-0.08	0.34	0.53	0.21	1.00	-	-
SMB UMD	0.08	0.56	-0.07	0.83	0.15	-0.03	0.35	0.11	1.00	-
HML UMD	-0.04	0.21	0.52	0.54	-0.06	0.20	0.15	0.49	0.47	1.00

This table shows results on the systematic crash risk exposure of the industry portfolios with respect to our main risk factors (MKT, SMB, HML, and UMD). Panel A presents summary statistics on the time-series averages of CRASH and MCRASH estimates across industries. We include CRASH with respect to the four factors and MCRASH with respect to all factor pairs. We report the minimum (min), the 25%-quantile (q25), the median (med), the average (avg), the 75%-quantile (q75) and the maximum (max) of the time-series averages across industries. Panel B shows the equal-weighted average correlations of our CRASH and MCRASH coefficients over all industries. All risk measures are calculated based on the dynamic copula approach presented in Section 3. The sample period is from January 1970 to December 2015. CRASH and MCRASH values are in percent.

Table 4: Portfolio Sorts

Panel A: Univariate Portfolio Sorts

Portfolio	MCRASH MKT SMB	MCRASH MKT HML	MCRASH MKT UMD	ES	MLTD MKT UMD
1 Low	0.714%	0.599%	0.477%	0.683%	0.539%
2	0.715%	0.778%	0.704%	0.706%	0.684%
3	0.784%	0.767%	0.757%	0.812%	0.878%
4	0.754%	0.814%	0.801%	0.742%	0.802%
5 High	0.739%	0.752%	0.995%	0.766%	0.811%
(5) – (1)	0.025%	0.153%	0.518%***	0.083%	0.272%
	(0.11)	(0.84)	(2.64)	(0.40)	(1.60)

Panel B: Bivariate Portfolio Sorts

Measure	Average High - Low Spread of $\text{MCRASH}^{\text{MKT, UMD}}$
Controlling for $\text{CRASH}^{\text{MKT}}$	0.23%*
	(1.77)
Controlling for $\text{CRASH}^{\text{SMB}}$	0.22%**
	(1.97)
Controlling for $\text{CRASH}^{\text{HML}}$	0.21%*
	(1.89)
Controlling for $\text{CRASH}^{\text{UMD}}$	0.23%**
	(2.15)
Controlling for $\text{MCRASH}^{\text{MKT, SMB}}$	0.27%**
	(2.47)
Controlling for $\text{MCRASH}^{\text{MKT, HML}}$	0.23%**
	(2.01)
Controlling for $\text{MCRASH}^{\text{SMB, HML}}$	0.22%*
	(1.90)
Controlling for $\text{MCRASH}^{\text{SMB, UMD}}$	0.21%*
	(1.86)
Controlling for $\text{MCRASH}^{\text{HML, UMD}}$	0.22%*
	(1.83)

Panel A of this table reports the results of univariate portfolio sorts based on $\text{MCRASH}^{\text{MKT, SMB}}$, $\text{MCRASH}^{\text{MKT, HML}}$, $\text{MCRASH}^{\text{MKT, UMD}}$, ES, and $\text{MLTD}^{\text{MKT, UMD}}$. Each month t, we rank our test assets into quintiles (1-5) based on their estimated crash risk coefficients and form equal-weighted portfolios that we hold over the following month $t+1$. We report average monthly excess returns over the T-Bill. The rows labelled '(5) – (1)' report differences between the returns of portfolio 5 and portfolio 1 with corresponding T-statistics. Panel B reports the results of bivariate portfolio sorts. First, we form two portfolios sorted on a specific CRASH- or MCRASH measure. Then, within each of those portfolios, we sort test assets into two equal-weighted portfolios based on $\text{MCRASH}^{\text{MKT, UMD}}$. We only report the average high minus low spread of the $\text{MCRASH}^{\text{MKT, UMD}}$ portfolios controlling for a specific CRASH- or MCRASH measure. The sample period is from January 1975 to December 2015. t-statistics are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 5: Industry Characteristics and Industry Betas Associated with $MCRASH^{MKT,\ UMD}$

Panel A: Industry Characteristics

Portfolio	Size	Book-To-Market	Past Return	Volatility	Coskewness	Downside Beta
1 Low	6.802	0.668	4.380%	6.167%	-0.244	0.846
2	6.661	0.632	7.656%	5.999%	-0.259	0.909
3	6.587	0.610	8.484%	6.164%	-0.275	0.967
4	6.568	0.572	11.748%	6.363%	-0.278	1.022
5 High	6.580	0.530	14.568%	7.227%	-0.263	1.113
(5) – (1)	-0.222^{***}	-0.138^{***}	$10.188\%^{***}$	$1.060\%^{***}$	-0.019^{*}	0.267^{***}
	(-3.28)	(-6.80)	(3.29)	(6.14)	(-1.71)	(7.76)

Panel B: Industry Betas

Portfolio	β_{MKT}	β_{SMB}	β_{HML}	β_{UMD}	β_{RMW}	β_{CMA}	β_{STR}	β_{LTR}	β_{PSL}	β_{BAB}	β_{TR}	β_{QMJ}
1 Low	0.864	0.582	-0.276	-0.125	-0.474	-0.552	0.578	0.163	-0.055	0.102	0.205	-1.062
2	0.973	0.678	-0.354	-0.076	-0.357	-0.655	0.604	0.161	-0.045	0.117	0.319	-1.015
3	1.044	0.745	-0.397	-0.041	-0.384	-0.717	0.608	0.135	-0.039	0.113	0.367	-1.076
4	1.106	0.799	-0.446	0.038	-0.403	-0.805	0.602	0.120	-0.027	0.099	0.397	-1.128
5 High	1.246	0.934	-0.570	0.085	-0.426	-1.058	0.618	0.045	0.022	0.072	0.461	-1.300
(5) – (1)	0.382^{***}	0.352^{***}	-0.294^{***}	0.210^{***}	0.048	-0.506^{***}	0.040	-0.118^{*}	0.077^{*}	-0.030	0.256^{***}	-0.238^{***}
	(12.16)	(6.66)	(-5.10)	(4.57)	(0.61)	(-7.32)	(1.17)	(-1.88)	(1.95)	(-0.67)	(9.28)	(-3.36)

This table characterizes the equal-weighted quintile portfolios sorted on $MCRASH^{MKT,\ UMD}$. In Panel A, we report equal-weighted industry characteristics for each of the quintile portfolios. In particular, we report an industry's average size, book-to-market ratio, past yearly return, volatility, coskewness, and downside beta. In Panel B, we display average linear industry betas with regard to the Carhart (1997) market (MKT), size (SMB), book-to-market (HML), and momentum (UMD) factors, the Fama and French (2015) investment (RMW) and profitability (CMA) factors, the Fama and French short-term and long-term reversal factors, the Pástor and Stambaugh (2003) traded liquidity risk (PSL) factor, the Frazzini and Pedersen (2014) betting-against-beta factor, the Kelly and Jiang (2014) tail risk (TR) factor, the Asness et al. (2017) Quality-Minus-Junk (QMJ) factor. All industry characteristics and industry betas are defined in the Appendix. The sample period is from January 1975 to December 2015. t-statistics are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 6: Fama and MacBeth (1973) Regressions: Controlling for Industry Characteristics

	(1) future return 1-month	(2) future return 1-month	(3) future return 1-month	(4) future return 1-month	(5) future return 1-month	(6) future return 1-month	(7) future return 3-months	(8) future return 6-months
MCRASH$^{\text{MKT, UMD}}$	0.217***	0.206***	0.232***	0.243***	0.212***	0.189***	0.379**	0.820***
	(2.94)	(2.81)	(3.25)	(3.46)	(3.24)	(2.85)	(2.15)	(2.62)
Size		0.0000548	0.000536	0.000286	0.000102	0.000252	0.000332	0.000800
		(0.08)	(0.88)	(0.49)	(0.18)	(0.46)	(0.22)	(0.27)
Book-To-Market			0.00509**	0.00670***	0.00700***	0.00654***	0.00872**	0.0117***
			(2.41)	(3.15)	(3.44)	(3.20)	(2.21)	(3.09)
Past Return				0.0322**	0.0301**	0.0217*	0.0466**	0.0833**
				(2.33)	(2.29)	(1.67)	(2.27)	(2.06)
Volatility					0.0486	0.0366	0.0662	0.120
					(1.04)	(0.60)	(0.41)	(0.40)
Coskewness					-0.000323	-0.00101	-0.00839	-0.0196
					(-0.12)	(-0.27)	(-0.83)	(-1.11)
Downside Beta						0.000590	-0.00406	-0.00753
						(0.20)	(-0.55)	(-0.59)
Constant	0.00229	0.000669	-0.00630	-0.00455	-0.00390	-0.00294	0.00531	0.0152
	(1.03)	(0.11)	(-1.20)	(-0.91)	(-0.67)	(-0.47)	(0.32)	(0.47)
N	24059	24059	24059	24059	24059	24059	23961	23814
R^2	0.102	0.164	0.197	0.248	0.330	0.368	0.373	0.384

This table presents the results of multivariate Fama and MacBeth (1973) regressions of future excess returns over the risk-free rate on MCRASH$^{\text{MKT, UMD}}$ and different industry characteristics. As industry characteristics, we use an industry's average size, book-to-market ratio, past yearly return, volatility, coskewness, and downside beta. All characteristics are defined in the Appendix. In regressions (1) - (6), we use the one-month ahead future excess return as the dependent variable. Regressions (7) - (8) apply three-months ahead and and six-months ahead future excess returns as dependent variables, respectively. The sample period is from January 1975 to December 2015. t-statistics are computed using Newey and West (1987) standard errors with 4 monthly lags in the specifications (1) - (6). In the regressions (7) and (8) with cumulative returns, we use 6 and 9 lags, respectively. standard errors ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 7: Fama and MacBeth (1973) Regressions: Controlling for Industry Betas

	(1) future return 1-month	(2) future return 1-month	(3) future return 1-month	(4) future return 1-month	(5) future return 1-month	(6) future return 1-month	(7) future return 1-month	(8) future return 1-month	(9) future return 3-months	(10) future return 6-months
$\text{MCRASH}^{\text{MKT, UMD}}$		0.148^{**}	0.155^{**}	0.170^{**}	0.148^{**}	0.132^{*}	0.124^{*}	0.145^{**}	0.529^{***}	1.325^{***}
		(2.11)	(2.19)	(2.48)	(2.18)	(1.93)	(1.78)	(2.14)	(2.86)	(3.78)
β_{MKT}	0.00179	-0.000104	-0.000887	0.00269	0.00109	0.00147	0.00109	0.00173	-0.00710	-0.0199
	(0.59)	(-0.03)	(-0.27)	(0.79)	(0.35)	(0.45)	(0.32)	(0.47)	(-0.79)	(-1.14)
β_{SMB}	0.00158	0.00225	0.00230	0.00345^{*}	0.00242	0.00336^{*}	0.00206	0.00454^{**}	0.00597	0.0110
	(1.00)	(1.37)	(1.12)	(1.92)	(1.44)	(1.80)	(1.16)	(2.26)	(1.33)	(1.28)
β_{HML}	0.00419^{**}	0.00438^{***}	0.00521^{**}	0.00320^{*}	0.00493^{**}	0.00646^{***}	0.00496^{***}	0.00605^{***}	0.00906^{**}	0.0165^{*}
	(2.56)	(2.74)	(2.56)	(1.71)	(2.56)	(3.33)	(3.06)	(3.35)	(1.97)	(1.75)
β_{UMD}	0.00602^{**}	0.00590^{**}			0.00483^{**}	0.00594^{**}	0.00737^{***}	0.00413	0.0100	0.0186
	(2.47)	(2.41)			(2.00)	(2.30)	(2.97)	(1.60)	(1.53)	(1.52)
β_{RMW}			0.00136							
			(0.68)							
β_{CMA}			-0.00187							
			(-0.93)							
β_{STR}				-0.00494^{**}						
				(-2.42)						
β_{LTR}				-0.000226						
				(-0.10)						
β_{PSL}					-0.00218					
					(-0.98)					
β_{BAB}						-0.00168				
						(-0.63)				
β_{TR}							-0.000139			
							(-0.09)			
β_{QMJ}								0.00179		
								(0.79)		
Constant	0.00406^{**}	0.00230	0.00267	0.00187	0.00210	0.00223	0.00228	0.00203	0.00752	0.0152
	(2.16)	(1.15)	(1.33)	(0.90)	(1.08)	(1.10)	(1.13)	(1.06)	(1.31)	(1.41)
N	24059	24059	24059	24059	24059	24059	22785	24059	23961	23863
R^2	0.268	0.310	0.344	0.341	0.347	0.340	0.335	0.345	0.311	0.336

Table 7: Continued

This table presents the results of multivariate Fama and MacBeth (1973) regressions of future excess returns over the risk-free rate on $\text{MCRASH}^{\text{MKT, UMD}}$ and different industry factor betas. As industry betas, we use the sensitivities to the Carhart (1997) market (MKT), size (SMB), book-to-market (HML), and momentum (UMD) factors, the Fama and French (2015) investment (RMW) and profitability (CMA) factors, the Fama and French short-term and long-term reversal factors, the Pástor and Stambaugh (2003) traded liquidity risk (PSL) factor, the Frazzini and Pedersen (2014) betting-against-beta factor (BAB), the Kelly and Jiang (2014) tail risk (TR) factor, and the Asness et al. (2017) Quality-Minus-Junk (QMJ) factor. All betas are defined in the Appendix. In regressions (1) - (8), we use the one-month ahead future excess return as the dependent variable. Regressions (9) and (10) apply three- and six-months ahead future excess returns as dependent variables. The sample period is from 1975 to 2015. *t*-statistics are computed using Newey and West (1987) standard errors with 4 monthly lags in the specifications (1) - (8). In the regressions (9) and (10) with cumulative returns, we use 6 and 9 lags, respectively. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 8: Out-of-Sample Tests

Panel A: Fama and MacBeth (1973) Regressions: Controlling for Industry Characteristics

	(1) future return 1-month	(2) future return 1-month	(3) future return 1-month	(4) future return 1-month	(5) future return 1-month	(6) future return 1-month	(7) future return 3-months	(8) future return 6-months
MCRASH$^{\text{MKT, UMD}}$	0.274***	0.277***	0.295***	0.280***	0.243***	0.199**	0.458*	1.088***
	(2.81)	(2.83)	(3.03)	(3.01)	(2.72)	(2.23)	(1.93)	(2.43)
Controls	-	Size	Size	Size	Size	Size	Size	Size
			BTM	BTM	BTM	BTM	BTM	BTM
				Past Ret	Past Ret	Past Ret	Past Ret	Past Ret
					Vola	Vola	Vola	Vola
					Coskew	Coskew	Coskew	Coskew
						Down Beta	Down Beta	Down Beta
N	13475	13475	13475	13475	13475	13475	13377	13230
R^2	0.094	0.150	0.183	0.239	0.321	0.361	0.357	0.368

Panel B: Fama and MacBeth (1973) Regressions: Controlling for Industry Betas

	(1) future return 1-month	(2) future return 1-month	(3) future return 1-month	(4) future return 1-month	(5) future return 1-month	(6) future return 1-month	(7) future return 1-month	(8) future return 3-months	(9) future return 6-months
MCRASH$^{\text{MKT, UMD}}$	0.162**	0.160**	0.190**	0.137*	0.146*	0.137*	0.127*	0.409*	0.991**
	(2.01)	(2.02)	(2.38)	(1.89)	(1.86)	(1.75)	(1.68)	(1.85)	(2.25)
Controls	β_{MKT}	β_{MKT}	β_{MKT}	β_{MKT}	β_{MKT}	β_{MKT}	β_{MKT}	β_{MKT}	β_{MKT}
	β_{SMB}	β_{SMB}	β_{SMB}	β_{SMB}	β_{SMB}	β_{SMB}	β_{SMB}	β_{SMB}	β_{SMB}
	β_{HML}	β_{HML}	β_{HML}	β_{HML}	β_{HML}	β_{HML}	β_{HML}	β_{HML}	β_{HML}
	β_{UMD}			β_{UMD}	β_{UMD}	β_{UMD}	β_{UMD}	β_{UMD}	β_{UMD}
		β_{RMW}	β_{STR}	β_{PSL}	β_{BAB}	β_{TR}	β_{QMJ}		
		β_{CMA}	β_{LTR}						
N	13475	13475	13475	13475	13475	12740	13475	13377	13279
R^2	0.304	0.338	0.333	0.340	0.335	0.326	0.341	0.294	0.319

Table 8: Continued

This table reports the results of out-of-sample tests based on multivariate Fama and MacBeth (1973) regressions. We estimate the parameters of the dynamic copula models used to compute $\text{MCRASH}^{\text{MKT, UMD}}$ over the first half of our sample period (i.e., from 1970 to 1992). Asset pricing tests are then subsequently run over the second half of our sample period (i.e., from 1993 to 2015). Panel A presents the results of multivariate Fama and MacBeth (1973) regressions of future excess returns over the risk-free rate on $\text{MCRASH}^{\text{MKT, UMD}}$ and different industry characteristics as in specifications (1) - (8) in Table 6. Panel B presents the results of multivariate Fama and MacBeth (1973) regressions of future excess returns over the risk-free rate on $\text{MCRASH}^{\text{MKT, UMD}}$ and different industry betas as in specifications (2) - (9) in Table 7. Control variables are included in the respective regressions, but coefficient estimates are suppressed. t-statistics are again computed using Newey and West (1987) standard errors with 4 lags for the regressions with 1-month returns and with 6 lags (9 lags) for the specifications with 3-months (6-months) returns. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 9: Additional Results

Panel A: Time Splits

	(1) Baseline	(2) 1975 to 1994	(3) 1995 to 2015	(4) MKT < 0%	(5) MKT > 0%	(6) UMD < 0%	(7) UMD > 0%	(8) MKT < 0% and UMD < 0%	(9) MKT > 0% and UMD > 0%
$MCRASH^{MKT, UMD}$	0.148^{**}	0.231^{**}	0.141^{*}	-0.105	0.317^{***}	-0.400^{***}	0.445^{***}	-0.692^{***}	0.587^{***}
	(2.19)	(2.05)	(1.82)	(-0.95)	(3.77)	(-4.02)	(5.17)	(-4.44)	(5.51)
N	24059	11760	12299	9604	14455	8771	14896	3577	8918
R^2	0.310	0.307	0.272	0.323	0.301	0.321	0.304	0.336	0.296

Panel B: Robustness

	(1) Estimation Log Returns	(2) Estimation Daily Data	(3) Marginal Distribution Empirical	(4) Marginal Distribution GJR	(5) Copula Student-t	(6) MLTD Cut-Off 5%	(7) Equal-Weighted Industry Returns	(8) Out-Of-Sample Daily Rolling Window
$MCRASH^{MKT, UMD}$	0.221^{***}	0.269^{*}	0.155^{**}	0.125^{**}	0.128^{*}	0.104^{**}	0.235^{***}	0.275^{**}
	(3.31)	(1.67)	(2.01)	(2.03)	(1.85)	(2.00)	(2.66)	(2.35)
N	24059	24059	24059	24059	24059	24059	24059	24059
R^2	0.305	0.309	0.303	0.309	0.305	0.308	0.271	0.301

Panel A of this table reports the result of the impact of $MCRASH^{MKT, UMD}$ on future returns during different subsamples. We investigate the impact of $MCRASH^{MKT, UMD}$ on future returns during the sample periods from 1975 to 1994 and 1995 to 2015, during periods of positive (negative) market returns, during periods of positive (negative) realizations of the UMD momentum risk factor, as well as during periods when both the market and the momentum risk factors are positive (negative). Panel B of this table reports the results of various robustness checks. We report the robustness of the relationship between $MCRASH^{MKT, UMD}$ and future returns when we use log returns instead of discrete returns for model estimation, change the frequency in the estimation of $MCRASH^{MKT, UMD}$ from weekly to daily, use the empirical distribution and GJR-GARCH models for the univariate margins, apply a Student-t copula when specifying the trivariate dependence structure between the individual stock return as well as MKT and UMD, estimate the crash risk coefficients using a 5% cut-off instead of using a 10% cut-off, apply equal-weighted instead of value-weighted industry returns in the empirical analysis, and when we only use a rolling window estimation with 1000 days and daily data to estimate the econometric models. In all tests, we apply the identical regression specification as in model (2) of Table 7. The sample period is from January 1975 to December 2015. t-statistics are computed using Newey and West (1987) standard errors with 4 monthly lags. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.

Table 10: Out-of-Sample Test with Individual Stocks

Panel A: Fama and MacBeth (1973) Regressions: Controlling for Stock Characteristics

	(1) future return 1-month	(2) future return 1-month	(3) future return 1-month	(4) future return 1-month	(5) future return 1-month	(6) future return 1-month	(7) future return 3-months	(8) future return 6-months
$MCRASH^{MKT, UMD}$	0.559^{**}	0.531^{**}	0.501^{**}	0.496^{*}	0.470^{*}	0.456^{*}	1.368^{**}	3.290^{***}
	(2.09)	(2.13)	(2.28)	(1.86)	(1.81)	(1.72)	(2.10)	(2.82)
Controls	-	Size	Size BTM	Size BTM Past Ret	Size BTM Past Ret Vola Coskew	Size BTM Past Ret Vola Coskew Down Beta	Size BTM Past Ret Vola Coskew Down Beta	Size BTM Past Ret Vola Coskew Down Beta
N	23156	23156	23156	23156	22174	22146	21829	21514
R^2	0.066	0.089	0.098	0.106	0.185	0.211	0.217	0.222

Panel B: Fama and MacBeth (1973) Regressions: Controlling for Stock Betas

	(1) future return 1-month	(2) future return 1-month	(3) future return 1-month	(4) future return 1-month	(5) future return 1-month	(6) future return 1-month	(7) future return 1-month	(8) future return 3-months	(9) future return 6-months
$MCRASH^{MKT, UMD}$	0.502^{**}	0.435^{*}	0.573^{**}	0.550^{**}	0.478^{*}	0.598^{**}	0.502^{**}	1.426^{**}	3.313^{***}
	(2.06)	(1.71)	(2.34)	(2.24)	(1.93)	(2.36)	(1.99)	(2.36)	(3.42)
Controls	β_{MKT} β_{SMB} β_{HML} β_{UMD}	β_{MKT} β_{SMB} β_{HML} β_{RMW} β_{CMA}	β_{MKT} β_{SMB} β_{HML} β_{STR} β_{LTR}	β_{MKT} β_{SMB} β_{HML} β_{UMD} β_{PSL}	β_{MKT} β_{SMB} β_{HML} β_{UMD} β_{BAB}	β_{MKT} β_{SMB} β_{HML} β_{UMD} β_{TR}	β_{MKT} β_{SMB} β_{HML} β_{UMD} β_{QMJ}	β_{MKT} β_{SMB} β_{HML} β_{UMD}	β_{MKT} β_{SMB} β_{HML} β_{UMD}
N	22174	22174	22174	22174	22174	20477	22174	21856	21665
R^2	0.221	0.248	0.248	0.244	0.248	0.245	0.250	0.225	0.256

Table 10: Continued

This table reports the results of out-of-sample tests based on multivariate Fama and MacBeth (1973) regressions for individual stocks. We employ the sample of all stocks that have been included in the S&P 100 index until 2015. We restrict our sample to months after a stock has been included in the index to be not affected by a look-ahead selection bias. We estimate the parameters of the dynamic copula models using a rolling window approach with 1250 days and a dynamic (DCC) student t-copula. Panel A presents the results of multivariate Fama and MacBeth (1973) regressions of future excess returns over the risk-free rate on $\text{MCRASH}^{\text{MKT, UMD}}$ and different stock characteristics as in specifications (1) - (8) in Table 6. Panel B presents the results of multivariate Fama and MacBeth (1973) regressions of future excess returns over the risk-free rate on $\text{MCRASH}^{\text{MKT, UMD}}$ and different stock betas as in specifications (2) - (10) in Table 7. t-statistics are again computed using Newey and West (1987) standard errors with 4 lags for the regressions with 1-month returns. For the regressions with 3-months and 6-months returns, we use 6 and 9 lags, respectively. ***, **, and * indicate significance at the one, five, and ten percent levels, respectively.