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RISK-NEUTRAL MOMENTUM AND MARKET FEAR

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Abstract

This study models a link between ex-ante autocorrelation in expected returns and risk-neutral momentum, enabling a straightforward interpretation of market sentiment. Correspondingly, concepts of fractal Brownian motion are applied to option implied volatility term structures. Based on an empirical investigation of daily SP500 and Euro Stoxx 50 data (2006–2018), we find that the expected return momentum varies over time, as fear spreads much faster than investor confidence can be regained. Thus, we conclude that risk-neutral momentum is a novel perspective for further research in the fields of risk management, asset allocation, and behavioral finance.

Keywords: momentum, sentiment, implied volatility, long-term memory, fractal Brownian motion, market fear
JEL: C22, G01, G12

1. Introduction

If markets are efficient (Fama (1970)), all available information is reflected not only in the current stock price but also in the corresponding option prices. This finding is fundamental in the risk-neutral literature discussing option implied moments. While stocks only have one dimension of price at time t, options have the advantage of forming a three-dimensional surface with different strikes and times-to-maturity τ, reflecting investor expectations that are actually traded. By rewriting option prices as Black and Scholes (1973) (hereafter B/S) volatilities, the surface is proven to embed information regarding the skewness and kurtosis of the risk-neutral return distribution[1]. However, to the best of our knowledge, no study has discussed risk-neutral momentum. We consider risk-neutral momentum to be a combination of the expected return and ex-ante autocorrelation. Our idea builds on the concepts of fractal Brownian motion and long-term memory (Hurst (1956); Mandelbrot and Van Ness (1968)) such that corresponding beliefs are observable from the option implied volatility surface (e.g., Hu and Øksendal (2003)), with the implied Hurst exponent H as a crucial determinant of risk-neutral momentum. Within our model, $H > 0.5$ implies the following: (i) positive ex-ante autocorrelation and (ii) an upward-sloping implied volatility curve over times-to-maturity, which causes expected returns to increase in τ. The opposite holds for $H < 0.5$. In the special case of $H = 0.5$, markets face the expectation of no momentum, thus fulfilling the assumptions of standard Brownian motion; however, based on our empirical observations of U.S. and European markets, this is rarely observed in reality. Furthermore, we find patterns suggesting that implied momentum is related to market fear, allowing us to estimate the current levels of investor confidence without having to rely on backward-looking data. This finding may be useful for investment timing, asset allocation, or risk management. In its interpretation as market sentiment, it stems from similar dynamics as the CBOE VIX, but comes with a directional feature allowing distinction between investor optimism and pessimism.

2. Related Literature

Two streams of the finance literature are relevant to this work. The first concerns risk-neutral measurement of investor expectations, and the second concerns autocorrelation within a stochastic process. The

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field of risk-neutral distributional moments, such as ex-ante skewness or kurtosis, has enjoyed extensive academic discussion both from theoretical and empirical perspectives[1]. In contrast, research investigating the risk-neutral autocorrelation property of an expected price development is scarce. The concept we use is based on fractal Brownian motion (Mandelbrot and Van Ness (1968)), in which Hurst (1956)'s exponent plays a crucial role. Peters (1991, 1994) and Granger and Ding (1995) were among the first scholars to apply fractal analysis to financial data, concluding that the U.S. market indeed shows characteristics of long-term memory. Further support for this ex-post approach was provided by Granger and Hyung (2004); Alvarez-Ramirez et al. (2008), and Caporale et al. (2018). Differing from these studies, research such as that conducted by Chow et al. (1996); Caporale and Gil-Alana (2004) and Lu and Perron (2010) resulted in opposing observations, rejecting long-term memory in stock returns. In our opinion, the chief problem with ex-post investigation of long-term memory is that several different estimation methods are used for fractal analysis, including rescaled range (R/S; Hurst (1956)), modified R/S (Lo (1991)), wavelet (Veitch and Abry (1999)), rescaled variance (Giraitis et al. (2003)), and so on. Rea et al. (2009) provide an empirical comparison of 12 such different ex-post methods. Overcoming the backward-looking bias, Hu and Øksendal (2003), extended by Elliott and Van Der Hoek (2003), show how fractal Brownian motion can be substituted into Black and Scholes (1973) option pricing such that the Hurst exponent can be measured on an ex-ante basis. A detailed description of this estimation method is provided by Li and Chen (2014), who conduct an empirical investigation using SP500 index data. Another empirical work concerning option implied measurement of the Hurst exponent was conducted by Flint and Mare (2016). Funahashi and Kijima (2017) further contributes to this fractal literature by modeling the relationships between the short and long ends of the term structure of volatility to estimate the implied Hurst exponent.

3. The Idea of Ex Ante Momentum

Our starting point is the Black and Scholes (1973) option implied volatility surface. If investor expectations match the assumptions of normally distributed returns and zero autocorrelation, the implied volatility surface actually traded would be flat in all dimensions; however, empirically, this is rarely the case (cp. Figs. 1 and 2). Risk-neutral measures of the shape of ex-ante distribution typically fix the time to maturity and decompose the volatility surface over the degrees of moneyness. Similarly, we believe that for a given strike level, the implied volatility curve over maturities (i.e., the term structure of volatility) provides information regarding the expected momentum.

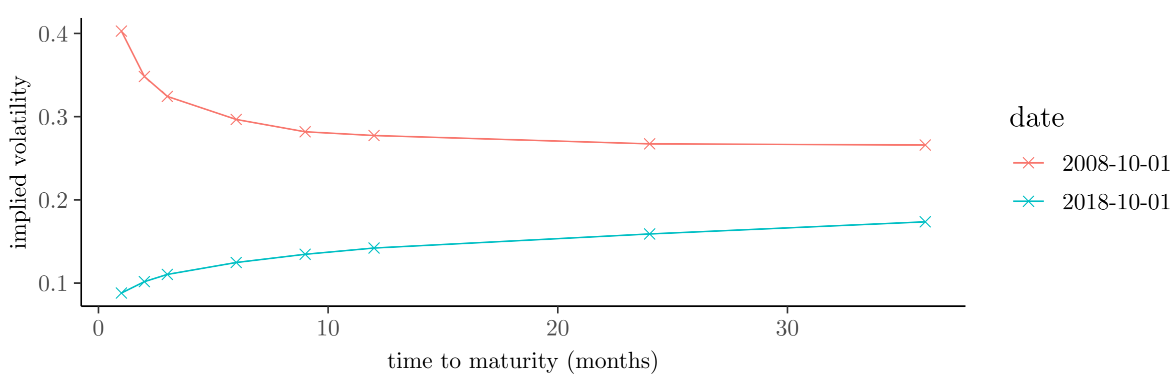

Figure 1
SP500's actually traded B/S at-the-money implied volatility curve over τ on two different dates. In theory, this curve should be flat. Empirically, in growing markets, this curve is typically upwards sloping (2018-10-01) and vice versa during pessimistic periods (2008-10-01).

[1] Examples include Jarrow and Rudd (1982); Backus et al. (2004); Bali and Hovakimian (2009); Xing (2010), Bali et al. (2019), etc.

Basic Relation

Let us begin with a simplified logic experiment involving two portfolios, that is, A and B.

Assume that we are at time t_0 and that the (annualized) expected risk $\mathbb{E}[\sigma]$ of A is greater than that of B; thus, no-arbitrage requires the expected return $\mathbb{E}[r]$ of A to be above that of B: $\mathbb{E}[\sigma_A] > \mathbb{E}[\sigma_B] \implies \mathbb{E}[r_A] > \mathbb{E}[r_B]$ – a very basic law. We suggest that each portfolio is simply a replicated pay-off of the same underlying built from a 1:1 relationship of at-the-money[2] Put and Call options having time-to-maturities of $\tau_A^{Call} = \tau_A^{Put}$ and $\tau_B^{Call} = \tau_B^{Put}$ but $\tau_A > \tau_B$. We derive two portfolios that are identical in all their fundamental characteristics but only distinguished by the time horizon τ. Under Black and Scholes (1973) assumptions, A and B should be priced at the same risk, but based on market data, where $\mathbb{E}[\sigma] = \sigma_{BS}$ is directly derived from the implied volatility surface, we observe that $\mathbb{E}[\sigma_A] > \mathbb{E}[\sigma_B]$ (cp. Fig. 1 '2018-10-01'). Thus, to re-establish the no-arbitrage principles, $\mathbb{E}[r(\tau_A)] > \mathbb{E}[r(\tau_B)]$ has to hold. Therefore, the underlying $\mathbb{E}[r]$ is dependent on the future time horizon τ and is not constant as assumed in classical theory. Since $\mathbb{E}[r]$ is a function of τ, at t_0, we can compare how the expected return is believed to develop for the future τ ahead. In the described case of $\frac{\partial \mathbb{E}[\sigma]}{\partial \tau} > 0$, investors expect returns to increase in the future or, following the fractal model below, believe in positive momentum. Thus, in contrast, $\frac{\partial \mathbb{E}[\sigma]}{\partial \tau} < 0$, would imply a negative expected momentum in $\mathbb{E}[r]$. Given that the observations are made with respect to the traded implied volatility surface, our concept is considered under risk-neutral beliefs. Simplified, the coherences evolve as follows:

$$\frac{\partial \mathbb{E}[\sigma]}{\partial \tau} \begin{cases} > 0 \implies \frac{\partial \mathbb{E}[r]}{\partial \tau} > 0 \implies \text{positive R.N. momentum} \\ = 0 \implies \frac{\partial \mathbb{E}[r]}{\partial \tau} = 0 \implies \text{no R.N. momentum} \\ < 0 \implies \frac{\partial \mathbb{E}[r]}{\partial \tau} < 0 \implies \text{negative R.N. momentum} \end{cases} \tag{1}$$

These basic relationships sketch the first idea that emerges from the fractal model, where ex-ante autocorrelation is directly linked to risk-neutral momentum.

Ex Ante Long-Term Memory

We define stock momentum as the autocorrelation in returns. Therefore, to quantify risk-neutral momentum, the assumptions of zero autocorrelation must be released within the stochastic process used. Here, we introduce fractal Brownian motion and long-term memory. While the fractal form is similar to classical Brownian motion, it has the advantage of being a self-similar process with allowance for correlation among increments[3]. Under this framework, the Hurst exponent H is a crucial determinant of the process autocorrelation. $H \in (0, 1)$ with 0.5 as an important level. If $H > 0.5$, returns will face positive autocorrelation; $H < 0.5$ represents negative autocorrelation, while an H equaling exactly 0.5 will again result in a standard Brownian motion.

Hu and Øksendal (2003) (and further extended by Elliott and Van Der Hoek (2003)) substitute the standard Brownian motion by its fractal form within the Black and Scholes (1973) option pricing formula such that the implied B/S volatility surface over maturity $\sigma_{BS}(\tau)$ can be divided into its (non-autocorrelated) fractal volatility σ_f and the Hurst exponent H, which is expressed as:

$$\sigma_{BS}(\tau) = \sigma_f \tau^{H - \frac{1}{2}}. \tag{2}$$

Using logarithms, Eq. 2 can be rewritten in a linear form such that

$$\underbrace{\ln(\sigma_{BS}(\tau))}_{\hat{y}} = \underbrace{\ln(\sigma_f)}_{\hat{\alpha}} + \underbrace{(H - 0.5)}_{\hat{\beta}} \cdot \ln(\tau). \tag{3}$$

[2]Strike is set to at-the-money such that the skewness and kurtosis effects are captured within the implied volatilities.

[3]Details are provided by Hurst (1956); Mandelbrot and Van Ness (1968). In fact, fractal theory is frequently discussed in disciplines of physics and mathematics, but is not yet that prominent in the field of finance.

Since $\sigma_{BS}(\tau)$ and τ are directly observable, the implied Hurst exponent H is to be estimated based on the coefficient $\hat{\beta}$ from fitting Eq. 3 by OLS regression.

To draw the connection between ex-ante autocorrelation and risk-neutral momentum, we refer to the stochastic return definition for which the functional form is the same under both the classical and fractal notations (see Eq. 4). Therefore, the underlying's expected return from t_0 to T is defined through the parameters of constant drift μ and volatility σ as

$$\mathbb{E}\left[\ln\left(\frac{S_T}{S_0}\right)\right] = \underbrace{\left(\mu + \frac{1}{2}\sigma^2\right)}_{\bar{r}}\tau. \tag{4}$$

Obviously, this notation implies that the expected rate of return $\bar{r}$ is constant and independent of the horizon τ: $\frac{\partial \bar{r}}{\partial \tau} = 0$. In the presence of autocorrelation beliefs, implementing fractality from Eq. 2 into the expected return notation of Eq. 4 releases this assumption:

$$\bar{r} = \mu + \frac{1}{2}\sigma_f^2\tau^{2H-1} \implies \frac{\partial \bar{r}}{\partial \tau} = (H - 0.5)\,\sigma_f^2\tau^{2(H-1)} \tag{5}$$

such that $\bar{r} = f(\tau)$. Regarding the first order derivative $\frac{\partial \bar{r}}{\partial \tau}$, our autocorrelation indicator H determines whether $\bar{r}$ is expected to remain, increase, or decrease in τ. Thus, since $\text{sgn}(H - 0.5) = \text{sgn}(\frac{\partial \bar{r}}{\partial \tau})$ [4], ex-ante autocorrelation maps into risk-neutral momentum analogously as demonstrated in Eq. 1. Consequently, our ex-ante momentum model suggests how investors expect the rate of return to evolve in the future.

4. Empirical Observations

Our sample covers SP500 and Euro Stoxx 50 data ranging from January 2006 to October 2018; this set is divided into the following three time-periodic subsamples: pre-crisis (01-2006 to 05-2007), crisis (06-2007 to 05-2009), and post-crisis (06-2009 to 10-2018). All data are derived from Bloomberg L.P. We draw the B/S implied volatility surface using at-the-money options with time-to-maturities from 3 to 18 months. The implied Hurst exponent H is estimated by daily OLS regression following Eq. 3. The corresponding results are displayed in Table 1 and Fig. 2. Generally, the regressions' goodness of fit measured by R^2 is considered high, ranging between 94.5% and 99.7% for SP500 and between 75.4% and 95.6% for Europe[5]. The time pattern of risk-neutral momentum is found to indicate investor confidence. While H is mainly above 0.5 when markets are growing, the momentum clearly drops during times of uncertainty. In Fig. 2, we can observe such critical times; examples include the financial crisis and early 2018, when the markets faced a substantial loss. Moreover, a comparison of Europe and the U.S. reveals the impact of the Euro crisis. While the risk-neutral momentum of SP500 recovered soon after 2009, that of Euro Stoxx 50 implied that the Hurst exponent continued oscillating around the critical 0.5 level. Thus, H provides insights that uncertainty remained in Europe after the financial crisis; this finding may relate to the weak growth of the Euro Stoxx 50 compared to that of the SP500. Overall, it appears that downward movements in H occur much faster than upward trends. The skewness in the log changes of H is significantly negative at -1.07*** (U.S.) and -0.17*** (Europe)[6]. This result suggests that fear spreads in a market much more quickly than trust in growth can be regained. To control the statement of H referring to investor sentiment, the correlations between H and broadly used volatility indices are measured[7]. The Pearson correlation between SP500's H and the VIX is substantially negative at -0.84^{***}. For Europe, there is a similar converse relationship between H and VSTOXX at -0.81^{***}. Such an opposing pattern suggests that if market fear (represented by VIX and

[4] sgn denotes the sign function. In fact, $\text{sgn}(\frac{\partial \ln \sigma_{BS}}{\partial \tau}) = \text{sgn}(\frac{\partial \sigma_{BS}}{\partial \tau}), \forall \tau$. Therefore, $\text{sgn}(\frac{\partial \sigma_{BS}}{\partial \tau}) = \text{sgn}(H - 0.5) = \text{sgn}(\frac{\partial \bar{r}}{\partial \tau})$ and is, thus, equal to the direction of risk-neutral momentum.

[5] The 1^{st} and 3^{rd} quartile of 3188 regressions per market index. The high goodness of fit supports the applicability of the fractal pricing model.

[6] $\Delta H_t = ln(\frac{H_t}{H_{t-1}})$; skewness is tested according to D'Agostino (1970). *** denotes a p-value below 1%.

[7] A large body of evidence confirms that CBOE's volatility index (VIX) reflects market fear, for example, Whaley (2000); Baker and Wurgler (2007), Caporale et al. (2018), and so on. In the following, the VIX and the Euro Stoxx 50 Volatility index (VSTOXX) are considered market fear gauges for the U.S. and Europe, respectively.

VSTOXX) is high, risk-neutral momentum H is low and vice versa. Therefore, the interaction between the implied Hurst exponent and market fear confirms our argument. Regarding the measurement of market sentiment, the advantage of using the fractal approach instead of volatility indices is clearly illustrated in Fig. 2 as follows: under H, one can directly observe whether the market mood is optimistic ($H > 0.5$), neutral ($H = 0.5$), or pessimistic ($H < 0.5$). − volatility indices per se do not allow for such a clear-cut interpretation.

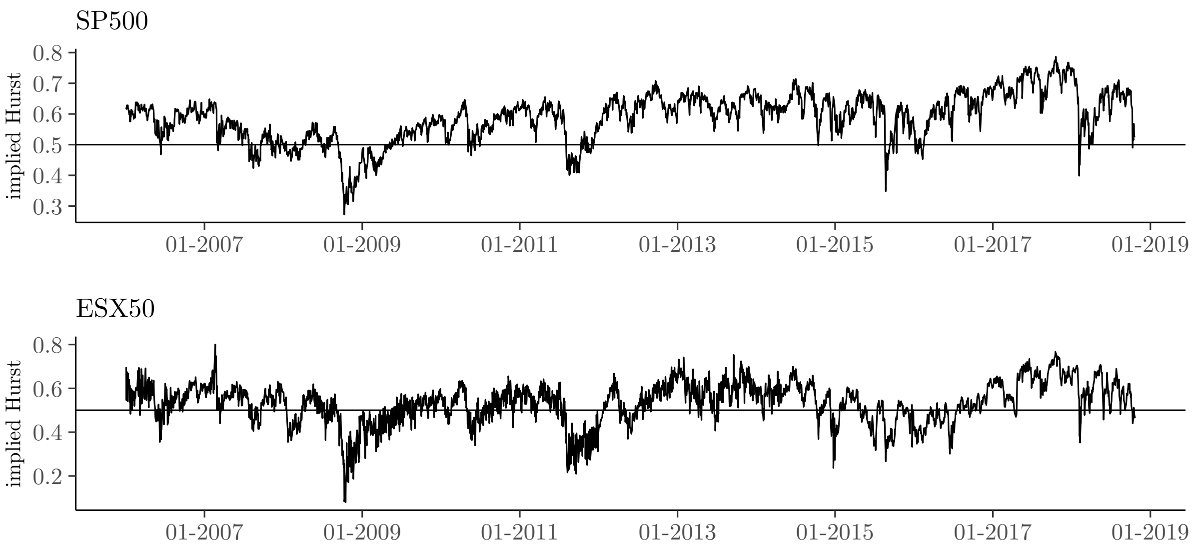

Figure 2
Daily risk-neutral momentum as implied by H, measured based on the SP500 and the Euro Stoxx 50 (ESX50). In times of market growth, investors trade on positive autocorrelation in expected returns with H above 0.5. During periods of nervousness, when the market is stagnating or in a crisis, H is primarily below 0.5, indicating an expected decline in returns. Generally, H's correlation between the U.S. and European markets is high at 0.76. Decreases in H are observed to occur faster than regrowth to higher levels. Generally, with a link to market fear, $H > 0.5$ suggests that investor sentiment is positive, and analogously, $H < 0.5$ reflects pessimistic times.

Table 1
Fractal analysis of SP500 and Euro Stoxx 50's implied volatility surface. Outside times of crisis, investors feel confident in trading on persistence in expected returns ($H > 0.5$). During the financial crisis, the volatility curve was downward sloping ($H < 0.5$) such that the risk-neutral momentum was negative. The risk-neutral momentum between the two markets was similar before and during the financial crisis. Following the crisis, when Europe additionally fell into the Euro crisis, we observe greater recovery in investor confidence in the U.S. market. σ_f denotes fractal volatility, which can be interpreted as the base level of non-autocorrelated volatility. The period is divided into pre-crisis (01-2006 to 05-2007), crisis (06-2007 to 05-2009), and post-crisis (06-2009 to 10-2018).

Panel A: SP500

	H		σ_f		return	
	mean	std. dev.	mean	std. dev.	mean	std. dev.
Pre-crisis	0.59	0.03	0.09	0.02	0.15	0.10
Crisis	0.48	0.06	0.34	0.25	-0.26	0.35
Post-crisis	0.61	0.07	0.11	0.07	0.12	0.15

Panel B: Euro Stoxx 50

	H		σ_f		return	
	mean	std. dev.	mean	std. dev.	mean	std. dev.
Pre-crisis	0.60	0.03	0.15	0.02	0.16	0.14
Crisis	0.48	0.06	0.28	0.11	-0.30	0.33
Post-crisis	0.54	0.09	0.20	0.06	0.03	0.20

From realized returns, it is already broadly verified that the no-autocorrelation assumption does not hold[8]. With our empirical investigation we find that this assumption does not even hold for return expectations since H is significantly different from 0.5. The fractal analysis of ex-ante data allows us to investigate current investor confidence, H is typically above 0.5 in times of smooth returns and low volatility, indicating positive expected autocorrelation. In times of nervousness, market expectations either revert to a random walk ($H = 0.5$) or lead to negative ex-ante momentum. Thus, substituting classical Brownian motion with its fractal form may cause improvements in tasks of equity and option pricing as the realistic component of momentum beliefs can be considered. Due to the clear interpretation and almost continuous data availability, we believe that information regarding the implied Hurst exponent embeds a high potential for investment timing and allocation, especially for VIX based strategies in which such a directional interpretability is not given.

5. Conclusion

Momentum is a known phenomenon in financial markets and is typically measured ex-post. To quantify momentum ex-ante, model assumptions of zero autocorrelation must be released, which can be achieved by introducing fractal Brownian motion. With fractal analysis, the option implied volatility term structure can be decomposed into its non-autocorrelated base level and Hurst exponent H. Technically, the Hurst exponents fulfill the following two functions: indicating the log slope of the volatility term structure and reflecting expected autocorrelation. Combining this ex-ante long-term memory with arbitrage free principles, we make the link to positive (negative) risk-neutral momentum as an expected increase (decline) in the rate of return. The empirical observations indicate that risk-neutral momentum highlights market fear as follows: during times of constant growth, the ex-ante momentum is positive, while during crises, momentum beliefs are negative. The correlations between H and volatility indices (as common market fear measures) are highly significant, but the Hurst exponent has the great advantage of a clear and directional interpretability in terms of investor sentiment. Through the feature of market mood, the advance measurement of actually traded momentum expectations may be useful for research in the field of behavioral finance, investment

[8]Empirical evidence goes back to Cowles and Jones (1937).

timing, or risk management. Thus, this work should contribute to the discourse related to market sentiment from a new perspective.

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