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Abstract

We study first- and second-order subjective expectations (beliefs) in strategic decision-making. We propose a method to elicit probabilistically both first- and second-order beliefs and apply the method to a Hide-and-Seek experiment. We study the relationship between choice and beliefs in terms of whether observed choice coincides with the optimal action given elicited beliefs. We study the relationship between first- and second-order beliefs under a coherence criterion. Weak coherence requires that if an event is assigned, according to first-order beliefs, a probability higher/lower/equal to the one assigned to another event, then the same holds according to second-order beliefs. Strong coherence requires the probability assigned according to first- and second-order beliefs to coincide. Evidence of heterogeneity across participants is reported. Verbal comments collected at the end of the experiment shed light on how subjects think and decide in a complex environment that is strategic, dynamic and populated by potentially heterogeneous individuals.

Keywords

Decision-making, beliefs, subjective expectations, experiments.

JEL Classification

D81, D83, D84, C92.

1 Introduction

Sherlock Holmes: 'My dear Mr Watson, you evidently did not realize my meaning when I said that this man may be taken as being quite on the same intellectual plane as myself. You do not imagine that if I were the pursuer I should allow myself to be baffled by so slight an obstacle.'
Mr Watson: 'What will he do?'
Sherlock Holmes: 'What I should do.'
Mr Watson: 'What would you do then?'

from 'The Final Problem' in 'The Memoirs of Sherlock Holmes' by Arthur Conan Doyle (1894)

Sherlock Holmes, pursued by his opponent, Moriarty, leaves London for Dover. The train stops at a station on the way, and he alights there rather than traveling on to Dover. He has seen Moriarty at the railway station, recognizes that he is very clever and expects that Moriarty will take a faster special train in order to catch him in Dover. [...] But what if Moriarty had been still more clever and had foreseen his actions accordingly? Then, obviously, he would have traveled to the intermediate station. Holmes, again, would have had to calculate that and he himself would have decided to go on to Dover. Whereupon, Moriarty would again have reacted differently.

from 'Economic Prediction' by Oskar Morgenstern (1928)[1]

In his first book, published in 1928, Oskar Morgenstern turns to the adventures of Sherlock Holmes for a vivid description of the 'chain of reciprocally conjectural reactions and counter-reactions' that arises in a strategic decision problem. Within the chain, subjective beliefs play a crucial role. Sherlock Holmes holds subjective beliefs about the behavior of his opponent Moriarty and chooses his best course of action based on his conjectures. Conjectures are reciprocal. Acknowledging that Moriarty (i) holds beliefs about his (i.e. Sherlock Holmes's) behavior and (ii) chooses his best course of action based on those beliefs, Sherlock Holmes holds in turn beliefs about Moriarty's beliefs. Restricting attention to the first two steps of the chain, we refer to Sherlock Holmes's conjectures about Moriarty's behavior as his first-order beliefs and to his conjectures about Moriarty's conjectures about his behavior as his second-order beliefs. Higher-order beliefs are likely to play a role in virtually all strategic situations. Examples include situations involving deception, social preferences, and more generally any environment with asymmetric information.

This paper studies first- and second-order subjective beliefs in a strategic decision-making experiment. Experiment participants play in pairs a simple Hide-and-Seek game, in which the hider needs to choose between two hiding places and the seeker needs to guess the hiding place chosen by the hider. While they play, their first- and second-order beliefs are elicited probabilistically. We have three main objectives. First, we assess the feasibility of eliciting both first- and second-order beliefs probabilistically. Given that decision makers are likely to feel some uncertainty about their own conjectures, being aware that they may or may not turn out to be correct, first- and second-order beliefs should be treated and elicited probabilistically. Second, we examine the relationship between observed choice and elicited first- and second-order beliefs. Finally, we investigate heterogeneity among decision-makers in terms of choice behavior and stated beliefs.

Probabilistic vs non-probabilistic belief elicitation. Throughout the paper, we distinguish probabilistic (or distributional) forecasts from non-probabilistic (or deterministic, or point) forecasts.

[1] Original full title 'Wirtschaftsprognose, Eine Untersuchung ihrer Voraussetzungen und Möglichkeiten', translated as 'Economic Prediction: An Examination of its Conditions and Possibilities'.

While a probabilistic forecast allows the forecaster to express any uncertainty she may perceive about her own forecast, a non-probabilistic forecast does not allow for it. If beliefs are reported as probabilistic forecasts, we refer to beliefs as being elicited probabilistically (in short, probabilistic beliefs). Analogously, if beliefs are reported as non-probabilistic forecasts, we refer to beliefs as being elicited non-probabilistically (in short, non-probabilistic beliefs).

The formats to be employed to elicit beliefs depend on whether the variable to be forecasted is discrete or continuous. To illustrate this point, we describe in turn the probabilistic and non-probabilistic forecasts of a binary discrete variable and of a continuous variable. Consider forecasting which of two mutually exclusive and jointly exhaustive binary events $\{A, B\}$ will occur. A non-probabilistic forecast is either of the statements 'I believe that A will occur' or 'I believe that B will occur'. A probabilistic forecast, instead, is a statement such as 'I believe with probability x percent that A will occur and with probability $100 - x$ percent that B will occur', with $x \in [0, 100]$.

Consider now forecasting the value taken by a continuous variable a. A non-probabilistic forecast is a statement such as 'I believe that a will equal a^*'. A probabilistic forecast, instead, is a statement such as 'I believe that a cannot be smaller than $\underline{a}$ nor larger than $\overline{a}$ and that, for any subset A of the support, the probability that a lies in A is $P(A)$. Thus, when forecasting a continuous variable, a non-probabilistic forecast is a specific and unique number, while a probabilistic forecast is a probability distribution over a support.

Table 1 summaries the above discussion illustrating probabilistic and non-probabilistic elicitation of first- and second-order beliefs in a two-player game such as the Hide-and-Seek game, in which player i chooses one action in $\{C, D\}$ and player j chooses one action in $\{A, B\}$.

Contributions The main contribution of this paper is to show that elicitation of probabilistic second-order beliefs, along with first-order beliefs, is feasible and to propose a methodology to achieve the elicitation. The proposed method consists of two steps. In the first step, which serves as an introduction for participants to better understand the second step, subjects report a point forecast (i.e. non-probabilistic), stating what they think the most likely value is for their opponent's probabilistic first-order beliefs. In the second step subjects report a probabilistic forecast, stating the probabilities with which they think their opponent's probabilistic first-order beliefs fall within several intervals. As discussed in Section 3, the appropriate information about second-order beliefs beliefs, in order to analyze players' decision-making, is provided by the probabilistic responses collected in the second step, but not by the non-probabilistic responses collected in the first step. The question format we designed allows us to elicit the appropriate (albeit still partial) information about probabilistic second-order beliefs, by collecting several values of $P([a_j, a_{j+1}])$ for a series of $a_j \in [0, 100]$ percent.

The feasibility of eliciting second-order beliefs as a probabilistic forecast would be undermined were the point and the probabilistic forecasts not coherent, in the sense that the point forecast is not some measure of central tendency for the probabilistic forecast. It is reasonable to expect that, if subjects were unable to state their second-order beliefs as a probabilistic forecast while they found it quite natural to state them as a point forecast, requiring them to submit both types of forecast would likely result in incoherent answers.[2] Evidence from both nonparametric and parametric analysis suggests that the point and probabilistic forecasts exhibit coherence.

[2] While surveys are often plagued by nonresponse, the experiment conducted in this study is not subject to nonresponse, since experiment participants were required to complete all forecast tasks. Therefore, we refer to subjects being possibly 'unable' to state second-order beliefs probabilistically, and don't refer to them being possibly 'unwilling'.

Table 1: Probabilistic vs non-probabilistic beliefs elicitation of first- and second-order beliefs.

agent i's beliefs	variable to be forecasted	beliefs elicitation probabilistic	 non-probabilistic
1st-order	agent j's binary action: $y_j = \{A, B\}$	discrete. prob. distr. over y_j $Prob^i(y_j = A)$ and $Prob^i(y_j = B)$ *Bellemare et al 2011*	$y_j^i = \{A, B\}$ *Bhatt and Camerer 2005*
2nd-order	agent j's probabilistic 1st-order beliefs: $Prob^j(y_i = C)$	continuous prob. distr. over $Prob^j(y_i = C)$ *this paper*	point-forecast of $Prob^j(y_i = C)$ *Bellemare et al 2011*
	agent j's non-probabilistic 1st-order beliefs: $y_i^j = \{C, D\}$	discrete. prob. distr. over y_i^j $Prob^i(y_i^j = C)$ and $Prob^i(y_i^j = D)$	${y_i^j}^i = \{C, D\}$ *Bhatt and Camerer 2005*

Notes to Table 1. Consider a simultaneous two-player game in which player i chooses between $\{C, D\}$ and player j between $\{A, B\}$. It is not necessary to define payoffs. Consider player i's 1st- and 2nd-order beliefs. Player i's 1st-order beliefs are her beliefs about player j's action. Player i's 2nd-order beliefs are her beliefs about player j's 1st-order beliefs, i.e. about what player j believes player i's action itself to be. With y_j we denote player j's chosen action. With y_j^i we denote player i's non-probabilistic 1st-order beliefs about the action chosen by player j. With $Prob^i(y_j = A)$ we denote player i's probabilistic 1st-order beliefs about A being the action chosen by player j. With ${y_i^j}^i$ we denote player i's non-probabilistic 2nd-order beliefs about player j's non-probabilistic 1st-order beliefs y_i^j. With $Prob^i(y_i^j = C)$ we denote player i's probabilistic 2nd-order beliefs about player j's non-probabilistic 1st-order beliefs y_i^j being equal to C.

We study the relationship between observed choice and elicited beliefs in terms of whether choice coincides with the optimal action given beliefs. While verifying whether choice coincides with the optimal action given beliefs is straightforward when considering 1st-order beliefs, it becomes more involved and subtle when considering 2nd-order beliefs. This occurs because, in order to engage in second-order conjectures, a player needs to put herself in the shoes of the opponent and think about how the opponent thinks and behaves. This requires thinking about the decision rule which the opponent employs. Throughout the paper we maintain the assumption that a player is certain that the opponent uses her beliefs to make an optimal choice. Allowing player i to be uncertain about the nature of player j's decision rule would require the support of player i's 2nd-order beliefs to consists of pairs $R_j \times [0, 100]$, where R_j is the set of player j decision rules that player i thinks feasible, further complicating the elicitation of 2nd-order beliefs. The maintained simplified view corresponds to what is also reflected in the fictional quote reported at the beginning of this section. Sherlock Holmes thinks that Moriarty 'may be taken as being quite on the same intellectual plane as myself', therefore believing (with certainty) that his opponent does choose the optimal action given his beliefs.

We study the relationship between first- and second-order beliefs under the following criterion of *coherence*. *Strong-coherence* requires that an event's probability evaluated according to 1st-order beliefs and 2nd- order beliefs coincides. *Weak-coherence* instead only requires that if an event is assigned, according to 1st-order beliefs, a probability higher/lower/equal to the one assigned to another event, then the same also holds according to 2nd-order beliefs.

The investigation uncovers a high degree of heterogeneity across experiment participants, in terms of both observed choices and elicited beliefs. In fact, it is reasonable to think that participants are far from being a homogeneous pool of individuals, in terms of the subjective beliefs they hold and the decision rules they employ. The collection of written verbal comments, which participants reported at the end of the experimental session, provides a rare opportunity to shed light on how subjects reason about and deal with concepts such as randomness, indifference, heterogeneity/homogeneity, aggregation, and learning/dynamics. Throughout the analysis, we devote attention to the interpretation of indifference, which occurs when a decision maker feel indifferent between two alternatives she can choose from. Although the evidence collected through the verbal comments is anecdotal, we believe that the use of final questionnaires/interviews may prove useful to improve our understanding of how subjects think and decide in a complex environment, which is strategic, dynamic and populated by a potentially heterogeneous population.

Implications and limitations The results of this paper provide encouraging evidence in favor of the feasibility of measuring second-order beliefs probabilistically. Moreover, while the proposed elicitation method is implemented within a lab experiment, we believe that its format and wording could prove useful also outside the lab. The feasibility of measuring second-order beliefs probabilistically represents a step forward in understanding the process of thinking that subjects experience when facing a strategic situation, and in turning the game-theoretic concept of higher-order beliefs into an observable variable.

Some reader may perceive as a limitation the fact that the paper focuses uniquely on decision-making under uncertainty, and does not extend in an obvious way to decision-making under ambiguity. In fact, throughout the paper, we work under the assumption that subjects hold a unique subjective probabilistic forecast for an unknown event, not allowing subjects to possibly hold a set of such forecasts. We certainly consider exploring the elicitation of beliefs under ambiguity an interesting topic for further research.

Related literature This paper is closely related to the literature on elicitation of probabilistic first-order beliefs. Researchers have elicited probabilistic beliefs for over a century and the practice has become common in survey research since the early 1990s. See the review article of Manski (2004). Elicitation of first-order beliefs in experimental economics is much more recent. Nyarko and Schotter (2002) studied a 2x2 normal-form game and show how the elicitation of first-order probabilistic beliefs can improve the prediction of choice behavior compared to the use of unverifiable proxies for beliefs. Manski (2002) showed how probabilistic beliefs data enable one to overcome the identification problem that arises when choice data alone are used to make inference about decision rules. Since then, elicitation of first-order beliefs in experiments has grown rapidly.[3]

[3] A rapidly-growing literature analyzes the relation between choice behavior and beliefs. Rutström and Wilcox (2009) and Palfrey and Wang (2009) focus on the effects that the elicitation of beliefs may have on choice behavior, including the possibility of more strategic behavior, lower risk aversion and overconfidence. Fehr, Kübler and Danz (2011) focus on the role that participants' matching mechanism, feedback about previous outcomes and information about opponent's payoff may have on the relation between choice behavior and beliefs, arguing that feedback about previous outcomes is the driving force of learning. Costa Gomes and Weizsacker (2008) study one-shot games with no feedback about outcomes nor opponent's behavior and find that choices are often not best response to first-order beliefs. Most of this

While elicitation of first-order beliefs has received growing attention, attempts to elicit second-order beliefs have been limited. Not only have the attempts been limited, but, as far as we are aware, previous research has measured second-order beliefs non-probabilistically, even though probabilistic elicitation of first-order beliefs has become standard practice. Bhatt and Camerer (2005) and Vanberg (2008) elicit 2nd-order beliefs non-probabilistically (along with non-probabilistic 1st-order beliefs), while Bellemare, Sebald and Strobel (2011) elicit 1st-order beliefs probabilistically and 2nd-order beliefs non-probabilistically.[4] Table 1 lists the above-mentioned papers according to whether they employed probabilistic or non-probabilistic elicitation and Table 2 reports the exact wording used. Finally, we mention also the unpublished version which preceded Costa Gomes and Weizsacker (2008) as also containing elicitation of second-order beliefs, albeit as point forecasts.[5] To our knowledge, our paper represents the first attempt to measure second-order beliefs probabilistically.

Plan of the paper The remainder of the paper is organized as follows. Section 2 presents the experimental design and the method used to elicit subjective first- and second-order beliefs probabilistically. Section 3 describes the decision problem and defines the relevant beliefs variables, together with the concepts of *strong-coherence* and *weak-coherence*. Section 4 presents the main discussion. Finally, Section 5 concludes, suggesting directions for further research.

2 Experimental Procedure

The experiment was conducted in the Computer Laboratory of the Main Library at Northwestern University in Evanston, IL. Participants were undergraduate students from Northwestern University. Subjects were recruited using the online recruitment system ORSEE (Greiner (2004)) and the experiment was programmed and conducted with the software Z-Tree (Fischbacher (2007)).

Each experimental session lasted for approximately 30 minutes, including the time for reviewing the instructions, and was identically administered by the same experimenter. When the subjects first arrived at the Computer Lab, they were randomly assigned to one of the 30 computer terminals in the Lab. A welcoming speech was then given, describing the structure and timing of the experiment. Finally, a three-page copy of the instructions was distributed to all participants, who then had 5 minutes to read the instructions and ask questions. Students who wished to ask questions would raise their hand and their questions would be answered privately. Students were allowed to keep a copy of the instructions during the entire session.

The participants play a Hide-and-Seek game. The game is played in pairs: one subject is given the role of Hider and the other subject the role of Seeker. Henceforth, the Hider and Seeker are called agents H and S respectively. The Hider has to hide a prize (a $10 banknote) in one of two locations. The Seeker has to guess where the prize has been hidden. If the Seeker guesses correctly, she wins the

literature has focused on normal-form games. Belief elicitation in extensive-form games is implemented by Dominitz and Hung (2004) and Ziegelmeyer, Bracht, Koessler and Winter (2010) in order to study social learning in information cascade games.

[4] We restrict our attention to quantitative statements and consider purely verbal ones (such as 'very likely', 'likely', 'somewhat likely', 'somewhat unlikely', 'unlikely', 'very unlikely') as non-probabilistic forecasts. Therefore, we consider the elicitation of 1st-order beliefs in Vanberg (2008) as non-probabilistic.

[5] Costa-Gomes and Weizsäcker (2008) state that they 'elicited point estimates of players' second-order beliefs, and not unrestricted probabilistic second-order beliefs.'

Table 2: First- and second-order beliefs elicitation methods used in previous works.

Bhatt and Camerer (2005)

1st-order beliefs: 'What do you think the other player will choose?'

A	*B*

2nd-order beliefs: 'What do you think the other player believes you will choose?'

A	*B*

Vanberg (2008)

1st-order beliefs: 'What do you think the other player will choose?'

certainly A	*probably A*	*unsure*	*probably B*	*certainly B*

2nd-order beliefs: 'What do you think the other player believes you will choose?'

certainly A	*probably A*	*unsure*	*probably B*	*certainly B*

Bellemare et al. (2011)

1st-order beliefs (of type-1-person):
How many type-2-persons out of 100 will choose A and how many B?

Number of type-2-persons out of 100 that will choose A: . . .
Number of type-2-persons out of 100 that will choose B: . . .

2nd-order beliefs (of type-2-person):
What do you think about type-1-person's beliefs about the behavior of type-2-persons?

Type-1-person believes that . . . type-2-persons out of 100 choose A.
Type-1-person believes that . . . type-2-persons out of 100 choose B.

prize. Otherwise, the Hider keeps the prize. The two locations are two zones in which a square field has been divided. The two zones have the same area, while they differ in shape and labeling. There is an inner square field, labeled A, and an outer contour-shaped field, labeled B. The instructions in Appendix D contain the figure representing the two zones.

Several designs of Hide-and-Seek games have been studied in situations with nonneutral payoffs and/or framing of locations. In the design used by Rubinstein, Tversky and Heller (1996), the Hider has to choose to hide a prize in one of four identical boxes, lined one next to each other and labeled, from left to right, as box A, B, A, A. In the game studied by Ayton and Falk (1995), "hide a treasure in a $5X5$ table", the Hider has to hide a treasure in one of the table's 25 boxes. The game used in this paper, by having a design with only two alternatives (A and B), simplifies considerably the elicitation of probabilistic beliefs. At the same time, as in the design with more than two alternatives, the game preserves a nonneutral framing: the inner region is the focal location, despite the fact that the inner and outer regions have the same area.

Six different treatments were implemented. The treatments are labelled C-1-2, C-2-1, 1-C-2, 1-2-C, 2-1-C and 2-C-1. Each treatment differs from the others in the order in which subjects are asked to report their choice (task 'C'), first-order beliefs (task '1') and second-order beliefs (task '2'). Each treatment is assigned randomly to one session. Every other element, except the order of decisions, is identical among treatments.

At the beginning of each round, subjects were matched randomly into pairs and roles were assigned randomly within each pair. Each participant was informed of the role assigned to him/her for that round: information would appear on the computer screen for the entire duration of the round. At the end of each round, each subject received feedback information consisting of: (i) whether or not she won the prize, (ii) the sum of the money earned in forecasting her opponent's choice *plus* the money earned in forecasting her opponent's first-order beliefs. The scoring rules used to reward forecasts are presented in Sections 2.1.1 and 2.1.2.

The subjects played for 4 rounds. When the last round ended, the computer randomly drew one of the rounds and the participants were paid according to their performance in that round only. Once the experiment was over, subjects filled in a questionnaire while waiting to be paid. The questionnaire consisted of questions about each participant's gender, age, major, year of graduation, familiarity with the game, and number of classes taken in (i) economics, finance or accounting, (ii) mathematics, and (iii) psychology. The participants were also given the option to leave specific comments about the way they played the game and/or general comments about the experimental session. Subjects were paid individually in a sealed envelope. Payments included $5 for attending the session, plus the amount earned in the experiment itself. On average, subjects earned approximately $13 for their participation ($5 from the show-up fee, $5 from the choice task and $2.93 from the first- and second-order beliefs tasks).

2.1 Beliefs Elicitation

In this section we describe the methods used to elicit first- and second-order beliefs.

2.1.1 First-Order Beliefs Elicitation

The wording of the question used to elicit first-order beliefs is reported below. Question $1H$ elicits what the Hider believes to be the probability that her opponent will choose A and B. Question $1S$, not shown here, is the analogous question presented to the Seeker.

QUESTION (1H)

What do you think the percent chance is that the Seeker will look for the prize in A? And in B?
Write your answers in the spaces provided below.
You can choose values between 0 and 100.
The values you choose should sum to 100.

Percent chance that the Seeker will look for the prize in A: . . .

Percent chance that the Seeker will look for the prize in B: . . .

The answers to questions $1H$ and $1S$ are remunerated using a quadratic scoring rule. The reward is paid in dollars. In order to illustrate the rule, consider a subject with the role of Hider who has reported probabilities P_H and $1 - P_H$ as the probabilities with which the Seeker will choose A and B respectively. Then, the Hider's reward according to the quadratic scoring rule will be:

$$S_H(P_H, I_A) = 2 - \left\{[I_A - P_H]^2 + [(1 - I_A) - (1 - P_H)]^2\right\} \tag{1}$$

where I_A is an indicator function equal to 1 if the Seeker chooses A and 0 if the Seeker chooses B.

Therefore, if the Seeker chooses A, then the Hider would earn the highest reward by assigning all the probability weight on A, i.e. $P_H = 1$. If the Hider assigns $P_H < 1$ to alternative A and $1 - P_H > 0$ to alternative B, then she will be penalized for both mistakes: for assigning a probability smaller than 1 to A and for assigning a probability larger than 0 to B. The first mistake will cause a penalty of $[1 - P_H]^2$ and the second mistake will cause a penalty of $[0 - (1 - P_H)]^2$. Both penalties will be subtracted from the maximum possible reward of \$2. The minimum possible reward is \$0.

2.1.2 Second-Order Beliefs Elicitation

A player's second-order beliefs are her forecast of her opponent's first-order beliefs. Since the Hider's and Seeker's probabilistic first-order beliefs are represented by probabilities, P_H and P_S respectively, the Hider's and Seeker's probabilistic second-order beliefs are to be represented by continuous probability distributions, which we label q_H and q_S respectively. We denote Q_H and Q_S the corresponding subjective cumulative distributions. Thus, $Q_H(x)$ denotes the subjective probability that the Hider assigns to the event that the Seeker's first-order beliefs P_S are smaller or equal than x. Similarly, $Q_S(x)$ denotes the subjective probability that the Seeker assigns to the event that the Hider's first-order beliefs P_H are smaller or equal than x. We opt for eliciting only partial information about Q_H and Q_S, which are therefore only partially identified. The procedure has the advantage of being simple, while still providing us with the data necessary to the analysis of decision-making conducted in Section 3.

The elicitation of second-order beliefs is divided into two steps, of which the first is meant only as an introduction for participants to better understand the second step. In the first step, subjects report a point forecast (i.e. non-probabilistic), stating what they think the most likely value is for their opponent's first-order beliefs. In the second step, subjects report a probabilistic forecast, stating what they think are the probabilities with which their opponent's first-order beliefs fall within several intervals. The intervals are: $[0, 5]$, $(5, 20]$, $(20, 50]$, $(50, 80]$, $(80, 95]$ and $(95, 100]$ percent.[6] If an experiment participant playing as a Hider assigns to the above intervals the probability vector $p = (p_{[0,5]}, p_{(5,20]}, p_{(20,50]}, p_{(50,80]}, p_{(80,95]}, p_{(95,100]})$, she is interpreted as reporting second-order beliefs characterized by $Q_H(5) = p_{[0,5]}$, $Q_H(20) = p_{[0,5]} + p_{(5,20]}$, $Q_H(50) = p_{[0,5]} + p_{(5,20]} + p_{(20,50]}$, $Q_H(80) = p_{[0,5]} + p_{(5,20]} + p_{(20,50]} + p_{(50,80]}$, and $Q_H(100) = p_{[0,5]} + p_{(5,20]} + p_{(20,50]} + p_{(50,80]} + p_{(95,100]}$. Since the experimental software checks whether the assigned probabilities sum to 100 percent, and reports an error message until the condition is satisfied, $Q_H(100) = 100$ percent always holds.

The wording of each question used in the experiment is reported below. Questions $2H$ and $3H$ elicit the Hider's second-order beliefs and questions $2S$ and $3S$, not shown here, elicit the Seeker's second-order beliefs.

[6] Notice that what a subject reports as point forecast (the 'most likely value') does not influence the definition of the intervals over which she is later asked to place probabilities, which are fixed.

QUESTION (2H)

You are the Hider.

For sure your opponent wants to find the prize, so he or she must be trying to guess where you will hide it.

We've just asked your opponent to tell us what he or she thinks. The question we asked was: What do you think the percent chance is that the Hider will hide the prize in A?

Your opponent has answered this question. You don't know the answer. How do you think your opponent has answered?

Tell us what you think the most likely value is for the answer given by your opponent.

I think the most likely value for the Seeker's answer is: ...

QUESTION (3H)

Now tell us something more. Please complete the following sentences.

I think that the percent chance that the Seeker's answer is not larger than 5 is: ...

I think that the percent chance that the Seeker's answer is larger than 5 and not larger than 20 is: ...

I think that the percent chance that the Seeker's answer is larger than 20 and not larger than 50 is: ...

I think that the percent chance that the Seeker's answer is larger than 50 and not larger than 80 is: ...

I think that the percent chance that the Seeker's answer is larger than 80 and not larger than 95 is: ...

I think that the percent chance that the Seeker's answer is larger than 95 is: ...

The answers to questions $2H$ and $2S$ are remunerated using a zero-one scoring rule[7]. The reward is paid in dollars. The zero-one scoring rule rewards an agent if her reported 'most likely value' coincides with the first-order beliefs stated by her opponent. In order to illustrate the rule, consider a subject, with the role of Hider, reporting m as the 'most likely value' for the Seeker's first-order beliefs P_S. Then, the Hider's reward according to the zero-one scoring rule will be:

$$S_H(m, P_S) = \begin{cases} 2 & \text{if } P_S = m \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The answers to questions $3H$ and $3S$ are remunerated using a quadratic scoring rule. In order to illustrate the rule, consider a subject, with the role of Hider, assigning the probability vector $p = (p_{[0,5]}, p_{(5,20]}, p_{(20,50]}, p_{(50,80]}, p_{(80,95]}, p_{(95,100]})$ to the intervals $[0, 5]$, $(5, 20]$, $(20, 50]$, $(50, 80]$, $(80, 95]$, and $(95, 100]$. Then, the Hider's reward will be:

[7] A zero–one scoring rule rewards a point forecast if the mode of the underlying probabilistic forecast materializes. We don't intend to stress the linkage between the point forecast and the mode, since we don't make any argument that subjects are in fact expressing the mode of their probabilistic beliefs. We chose the zero–one scoring rule mainly for its simplicity.

$$S_H(p, I) = 2 - \left(\sum_{j=1}^{6} (I_{[x_j, y_j]} - p_{[x_j, y_j]})^2 \right) \tag{3}$$

where $I_{[l,r]}$ is an indicator function that takes value 1 if the Seeker has reported as her first-order belief a percentage chance which lies in the interval $[l, r]$ and 0 otherwise, and $p_{[l,r]}$ is the Hider's belief that the first-order belief reported by the Seeker lies in the interval $[l, r]$. The score ranges between \$0 and \$2. The worst possible guess, i.e. assigning 100% chance to an interval while the correct value lies in another interval, yields a payoff of \$0. The best possible guess, i.e. assigning 100% chance to the interval where the correct value lies, yields a payoff of \$2.

2.1.3 Remarks

As mentioned above, the feedback information provided to each participant i at the end of each round consists of: *(a)* whether or not she won the prize, *(b)* the sum of the money earned in forecasting her opponent j's choice plus the money earned in forecasting her opponent j's first-order beliefs P_j. The sum corresponds to $S_i(P_i, I_A) + S_i(m_i, P_j) + S_i(p_i, I)$.

By learning *(a)*, subject i can immediately infer j's choice, without any need of back-engineering from knowledge of the scoring rules and of *(b)*. Subject i simply needs to remember her own choice. On the contrary, subject i usually cannot exactly infer j's first-order beliefs P_j.[8] Even if subject i recovered $S_i(P_i, I_A)$ by remembering her own first-order beliefs P_i, inferring j's choice from *(a)*, and using scoring rule (1), and then tried to back-engineer j's first-order beliefs P_j from knowledge of scoring rules (2)-(3) and of *(b)*, different values of P_j would be possibly consistent with the same combination of i's forecasts m_i and p_i and her forecast earnings.[9]

Finally, since third-order beliefs are not elicited, no feedback information is provided on one's opponent's second-order beliefs.

3 Beliefs and Choices

In this section we illustrate the Hider's and Seeker's decision problem and the relationship between choices, first-order beliefs and second-order beliefs. Given the beliefs P_H, the Hider's optimal decision is to choose A if $P_H < 0.5$ and B if $P_H > 0.5$. If instead $P_H = 0.5$, then the Hider's optimal decision is undefined since she is indifferent between A and B. Analogously for the Seeker: given the beliefs P_S, the Seeker's optimal decision is to choose A if $P_S > 0.5$ and B if $P_S < 0.5$. If instead $P_S = 0.5$, then the Seeker's optimal decision is undefined since she is indifferent between A and B. Therefore, we can denote the Hider's and Seeker's optimal response (or reaction) to second-order beliefs, denoted $r^*_H(P_H)$ and $r^*_S(P_S)$ respectively, as:

[8] An exception consists of when subject i learns that $S_i(P_i, I_A) + S_i(m_i, P_j) + S_i(p_i, I)$ equals \$6. Given scoring rules (1)-(3), this situation corresponds to the maximum total forecast earnings and to perfectly accurate forecasts of both j's choice and j's first-order beliefs.

[9] Recall that $S_i(P_i, I_A)$ is only one of the components of *(b)*.

$$r_H^*(P_H) = \begin{cases} A & \text{if } P_H < 0.5 \\ B & \text{if } P_H > 0.5 \\ A \text{ or } B & \text{if } P_H = 0.5 \end{cases} \tag{4}$$

$$r_S^*(P_S) = \begin{cases} A & \text{if } P_S > 0.5 \\ B & \text{if } P_S < 0.5 \\ A \text{ or } B & \text{if } P_S = 0.5 \end{cases} \tag{5}$$

Notice that saying that a subject is indifferent between A and B does not allow us to expect any specific likelihood of A or B actually been chosen. Attaching a likelihood of 0.5 to A and 0.5 to B would be arbitrary, since an indifferent subject can randomize between A and B with any probability weights. Section 4 will assess the experimental evidence of occurrence of first- and/or second-order beliefs that imply indifference.

We now turn to defining what the optimal response to second-order beliefs should be. We illustrate the Hider's case only, since the Seeker's case is analogous. In order to engage in second-order conjectures, the Hider needs to put herself in the shoes of the opponent and think about how the opponent thinks and behaves. Specifically, the Hider needs to think what decision rule her opponent employs. If the Hider thinks that the Seeker chooses the optimal response to beliefs P_S (i.e., that she follows decision rule (5)), then, whenever the Hider believes that the Seeker's first-order beliefs P_S are larger than 0.5, the Hider also believes that the Seeker chooses A.[10] Given the definition of second-order beliefs Q_H in Section 2, the Hider's second-order beliefs that the Seeker's first-order beliefs P_S are larger than 0.5 is $1 - Q_H(0.5)$. Thus, the Hider's optimal response to her second-order beliefs is A if $1 - Q_H(0.5) < 0.5$, i.e. $Q_H(0.5) > 0.5$. We can rewrite the Hider's and Seeker's optimal response to second-order beliefs, denoted $r_H^*(Q_H)$ and $r_S^*(Q_S)$ respectively, as:

$$r_H^*(Q_H) = \begin{cases} A & \text{if } Q_H(0.5) > 0.5 \\ B & \text{if } Q_H(0.5) < 0.5 \\ A \text{ or } B & \text{if } Q_H(0.5) = 0.5 \end{cases} \tag{6}$$

$$r_S^*(Q_S) = \begin{cases} A & \text{if } Q_S(0.5) > 0.5 \\ B & \text{if } Q_S(0.5) < 0.5 \\ A \text{ or } B & \text{if } Q_S(0.5) = 0.5. \end{cases} \tag{7}$$

When assessing how experimental choice data compare with the behavior prescribed by optimal response to the elicited first-order beliefs and optimal response to the elicited second-order beliefs, we

[10] Notice that we wrote 'If the Hider thinks that the Seeker chooses optimally' because we maintain the simplifying assumption that a player does not doubt if the opponent chooses the best action given her beliefs. A different approach, which we do not pursue in this paper, would allow a player to hold also subjective beliefs about the nature of the opponent's decision rule. Allowing for this further source of uncertainty would then require that the support of player i's 2nd-order beliefs consists of pairs $R_j \times [0, 100]$ where R_j is the set of player j's decision rules. Such an approach could not be followed if Player i's 2nd-order beliefs only conveyed information about player i's beliefs about player j's 1st-order beliefs. See Siniscalchi (2008) for a related discussion.

employ the following concept of *consistency*. We define the Hider's observed choice C_H as *consistent with the optimal response to elicited first-order beliefs* P_H if $C_H = r^*_H(P_H)$ and the Seeker's observed choice C_S as consistent with the optimal response to her elicited first-order beliefs P_S if $C_S = r^*_S(P_S)$. Analogously, we define the Hider's observed choice C_H as *consistent with the optimal response to elicited second-order beliefs* Q_H if $C_H = r^*_H(Q_H)$ and the Seeker's observed choice C_S as consistent with the optimal response to her elicited second-order beliefs Q_S if $C_S = r^*_S(Q_S)$. Given the definition of optimal response in (4)-(7), probabilistic belief elicitation, is necessary to measure variables P_H, P_S, Q_H and Q_S and to compare experimental choice data to the behavior predicted by optimal response to beliefs.

In order to investigate how the Hider's first-order beliefs P_H are related to her second-order beliefs Q_H (and analogously how the Seeker's first-order beliefs P_S are related to her second-order beliefs Q_S), we define *coherence* of first- and second-order beliefs as follows. *Strong coherence* requires that an event's probability evaluated by 1st-order beliefs and 2nd-order beliefs coincides.

Definition

- A Hider's first- and second-order beliefs are *strongly-coherent* if $P_H = 1 - Q_H(0.5)$.
- A Seeker's first- and second-order beliefs are *strongly-coherent* if $P_S = Q_S(0.5)$.

Thus, the Hider holds strongly coherent first- and second-order beliefs if the probability P_H, which the Hider assigns to the event that the Seeker chooses A, coincides with the probability $1 - Q_H(0.5)$, which the Hider assigns to the event that the Seeker considers A more likely to be chosen by the Hider ($P_S > 0.5$) and thus chooses A (since A is the optimal response to $P_S > 0.5$). The Seeker holds strongly coherent first- and second-order beliefs if the probability P_S, which the Seeker assigns to the event that the Hider chooses A, coincides with the probability $Q_S(0.5)$, which the Seeker assigns to the event that the Hider considers A less likely to be chosen by the Seeker ($P_H < 0.5$) and thus chooses A (since A is the optimal response to $P_H < 0.5$).

Weak coherence instead only requires that if an event is assigned, according to 1st-order beliefs, a probability higher/lower/equal to the one assigned to another event, then the same also holds according to 2nd-order beliefs.

Definition

- A Hider's first- and second-order beliefs are *weakly-coherent* if any of these conditions holds:
  (i) $P_H \geq 0.5$ and $Q_H(0.5) \leq 0.5$,
  (ii) $P_H \leq 0.5$ and $Q_H(0.5) \geq 0.5$.
- A Seeker's first- and second-order beliefs are *weakly-coherent* if any of these conditions holds:
  (I) $P_S \geq 0.5$ and $Q_S(0.5) \geq 0.5$,
  (II) $P_S \leq 0.5$ and $Q_S(0.5) \leq 0.5$.

Notice that strong-coherence implies weak-coherence, but not vice versa. Also notice that verifying weak-coherence coincides with verifying whether first- and second-order beliefs lead to best-response prescriptions that are non-contradictory.[11] Table 3 summarizes the cases that can arise empirically when assessing the relationship between an Hider's observed choice and elicited beliefs, in terms of whether (a) the observed choice is consistent with optimal response to elicited 1st-order beliefs, (b) the observed choice is consistent with optimal response to elicited 2nd-order beliefs, and (c) 1st- and 2nd-order beliefs are weakly coherent.

Table 3: Possible cases for the relationship between and Hider's observed choice and her elicited 1st- and 2nd-order beliefs, in terms of whether (a) the observed choice is consistent with optimal response to elicited 1st-order beliefs, (b) the observed choice is consistent with optimal response to elicited 2nd-order beliefs, and (c) 1st- and 2nd-order beliefs are weakly coherent. An X stands for a possible case.

observed choice consistent with:	**coherence of 1st- and 2nd-order beliefs** weak coherence	strong coherence	no weak coherence
neither optimal response to 1st-order beliefs nor to 2nd-order beliefs	B, $P_H < 0.5$, $Q_H(0.5) > 0.5$ A, $P_H > 0.5$, $Q_H(0.5) < 0.5$	B, $P_H = 1 - Q_H(0.5) < 0.5$ A, $P_H = 1 - Q_H(0.5) > 0.5$	
optimal response to 1st-order beliefs only	A, $P_H = 0.5$, $Q_H(0.5) < 0.5$ B, $P_H = 0.5$, $Q_H(0.5) > 0.5$		A, $P_H < 0.5$, $Q_H(0.5) < 0.5$ B, $P_H > 0.5$, $Q_H(0.5) > 0.5$
optimal response to 2nd-order beliefs only	A, $P_H > 0.5$, $Q_H(0.5) = 0.5$ B, $P_H < 0.5$, $Q_H(0.5) = 0.5$		B, $P_H < 0.5$, $Q_H(0.5) < 0.5$ A, $P_H > 0.5$, $Q_H(0.5) > 0.5$
both optimal response to 1st-order beliefs and to 2nd-order beliefs	A, $P_H \leq 0.5$, $Q_H(0.5) > 0.5$ B, $P_H \geq 0.5$, $Q_H(0.5) < 0.5$ A, $P_H < 0.5$, $Q_H(0.5) \geq 0.5$ B, $P_H > 0.5$, $Q_H(0.5) \leq 0.5$	A, $P_H = 1 - Q_H(0.5) \leq 0.5$ B, $P_H = 1 - Q_H(0.5) \geq 0.5$	

We conclude this section highlighting the major differences between our approach and previous experimental work. While comparing methods, we intentionally avoid comparing results, given that the interpretation of each set of results is affected by the different manner in which beliefs are defined and elicited. First, both Bhatt and Camerer (2005) and Costa-Gomes and Weizsäcker (2008) use terms such as 'x and y are consistent' or 'x coincides with the best response to y' both when investigating the relationships between choice and beliefs (either first- or second-) and when investigating the relationship between first- and second-order beliefs.[12] We instead use only the term 'x is/is not equal to the optimal response to y' uniquely when referring to the observed choice coinciding or not with the optimal action given the elicited beliefs. We do not use phrases such as 'first-order beliefs are a best

[11] For example, 1st-order beliefs $P_H = 0.5$ and 2nd-order beliefs $1 - Q_H(0.5) > 0.5$ lead to two non-contradictory best-response prescriptions: the optimal response to 1st-order beliefs $P_H = 0.5$ is *A or B* and the optimal response to 2nd-order beliefs $1 - Q_H(0.5) > 0.5$ is *B*.

[12] Terms used include: 'consistency between actions and beliefs', 'actions are best response to stated beliefs', 'first-order beliefs are a best response to second-order beliefs'.

response to second-order beliefs', given that we intend the concept of 'best response to' as applicable uniquely to an action possibly being a best action given the held beliefs. Therefore, we view coherence of 1st- and 2nd-order beliefs as unrelated to 'first-order beliefs being a best response to second-order beliefs'.

Secondly, by eliciting second-order beliefs as a probabilistic forecast we overcome the limitations that would otherwise arise in verifying whether observed choice coincide with the optimal response to elicited second-order beliefs and whether first- and second-order beliefs are coherent. Costa-Gomes and Weizsäcker (2008) seem to refer to this kind of limitations when they acknowledge (referring to the unpublished version of their paper) the difficulties they encountered having elicited second-order beliefs non-probabilistically.[13] Moreover, non-probabilistic belief elicitation provides no way for respondents to express uncertainty. As a result, respondents cannot express indifference between choosing one action or the other and empirical researchers cannot detect whether any choice is generated by indifference. With indifference undetected, whether the choice made by a subject, who is indifferent between alternative actions, is labeled or not as best response depends on the non-probabilistic beliefs actually reported. Since respondents could use any rule to 'translate' probabilistic beliefs into non-probabilistic ones, it would be impossible to identify correctly the occurrence of a best response.

Finally, by eliciting second-order beliefs as a probabilistic forecast we can address questions that previous work cannot address unequivocally. Allowing for uncertainty would have enabled Bhatt and Camerer (2005) to address their conjectures that 'as players reason further up the hierarchy from choices, to beliefs, to iterated beliefs, their beliefs become less certain' and that, 2nd-order beliefs being more uncertain than 1st-order beliefs, 2nd-order beliefs should be less consistent with 1st-order beliefs than 1st-order beliefs are with choices, and 2nd-order beliefs and choices should be least consistent.[14] Neither of their conjectures can be addressed unequivocally using non-probabilistic beliefs.

Throughout this paper we argue in favor of probabilistic elicitation as it allows for expressing uncertainty and we view the uncertainty inherent in 2nd-order belief as the spread of the elicited subjective distribution. An alternative formal meaning of uncertainty of 2nd-order beliefs can be the presence of ambiguity. Allowing for ambiguity would mean viewing respondents as holding not a unique subjective distribution for an unknown event but possibly a set of subjective distributions, and asking them accordingly to report some information about those distributions.

[13] Costa-Gomes and Weizsäcker (2008) state that 'a limitation of the analysis of these second-order belief statements is that we elicited point estimates of players' second-order beliefs, and not unrestricted probabilistic second-order beliefs.'

[14] Contrary to the conjecture, they find that consistency of choice with 2nd-order beliefs occurs more often than consistency of 1st-order beliefs with 2nd-order beliefs. Over all games, they find that condition (i) holds in 66% of the trials, condition (ii) in 63% of the trials, condition (iii) in 75% of the trials and all conditions hold simultaneously in 23% of the trials. They interpret consistency of choice with 2nd-order beliefs occurring more often than consistency of 1st-order beliefs with 2nd-order beliefs as suggesting that 'the process of generating a [...] iterated belief might be similar to the process of generating a choice, rather than simply iterating a process of forming beliefs to guess what another player believes about oneself'. They report evidence from functional magnetic resonance imaging (fMRI) showing that forming 2nd-order beliefs, compared to forming 1st-order beliefs, activates the *anterior insula* region of the brain, which previous studies have shown to be activated by a sense of agency and self-causation. They interpret this as consistent with people anchoring on their own likely choice and then guessing whether other players will figure their choice out.

4 Discussion

In this section we present the main results. Section 4.1 is an overview of the participants and the treatments. In Section 4.2 we describe the observed choices and in Section 4.3 and 4.4 the elicited first- and second-order beliefs. In Section 4.5 we turn to describe the relationship between choice, first- and second-order beliefs. Finally, Section 4.6 presents a discussion on participants' heterogeneous decision rules.

4.1 Participants and Treatments

The experiment was conducted under 6 treatments, each one with a different order in which questions about choice, first- and second-order beliefs were asked. Each session corresponds to a treatment. Each session was initially scheduled to have about 26 students signed up, expecting about 18-20 students to show up on time and participating. As Table 4 reports, the sessions turned out to have 20, 16, 18, 16, 26, 18 participants respectively, for a total of 114 participants and 456 observations.

Table 4: Participants and Treatments.

Session Name	**Number of Participants**	**Number of Rounds**	**Number of observations**	**Task Order**
C 1 2	20	4	80	choice, 1st-order beliefs, 2nd-order beliefs
C 2 1	16	4	64	choice, 2nd-order beliefs, 1st-order beliefs
1 C 2	18	4	72	1st-order beliefs, choice, 2nd-order beliefs
1 2 C	16	4	64	1st-order beliefs, 2nd-order beliefs, choice
2 C 1	26	4	104	2nd-order beliefs, choice, 1st-order beliefs
2 1 C	18	4	72	2nd-order beliefs, 1st-order beliefs, choice
all	114	4	456	

Table 22 in Appendix B reports the sample distribution of participants in each treatment according to their field of studies and gender.[15] Females make up 60% of all participants. Participants with a major in the Social Sciences make up 44% of the participants, while participants with a major in the Sciences or in the Humanities make up 26% of the participants respectively. These ratios are stable across treatments, except for treatment 2C1, which has a lower ratio of female participants (38% vs. 60%) and a lower ratio of social-sciences-major participants (27% vs. 44%) compared to other treatments.

[15] Fields of studies are categorized as social sciences, humanities, or sciences. Majors in the social sciences include: economics, social policy, human development and psychological services, learning and organizational change, political science, psychology, sociology, education. Majors in the humanities include: art history, art theory, theatre, art theory and practice, classics, english, spanish, philosophy, legal studies, anthropology, gender studies, history, communication studies, journalism, comparative literary studies, middle eastern language and civilization, european studies, international studies, religious studies, music. Majors in the sciences include: biology, biochemistry, chemistry, environmental studies, mathematics, statistics, material science, chemical/civil/electrical/mechanical/computer/industrial engineering.

4.2 Choices and Outcomes

Table 5 pools all observations together and reports the empirical frequency, in percentages, of alternative A being chosen, distinguishing between choices made by Hiders and choices made by Seekers. Across all periods, the frequency of A being chosen is 60% among Hiders and 41% among Seekers.[16] The frequency is stable around 60% for choices made by Hiders. For choices made by Seekers, however, there is an increase from 35-39% in period 1-3 to 54% in period 4.

Table 5: Frequency of choice A, choice outcome, and occurrence of Seeker's wins. (X,Y)=(Hider's choice, Seeker's choice).

	Period				
	1	**2**	**3**	**4**	**all**
	%	%	%	%	%
Choice A among Hiders	60	60	61	60	60
Choice A among Seekers	35	39	37	54	41
Choice outcome					
(A,A)	18	23	26	32	25
(A,B)	42	37	35	28	36
(B,A)	18	16	11	23	17
(B,B)	23	25	28	18	23
Seeker wins	40	47	54	49	48

Table 5 also reports the frequency of the possible choice outcomes. The most common outcome, occurring in 36% of the Hider-Seeker matched observations, is (A, B), which corresponds to a Hider choosing A and a Seeker choosing B. The least common outcome, occurring in 17% of the observations, is (B, A). Across all periods, the frequency with which the game is won by a Seeker, by successfully matching the opponent's choice, is 48%. Changing the order in which the tasks are presented seems to have no discernible effect. Results are omitted for brevity.

4.3 First-Order Beliefs

The experimental design elicits participants' first-order subjective beliefs (i.e., beliefs about opponent's choice) by eliciting the probability that a player assigns to the event that her opponent chooses A. Table 23 in Appendix B pools all observations together and reports the empirical distribution of the answers given in the first-order beliefs elicitation task. Responses are most often multiples of 5 percent

[16] In their hide-and-seek experiment with four identical boxes labeled, from left to right, A, B, A, A, Rubinstein, Tversky and Heller (1996) finds that the distribution of seekers' answers (13%, 31%, 45%, 11%) is strongly biased towards the central A box, avoiding the edges. Similarly, the distribution of hiders' answers (9%, 36%, 40%, 15%) is biased toward the central A box. See Crawford and Iriberri (2007) for a nonequilibrium model based on 'level-k' thinking that can explain the distribution of answers. The strong tendency to avoid the edges was also observed by Ayton and Falk (1995) in their experiment 'hide a treasure in a $5X5$ table', where the subject hides a treasure in one of the table's 25 boxes.

and common values are 0, 25, 40, 50, 60, 75, 100 percent. The most common answer is 50 percent ($P = 0.50$), which occur in 46% of all observations.

A subject reporting first-order beliefs $P = 0.50$ can be interpreted as reporting indifference between choosing A or choosing B. Since one of the objectives of this paper is to cast light on the relationship between the choices participants make and the beliefs they report, we also plan to specifically investigate the empirical occurrence of indifference between alternatives and the corresponding choice behavior. A more general analysis of the relationship between choices and beliefs will be conducted in Section 4.5.

Table 6: Frequency of 1st-order beliefs $P = 0.50$ among Hiders and among Seekers in the pooled dataset.

frequency of 1st-order beliefs P=0.5	**Period**				
	1	**2**	**3**	**4**	**all**
	%	%	%	%	%
Hiders	39	53	53	49	48
Seekers	49	51	39	40	45
any role	44	52	46	45	46

Table 6 uses the pooled dataset and reports the empirical frequency of first-order beliefs $P = 0.50$, distinguishing according to the role played by the subject reporting $P = 0.50$ and according to the round when $P = 0.50$ is reported. The empirical frequency does not appear to change across roles or periods. Pooling all observations together does not allow us to uncover possible heterogeneity across subjects, specifically the possibility that some subjects reported $P = 0.50$ most of the time while other subjects reported $P \neq 0.50$ most of the time. In order to explore heterogeneity across subjects, in Table 7, instead of pooling all observations together, we consider each individual subject as the unit of observation. For each subject, we compute the number of periods (out of the total of four rounds) in which the subject reported first-order beliefs $P = 0.50$. Two main groups stand out, each consisting of approximately 1/3 of all participants. One group consists of subjects always reporting $P = 0.50$, and the other group consists of subjects never doing so.

Table 7: Distribution of the individual subjects according to the number of periods when the subject reports 1st-order beliefs $P = 0.50$.

	No. rounds					
	4 always	**3**	**2**	**1**	**0 never**	**all**
No.	38	6	12	18	40	114
%	33	5	11	16	35	100

Because of the high occurrence of first-order beliefs $P = 0.50$ in the pooled data (46% of all observations) and the existence of a large group of participants who report first-order beliefs $P = 0.50$ in all rounds (33% of all subjects), we turn to investigate the choices made by subjects reporting such first-order beliefs. Table 8 uses the pooled dataset and reports the empirical frequency of choice A when first-order beliefs $P = 0.50$, distinguishing according to the role played by the subject choosing A and reporting $P = 0.50$ and according to the round when such choice and beliefs occur. The frequency of choice A among players with the role of Hider is in all periods approximately 60%. The analogous frequency among players with the role of Seeker is instead approximately 40% in all periods.

Table 8: Choice made by subjects holding 1st-order belief $P = 0.50$ (i.e., assigning 50 percent probability to the event of the opponent choosing A).

frequency of choice A (if P=0.50)	**Period**				
	1 %	**2** %	**3** %	**4** %	**all** %
Hiders	68	63	57	64	63
Seekers	39	38	45	52	43
any role	52	51	52	59	53

4.4 Second-Order Beliefs

The experimental design elicits participants' second-order subjective beliefs (i.e., beliefs about opponent's beliefs) by asking two types of questions. First, participants are asked what they expect to be the *most likely value* of their opponent's answer to the first-order beliefs task. In other words, they are asked what they expect their opponent to report as the probability that they themselves will choose alternative A. As already argued in the previous sections, this is a point forecast (or non-probabilistic forecast). Second, participants are asked to place probabilities to the intervals [0,5], (5,20], (20,50], (50,80], (80,95] and (95,100] percent, where each interval represents a possible range of values for the opponent's answer to the first-order beliefs task. This is a probabilistic forecast.

In this section we describe the elicited probabilistic forecasts and omit a description of the elicited point forecasts. We do this not only for brevity, but also because, as argued in Section 3, probabilistic forecasts are the appropriate beliefs variable when analyzing the relationship between observed choice and optimal choice with respect to second-order beliefs. In Appendix A we perform both a nonparametric and a parametric analysis to compare elicited point and probabilistic forecasts. We refer to Engelberg, Manski and Williams (2009) for an introduction to how point forecasts and probabilistic forecasts compare.[17]

[17] While Engelberg, Manski and Williams (2009) studied the first-order subjective beliefs elicited in a survey, the parametric and nonparametric analysis, which they proposed, can be readily extended to the present analysis of second-order beliefs elicited in a laboratory experiment.

Since reporting probabilistic second-order beliefs requires subjects to assign probabilities over six intervals which span the range [0,100] percent, subjects can potentially assign probability zero to one or more intervals. Table 9 reports the empirical distribution of the number of intervals assigned with positive probability. When applicable (i.e., for a number of intervals larger than 1 and not larger than 5), the table also distinguishes whether the intervals assigned with positive probability are adjacent to one another or not. The most common answers consist of assigning positive probability to six, four or two intervals (28%, 26% and 18% of observations). Moreover, probability is mostly assigned to adjacent intervals rather than to non adjacent intervals.

Table 9: Empirical distribution, over the pooled dataset, of the number of intervals assigned with a positive probability and whether those intervals are adjacent to one another or not.

Intervals	**non adjacent**		**adjacent**		**all**	
	No.	%	No.	%	No.	%
1					43	9
2	20	4	64	14	84	18
3	17	4	33	7	50	11
4	13	3	107	23	120	26
5	3	1	28	6	31	7
6					128	28
all	53		232		456	100

As illustrated in Section 3, the decision taken by a subject who chooses the optimal alternative given her second-order beliefs should be based on the value of $Q(0.5)$. $Q(0.5)$ is the probability that a subject assigns to her opponent's first-order beliefs being not higher than 50 percent. Being '50 percent' the right endpoint of one of the intervals presented to the subjects in the elicitation task (interval (20,50] percent), the value of $Q(0.5)$ is readily available from participants' responses, by simply summing the probabilities that a subject assigned to the intervals [0,5], (5,20] and (20,50] percent. Using the pooled dataset, Table 10 report summary statistics for $Q(0.5)$ and Table 24 in Appendix B reports its empirical distribution. Both tables reveal the preponderance of second-order beliefs for which $Q(0.5) = 0.5$, which occur in approximately 40 percent of the observations.[18]

As illustrated in Section 3, a subject reporting second-order beliefs such that $Q(0.5) = 0.5$ can be interpreted as reporting indifference between choosing A or choosing B. As we reported about indifference generated by first-order beliefs $P = 0.50$ in Section 4.3, we report here about indifference generated by second-order beliefs for which $Q(0.5) = 0.5$. Table 11 uses the pooled dataset and reports the empirical frequency of second-order beliefs for which $Q(0.5) = 0.5$, distinguishing according to the role played by the subject reporting $Q(0.5) = 0.5$ and according to the round when $Q(0.5) = 0.5$ is reported. The empirical frequency does not appear to change across roles or periods.

In order to explore heterogeneity across subjects, in Table 12, instead of pooling all observations together, we consider each individual subject as the unit of observation. For each subject, we compute the number of periods (out of the total of four rounds) in which the subject reported second-order beliefs such that $Q(0.5) = 0.5$. There is evidence of heterogeneity. While the largest group, making

[18]Table 25 in Appendix B reports the empirical distribution of the answers reporting the most likely value of one's opponent's 1st-order beliefs. The value of '50 percent' is the most commonly reported answer, reported in approximately half of all observations.

Table 10: Summary statistics, over the pooled dataset, of $Q(0.5)$: the probability that a subject assigns to her opponent's first-order beliefs being not higher than 50 percent.

No	456
min	0
mean	0.45
10th percentile	0
25th percentile	0.35
median	0.50
75th percentile	0.50
90th percentile	0.70
max	1

Table 11: Frequency of 2nd-order beliefs $Q(0.5) = 0.5$ among Hiders and among Seekers in the pooled dataset.

frequency of 2nd-order beliefs Q(0.5)=0.5	**Period**				
	1	**2**	**3**	**4**	**all**
	%	%	%	%	%
Hiders	33	40	44	56	43
Seekers	40	46	39	37	40
any role	37	43	41	46	42

up approximately 39 percent of all participants, consists of participant whose second-order beliefs are *never* such that $Q(0.5) = 0.5$ the second largest group, making up approximately 23 percent of all participants, consists instead of participant whose second-order beliefs are *always* such that $Q(0.5) = 0.5$.

Table 12: Distribution of individual subjects according to the number of periods when subject reports 2nd-order beliefs such that $Q(0.5) = 0.5$.

	No. rounds					
	4 always	**3**	**2**	**1**	**0 never**	**all**
No.	26	15	14	14	45	114
%	23	13	12	12	39	100

Because of the high occurrence in the pooled data of second-order beliefs for which $Q(0.5) = 0.5$ (42% of all observations) and the existence of a sizable group of participants who report in all rounds second-order beliefs for which $Q(0.5) = 0.5$ (23% of all subjects), we investigate the choices made by subjects reporting such second-order beliefs. Table 13 uses the pooled dataset and reports the empirical frequency of choice A when second-order beliefs $Q(0.5) = 0.5$, distinguishing according to the role played by the subject choosing A and reporting $Q(0.5) = 0.5$ and according to the round when such choice and beliefs occur. The frequency of choice A among players with the role of Hider is in all periods approximately 60%. The analogous frequency among players with the role of Seeker is instead approximately 40% in all periods. Both percentages are similar to the ones found for the choice frequency of players with first-order beliefs $P = 0.50$, reported in Section 4.3.

Table 13: Choice made by subjects holding a second-order belief $Q(0.5) = 0.5$ (i.e., assigning a equal probability mass to the event of the opponent having a first-order belief lower or higher than 50 percent).

frequency of choice A if Q(0.5)=0.5	**Period**				
	1	**2**	**3**	**4**	**all**
	%	%	%	%	%
Hiders	63	65	60	53	60
Seekers	39	42	41	48	42
any role	50	53	51	51	51

4.5 Choice, First- and Second-Order Beliefs

In this section we present descriptive evidence on the relationship between choice, 1st- and 2nd-order beliefs. When assessing the relationship between 1st- and 2nd-order beliefs, we proceed according to the criteria of *strong coherence* and *weak coherence* introduced in Section 3. We relabel the original definition of strong coherence as 0%-*strong coherence*, in order to emphasize that it requires an exact equivalence: $P_H = 1 - Q_H(0.5)$ or $P_S = Q_S(0.5)$. The requirement being very stringent, 0%-strong coherence can fail not because of beliefs lacking coherence, but simply because of participants rounding their beliefs up or down. For this reason, we introduce two less stringent definitions of strong coherence, which do not require an exact equivalence: 5%-*strong-coherence* requires $|P_H - (1 - Q_H(0.5))| \leq 0.05$ or $|P_S - Q_S(0.5)| \leq 0.05$, and 10%-*strong-coherence* requires $|P_H - (1 - Q_H(0.5))| \leq 0.10$ or $|P_S - Q_S(0.5)| \leq 0.10$. Notice that while 0%-strong coherence implies weak coherence, 5%- and 10%-strong coherence don't.

Throughout this section, we inspect the *empirical* percentage frequencies of observations for which: *(i)* the observed choice is consistent with optimal response to 1st-order beliefs, *(ii)* the observed choice is consistent with optimal response to 2nd-order beliefs, *(iii)* 1st- and 2nd-order beliefs are coherent (according to 0%-, 5%-, 10%-strong coherence or weak coherence), *(i)-(iii)* hold simultaneously. Along with empirical frequencies, we also inspect the *theoretical* percentage probabilities with which conditions *(i)*, *(ii)*, and/or *(iii)* would hold under the assumption that participants' choice, 1st- and 2nd-order beliefs are submitted randomly and independently the one of the others. Thus, theoretical probabilities are computed assuming that:

- choice over $\{A, B\}$ is drawn from the distribution $Prob(A) = Prob(B) = 0.5$,
- 1st-beliefs P are drawn from the uniform distribution over support $[0, 100]$ percent,

- 2nd-order beliefs $Q(0.5)$ are drawn from the uniform distribution over support $[0, 100]$ percent.[19]

Comparing empirical frequencies with theoretical probabilities allows us to assess whether and to what extent empirical frequencies reveal something about the participants' decision-making process which would not be equally generated by randomness.

In Section 3 we showed how a subject, choosing the best action given her beliefs, is indifferent between A and B whenever her 1st-order beliefs P or her 2nd-order beliefs $Q(0.5)$ equal 0.5. Thus, any choice made by a participant reporting $P = 0.5$ trivially satisfies condition (i) and any choice made by a participant reporting $Q(0.5) = 0.5$ trivially satisfies condition (ii). Whether $P = 0.5$ and/or $Q(0.5) = 0.5$ also affects whether condition (iii) holds. If $P = Q(0.5) = 0.5$, then weak coherence and 0%-, 5%-, 10%-strong coherence are all trivially satisfied. If $P = 0.5$ and $Q(0.5) \neq 0.5$ (and analogously if $Q(0.5) = 0.5$ and $P \neq 0.5$), then weak coherence is trivially satisfied, 5%- and 10%-strong coherence can possibly hold, but 0%-strong coherence cannot hold. Throughout this section, when we present either empirical frequencies or theoretical probabilities, we compare results obtained over the entire sample with results obtained over several subsamples which rule out $P = 0.5$ and/or $Q(0.5) = 0.5$. Therefore, theoretical probabilities for each of these subsamples are conditional on 1st- and 2nd-order beliefs falling in each of the categories: $P \neq 0.5$ *and* $Q(0.5) \neq 0.5$, $P \neq 0.5$ *and* $Q(0.5) = 0.5$, $P = 0.5$ *and* $Q(0.5) \neq 0.5$, $P = 0.5$ *and* $Q(0.5) = 0.5$. The comparison allows us to assess whether and to what extent empirical frequencies and theoretical probabilities are affected by indifference.

In Table 14, Panel 1 presents the *empirical* percentage frequencies and Panel 2 the *theoretical* percentage probabilities. The first row of each panel reports frequencies computed over the entire sample, while the remaining rows divide the data in subsamples according to whether 1st and/or 2nd-order beliefs imply or not indifference.

We start by comparing, over the entire sample, the empirical frequencies in Panel 1 with the corresponding theoretical probabilities in Panel 2. Observed choice is consistent with optimal response to 1st-order beliefs in 89% of the observations and consistent with optimal response to 2nd-order beliefs in 75% of the observations. The corresponding theoretical probabilities are 73% and 71%, respectively. Thus, while observed choice is more likely to be consistent with optimal response to 1st-order beliefs than what would be simply due to random answers (89% > 73%), the same cannot be said for observed choice being consistent with optimal response to 2nd-order beliefs (75% $\sim$ 73%). Assessing coherence of 1st- and 2nd-order beliefs, the empirical frequencies for 0%-, 5%-, 10%-*strong coherence* or *weak coherence* are only slightly higher than the corresponding theoretical probabilities. When assessing joint occurrence of *(i)-(ii)-(iii)*, only slight differences exist between empirical frequencies and theoretical probabilities when 0%-, 5%-, or 10%-*strong coherence* is employed as criterion. When instead *weak coherence* is employed, empirical frequencies of the joint occurrence of *(i)-(ii)-(iii)* are higher than the theoretical probabilities (68% > 54%).

Comparing, for Panel 1 and Panel 2 separately, the subset of observations with $P \neq 0.5$ *and* $Q(0.5) \neq 0.5$ with the entire sample, we can assess how the exclusion of observations corresponding to participants who are indifferent affects both the empirical frequency and the theoretical probability with which conditions *(i)*, *(ii)* and/or *(iii)* hold. Not surprisingly, both empirical frequencies and theoretical probabilities are lower for the subset compared to the entire sample.

Focusing on the subset of observations with $P \neq 0.5$ *and* $Q(0.5) \neq 0.5$ and comparing empirical frequencies (Panel 1) with theoretical probabilities (Panel 2), we assess if and to what extent conditions

[19] See Appendix C for the computation of theoretical probabilities.

(i), *(ii)* and/or *(iii)* hold more often that what would be implied by random answers. Over the subset, condition *(i)* holds in 81% of the observations and condition *(ii)* in 57% of the observations. The corresponding theoretical probabilities are both 50%. Thus, while observed choice is more likely to be consistent with optimal response to 1st-order beliefs than what would be simply due to random answers (81% $>$ 50%), the same cannot be said for observed choice being consistent with optimal response to 2nd-order beliefs (57% $\sim$ 50%). For condition *(iii)*, the empirical frequencies of 0%-, 5%-, 10%-*strong coherence* are all higher than the corresponding theoretical probabilities (19%, 28%, 37% versus 0%, 9.75% 19%), while the empirical frequency of *weak coherence* is only slightly higher than the theoretical probability (55% $\sim$ 50%). For joint occurrence of *(i)*-*(ii)*-*(iii)*, both under 0%-, 5%-, or 10%-*strong coherence* and *weak coherence*, the empirical frequencies are higher than the corresponding theoretical probabilities (17%, 25%, 32%, 47% versus 0%, 2.4375%, 4.75%, 25%).

Summing up, the above evidence suggests that there are behavioral reasons that lead choices to coincide with the optimal response to 1st-order beliefs, and also simultaneously lead choices to coincide with the optimal response to 2nd-order beliefs and lead 1st- and 2nd-order beliefs to be weakly and strongly coherent. While this is suggested for the entire dataset as well as for the subsample with $P \neq 0.5$ *and* $Q(0.5) \neq 0.5$, the extent to which empirical frequencies are higher than theoretical probabilities is stronger for the subset than it is for the entire dataset. For example, while for the entire dataset the empirical frequency with which condition *(i)* holds is approximately 15 percentage points higher than the theoretical probability (89% vs 73%), in the subset with $P \neq 0.5$ *and* $Q(0.5) \neq 0.5$ the empirical frequency is up to 30 percentage points higher than the theoretical probability (81% vs 50%). Differences between empirical frequencies and theoretical probabilities with which condition *(iii)* or conditions *(i)*-*(ii)*-*(iii)* hold are also substantially larger in the subset than in the entire sample.

For the subsample of observations with $P \neq 0.5$ *and* $Q(0.5) = 0.5$, condition *(i)* holds in 79% of the observations, which is a higher frequency than what would be simply due to random answers (79% > 50%). On the contrary, condition *(ii)* is satisfied trivially and thus both empirical frequency and theoretical probability with which *(ii)* holds are 100%. For joint occurrence of *(i)*-*(ii)*-*(iii)* under *weak coherence*, the empirical frequency is 79% and higher than the 50% theoretical probability.

For the subsample of observations with $P = 0.5$ *and* $Q(0.5) \neq 0.5$, condition *(i)* holds in 59% of the observations, which is only slightly higher than what would be simply due to random answers (59% $\sim$ 50%). Similarly, the empirical frequency with which conditions *(i)*-*(ii)*-*(iii)* jointly hold under *weak coherence* is 59% and only slightly higher than the 50% theoretical probability.

The comparison between empirical frequencies and theoretical probabilities has allowed us to detect that the relationship between choice and 1st-order beliefs is stronger-than-random, while the relationship between choice and 2nd-order beliefs is not. Moreover, a stronger-than-random relationship between choice and 1st-order beliefs appears to correlate not only with 1st- and 2nd-order beliefs being strongly coherent but also, jointly with coherence, with choice coinciding with the optimal response to 2nd-order beliefs too.

4.6 Heterogeneous Decision Rules

The discussion in Section 4.5 and the results reported in Table 14 pool participants together. It is reasonable to think that participants are far from being a homogeneous pool of individuals, in terms of the beliefs they hold and the decision rule(s) they employ. The approach of this section is to treat each participant as a unit of observation, in order to provide descriptive evidence of how individuals differ in terms of the relationship between choice, 1st- and 2nd-order beliefs.

Table 15 reports the distribution across subjects of the frequency with which choice coincides with the optimal response to 1st-order beliefs or with the optimal response to 2nd-order belief. Out of the 114 participants, 38 subjects always report $P = 0.5$ and their choice therefore always trivially coincide with the optimal response to 1st-order beliefs, and 26 subjects always report $Q(0.5) = 0.5$ and their choice therefore always trivially coincide with the optimal response to 2nd-order beliefs. For subjects who report $P \neq 0.5$ in at least one period, Table 15 reports the fraction of periods out of those with $P \neq 0.5$ when choice coincides with the optimal response to 1st-order beliefs. Analogously, for subjects who report $Q(0.5) \neq 0.5$ in at least one period, Table 15 reports the fraction of periods out of those with $Q(0.5) \neq 0.5$ when choice coincides with the optimal response to 2nd-order beliefs.

Both columns of Table 15 report a distribution across participants: from subjects who are indifferent in all rounds, to subjects who are non-indifferent in most rounds and to subjects who in all rounds not only are non-indifferent but also make a choices that coincides with the optimal action given their beliefs. With regard to how often the observed choice coincides with the optimal response to 1st-order beliefs, the distribution is clustered across two large groups: subjects who are indifferent in all rounds and subjects who in most rounds are non-indifferent and whose choice in those rounds coincides with the optimal action given their beliefs. If we define the latter group as those subjects who are non-indifferent in more than half of the rounds and whose choice coincides with the optimal response to 1st-order beliefs in more than half of those rounds, then we can count 5+12+15+18= 50 participants in this cluster. The second column of Table 15 presents a less clustered distribution. There are 4+3+15+7 = 29 subjects in the cluster of those who are non-indifferent in more than half of the rounds and whose choice coincides with the optimal response to 2nd-order beliefs in more than half of those rounds.

How can we interpret heterogeneity across participants? In order to address this question we turn to inspect the data collected in the questionnaire at the end of the experimental session. The questionnaire, which participants filled out at the end of the session, consisted not only of questions about personal background, but also questions about the decision rule(s) employed in the course of the experiment[20]. The specific questions were:

- How did you choose your actions in the game? Please describe briefly.
- How did you choose your forecasts in the game? Please describe briefly.
- Do you have any comments about this experiment?[21]

We found that the answers, especially to the first question, provide an insight into the decision rules employed by the participants. As far as we know, we are the first to explore the possibility and the usefulness of asking directly participants to explain how they reached decisions. We believe

[20]Question about personal background included questions about gender, age, major, year of graduation, familiarity with the game, classes taken in specific fields such as economics/finance/accounting, mathematics, and psychology.

[21]Answering these three questions was not mandatory. However, the response rate was extremely high. All subjects except one (113 out of 114) answered both first and second questions, and more than half of the subjects (61 out of 114) answered also the third question. The answers to the third question were mainly comments about enjoying participating in the experiment. No participants expressed concerns about not understanding the instructions or the tasks (either choice or forecast tasks). This is encouraging evidence that participants felt comfortable enough with the format/wording used to elicit subjective beliefs.

that including questions of this nature at the end of experiment questionnaires can provide valuable information to researchers.

Making decisions in the Hide-and-Seek game requires taking decisions in an unknown complex environment that is strategic and dynamic and that consists of possibly heterogeneous agents. Table 16 helps visualize the complexity of the environment according to how subjects deal with strategy, dynamics, and heterogeneity. The environment is unknown because each participant does not know whom she is randomly paired with in each round, nor does she know the characteristics of the population of all same-session participants, from which her opponent is drawn.

The environment is strategic because the outcome of the interaction of two players is determined by the decision that each of them makes, which leads each player to think ahead to what the decision of her opponent is likely to be. *Subject 66*, for example, mentions thinking that there are reasons why the most likely decision made by her potential opponent playing as Seeker would be B and that therefore the best action for herself as Hider is A. We could loosely interpret the differences across participants' strategic arguments as differences in 'levels of thinking', as the growing literature of 'level-k' models does.

The environment is dynamic not simply because interaction is repeated over several rounds and subjects can change their beliefs and behavior from one round to the next, but also because at the end of each round information about the interaction which just took place is provided to subjects, who may react to the information by changing their beliefs and behavior. A subject that acknowledges the environment dynamics may or may not think that, as much as she might change beliefs and behavior across time, other individuals in the population can change their beliefs and behavior too. While the environment is inherently non-stationary, a subject can reason and behave as if the environment were stationary. Learning in a non-stationary environment is an extremely demanding task. Therefore, in our review of participants' comments we did not expect to find mention of specific (nor let alone optimal) rules to deal with learning in a non-stationary environment. Our aim was simply to verify whether participants acknowledge the complexity of the decision task and whether they the decision rules they describe implicitly assume a stationarity condition. *Subject 98*, for example, mentions changing her behavior in any round based on the results experienced in the previous round, thinking that other participants do not changes their behavior across time (while she does!).

Granted that all participants belong to the same student community, it is reasonable to think that such community is not perfectly homogeneous in terms of decision rules, as Table 15 revealed in terms of beliefs and behavior and as Table 17 reveals (for a selection of participants) also in terms of comments. A subject may or may not think that the pool of subjects from which her opponent is drawn consists of heterogeneous individuals, who hold different beliefs, follow different decision rules and update their beliefs and behavior differently. If heterogeneity is considered, then a subject may think about a way in which to reason about a heterogeneous population in an aggregate manner. *Subject 111*, for example, mentions that the likely result of aggregating the behavior of many heterogenous potential opponents is very random. *Subject 105* mentions behaving as if playing against the same opponent, implicitly implying the existence of a sort of 'representative agent'. *Subject 66*'s comments that she would choose A when playing as Seeker because she would choose A when playing as Hider implies that she reasons as if other participants would behave as she behaves herself when playing as Hider.

The collected comments also provide an insight into how widely subjects differ in the way in which they perceive indifference and make decisions when indifferent. *Subject 9*, *111* and *75* are three participants who report indifference in the form of 1st-order beliefs $P = 0.5$ and/or 2nd-order beliefs

$Q(0.5) = 0.5$. In their comments they report always choosing the same action in face of indifference and motivate indifference as due to no 'background knowledge necessary to accurately ascertain' how other participants act, or due to facing a 'very random' aggregate process of other participants' decisions, or due to facing independent 'draws' from two alternatives assumed to have 'equal probability'.

5 Conclusion

In this paper we proposed a method to elicit probabilistic 2nd-order beliefs, along with 1st-order beliefs, and we examined choices and 1st- and 2nd- beliefs among participants to a Hide-and-Seek experiment. Throughout the paper we highlighted how eliciting second-order beliefs as a probabilistic forecast helps overcome the limitations that would otherwise arise in interpreting the relationship between observed choices and elicited beliefs in terms of best-response and coherence requirements. Thus, we view measuring second-order beliefs probabilistically as a step forward in understanding the process of thinking that subjects experience when facing a strategic situation, and in turning the game-theoretic concept of higher-order beliefs into an observable variable. In addition to choice and beliefs data, the collection of participants' written comments provided us with a rare further glimpse into how subjects think and decide in a complex environment and how they deal with concepts such as randomness, indifference, heterogeneity/homogeneity, aggregation, and learning/dynamics.

While this paper documents the ability of subjects to report their subjective beliefs in probabilistic form, we are aware that an alternative to the study of decision under uncertainty, and to the corresponding focus on subjective beliefs, is represented by the study of decision under ambiguity. In decision under ambiguity subjects do not hold a unique subjective distribution for an unknown event but may hold a set of subjective distributions. We consider exploring the elicitation of beliefs under ambiguity an interesting topic for further research.

Table 14: Relationship between choice, 1st- and 2nd-order beliefs, according to whether: *(i)* the observed choice is consistent with optimal response to 1st-order beliefs, *(ii)* the observed choice is consistent with optimal response to 2nd-order beliefs, *(iii)* 1st- and 2nd-order beliefs are coherent (according to 0%-, 5%-, 10%-*strong coherence* or *weak coherence*), *(i)-(ii)-(iii)* hold simultaneously. Panel 1 presents the *empirical* percentage frequencies of observations for which conditions *(i)*, *(ii)*, and/or *(iii)* hold. Panel 2 presents the *theoretical* percentage probabilities of observations for which conditions *(i)*, *(ii)*, and/or *(iii) would* hold, computed under the assumption that participants' answers (choice, 1st- and 2nd-order beliefs) are random. The first row of each panel reports frequencies computed over the entire sample, while the remaining rows divide the data in subsamples according to whether 1st and/or 2nd-order beliefs imply or not indifference.

	(i)	(ii)	(iii)				(i)-(ii)-(iii)					
	choice and 1st-order beliefs	**choice and 2nd-order beliefs**	**1st- and 2nd-order beliefs**				**choice, 1st- and 2nd-order beliefs**				**obs**	
			strong-coherence			*weak coherence*	*strong-coherence*			*weak coherence*		
			0%	5%	10%		0%	5%	10%			
Panel 1: Empirical percentage frequencies												
	%	%	%	%	%	%	%	%	%	%	no.	%
all obs	89	75	34	40	52	83	33	38	46	68	456	100
$P \neq 0.5$, $Q(0.5) \neq 0.5$	81	57	19	28	37	55	17	25	32	47	173	38
$P \neq 0.5$, $Q(0.5) = 0.5$	79	100	0	10	39	100	0	10	32	79	71	16
$P = 0.5$, $Q(0.5) \neq 0.5$	100	59	0	9	26	100	0	5	12	59	92	20
$P = 0.5$, $Q(0.5) = 0.5$	100	100	100	100	100	100	100	100	100	100	120	26
Panel 2: Theoretical percentage probabilities assuming participants' answers to be random												
	%	%	%	%	%	%	%	%	%	%	no.	%
all obs	73	71	26	34	41	81	26	29	32	54	456	100
$P \neq 0.5$, $Q(0.5) \neq 0.5$	50	50	0	9.75	19	50	0	2.4375	4.75	25	173	38
$P \neq 0.5$, $Q(0.5) = 0.5$	50	100	0	10	20	100	0	5	10	50	71	16
$P = 0.5$, $Q(0.5) \neq 0.5$	100	50	0	10	20	100	0	5	10	50	92	20
$P = 0.5$, $Q(0.5) = 0.5$	100	100	100	100	100	100	100	100	100	100	120	26

Table 15: Distribution across subjects of the frequency with which choice coincides with the optimal response to 1st-order beliefs or with the optimal response to 2nd-order belief.

How often choice coincides with optimal response to ...	**... 1st-order beliefs**	**... 2nd-order beliefs**
	No. subjects	No. subjects
always (subject is always indifferent)	38	26
fraction of periods out of those when the subject is not indifferent		
0/1	3	5
1/1	3	10
0/2		4
1/2	4	5
2/2	8	5
0/3	1	1
1/3		6
2/3	5	4
3/3	12	3
0/4		4
1/4	4	6
2/4	3	13
3/4	15	15
4/4	18	7
all subjects	114	114

Note: For both 1st- and 2nd-order beliefs, the table distinguishes between subjects who report to be indifferent in all four periods (and for whom therefore choice always coincides with the optimal response to beliefs), and subjects who report being non-indifferent in at least one period. For the latter, the table reports the fraction of periods, out of those characterized by non-indifference, when choice coincides with the optimal response to beliefs.

Table 16: A complex environment requiring participants to handle strategy, dynamics and heterogeneity.

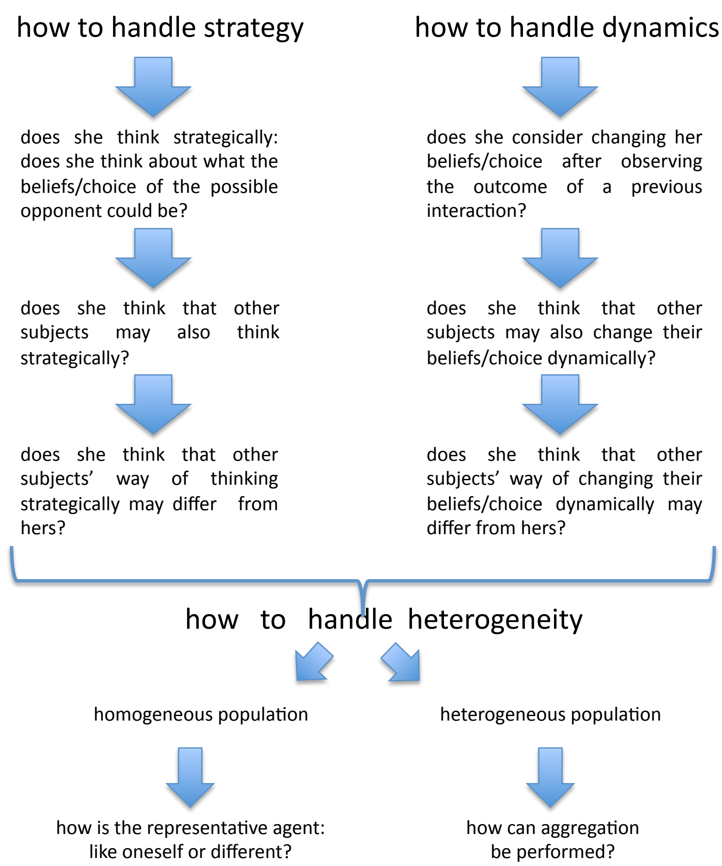


Table 17: A selection of participants' comments.

subject ID	How did you choose your actions in the game? Please describe briefly.	interpretation
66	I figured people would be most likely to look in B for the prize because it looks smaller and is a better hiding place, so I hid in A. I looked in A because I would hide in A	thinking strategically, as if reacting to someone similar to oneself
9	I clicked B each time because I didn't want to try to overthink how people act in this game, knowing that I don't have the background knowledge necessary to accurately ascertain that.	choosing always the same action, stating no knowledge
111	I think people end up doing a lot of reverse psychology on the decisions and will probably ends up very random when aggregated. Therefore I just stuck with choosing A all along.	choosing always the same action, stating an aggregate random process
75	I knew that B and A had equal area, so I assumed that there were equal probability of being chosen. Thus, I chose A every time because all the draws are independent.	choosing always the same action, as if facing an equal probability event
98	I based them on the results of my last period, because I thought in general people choose the same way.	based on previous round, thinking others are similar/don't change behavior
105	Chose my first round randomly. Then based subsequent choices on the first round, as if I were playing the same person.	based on previous round, as if playing against the same opponent who does not change behavior

References

[1] P. Ayton and R. Falk. Subjective randomness in hide-and-seek games. In *Books of Abstracts of the 15th Bi-annual Conference on Subjective Probability, Utility and Decision-Making*, 1995.

[2] C. Bellemare, A. Sebald, and M. Strobel. Measuring the willingness to pay to avoid guilt: estimation using equilibrium and stated belief models. *Journal of Applied Econometrics*, 26:437–453, 2011.

[3] M. Bhatt and C. Camerer. Self-referential thinking and equilibrium as states of mind in games: fmri evidence. *Games and Economic Behavior*, 52(2):424–459, 2005.

[4] M. Costa-Gomes and G. Weizsäcker. Stated beliefs and play in normal-form games. *Review of Economic Studies*, 75(3):729–762, 2008.

[5] V. Crawford and N. Iriberri. Fatal attraction: salience, naïveté, and sophistication in experimental 'hide-and-seek' games. *The American Economic Review*, 97(5):1731–1750, 2007.

[6] J. Dominitz and A. Hung. Homogeneous actions and heterogeneous beliefs: Experimental evidence on the formation of information cascades. *Carnegie Mellon University*, 2004.

[7] A.C. Doyle. The final problem. In *The memoirs of Sherlock Holmes*. George Newnes, 1894.

[8] J. Engelberg, C. Manski, and J. Williams. Comparing the point predictions and subjective probability distributions of professional forecasters. *Journal of Business and Economic Statistics*, 27(1):30–41, 2009.

[9] D. Fehr, D. Kübler, and D. Danz. Information and beliefs in a repeated normal-form game. *Experimental Economics*, forthcoming 2011.

[10] U. Fischbacher. z-tree: Zurich toolbox for ready-made economic experiments. *Experimental Economics*, 10(2):171–178, 2007.

[11] Ben Greiner. The online recruitment system orsee 2.0 - a guide for the organization of experiments in economics. 2004.

[12] C. Manski. Identification of decision rules in experiments on simple games of proposal and response. *European Economic Review*, 46(4-5):880–891, 2002.

[13] C. Manski. Measuring expectations. *Econometrica*, 72(5):1329–1376, 2004.

[14] O. Morgernstern. *Wirtschaftsprognose: eine Untersuchung ihrer Voraussetzungen und Möglichkeiten*. Springer, 1928.

[15] Y. Nyarko and A. Schotter. An experimental study of belief learning using elicited beliefs. *Econometrica*, 70(3):971–1005, 2002.

[16] T. Palfrey and S. Wang. On eliciting beliefs in strategic games. *Journal of Economic Behavior & Organization*, 71(2):98–109, 2009.

[17] A. Rubinstein, A. Tversky, and D. Heller. Naive strategies in zero-sum games. In *Understanding Strategic Interaction – Essays in Honor of Reinhard Selten*, pages 394–402, Berlin, 1996. Springer.

[18] E. Rutström and N. Wilcox. Stated beliefs versus inferred beliefs: a methodological inquiry and experimental test. *Games and Economic Behavior*, 67(2):616–632, 2009.

[19] M. Siniscalchi. Epistemic game theory: beliefs and types. In *The New Palgrave Dictionary of Economics*. Palgrave Macmillan, 2008.

[20] C. Vanberg. Why do people keep their promises? an experimental test of two explanations. *Econometrica*, 76(6):1467–1480, 2008.

[21] A. Ziegelmeyer, F. Koessler, J. Bracht, and E. Winter. Fragility of information cascades: an experimental study using elicited beliefs. *Experimental Economics*, 13(2):121–145, 2010.

Appendix A
Comparison of point and probabilistic 2nd-order beliefs

In this appendix we compare the two measures for 2nd-order beliefs elicited in the experiment: the point forecast expressed by means of the 'most likely value' and the probabilistic forecast expressed by means of probabilities over intervals. We refer to Engelberg, Manski and Williams (2009) for an introduction of how point forecasts and probabilistic forecasts compare. While Engelberg, Manski and Williams (2009) studied 1st-order beliefs, the parametric and nonparametric analysis, which they proposed, can be readily extended to the present analysis of 2nd-order beliefs.

As argued in the paper, we can interpret the coherence of the point forecast and the probabilistic forecast as the elicited 'most likely value' being a measure of central tendency for the probabilistic forecast. Which measure of central tendency? The mode, the mean or the median? While the actual wording used in the experiment (the 'most likely value') may suggest that the mode of a subjective probability distribution is in fact the measure elicited from the experimental subjects, we argue that we cannot make any inference on the subjective mode.

The limitation stems from the way the data for the subjective probability distribution was elicited. The experimental setting constrained to elicit probabilities only within certain intervals. Being the mode a local concept, we cannot tell which interval it is in. On the one hand, assuming that the mode is in the interval assigned with the highest probability is not a reasonable assumption, because the intervals have different widths. On the other hand, assuming that the density is uniform within each interval is not a reasonable assumption either, because the middle intervals have a large width. Therefore, we cannot make any inference on the subjective mode, and focus instead on inference on the subjective mean and median.

Nonparametric Analysis

In this section we use the elicited probabilities assigned to each interval [0,5], (5,20], (20,50], (50,80], (80,95] and (95,100] percent to compute bounds on the mean and median of the subjective distribution representing a subject's 2nd-order beliefs.

Suppose that a subject assigns probability 0.30 to interval (20,50]%, 0.60 to interval (50,80]%, 0.05 to interval (80,95]%, 0.05 to interval (95,100]% and zero probability to all other intervals. Then we can conclude that the subjective median lies in the interval (50,80]%. Lower and upper bounds on subjective means are computed by placing the probability mass assigned to each interval at the interval's lower and upper endpoints respectively. In our example the lower bound is $0.30 \times 20\% + 0.60 \times 50\% + 0.05 \times 80\% + 0.05 \times 95\% = 44.75\%$ and the upper bound is $0.30 \times 50\% + 0.60 \times 80\% + 0.05 \times 95\% + 0.05 \times 100\% = 72.75\%$ The resulting bounds are (44.75, 72.75]%.

Table 18 reports the 25th, 50th and 75th percentiles of the sample distribution of the width of the bounds. The width of the bounds is closely dependent on the specific definition of the intervals used in the experiment. Specifically, the width of the bounds on the subjective median is usually 30 because the lower and upper bounds are usually 20 or 50 and 50 or 80, respectively. No stark difference stands out either across periods or across treatments. Results are omitted for brevity.

Table 19 reports the percentage of observations for which the elicited 'most likely value' lies within the bounds on the subjective mean (69%) or within the bounds on the subjective median (79%). For

Table 18: Percentiles of the width of the bounds on the subjective mean and on the subjective median.

	percentiles of the width 25th	50th	75th
bounds on subj. mean	0.21	0.25	0.28
bounds on subj. median	0.30	0.30	0.30

Table 19: Percentage of observations for which the elicited 'most likely value' lies within the bounds on the subjective mean or the subjective median.

	within the bounds on subj. mean %	subj. median %	obs.
all obs.	69	79	456
by treatment			
12C	75	81	64
1C2	75	86	72
21C	60	67	72
2C1	66	76	104
C12	79	81	80
C21	61	83	64
21C and 2C1	64	72	176
all others	73	83	280

most observations, the elicited 'most likely value' is consistent with the hypothesis that subjects report their subjective mean or median belief. The fact that the elicited 'most likely value' lies more often within the bounds on the subjective median than within the bounds on the subjective mean (79% versus 69%) is possibly due to the bounds on the subjective median being wider than the bounds on the subjective mean. Therefore, we cannot interpret this result as evidence in favor to the argument that subjects report their subjective median versus their subjective mean.

Third, the treatments in which subjects are asked to report their 2nd-order beliefs as their first task, labeled 21C and 2C1, correspond to the treatments in which coherence with respect to the bounds on the subjective mean and median occurs less compared to all other treatments. (64% vs 73% for the subjective mean and 72% vs. 83% for the subjective median).

This result could be due to a difference in the width of bounds in treatments 21C and 2C1 compared to all other treatments. A less spread-out subjective distribution, characterized by narrower bounds on subjective mean and median, could make it less likely for the 'most likely value' to fall inside the bounds. This explanation however can be ruled out, since we find that the width of the bounds does not vary significantly across treatments. Thus, it could be the case that treatments other than 21C and 2C1 lead to a more thoughtful response of the 2nd-order beliefs task because this is not the first task, and subjects have already gone through a choice task or a forecast task (or both). In turn, a more thoughtful response of the 2nd-order beliefs task could make it more likely for the elicited 'most likely value' to fall inside the bounds.

Finally, the percentage of observations for which the elicited 'most likely value' lies within the bounds on the subjective mean or median does not vary significantly across periods nor across player roles (hider and seeker). Results are omitted for brevity.

Parametric Analysis

In this section we use the elicited probabilities assigned to each interval [0,5], (5,20], (20,50], (50,80], (80,95] and (95,100] percent to fit the subjective cumulative distribution functions (CDF) Qs that represent subjects' 2nd-order beliefs. From the knowledge of the probabilities assigned to each interval, the value of the subjective CDF at the right endpoints of the six intervals can be inferred.

In what follows, instead of using percentage points between 0 and 100, we use values between 0 and 1. Therefore, lets relabeled the right endpoints of the six intervals as $r_1 = 0.05$, $r_2 = 0.20$, $r_3 = 0.50$, $r_4 = 0.80$, $r_5 = 0.95$ and $r_6 = 1$. Lets denote the values of the subjective CDF at these points as $Q(r_1), ..., Q(r_6)$. Finally, lets denote the lower bound of the first interval and the upper bound of the last interval over which a subject places positive probability as L and R, respectively. Therefore, we denote the support of the subjective distribution as $[L, R]$.

Following Engelberg, Manski and Williams (2009), we maintain the assumption that the subjective distribution is a member of the generalized Beta family, provided that it is possible to fit a unique Beta distribution to the data. It is possible to fit a unique Beta distribution to the data only when a subject assigns positive probability to at least three intervals. As reported in Table 9 (see Section 4), this occurs in 329 out of 456 observations (72%)[22]. In the remaining 127 observations, positive probability is assigned to one or two intervals. In the cases with one interval (43 observations, 9% of the total), we assume that the subjective distribution has the shape of an isosceles triangle whose base coincides with the interval. In the cases with two intervals (84 observations, 18% of the total), the two are adjacent to one another in 64 observations and non-adjacent in 20 observations.

Consider the case of two adjacent intervals being assigned with positive probability. Suppose that a subject assigns probability a and $1 - a$ to the intervals $[x, y)$ and $[y, z)$ respectively, where the intervals have possibly different width. Denote with b and $1 - b$ the probability mass that interval $[x, y)$ and $[y, z)$ respectively would have, were the probability mass of each interval proportional to their width. Therefore $b = \frac{y-x}{z-x}$ and $1 - b = \frac{z-y}{z-x}$. In the case of intervals of equal width, $b = 1/2$. We assume that the subjective distribution has the shape of an isosceles triangle whose base includes all of the interval with a probability mass more than proportional to its width and part of the other interval. If $a < b$ (i.e., interval $[x, y)$ has a probability mass less than proportional to its width), then we assume that the subjective distribution has the shape of an isosceles triangle whose base includes all of the interval $[y, z)$ and part of the interval $[x, y)$. Letting $t = \frac{(z-y)\sqrt{\frac{a}{2}}}{1-\sqrt{\frac{a}{2}})}$, it is straightforward to show that the isosceles triangle with height $h = \frac{2}{z-y+t}$ and endpoints $y - t$ and z defines a subjective probability density function that is consistent with the subject's reported beliefs.[23] This procedure generalizes Engelberg, Manski and Williams (2009) to the case of positive probability being assigned to two adjacent intervals that have unequal width.[24]

[22]Notice that in 33 out of 329 observations the intervals are not all adjacent to each other.

[23]If instead $a > b$ (i.e., interval $[x, y)$ has a probability mass more than proportional to its width), then we assume that the subjective distribution has the shape of an isosceles triangle whose base includes all of the interval $[x, y)$ and part of the interval $[y, z)$. Letting $t = \frac{(y-x)\sqrt{\frac{1-a}{2}}}{1-\sqrt{\frac{1-a}{2}})}$, it is straightforward to show that the isosceles triangle with height $h = \frac{2}{y-x+t}$ and endpoints x and $y + t$ defines a subjective probability density function that is consistent with the subject's reported beliefs.

[24]Out of the 64 observations with two adjacent intervals, only 6 observations have two intervals with unequal width. This case is not present in the dataset analyzed by Engelberg, Manski and Williams (2009), where all (bounded) intervals

Finally, if the two intervals assigned with positive probability are non-adjacent, then fitting a Beta distribution is not possible and assuming that the subjective distribution has the shape of a isosceles triangle ranging over both intervals does not seem reasonable. This occurs in 20 out of 456 observations (4%).[25] We decide to exclude these observations from the analysis.

When fitting a Beta distribution, the CDF defined over $[L, R]$ and evaluated at x is denoted $Beta(x, \alpha, \beta, L, R)$, where α and β are the shape parameters and L and R are the location parameters. We put no constraint on the value of α and β, thus allowing for unimodal, uniform, U-shaped, strictly increasing or strictly decreasing distributions. For each subject i and period t, we use the elicited $Q_{i,t}(r_j)$ to find the parameters $\alpha_{i,t}$ and $\beta_{i,t}$ that solve the least-squares problem:

$$\min_{\alpha,\beta} \left\{ \sum_{j=1}^{6} [Beta(r_j, \alpha_{i,t}, \beta_{i,t}, L_{i,t}, R_{i,t}) - Q_{i,t}(r_j)]^2 \right\}. \tag{8}$$

There is inevitably some arbitrariness in using a specific criterion to fit the experimental subjects' answers to a distribution, but the obtained beta CDFs fit the answers well: the 25th, 50th and 75th percentiles of the sample distribution of the minimized objective function are 0, 0.0013 and 0.0045, respectively. The sample average is 0.0052 and the largest value is 0.0654.

Table 20 reports summary statistics of the empirical distribution of the absolute differences $|A_{i,t} - M_{i,t}|$ and $|A_{i,t} - Me_{i,t}|$, distinguishing between the cases in which a Beta distribution or a triangle distribution is fitted to the data. As mentioned above, we compute the fitted mean M and the fitted median Me for all observations except those in which positive probability is assigned to two non-adjacent intervals.[26] The empirical distributions of both $|A_{i,t} - M_{i,t}|$ and $|A_{i,t} - Me_{i,t}|$ have 25th, 50th and 75th percentiles equal to 0, 5 and 15 percent respectively.[27] Therefore, both the fitted mean and the fitted median seem to provide a good proxy for the elicited *most likely value*.

One observation that was made under nonparametric analysis holds in an analogous way under parametric analysis too. As table 21 reports, the treatments in which subjects are asked to report their 2nd-order beliefs as their first task, labeled 21C and 2C1, are associated with a larger absolute difference between the elicited $A_{i,t}$ and the fitted mean $M_{i,t}$ and a larger absolute difference between the elicited $A_{i,t}$ and the fitted median $Me_{i,t}$, compared to all other treatments.

Conclusion

Using the answers reported by subjects in terms of probabilities over intervals, we compute nonparametric bounds on the the subjective mean and the subjective median. Using the same answers, we also fit a parametric distribution and determine the mean and the median of the fitted subjective distribution. Thus, we have both nonparametric and parametric information about the subjective distribution and we can answer the following question: are the answer in terms of 'most likely value' and the answer in terms of probabilities over intervals coherent?

have equal width.

[25] This case is not present in the dataset analyzed by Engelberg, Manski and Williams (2009).

[26] For the Beta distribution, the fitted mean is $M = \frac{\alpha}{\alpha+\beta}$. For the fitted median there is no close form solution.

[27] The empirical distributions of both $A_{i,t} - M_{i,t}$ and $A_{i,t} - Me_{i,t}$ have 25th, 50th and 75th percentiles equal to -4, 0 and 7 percent respectively.

Table 20: Sample distribution (by fitting method) of the absolute difference between the elicited 'most likely value' and the fitted mean or median of the subjective 2nd-order beliefs. Percentage points.

	|'most likely value'-fitted mean|				
	mean	25th perc.	50th perc.	75th perc.	obs.
Beta distribution	11	0	7	16	329
of which					
unimodal ($\alpha > 1, \beta > 1$)	8	0	5	12	155
uniform ($\alpha = 1, \beta = 1$)	8	0	0	12	29
U-shaped ($\alpha < 1, \beta < 1$)	14	0	7	23	108
strictly decreasing ($\alpha < 1, \beta > 1$)	15	9	16	20	11
strictly increasing ($\alpha > 1, \beta > 1$)	11	4	10	14	26
Triangle distribution	8	0	2	9	107
Beta and Triangle distr.	10	0	5	15	436
	|'most likely value'-fitted median|				
	mean	25th perc.	50th perc.	75th perc.	obs.
Beta distribution	11	0	8	16	329
of which					
unimodal ($\alpha > 1, \beta > 1$)	9	0	5	13	155
uniform ($\alpha = 1, \beta = 1$)	8	0	0	12	29
U-shaped ($\alpha < 1, \beta < 1$)	16	0	9	28	108
strictly decreasing ($\alpha < 1, \beta > 1$)	15	10	13	26	11
strictly increasing ($\alpha > 1, \beta > 1$)	11	6	11	17	26
Triangle distribution	8	0	2	9	107
Beta and Triangle distr.	10	0	5	15	436

Table 21: Sample distribution (by experimental treatment) of the absolute difference between the elicited 'most likely value' and the fitted mean or median of the subjective 2nd-order beliefs. Percentage points.

	|'most likely value'-fitted mean|				|'most likely value'-fitted median|				
	mean	25th perc.	50th perc.	75th perc.	mean mean	25th perc.	50th perc.	75th perc.	obs.
all obs.	10	0	5	15	10	0	5	15	436
by treatment									
12C	8	0	2	12	8	0	2	13	62
1C2	10	0	5	15	11	0	5	14	71
21C	15	4	10	24	16	5	12	25	68
2C1	11	0	4	15	12	0	7	16	102
C12	7	0	2	9	7	0	2	9	74
C21	8	0	7	15	9	0	5	13	59
21C and 2C1	13	1	7	18	13	2	8	20	170
all others	8	0	4	13	9	0	4	13	266

Using nonparametric methods, we find that the elicited 'most likely value' lies within the bounds on the mean and the median of the subjective distribution between 70% and 80% of the time. Using parametric fitting, we find that the elicited 'most likely value' matches very closely both the mean and the median of the fitted subjective distribution. Thus, evidence suggests that the answers provided in probabilistic form by experimental subjects exhibit coherence. By conducting treatments that differ in the order in which subjects are asked to report choices, 1st- and 2nd-order beliefs, we examine whether task order has an impact on coherence. Both nonparametric and parametric analysis suggest that in those treatments, in which the 2nd-order beliefs question is presented as first task, coherence of the answer in terms of 'most likely value' and the answer in terms of probabilities over intervals occurs less often than in the other treatments. In addition, we do not find that in those treatments, in which the 2nd-order beliefs question is presented as first task, 2nd-order beliefs are characterized by a distribution with a higher spread. In other words, we do not find evidence that subjects feel more uncertain about their 2nd-order beliefs in those treatments compared to the others. The evidence of lower coherence without higher uncertainty may suggest that it is inherently more difficult for subjects to form their 2nd-order beliefs before expressing their 1st-order beliefs and choices, independently of how uncertain 2nd-order beliefs may be.

Appendix B. Additional tables and figures

Table 22: Sample distribution of participants according to field of studies and gender. Percentage sample frequency in each treatment.

	Treatment						
	12C	1C2	21C	2C1	C12	C21	all
	%	%	%	%	%	%	%
Studies							
Humanities	19	28	22	35	20	31	26
Sciences	19	17	33	35	25	25	26
Social Sciences	57	50	45	27	50	44	44
of which							
Economics	*38*	*22*	*39*	*23*	*45*	*25*	*32*
Undecided	6	6	0	4	5	0	4
Gender							
Female	62	78	61	38	55	75	60
Male	38	22	39	62	45	25	40

Table 23: Sample distribution of 1st-order beliefs P, i.e. the subjective probability that the opponent chooses A.

1st-order beliefs P (%)	**Period**				
	1	**2**	**3**	**4**	**all**
	%	%	%	%	%
0	9	10	12	7	9
10			1		
15		1			
20	1	1	3	3	2
25	4	4	2	3	3
30	4	4	4	5	4
35	1	1			
40	9	6	11	4	7
42			1		
45	2	1	1		1
48		1	0		
49				1	
50	44	52	46	45	46
52				1	
55		1	1	2	1
58				1	
60	7	7	8	4	7
61		1			
65	1	1			
70	1	1	1	4	2
75	3	1	3	2	2
80	2	1	2	4	2
85	1				
90				2	
95	1				
100	11	9	7	15	11
all	100	100	100	100	100

Table 24: Sample distribution of the *most likely value* of the opponent's 1st-order beliefs.

most likely value of opponent's 1st-order beliefs (%)	**Period** **1** %	**2** %	**3** %	**4** %	**all** %
0	6	2	4	7	5
1	1				
10			1		
20		3	3	4	2
25				1	
30	4	2	3	4	3
35			1	2	1
40	4	8	2	3	4
42				1	
45		1	2		1
50	50	53	55	59	54
52			1		
55		1		1	
60	5	9	9	7	7
65	1	2	2		1
67		1			
70	4	4	5	4	4
72	1		1		
75	6	4	3	2	4
80	1	2	1	1	1
90	1		1	1	1
100	16	11	9	4	10
all	100	100	100	100	100

Table 25: Sample distribution of $Q(0.5)$, i.e. the subjective probability that the opponent's 1st-order beliefs are smaller than or equal to 50%.

2nd-order beliefs Q(0.5) (%)	**Period 1** %	**2** %	**3** %	**4** %	**all** %
0	12	11	11	8	11
5	1				
10	1	2			1
15	2	2		1	1
20	1	3	1	2	2
21			1		
25	4	4	4	4	4
30	9	3	4	1	4
33	1	1	1	1	1
35	3	2	4	2	2
36		1			
38	1				
40	5	9	11	6	8
45	5	1	2	4	3
50	37	43	41	46	42
51			1	1	
52	1				
53		1			
55	1	1	4	4	2
56		2			
58		1			
60	2	5	2	4	3
62				1	
65	4	4	2		2
70	2		6	3	3
74	1			1	
75	2	1	1	2	1
76			2		
77			1		
80	1	2	2	3	2
85				1	
86	1	1			
90	1	1	1	1	1
98				1	
100	4	3	2	6	4
all	100	100	100	100	100

Table 26: Participants' answers to the question 'How did you choose your actions in the game? Please describe briefly.'

subject	comments
1	went with gut
2	I chose the same outcome each time since statistically it should happen 50%
3	I thought people will be consistent in their choices
4	I chose B the first time and stuck with it every time.
5	At first randomly (1 and 2), then I based my decisions on the first two rounds, minus in the last one I thought I was the seeker but I was the hider, or I would have hid it in B, because seemingly everyone searches in A first.
6	picked randomly
7	I knew the areas of the regions were the same so there was a 50/50 chance.
8	I chose the same action each time.
9	I clicked B each time because I didn't want to try to overthink how people act in this game, knowing that I don't have the background knowledge necessary to accurately ascertain that.
10	I didn't expect the hider to place the object in a because even though it didn't have more area, it seemed to have more area and was a more likely place the seeker would look
11	More or less based on the belief that a person is equally likely to hide/seek in A or B, because the areas are the same.
12	After the first round, I realized I didn't forecast in the way I wanted to. I took an all or nothing approach, and once that worked, I continued.
13	Tried to predict what the other player predicted, and then did the opposite.
14	I originally just planned to pick A and stick to it, but after I got burned twice as the hider I switched it to B on t he third hider attempt.
15	randomly
16	I chose on the basis of rational thinking. Since the area of the two squares was equivalent I thought it would make sense to assume that any rational person would give equal mind to each square's possibility as a hiding/seeking spot.
17	I guessed.
18	There wasn't much hint, so I followed my guts.
19	I picked 50% every time because that maximized my expected payoff based on your formula. I also assumed the hiding/seeking process was random, so I just picked A every time.
20	Predicted that most people would hide the prize in zone B, because it looks optically smaller and safer to hide, even though the area is the same.
21	I picked section B as my answer every single time.
22	I wanted to keep my actions consistent but I found myself going closer and closer to 50/50 percent chances towards the end.
23	think about how other people think based on each previous game played
24	I chose B to hide/seek every time and assumed that my partner would hide/seek in A.
25	Chose A consistently
26	I made the same decision each time
27	Assumed everything was 50-50, so i maintained my choice throughout the game.
28	Since A was most prominent on the paper i figured it wouldn't be hidden there.; But on the last try I figured they would hide the prize in A because they would expect me to chose b.
29	randomness
30	Even though areas of A and B is the same, A appeared to be larger than B. So as the seeker I chose B consistently.
31	I tried to predict where the other participant would hide the object and stuck with that prediction. The probability of the participant placing the object in A or B was 50%, so I thought I had a higher chance of finding it if I stuck with the same guess in each round.
32	Randomly

continued

subject	comments
33	While the area for A was bigger, it eventually came down to a 50% chance between A or B. I just picked one and went with it, usually B because I felt like people would expect you to pick A (because it is first and the difference in area)
34	By putting myself in the shoes of the other people
35	Trying to predict others
36	Found success in 2nd round, repeated in 3rd round, found success, then did for 4th round.....trial and error between 1st and 2nd rounds
37	i was always the hider. I first chose to hide in A because it looks like a larger area so people think that others would be less likely to hide in that area. It worked the first round, so I kept on hiding it in A.
38	I chose my actions based on the fact that I thought most people would NOT choose A, since the questions were all based around A.
39	Selecting A or B was a random choice
40	intuitively
41	I tried to think about what the other person would do and planned accordingly. As the hider, I thought that the seeker would look in A, so I chose to hide in B, etc.
42	I chose mostly block A because it seemed like most people would choose block B.
43	I thought people would pick B, so I always picked A. Just an intuition. Stayed with it the whole time.
44	I chose my actions usually by playing it safe and trying to think like the other person and how I would have played.
45	Completely random. Since it isn't about area, and just based on whether someone was going to choose between 2 different choices, it's all just a guessing game
46	I choose the box that I thought the other person would think would be too obvious to choose.
47	Always either hid or guessed A, until round 3 when I had some random inkling that it might be B (and I was right)
48	50/50; didn't think much about it
49	To place the token, I made the same choice (A) each time, and I always guessed B when I was the seeker. Since I have no information about my partner, the process was random, so switching guesses doesn't help.
50	I didn't have any system to choose which box to hide/look for the prize in. I just randomly chose a box to put the prize in when I was the hider, and when I was the seeker, I just guessed about which box it would be in.
51	Just whatever, cause it didn't matter. depended on another person
52	Areas of two squares were equal so I predicted 50% chance of picking either one, assumed that other participants would act accordingly
53	Randomly choosing between A and B
54	Randomly
55	I tried to not go with the most obvious answers
56	I decided beforehand that I would always hide it in A and search in B
57	Randomly pick by heart
58	Half half
59	Expected value. There were 9 possible opponents, so despite the low sample size, it was still the smartest move to go for par.
60	I changed where I hid it or looked for it
61	I chose based on the fact that zone a looks bigger than zone b so people would be more likely to choose zone A even though there was no advantage over zone B in terms of hiding.
62	I decided to always hide in A, no matter what.
63	Picked b the first couple of times, but then switched for the last one because it kept losing.
64	I choose the outside blue area first because the red was more attractive to the eye. But my teammate had the same idea. I then just alternated based on a habit of variation.
65	I chose the same actions every time, because I thought that some people would choose different answers.
66	I figured people would be most likely to look in B for the prize because it looks smaller and is a better hiding place, so I hid in A. I looked in A because I would hide in A

continued

subject	comments
67	I used the same numbers and box choice, because I figured that if I completed the game enough times with the same answers, I would eventually win.
68	As a seeker for all four rounds, I thought about the thought process the hider would go through and thus which zone they would hide the money under.
69	I thought that the hider would be more likely to hide the prize in B so I looked there.
70	i put all the eggs in one basket
71	I just wanted to maximize my earnings so I took the greatest risk.
72	Based on logic and math
73	Randomly
74	Reverse Psychology.
75	I knew that B and A had equal area, so I assumed that there were equal probability of being chosen. Thus, I chose A every time because all the draws are independent.
76	I hid it in square B because it was less likely that the seeker would look there, since square A was so much brighter. I looked in square B because it seemed more likely that the hider would put it there since it seemed smaller.
77	Just made a decision and put all confidence behind it.
78	At first i just chose pretty randomly with some idea that values around 50 were a safe bet. And i just picked A first and then B.
79	Pretty randomly. But I always matched where I thought they were hiding it with where I would look for it.
80	I wanted to get it over with quickly
81	Picked A every time. Switching between A and B would not increase my likelihood
82	I played conservatively. I made my predictions so that they tended toward average.
83	I played conservatively rather than high risk-high gain.
84	I was consistent with my actions throughout. When I was a hider I placed the $ into the B zone and when I was the seeker I guessed A.
85	I initially chose to hide the prize in A because even though the areas of the two spaces are the same, psychologically it just seems ‘safer’ to hide it in B and I guessed that was what my opponent would guess where it was. Then I decided not to change my answer.
86	I just picked randomly - overall it’s 50-50 when it comes down to it.
87	zone a is larger in area and just sticked to it since the other player’s choice is random..
88	randomly.
89	I almost always put down extreme percentages and randomly chose A or B. I didn’t put much thought into it.
90	I decided to hit it in B for the majority of the time because there was really just a 50/50 chance that the person would look there.
91	Randomly
92	Based everything on a 50/50 chance that the prize would be hidden in A or B pretty much.
93	I just thought about the percentages for forecasts. As for deciding which square to choose, it was pretty much random.
94	not too greedy and consistency with my choices
95	Went 50/50 across the board and choose A twice and B twice
96	area is the same, so I just choose randomly. Plus the seeker/hider has only two choice so half-half.
97	Random
98	I based them on the results of my last period, because I thought in general people choose the same way.
99	I think although A and B are the same in size, B looks more safe to hide, and seekers are more likely to check B too.
100	I try to make half-half decision at first, then I bet 100% decision once.
101	By the choices that put in the statements. If they said, ‘how likely do you think they’ll choose choice A?’ I knew that just by mentioning A, it would bring more attention to that choice so I increased the percent for that option, but knew that people would sometimes think that was too obvious so I would alternate by which area I would hide the prize in.

continued

subject	comments
102	I went with the most psychologically expected action, hoping that people would go against it.
103	Divided percentages up equal
104	A seemed like a bigger area so I thought the hider would choose that to hide.
105	Chose my first round randomly. Then based subsequent choices on the first round, as if I were playing the same person.
106	thought about what people would most likely choose. seeing how region b was smaller and less noticeable, I figured many of the seekers would assume that i would hide it there. So I hid the prize in region A.
107	Figured people wouldn't go for A because they expected that was the obvious choice. Even though they knew the areas were the same, they'd figure I wouldn't guess a because it looked bigger.
108	Randomly
109	just intuition- i don't think i'm good at these type of games
110	I thought the red square was the most obvious and it seems to have a larger area, so most people would not choose to look for the prize there.
111	I think people end up doing a lot of reverse psychology on the decisions and will probably ends up very random when aggregated. Therefore I just stuck with choosing A all along.
112	
113	Gut reaction.
114	randomly

Appendix C

This appendix contains the computation of the theoretical probabilities reported in the lower panel of Table 14. The theoretical probabilities are computed conditional on first- and second-order beliefs falling in each of the categories in which Table 14 is divided: $P \neq 0.5$ *and* $Q(0.5) \neq 0.5$, $P \neq 0.5$ *and* $Q(0.5) = 0.5$, $P = 0.5$ *and* $Q(0.5) \neq 0.5$, $P = 0.5$ *and* $Q(0.5) = 0.5$. To compute the theoretical probabilities, we assume that choice, 1st-order beliefs and 2nd-order beliefs are submitted randomly and independently the one of the others. Thus, it's as if:

- choice over $\{A, B\}$ is drawn from the distribution $Prob(A) = Prob(B) = 0.5$,
- 1st-beliefs P are drawn from the uniform distribution over support [0,100] percent,
- 2nd-order beliefs $Q(0.5)$ are drawn from the uniform distribution over support [0,100] percent.

We compute the probabilities from the point of view of the choices and beliefs of a subject playing as Hider. We would obtain the same values were we to compute the probabilities from the point of view of the choices and beliefs of a subject playing as Seeker. Finally, to simplify notation, we write Q_H for $Q_H(0.5)$.

$$\begin{aligned}&Prob(\textit{choice optimal wrt 1st order beliefs} \mid P_H \neq 0.5) = Prob(B, P_H > 0.5) + Prob(A, P_H < 0.5)\\&= Prob(B)Prob(P_H > 0.5) + Prob(A)Prob(P_H < 0.5) = \tfrac{1}{2}\tfrac{1}{2} + \tfrac{1}{2}\tfrac{1}{2} = \tfrac{1}{2}\end{aligned} \tag{9}$$

$$\begin{aligned}&Prob(\textit{choice optimal wrt 2nd order beliefs} \mid Q_H \neq 0.5) = Prob(B, Q_H < 0.5) + Prob(A, Q_H > 0.5)\\&= Prob(B)Prob(Q_H < 0.5) + Prob(A)Prob(Q_H > 0.5) = \tfrac{1}{2}\tfrac{1}{2} + \tfrac{1}{2}\tfrac{1}{2} = \tfrac{1}{2}\end{aligned} \tag{10}$$

$$\begin{aligned}&Prob(\textit{weak coherence} \mid P_H \neq 0.5, Q_H \neq 0.5) = Prob(P_H > 0.5, Q_H < 0.5) + Prob(P_H < 0.5, Q_H > 0.5)\\&= Prob(P_H > 0.5)Prob(Q_H < 0.5) + Prob(P_H < 0.5)Prob(Q_H > 0.5) = \tfrac{1}{2}\tfrac{1}{2} + \tfrac{1}{2}\tfrac{1}{2} = \tfrac{1}{2}\end{aligned} \tag{11}$$

$$\begin{aligned}&Prob(\textit{choice optimal wrt 1st and 2nd order beliefs, weak coherence} \mid P_H \neq 0.5, Q_H \neq 0.5)\\&= Prob(B, P_H > 0.5, Q_H < 0.5) + Prob(A, P_H < 0.5, Q_H > 0.5)\\&= Prob(B)Prob(P_H > 0.5)Prob(Q_H < 0.5) + Prob(A)Prob(P_H < 0.5)Prob(Q_H > 0.5)\\&= \tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2} + \tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2} = \tfrac{1}{4}\end{aligned} \tag{12}$$

$$\begin{aligned}&Prob(\textit{choice optimal wrt 1st and 2nd order beliefs, weak coherence} \mid P_H \neq 0.5, Q_H = 0.5)\\&= Prob(B, P_H > 0.5) + Prob(A, P_H < 0.5) = Prob(B)Prob(P_H > 0.5) + Prob(A)Prob(P_H < 0.5)\\&= \tfrac{1}{2}\tfrac{1}{2} + \tfrac{1}{2}\tfrac{1}{2} = \tfrac{1}{2}\end{aligned} \tag{13}$$

$$\begin{aligned}&Prob(\textit{choice optimal wrt 1st and 2nd order beliefs, weak coherence} \mid P_H = 0.5, Q_H \neq 0.5)\\&= Prob(B, Q_H < 0.5) + Prob(A, Q_H > 0.5) = Prob(B)Prob(Q_H < 0.5) + Prob(A)Prob(Q_H > 0.5)\\&= \tfrac{1}{2}\tfrac{1}{2} + \tfrac{1}{2}\tfrac{1}{2} = \tfrac{1}{2}\end{aligned} \tag{14}$$

$$\begin{aligned}&Prob(\alpha\%\textit{strong coherence} \mid P_H = 0.5, Q_H \neq 0.5) = Prob(|P_H - (1 - Q_H)| \leq \tfrac{\alpha}{100} \mid P_H = 0.5)\\&= Prob(|Q_H - 0.5| \leq \tfrac{\alpha}{100}) = Prob(0.5 - \tfrac{\alpha}{100} \leq Q_H \leq 0.5 + \tfrac{\alpha}{100}) = \tfrac{2\alpha}{100}\end{aligned} \tag{15}$$

$$\begin{aligned}&Prob(choice\ optimal\ wrt\ 1st\ and\ 2nd\ order\ beliefs,\ \alpha\%strong\ coherence \mid P_H = 0.5, Q_H \neq 0.5)\\&= Prob(B, Q_H < 0.5, 0.5 - \tfrac{\alpha}{100} \leq Q_H \leq 0.5 + \tfrac{\alpha}{100}) + Prob(A, Q_H > 0.5, 0.5 - \tfrac{\alpha}{100} \leq Q_H \leq 0.5 + \tfrac{\alpha}{100})\\&= Prob(B)Prob(0.5 - \tfrac{\alpha}{100} \leq Q_H < 0.5) + Prob(A)Prob(0.5 < Q_H \leq 0.5 + \tfrac{\alpha}{100}) = \tfrac{1}{2}\tfrac{\alpha}{100} + \tfrac{1}{2}\tfrac{\alpha}{100} = \tfrac{\alpha}{100}\end{aligned} \tag{16}$$

$$\begin{aligned}&Prob(5\%strong\ coherence \mid P_H \neq 0.5, Q_H = 0.5) = Prob(|P_H - (1 - Q_H)| \leq \tfrac{\alpha}{100} \mid Q_H = 0.5)\\&= Prob(|P_H - 0.5| \leq \tfrac{\alpha}{100}) = Prob(0.5 - \tfrac{\alpha}{100} \leq P_H \leq 0.5 + \tfrac{\alpha}{100}) = \tfrac{2\alpha}{100}\end{aligned} \tag{17}$$

$$\begin{aligned}&Prob(choice\ optimal\ wrt\ 1st\ and\ 2nd\ order\ beliefs,\ \alpha\%strong\ coherence \mid P_H \neq 0.5, Q_H = 0.5)\\&= Prob(B, P_H > 0.5, 0.5 - \tfrac{\alpha}{100} \leq P_H \leq 0.5 + \tfrac{\alpha}{100}) + Prob(A, P_H < 0.5, 0.5 - \tfrac{\alpha}{100} \leq P_H \leq 0.5 + \tfrac{\alpha}{100})\\&= Prob(B)Prob(0.5 < P_H \leq 0.5 + \tfrac{\alpha}{100}) + Prob(A)Prob(0.5 - \tfrac{\alpha}{100} \leq P_H < 0.5) = \tfrac{1}{2}\tfrac{\alpha}{100} + \tfrac{1}{2}\tfrac{\alpha}{100} = \tfrac{\alpha}{100}\end{aligned} \tag{18}$$

$$\begin{aligned}&Prob(\alpha\%strong\ coherence \mid P_H \neq 0.5, Q_H \neq 0.5)\\&= Prob(\alpha\%strong\ coherence \mid P_H > 1 - Q_H)Prob(P_H > 1 - Q_H)\\&+Prob(\alpha\%strong\ coherence \mid 1 - Q_H > P_H)Prob(1 - Q_H > P_H)\\&= Prob(P_H - (1 - Q_H) \leq \tfrac{\alpha}{100} \mid P_H > 1 - Q_H)\tfrac{1}{2} + Prob(1 - Q_H - P_H \leq \tfrac{\alpha}{100} \mid 1 - Q_H > P_H)\tfrac{1}{2}\\&= Prob(P_H \leq 1 - Q_H + \tfrac{\alpha}{100} \mid P_H > 1 - Q_H)\tfrac{1}{2} + Prob(1 - Q_H \leq P_H + \tfrac{\alpha}{100} \mid 1 - Q_H > P_H)\tfrac{1}{2}\\&= [1 - Prob(P_H \geq 1 - Q_H + \tfrac{\alpha}{100} \mid P_H > 1 - Q_H)]\tfrac{1}{2} + [1 - Prob(1 - Q_H \geq P_H + \tfrac{\alpha}{100} \mid 1 - Q_H > P_H)]\tfrac{1}{2}\\&= 1 - [1 - \tfrac{2\alpha}{100}(1 - \tfrac{1}{2}\tfrac{\alpha}{100})] = \tfrac{2\alpha}{100}(1 - \tfrac{1}{2}\tfrac{\alpha}{100}) = \tfrac{2\alpha}{100} - (\tfrac{\alpha}{100})^2\end{aligned} \tag{19}$$

since

$$\begin{aligned}&Prob(P_H \geq 1 - Q_H + \tfrac{\alpha}{100} \mid P_H > 1 - Q_H) = \frac{Prob(P_H \geq 1 - Q_H + \frac{\alpha}{100},\ P_H > 1 - Q_H)}{Prob(P_H > 1 - Q_H)} = \frac{Prob(P_H \geq 1 - Q_H + \frac{\alpha}{100})}{\frac{1}{2}}\\&= \frac{\frac{1}{2} - \frac{\alpha}{100}(1 - \frac{1}{2}\frac{\alpha}{100})}{\frac{1}{2}} = 1 - \tfrac{2\alpha}{100}(1 - \tfrac{1}{2}\tfrac{\alpha}{100})\end{aligned} \tag{20}$$

and

$$\begin{aligned}&Prob(P_H \geq 1 - Q_H + \tfrac{\alpha}{100}) = \int_0^1 Prob(P_H \geq 1 - Q_H + \tfrac{\alpha}{100} \mid Q_H = x)dx\\&= \int_0^1 Prob(P_H \geq -x + 1 + \tfrac{\alpha}{100})dx = \int_0^{\frac{\alpha}{100}} Prob(P_H \geq -x + 1 + \tfrac{\alpha}{100})dx + \int_{\frac{\alpha}{100}}^1 Prob(P_H \geq -x + 1 + \tfrac{\alpha}{100})dx\\&= 0 + \int_{\frac{\alpha}{100}}^1 Prob(P_H \geq -x + 1 + \tfrac{\alpha}{100})dx = \int_{\frac{\alpha}{100}}^1 [1 - (-x + 1 + \tfrac{\alpha}{100})]dx = \int_{\frac{\alpha}{100}}^1 (x - \tfrac{\alpha}{100})dx\\&= \tfrac{1}{2}(1)^2 - \tfrac{\alpha}{100}1 - [\tfrac{1}{2}(\tfrac{\alpha}{100})^2 - \tfrac{\alpha}{100}\tfrac{\alpha}{100}] = \tfrac{1}{2} - \tfrac{\alpha}{100}(1 - \tfrac{1}{2}\tfrac{\alpha}{100})\end{aligned} \tag{21}$$

and

$$Prob(P_H \geq 1 - Q_H + \tfrac{\alpha}{100} \mid P_H > 1 - Q_H) = Prob(1 - Q_H \geq P_H + \tfrac{\alpha}{100} \mid 1 - Q_H > P_H) \tag{22}$$

$$\begin{aligned}&Prob(choice\ optimal\ wrt\ 1st\ and\ 2nd\ order\ beliefs,\ \alpha\%strong\ coherence \mid P_H \neq 0.5, Q_H \neq 0.5)\\&= Prob(B)Prob(P_H > 0.5, Q_H < 0.5, |P_H - (1 - Q_H)| \leq \tfrac{\alpha}{100})\\&+Prob(A)Prob(P_H < 0.5, Q_H > 0.5, |P_H - (1 - Q_H)| \leq \tfrac{\alpha}{100})\\&= \tfrac{1}{2}Prob(P_H > 0.5, Q_H < 0.5, |P_H - (1 - Q_H)| \leq \tfrac{\alpha}{100})\\&+\tfrac{1}{2}Prob(P_H < 0.5, Q_H > 0.5, |P_H - (1 - Q_H)| \leq \tfrac{\alpha}{100})\\&= \tfrac{1}{2}Prob(P_H > 0.5, Q_H < 0.5, P_H - (1 - Q_H) \leq \tfrac{\alpha}{100}, P_H > 1 - Q_H)\\&+\tfrac{1}{2}Prob(P_H > 0.5, Q_H < 0.5, 1 - Q_H - P_H \leq \tfrac{\alpha}{100}, 1 - Q_H > P_H)\\&+\tfrac{1}{2}Prob(P_H < 0.5, Q_H > 0.5, P_H - (1 - Q_H) \leq \tfrac{\alpha}{100}, P_H > 1 - Q_H)\\&+\tfrac{1}{2}Prob(P_H < 0.5, Q_H > 0.5, 1 - Q_H - P_H \leq \tfrac{\alpha}{100}, 1 - Q_H > P_H)\\&= \tfrac{1}{2}4\tfrac{1}{4}\tfrac{\alpha}{100}(1 - \tfrac{1}{2}\tfrac{\alpha}{100}) = \tfrac{1}{2}\tfrac{\alpha}{100}(1 - \tfrac{1}{2}\tfrac{\alpha}{100})\end{aligned} \tag{23}$$

since

$$
\begin{aligned}
&Prob(P_H > 0.5, Q_H < 0.5, P_H - (1-Q_H) \le \tfrac{\alpha}{100}, P_H > 1-Q_H) \\
&= Prob(P_H > 0.5, Q_H < 0.5, P_H > 1-Q_H) - Prob(P_H > 0.5, Q_H < 0.5, P_H > 1-Q_H, P_H - (1-Q_H) \ge \tfrac{\alpha}{100}) \\
&= Prob(P_H > 0.5, Q_H < 0.5, P_H > 1-Q_H) - Prob(P_H > 0.5, Q_H < 0.5, P_H > 1-Q_H, P_H \ge 1-Q_H + \tfrac{\alpha}{100}) \\
&= Prob(P_H > 0.5, Q_H < 0.5, P_H > 1-Q_H) - Prob(P_H > 0.5, Q_H < 0.5, P_H \ge 1-Q_H + \tfrac{\alpha}{100}) \\
&= \tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2} - \tfrac{1}{2}\tfrac{1}{2} Prob(P_H \ge 1-Q_H + \tfrac{\alpha}{100}) = \tfrac{1}{8} - \tfrac{1}{4}[\tfrac{1}{2} - \tfrac{\alpha}{100}(1 - \tfrac{1}{2}\tfrac{\alpha}{100})] = \tfrac{1}{4}\tfrac{\alpha}{100}(1 - \tfrac{1}{2}\tfrac{\alpha}{100})
\end{aligned}
\qquad (24)
$$

and

$$
\begin{aligned}
&Prob(P_H > 0.5, Q_H < 0.5, P_H - (1-Q_H) \le \tfrac{\alpha}{100}, P_H > 1-Q_H) \\
&= Prob(P_H > 0.5, Q_H < 0.5, 1-Q_H - P_H \le \tfrac{\alpha}{100}, 1-Q_H > P_H) \\
&= Prob(P_H < 0.5, Q_H > 0.5, P_H - (1-Q_H) \le \tfrac{\alpha}{100}, P_H > 1-Q_H) \\
&= Prob(P_H < 0.5, Q_H > 0.5, 1-Q_H - P_H \le \tfrac{\alpha}{100}, 1-Q_H > P_H)
\end{aligned}
\qquad (25)
$$

Appendix D
Instructions

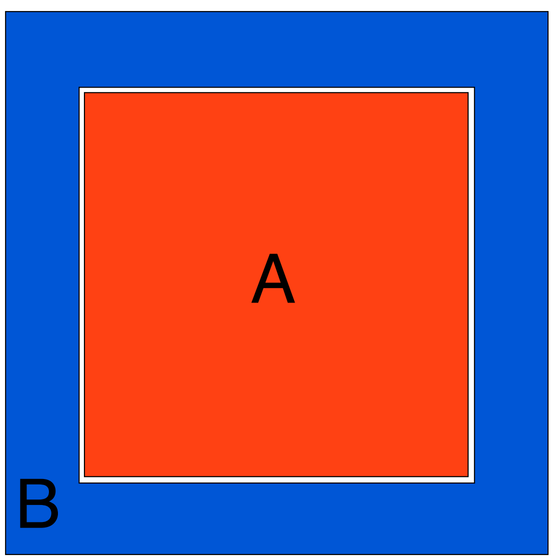
Welcome! Thank you for participating in this experiment. You are going to take part in a study of decision making. Please follow these instructions carefully. You will be paid according to your performance. In case you have questions, please raise your hand at any time. Please do not speak to other participants during the experiment.

You are going to be randomly paired with another student in the room to play a hide-and-seek game. In the hide-and-seek game, one person chooses a place where to hide a prize, and the other person needs to predict where the prize is hidden. The prize is a $10 banknote.

The prize must be hidden somewhere in a field. The field is divided into two zones: an inner zone and an outer zone. The two zones are identical in area. The figure below represents the field. In the figure, the inner zone is labelled A and colored in red and the outer zone is labelled B and colored in blue.

The hider has to choose between hiding the prize in zone A or in zone B. The seeker has to choose between predicting that the prize is hidden in zone A or in zone B. If the seeker does not predict correctly, then the hider wins the $10 prize. If instead the seeker predicts correctly, then he or she wins the $10 prize.

You will play the game for four rounds. In each round you will be randomly paired with another student. Therefore, you will not necessarily play the game every time with the same person. In each round the roles of hider and seeker will be assigned randomly to you and the other student who is paired with you. You will see the information on the screen. At the end of the session, the computer will randomly select one of the rounds, and you will be paid according to your performance in that round only.



Additional tasks

Besides having the opportunity to earn the $10 prize, you will also be given the opportunity to earn extra money by making forecasts. You will be asked to forecast the choice made by your opponent. You may be asked to assign a percent chance to each possible outcome and/or you may be asked to specify what you think the most likely outcome is.

A percent chance is a number between 0 and 100 percent, where 100 percent chance assigned to an outcome means that you are certain that such outcome is going to be the correct one, and 0 percent chance means that you are certain that such outcome is *not* going to be the correct one.

You will be paid based on the accuracy of your forecasts.

When forecasts are expressed in terms of what you think the most likely outcome is, you will earn nothing is your forecast is wrong (i.e., if the correct answer does not coincide with the one you chose), while you will earn $2 if your forecast is correct (i.e., if the correct answer coincides with the one you chose).

When forecasts are expressed in terms of a percent chance, then the payoff is determined as follows. Suppose that you are asked to assign a percent chance to two possible outcomes. For convenience let's label the two outcomes X and Y. Suppose that you assign percent chance p_X to outcome X and percent chance p_Y to outcome Y. We will give you $2 from which we will subtract an amount which depends on how inaccurate your answer was.

If outcome X turns out to be the correct one, the amount $(1 - \frac{p_X}{100})^2 + (\frac{p_Y}{100})^2$ is subtracted from the initial $2.

If outcome Y turns out to be the correct one, the amount $(\frac{p_X}{100})^2 + (1 - \frac{p_Y}{100})^2$ is subtracted from the initial $2.

Lets consider an example. Suppose that the correct outcome is X. Then **the worst you can do** is to assign a 0 percent chance to X and a 100 percent chance to Y. In this case your payoff is:

$$\$2 - \$\left(1 - \frac{0}{100}\right)^2 - \$\left(\frac{100}{100}\right)^2 = \$2 - \$(1)^2 - \$(1-0)^2 = \$2 - \$1 - \$1 = \$0$$

The best you can do is instead to assign a 100 percent chance to X and 0 percent chance to Y. In this case your payoff is:

$$\$2 - \$\left(1 - \frac{100}{100}\right)^2 - \$\left(\frac{0}{100}\right)^2 = \$2 - (1-1)^2 - (0)^2 = \$2 - \$0 - \$0 = \$2$$

Therefore your payoff in this task is between $0 and $2.

The same rule is applied when more than two possible outcomes exist. Your payoff in this task is always between $0 and $2.

Note Since your forecasts are made when you don't know what your opponent has chosen, **the best thing you can do to maximize the expected size of your payoff is to simply state what you think.**

Final comments

By participating to the game you will receive a show-up fee of $5, plus the $10 prize if you are the winner in the game, plus payment based on the accuracy of your forecasts. Payment for the forecasts ranges between $0 and $6.

Therefore, the total payment you will receive will be between $5 and $21.